18.02 Multivariable Calculus
Fall 2007

18.02 Practice Exam 3 B

Problem 1. a) Draw a picture of the region of integration of $\int_0^1 \int_x^{2x} dydx$.

b) Exchange the order of integration to express the integral in part (a) in terms of integration in the order $dxdy$. Warning: your answer will have two pieces.

Problem 2. a) Find the mass M of the upper half of the annulus $1 < x^2 + y^2 < 9$ $(y \geq 0)$ with density $\delta = \dfrac{y}{x^2 + y^2}$.

b) Express the x-coordinate of the center of mass, $\bar{x}$, as an iterated integral. (Write explicitly the integrand and limits of integration.) Without evaluating the integral, explain why $\bar{x} = 0$.

Problem 3. a) Show that $\mathbf{F} = (3x^2 - 6y^2)\hat{\mathbf{i}} + (-12xy + 4y)\hat{\mathbf{j}}$ is conservative.

b) Find a potential function for $\mathbf{F}$.

c) Let C be the curve $x = 1 + y^3(1 - y)^3$, $0 \leq y \leq 1$. Calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

Problem 4. a) Express the work done by the force field $\mathbf{F} = (5x + 3y)\hat{\mathbf{i}} + (1 + \cos y)\hat{\mathbf{j}}$ on a particle moving counterclockwise once around the unit circle centered at the origin in the form $\int_a^b f(t)dt$. (Do not evaluate the integral; don't even simplify $f(t)$.)

b) Evaluate the line integral using Green's theorem.

Problem 5. Consider the rectangle R with vertices $(0, 0)$, $(1, 0)$, $(1, 4)$ and $(0, 4)$. The boundary of R is the curve C, consisting of C_1, the segment from $(0, 0)$ to $(1, 0)$, C_2, the segment from $(1, 0)$ to $(1, 4)$, C_3 the segment from $(1, 4)$ to $(0, 4)$ and C_4 the segment from $(0, 4)$ to $(0, 0)$. Consider the vector field

$$\mathbf{F} = (xy + \sin x \cos y)\hat{\mathbf{i}} - (\cos x \sin y)\hat{\mathbf{j}}$$

a) Find the flux of $\mathbf{F}$ out of R through C. Show your reasoning.

b) Is the total flux out of R through C_1, C_2 and C_3, more than, less than or equal to the flux out of R through C? Show your reasoning.

Problem 6. Find the volume of the region enclosed by the plane $z = 4$ and the surface

$$z = (2x - y)^2 + (x + y - 1)^2.$$

(Suggestion: change of variables.)