16.001 - Materials & Structures
Problem Set #10

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Question	Points
1	6
2	10
3	12
Total:	28

○ **Problems M-10.1** [6 points]

A turbine blade of length $L = 0.5m$ in operation rotates around $\mathbf{e_2}$ with an angular velocity $\omega = 5000$rpm, as shown in Figure 1. The blade is constrained from extensional motion by a ring housing (i.e. the displacement $u(L) = 0$ at the extremity A), but there is frictionless sliding between the rotating blade and the ring surface. The blade is made of a titanium alloy with a mass density $\rho = 8470 kg \cdot m^{-3}$, a Young's modulus E = 80GPa, a cross-sectional area of $A_0 = 10cm^2$ and a yield stress of $\sigma_y = 400MPa$.

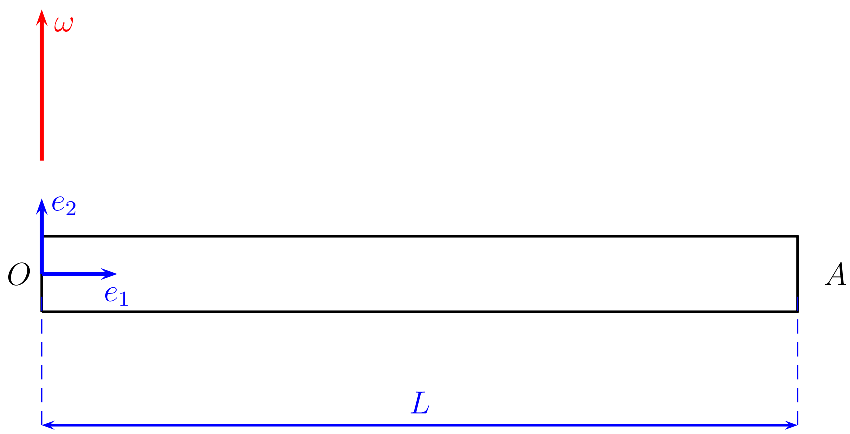


Figure 1: Rotating blade

1.1 (1 point) Write down the applicable governing equations and boundary conditions for this problem. What principles do they represent? Can this problem be solved by static considerations alone?

1.2 (3 points) Integrate the resulting equation(s) and apply the boundary conditions to obtain the following solution field distributions along the axis of the bar: displacement, strain, stress.

1.3 (1 point) What is the maximum stress and where does it happen? Will the material yield plastically for the data given?

1.4 (1 point) What is the maximum displacement and where does it happen?

○ **Problems M-10.2** [10 points]
(M14, M15)

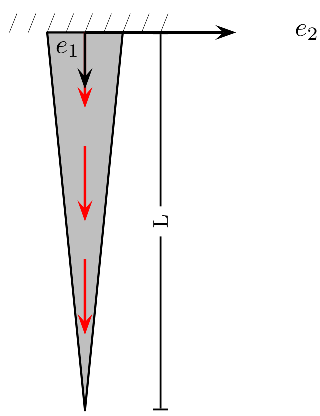
Consider the schematic below which depicts a nail being driven into a piece of wood. The nail has an elastic modulus E, length L, and a radius which varies linearly along its length according to the formula

$$R(x_1) = R_0 \left(1 - \frac{x_1}{L}\right) \tag{12}$$

Along its length, friction between the nail and the wood creates a distributed load per unit length $p_{dist}(x_1)$. The wood exerts a pressure normal to the surface of the nail proportional to the depth x_1 according to the following expression:

$$p(x_1) = \frac{p_0 x_1}{L}$$

The friction coefficient between the nail and wood is μ.



2.1 (2 points) Find an expression for $p_{dist}(x_1)$ in terms of the problems parameters.

2.2 (2 points) Integrate the equilibrium equation in closed form to obtain the load distribution $N(x_1)$. Determine the force N required to drive the nail into the ground farther.

2.3 (1 point) Determine the stress field along the nail just before it moves farther.

2.4 (2 points) Determine the displacement field in the nail as it starts to move.

2.5 (3 points) Find the maximum values of stress and displacement and their locations.

Problems M-10.3 [12 points]

The vertical rod shown in Figure 2 is made from an isotropic homogeneous linear elastic material (Young's modulus E, coefficient of thermal expansion α). It features a circular cross-section with varying radius $r(x_1)$ given by:

$$r(x_1) = R\sqrt{4 - 3\frac{x_1}{L}}$$

The rod is constrained at both its ends. It has the length L in the undeformed configuration at the reference temperature T_{ref}. It is subjected to a temperature change $\Delta T(x_1) = \Delta T_0 + kx_1$ which varies linearly with the spatial coordinate x_1.

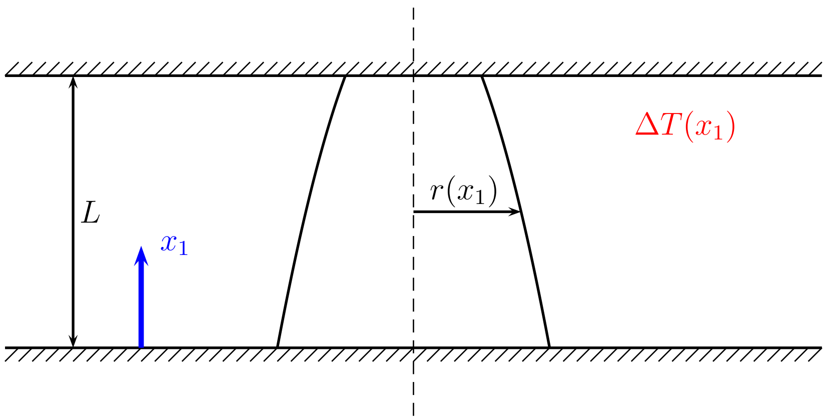


Figure 2: Constrained rod subjected to a temperature change $\Delta T(x_1)$.

3.1 (1 point) Is this system statically determinate?

3.2 (3 points) State the equation governing the axial displacement $\bar{u}_1(x_1)$ of the rod.

3.3 (1 point) State the boundary conditions at $x_1 = 0$ and $x_1 = L$.

3.4 (2 points) Determine the general solution to your governing equation. (You can use mathematical software like Mathematica or MATLAB to do this if you wish.)

3.5 (2 points) Find the displacement field $\bar{u}_1(x_1)$ in the rod by specializing the general solution you found in the previous part to the boundary conditions.

3.6 (3 points) Compute the axial force field $N_1(x_1)$, the stress field $\sigma_{11}(x_1)$, and the strain field $\varepsilon_{11}(x_1)$ in the rod.