Warm-Ups 03

ⓘ This is a preview of the published version of the quiz

Started: Mar 4 at 12:51pm

Quiz Instructions

Please attend/watch Lecture 03 and optionally read the **Recommended Lecture Readings** , and then answer these questions.

Question 1 3 pts

The Fibonacci numbers 0,1,1,2,3,5,8,13,... are defined as follows: let $F(n)$ be the nth Fibonacci number. Then:

$F(0) = 0$
$F(1) = 1$
$F(n) = F(n-1) + F(n-2)$ for $n \geq 2$.
In other words, each term in the sequence is the sum of the two previous terms.

Bogus Claim: Every Fibonacci number is even.

Which step in the proof contains the crucial *logical* error, i.e., which is the **first false statement**?

- Proof by strong induction.
- Induction hypothesis $P(n)$: "$F(n)$ is even"
- (Base Case) $F(0) = 0$, which is even.
- (Induction step) Suppose $n \geq 2$, and assume $F(0), F(1), \ldots, F(n-1)$ are all even. We must prove that $F(n)$ is even.
- By the assumption, both $F(n-1)$ and $F(n-2)$ are even.

○ Therefore $F(n) = F(n-1) + F(n-2)$ is also even, as claimed.

○ Conclusion: by the Strong Induction principle, $F(n)$ is even for all $n > 0$.

Question 2 2 pts

Which of the following are correct about Ordinary and Strong Induction?

☐ Strong induction and ordinary induction are technically equivalent.

☐ Ordinary induction can be seen as a special case of strong Induction where some of the assumptions are not used.

☐ A strong induction proof can be turned into ordinary induction by copying the strong induction proof but omitting the assumptions about $P(k)$ for $k < n$.

☐ A strong induction proof can be turned into an ordinary induction by revising the induction hypothesis from $P(n)$ to $\forall k \leq n.\ P(k)$.

Question 3 3 pts

Alice wants to prove by induction that predicate P holds for certain nonnegative integers. She has proven that $P(n) \implies P(n+3)$ for all nonnegative integers $n = 0, 1, 2, 3 \ldots$

Suppose Alice also proves $P(5)$. Which of the following propositions can she infer? Select all that apply. The domain of definition for n is the nonnegative integers.

☐ $P(n)$ holds for all $n \geq 5$

☐ $P(3n)$ holds for all $n \geq 5$

☐ $P(n)$ holds for all $n = 5, 8, 11, 14, \ldots$

☐ $P(n)$ does not hold for $n \leq 4$

Question 4 2 pts

Using the same setup as the previous question, what other propositions can Alice infer? Select all that apply.

(Reminder: Alice has proven that $P(n) \implies P(n+3)$ for all nonnegative integers $n = 0, 1, 2, 3 \ldots$ and has also proven $P(5)$.)

☐ $\forall n.\ P(3n+5)$

☐ $\forall n.\ \left[(n > 10) \text{ IMPLIES } P(3n-1)\right]$

☐ $P(0) \text{ IMPLIES } \forall n.\ P(3n)$

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