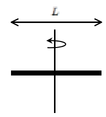
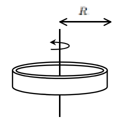
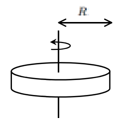
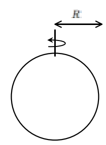
8.012 Physics I: Classical Mechanics
Fall 2008

USEFUL EQUATIONS

Velocity in polar coordinates	$\vec{r} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
Acceleration in polar coordinates	$\vec{r} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$
Center of mass (COM) of a rigid body	$\vec{R} = \frac{\sum m_i\vec{r}_i}{\sum m_i} = \frac{1}{M}\int \rho\vec{r}dV$
Volume element in cylindrical coordinates	$dV = rdrd\theta dz$
Kinetic energy	$K = \frac{1}{2}M(\vec{v}\cdot\vec{v}) + \frac{1}{2}\vec{\omega}\cdot\mathbf{I}\cdot\vec{\omega}$
Work	$W = \Delta K = \int \vec{F}\cdot d\vec{r} = \int \vec{\tau}\cdot d\vec{\theta}$
Potential Energy (for conservative forces)	$U = -\int \vec{F}_c \cdot d\vec{r}$ where $\vec{\nabla}\times\vec{F}_c = 0$
Angular momentum	$\vec{L} = \vec{r}\times\vec{p} = \mathbf{I}\cdot\vec{w}$
Torque	$\vec{\tau} = \vec{r}\times\vec{F} = \frac{d\vec{L}}{dt}$ Fixed axis rotation: $\tau_z = I_{zz}\dot{\omega}$

COM Moment of inertia for a uniform bar	 $I_{zz} = \frac{1}{12} M L^2$
COM Moment of inertia for a uniform hoop	 $I_{zz} = M R^2$
COM Moment of inertia for a uniform disk	 $I_{zz} = \frac{1}{2} M R^2$
COM Moment of inertia for a uniform sphere	 $I_{zz} = \frac{2}{5} M R^2$
Scalar parallel axis theorem	$I = I_{COM} + M R^2$
Moments of inertia tensor (permute x→y→z)	$I_{xx} = \sum_i m_i (y_i^2 + z_i^2) = \int dV \rho (y^2 + z^2)$
Euler's Equations (permute 1→2→3)	$\tau_1 = I_1 \dot{\omega}_1 + (I_3 - I_2)\omega_2 \omega_3$
Time derivative between inertial and rotating frames	$\left(\frac{d\vec{B}}{dt}\right)_{inertial} = \left(\frac{d\vec{B}}{dt}\right)_{rotating} + \vec{\Omega} \times \vec{B}$
Fictitious force in an accelerating frame	$\vec{F}_f = -m\vec{A}$
Fictitious force in a rotating frame (Ω constant)	$\vec{F}_f = -2m\vec{\Omega} \times \vec{v}_{rot} - m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$
Taylor Expansion of f(x)	$f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}\|_a (x-a) + \frac{1}{2!}\frac{d^2 f}{dx^2}\|_a (x-a)^2$