MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Mechanical Engineering
2.71/2.710 Optics
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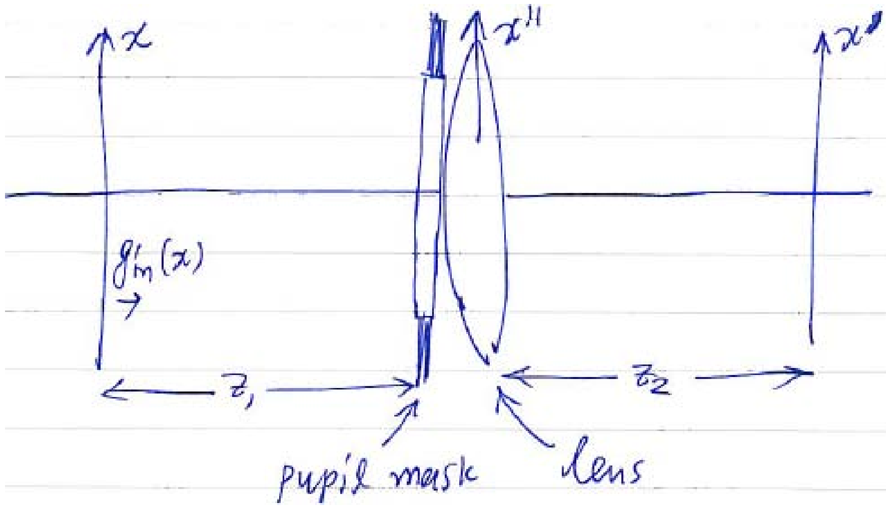
Supplement to Lecture 23
Single lens imaging



$$
\begin{aligned}
g_{\text{in}}(x,y) =& g_{\text{illum}}(x,y)\cdot g_t(x,y) \\
g_{pp-}(x'',y'') =& \frac{e^{i2\pi z_1/\lambda}}{i\lambda z_1}\iint g_{\text{in}}(x,y) e^{i\pi\frac{(x''-x)^2+(y''-y)^2}{\lambda z_1}}\,dx'dy \\
g_{pp+}(x'',y'') =& g_{pp-}(x'',y'')\times \underbrace{e^{-i\pi\frac{x''^2+y''^2}{\lambda f_0}}}_{\substack{\text{lens complex}\\ \text{transmissivity}}}\times \underbrace{g_{\text{pm}}(x'',y'')}_{\substack{\text{pupil mask}\\ \text{complex}\\ \text{transmissivity}}} \\
g_{\text{out}}(x',y') =& \frac{e^{i2\pi z_2/\lambda}}{i\lambda z_2}\iint g_{pp+}(x'',y'') e^{i\pi\frac{(x'-x'')^2+(y'-y'')^2}{\lambda z_2}}\,dx''dy'' \\
=& \frac{e^{i2\pi\frac{z_1+z_2}{\lambda}}}{-\lambda^2 z_1 z_2}\iint dxdy\iint dx''dy'' g_{\text{in}}(x,y)g_{\text{pm}}(x'',y'') \\
&\times e^{i\pi\frac{(x''-x)^2+(y''-y)^2}{\lambda z_1}-i\pi\frac{x''^2+y''^2}{\lambda f}+i\pi\frac{(x'-x'')^2+(y'-y'')^2}{\lambda z_2}} \\
=& \frac{e^{i2\pi\frac{z_1+z_2}{\lambda}}}{-\lambda^2 z_1 z_2}\iint dxdy\; g_{\text{in}}(x,y)e^{i\pi\frac{x^2+y^2}{\lambda z_1}}\,e^{i\pi\frac{x'^2+y'^2}{\lambda z_2}} \\
&\times\iint dx''dy'' e^{i\pi\frac{x''^2+y''^2}{\lambda}\left(\frac{1}{z_1}+\frac{1}{z_2}-\frac{1}{f}\right)} \\
&\times g_{\text{pm}}(x'',y'')e^{-i2\pi\left[\frac{x''}{\lambda}\left(\frac{x}{z_1}+\frac{x'}{z_2}\right)+\frac{y''}{\lambda}\left(\frac{y}{z_1}+\frac{y'}{z_2}\right)\right]}
\end{aligned}
$$

By choosing $\frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{f}$ (lens law) we eliminate one quadratic:

$$g_{\text{out}}(x', y') = \frac{e^{i2\pi\frac{z_1+z_2}{\lambda}}}{-\lambda^2 z_1 z_2} e^{i\pi\frac{x'^2+y'^2}{\lambda z_2}} \iint g_{\text{in}}(x,y) e^{i\pi\frac{x^2+y^2}{\lambda z_1}} \times \left[\iint g_{\text{pm}}(x'', y'') e^{-i2\pi[\frac{x''}{\lambda}(\frac{x}{z_1}+\frac{x'}{z_2}) + \frac{y''}{\lambda}(\frac{y}{z_1}+\frac{y'}{z_2})]} dx''dy'' \right] dxdy$$

To evaluate the inner integral, we perform the coordinate transform $x'' = \lambda z_1 u, y'' = \lambda z_1 v$, then:

$$\left[\iint \cdots dx''dy'' \right] = (\lambda z_1)^2 \iint g_{\text{pm}}(\lambda z_1 u, \lambda z_1 v) e^{-i2\pi[\frac{\lambda z_1 u}{\lambda}(\frac{x}{z_1}+\frac{x'}{z_2}) + \frac{\lambda z_1 v}{\lambda}(\frac{y}{z_1}+\frac{y'}{z_2})]} du\, dv$$

We rename the scaled $g_{\text{pm}}(\lambda z_1 u, \lambda z_1 v) \equiv H(u,v)$ i.e. the Amplitude Transfer Function (ATF) so we have

$$\left[\int \int \cdots dx''dy'' \right] = (\lambda z_1)^2 \iint H(u,v) e^{-i2\pi[(x+\frac{z_1}{z_2}x')u + (y+\frac{z_1}{z_2}y')v]} dudv \equiv h(x + \frac{z_1}{z_2}x', y + \frac{z_1}{z_2}y')$$

where $h(x_0', y_0') = \mathcal{F}\left\{H(u,v)\right\}|_{x_0', y_0'}$ is the PSF in the reduced coordinates (without inversion and magnification - see slide 7 in Lecture 20)

$$g_{\text{out}}(x', y') = -e^{i2\pi\frac{z_1+z_2}{\lambda}} \left(\frac{z_1}{z_2}\right) e^{i\pi\frac{x'^2+y'^2}{\lambda z_2}} \times \iint g_{\text{in}}(x,y) \underbrace{e^{i\pi\frac{x^2+y^2}{\lambda z_1}}}_{\text{blur term*}} h\left(x + \frac{z_1}{z_2}x', y + \frac{z_1}{z_2}y'\right) dx\, dy$$

*see Goodman p. 113, equation 5.32 and reference 303