16.001 Unified Engineering
Materials and Structures

Measurement of physical quantities, Units and Systems of Units

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References:

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We express *physical quantities* in terms of numbers. These numbers are obtained from *measurements*. It is clear right from the start that we need precise definitions for these terms:

Definitions

- A **quantity in the general sense** is a property ascribed to phenomena, bodies, or substances that can be quantified for, or assigned to, a particular phenomenon, body, or substance. Examples are mass and electric charge.
- A **quantity in the particular sense** is a quantifiable or assignable property ascribed to a particular phenomenon, body, or substance. Examples are the mass of the moon and the electric charge of the proton.
- A **physical quantity** is a *quantity* that can be used in the mathematical equations of science and technology.
- A **unit** is a particular *physical quantity*, defined and adopted by convention, with which other particular *quantities* of the same kind are compared to express their *value*.
- A **measurement** is a direct or indirect comparison of a certain *quantity* with an appropriate standard or **unit of measurement**.
- The **value** of a *physical quantity* is the quantitative expression of a particular *physical quantity* as the product of a number and a *unit*, the number being its **numerical value**. Thus, the *numerical value* of a particular *physical quantity* depends on the *unit* in which it is expressed.

Example:

The value of the height h_W of the Washington Monument is $h_W = 169\,\mathrm{m} = 555\,\mathrm{ft}$. Here h_W is the *physical quantity*, its *value* expressed in the *unit* "meter", unit symbol m, is $169\,\mathrm{m}$, and its *numerical value* when expressed in meters is 169. However, the value of h_W expressed in the *unit* "foot", symbol ft, is $555\,\mathrm{ft}$, and its *numerical value* when expressed in feet is 555.

$$\underbrace{h_W}_{\text{value of the physical quantity}} = \underbrace{\underbrace{169}_{\text{numerical value}}\ \underbrace{\mathrm{m}}_{\textit{unit}}}_{\text{value}} = \underbrace{\underbrace{555}_{\text{numerical value}}\ \underbrace{\mathrm{ft}}_{\textit{unit}}}_{\text{value}}$$

Fundamental and derived units

We said we need units to measure physical quantities. This process usually goes as follows:

1. Single out (Identify) the class of phenomena of interest (mechanics, electromagnetics, etc)
2. List *physical quantities* involved.
3. Adopt standard reference values for **fundamental** quantities. Realize that mathematical expressions relate these *quantities*, therefore they cannot all have independent reference standards.
4. Once *fundamental units* have been decided upon, **derived units** are obtained from the fundamental units using the mathematical definitions relating the quantities involved.

Example

- *Phenomenon*: Geometry of objects
- *Physical quantities*: Length, Area, Volume
- *Fundamental units*: meter (symbol m).
- *Derived units*:
  - Area: meter squared (symbol m^2)
  - Volume: meter cubed (symbol m^3)

Fundamental and derived units

Here are some other examples to think about:

Example

1. *Phenomenon*: kinematics of bodies
2. *Physical quantities*: Distance, Time, Velocity, Acceleration
3. *Fundamental units*:
   - Distance: meter (symbol m).
   - Time: second (symbol s or sometimes sec).
4. *Derived units*:
   - Velocity: meter per second (symbol $\mathrm{m.s}^{-1}$ or $\mathrm{m.sec}^{-1}$)
   - Acceleration: meter per second squared (symbol $\mathrm{m.s}^{-2}$ or $\mathrm{m.sec}^{-2}$).

Example

1. *Phenomenon*: Dynamics of bodies
2. *Physical quantities*: all kinematics + Mass, Force, Work, Momentum, ...
3. *Fundamental units*:
   - Distance: meter (symbol m).
   - Time: second (symbol s or sometimes sec).
   - Mass: kilogram (symbol kg).
4. *Derived units*:
   - Force: Newton (symbol $\mathrm{N} = \mathrm{kg.m.s}^{-2}$)
   - Work: Joule (symbol $\mathrm{J} = \mathrm{N.m}$)
   - ...

We observe that it is the class of phenomena under consideration, i.e., the complete set of physical quantities in which we are interested that ultimately determines if a given set of *fundamental units* is sufficient for its measurement. For example, it is not possible to define a unit of density from the fundamental units of length and time. It is necessary to add the unit of mass.

System of units:

Definition

A set of fundamental units that is **sufficient** for measuring the properties of the class of phenomena under consideration is called a **system of units**.

The International System of Units

The International System of Units, universally abbreviated SI (from the French Le Système International d'Unités), is the modern metric system of measurement. Long the dominant system used in science, the SI has rapidly become the dominant measurement system used in international commerce. The definitive international reference on the SI is a booklet published by the International Bureau of Weights and Measures (BIPM, Bureau International des Poids et Mesures).

In recognition of this fact and the increasing global nature of the marketplace, the **Omnibus Trade and Competitiveness Act of 1988**, which changed the name of the National Bureau of Standards (NBS) to the National Institute of Standards and Technology (NIST) and gave to NIST the added task of **helping U.S. industry increase its competitiveness**, **designate the metric system of measurement as the preferred system of weights and measures for United States trade and commerce**.

The official U.S. website with information on the SI is the Physics Laboratory of NIST.

You may download the authoritative publications for future reference directly from these links:

- NIST Special Publication 811. Guide for the Use of the International System of Units (SI)
- NIST Special Publication 330. The International System of Units (SI), 2019 Edition

SI base units

The SI is founded on seven SI base units for seven base quantities assumed to be mutually independent:

SI base Units

	SI base unit	
Base quantity	**Name**	**Symbol**
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Definitions of the SI base units		
Unit of length	meter	The meter is the length of the path travelled by light in vacuum during a time interval of $1/299,792,458$ of a second.
Unit of mass	kilogram	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. In 2018, this was redefined based on the Planck constant
Unit of time	second	The second is the duration of $9,192,631,770$ periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.
Unit of electric current	ampere	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
Unit of thermodynamic temperature	kelvin	The kelvin, unit of thermodynamic temperature, is the fraction $1/273.16$ of the thermodynamic temperature of the triple point of water.
Unit of amount of substance	mole	1. The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is "mol". 2. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
Unit of luminous intensity	candela	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×1012 hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian.

Other quantities, called **derived quantities**, are defined in terms of the seven base quantities via a system of **quantity equations**. The **SI derived units** for these derived quantities are **obtained from these equations and the seven SI base units**.

Examples of SI derived units

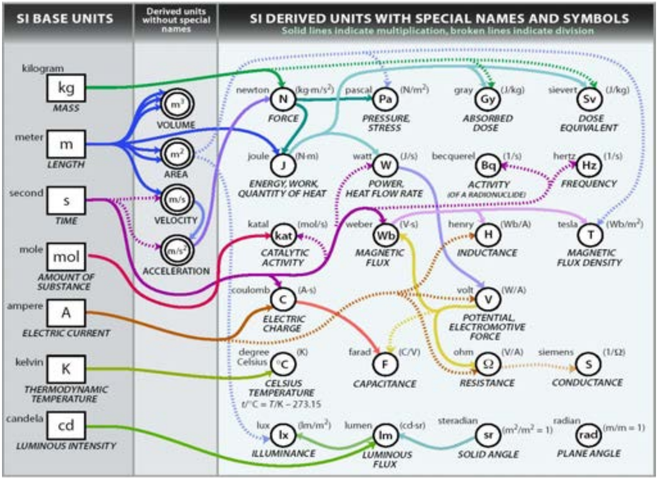
	SI derived unit	
Derived quantity	**Name**	**Symbol**
area	square meter	m^2
volume	cubic meter	m^3
speed, velocity	meter per second	$m.s^{-1}$
acceleration	meter per second squared	$m.s^{-2}$
mass density	kilogram per cubic meter	$kg.m^{-3}$
specific volume	cubic meter per kilogram	$m^3.kg^{-1}$
current density	ampere per square meter	$A.m^{-2}$

For ease of understanding and convenience, 22 SI derived units have been given special names and symbols:

22 SI derived units with special names and symbols				
	SI derived unit			
Derived quantity	**Name**	**Symbol**	**Expression in terms of other SI units**	**Expression in terms of SI base units**
plane angle	radian	rad	-	$m \cdot m^{-1} = 1$
solid angle	steradian	sr	-	$m^2 \cdot m^{-2} = 1$
frequency	hertz	Hz	-	s^{-1}
force	newton	N	-	$m \cdot kg \cdot s^{-2}$
pressure, stress	pascal	Pa	$N.m^{-2}$	$m^{-1} \cdot kg \cdot s^{-2}$
energy, work, quantity of heat	joule	J	$N \cdot m$	$m^2 \cdot kg \cdot s^{-2}$
power, radiant flux	watt	W	$J.s^{-1}$	$m^2 \cdot kg \cdot s^{-3}$
electric charge, quantity of electricity	coulomb	C	-	$s \cdot A$
electric potential difference, electromotive force	volt	V	$W.A^{-1}$	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
capacitance	farad	F	$C.V^{-1}$	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
electric resistance	ohm	Ω	$V.A^{-1}$	$m^2 \cdot kg \cdot s^{-3} \cdot A$
electric conductance	siemens	S	$A.V^{-1}$	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$

22 SI derived units with special names and symbols				
	SI derived unit			
Derived quantity	Name	Symbol	Expression in terms of other SI units	Expression in terms of SI base units
magnetic flux	weber	Wb	$V \cdot s$	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
magnetic flux density	tesla	T	$Wb.m^{-2}$	$kg \cdot s^{-2} \cdot A^{-1}$
inductance	henry	H	$Wb.A^{-1}$	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
Celsius temperature	degree Celsius	^oC	-	K
luminous flux	lumen	lm	$cd \cdot sr$	$m^2 \cdot m^{-2} \cdot cd = cd$
illuminance	lux	lx	$lm.m^{-2}$	$m^2 \cdot m^{-4} \cdot cd = m^{-2} \cdot cd$
activity (of a radionuclide)	becquerel	Bq	-	s^{-1}
absorbed dose, specific energy (imparted), kerma	gray	Gy	$J.kg^{-1}$	$m^2 \cdot s^{-2}$
dose equivalent	sievert	Sv	$J.kg^{-1}$	$m^2 \cdot s^{-2}$
catalytic activity	katal	kat	-	$s^{-1} \cdot mol$

SI base and derived units with special names and symbols



SI base and derived units

The diagram above shows graphically how the 22 SI derived units with special names and symbols are related to the seven SI base units.
In the first column the symbols of the SI base units are shown in rectangles, with the name of the unit shown toward the upper left of the rectangle and the name of the associated base quantity shown in italic type below the rectangle.
In the third column the symbols of the derived units with special names are shown in solid circles, with the name of the unit shown toward the upper left of the circle, the name of the associated derived quantity shown in italic type below the circle, and an expression for the derived unit in terms of other units shown toward the upper right in parenthesis.
In the second column are shown those derived units without special names [the cubic meter (m^3) excepted] that are used in the derivation of the derived units with special names. In the diagram the derivation of each derived unit is indicated by arrows that bring in units in the numerator (solid lines) and units in the denominator (broken lines), as appropriate.

SI prefixes

The 20 **SI prefixes** used to form decimal multiples and submultiples of SI units are

SI Prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deka	da	10^{-24}	yocto	y

Units outside the SI I

Certain units are not part of the International System of Units, that is, they are outside the SI, but are important and widely used. Examples are:

- the minute (time), symbol: min, value in SI units: $1\,\mathrm{min} = 60\,\mathrm{s}$
- the hour (time), symbol: hr, value in SI units: $1\,\mathrm{hr} = 3600\,\mathrm{s}$
- units of angle: degree, minute, second
- the liter
- the electronvolt
- the hectare
- the Angstrom
- etc

Dimensions of quantities I

By convention physical quantities are organized in a system of dimensions. Each of the seven base quantities used in the SI is regarded as having its own dimension, which is symbolically represented by a single sans serif roman capital letter. The symbols used for the base quantities, and the symbols used to denote their dimension, are given as follows

Base quantities and dimensions used in the SI

Base quantity	Symbol for dimension
length	L
mass	M
time	T
electric current	I
thermodynamic temperature	Θ
amount of substance	N
luminous intensity	J

All other quantities are derived quantities, which may be written in terms of the base quantities by the equations of physics. The dimensions of the derived quantities are written as products of powers of the dimensions of the base quantities using the equations that relate the derived quantities to the base quantities. In general the dimension of any quantity Q is written in the form of a dimensional product:

$$\mathrm{dim}(Q) = [Q] = \mathsf{L}^{\alpha}\mathsf{M}^{\beta}\mathsf{T}^{\gamma}\mathsf{I}^{\delta}\Theta^{\epsilon}\mathsf{N}^{\xi}\mathsf{J}^{\eta}$$

In this sense, the dimension of the quantity Q, can be thought of as the relation that describes the *derived unit* for this quantity in terms of the *fundamental units*. It is customary (following a suggestion of Maxwell) to denote the **dimension** of a quantity Q by $[Q]$.
For example, the dimension of density ρ is:

$$[\rho] = \mathrm{ML}^{-3}$$

Classes of systems of units

Definition

Two systems of units are said to be in the same **class of systems of units** if both systems use standard quantities of the same physical nature as *fundamental units*.

Example

For mechanics phenomena, the SI uses **length, mass and time** as the standard quantities defining the fundamental units. **We denote this class of system of units as the** LMT **class**. The fundamental SI units for the LMT class are the meter, the Kilogram and the second (and sometimes referred to as the MKS system). Any system of units that uses the same standard quantities will be in the same class. The CGS system is in the same class as the SI but the fundamental units are the centimeter, the gram and the second.

Definition

Dimensional and dimensionless quantities Quantities whose numerical values are identical in all systems of units within a given class are called **dimensionless**. All other quantities are called **dimensional**.

Dimensional consistency

Dimensional consistency of equations **The *dimensions* of each and every term in both sides of any equation with physical sense must be identical.** Otherwise, an equality in one system would be broken upon conversion to another system. This fact is used to obtain *derived units* from *fundamental units*.

Example

In the LMT class, the dimension of mass is M, the dimension of acceleration is LT^{-2}, the dimension of force can be obtained (derived) from Newton's second law:

$$F = m\,a$$

$$[F] = [m]\;[a] = \mathrm{MLT}^{-2}$$

In other words, in the LMT class, the *dimension* of force is LMT^{-2}.

An example of a relation that does not satisfy this requirement: The time t (in minutes) to drive to a point at a distance d (in miles) is equal to the number of miles plus the number of traffic lights n (dimensionless). $t = d + n$
Although this may be a good approximation with this specific choice of units, it would clearly fail with any other choice of units. That is because the relation is not dimensionally consistent. $[t] = \mathrm{T} \neq \mathrm{L} + 1 = [\mathrm{d}] + [\mathrm{n}]$

Dimensional Analysis

We can determine the unknown exponent p in the following equation by requiring the same units on both sides:

$$\begin{aligned} E &= m\,c^p \\ \mathrm{ML^2T^{-2}} &= \mathrm{M\left(LT^{-1}\right)^p} \\ \rightarrow \quad p &= 2 \end{aligned}$$

This is one technique of Dimensional Analysis, which can allow us to identify the controlling physical quantities in unfamiliar or complicated quantities.
Fluid Dynamics, Solid Mechanics and Thermodynamics make extensive use of Dimensional Analysis.
We will have an entire class dedicated to Dimensional Analysis in one of the Unified Lectures in the Spring.

Some conversion factors between SI and English systems

dimension	SI unit	English Unit	conversion
Length	m	ft	0.305 m/ft
Time	s	sec	—
Mass	kg	slug	14.6 Kg/slug
Force	N	lb	4.45 N/lb
Pressure	Pa	psi	6900 Pa/psi

In the SI system, mass (kg) is a fundamental unit, and force (N) is the derived unit, while in the English system, force (lb) is a fundamental unit, and mass (slug) is derived. This distinction is not too important in practice.

Final recommendations regarding Units and Dimensions

- Be VERY careful to give units with any numerical quantity.
- Be VERY careful not to mix units — "do not compare apples & oranges"
- Use dimensions as a check on equation correctness.