MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

Physics 8.022 ***Spring 2002***

ELECTRICITY AND MAGNETISM

Practice QUIZ #3

There are four questions. Answer all of the questions. This is a closed book quiz

Problem 1 (Propagation of an EM wave) (25 pts)

In a region of free space having no charges or currents, a magnetic field has the form $\vec{B}(\vec{r},t) = \hat{x}B_0 \sin(ky + \omega t)$. Using cgs units throughout:

a) Calculate $\vec{\nabla} \times \vec{B}$ and using this result in one of Maxwell's equations, solve for $\vec{E}(\vec{r},t)$.

b) Calculate $\vec{\nabla} \times \vec{E}$ and using this result in another of Maxwell's equations, find a relation between k and ω.

c) Show that $\vec{E}(\vec{r},t)$ and $\vec{B}(\vec{r},t)$ satisfy the remaining two Maxwell's equations.

Problem 1 continues on next page

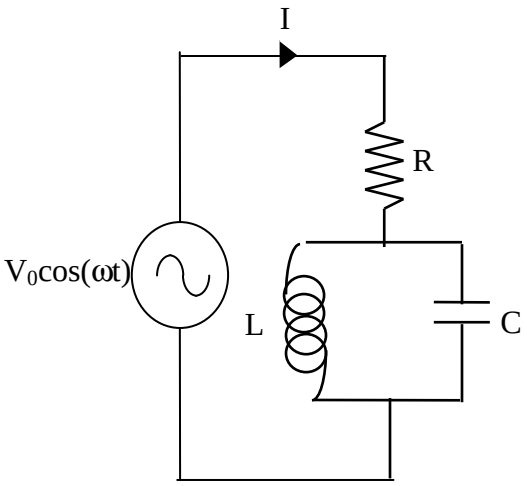
d) In which direction is the wave traveling? What is it's speed? How is it polarized?

e) Calculate the average energy density in this wave and the average energy flow per unit area. It will help you to know that $\frac{1}{\pi}\int_0^{\pi} \sin^2(\theta) d\theta = \frac{1}{2}$.

f) Write down any vector potential $\vec{A}$ corresponding to the magnetic field $\vec{B}(\vec{r},t)$.

Problem 2 (Impedance and Resonance) (25 pts)

a) What is the complex impedance of the combination of circuit elements, R, L, and C shown on the right? Please rationalize the expression into separate real and imaginary parts. (Hint: impedances in parallel behave in just the same way as resistors in parallel).



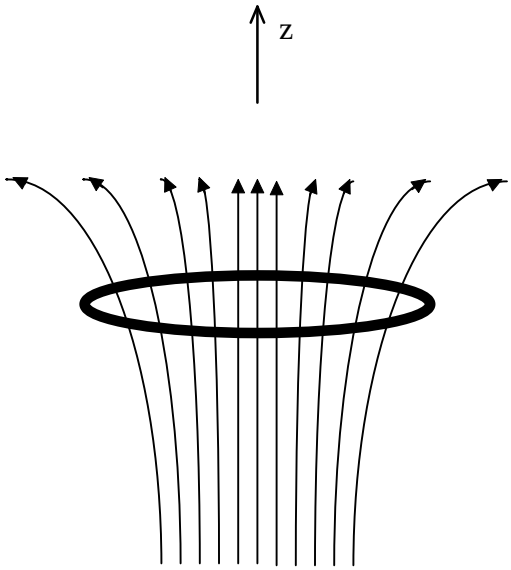
b) What is the current, I (the actual and not the complex current,) flowing through the circuit? Give an expression for the phase angle.

c) Explain the low and high frequency behavior of the phase shift of the current in terms of the currents through each of the circuit elements.

Problem #3 (Levitating ring demonstration) (30 pts)

A conducting circular loop of radius *a* is levitated by a magnetic field (created by a coil which is not shown). The loop is centered on the axis of symmetry of the coil, so you may consider the component of the field pointing radially outward from the center of the loop, B_r as being constant around the loop.

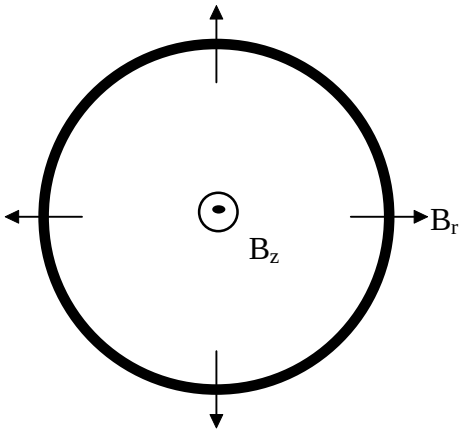
Side View



Two cases are considered here. First you will consider what happens when the field strength is increased at a constant rate. Second, you will determine what happens in the AC case.

a) **(Thumbology 101)** If the field strength is increasing at a constant rate, a current will start to flow in the loop. On the diagram labeled "Top View", draw an arrow showing the direction of this current.

Top View



b) The increasing magnetic field creates an increasing magnetic flux $d\Phi/dt$ through the loop. The inductance of the loop is *L*, and the resistance of the loop *R*. Write down the correct differential equation describing the current flow in the loop. Your equation should involve $\frac{d\Phi}{dt}, \frac{dI}{dt}, I, L, R,$ and *c*. (Hint: consider the EMF induced in the loop to be a battery which drives the current in the direction you determined in part (a)).

Problem 3 continues on next page

c) Write down the value for the current around the loop as $t \to \infty$. (Yes, we are pretending that the field can keep increasing forever.)

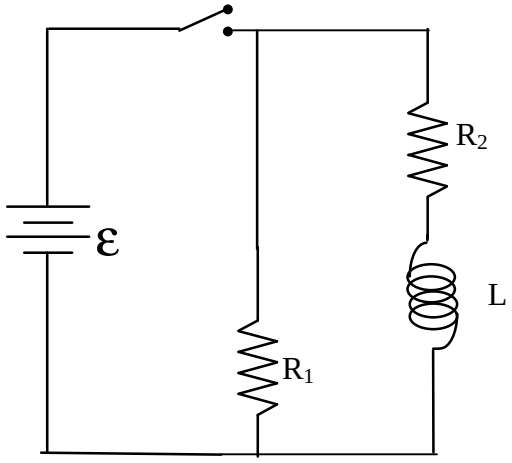
d) Now consider that the magnetic field oscillates in time. This means that the flux through the loop oscillates as $\Phi = \Phi_0 \cos(\omega t)$ .What is the current in the loop? What is the phase of the current compared to the phase of the flux? Again, think of the EMF induced in the loop as a battery which drives current in the direction that you determined in part (a).

HINT: Use complex numbers and note that $\Phi = Re\left[\Phi_0 e^{i\omega t}\right] = Re\left[\tilde{\Phi}\right]$. Also, use the fact that $\frac{d\tilde{\Phi}}{dt} = i\omega\tilde{\Phi}$.

e) What is the force on the loop? Which way does it point? Use the expression for the current that you derived in part (d). Remember that the magnetic field is oscillating so that B_z oscillates as $B_z = B_{z0}\cos(\omega t)$, and B_r oscillates as $B_r = B_{r0}\cos(\omega t)$. For what value of the resistance is the force maximized?

Problem #4 (Power and inductors) (20 pts)

In the schematic on the left, the switch S has been closed for a very long time, and then it is opened at time t=0.



(a) Calculate the power dissipated in the resistors R_1 and R_2 before the switch is opened.

(b) Find the energy in the inductor immediately after the switch is opened.

(c) Calculate the power dissipated in the resistors R_1 and R_2 after the switch is opened and as a function of time.

(d) Show that the total energy dissipated equals the result in (b).