Problem Set 3

Thermodynamics and Climate Change

MOSTEC 2021

1. **Concept questions:** Answer the following with a brief explanation.

   (a) For a system undergoing a reversible *process*: (i) Can there be a total positive change in entropy? (ii) Can there be a total negative change in entropy? (iii) Answer parts (i) and (ii) if instead the system is undergoing an *irreversible* process.

   (b) For a system undergoing a reversible *cycle*: (i) Can there be a total positive change in entropy? (ii) Can there be a total negative change in entropy? (iii) Answer parts (i) and (ii) if instead the system is undergoing an *irreversible* cycle.

   (c) On a T-V diagram (V on x-axis and T on y-axis), sketch the following curves for an ideal gas, each starting from the same initial temperature and volume: (i) Isochoric heating, (ii) isobaric heating, (iii) isothermal compression, (iv) adiabatic expansion

   (d) Air conditioners and heat pumps are engines in reverse and can have a COP > 1 (i.e. more heat can be transferred than work is put in). How is this possible?

   (e) Describe generally how to have (i) a reversible process that expands a gas from V_1 to V_2, (ii) an irreversible process that does the same. Where does the irreversibility come from?

   (f) For each of the following scenarios, state whether or not the Second Law is violated and why: (i) A thermodynamic cycle in which net heat is transferred from a cold object to a hotter object. (ii) A system in communication with only one thermal reservoir and that does work. (iii) A reversible process in which a gas expands isothermally and adiabatically. (iv) Heat flowing from a cold object to a hotter object without any work being done. (v) An irreversible cycle in which the net entropy generated is negative. (vi) A reversible cycle in which the net entropy generated is negative.

2. **Entropy of a Perfect Gas:** We know that the change in entropy for a perfect gas is given by

$$\Delta s = s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1} \tag{1}$$

   Show that this expression is equivalent to:

   (a)
$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \tag{2}$$

   (b)
$$\Delta s = c_p \ln \frac{V_2}{V_1} + c_v \ln \frac{P_2}{P_1} \tag{3}$$

3. **Carnot Solar-Powered Heat Pump:** As the Earth warms, efficient cooling systems are becoming ever more important so as to not compound the problem and create a positive feedback loop between energy usage and carbon emissions driving that energy usage. We want to design a simple solar-powered heat pump based on the ideal reversible Carnot Cycle that can keep a house cool in the summer. Our house can be represented by a thermal reservoir at $T_L = 20$ °C and the outside by a thermal reservoir at $T_H = 35$ °C.

   In our system, as shown in Fig. 1 we have a solar panel powering a small electric motor driving an *isentropic* (reversible and adiabatic) compressor. Our working fluid is assumed to be an ideal gas. From states $1 \rightarrow 2$ in this Reverse Carnot Cycle, our gas flows smoothly past our cold reservoir (house) absorbing thermal energy isothermally at T_L. Then from states $2 \rightarrow 3$ the gas is compressed isentropically, bringing its temperature to T_H. From states $3 \rightarrow 4$, heat is rejected isothermally at T_H to the environment. Finally, from states $4 \rightarrow 1$, the gas expands isentropically, doing some

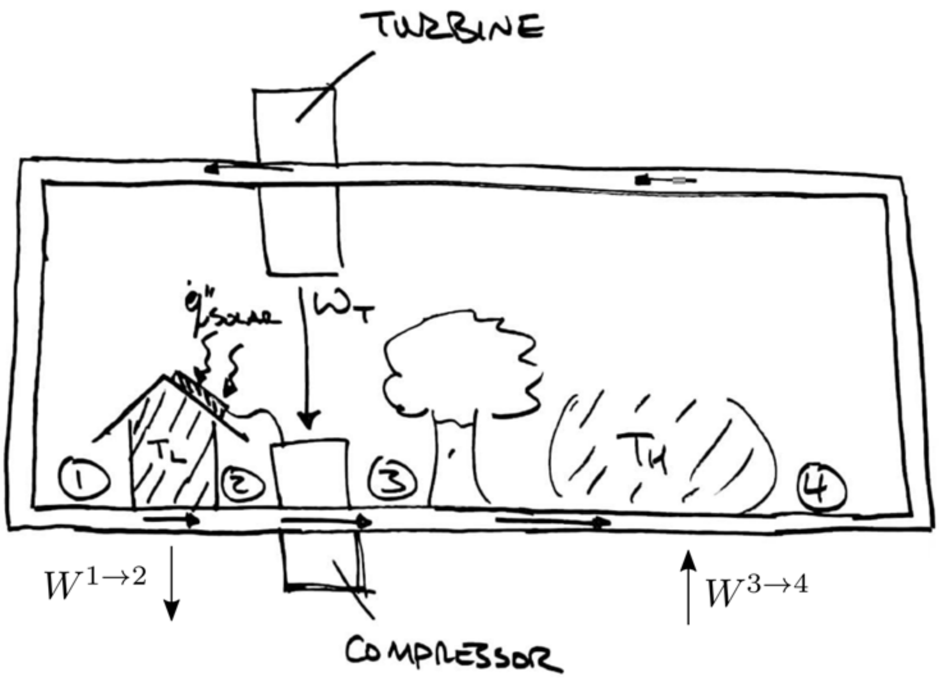

Figure 1: Solar-powered Carnot heat pump for Problem 3.

work and bringing the gas back to T_L. The work done in this expansion is used to partially drive the compressor, though some additional solar energy will be needed. Assume the working fluid is air, modeled as a perfect gas with a specific heat at constant volume, $c_v = 718$ J/kg-K, and a molar mass of 28.97 g/mol. For this problem, $P_1 = 10$ bar and $P_2 = 1$ bar.

(a) Sketch this cycle on a P-V diagram. Clearly label states 1-4 and sketch the curves between them. Be sure to include arrows on the curves to show direction of processes.

(b) For each process, what is the total work and heat transfer to/from the gas per unit mass?

(c) If 500 W of heat consistently during the day is being added to the house, what mass flow rate is needed for our system to keep the house at a constant temperature?

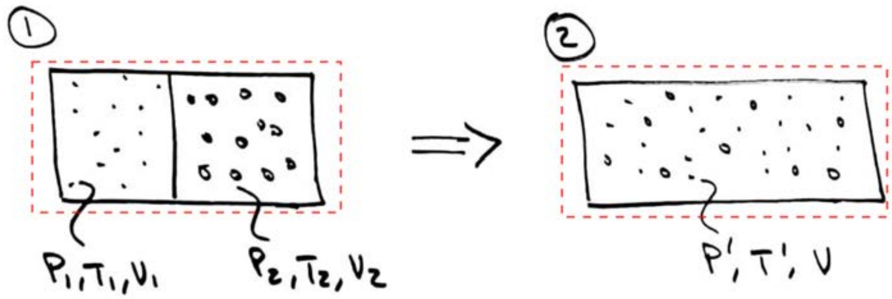

Figure 2: Mixing of argon and nitrogen gas for Problem 5.

(d) If we want to minimize the amount of working fluid used, should it have a high or low heat capacity? Why?

(e) The real system will not be perfectly reversible. Identify a few sources of irreversibility and explain how we can help reduce entropy generation for the real system.

4. **Exploring Entropy:** (Coding) Using the nasaPoly library, what is the change in entropy for the gas per mole in the following cases for carbon dioxide? *Assume imperfect gas (i.e. $c_p = c_p(T)$).* Start by copying the Google Colab template here.

   (a) An expansion that brings the gas temperature from 600 to 400 K with an accompanying pressure drop from 10 to 1 bar.

   (b) A compression that brings the gas from a specific volume of 4 to 2 m^3/mol with an accompanying temperature rise of 300 to 700 K.

   (c) Isobaric heat transfer at 1 bar raising the gas temperature from 600 to 900 K.

5. **(Challenge) Non-Isothermal Gas Mixing:** (Coding) As shown in Fig. 2, we have two gases that are mixed non-isothermally. In state 1, we have 2 kg of nitrogen gas (N_2) at an initial temperature of 400 K and pressure of 5 bar and 1 kg of carbon dioxide gas at an initial temperature of 600 K and pressure of 10 bar. They are separated by an adiabatic membrane at first. Suddenly, this membrane vanishes and the system proceeds to state 2 with the gases evenly mixed throughout the total

volume. Throughout this process, there is no heat or work exchanged with environment.

(a) What volumes do the gases initially occupy in state 1?

(b) What is the final temperature and pressure of the gas mixture at equilibrium in state 2?

Hint: You will need to solve this part iteratively in code to find a value of temperature that satisfies one of your equations.

(c) What is the entropy generated in this process?

(d) What would be the minimum work required to separate these gases isothermally at 300 K?

Resource: Thermodynamics and Climate Change
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