2.710 Optics

Light Propagation in Sub-wavelength Modulated Media

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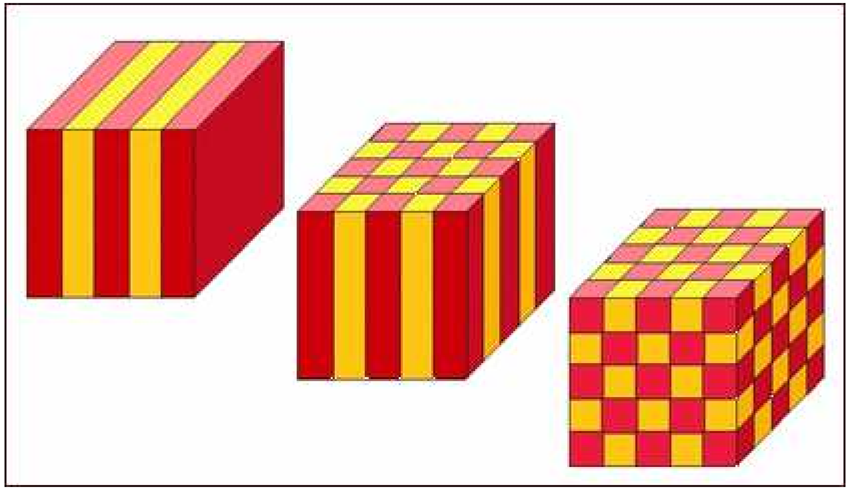
Outline

- Photonic Crystals and Electromagnetism
- Understanding FDTD
- Simulation results

Photonic Crystals

- Photonic crystals are artificial media with periodic index contrast.
- The periodicity or spacing determines the relevant light frequencies.



Maxwell's Equations

$$\nabla \bullet B = 0$$

$$\nabla \bullet D = 4\pi\rho$$

$$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} = 0$$

$$\nabla \times H - \frac{1}{c}\frac{\partial D}{\partial t} = \frac{4\pi}{c} J$$

In the absence of free charges and currents, we can set $\rho = J = 0$.

Electromagnetism as an Eigenvalue Problem

$$E(r,t) = E(r)e^{i\omega t} \qquad H(r,t) = H(r)e^{i\omega t}$$

$$\nabla \times \left(\frac{1}{\varepsilon(r)} \nabla \times H(r)\right) = \left(\frac{\omega}{c}\right)^2 H(r)$$

$$\Theta H_n = \lambda_n H_n$$

General Properties of the Harmonic Modes

- ω^2 is real and positive
- Two modes $H_1(r)$ and $H_2(r)$ at different frequencies ω_1 and ω_2 are orthogonal
- Scale invariance – the solution at one scale determines the solution at all other length scale

Bloch Theorem for electromagnetism

In a periodic dielectric medium,

$$\text{i.e. } \varepsilon(r+a) = \varepsilon(r),$$

then the solution to the Master's Equation has to satisfy:

$$H(r) = e^{i(k \cdot r)} u_k(r)$$

where $u_k(r) = u_k(r+a)$ is a periodic function.

FDTD

- Finite-difference Time-domain methods are widely used in computational electromagnetics to analyze interactions between electromagnetic waves and complex dielectric or metallic structures.
- Able to compute the respond of a linear system at many frequencies with a single computation by just taking the Fourier transform of the response to a short pulse.

FDTD

- Approximating Maxwell's equation in real space using finite differences
- Imposing appropriate boundary conditions
- Explicitly time-marching the fields to obtain the direct time-domain response

Yee's Lattice

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Difference Equations

$$\frac{\partial f(x,t)}{\partial t} = \lim_{\Delta t \to 0} \frac{f(x,t_2) - f(x,t_1)}{\Delta t} \approx \frac{f(x,t_2) - f(x,t_1)}{\Delta t}$$

$$\frac{\partial f(x,t)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x_2,t) - f(x_1,t_1)}{\Delta x} \approx \frac{f(x_2,t) - f(x_1,t)}{\Delta x}$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \qquad \frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

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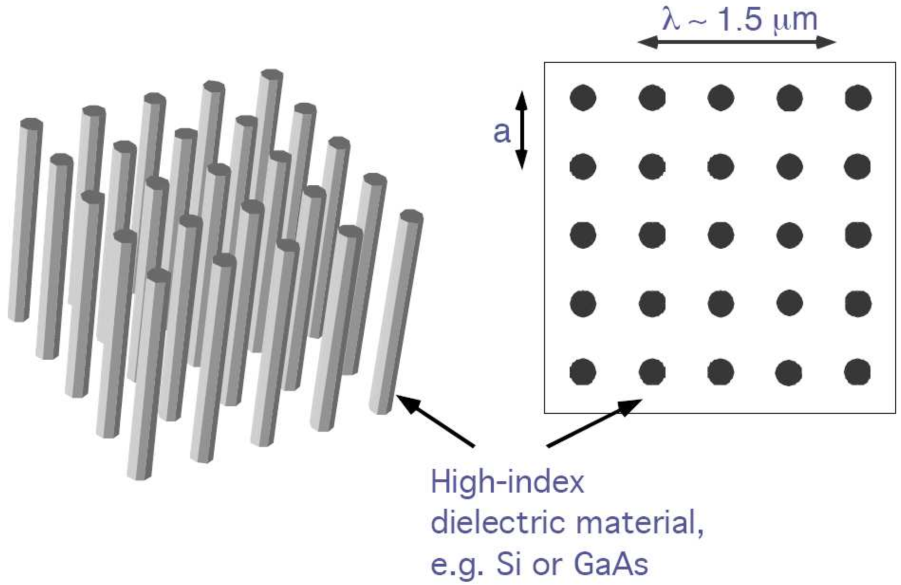
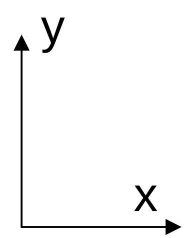
$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \quad \Rightarrow \quad \frac{E_x^n - E_x^{n-1}}{\Delta t} = \frac{1}{\varepsilon}\left(\frac{\Delta H_z^{n-1/2}}{\Delta y} - \frac{\Delta H_y^{n-1/2}}{\Delta z}\right)$$

$$\frac{\partial H_x}{\partial t} = -\frac{1}{\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) \quad \Rightarrow \quad \frac{H_x^{n+1/2} - H_x^{n-1/2}}{\Delta t} = \frac{1}{\mu}\left(\frac{\Delta E_z^n}{\Delta y} - \frac{\Delta H_y^n}{\Delta z}\right)$$

Photonic Cyrstals

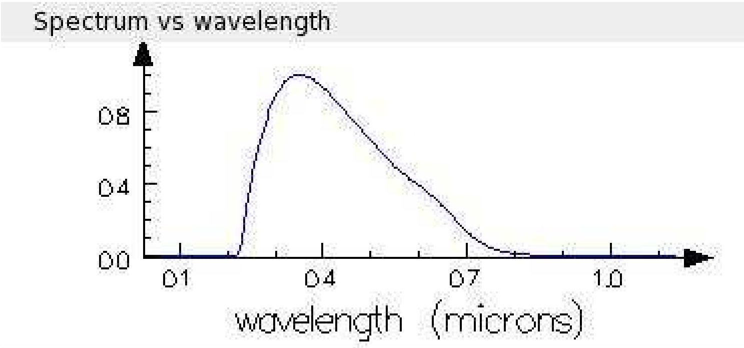
- Periodic arranged rods of aluminium
- Spacing between adjacent rods a = 0.07 micron

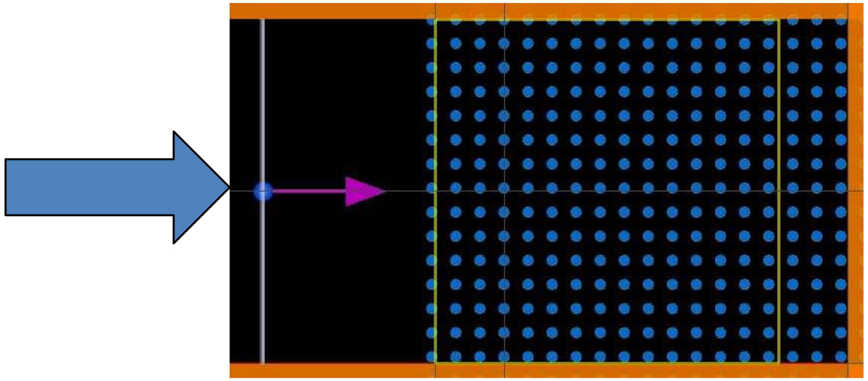


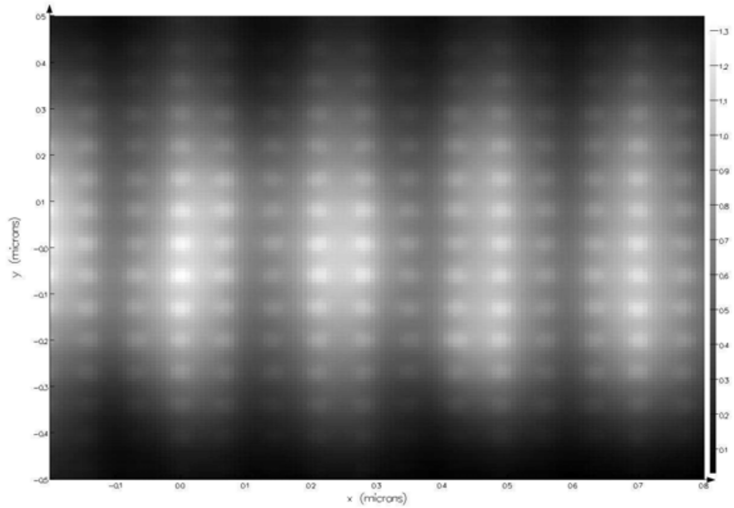


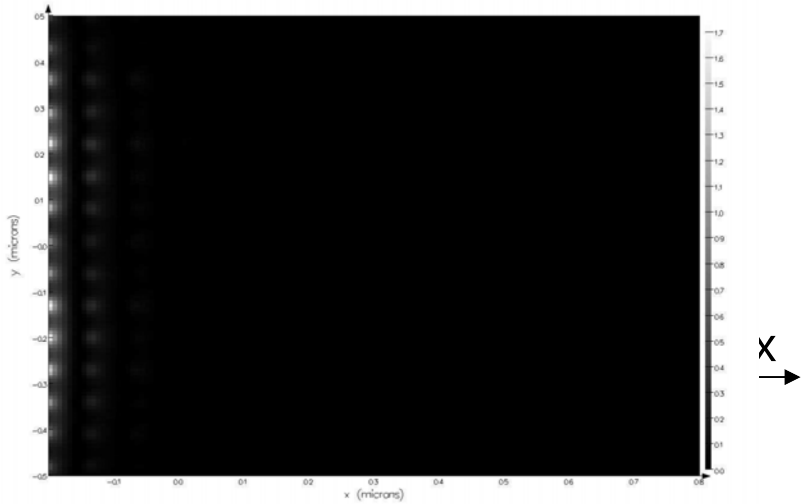
Behavior at different frequencies

- Start wavelength = a/0.3
- Stop wavelength = a/0.1

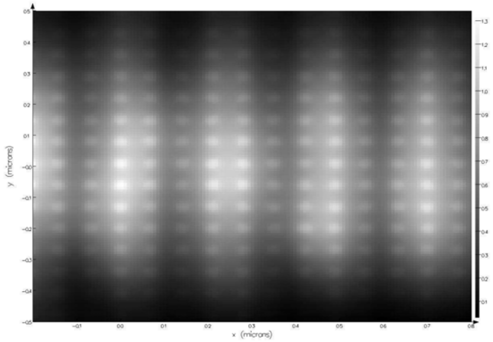


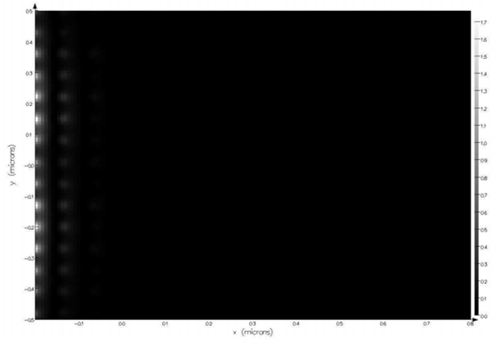


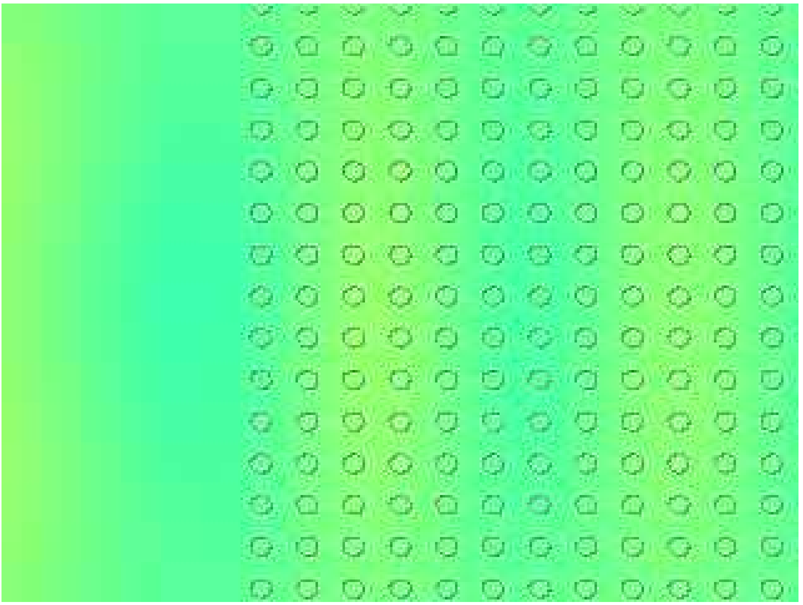


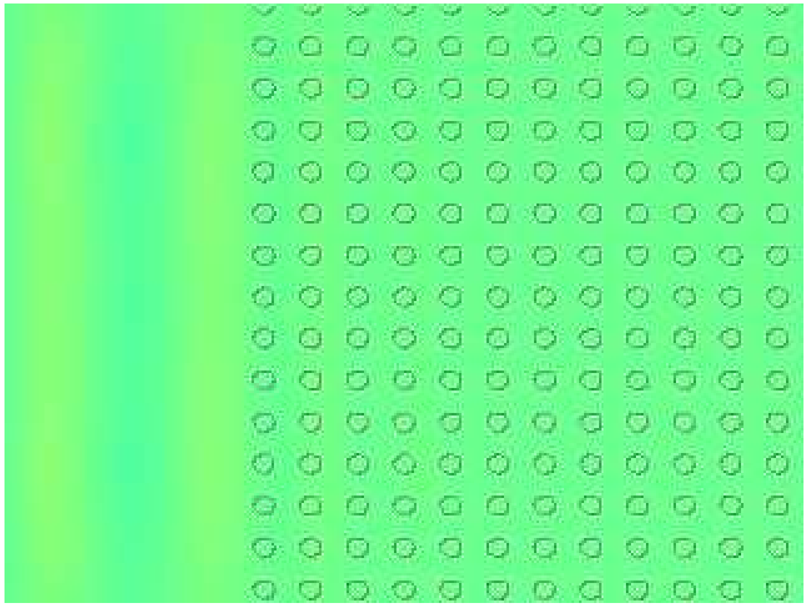


Behavior at different frequencies

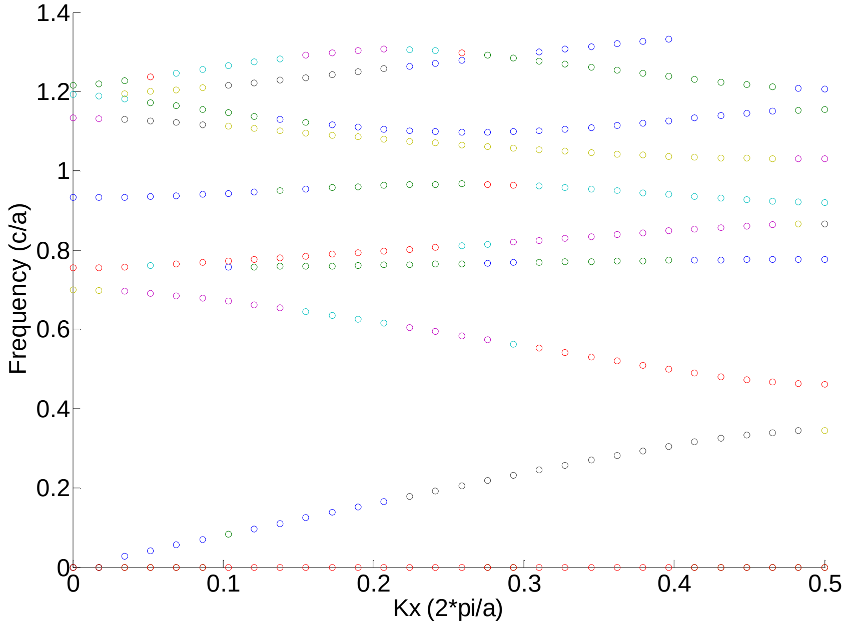


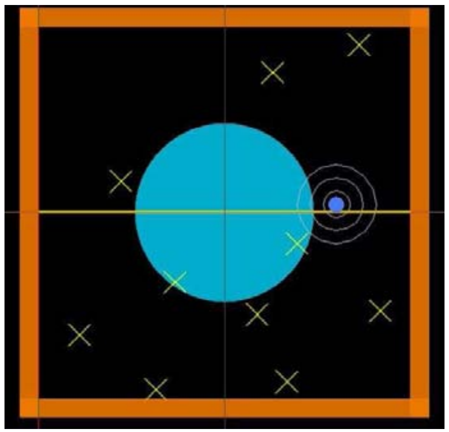




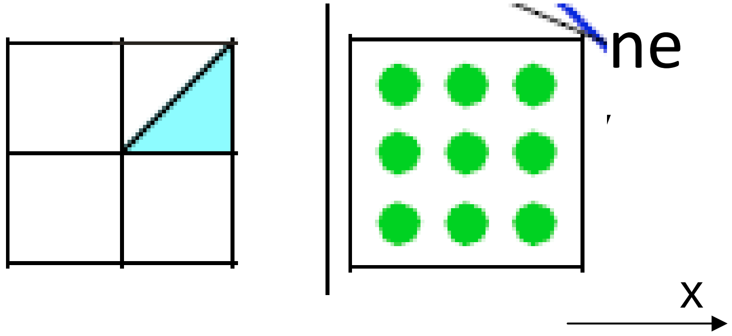


Dispersion diagram



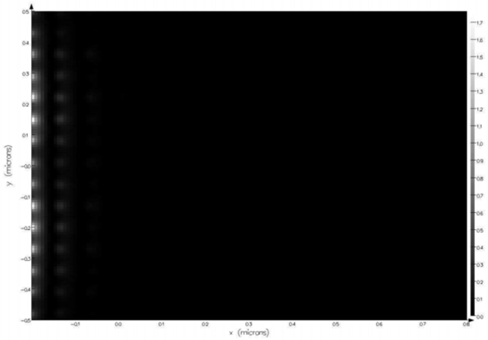


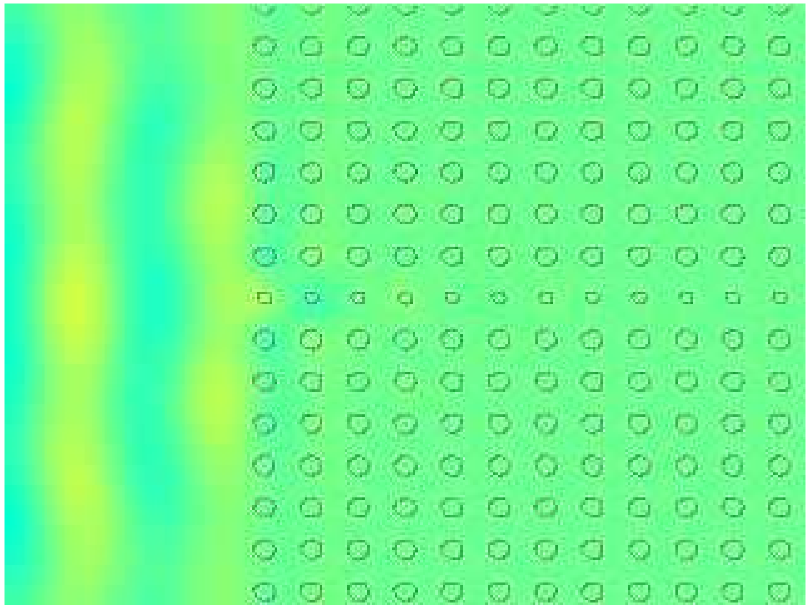
- $H(r) = e^{i(k \cdot r)} u_k(r)$
- If we take k = ko+2pi/a
- We will get similar profile
- Thus we just need to consider the k values



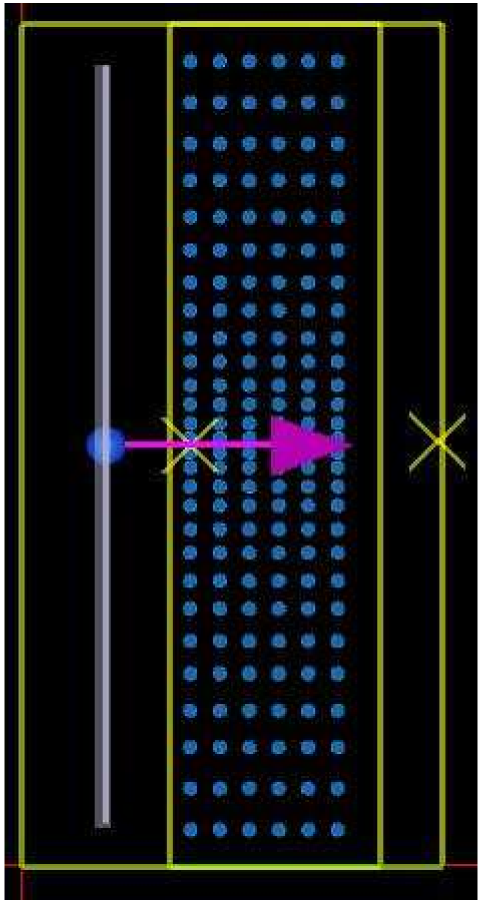
Lumerical example file

Controlling the light





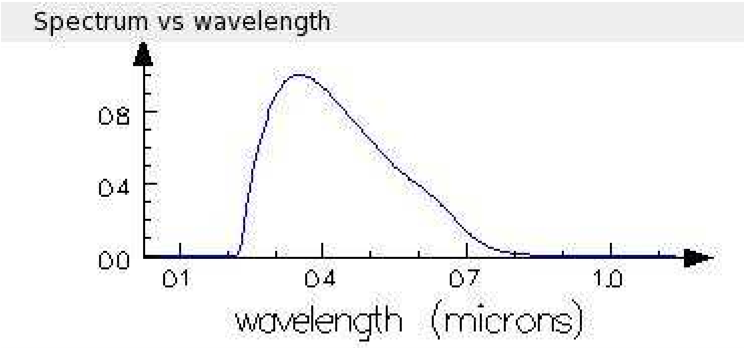
Focusing effect

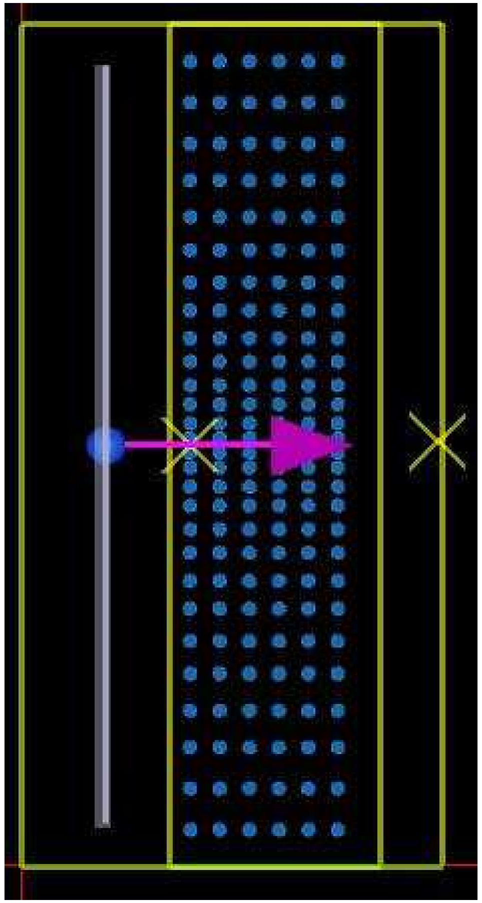


- At center spacing between two adjacent rows=a
- We increased the spacing at the rate 0.15a, as we go further away from the central line

The focusing effect of graded index photonic crystals
H Kurt et al. Applied physics letters 93, 171108 (2008)

Focusing effect





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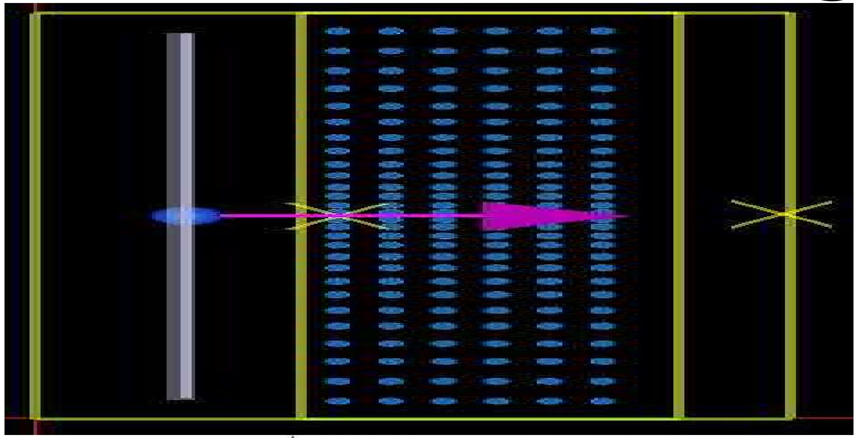
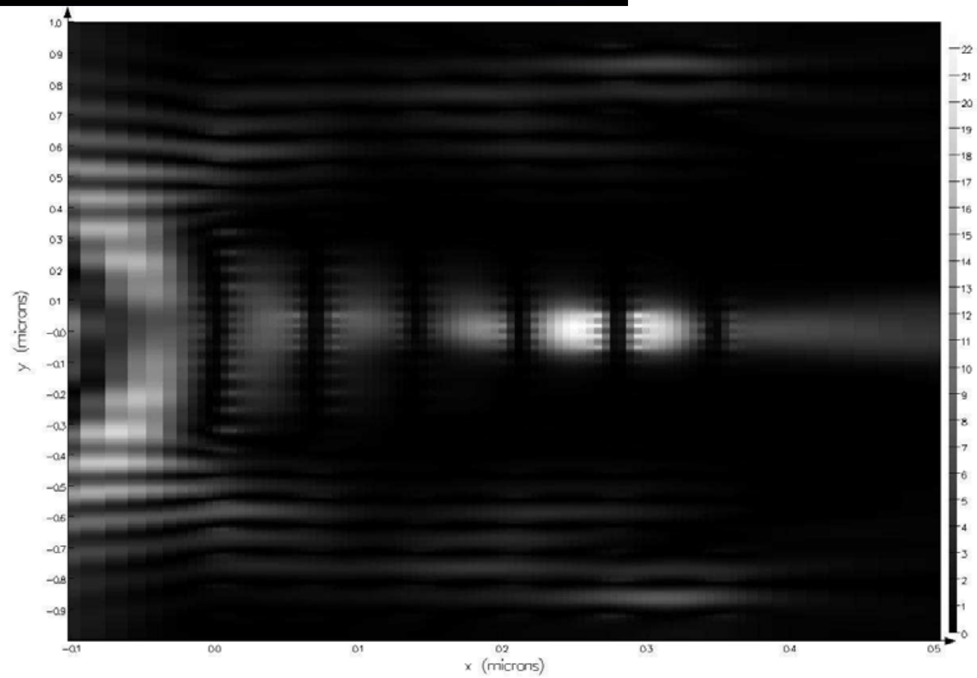


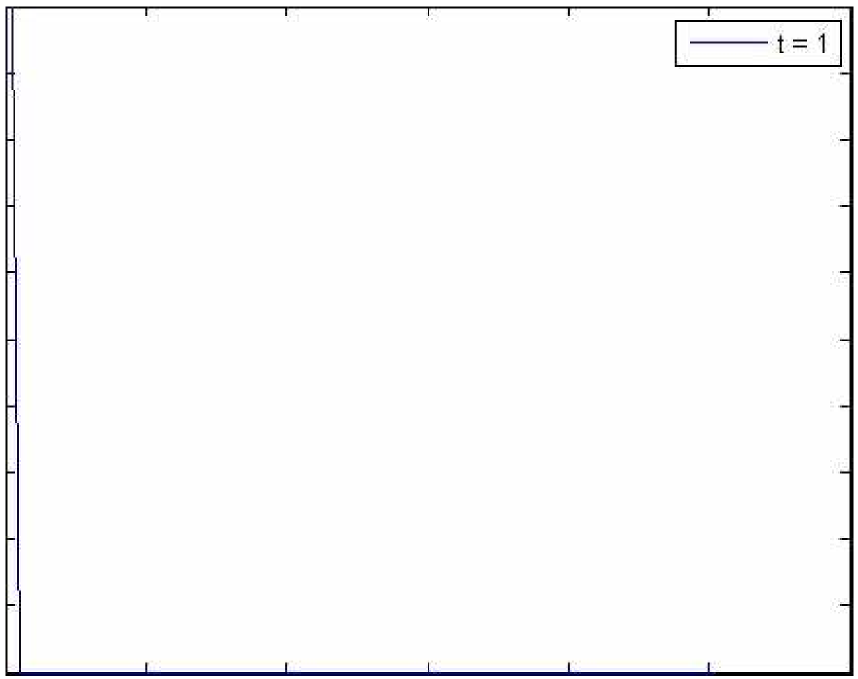


FDTD 1D Example

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\left.\frac{\partial^2 u}{\partial t^2}\right|_{x,t} = \frac{u_x^{t+dt} - 2u_x^t + u_x^{t-dt}}{dt^2}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x,t} = \frac{u_{x+dx}^t - 2u_x^t + u_{x-dx}^t}{dx^2}$$



$$u_x^{t+dt} = (cdt)^2 \left[\frac{u_{x+dx}^t - 2u_x^t + u_{x-dx}^t}{dx^2} \right] + 2u_x^t - u_x^{t-dt}$$

Yee's Lattice

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Difference Equations

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Summary

- Photonic crystals have various interesting properties and can be used to control the behavior of the light at micro-scale
- Characterization of it requires use of numerical techniques

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- H. Kurt, E. Colak, O.Cakmak, H. Caglayan and E. Ozbay. "The Focusing Effect of Graded Index Photonic Cyrstals" Appl. Phys. Lett. 93, 171108(2008)

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