SE BAEK OH: So today's class, we are going to talk about-- called coherence. And basically Fourier system with incoherent imaging. So particularly, this topic is very important if you want to take our optics course because We're dealing with coherence and incoherence.

So far, we have dealt with the Fourier system with a coherent illumination. Basically, laser light from a point source-- I mean, pinhole. So the coherent plane waves illuminating the Fourier system, and the frequency content appears at the plane. And you can put the [INAUDIBLE] mask to play with the spatial filtering, and you're going to get some image at the alpha plane.

But in everyday life, if you use your digital camera, you're not using the really laser illumination. You mostly use this ambient light, or if it's too dark, then you're going to use the flashlight. So today, we are going to talk about more realistic case. We use the incoherent light.

By the way, the difference between coherent and incoherent is basically, you can make the interference with the coherent light. So if you're not make interference-- you're using laser, but in most case, like this white light source, you're not going to see interference. So we're going to first define what coherence is.

And actually, there are two kinds of coherence, temporal coherence and spatial coherence. And the rest of the topic is pretty straightforward. We will define incoherent point spread function and its Fourier transform which is optical transfer function and modulation transfer function, which is modulus of OTF.

And we're going to see what MTF means. And we want to compare the difference between spatially coherent and incoherent images. So as I just described, the coherence is basically ability to make interference.

So if you have incoherent light, you cannot make interference. And if you have coherent light, then you can make the interference. And I'm going to describe it later, but mathematically-- actually, the coherence is nothing but the correlation of wave at two time space points, which means we have some light, and you just sample two points.

It could be different time points, but you combine them. And if you see interference, then you can say it's coherent. But if you don't see interference, then it is incoherent. So first, let me describe what's going to happen if I use the incoherent light in Michelson interferometer.

So here, we have the Michelson interferometer. So from the point source, the light-- oops-- is coming in this direction, this direction. And at the beam splitter, half of the light just transmit the beam splitter, but the other half is reflected. So this path gets reflected again at the mirror, and passed through the beam splitter, and arrives at the detector.

And the other path is reflected again, and reflected at the beam splitter. And again you get-- I mean, it arrives at the detector. So let's just suppose that we have the laser source here. Then what you're going to see at the detector is a nice sinusoidal pattern if I move one of the mirror.

Because I can tune the optical path length that light travels. And if the distance match, then I get the constructive interference. And if the phase delay pi, then I get disruptive interference. So as I move the one with the mirror, I get nice sinusoidal pattern with the full contrast.

And next, let's suppose instead of the laser, I have incoherent source, which means-- so the horizontal axis is time, and the vertical axis is amplitude. But the phase or amplitude is varying randomly. So over time, the amplitudes are varying randomly.

And even though it's random, this wave train will travel through the interferometer. So in this path, the optical path length is actually constant, which is-- I mean, these paths, they're common in two path. So constants plus 2d1 over c, which is time-- speed of light.

And in the second path, I have the same-- this wave train also travels. But due to the difference between d1 and d2, at the detector, essentially, I'm going to sample a different time point. So even though it's random, I basically choose two time points here and here, and combine them at the detector.

So if the d1 and d2 are same, then basically, identical two waves are coming together, so I always get constructive interference. But what if d1 and d2 are not same? And then basically, they get different phase shift.

And since it's random, they kind of average out. So d1 and d2 are getting bigger. Then basically, the degree of correlation is getting decreased, which means at the detector, the contrast will also get decreased.

So if I plot the intensity at the detector with respect to d2 minus d1, which is the distance difference of two arms, then the intensity is going to be like this. So remember that if I have the laser at point source, then I will get the sinusoidal pattern with the full contrast.

But since now I have the randomly varying source, this envelop is getting attenuated like this. So if I have the rapid decay curve, which means I have more incoherent light. And if I have the slowly decaying envelop, then it's more coherent.

And there is another kind of coherence which is spatial coherence. So basically, the concept is the same, but we only define the correlation in space domain. So now I have the Young interferometer, which I have two pinholes, x1 and x2. And I put a detector at the center of the two pinholes.

So from this pinhole, we see-- I mean, the spherical wave is going to be emanating, but since it is at the center, these paths are exactly same. And first, let's suppose we have laser illumination. So I have the plane wave coming in.

Then this spherical wave will make the Young's interference. So I'm going to have the sinusoidal pattern with the full contrast at this plane, but I'm just seeing at one point. And again, if I have the randomly varying wave here-- so in this case, this horizontal axis amplitude and vertical axis is space instead of time.

So this amplitude and phase are randomly varying over space. Since I have two pinholes, so I essentially sample at two points here and there, and combine them. So again, if these two pinholes are very close, then these sampling points are also very close. So the amplitude and phase are more likely similar. [INAUDIBLE].

But if I move these two pinholes in the upward and downward like this, then the spacing between two pinholes are getting bigger, and basically, I lose the correlation, which means the spatial coherence is getting attenuated-- getting smaller. So again, if I plot the intensity at the detector with respect to x2 minus x1, which is the spacing between two pinholes, then intensity varying like this.

Again, remember, if I have the laser, then it doesn't really matter. I mean, I always get a full contrast. Even though those frequencies-- I mean, [INAUDIBLE] plane-- the frequency of the interference is changing. But I always get the full contrast, but contrast getting smaller if you have the incoherent source.

So let me describe mathematically why this happens. It doesn't really matter whether I use the Michelson interferometer or Young's interferometer. Basically, interferometer, we have two waves. So a1, e to the pi 1, and a2, d to the pi 2.

Basically, we have two waves I could sample in space or time domain. But anyway, I'm going to have two waves and just add them together, and take the magnitude square, which is intensity. So Michelson interferometer, this is-- I mean, these two waves came from the different-- the path length, d1 and d2.

And Young's interferometer, basically, they came from the two different pinholes, x1 and x2. But anyway, I can always write the interference of two waves like this. So wave one and wave two, it's negative square is going to be i0 and 1 plus m cosine, the phase difference.

AUDIENCE: [INAUDIBLE]

SE BAEK OH: We are assuming monochromatic, yes. I mean, you can write-- [INAUDIBLE] we use assuming monochromatic because we're using phase notation. So we write this and we write phaser, but actually, there is e to the i or negative i.

So yeah, we assume it's monochromatic. But even though it's not chromatic, you can always write just addition of two waves. It really doesn't matter. So interference is basically the dc term which is i0. So i0 is actually a1 squared plus a2 squared, which is intensity of wave 1 and intensity of wave 2.

So dc term is just addition of the intensity, and we have some cosine modulation term. So this is basically interference. So if we have coherent light, then we have some dc term, and have a sinusoidal modulation.

And as the phase varies, we're going to see the sinusoidal pattern is changing. But remember that this optical frequency is very fast-- like tens of gigahertz [INAUDIBLE]. So actually, what we measure-- I mean, intensity is time average of this guy.

So we're going to take the time average of intensity. And if you do the math-- actually, we have the dc term and this kind of modulation term, which is time average of cosine with a phase difference. But remember that if we have incoherent light, this phase difference is random.

So what is the average of cosine of random phase? [INAUDIBLE]. What is the average of cosine or random phase?

AUDIENCE: Q.

SE BAEK OH: Yeah, Q. Cosine is oscillating between plus 1 and to minus 1. So if you have random phase and average, then you get 0.

So if you have incoherent light, then basically intensity is just addition of-- the intensity of 2 waves, and we don't have any interference term. So that's the difference between coherent and incoherent light.

So coherent light, you always have a modulation term. I mean, it could be 0 depending on phase difference. But in [INAUDIBLE], you always have just addition of dc term-- I mean addition of intensity.

So if it's perfectly incoherent, then we have just addition of intensity. And actually, coherent and incoherent is very-- actually two opposite extreme cases. So it's just like monochromatic. In real case, we are not going to have really monochromatic source. Our source is always finite bandwidth. It's not going to be delta function.

So it's same thing. We are not going to have the perfectly coherent or perfectly incoherent. Everything is partially coherent. So if you want to deal with it, then you have to apply basically statistics to all these Fresnel and [INAUDIBLE]-- I mean, this kind of [INAUDIBLE] optics.

So it's called statistical optics, and it's covered in 2.707. So if you want to learn more about it, then you can take 2.7 or 1.7. But nevertheless, the coherent and incoherent case are enough to describe many, many phenomena. So we are focused on coherent and incoherent state detection.

Going a little bit fast. So-- oops. Sorry. So since we have two kinds of coherence-- so temporal coherence and spatial coherence. So actually, we have four different combinations.

So temporarily and spatially coherent, and temporarily/spatially incoherent. And temporal incoherent, spatially coherent, temporarily coherent, spatially incoherent. So the rule of thumb-- so temporal coherence is related to the bandwidth of your source-- basically, the color of your source. So if you have one color in monochromatic, then it's-- question?

AUDIENCE: Yeah, where do diode lasers go in that chart?

SE BAEK OH: Diode laser is more like temporally and spatially coherent. So yeah, let me just continue. So temporal coherence is related to color. So if you have laser, like single-color monochromatic, then it's more like coherent.

But if you have white light, like this fluorescent lamp or halogen lamp, then it's broadband, so it's temporally incoherent. And spatial coherence is related to actually the size of the source and the distance. So everything coming from the point source is spatially coherent because they're basically coming from the same point.

But if your source is extended like-- or you have the filament in a bulb, or this kind of fluorescent light, then it's spatially incoherent. So the monochromatic laser source, laser is very monochromatic, and the output is very small. So laser is typically very temporarily and spatially incoherent, and that's why people use it in interferometry.

And white light source-- and if you are nearby the white light source, then it's more like temporally and spatially incoherent. And actually, these two are a little bit interesting. So temporally incoherent or spatially coherent.

So if you put a small pinhole in front of the white light, then it's temporally incoherent because it's broadband, but it's coming from a pinhole, so it's spatially coherent. At the same reason, the sunlight is very spatially coherent, but temporally incoherent. But the sun is very huge, but also the distance is very huge.

So if you see the sun, then it looks-- sunlight is coming from point source, so that's why it's spatially coherent. And yeah, the light from-- yeah, press the button, please. It doesn't work?

AUDIENCE: Yeah. But anyway, so one of the things that [INAUDIBLE] to measure coherence, but what exactly is coherent [INAUDIBLE]?

SE BAEK OH: Yeah, that is a good question. So actually, it's a little bit lengthy to explain, but what essentially the light-- I mean, this kind of light, it excites some atoms. And when they are going down, you get the energy.

Basically, light energy by e to the-- I mean energy is H mu. But it's not-- I mean, this kind of spontaneous emission is not correlated. You have many, many atoms, but they are not synchronized. They are emitting randomly.

So basically, you have some finite-- there are wave train. So in that wave train, you get basically coherent. But you have many, many of them with the random phase. That's why you add them together, then you get incoherent.

So coherent length is essentially what is the length of a finite wave train that you can define deterministic phase. So d1-- so in just imagine the Michelson interferometer, you have the optical path first, d1 minus d2. But that difference is within the distance, then you get interference. That's why you call the coherence length and coherence time. And yeah, essentially, that's it. So remember that if we had--

AUDIENCE: Excuse me.

SE BAEK OH: Yep?

AUDIENCE: Here I also will have one question. So I want to check whether this understanding is correct. So let's say if the light comes at a perfect parallel direction, even though they have different wavelengths, it's considered especially coherent. And even if light go into different directions, but [INAUDIBLE] perfect-- the single wavelength, they're considered time coherent. Is this understanding correct?

SE BAEK OH: So actually, if you have plain wave, then you already assume your wave is spatially coherent. Because plane wave, it doesn't really matter where you sample. They have always same phase. So your assumption is spatially coherent if you assume spatially coherent light.

AUDIENCE: You mean even if they have different wavelengths?

SE BAEK OH: Yeah. Even though you have different wavelengths, that's temporally incoherent. But in space domain, you have spatially coherent. Another way to think of spatially incoherent is you have many, many plane waves propagating all the different directions, but they are randomly-- they have random initial phase.

So especially in coherent light, you cannot really define a unique propagation direction. But spatially coherent light, you can always define direction. Even though spherical wave it has all the directions, but you can always say-- if you put-- if you define the position, then that position, you have a k vector. So you can define the direction.

So let me just finish this thing. So if d1 and d2 are within the coherence length or coherence time, then you get some interference. But after that-- so basically, this is essentially the coherence length of this particular source.

And remember that-- I mean, this always happens. So even though you have a white light, you can get interference, but within a very limited range. Where did I stop?

[CLEARS THROAT]

Excuse me. So another interesting thing is there is a laser that has a broadband which is [INAUDIBLE] laser. So it's spatially coherent, but it's broadband, so you have temporally incoherence.

And the other kind of this combination is temporally coherent, but it's spatially incoherent. And it's typically referred to as quasi-monochromatic-- temporally coherent and spatially incoherent. So if you put a rotating diffuser, which is when you rotate the ground glass or milky glass in front of the laser, then basically, it induces some random phase to the laser beam where we can say it destroyed the correlation of the related.

So it's monochromatic, so it temporally coheres. But in space domain, it is a random phase, so you can have the spatially incoherent light. So these are a list of optical instruments using spatial coherence on imaging.

The Michelson stellar interferometer and the radio telescope, they are basically measuring spatial coherence of the light coming from a distant star, or sun, or-- yeah, they're basically for astronomy. So they can measure, for example, the diameter of a distant star. Remember, the spatial coherence is related to the source size and the distance.

So if you properly measure spatial coherence-- actually, the envelope of the spatial coherence, then you can estimate the diameter of the star. I believe Michelson got a Nobel Prize for this. And optical coherence tomography is actually utilizing temporal incoherence.

So as I just described, even though you have white light, if you properly match the optical path difference, then you can get interference within a very small range. So let's say you have multiple layers like tissue, and you construct the Michelson interferometer with the white light, and as you move the other mirror, then if the optical path difference matches, then you get interference.

But if you slightly off that point, then you don't get the interference. So if you plot the interference versus the distance of the mirror, then you can see kind of the peak whenever you have the right position at each layer. So you can kind of do the optical sectioning.

And in the lithography replication technique, people use some spatial coherence to basically make some nice pattern. Because as we described spatial coherence is essentially affecting this resolution and the contrast of your image. So there are these kind of things

Even though we deal with the spatially coherent with the first system, so what if we change the illumination to a spatially incoherent or temporally incoherent? And obviously, my image will be different depending on the coherent state. Actually, that's the bottom line of today's lecture.

So first, if we have the temporal incoherent-- basically, if you have the broadband for many, many colors-- then the chromatic aberration is typical evidence that I have temporal incoherent source. Then how to deal with it if you have many, many colors?

That's pretty straightforward. So you can define point spread function, or ATF, or out-- or whatever it is. You can define that thing for each wavelength. So for example, if I can define point spread function for red wavelength, I can define point spread function for blue wavelengths.

So it's just-- I can do with a straightforward equation. And spatial coherence is a little bit-- and you need to be careful. So the bottom-- actually, this is the take-home message of the lecture. If the illumination is spatially coherent, then input field is conversion of the-- actually, output field is a convolution of input field with a coherent point spread function, as we all know.

But if the illumination is spatially incoherent, then output intensity, not the output field. So output intensity is a convolution of the input intensity with the incoherent point spread function. So that's the-- which means if you have spatial coherent illumination, then-- fields are linear systems. You can do the convolution in field.

But if the illumination is incoherent, then intensity is a linear system, so you can deal with the intensity with the convolution. So rest of the lecture, we are going to deal with the spatial incoherent illumination. So how we can define point spread function, and transfer function, and et cetera.

So first, let's review what we have learned so far. So we have fourth system with a two length whose focal lengths are f1 and f2. And typically, we put our input transparency grating at the input plane. And we illuminate with the spatially coherent illumination and also temporally coherent.

So the input field just right after the input transparency is going to be just a product of the illumination field and the complex transparency. That's the input field. And you're going to take the Fourier transform to get the field at the wave plane.

And another Fourier transform, you get the final image. And from that, we all know that amplitude [INAUDIBLE] function is nothing but the [INAUDIBLE] function with the coordinate scaling. So ATF, H of u [INAUDIBLE] some constant and [INAUDIBLE] mask function, and lambda f1u and lambda f1v.

And the point spread function in field is just Fourier transform of the ATF. Or you can say it's field transform [INAUDIBLE] function. So this is the coherent point spread function.

And you can define [INAUDIBLE] function reduce coordinate. And the relation between input and output field is this convolution. So G output-- I mean, output field is convolution of input field, and this is coherent point spread function which is defined in field.

So this is the stuff that we all know. So the next thing is let me put the two pinholes at the input plane. And illuminate with the spatially coherent light. So basically, we have Young interferometer here, and I'm going to image that pattern.

And since it is coherent-- it is illuminated with the coherent light. So essentially, I have-- at the image plane, there's actually two point spread functions coming from two pinholes. So in the upper image is actually coming from the lower pinhole, and the lower image actually coming from the top pinhole.

But I have the addition of two point spread functions which is shifted by the position of the two pinholes. And I may have some initial phase, a1, e to the phi 1, and a to-- e to the phi 2. But the bottom line is in coherent case, I just have coherent addition of two point spread functions like this.

And to get the intensity, as you all know, we just take the negative square of the sum, and if we expand it, then we have [INAUDIBLE] which is the intensity of the first point spread function and intensity of the second point spread function. And we have some additional modulation term, which means I have some interference. So it's a1, a2, and rear path of this guy.

But rear path is always cosine. So if you change the phase, you have some sinusoidal modulation term. So this is sometimes called interference term or [INAUDIBLE] term. It implies that you have interference.

So with these parameters-- so initial phase is just d to the negative pi over 3, and the other one is the positive pi over 3. Then if I plot this intensity, then I'm going to have this blue line. So this is the case of spatially coherent illumination.

And next, let's suppose I illuminate with the incoherent illumination. So as I described earlier, the difference between incoherent and coherent is I don't have interference term if I illuminate with the incoherent source. So at the intensity, I only have these two terms-- just intensity of the first point spread function and intensity of the second point spread function. And I don't have the extra cosine term.

So this is just intensity. So if I plot the intensity with the same parameter, then this is how it looks. And if I put together, then the upper left is the two intensity profiles for coherent and incoherent illumination. So they are slightly different.

And if I play with a different parameter, like the different initial phase. And in this case, my point spread function has extra linear phase term, then the difference is quite dramatic. So you always have to be careful what kind of coherence your illumination is. Any questions?

So in previous case, which is spatially coherent, then the input field and output field is linear, so you can use the convolution. But in spatially incoherent case, we deal with the input intensity which is-- so we're going to deal with the input intensity and output intensity.

And at the image plane, the intensity will be [INAUDIBLE] incoherently. So I illuminate with the intensity-- I mean, incoherent illumination with the intensity [INAUDIBLE], and g sub t is the complex transparency. But this is field, so you have to take negative square.

So product of this guy is basically your input intensity. So this is illumination intensity and-- intensity of your grating-- and this is point spread function-- actually, coherent point spread function where you take the negative square because you take the negative square to get intensity. [CLEARS THROAT] Excuse me.

We define incoherent point spread function, which is ipsf-- it's actually negative square over the coherent point spread function. It's pretty simple, right? You have point spread function in field. And if you just take the intensity and test the intensity point spread function. But you probably already noticed the difference. So if you use the coherent illumination, then the fields are related to convolution, but in coherent case, intensities are related to convolution, and point spread function are different.

Any questions? I'm going to [INAUDIBLE]. So we just defined the intensity point spread function, which is negative square coherent point spread function. And by analogy, I mean in the coherence case, we had the coherent point spread function.

And if you take the Fourier transform of the coherent point spread function, then that is ATF-- amplitude transfer function. So we could define the same transform function for incoherent point spread function, which is OTF here. So [INAUDIBLE] says that-- so actually, the small h is the coherent point spread function, and the capital H is ATF.

So they are a Fourier transform pair. And hI, which is intensity point spread function, is the negative square of the coherent point spread function. But hI and OTF which is this weird h, they are also a Fourier transform pair.

So if you go through this derivation, then it will prove actually they are-- that OTF is actually this integration of ATF. So once you have ATF, and after you compute this information, then you get OTF which is auto correlation.

And I'm going to derive in a different way here. [INAUDIBLE] please. So we start with hI of x, which is intensity point spread function is negative square of the coherent point spread function. And-- yes?

AUDIENCE: Can you write much bigger? [INAUDIBLE].

SE BAEK OH: All right.

[LAUGHS]

Let me try. That's too big, right?

AUDIENCE: [INAUDIBLE]

[LAUGHS]

SE BAEK OH: It's too small? OK. A little bit smaller. We start with this thing, and we know that the OTF is Fourier transform of intensity point spread function.

And I can just plug into here. So it's Fourier transform of negative square of the coherent point spread function and Fourier transform. And since it is complex conjugate, I can write as h of x. And it's complex conjugate.

And since these two are products, then I can write as convolution of two guys. So it's Fourier transform of h of x, conversion with Fourier transform of a complex conjugate like this. [INAUDIBLE].

And Fourier transform of the H is actually H of u-- I mean this guy. ATF is just nothing but Fourier transform of the coherent point spread function. And what about this guy?

Fourier transform of complex conjugate of small h. So I can write this in integral form. So integral h, complex conjugate of x. e to the minus j2pi, xu, dx. And since it is complex conjugate, I can write as integral h of x, e to the I2pi xu, dx, x, and conjugate.

So I can write as OTF is H of u convolution, and complex conjugate of Fourier transform of-- like this. Actually, [INAUDIBLE] transform. So actually, this inverse Fourier transform, if you remember, the difference of the Fourier transform is actually integral h of x, e to the I2pi xu, dx. But

You can write as h of x, e to the minus 2pi x minus u, dx. So actually, it is Fourier transform, and h of x. But you change the spatial frequency variable as a minus u, So it's going to be H of minus u.

So at the end, we have H of u convolution, and complex conjugate of-- ah, sorry. H complex conjugate of minus u. [INAUDIBLE] where I can write as an integral form. Integration H of u prime, H of minus u minus u prime, complex conjugate, and du prime. So essentially, integration H complex conjugate-- sorry.

So I have H of u prime, complex conjugate u prime minus u, du prime, which is essentially this guy in [INAUDIBLE]. And sometimes, we denote by H of u [INAUDIBLE] H of u, which is auto-correlation. Any questions on the derivation? So before we move on, can anyone guess what this integration means? So you have-- you want to compute the OTF, which is auto-correlation of the ATF.

But this integration is basically you have H, and you take the complex conjugate, but you shift and take the integral. So next slide, I'm going to explain how we can compute-- so bottom line of this slide is OTF is nothing but auto-correlation of your ATF. But next slide is how to compute when-- you can simply compute this integration, but there is another easy way to guess the OTF.

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Conceptually, yes. I mean, actually, Fourier transform of the input intensity and Fourier transform of the output intensity, and the ratio of [INAUDIBLE]. So let's first compute the OTF for the simple case. I have the rectangular aperture in my [INAUDIBLE] plane. And from that, I know that the ATF-- H of u-- is just [INAUDIBLE] function, and u over umx. So umx is defined by the physical size of the [INAUDIBLE] function.

And so if it's just x, double prime x, and with the scaling vector of lambda F1. So this is the ATF. And I just put that OTF is auto-correlation of ATF. In integral form, I have this integration.

So let's just take a look at this [INAUDIBLE] actually. So OTF is function of u, but this integration-- I mean inside of this integration, I have function of H of u prime. And H complex conjugate, u prime minus u. So I have the first function, which is essentially same as my original function even though I change the variable from u to u prime.

But this guy is function of u prime where you shift by u. And [INAUDIBLE] integration. And this whole thing is function of u, which means you're going to compute this integration for every u, and then you get OTF. And this integration is basically you have two functions, one is shifted by u, and multiply them, and take the integration, which is essentially you compute the overlapped area of the two functions.

So graphically, if I have [INAUDIBLE] function and plug it in this integration, I have this integration. But I can imagine for a different situation, the first-- so this [INAUDIBLE] function is the coherent [INAUDIBLE] function. And the second one which is moving is a coherent [INAUDIBLE] function.

So if they are not overlapped, then I get simply 0 the first situation. And as soon as they are overlapped, and then the overlapped area is getting linearly increased. So I get this u plus umx until they are completely overlapped.

And then-- so this is [INAUDIBLE]-- and then once you move a little bit further, then basically this overlapped area is getting decreased. So this is third situation, which is this guy. And eventually, they are not going to overlap, so I get 0 again.

Remember that this OTF is functional of u, which is the amount of shift of this [INAUDIBLE] function. Any questions? So if you plot this overlapped area-- I mean this guy and this guy-- with respect to u, then actually we are going to get a triangular function, which looks like this one.

And of course, we normalize at the 0 frequency. So at the 0, we always have a 1. So you notice the difference between these two. So ATF-- my ATF is nothing but my [INAUDIBLE] function-- just left function. And the color frequency, umx, is defined by my physical size of the [INAUDIBLE] with the scaling factor.

But OTF is auto-correlation, so the shape is now the triangular shape. But also the color frequency-- the umx is twice of the umx, because basically you go through all this process. So you get twice of the color frequency.

So-- yeah? Oh. So whatever [INAUDIBLE] you have, if you compute this kind of overlapped area, then you can get OTF. So [INAUDIBLE] you have a rectangular square aperture, then you if you go through this auto-correlation, then you can get this kind of OTF.

So this is just completely linear in u and [INAUDIBLE] direction. Again, the fx and fy, they are [INAUDIBLE]. And if you have the circular aperture, it's almost linear, but it's slightly curved at the edge. But essentially, they are the same thing. So if you move around two circles in completely overlapped area, then you end up with this function. I guess this is a good time to take a break.

Any questions so far? So we discussed there is a lot of difference between temporal and spatial coherence, and defined the intensity point spread function, and also defined the OTF which is Fourier transform with the intensity point spread function. And at the last-- [CLEARS THROAT] excuse me-- OTF is auto-correlation of the ATF. And I visually showed how to compute the OTF.

So let's just summarize what we discussed so far. So first, we all know that coherent point spread function is nothing but an optical field generated by a point source at [INAUDIBLE] plane. And that's the field. And also you know that coherent point spread function is Fourier transform [INAUDIBLE] function with a coordinate scaling.

And incoherent point spread function is negative square of coherent point spread. Or another way to say is intensity point spread function is intensity of the field coherent point spread function. And we define the amplitude [INAUDIBLE] function, ATF, which is Fourier transform of the coherent point spread function.

And it is nothing but the [INAUDIBLE] function with the scaling vector-- I mean coordinate scaling. And we just derive the OTF, which is Fourier transform of the intensity point spread function is actually auto-correlation with the ATF. In the rest of the lecture, we're going to mostly deal with the MTF which is negative modulus of the OTF. Actually, it should be OTF.

So MTF is the negative component of OTF. In terms of input and output, we're dealing with a linear system approach. So if you have the spatially coherent illumination-- so [INAUDIBLE] is the illumination field. And I have the thin transparency-- the complex transparency which is sub t.

Then my input field is just product of my illumination field and complex transparency. So g sub n, that's the input field. And if I convert with the coherent point spread function, that's going to be my output field, which is g sub f.

And I kind of think this whole thing in frequency domain. So if I take a Fourier transform-- [INAUDIBLE] is actually Fourier transform of small gn, which is Fourier component of my input field. And convolution is multiplication in frequency domain.

So I multiply by ATF, then I'm going to get g sub-- [INAUDIBLE] which is Fourier inform of my output field. And if I have the spatially incoherent illumination, then I deal with the intensity. So I have the intensity of illumination, and I-- still complex transparency. So actually, my input intensity is illumination intensity times negative square of g sub t.

So remember that gt is complex transparency. So to make intensity, you have to take negative square. So this is my input intensity. And in current case, it's linear in intensity, so you have to convert with the intensity point spread function, which is negative square of the coherent point spread function. Then you're going to get in output intensity.

And again, in the frequency domain, we can take the Fourier transform-- so g sub i-- what can I say? Character gi input-- character g sub i in, which is Fourier transform of the input intensity. And you're going to multiply by OTF, which is what auto-correlation of the ATF. And you'll get the Fourier transform of the output intensity.

So actually, if you're familiar with the control theory, then you probably know the Bode plot. So Bode plot is nothing but you have-- in frequency domain, it describes the weighting function that is going to be multiplied by your input and produce your output. So this ATF, and OTF, and also MTF, they are basically the same thing.

You multiply your-- so basically, this transform function in frequency domain describes the weighting function here that is going to be multiplied by your input. And you can just compare the-- and also another way to say it is the ratio between input and output. So MTF-- so, yeah.

So as I described, this transform function is nothing but with weighting function in frequency domain. So let me show you how to use it. Because if you buy a lens for your digital camera-- I mean, typically the SLR camera-- so lets make us provide the MTF layer, which is this weighting function with the spatial frequency, and with the color frequency. It's cool-- you can say what kind of image quality I can expect with the lens.

So first, let's consider our [INAUDIBLE] grating whose MTF transform function is given by this equation. So the input intensity is going to be-- I mean intensity of the grating is going to be negative square, so I add 1 over 2. Then 1 plus n modulation, and cosine term.

And the phase [INAUDIBLE] it doesn't matter why, because if you take a negative square, it's always 1. The Fourier transform of this input intensity is-- we all know that it has three 3 orders.

So 0-th order, and the 1 over capital lambda, and minus 1 over capital lambda. We have three different orders. And it's going to be multiplied by the OTF, which is at this h. And since this is delta function, which means this one is 1 only at u equals 0, and when u equals 1 over capital lambda, and at minus 1 capital lambda.

So if you multiply the OTF-- and actually, you still have the three components. So at 0 component, you have the OTF at the 0 frequency. And these two delta functions are multiplied by OTF at the 1 over capital lambda and minus 1 over capital lambda. We still have the three [INAUDIBLE] orders.

And let me just go back. So since this is your Fourier transform of the output intensity, you can just take the inverse Fourier transform to compute the output intensity, right? But there is another-- there is an important property of the OTF. So this guy and this guy.

So OTF is actually-- intensity point spread function, which is h sub i, is negative square of the coherent point spread function. So it is always positive-- it's non-negative I should say. So it's non-negative.

So this Fourier transform is actually Hermitian, which mathematically, you can [INAUDIBLE] h of minus u-- is actually h complex conjugate of u. Or you can say its real path is [INAUDIBLE] function, but its simulated path is out function. So OTF of the physically realizable optical system must be Hermitian. So this condition should be satisfied always.

So in the previous slide, I had-- let me just-- [INAUDIBLE]. Capital G, I, out, was 1 over 2 OTF at 0, and delta u. Plus n over 2, H, 1 over capital lambda, and delta u, 1 over capital lambda. Plus H minus 1 over capital lambda, delta u, 1 over capital lambda.

Since the OTF is Hermitian, which means my OTF over minus 1 over capital lambda is actually complex conjugate of H, 1 over capital lambda. This is Hermitian. So G out-- I mean, the capital G, I, out, I can write as 1 over 2 OTF at 0 is still delta function. And m over 2, H, 1 over capital lambda, delta u, 1 over capital lambda.

But the last delta function I can write as H complex conjugate, 1 over capital lambda, decay u plus 1 over capital lambda. So the first delta function-- I mean, if you take the inverse Fourier transform [INAUDIBLE] transform, the first delta function [INAUDIBLE]-- just 1 or something. So it's pretty straightforward.

The question is still what happened in these two delta functions, because this one has H of-- I mean the OTF at one over capital lambda, and the other one is actually complex conjugate. So if I just write the two delta function part, and I'm going to take the inverse Fourier transform.

So inverse Fourier transform, or m over 2, [INAUDIBLE] OTF at 1 over capital lambda, delta u, 1 over capital lambda, plus H complex conjugate, capital lambda, delta u plus 1 over capital lambda. Then what is-- inverse Fourier transform [INAUDIBLE] function is going to be exponential.

And [INAUDIBLE] do we scale it? So [INAUDIBLE] just outside. And I'm going to have H of 1 over capital lambda, and exponent e to the I2pi, x over capital lambda, plus H complex conjugate, 1 over capital lambda. e to the minus I2pi, x over capital lambda.

And I can write as capital lambda-- 1 over capital lambda, e to the I2pi, x over lambda. Plus-- actually the same thing-- H, 1 over capital lambda, e to the I2pi, x over lambda. And complex conjugate like this.

So what happens if you add A and its complex conjugate? So for example, this is A, and this is A complex conjugate. And if you add them together, then you get only rear path, because two rear paths over A.

So then I'm going to have m, the rear path of OTF at 1 over capital lambda, to the I2pi x over lambda, which I can write as m, and negative-- this H is just non-zero. So it could have-- I mean, it can have only 0 or minus [INAUDIBLE] basically. So I can write it as negative here.

And this [INAUDIBLE] cosine 2pi, x over lambda. So that's why the output intensity is just 1 plus m, which is original modulation. But I add extra modulation due to OTF and cosine term.

Should I prove why OTF is Hermitian? Yeah, let me prove why OTF is Hermitian. So OTF is auto-correlation of ATF, which is like this. And integration with the integral form I can write as H of u prime, H complex conjugate, u prime minus u, and [INAUDIBLE] prime. And how to compute H-- I mean OTF and complex conjugate of u. So I can write as-- the total thing-- H of u prime, H complex conjugate u prime u, u prime, and complex conjugate.

AUDIENCE: Excuse me.

SE BAEK OH: Yes?

AUDIENCE: We're not able to see what you are writing.

SE BAEK OH: Oh, it's too small?

AUDIENCE: OK now.

SE BAEK OH: So I just take the complex conjugate on both sides. And I can chain the complex conjugate. So it's going to be H complex conjugate of u prime, H, and u prime and u, and du prime. So I just put the complex conjugate inside of the integration.

And here, I define the new variable. So u double prime is u prime minus u. So u prime is u plus u double prime. And du prime is actually du double prime.

So this H complex conjugate is actually H complex conjugate, u plus u double prime, h of u double prime, and du double prime, which I can write as integration h u double prime. A complex conjugate u double prime, minus u, du double prime. So it's H of minus u.

So this is complex conjugate Hermitian. I just proved you can easily do that. So then the bottom line is at the output plane-- I mean output intensity, I have extra modulation coming from the OTF. I mean, you can say intensity is negative, so you can say [INAUDIBLE].

So if you compare in terms of the contrast or visibility, so the contrast is defined by imx minus ime over imx plus ime. But at the output plane, we have the extra modulation term. So if you compute the same thing, then actually your contrast is going to be at the original modulation multiplied by MTF at the spatial frequency.

So MTF is just contrast at the spatial frequency relative to the transmission that we're discussing. So what does it mean? So for example, I have MTF looks like this. And I initially had the grating-- the intensity of the grating is actually 1 over 2, 1 plus n, cosine, 2pi, x over capital lambda.

So if I draw in grayscale-- so the white is 1 and black is 0. So I have nice sinusoid with a full contrast like this. But the output intensity, I'm going to have 1 plus m, and additional modulation, and cosine term.

And this factor, you can find [INAUDIBLE]. So if you have the 1 over capital lambda, then you can find the value at that point-- I mean MTF at this point. So if that value is 0.6 in this case, this sinusoidal pattern will look like this.

But if the modulation is 0.2, then the black becomes brighter and white becomes darker. So you kind of get the grayscale-- [INAUDIBLE] version of the gradient like this. So basically, this MTF tells you what kind of contrast I expect at this particular spatial frequency, and also it specifies the maximum spatial frequency you can resolve-- I mean, you can see. Any questions?

So so far, we just deal with the idea of thin lens with a clear aperture, like the rectangular aperture is a clear aperture. So in the case of the rectangular aperture, we know that the OTF is just auto-correlation of the two [INAUDIBLE] functions which is just triangular function. So it's linear from 1 to color frequency, which is twice of the [INAUDIBLE] frequency at OTF-- the ATF.

And if you have the circular aperture, it's almost linear except at the [INAUDIBLE] you have slightly curved line. But it's the same thing. So this blue line and the black line we call defraction limited OTF, because we assume the idea of a thin lens without any aberration.

So this is kind of best OTF you can achieve. But in practice, you always have aberration-- chromatic aberration, or any high aberration. Or you can have a defocus, or some imperfect factor in lens. So typically, the typical lens has this kind of distorted or degraded OTF like this. The red one right here or here.

So in terms of this defraction limited, you want to buy a lens that OTF is similar to this blue line, because this is defraction limited, which means the best OTF we can achieve. And [INAUDIBLE].

So I describe-- I mean, this is slope is linear, and circular aperture. I mean, locally, it can have a higher value, but the overall shape should be smaller than this blue line. So let's just apply all these concepts in [INAUDIBLE] system.

So we have first system whose focal length f1, f2. And I put the input transparency, which is a binary [INAUDIBLE] grading right here. And illuminate with the quasi-monochromatic which is on a single color, and spatial [INAUDIBLE] illumination, and [INAUDIBLE] we assume the intensity variation is uniform.

So the possible question is what kind of image we can expect at the image plane, or what is the OTF, or what is ATF, and blah, blah, blah. So let's go through it. So this is the binary amplitude grating. So it is still complex transparency in here.

So if you have [INAUDIBLE] illumination, you want to compute the intensity. So you have to take the negative scale. So this is just 0 or 1, so basically you have the same thing in intensity. So that's the input intensity at the upper right.

And this is the phase mask located at the rear plane. And we know that the ATF is basically the same function with different coordinates. So ATF is phase mask, but the function is the x double prime-- well, I mean you have this coordinate scaling vector.

So if you compute the OTF from this ATF, basically, what you do is you have two [INAUDIBLE] functions. So you will have actually 4 [INAUDIBLE] functions. And you move toward them, so whenever they overlap, you get some value.

So if could do the auto-correlation, then actually you're going to have three triangular. But remember, if you have the [INAUDIBLE] function, then you move two [INAUDIBLE] function. So you're going to have one triangular.

So since we have two [INAUDIBLE] functions, so whenever they first [INAUDIBLE], then we have the one triangle. But the second and the two [INAUDIBLE] functions overlap, then we have the center triangle. And like this, we another triangle, so we get three triangles.

So that's the OTF. And if you expand with the Fourier series of your input intensity, then we're going to have an infinite number of diffraction. But in ATF case, we have only first order and the minus first order, and minus first order going to be transmitted.

But in OTF, actually, they are blocked. [INAUDIBLE]. Actually, plus 2, and 0, and minus 2, they are going to be transmitted. So you just multiply these three delta functions by the value of the ATF-- this, this, this. And the Fourier transform will be your output intensity, which is this.

So 1 over 3 just came from the central [INAUDIBLE] function, because this is 1-- so just 1 over 3. And for the first order, at the 0.1 and minus 0.1 micrometer [INAUDIBLE], actually they have 0 value. So I multiply by 0. And for the second order, I have one over 2 which is this value here and here.

So that is the [INAUDIBLE] of the output intensity. So if you compute the inverse Fourier transform of this equation, then we are going to have 1 over 3, and 1 over 2, and two exponent here which is actually cosine like this. So in terms of grayscale image, we have the binary amplitude grating, which varies just black or white like this.

But if you go through this [INAUDIBLE] system at the image, we're going to have this kind of grayscale because contrast is just 0.1034. So it's very little. And the average value is about 0.33 or something.

So basically, it looks gray with a little wiggling. So ATF basically tells you the contrast of the particular spatial frequency. So, yeah. What happened-- I mean, the orders at this side.

So if my OTF is 0, then what kind of image I can expect? It's black? It's not light, right? So if it is 0-- I mean if the values are 0, then-- so I still have [INAUDIBLE] so some kind of grayscale. So I don't see any [INAUDIBLE].

OTF is auto-correlation, so you always have 1 at the dc frequency. Because if you move the two functions, so when they exactly overlap, you always have the maximum value. So you always have [INAUDIBLE] system.

And actually, that is very different in current case. So in ATF case, you can actually cancel [INAUDIBLE] system, right? But OTF, you can not cancel the system, because it's auto-correlation. So again, we have some relation with the [INAUDIBLE]. So this particular circular aperture, if I eliminate the coherent illumination, then this is the image--

AUDIENCE: Excuse me.

SE BAEK OH: Yes?

AUDIENCE: In Singapore slide, the [INAUDIBLE] slides are not [INAUDIBLE].

SE BAEK OH: Maybe it takes time or--

AUDIENCE: No, it even didn't show that input and output image when you explained about that contrast.

SE BAEK OH: Just let me ask [INAUDIBLE]. Hello, [INAUDIBLE]?

AUDIENCE: Yes?

SE BAEK OH: Yes, [INAUDIBLE]. It's frozen.

AUDIENCE: [INAUDIBLE]

SE BAEK OH: I think they're going to reset the system. So let me just take a few moments. Any questions so far? [INAUDIBLE].

AUDIENCE: [INAUDIBLE]

SE BAEK OH: So actually-- yeah?

AUDIENCE: What happens when you [INAUDIBLE]?

SE BAEK OH: Actually, I'm going to talk about it, but in general, if you use the incoherent light, then you don't see any phase effects because [INAUDIBLE] the negative square. So it doesn't really matter. So you don't see any-- I mean, you barely see some variation, but usually it's not dramatic as [INAUDIBLE].

And what was I about to say? Oh, yeah-- so MTF is just magnitude of the OTF. So we haven't talk about any phase of the OTF. So what's the effect of the-- I mean, what does it mean if I have some phase variation in OTF?

Can you guess? So for example, OTF is always non-zero. It's always 0 or positive. But its phase can be 0 or pi. So what if I have pinhole phase shift in OTF?

For example, I have sinusoidal grating, and at the output I have a sinusoidal grating. But what does it mean I have the pi phase shift?

AUDIENCE: [INAUDIBLE]

SE BAEK OH: Not really. What happens if you shift by pi-- I mean if you move to a cosine by pi? You get sine. So you have cosine pattern here. And if you have pi phase shift, then you will have the sine pattern, which means you flip the black and white of the image.

And I flip the [INAUDIBLE]. So I flip the-- but still I [INAUDIBLE] between the black and white, not the sine/cosine. [INAUDIBLE] can anybody see the slide right now, or?

AUDIENCE: Yes. I have a question. How do you introduce this pi phase shift in OTF?

SE BAEK OH: So if you have pi phase shift in OTF, then basically you flip the black and white. So your input image is black-- let's say the black, white, black, white. And your image is going to be white, black, white, black.

AUDIENCE: No, my question is physically, how do you introduce that controllable phase shift in OTF?

SE BAEK OH: So this ATF can be complex. So if you compute the auto-correlation, then actually, you can have negative value of OTF. And if you have the negative value in OTF, which means you have still positive and negative, but you have some negative-- I mean, basically pi phase.

So if you want to do that kind of thing, basically, you have some complex transparency at your [INAUDIBLE] mask. But if you have clear aperture, then you don't get that effect. So let me continue this slide. I mean, these three images we already seen maybe two or three lectures ago.

So this is MIT lecture-- I mean the image of MIT letters with the coherent emission. And the right two images obtained with the incoherent emission. So I can notice two main differences here.

So if I use the [INAUDIBLE] emission, basically, the image looks sharper than the one with the coherent. And that's because OTF has a higher-- twice of higher color frequency even though they get attenuated. But basically, it can pass through the higher spatial frequency, so that's why it preserves more higher frequency-- preserves higher frequency.

And it's same thing here. So with the coherent light, you barely see the letter "I", but you can still see the MIT. And another difference is actually if you see "M" or "T", there are some variations, like black or white. Some wiggling in MIT here.

But in coherent case, the intensity is more like uniform. You don't see some weird variation. So that's another difference. So here's the incoherent light. So generally, you don't have the ringing artifact as I described, and also no [INAUDIBLE].

So you're not going to see any interference. And it is higher bandwidth because the color frequency is twice higher than the color frequency in ATF. So you can see the smaller features-- I mean the two points closer. You can see the smaller feature.

But as I answered to [INAUDIBLE] question, incoherent image is insensitive to phase variation. So mostly, we just deal with the monochromatic, and especially incoherent. But if you have polychromatic, which is broadband, then you're going to have some chromatic aberrations.

But again, you can define all these OTF, or MTF, or point spread functions for each different wavelength. So it just extends this in a straightforward fashion. It's the end of show, so any questions?

So the take-home message of this [INAUDIBLE] lecture is if you have coherent illumination, then you have linear convolution in field. But if you have incoherent case, and you have the linear relation in intensity, and for incoherent light, you have to deal with the OTF, which is auto-correlation of ATF. And that's pretty much about it. If you don't have questions, then let's see next Monday.