MITOCW | S3:E9: Visualizing Calculus with Prof. Gigliola Staffilani

[MUSIC PLAYING]

SARAH

Today, on *Chalk Radio*, making calculus a little less abstract.

HANSEN:

GIGLIOLA

STAFFILANI:

We put together a lot of problems that come from concepts that we see in real life. If it's connected with something which is more real, you are more willing to work through that.

SARAH HANSEN: I'm your host, Sarah Hansen. Calculus has an intimidating reputation, but our guest today has been experimenting with creative new ways to teach this complex subject. Gigliola Staffilani is a Professor of Mathematics, and one of the lead instructors of introductory course, Calculus I. And she's found that with a little help, students often find calculus to be more intuitive than they first expect.

GIGLIOLA STAFFILANI: I feel like calculus actually gets introduced without students really knowing about it-- like when they start doing algebra, and they start talking about functions, and also by watching movies, documentaries, and so on. You often learn about function velocity and you learn that there is a direction, there is a magnitude, which is the speed. And it's kind of like going to a foreign country.

And if you just live there for a little bit, after a couple of weeks, you pick up a few words. But then, it's only when you actually take a class that you put all this together.

SARAH

But sometimes, students get stuck in translation when moving across disciplines.

HANSEN:

GIGLIOLA
STAFFILANI:

I think us mathematicians like to use way less common language like, for example, most common notation for a variable is x. Sometimes, we ask y, the function is always an f. We just are very familiar in using that, and it's very common among mathematicians.

But I realized, for example, that it's not a common notation for the same kind of question when you read books that come from chemistry. In math, you want to reduce everything to a mathematical concept and you don't want to be distracted with names and maybe cumbersome type of notation. You just don't want that to distract you. But in chemistry, the name of the variable is important because you probably need to remember what that means within a certain reaction or something like that.

It is very hard, I think, for a student to, when he or she starts taking a different class after calculus, to realize that actually, they know already what they are supposed to know. They just think that they don't because the notation is completely different.

SARAH HANSEN: To help students get grounded in the world of calculus, Gigliola and her colleagues have brought visual learning methods into the classroom.

GIGLIOLA
STAFFILANI:

When we put together the calculus class that we have on MITx, it was very important for us to give a tool to the students to actually draw pictures. In calculus, it's very important to understand the relationship between the formula-- so the analysis, what we call mathematical analysis, so there is a formula written with variables-- and what that formula represents.

So for example, we can write a line with an equation, but we also can draw a line. And we need to understand why the two objects are exactly the same, and if we change a parameter, how that line or other curve changes.

SARAH

Sometimes, those methods might look like a demonstration in class.

HANSEN:

GIGLIOLA STAFFILANI: One that I have used in my class at some point, which is really very hard to grasp, is Lagrange multipliers. This is a method that we use to find maximum minimum with constraints. Let's think about sphere, for example, when you have a surface that grows, so the maximum radius until the sphere touches the boundary of a square. When that touching happens, then the two orthogonal vectors are going to be parallel. That's really what the system tells you.

SARAH HANSEN: If, like me, you're not a mathematician, you might be a little lost in all these technical terms. But just wait until Gigliola describes how she turns these abstract ideas into a physical example.

GIGLIOLA STAFFILANI: So I usually bring a balloon that I start inflating in a box. And then, when it touches the box, it's clear that the side of the box is totally tangent to the side of my surface. And that's where the normal vectors are orthogonal to each other. So this is intuitively visible, and it tells you that, really, these vectors are parallel.

And then, setting up the system becomes more like writing down what it means to have these two vectors in a parallel way instead of just remembering the algorithm, having to take all these derivatives that equal to zero and things like that. So it is very important, whenever you have the opportunity to visualize an object, it is important to do that except that, as you become more and more sophisticated in mathematics, that becomes harder and harder. But whenever you can, anything you can use is good.

SARAH HANSEN: Gigliola and her colleagues have incorporated digital applets to bring these kinds of visual representations into learning experiences. Applets are also expanding the kinds of problems they're able to challenge students with.

GIGLIOLA STAFFILANI: So some problems are like a yes and no question or like a plug-in a number. And some other problems, in particular in calculus, have some kind of a complicated formula or a drawing or something which is not easy to grade or to decide whether it's correct or not. So the sketching tool was very important because there are problems in calculus in which you don't need to be extremely precise, but with a drawing, you can show us whether you understand the concept or not.

But then, of course, my drawing might be different than your drawing although they're both correct, they both reflects what we ask you to do. And this was a complicated type of problem and a very difficult problem to grade. So the sketching tool, I think, gave us a way of grading questions and the possibility to the student to solve questions in a way which was exactly what we want them to do instead of a complicated, cumbersome way for us to decide whether you understand or not.

SARAH HANSEN: As she experiments with new teaching tools for Calculus I, Gigliola has also grappled with inequities in the field of mathematics.

GIGLIOLA STAFFILANI:

We thought about also giving the chance to students to solve the problem kind of like a little video games or things like that. But it's just really complicated to do that. We also thought that will have been not a fair way of dealing with this because-- I think there are studies on that-- that the boys tend to do video games or work on video games much more than girls.

So if we set up something like that, then we were afraid that the girls will not be interested in it. And so we decided not to even go that way. So to make it fair and make it interesting for all, not just for a category of people.

SARAH HANSEN:

And as a mother, she's witnessed firsthand how biases are formed years before students even reach the college level.

GIGLIOLA STAFFILANI:

I have twins, a boy and a girl. And now, they are 16. And I witnessed from the time they were in middle school how different they were treated because one was a boy, one was a girl. And the reaction that they have to the way they are treated, right?

So my son, for example, is kind of-- I wouldn't say he shows off, but he likes to be thought as somebody that picks hard classes. And my daughter, who is just as good as he is and she likes her class, but she hides it completely. She doesn't say that she's taking hard classes because that's not considered to be attractive for a young woman or older woman. Why for boys it's OK?

So there is a lot of social pressure and also a lot of different reactions to the behavior from the teachers. So it's very acceptable if a girl is not really putting too much effort in science and math, and it's less acceptable for boys. So yeah, unfortunately, from that early on, women, young women and young men, are treated differently. And I think it really takes a lot of effort for the young women who want to do more science and more math because I think they just have to go against very high social pressure.

So then, by the time I see them, undergrad at MIT, we know it's about 49% women, 51% men. In the math department, I think it's around 30% women in math major. And then, when you go to grad school, there are maybe 18%, 20% women. And when you go to faculty, well, there are four of us.

SARAH HANSEN:

Is there one small thing that educators could do to try to make this situation better?

GIGLIOLA STAFFILANI:

Yeah, I think being aware that they themselves have implicit biases would be already a big step. And I do have implicit biases myself. I know that, but I'm aware of it. So I'm trying to go in the opposite direction I will actually be tempted to go without thinking about it.

So I think being aware of the issue, being aware that when you ask a question in your math class and you might be a woman teacher, you look at the boys more than the girls, which is something that happens very often. Being aware of that, that's already a big step. So next time you try to do 50-50.

If all the problems are about baseball players, and soccer, and-- well, now, the girls do soccer and they are even better than men. So that's great. But there were problems that were written 50 years ago and they were all for boys because those are the people that would take those classes. So being aware of this and what difference could make if you just phrase the problem in a different way, I think it's very important.

SARAH

At the end of the day, Gigliola hopes that students will feel invested in the questions they're answering in

HANSEN: Calculus I.

GIGLIOLA

STAFFILANI:

We put together a lot of problems that come from concepts that we see in real life. That really helps, I think, also

in accepting the difficulties that you face when you actually have to solve a problem. If it's connected with

something which is more real, you are more willing to work through that.

So for example, one problem that I like, which actually has to do with solving some linear equations, is a question

on how many bagels I'm supposed to buy versus how many croissants and how many-- if the classes want a

certain-- or likes more sweets or salty or things like that. So it's a simple problem, but then when you have to

translate into math and find a solution, you have to use relatively sophisticated tools to do that if there's more

than one variable.

SARAH

With an important outcome, too.

HANSEN:

GIGLIOLA

STAFFILANI:

Yeah. Right. I usually bring that stuff in this class afterwards-- croissants, and bagels, and donuts. And then, I try

to say, how would you attack this? How would you do this?

So maybe some questions from biology or questions from chemistry or something that has at least a first step

which one can understand using calculus to give you the idea that this is not like some abstract nonsense that we

forced you to take at MIT, but it's something that's very important to, like I said from the very beginning, it's a

language that you need to acquire in order to attack any basic question that might come up in science in general

which has not already been answered. Which is something that they would love to be in the position of doing in

your life, right? Answer a new question.

SARAH

HANSEN:

If you're interested in teaching with Gigliola's calculus materials, head over to MIT's Open Learning library site

where you'll find her single variable calculus MOOCs. Thank you so much for listening. Until next time, signing off

from Cambridge, Massachusetts, I'm your host, Sarah Hansen from MIT OpenCourseWare.

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