

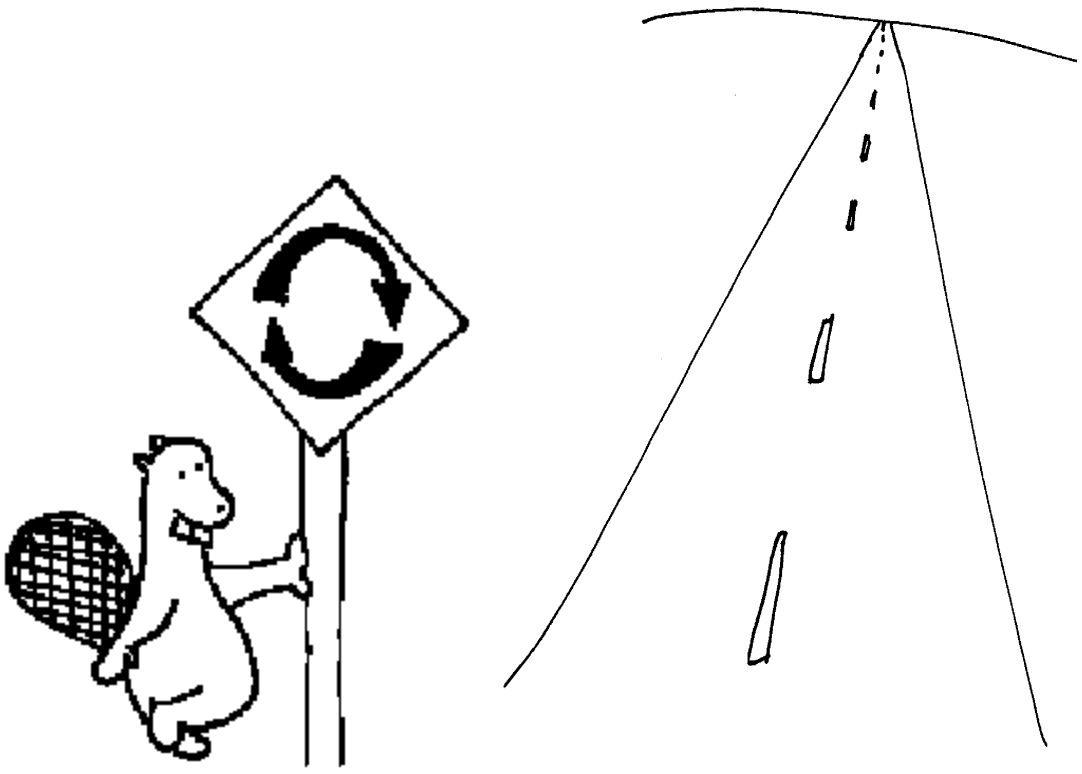
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A Guide to Learning System Dynamics

Prof. Jay Forrester

Road Maps 0

A Guide to Learning System Dynamics



System Dynamics in Education Project

Road Maps 0

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

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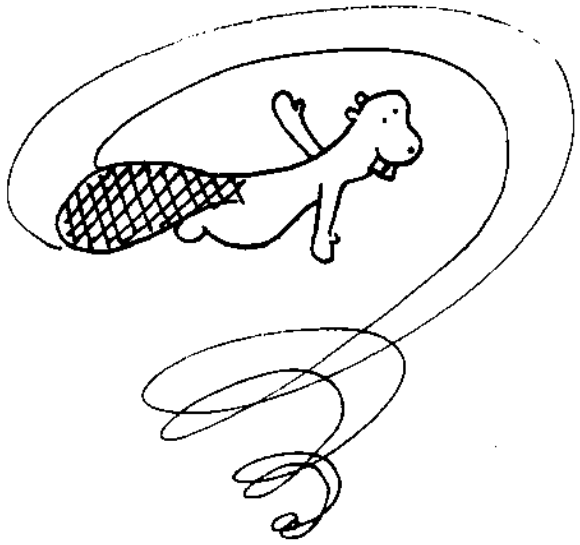
Welcome to Road Maps!

Many books and thousands of papers cover the field of system dynamics. With all of these resources available, it is difficult to know where to begin learning about system dynamics. The System Dynamics in Education Project at MIT is putting together Road Maps to help sort through this vast library of books and papers. Road Maps is a series of self-study guides that use modeling exercises and selected literature to provide a resource for learning about the principles and practices of system dynamics. In its present form, Road Maps is not meant to be a teaching curriculum for classroom use.

The Spiral Learning Approach

A new concept is usually not fully mastered after reading about it only once. Also, learning usually involves building one's understanding up from one level to a higher level. For this reason, Road Maps utilizes a spiral learning approach, in which each new concept is repeatedly reinforced in successive chapters. This repetition moves upward along a spiral as more advanced concepts build upon basic ones.

In constructing this spiral, Road Maps has been divided into chapters. The end of each chapter marks the completion of a particular subset of concepts, with the next chapter building on the material just learned. Each chapter comes as a separate document so the reader can request them as needed.



Organization



The Road Maps series begins with **Road Maps One**, which introduces the concepts of causal loops and circular feedback in systems. In addition, some of the applications and history of system dynamics are presented.

Road Maps Two takes you through your first system dynamics models, introducing computer simulation and the concept and methods of graphical integration. Two basic elements of system dynamics models, the stock and flow, are presented. You will need a computer and STELLA™ or Vensim™ software for this and subsequent chapters of Road Maps.

Road Maps Three presents two of the most common structures in system dynamics: the first-order positive and negative feedback loops. Systems exhibiting exponential growth and decay are examined, and the idea of loop dominance in systems is introduced

Road Maps Four introduces generic structures and discusses the use of computer simulation games in teaching system dynamics. After playing the *Fish Banks* game, you will construct a model of the scenario and learn about the tragedy of the commons. Using your computer model, you will simulate and analyze several policies for this system.

Road Maps Five probes the structure of system dynamics models, emphasizing some important features that you may not have noticed, and explains the spread of an epidemic. Road Maps Five also introduces testing for model validity in the system dynamics context. How do you know that your model is a good representation of the real system?

In **Road Maps Six**, the dynamics of economic supply and demand are explored and more modeling exercises are provided. Also, oscillatory systems are used to show the transferability of structures. More modeling exercises sharpen your modeling skills and develop intuition about systems.

Road Maps Seven presents some unexpected behaviors that can occur in higher-order positive feedback loops. It also points out mistakes commonly found in system dynamics models and provides more independent modeling exercises.

Road Maps Eight continues to improve your modeling skills by analyzing the first stage of the process of building a model: conceptualization, and by warning you against other mistakes commonly made in models. Road Maps Eight

also increases your understanding of oscillating systems and introduces sensitivity analysis.

Road Maps Nine explores the dynamics of credit card spending, illustrates how to correctly formulate table functions, and introduces the behavior known as overshoot and collapse. More graphical integration exercises develop your intuitive understanding of the process of graphical integration.

Road Maps Ten provides further instruction on how to build a model from scratch with exercises for the second step, formulation. There are a number of other papers in Road Maps Ten which will continue to build skill in working with system dynamics models and continue the reader's understanding of system dynamics principles.

Road Maps discusses the principles and characteristics of systems, explores the interrelationship between structure and behavior of dynamic models, provides guidelines for good modeling practice, and discusses the applications of system dynamics. From Road Maps One on, the spiral of learning returns to these primary areas several times by introducing more advanced material while building on fundamental concepts. After completing Road Maps, you should have a strong working knowledge of system dynamics, have developed intuition about the fundamental principles of systems and be ready to explore the application of system dynamics in any area of interest.

How to Use Road Maps

First, you need to determine where you should begin in Road Maps. Some readers have had more background in system dynamics than others. This introduction should help you find a good starting point in Road Maps.

Road Maps explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most

of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Things You'll Need for Road Maps

Modeling Software

In order to complete Road Maps Two and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is

located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

Books

1) Goodman, Michael R, 1974. *Study Notes in System Dynamics*. Portland, Oregon: Pegasus Communications, 388 pp. (Road Maps 2 onwards)

2) Forrester, Jay W., 1969. *Urban Dynamics*. Portland, Oregon: Pegasus Communications, 285 pp. (Road Maps 3 onwards)

If you have any problems in getting the above three books, contact Pegasus Communications (see Appendix).

3) Meadows, Donella H., Dennis Meadows, Jorgen Randers, 1992.
Beyond the Limits: Confronting Global Collapse, Envisioning a Sustainable Future.
Post Mills, VT: Chelsea Green Publishing Co., 300 pp. (Road Maps 5)
To order a copy of this book, contact Chelsea Green Publishing Co. (see Appendix).

Good luck with Road Maps!

System Dynamics in Education Project

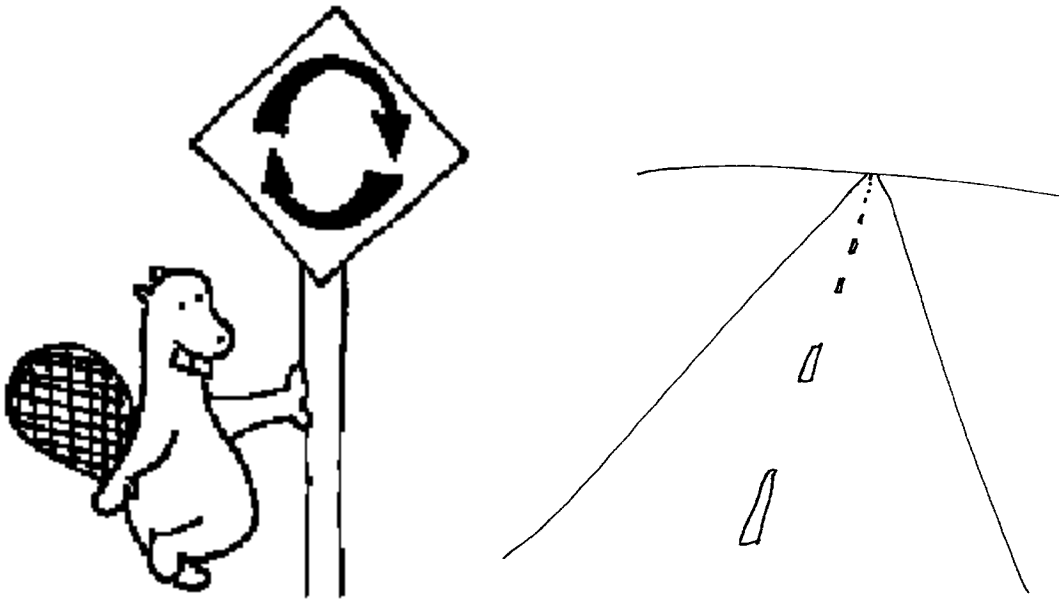
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Road Maps 1

A Guide to Learning System Dynamics



System Dynamics in Education Project

Road Maps 1

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps One!

Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps One is the first in the series of chapters in Road Maps and serves as a general introduction to the field of system dynamics and its applications. We include readings that explain what a 'system' is and show how the concept of 'feedback' is critical to understanding how the components of a system are interconnected. We explore the use of system dynamics in K-12 education, and provide an example from the classroom. Finally, we illustrate the use of system dynamics in understanding the behavior of social systems.

Topics Covered in Road Maps One

An Introduction to Systems

- *System Dynamics and K-12 Teachers* (D-4665-4)
  by Jay W. Forrester

System Dynamics in Education

- *System Dynamics and Learner-Centered Learning in Kindergarten through 12th Grade Education* (D-4337)
  by Jay W. Forrester
- *Simulating* Hamlet *in the Classroom* (D-4540-1)
  by Pamela L. Hopkins

System Dynamics and the Media

- *System Dynamics Meets the Press,* (D-4143-1)
  an excerpt from *The Global Citizen*
  by Donella H. Meadows

Policy Analysis in Social Systems

- *Counterintuitive Behavior of Social Systems* (D-4468-1)
  by Jay W. Forrester

Things You'll Need for Road Maps One

Books

All five introductory papers to read to complete Road One Maps are included in the chapter materials to download from the web site. There are no books to purchase.

How to Use Road Maps One

Road Maps One explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

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As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

An Introduction to Systems

The idea of circular feedback in systems is one of the most basic concepts in system dynamics. Systems are built from interlocking feedback loops. The first reading in Road Maps One provides a good introduction to these concepts written by the founder of the field of systemn dynamics..

- System Dynamics and K-12 Teachers

by Jay W. Forrester

System Dynamics and K-12 Teachers introduces systems and feedback loops, two important ideas which will be further developed in more readings throughout Road Maps. A strong grasp of systems and feedback loops will build a good foundation for learning more advanced principles in system dynamics.

Secton 4 discusses feedback loops while Section 7 gives some important examples of different kinds of complex systems. Section 13 explains the importance of generic structures to the study and understanding of system dynamics.

Throughout the paper, Professor Forrester quotes grade school, middle school and high school teachers with how they use system dynamics to teach children.

Please read *System Dynamics and K-12 Teachers* now.

After reading *System Dynamics and K-12 Teacher:*

Many of the diagrams used in early systems study are known as **causal loops diagrams** or **influence diagrams**. We will see causal loops fairly often during the first few chapters of Road Maps. However, causal loop diagrams are not the only way to represent systems, and in later Road Maps, we will introduce the more powerful method of representing systems through the use of stocks and flows shown on page 7.

Now it's time to introduce our first System Principle. Basic concepts in system dynamics are organized as System Principles in Road Maps. We will be revisiting this system principle later in Road Maps.



System Principle #1:
The feedback loop is the basic structural element of systems.
A simple system is composed of a positive and a negative feedback loop. Notice the circular structure of the feedback loops. Feedback loops are the building blocks of systems that are linked together to build more complex systems.

Let's move on to some of the applications of system dynamics.

System Dynamics in Education

System dynamics is now extensively used in education. System dynamics and learner-centered learning motivate children to become actively involved in their own education. The next two readings illustrate the use of system dynamics in the classroom.

- ***System Dynamics and Learner-Centered Learning in Kindergarten through 12th-Grade Education***[1]

by Jay W. Forrester

In this paper, pre-college education has been described as "poorly serving the needs of society." Most students are resistant to the methods of teaching currently used by their schools. Forrester, the founder of system dynamics, presents his views on how system dynamics may improve classroom learning. The paper illustrates how the content of education could be changed through the use of system dynamics.

Please read *System Dynamics and Learner-Centered Learning* now.

After reading *System Dynamics and Learner-Centered Learning...*

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[1] Forrester, Jay W., 1993. *System Dynamics and Learner-Centered-Learning in Kindergarten through 12th Grade Education* (D-4337), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, December 21, 20 pp.

The paper described the benefits as well as the feasibility of implementing a system dynamics framework in pre-college education. How does system dynamics encourage learning? System dynamics provides a framework for the instruction and integration of all subjects. Schools should encourage the students in seeing the world as interconnected, not compartmentalized! What obstacles must be overcome before system dynamics can become an accepted learning tool in our schools? These questions are addressed by the paper.

A Note on the System Dynamics in Education Project

The System Dynamics in Education Project (SDEP) team is a group of students and staff at the Massachusetts Institute of Technology, working under the guidance of Professor Forrester. Road Maps is the current project of SDEP to help beginners as well as more advanced learners in learning about systems and system dynamics. If you would like to know more about SDEP or would like to send us your comments, contact information is provided in the Appendix. We look forward to hearing from you!

The previous paper referred to some of the actual progress being made with system dynamics in the classroom. The next section talks about one such example.

- ***Simulating* Hamlet *in the Classroom*[2]**

by Pamela Hopkins

This paper describes how the concepts of system dynamics were successfully used in a high school literature class. The approach which Hopkins used focuses on learner-centered learning, as described in the preceding paper. This article includes the computer model of *Hamlet* which was used in the classroom; however, understanding the structure of the model is not essential.

Please read *Simulating* Hamlet *in the Classroom* now.

After reading *Simulating* Hamlet *in the Classroom...*

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2 Hopkins, Pamela L., 1992. "Simulating *Hamlet* in the Classroom," *System Dynamics Review*, Vol. 8, No. 1, Winter 1992, System Dynamics Society, pp. 91-98.

In this case, system dynamics and learner-centered learning have helped to transform the classroom environment. System dynamics is being increasingly used in K-12 education. Organizations such as the System Dynamics in Education Project and the Creative Learning Exchange were founded for the purpose of integrating system dynamics into education.

How does this actual classroom experience compare with the predictions and proposals presented in *System Dynamics and Learner-Centered Learning in Kindergarten Through 12th Grade Education*? How might these techniques affect the quality of education? These experiments clearly show the impact that system dynamics and systems thinking could have on education.

System Dynamics and the Media

This next section discusses how the public and the media have responded to system dynamics.

- ***System Dynamics Meets the Press,***[3]
  an excerpt from ***The Global Citizen***
  by Donella H. Meadows

Donella Meadows writes a syndicated newspaper column entitled *The Global Citizen*. She has tried to challenge popular but incorrect ideas about various topics. From discussing global warming to examining the situation that led to the space shuttle accident, the author addresses some of our problematic assumptions about the systems we live in. Many of these problems arise from a lack of understanding of basic system dynamics concepts. Her efforts have produced some interesting results. In this reading, some of her experiences in dealing with the public, the mass media, and system dynamics are described.

Please read *System Dynamics Meets the Press* now.

After reading *System Dynamics Meets the Press...*

The article makes a strong issue of challenging people's internal models, or paradigms. Through the newspaper column, Meadows has tried to challenge some of the common misunderstandings people have about economics, the government,

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[3] Meadows, Donella, 1991. "System Dynamics Meets the Press," *The Global Citizen*, pp. 1-12, Washington, DC, Island Press.

and other social and natural systems. Why is it so important to challenge these paradigms?

Thinking back to Forrester's paper, do you see how a K-12 education in systems could help accomplish this very same goal? How does a system dynamics background help one to understand these areas? How could understanding system dynamics help the public make more informed choices in their own lives?

Social Systems, Policies, and Misunderstandings

The next section addresses the problems that arise in social systems, and explains why people are often mislead by the behavior of complex systems.

- ***Counterintuitive Behavior of Social Systems***[4]

by Jay Forrester

This paper explains that well-intentioned policies intended to help cure social problems often fail to do so. On the contrary, they sometimes worsen the situation they were intended to improve. How can such contrary results happen? Such policies are derived from unclear mental models. These mental models are insufficient for understanding complex social systems, and decisions based on them are likely to do more harm than good. Computer simulation models is then offered as a method of understanding and approaching these systems.

The paper takes a look at two system dynamics models and discusses the unexpected behavior that can arise in multiple-loop feedback systems. It is not necessary to understand the intricacies of the models nor the hypothetical situations explored in the middle of the paper. However, it is important to observe that these computer models can shed light on important social problems, and may have significant impact on how people view the systems around them.

Please read *Counterintuitive Behavior of Social Systems* now.

After reading *Counterintuitive Behavior of Social Systems*...

System dynamics has much to offer in social sciences such as economics, urban planning, and politics. Policy makers have historically drawn from their own

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4 Forrester, Jay W., 1971. "Counterintuitive Behavior of Social Systems," *Technology Review*, Vol. 73, No. 3, pp. 53-68. Also appears as Chapter 14, pages 211-244, in the author's *Collected Papers* 1975; and as Chapter 1, pp. 3-30, in *Toward Global Equilibrium: Collected Papers*, 1973. Dennis L. Meadows, ed., both from Portland, Oregon: Productivity Press.

intuitions for their policies, but their intuitions often point to the wrong solutions. These mistakes are not necessarily caused by incompetence or malice, but because people cannot accurately simulate a complex system in their heads. With the world becoming increasingly complex, the need for new tools is clear. System dynamics can become such an effective tool to deal with the complex systems.

Finishing off Road Maps One

Road Maps One was an introduction to systems, to the discipline of system dynamics, and to the concept of feedback loops. What is a positive loop? A negative loop? Why are they so important in system dynamics? Road Maps One also presented our first system principle, that the feedback loop is the basic structural element of systems.

We also covered readings that discussed some of the applications of system dynamics. Its uses in education and the social sciences are becoming widespread. How does it help students learn? How does system dynamics achieve learner-centered learning?

To learn more about system dynamics, please continue on to Road Maps Two. Road Maps Two will get you started on creating your first actual model, and start discussing the basics of computer simulation and system models.

Key Terms and Concepts:

Causal Loop Diagrams
Circular Feedback
Computer Simulation
Influence Diagrams
Learner-Centered Learning
Mental Model
Negative Loop
Paradigm
Policy
Positive Loop
System
System Dynamics



System Dynamics and K-12 Teachers

a lecture at the
University of Virginia School of Education
by
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Massachusetts Institute of Technology
Cambridge, MA, USA

May 30, 1996

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System Dynamics and K-12 Teachers

by
Jay W. Forrester

1. SYSTEM DYNAMICS

System dynamics, which has been under development at MIT since 1956, and now at many other places, deals with how things change through time. That covers a great deal of what most people find important.

System dynamics first developed as a management discipline to understand how the policies of corporations produce successes and failures. What is it in decision-making policies that produce growth or decay; what policies make huge fluctuations of employment, as seen in some corporations that work seven days a week and overtime one year, and two or three years later have half the people laid off?

Later we realized that system dynamics was not limited to the field of management, and there were excursions into larger social systems. Work I did with former mayor John Collins of Boston became my *Urban Dynamics*[1] book dealing with how cities grow and then stagnate, and how national policies for dealing with cities often lie between neutral and highly detrimental, both for the city as an institution and also for the low income, unemployed residents.

Some of you may have encountered the branching of system dynamics into work with an organization called the Club of Rome that resulted in my *World Dynamics*[2] book and *The Limits to Growth*[3] by Meadows. These two books produced vastly more public reaction than any other computer modeling in the social sciences. More recently, I have applied system dynamics to better understanding of the dynamics of national economies.

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[1] Forrester, Jay W. (1969). *Urban Dynamics*. Waltham, MA: Pegasus Communications. 285 pp.

[2] Forrester, Jay W., 1973. *World Dynamics*, (2 ed.). Waltham, MA: Pegasus Communications. 144 pp.

[3] Meadows, Donella H., Dennis L. Meadows, Jørgen Randers, and William W. Behrens, III. (1972). *The Limits to Growth.* New York: Universe Books. 205 pp.

2. SYSTEM DYNAMICS IN EDUCATION

This morning we discuss the most recent frontier in system dynamics--its use as a foundation underlying education in kindergarten through 12th grade.

The dynamic, computer-modeling approach provides a foundation that is transferable from field to field. There are several dozen K-12 schools now doing excellent work, and several hundred doing something. Pioneering schools are scattered over the United States, extend into the Scandinavian countries and Germany, and other places from which we have little information.

System dynamics uses computer simulation models to reveal how known structures and policies often produce unexpected and troublesome behavior. The computer models are constructed from descriptive information that usually is already known. Such information relates to who is striving to do what, the information that each person has available, time delays in taking action, and what individuals will do under a variety of pressures. The same approach carries over to nonhuman systems in nature and physical change.

In K-12 education, system dynamics modeling has been applied to mathematics, physics, social studies, history, economics, biology, and literature.

3. LEARNER-CENTERED LEARNING

In the more successful schools, system dynamics is combined with a classroom reorganization we call “learner-centered learning.” Such a project-oriented approach goes by various names in other proposals for K-12 education, but is especially powerful when coupled with system dynamics. Learner-centered learning focuses on solving substantial problems. Teachers are no longer lecturers, no longer the source of all wisdom, not even necessarily authority figures. Teachers becomes advisors and coaches to students who are doing projects that may lie beyond the teacher’s experience. Indeed, moving to learner-centered learning can be traumatic for some teachers who feel they must be in command of all that the students are doing and learning.

A junior-high classroom can become much like a university research laboratory. Students address projects with real-world significance for which they have not been given the necessary background and techniques. They start by facing the challenge of learning what they need to know in order to accomplish the

project. Such an approach departs from the highly unrealistic format of most education that is found all the way up through college.

Education in most schools could hardly be more unrealistic. Students, when they are given a problem, can usually assume they have been taught everything needed to solve the problem. How many of you in your own affairs find that challenges come pre-equipped with everything for a solution?

In the full use of learner-centered learning, if a student asks a specific question, the teacher may not know the answer, but, even if the teacher does know the answer, rather than answering, the better approach is to discuss how the student might find the answer.

4. THE NATURE OF SYSTEMS

In its full development, system dynamics is a profession with the scope of science, education, law, engineering, or medicine. On the other hand, it is becoming clear that teachers in ordinary K-12 schools can make enough progress in two or three years to achieve major improvement in students' thinking, self reliance, and enthusiasm for learning.

Many principles form the foundation of system dynamics. Such principles become a basis for thinking in all endeavors.

4.1 Feedback Loops

Here we touch on the nature of feedback loops. People seldom realize the pervasive existence of feedback loops in driving everything that changes through time. Most people think in linear, nonfeedback terms. For example, in Figure 1, people see a problem, decide on an action, expect a result, and believe that is the end of the issue. Figure 1 illustrates the framework within which most discussions are framed in the press, business, and government.

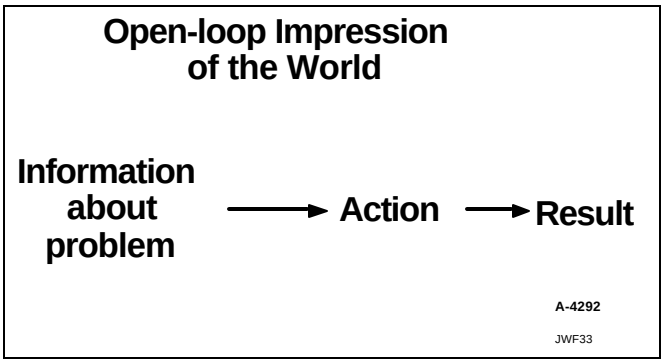


Figure 1.

However, a far more realistic perception would be Figure 2 in which a problem leads to action that produces a result that creates future problems and actions. There is no beginning or end.

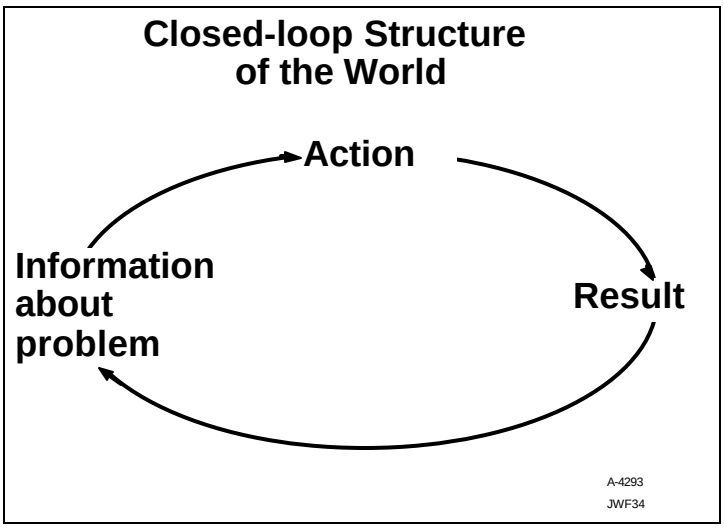


Figure 2.

We live in a complex of nested feedback loops. Every action, every change in nature, is set within a network of feedback loops. Feedback loops are the structures within which all changes occur.

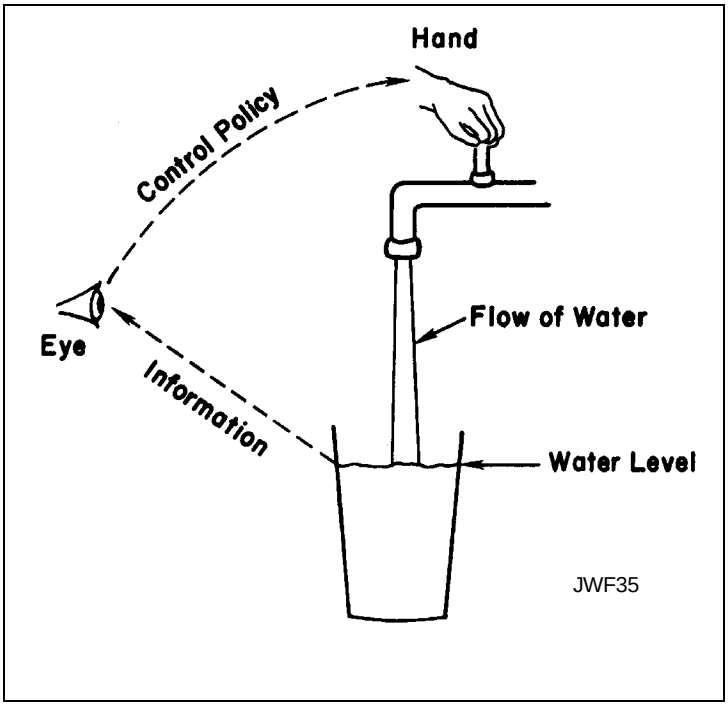


Figure 3.

Filling a glass of water is not merely a matter of water flowing into the glass. There is a control of how much water. That control is the feedback loop from water level to eye to hand to faucet and water flow. Such closed loops control all action everywhere.

4.2 Simplest Feedback Loop

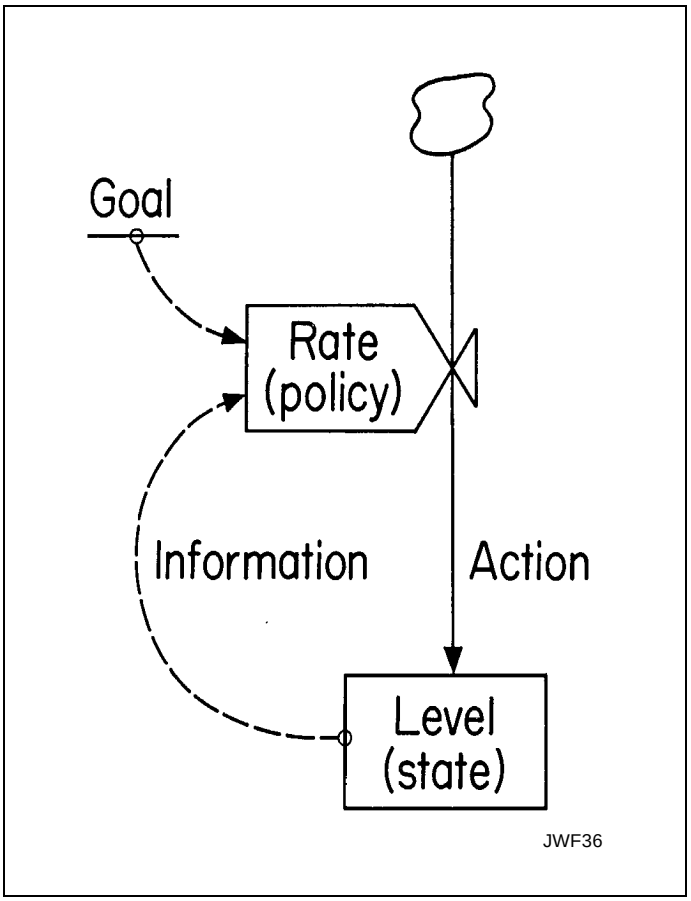


Figure 4.

Figure 4 shows the simplest possible feedback system. In spite of its simplicity, many universal truths are evident:[4]

a. In the figure are two symbols--a level, and a rate. The level is an accumulation, or integration, or stock, to choose terminology from different fields. The rate is a flow that changes the amount in the level. The rate is defined by a "policy" statement that tells how the rate is controlled by the value of the level in comparison to a goal. All systems, everywhere, consist of these two kinds of concepts --levels and rates--and none other. Such a statement, that there two and only two kinds of variables in a system, is powerful in simplifying our view of the world. People familiar with accounting statements, as in annual reports of corporations, will recognize the two classes of variables. A financial report is presented on two different pages--the balance sheet and the profit and loss statement. All numbers on the balance sheet are levels representing accumulations that have evolved over time. The profit and loss statement represents the flows that cause the levels to change. There is no third page, only the page representing levels and the page representing rates. That structure of an accounting statement represents a fundamental truth about all systems. Water in a bathtub is a level, the flow of water is the rate that changes the level. A person's reputation is a level that is changed by the flow of good and bad actions by that person. The degree of frustration in a group is a level that gradually changes in response to surrounding pressures.

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[4] Forrester, Jay W. (1968). *Principles of Systems.* (2nd ed.). Waltham, MA: Pegasus Communications. 391 pp.

b. As in Figure 4, levels are changed only by rates of flow, never directly by other levels.

c. Rates of flow are controlled only by levels, never directly by other rates of flow.

d. At the point where a rate of flow is being determined, the condition of the system, as indicated by the values of the levels, is compared with a goal to determine control of the flow.

5. QUOTATIONS FROM TEACHERS

When I discuss K-12 education, you might be justified in feeling that, because I have not taught pre-college students, I lack credibility. So, in preparing this talk, I sent out email messages to some 20 teachers asking each for a quotable paragraph that captures how system dynamics is being used in their schools. I will read some of the responses to illustrate my message.

The basic concepts about systems, illustrated in Figure 4, can be introduced as early as kindergarten, as described by Tim Lucas, elementary school principal, Ridgewood, NJ:

> "We are introducing kindergartners to the concepts of stocks and flows and the idea that behaviors can be graphed over time. Beginning in first grade students are mapping larger sets of information and working with causal loops to explain cycles in nature and everyday events. Students continue working across the curriculum, strengthening their understandings of behaviors over time, causal loops, and simulations mediated through a systems approach. By fifth grade, students are manipulating simple computer models that integrate into their curriculum.
>
> "Every one of the 40 Macintosh computers in the building has STELLA, SemNet, and Inspiration on the hard drive. We introduce the software across the curriculum starting with Inspiration in grade 2."

STELLA is the system dynamics software most commonly used in schools, although Powersim and Vensim are also being adopted.[5]

---

[5] STELLA from High Performance Systems, Hanover, NH; Powersim from the PowerSim Corporation, Reston, VA; and Vensim from Ventana Systems, Belmont, MA.

In the above quote from Lucas, kindergarten students go through their environments identifying levels and rates (that is, stocks and flows). They are beginning to see the fundamental structure of all systems in their surrounding world.

6. FROM SIMPLE TO COMPLEX SYSTEMS

The basic feedback loops in Figures 3 and 4 appear too simple to represent real-world situations. They are indeed simple, but they have a more serious shortcoming--they are misleading and teach the wrong lessons. Most of our intuitive learning comes from very simple systems. The truths learned from simple systems are often completely opposite from the behavior of more complex systems. A person understands filling a water glass, as in Figure 3. But, if we go to a system that is only five times as complicated, as in Figure 5, intuition fails. A person can not look at Figure 5 and anticipate the behavior of the pictured system.

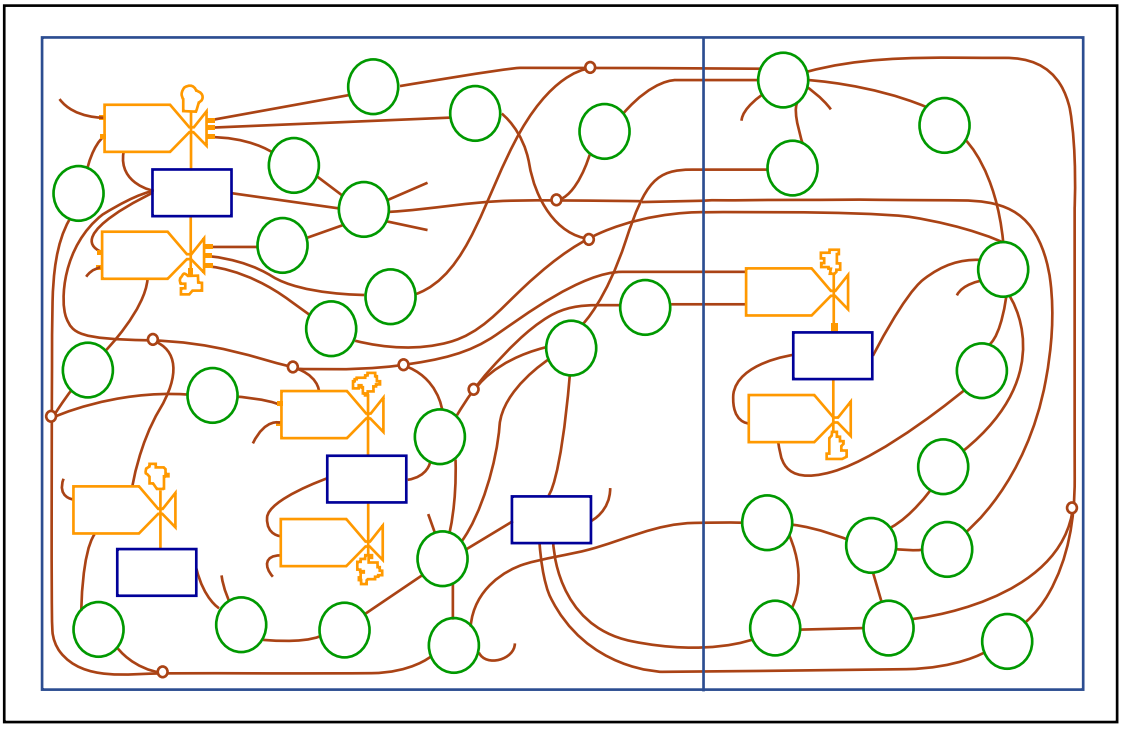


Figure 5.

Figure by MIT OpenCourseWare.

Figure 5 is five times more complicated than Figure 4 in the sense that it has five levels--the rectangles in the figure. The figure shows how rapidly apparent complexity increases as more system levels are added. In Figure 5 from the *World*

Dynamics book, each level is controlled by more than a single flow. Each flow can be influenced by several of the system levels. The circles in the diagram do not contradict the earlier statement that there are only two kinds of variables: the circles are subdivisions of the equations (policies) for the rates of flow; such subdivisions define separately identifiable concepts within the control of rates.

Mathematicians would describe Figure 5 as a fifth-order, nonlinear, dynamic system. No one can predict the behavior by studying the diagram or its underlying equations. Only by using computer simulation can the implied behavior be revealed.

Figure 5 displays interactions between population, capital equipment, agriculture, resources, and pollution. The diagram links multiple disciplines. A proper study of systems must usually break down the boundaries between academic disciplines. As stated by Gordon S. Brown, former dean of engineering at MIT, "The message is in the feedback, and the feedback is inherently interdisciplinary."

Complex systems behave in ways entirely different from our expectations derived from experience with simple systems. Because intuition is based on simple systems, people are misled when making decisions about complex systems. Six examples show the dangers in judging real-life systems based on a lifetime of conditioning from simple systems:

6.1 Cause and Effect not Closely Related

> From simple systems we learn that cause and effect are closely related in time and space. When we touch a hot stove, we are burned here and now. Experiences that are understandable almost always drive home the lesson that the cause of a symptom is to be found nearby and immediately before the observed consequence. But in complex systems the cause of a symptom is usually far back in time and arises from an entirely different part of the system. To make matters even more misleading, complex systems usually present what appear to be causes that are close in time and space to the immediate problem, but those apparent causes are only coincident symptoms. Learning ever since childhood teaches lessons that cause people to misjudge and mismanage complex systems.

6.2 Long-Term vs. Short-Term tradeoffs

In a simple system, a goal can be accomplished and a task finished. When the water glass is full we turn off the water, the objective has been met, and there are probably no direct unpleasant consequences. However, in complex systems there is nearly always a tradeoff. If the short-term goal is maximized, the result is a longer-term undesirable consequence. A child takes a toy from a playmate, the goal of having another toy is achieved, but a fight is likely to ensue. Borrowing on credit cards enhances the short-term standard of living with the longer consequence that the standard of living must be lowered to pay interest and refund the loan. Excessive welfare programs in many countries relieved immediate social pressures but led to mounting governmental debt and severe political consequences as expenditures had to be curtailed. Gratification of immediate desires may lead some to stealing with the later consequence of jail.

6.3 Ineffective Actions

In simple systems, it is clear what must be done to achieve a goal. One turns the steering wheel in the right direction to keep a car on the road. But in complex systems, the obvious decisions are often ineffective. An exceedingly large fraction of policies in complex systems have very little effect. Nevertheless those low-leverage policies receive most of the attention in business and government. Debate centers around decisions that will be defeated by consequences emerging from other feedback loops in the system. Taxes may be raised to balance the budget, but the extra governmental income can become an excuse for more expenditure and the budget remains in deficit. Experience from simple systems misguides people to take actions that the system itself can defeat.

6.4 Wrong Directions for Effective Actions

In simple systems, the direction of action to achieve a goal is obvious. Diligent work and longer hours will increase income. In complex systems, even when a rare high-leverage policy has been chosen, the desirable direction to change that policy is often unclear, or worse, may usually be misjudged and the policy moved in the wrong

direction. In the *Urban Dynamics* model, low-cost housing was found to be a high-leverage policy for affecting the vitality of a city and well being of its residents. Governments had been constructing low-cost housing, but that is the wrong direction. Old and decaying housing, which is the principal stock of low-cost housing, should be removed, not augmented. Low-income housing uses land space that could instead be used for job-creating industrial structures, while the housing draws in people who need jobs. The additional housing reduces jobs while increasing the number of people who need jobs. Additional housing is not a way to alleviate poverty but instead is an active force for increasing poverty. The validity of the model has been verified by urban trends. It is only through comprehensive modeling of complex systems that we can hope to overcome the policy errors that arise from a lifetime of learning the wrong lessons from simple systems.

6.5 Blaming Others

In simple systems, the cause of a failure is clear. One trips over a rock because the foot was not raised high enough; it is obvious that the fault was our own. In complex systems, causes are more obscure; it is not evident that we have caused our own crises, so, there is a strong tendency to blame others. However, the practice of blaming others diverts attention from the real cause of trouble, which is usually our own actions. By looking to others as the culprits, we take attention off the more embarrassing, but more productive, need to change our own actions. A management will blame the competition, or bankers, or its employees for low profits or falling market share, even though other companies in the same business, that deal with the same customers and bankers, are successful. The difference must lie in the policies of the failing company. The United States has a problem of illegal drugs, so drug supplying countries are blamed, rather than asking why our country is the largest market for drugs. There would be no suppliers if there were no users. In simple systems, the source of a problem is evident and lies in our own actions. In complex systems, causes are hidden and blame can be attributed to scapegoats through which correction is not possible.

6.6 Collapse of Goals

> In simple systems, goals are reinforced and maintained. The goal of staying in the proper highway lane is sustained by the threat of an accident. In less obvious systems, goals can gradually erode. One's goal of maintaining a sound financial condition can yield to pressure to borrow for a vacation or to purchase a fancier automobile. The goal can gradually decline from a safe financial condition, to wanting to fall no farther into debt, to striving to meet debt payments, to hoping to avoid foreclosure on one's house. In some countries there was a time when unemployment rates of 4% were considered high, gradually the goal eroded until 8% or more became the norm.

Unless a public emerges from our schools with an understanding of the ways in which real-life systems in families, schools, business, and politics must be handled in ways very different from the lessons learned from simple systems, we will continue to have stress-creating failures.

7. EVERYONE USES MODELS

I sometimes ask an audience how many use models for all their decisions. No one responds. How then, I ask, do they make decisions? They quickly see that all decisions are made on the basis of mental models. No one's head contains a family, city, school, country, or business. Decisions are based only on assumptions about separate parts of real systems, and trying by intuition to fit those fragments of knowledge into an estimate of how things change and what will be the consequences of a proposed action.

Such mental models belong to the same class as the computer models used in system dynamics. In fact a system dynamics model is often built from assumptions in the mental models. Mental models are rich and often sufficiently accurate about the pieces of a system--what information is available, who is connected to whom, what are different people trying to achieve. But mental models are entirely unreliable in deducing what behavior will result from the known pieces of a complex system. On the other hand, a computer simulation can, without doubt, reveal the behavior implicit in the structure from which it is constructed.

8. MODELS OF PLOTS IN LITERATURE

Almost every activity in K-12 education can be enhanced by system dynamics modeling. Some applications have surprised even those of us who already had high optimism.

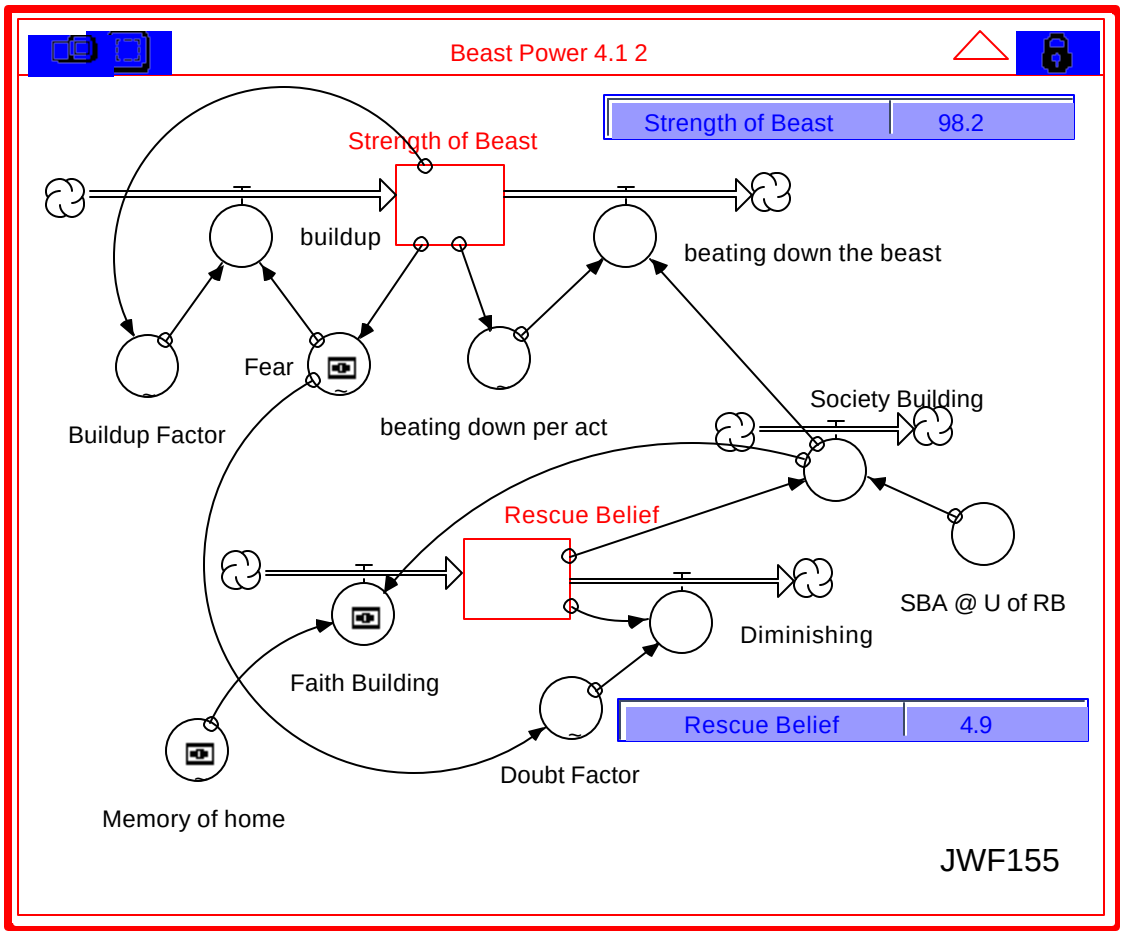


Figure 6.

To illustrate going from a written description to a system dynamics model, English teachers have created models to show social and psychological pressures that cause plots in literature to unfold. Thus far, most such models may be overly simple, maybe even naive, nevertheless, they point a direction and provide a vehicle for intensified discussion of a book.

One such model by Timothy Joy, a Catholic high school English teacher in Milwaukie, Oregon, arose from Golding's *Lord of the Flies.* Figure 6 shows the model structure and Figure 7 a simulation output.

Image unavailable.

Figure 7.

About this model, Tim Joy writes:

> "I honed two models for Golding's novel; one based on the boys' declining civility, another describing how the boys' loss of hope drives the increasing power of the beast.
>
> "I was left with how to introduce system dynamics and the STELLA software to 135 sophomores within the guise of an English class.
>
> "Graphs in hand, the students were arguing positions before I could take attendance, peering over books and tables, pointing out misjudgments and omissions.
>
> "This simple model was readily understood by most students. An amount of civility is transformed, through isolation and turmoil,

into savagery. It was not until after running the model that some sophomores saw the model's weakness.

"Students were not ready to accept their first attempt. Indeed, quite a few wanted to redo it. Some asked for more time to run another graph on a different boy. Others were just plain dissatisfied and came after school, on Friday no less, or the following Monday to finish the work. This was a new experience for me."

Timothy Joy on his experiences in teaching:

"I taught writing and literature for 13 years and always suspected I was party to some intellectual crime. Why is it that so many students thought the world of language began and ended at the door of the classroom? Then I discovered system dynamics.

"System dynamics has a logic-based grammar, a universal language that students can readily learn and manipulate to create meanings. What have I found? Creating "meaning" results in bolder QUESTIONS, whole new views which do not house traditional understandings.

"In some ways, it's been terrifying. I have to give over some portion of the direction and instruction of the material to their own instincts and inquiry. It's slower at the start, but the curve steepens as we discuss and build models. For instance, I have changed the phrasing of questions in literature--we speak of influences (change of rate) to character motivations (levels or stocks), we focus on whether one defining moment is greater than another (in essence, it's a mathematical discussion on a literary event). Let's face it, reading a Charles Dickens' novel is to plunge headlong into a very complex system; STELLA provides a means through it.

"I'll never be the same teacher, never go back to teaching only one thing. When I think of living, I just can't recall a time when ONLY ONE THING happened; lots of things happen, all the time, each in differing degrees. What happens in a classroom ought to prepare people for those realities.

Referring to his church-sponsored high school Tim Joy continues:

"I still teach writing and literature but it's changing radically. We use STELLA (software) at all levels and in many disciplines: Physics, English, Government, Economics. This year we'll add models in Health and Biology. We're still hoping to convert the Religious Studies Department."

9. COMPUTER MODELS AND MENTAL MODELS

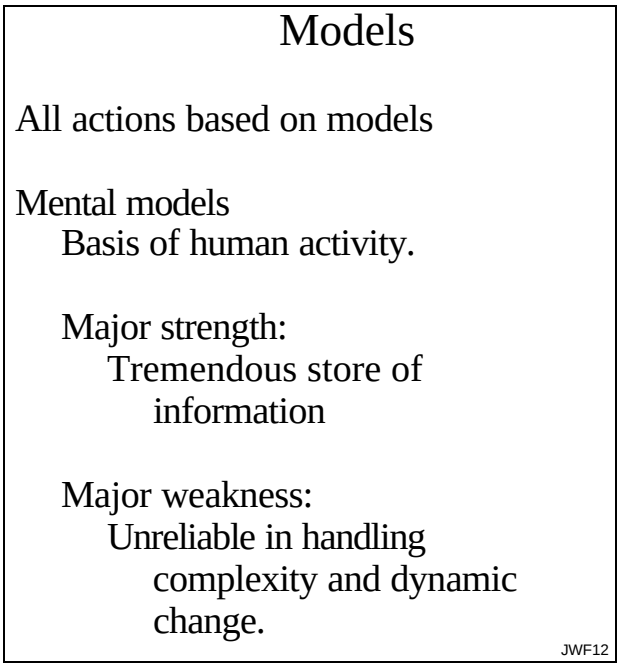


Figure 8.

System dynamics builds two-way communication between mental models and simulation models. Mental models are the basis for every-day decisions. Mental models contain tremendous stores of information. But the human mind is unreliable in understanding what the available information means in terms of behavior. Computer simulation meshes nicely with mental models by taking the mentally stored information and then displaying the dynamic consequences.

From the report on a teachers' conference at the System Dynamics Group, Trinity College, Vermont:

"Models provide a common language with which to engage learners with diverse learning styles and interests. Simulations are especially engaging, and draw out many who might not otherwise participate in more traditional discussions and activities.

"Models are extraordinarily powerful for helping to convert abstractions into concrete realities. A learner's ability to 'see' a system—what goes into a stock, where feedbacks exist—and then to run a model and ascertain how the system operates under varied conditions, renders abstractions into real meaningful, concrete terms. This discovery is true for students at all levels.

> "Models provide a vehicle through which students see issues as involving trade-offs rather than having one or two sides (as in the debate format). Positions move from being simplistically labeled 'good' and 'bad' to being seen as much more multi-faceted.
>
> "Models which deal with real issues in student's lives—such as financial and interpersonal—render learning especially meaningful.
>
> "Participants agreed that many of the greatest benefits of a systems approach are in disciplines like literature, the arts, and history. A great work like 'Animal Farm' expands from a piece of writing into a powerful vehicle for learning more about general human behavior.
>
> "Kids' enthusiasm for learning carries over into other classes and leads other teachers to express interest in learning about systems."

The above report mentions discovery being true "for students at all levels." And one can add for students of all academic rankings. Evidence is accumulating that there is no correlation between how students have previously been ranked as fast or slow learners and how they do in the system dynamics setting. Some top students remain at the top. But some top students have ranked high only because they learn and repeat facts but without deep understanding of the meaning of the facts; such students may do poorly in the systems framework. On the other hand, some students who have been graded at the bottom of their classes may be there not for stupidity but because they see the academic work as irrelevant and not engaging; they often blossom when given the chance to bring their real world into a focused systems context.

10. WORKING WITH COMPUTER MODELS

The translation of a mental model to a system dynamics simulation model moves through several stages.

> 1. A model must be created with no logical inconsistencies. All variables must be defined. None can be defined more than once, Equations must be unambiguous. Units of measure should be the same on both sides of an equation. Most system dynamics software applications check for and find such logical errors.

2. When a model is first simulated, the results may be absurd. Simulated behavior may be impossible. Inventories, or water in a bath tub, or students in the school may go negative; negative values often have no real-world meaning. One goes back to refine the model and make the structure more realistic and more robust.

3. As a model becomes better, surprising behavior often does not reveal model errors but instead begins to tell something about real life that was not previously realized. I have usually had such new insights from models. One example arose from the model in my *Urban Dynamics* book dealing with the growth and stagnation in cities. One weekend I added a job-training program to the model. It was a perfect job-training program in the sense that it simply took people out of the unskilled category and put them in skilled labor; and, furthermore, no charge was assigned so it cost nothing.

The perfect job-training program caused unemployment in the model to go up. The increase in unemployment surprised me until I spent a day discovering what the model was doing, after which the result seemed plausible. I took the computer runs back to former mayor John Collins and the several people from Boston business and politics who had been working with me. They looked at the rising unemployment in silence for several minutes until one said, "Oh!, Detroit has the best job-training program in the country and the most rapidly rising unemployment rate." Later I went to people in the business of running job-training programs and asked if they had ever heard of a situation where job training could increase unemployment. Their answer, "Of course, when that happens we go to another city."

The job-training program in the model was defeated by three forces: 1) before the program, businesses had been dipping into the unskilled and unemployed pool as necessary to obtain employees. The job-training program substituted for the training that businesses would have done, so training by businesses stopped. About half of the training program was neutralized by such substitution; 2) the program increased the number of skilled workers thereby increasing unemployment among skilled workers and resulted in increased downward flow back to the unskilled-unemployed pool. Nearly another half of the training was lost through the increased downward mobility; 3) and last, the training program had high public visibility and attracted unemployed from other cities, even though the program had

not created significant new jobs. Forces within the system neutralized the training program and the public visibility of the program attracted additional people who would remain unemployed.

Another situation of first learning about real life from a simulation model arose when a student modeled the behavior of insulin and glucose in various aspects of diabetes. In response to an experiment simulated within the model, he got a result from his computer patient that had never been reported in the medical literature. Was there something wrong with the model? He showed the results to doctors doing diabetes research. After studying what was happening in the model, their response was, "We had a patient like that once, but always thought there was a mistake in the measurements." By this process a new medical syndrome had been identified.

11. SOURCES OF INFORMATION

Consider the available databases, or sources of information from which we can build computer simulation models.



Figure 9.

I suggest that the world's store of information lies primarily in people's heads--the mental database. As a test of that statement, consider any institution, for example your school system. Imagine that at 10 o'clock some morning every teacher and administrator suddenly leaves and is replaced by a person who can read but has no experience with a school system. You instruct your replacement to follow the instructions and policy statements in your office and carry on for you. Chaos would result. Our families, schools, businesses and countries operate on the information in people's heads gained from participation, apprenticeship, and on-the-job learning. The mental database is vastly richer that the written database in the form of books, magazines, and newspapers. In

turn, the written database is far more informative about how society operates than the numerically recorded information.

System dynamics modeling should build on all available information, including the voluminous mental database. By contrast, most analyses in the social sciences have been limited to information that has been numerically recorded. The numerical information is an extremely small part of all the information that is available.

Pamela Hopkins, 11th grade English literature teacher, in the Desert View High School, Tucson, AZ:[6]

> "The *Hamlet* model was used with my students… Teaching Shakespearean drama has traditionally been difficult… a high crime area… drug-related activities are ever-present… many of our students are members of dysfunctional families… The lower socioeconomic level of the students further complicates the issue… students are distracted by financial difficulties… tired from working part time… Shakespeare fails to draw their attention.
>
> "When we used a STELLA model which analyzed the motivation of Shakespeare's Hamlet to avenge the death of his father in HAMLET… The students were engrossed throughout the process… The amazing thing was that the discussion was completely student dominated. They were talking directly to each other about the plot events and about the human responses being stimulated. They talked to each other about how they would have reacted and how the normal person would react. … My function became that of listening to their viewpoints and entering their decisions into the computer. It was wonderful! It was as though the use of precise numbers to talk about psychological motives and human responses had given them power, had given them a system to communicate with. It had given them something they could handle, something that turned thin air into solid ground. They were directed and in control of learning, instead of my having to force them to keep their attention on the task."

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[6] Hopkins, Pamela Lee. (1992). Simulating *Hamlet* in the Classroom. *System Dynamics Review*, Vol. 8 (1, Winter), pp. 91-98.

Hopkins also said the students would tell her in quantitative terms how to change the personality of a character to alter the plot and to see who got killed instead. Students found that fascinating.

12. MODELS DETERMINE ALL HUMAN ACTIVITIES

Models are central to all human activities.

Mental models and computerized models should not be considered polar extremes, but rather as overlapping and mutually reinforcing ways to understand reality.

JWF67

Figure 10.

Again from Tim Lucas, in Ridgewood, NJ

> "teachers, especially at the elementary level, have found it very easy to integrate systems thinking across their curricula. It is not another subject to be added, but instead it integrates and supports our work with… composition… use of technology… science and mathematics… and … interdisciplinary projects in all areas.
>
> "At this point we have students, teachers, parents, and the larger community working toward a better understanding of the use of systems thinking in school programs, parenting issues, and as a tool for understanding the changes we are facing.… This has been a grass-roots effort with support that continues to grow as we learn more."

One can include in a formal model anything that can be described explicitly in words. To say, "We want life to be better," is not explicit, it has no operational content. However, to say, " We want more income," or, "We need more free time," is coming closer to a concrete statement. Modeling forces a person to go from empty statements to ones with operational content.

From Jan Mons, who is the systems mentor to assist other teachers in incorporating systems concepts into their classrooms in Glynn County, Georgia:

> "My most fruitful experiences occur when I discuss classroom discipline systems. We have both students and teachers build a discipline system together so that all parties will know what the system is capable of producing. When we do this many students have an

"Aha!" experience and state that they now understand how a teacher's frustration can accumulate over time. Teachers have their own insights as well--they begin to understand how they have often built discipline systems that were "preprogrammed" to result in unpleasant situations."

Albert Powers, science high-school teacher, Concord, MA:

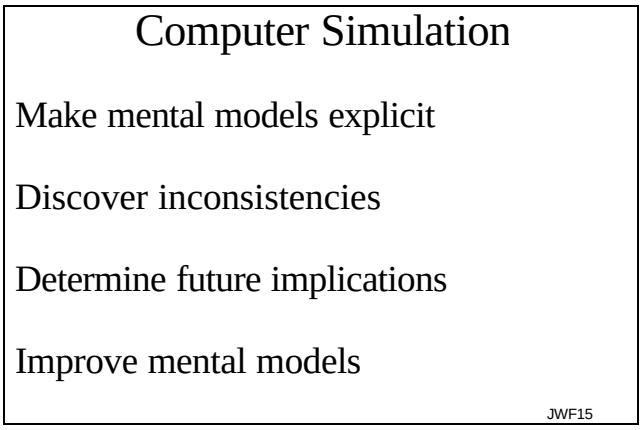


Figure 11.

"My favorite vignette is one that occurred while three girls were working in a chemistry lab predicting shifts in reaction rates, concentration, and mass action values as a chemical system is repeatedly stressed. I overheard one student ask her partner what would happen if they plotted all of these variables together on a single graph. Her partner said, 'Let's do it!'.... and they did. Upon observing the graphs evolving together, one said, 'What a mess!' Her partner then exclaimed, 'But look! Everything's happening at once!' This last remark piqued my curiosity. I asked her what she meant by it. Her reply: 'First we studied rate changes. Then we looked at concentration changes. Then we did the mass action behavior. I thought that first one thing happened… then the next: and then the next. But it all happens at once! Everything depends on everything else!'

"Here we had a linear thinker who made a quantum leap towards more systemic thinking. It's interesting to note that her overall class performance also took a quantum leap from that moment forward (C all the way to an A).

Those comments from teachers remind me of a TV producer who was filming opinions, insights, and discussions among parents, teachers, and students who were involved is a school using system dynamics. The TV producer turned to one eighth-grade boy and asked, "What has all this meant to you." The immediate answer, "I am much better able to deal with my mother."

13. GENERIC OR TRANSFERABLE STRUCTURES

Many structures of levels and rates are found repeatedly. They are "generic structures" because they are found in many different situations, even in entirely different fields of application. If a particular structure is understood in one setting, it is understood in all settings. Generic structures provide the student with power to move between subjects with the learning on one subject being applied to other subjects.

After understanding a collection of basic dynamic structures, a student can quickly draw on one to understand a new situation if its structure has been encountered previously.

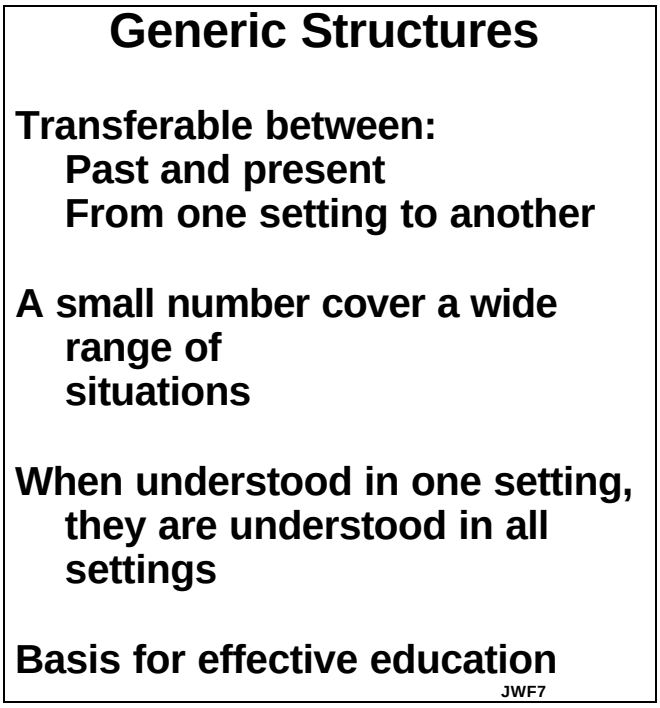


Figure 12.

13.1 Generic Structure of a Clock and Economic Business Cycles

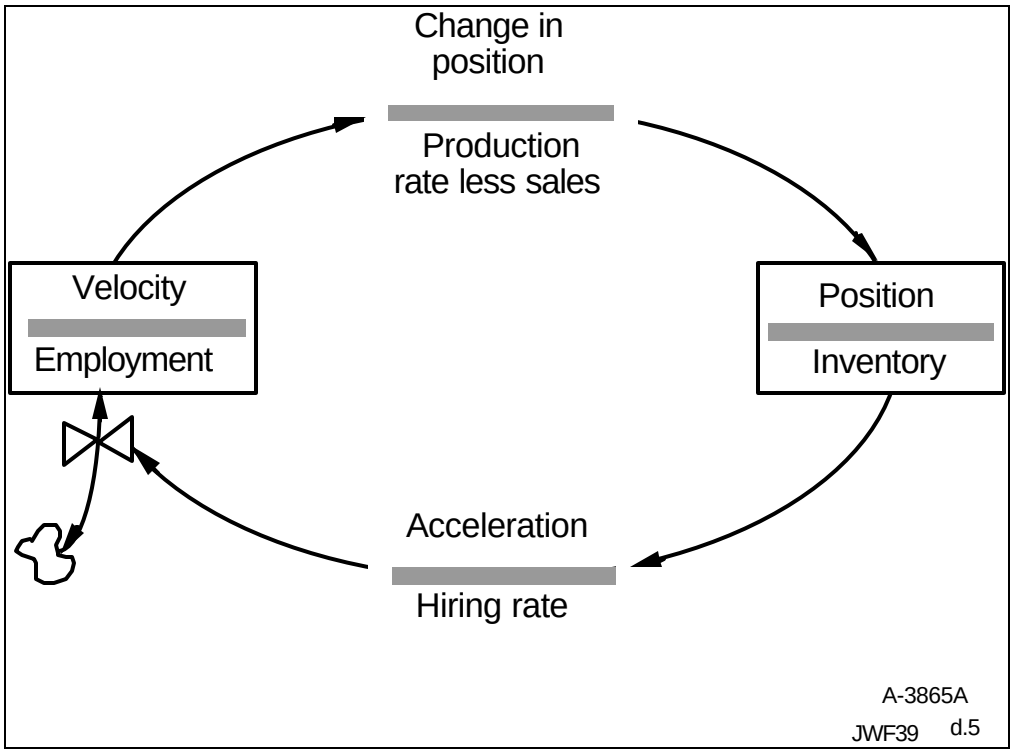


Figure 13.

Figure 13 shows two sets of nomenclature. The upper definitions relate to the swinging pendulum of a clock. The lower definitions describe inventory and employment in manufacturing. With appropriate choices of parameter values, the structure will exhibit the oscillation of a one-second clock pendulum, or alternatively the several-year interval between peaks of a business cycle. The single loop with two levels as in Figure 13 results in only a sustained oscillation. Additional structure in necessary to represent friction in a pendulum or the forces that might change the amplitude of business cycles.

A swinging pendulum and the central core of the production-inventory business cycle have the same oscillatory structure.

In my own experience with transferability of structure, I was meeting with a group of medical doctors and pharmacologists in Palo Alto one day. A topic came up that was not on the agenda. They began to talk about a doctor who was doing an experimental treatment, which clearly those present did not approve. They had described the experiment but had said nothing of the results when I suggested an outcome. I told them that if I were to judge that medical experiment on the basis of

the kind of behavior that we had seen in the *Urban Dynamics* model of cities, I would guess the treatment would cause atrophy of the pancreas. They said, "You're right. That's exactly what's happening." There was enough transferability of structure between the two systems to justify suggesting the medical outcome.

For dramatic, personal-experience learning, computer structures can be converted into games with people making the decisions that are made by the rates (policy statements) in a model. A distribution system from manufacturer, through distributor and retailer, to customers has been played by hundreds of thousands of people around the world to drive home the way in which people can interact to create instability. Other games show the dynamics of producing great depressions some 45 to 70 years apart, and still others show how companies can grow so rapidly that they cause their own failure.

14. OTHER COUNTRIES

System dynamics is entering K-12 education in other countries. We are now translating a book from German that uses system dynamics for teaching high school physics.

Pal Davidsen, professor at the University of Bergen in Norway, has led of a Scandinavian consortium for introducing system dynamics into high schools:

> "In Norwegian and Nordic schools, we have chosen to utilize the conceptual framework offered by system dynamics for our educational purposes… Academic boundaries no longer constitute the boundaries of our imagination or our investigation. Historic and economic considerations are merged with physics and chemistry in our study of ecological issues."

Michael Radzicki, professor at Worcester Polytechnic Institute, in Massachusetts writes of a trip to Japan:

> "My most recent experience with K-12 education and system dynamics was this summer in Japan. We did a two-day seminar for Japanese high school students (15 year olds), their teachers, and observers from Japanese industry. We were asked to focus on teaching physics and biology using system dynamics. Students worked in groups of three. In two days we were able to cover the following:

velocity, acceleration, and position
the basics of stocks and flows
growth of yeast cells, adding feedback and exponential growth
radioactive decay
mass on a spring with damping

"Students were VERY engaged in the activities. They refused to take breaks; they preferred to keep working on the problems or to try building their own models."

15. DIFFERENTIAL EQUATIONS VS. INTEGRATION

One might ask how it is now possible to teach behavior of complex dynamic systems in K-12 when the subject has usually been reserved for college and graduate schools. The answer lies in having realized that the mathematics of differential equations has been standing in the way.

Differential equations are difficult, confusing, weak, and unrealistic. They often mislead students as to the nature of systems. Mathematicians have had difficulty defining a derivative and there is a reason. Derivatives do not exist except in a mathematician's imagination. No where in nature does nature take a derivative. Nature only integrates, that is, accumulates. Casting behavior in terms of differential equations leaves many students with an ambiguous or even reversed sense of the direction of causality. I have had MIT students argue that water flows out of the faucet because the level of water in the glass is rising; that seems natural to them if the flow has been defined as the derivative of the water level in the glass.

Any child who can fill a water glass or take toys from a playmate knows what accumulation means. The levels (stocks) in a system dynamics model (the rectangles in Figures 4 and 5) are the integrations (accumulations). By approaching dynamics through the window of accumulation, students can deal with high-order dynamic systems without ever discovering that their elders consider such to be very difficult.

Helen Zhu, an MIT undergraduate working to develop system dynamics materials for K-12 education observed:

"In my differential equations class we used calculus to figure out the behavior of populations. I realized just how much simpler system dynamics made that thought process. Whereas only college students can understand

such phenomena using math, elementary schoolers can understand the same things by using system dynamics modeling. It's really amazing."

16. REPORTS FROM CLASSROOMS

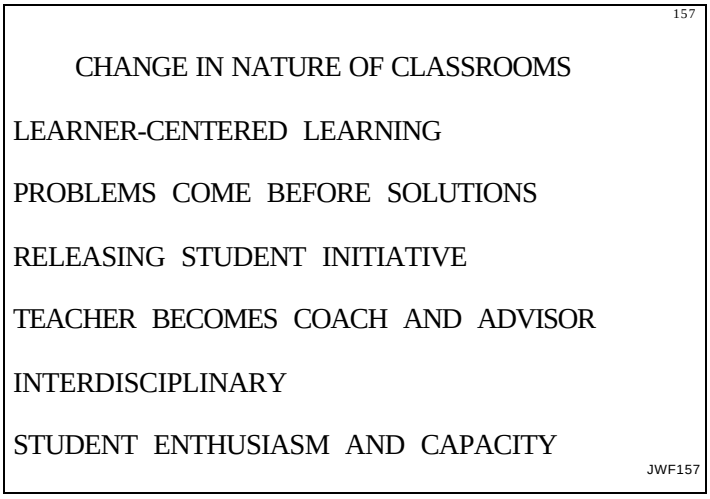


Figure 14.

The project-oriented research organization of the learner-centered-learning classroom shares with students the role of teaching. One of the best ways to learn is to explain a subject to another person. Al Powers reports student reactions in his chemistry classes:

"Working in groups was incredibly effective. Often it is easier to understand concepts when they are explained by a peer."

"I feel that everyone is heard and, therefore, the people are more willing to contribute to the discussions and admit to being uncertain about a concept."

"The graphs and simulations brought the concepts to life."

"I found myself explaining concepts to people in other classes. This has never happened to me before. I think they would have benefited just as much as I did if they had had the opportunity."

"Being a visual learner, it really helped to see the reactions in easy schematics. The graphs produced were even more helpful. My father and I spent long hours discussing the graphs and talking about what was the initial change and what was the reaction to that change."

" This was a great lab! Using the computer made it easier for me to understand what was going on in the reaction."

From Robert Gotwals, education specialist, North Carolina Supercomputing Center:

"Last year I was asked to help with a large 6th-grade class project on Africa. The class had discovered that diseases play a major role in the culture and customs of the peoples. I was asked to build epidemiological template models of several diseases for the students to use. The students wanted no part of that, so they, with help of myself and a graduate student, researched the literature and built models from scratch. One student even located one of the authors on the Internet and corresponded with him through email. The project was so successful that the 6th grade kids presented an 'epidemiology conference' to a group of epidemiologists from Duke University. Without a doubt, the kids clearly understood the mechanisms involved in the various diseases, and used the models to make some interesting observations about Africa's future."

Frank Draper, 8th grade biology teacher, Tucson, AZ

"Our classrooms have undergone an amazing transformation. Not only are we covering more material than just the required curriculum, but we are covering it faster (we will be through with the year's curriculum this week and will have to add more material to our curriculum for the remaining 5 weeks) and the students are learning more useful material than ever before. Facts are now anchored to meaning through the dynamic relationships they have with each other. In our classroom students shift from being passive receptacles to being active learners. Our jobs have shifted from dispensers of information to producers of environments that allow students to learn as much as possible.

"We now see students come early to class (even early to school), stay after the bell rings, work through lunch and work at home voluntarily (with no assignment given). There are essentially no motivation or discipline problems in our classrooms."

From Ellen Mandinach, Educational Testing Service, Princeton, NJ and Director of the Systems Thinking and Curriculum Innovation Project involving some eight schools in different parts of the country:

"The teachers in the STACI Project perceive the systems thinking approach to be both an effective instructional strategy and a professional development opportunity. Many of the project teachers

> have altered completely the way they structure their classroom activities. Both students and teachers benefit from the systems approach. For teachers, it is the stimulus for changing the fundamental role of the teacher to facilitator, coach, and mentor rather than the purveyor of information and facts."

Jeff Potash, history teacher, Trinity College, Vermont, after a summer training workshop for teachers:

> "Last summer we assembled middle and high school teachers who use system dynamics and STELLA in their classrooms. Again, we learned a phenomenal amount from them. Perhaps the most important message relates to the degree that middle school and high school teachers are at fundamental odds with one another. Innovative middle school people, like those in Glynn County Georgia, seek to expose their students to 'real world' models which demand involvement and choice, with the conscious desire to 'empower' kids to view themselves as active players in decision-making. High school teachers, sadly, seek to inculcate discipline-based knowledge. We need to rethink this system.
>
> "When we engage in modeling, we move beyond the traditional boundaries of disciplinary 'turf' and seek to develop common ground upon which to better communicate the workings of the real world, a world that too often refuses to respect the simple disciplinary borders of the academic specialist."

I attended one conference of some 200 K-12 teachers involved in system dynamics. Widely divergent views were evident. Some teachers felt they would need to devote a full introductory term to teaching system dynamics and modeling before the ideas could be introduced into normal subjects. Others had already demonstrated that students could pick up what they needed on their own in a few days. Several teachers commented to me, "I never realized that the students could do so much."

In fact, the students, if given a chance, may be the leaders in an educational revolution. Lola Piper, computer science teacher, Blair Magnet Program Silver Spring, MD, describes their experience:

> "We have a group of experienced student modelers and a collection of student-developed models. Our next step is to get these models actually integrated into the curriculum of other courses.
>
> "Our biggest problem to date is getting our staff to integrate system dynamics into their curriculum.… Our biggest success to date is having our students reach out to the teachers by asking the teachers to be their mentors in the development of models that could then be used within the teachers' classes. Given the personal touch of a student-teacher mentorship we hope to generate the teachers' interest and through one-on-one meeting we hope to demystify modeling for the teachers."

At the Silver Springs school, students and teachers have modeled:

> Ecological Succession from Sand Dune to Maritime Forest
> A Comet's Flight Through the Earth's Atmosphere
> Oxygen Levels in a River Resulting from various Dumped Wastes
> Squirrel Population Dynamics

17. FUNDING FOR EDUCATIONAL INNOVATION

Some people have expressed concern for how the transition to education based on system dynamics can be funded. Teachers must have released time to learn. Teachers should attend the conferences that allow them to hear what is being done in other schools. Some schools will need to add computers and software. A school system will make the most rapid progress if a more experienced system dynamics modeler can be hired as a mentor to assist teachers. Schools have solved these funding demands in various ways.

Government can sometimes be a source of funds, but that is often blocked by unresponsive bureaucracies. However, the National Science Foundation provided more than a half million dollars for teacher training in system dynamics in Oregon. In Massachusetts, one of 25 charter schools was approved to be based on system dynamics.

Many schools are overlooking the strong appeal that a persuasive new approach to education can have with private individuals. Several million dollars have been given by individuals to foster system dynamics in a number of schools.

Private funding is often easier to arrange, and is more flexible than money from government or large foundations.

A great national concern drives the search for better K-12 education. But many of the interventions have been futile, or even have contributed to making matters worse. Very often, the misplaced efforts for improvement have put additional stress on doing more of what is already causing the decline in education--more authoritarian control, more accountability, more testing, more pressure. Such misplaced emphasis removes the fun and challenge from learning.

One New York business man who thought he might devote personal time and money to system dynamics in education became involved after visiting a junior high classroom on a Friday afternoon. He was convinced when the final bell of the day and final bell of the week rang and not a single student got up to leave.

18. MORE ON SYSTEM DYNAMICS

Study materials are available from the Creative Learning Exchange and the web site at MIT. The Internet discussion group has several hundred participants. The web site at MIT receives one or two thousand accesses per week for downloading material. About two-thirds of those accesses come from corporations that use the material for internal training. Exactly the same material can be used anywhere from the 5th grade to chief executive officers; it is new to all.

System Dynamics books:

Pegasus Communications, Inc.
One Moody Street
Waltham, MA 02453-5339

tel: 781-398-9700
fax: 781-894-7026

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System Dynamics and Learner-Centered-Learning in Kindergarten through 12th Grade Education

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Abstract: *Pre-college education is under attack for poorly serving the needs of society. Unless a superior concept for improving education emerges, public displeasure is apt to result in still more of what is already not working. But now, a fundamentally new and more effective approach to education is emerging from advances in system dynamics. System dynamics offers a framework for giving cohesion, meaning, and motivation to education at all levels from kindergarten upward. A second important ingredient, "learner-centered learning," imports to pre-college education the challenge and excitement of a research laboratory. Together, these two innovations harness the creativity, curiosity, and energy of young people. System dynamics allows reversing the traditional educational sequence in which deadening years of learning facts have preceded use of those facts by introducing synthesis (putting it all together) at an early stage in a student's experience. Such synthesis can be based on facts that even elementary school students already have gleaned from life. Learner-centered learning reverses the process of a teacher lecturing facts to resistant students. Learners have the opportunity to explore, gather information, and create unity out of their educational experiences. A "teacher" in the new setting acts as a guide and participating learner, rather than as an authoritarian source of all wisdom.*

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System Dynamics and Learner-Centered-Learning in Kindergarten through 12th Grade Education

by
Jay W. Forrester

Secondary education is under increasing attack for not preparing students to cope with modern life. Failures appear in the form of corporate executives who misjudge the complexities of growth and competition, government leaders who are at a loss to understand economic and political change, and publics that support inappropriate responses to immigration pressures, changing international conditions, rising unemployment, the drug culture, governmental reform, and inadequacies in education.

Growing criticism of education may direct attention to incorrect diagnoses and ineffective treatments. Weakness in education arises not so much from poor teachers as from inappropriateness of material that is being taught. Students are stuffed with facts without having a frame of reference for making those facts relevant to the complexities of life. Responses to educational deficiencies are apt to result in public demands for still more of what is causing the present educational failures. Pressures will increase for additional science, humanities, and social studies in an already overcrowded curriculum, a curriculum that fails to instill enthusiasm and a sense of relevance. Instead, an opportunity exists for moving toward a common foundation that pulls all fields of study into a more understandable unity.

1. Sources of Educational Ineffectiveness

Much current dissatisfaction with pre-college education arises from past inability to show how people interact with one another and with their physical environment, and to reveal causes for what students see happening. Because of its fragmentary nature,

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traditional education becomes less relevant as society becomes more complex, crowded, and tightly interconnected.

Education is compartmentalized into separate subjects that, in the real world, interact with one another. Social studies, physical science, biology, and other subjects are taught as if they were inherently different from one another, even though behavior in each rests on the same underlying concepts. For example, the dynamic structure that causes a pendulum to swing is the same as the core structure that causes employment and inventories to fluctuate in a product-distribution system and in economic business cycles. Humanities are taught without relating the dynamic sweep of history to similar behaviors on a shorter time scale that a student can experience in a week or a year.

High schools teach a curriculum from which students are expected to synthesize a perspective and framework for understanding their social and physical environments. But that framework is never explicitly taught. Students are expected to create a unity from the fragments of educational experiences, even though their teachers have seldom achieved that unity.

Missing from most education is direct treatment of the time dimension. What causes change from the past to the present and the present into the future? How do present decisions determine the future toward which we are moving? How are lessons of history to be interpreted to the present? Why are so many corporate, national, and personal decisions ineffective in achieving intended objectives? Conventional educational programs seldom reveal the answers. Answers to such questions about how things change through time lie in the dynamic behavior of social, personal, and physical systems. Dynamic behavior, common to all systems, can be taught as such. It can be understood.

Education has taught static snapshots of the real world. But the world's problems are dynamic. The human mind grasps pictures, maps, and static relationships in a wonderfully effective way. But in systems of interacting components that change through time, the human mind is a poor simulator of behavior. Mathematically speaking, even a simple social system can represent a tenth-order, highly nonlinear, differential equation.

Mathematicians can not solve the general case for such an equation. No scientist, citizen, manager, or politician can reliably judge such complexity by intuition. Yet, even a junior high school student with a personal computer and coaching in computer simulation can advance remarkably far in understanding such systems.

Education faces the challenge of undoing and reversing much that people learn by observing simple dynamic situations. Experiences in everyday life deeply ingrain lessons that are deceptively misleading when one encounters more complex social systems (Forrester, 1971). For example, from burning one's fingers on a hot stove, one learns that cause and effect are closely related in both time and space. Fingers are burned here and now when too close to the stove. Almost all understandable experiences reinforce the belief that causes are closely and obviously related to consequences. But in more complex systems, the cause of a difficulty is usually far distant in both time and space. The cause originated much earlier and arose from a different part of the system from where the symptoms appear.

To make matters even more misleading, a complex feedback system usually presents what we have come to expect, an apparent cause that lies close in time and space to the symptom. However, that apparent cause is usually a coincident symptom through which little leverage exists for producing improvement. Education does little to prepare students for succeeding when simple, understandable lessons so often point in exactly the wrong direction in the complex real world.

2. Cornerstones for a More Effective Education

Two mutually reinforcing developments now promise a learning process that can enhance breadth, depth, and insight in education. These two are system dynamics and learner-centered learning.

2.1. Precursors of System Dynamics

System dynamics evolved from prior work in feedback-control systems. The history of engineering servomechanisms reaches back several hundred years. Popular writing, religious literature,

and the social sciences have grappled with the closed-loop circular nature of cause and effect for thousands of years (Richardson, 1991). In the 1920s and 1930s, understanding the dynamics of control systems accelerated. New theory evolved during development of electronic feedback amplifiers for transcontinental telephone systems at the Bell Telephone Laboratories and work at MIT on feedback controls for analog computers and military equipment.

After 1950, people became more aware that feedback control applies not only to engineering systems but also to all processes of change—biological, natural, environmental, and social.

2.2. System Dynamics in Pre-College Education

During the last 30 years, those in the profession of system dynamics have been building a more effective basis than previously existed for understanding change and complexity. The field rests on three foundations:

> 1. Growing knowledge of how feedback loops, containing information flows, decision making, and action, control change in all systems. Feedback processes determine stability, goal seeking, stagnation, decline, and growth. Feedback systems surround us in everything we do. A feedback process exists when action affects the condition of a system and that changed condition affects future action. Human interactions, home life, politics, management processes, environmental changes, and biological activity all operate on the basis of feedback loops that connect action to result to future action.
>
> 2. Digital computers, now primarily personal computers, to simulate the behavior of systems that are too complex to attack with conventional mathematics, verbal descriptions, or graphical methods. High school students, using today's computers, can deal with concepts and dynamic behavior that only a few years ago were restricted to work in advanced research laboratories. Excellent user-friendly software is now available (High Performance Systems, 1990; Pugh, 1986).[1]

---

[1] For most work at the pre-college level, STELLA™ on Macintosh computers is easiest to use. It includes an excellent manual with learning exercises and an introduction to the philosophy of system dynamics. Some other system dynamics software packages are being developed with

> 3. Realization that most of the world's knowledge about dynamic structures resides in people's heads. The social sciences have relied too much on measured data. As a consequence, academic studies have failed to make adequate use of the data base on which the world runs—the information gained from living experience, apprenticeship, and participation. Students, even as early as kindergarten, already have a vast amount of operating information about individuals, families, communities, and schools from which they can learn about social, business, economic, and environmental behavior.

The system dynamics approach has been successfully applied to behavior in corporations, internal medicine, fisheries, psychiatry, energy supply and pricing, economic behavior, urban growth and decay, environmental stresses, population growth and aging, training of managers, and education of primary and secondary school students.

Nancy Roberts first demonstrated system dynamics as an organizing framework at the fifth and sixth grade levels (Roberts, 1975). Her work (Roberts, 1978) showed the advantage of reversing the traditional educational sequence that normally progresses through five steps:

1) learning facts
2) comprehending meaning
3) applying facts to generalizations
4) analyzing to break material into constituent parts
5) synthesizing to assemble parts into a whole.

Most students never reach that fifth step of synthesis. But, synthesis—putting it all together—should be placed at the beginning of the educational sequence. By the time students are in school they already possess a wealth of observations about family, interpersonal relations, community, and school. They are ready for a framework into which the facts can be fitted. Unless that framework exists, teaching still more facts loses significance.

---

special attention to use in secondary schools. For more advanced professional use, software exists for system dynamics modeling, such as DYNAMO™ from Pugh-Roberts and Vensim™ from Ventana Systems.

In his penetrating discussion of the learning process, Bruner states, "the most basic thing that can be said about human memory... is that unless detail is placed into a structured pattern, it is rapidly forgotten" (Bruner, 1963, p. 24). For most purposes, such a structure is inadequate if it is only a static framework. The structure should show the dynamic significance of the detail—how the details are connected, how they influence one another, and how past behavior and future outcomes arise from decision-making policies and their interconnections.

System dynamics can provide that dynamic framework to give meaning to detailed facts. Such a dynamic framework provides a common foundation beneath mathematics, physical science, social studies, biology, history, and even literature.

In spite of the potential power of system dynamics, it could well be ineffective if introduced alone into a traditional educational setting in which students passively receive lectures. System dynamics can not be acquired as a spectator sport any more than one can become a good basketball player by merely watching games. Active participation instills the dynamic paradigm. Hands-on involvement is essential to internalizing the ideas and establishing them in one's own mental models. But traditional class rooms lack the intense involvement so essential for deep learning.

2.3. Learner-Centered Learning

Those who have experienced the excitement and intensity of a research laboratory know the involvement accompanying new discoveries. Why should not students in their formative years experience similar exhilaration from exploring new challenges? That sense of challenge exists when a classroom operates in a "learner-centered-learning" mode.

Learner-centered learning, is a term I first encountered from Mrs. Kenneth Hayden of Ideals Associated.[2] It substantially alters the role of a teacher. A teacher is no longer a dispenser of knowledge

---

[2] Ideals Associated, 2570 Avenida de Maria, Tucson, AZ 85718 USA is a small foundation that for two decades has fostered an approach to learning that enlists students themselves in an active participation that contributes to the momentum of the educational process.

addressed to students as passive receptors. Instead, where small teams of students explore and work together and help one another, a "teacher" becomes a colleague and participating learner. Teachers set directions and introduce opportunities. Teachers act as guides and resource persons, not as authoritarian figures dictating each step of the educational process. The relationship is more like being a thesis adviser than a lecturer.

3. The Gordon Brown Influence

The thread leading to system dynamics started when I was introduced to feedback systems in the early 1940s by Gordon S. Brown, then director of the MIT Servomechanisms Laboratory. Later, Brown became head of the MIT Electrical Engineering Department and then Dean of Engineering before retiring in 1973. In the late 1980s, he completed the circle he had originally launched by picking up system dynamics and introducing it into the Orange Grove Junior High School in Tucson, Arizona (Brown, 1992).

Friends of Brown have established the “Gordon Stanley Brown Fund,” administered through the System Dynamics Society. The fund will support released time and summer time for teachers who have applied system dynamics, so that they can put into transmittable and usable form the materials and methods that can help others. It will also support communication of experiences that did not meet expectations so that others can be forewarned of difficulties and paths to be avoided.

Brown describes his role as the “citizen champion” engaged in drawing all participants in the school system together in their search for a new kind of education:

> "the use of computers in the classroom (not in a computer lab) has, for us in Tucson, resulted in a very unique learning environment... (students) learn what they need to know as the teacher guides them in conducting a simulation in class. They work in groups, two or three to a computer—certainly not one per computer—and thereby help one another. Dr. Barry Richmond says that this situation, in effect, multiplies the number of teachers by the number of students. Before doing a simulation the students spend several class periods gathering information about the topic; they take notes during lectures, learn about a library and read references, and, working as a

> group, plan the simulation. By working this way Draper's students do not merely try to remember the material for a test but actually have to use it in a project simulating real life situations. This has led us to identify a new teaching paradigm which we define as SYSTEM THINKING with LEARNER-CENTERED LEARNING." (Brown, 1990)

Gordon Brown started by loaning the STELLA software for a weekend to Frank Draper, an 8th grade biology teacher. Draper returned with the comment, “This is what I have always been looking for, I just did not know what it might be.” At first, Draper expected to use system dynamics and computer simulation in one or two classes during a term. Then he found they were becoming a part of every class. With so much time devoted to system dynamics and simulation, he feared he would not have time to cover all the required biology. But, two thirds of the way through the term, Draper found he had completed all the usual biology content. He had a third of the term left for new material. The more rapid pace had resulted from the way biology had become more integrated and from the greater student involvement resulting from the systems viewpoint. Also, much credit goes to the “learner-centered learning” organization of student cooperative study teams within the classroom. To quote Draper, “There is a free lunch.” He writes of his classroom experience:

> "Since October 1988 our classrooms have undergone an amazing transformation. Not only are we covering more material than just the required curriculum, but we are covering it faster (we will be through with the year's curriculum this week and will have to add more material to our curriculum for the remaining 5 weeks) and the students are learning more useful material than ever before. 'Facts' are now anchored to meaning through the dynamic relationships they have with each other. In our classroom students shift from being passive receptacles to being active learners. They are not taught about science per se, but learn how to acquire and use knowledge (scientific and otherwise). Our jobs have shifted from dispensers of information to producers of environments that allow students to learn as much as possible.
>
> "We now see students come early to class (even early to school), stay after the bell rings, work through lunch and work at home voluntarily (with no assignment given). When we work on a systems project—even when the students are working on the book research leading up to system work—there are

> essentially no motivation/discipline problems in our classrooms." (Draper, 1989)

A dynamic framework can even organize the study of literature (Hopkins, 1992). Classes taught by Pamela Hopkins are from an underprivileged section of the city and many had been labeled as slow learners. Simulation opened the door to a new way of capturing student interest and involvement. In a seminar for teachers taught by Barry Richmond and Steve Peterson of High Performance Systems, she participated in developing a model of psychological dynamics in Shakespeare's *Hamlet*:

> "(when we used) a STELLA model which analyzed the motivation of Shakespeare's Hamlet to avenge the death of his father in HAMLET... The students were engrossed throughout the process... The amazing thing was that the discussion was completely student dominated. For the first time in the semester, I was not the focal point of the class. I did not have to filter the information from one student back to the rest of the class. They were talking directly to each other about the plot events and about the human responses being stimulated. They talked to each other about how they would have reacted and how the normal person would react. They discussed how previous events and specific personality characteristics would affect the response to each piece of news, and they strove for precision in the values they assigned for the power of each event. My function became that of listening to their viewpoints and entering their decisions into the computer. It was wonderful! It was as though the use of precise numbers to talk about psychological motives and human responses had given them power, had given them a system to communicate with. It had given them something they could handle, something that turned thin air into solid ground. They were directed and in control of learning, instead of my having to force them to keep their attention on the task." (Hopkins, 1990)

Several months after the experience related in the Hopkins article, I received a letter from Louise Hayden, director of Ideals Associated:

> "Pam and I are so pleased and surprised at the ongoing involvement and depth of interest the high school students in her workshop of last June are showing. They are meeting with her weekly after school, eager to learn more about system dynamics and to use their advances to help younger students

> learn. They are arousing considerable teacher interest as they try to use causal loops in all their class rooms. Information is flowing upward—and from students who varied in achievement from high to very low.
>
> We attribute the enthusiasm and commitment to their sense of the potential of systems thinking, and to the feelings of self-worth from being regarded as educational consultants. It is their first experience in learner-centered learning. This may well be the first time they have considered themselves a responsible part of the social system." (Hayden, 1990)

Many people assume that only the "best" students can adapt to the style of education here suggested. But who are the best students? Results so far indicate no correlation between students who do well in this program and how they had been previously labeled as fast or slow learners. Some of the so-called slow learners find traditional education lacks relevance. They are not challenged. In a different setting they come into their own and become leaders. Some of the students previously identified as best are strong on repeating facts in quizzes but lack an ability to synthesize and to see the meaning of their facts. Past academic record seems not to predict how students respond to this new program.

4. The Present Status

System dynamics is developing rapidly, but does not yet have widespread public visibility. The international System Dynamics Society was formed in 1985. Membership has grown to some 300. Annual international meetings have been held for fifteen years in locations as widely spread as Norway, Colorado, Spain, China, California, Germany, and Thailand. System dynamics books and papers are regularly translated into many languages including Russian, Japanese, and Chinese.

Six hundred people attended a recent conference on systems thinking organized by Pegasus Communications.[3]

After 30 years of development, several dozen books present the theory, concepts, and applications of system dynamics. Some have exerted surprising public impact (Forrester, 1969; Forrester, 1971).

---

[3] Pegasus Communications, 1696 Massachusetts Ave., Cambridge, MA 02138, publisher of the monthly *The Systems Thinker.*

The Limits to Growth book (Meadows, et al., 1972), showing interplay among population, industrialization, hunger, and pollution, has been translated into some 30 languages and has sold over three million copies. Such wide-spread readership of books based on computer modeling testifies to a public longing to understand how present actions influence the future. *Limits to Growth* has been recently updated as *Beyond the Limits*. (Meadows, et al., 1992)

Early leaders in system dynamics were educated at M.I.T. But competence is now appearing in many places. Talent exists on which to build a new kind of education, even though system dynamics is so broadly applicable throughout physical, social, biological, and political systems that the present small number of experts are thinly dispersed over a wide spectrum of activities.

System dynamics is now becoming well established in some thirty junior and senior high schools. Several hundred schools have started exploratory activity.

Part of the educational emphasis focuses on "generic structures." A rather small number of relatively simple structures appear repeatedly in different businesses, professions, and real-life settings. Students can transfer insights from one setting to another. For example, one of Draper's eighth grade students grew bacteria in a culture dish, then looked at the same pattern of environmentally limited growth through computer simulation. From the computer, the student looked up and observed, "This is the world population problem, isn't it?" Such transfer of insights from one setting to another will help to break down barriers between disciplines. It means that learning in one field becomes applicable to other fields.

There is now promise of reversing the trend of the last century toward ever greater fragmentation in education. There is real hope of moving back toward the "Renaissance man" idea of a common teachable core of broadly applicable concepts. We can now visualize an integrated, systemic, educational process that is more efficient, more appropriate to a world of increasing complexity, and more supportive of unity in life.

Several high schools, curriculum-development projects, and colleges are using a system dynamics core to build study units in mathematics, science, social studies, and history. But such programs have not yet reached the point of becoming fully integrated educational structures.

The most advanced United States experiment in bringing system dynamics and learner-centered learning together into a more powerful educational environment appears to be in the Catalina Foothills School District of Tucson, Arizona. In that community the necessary building blocks for successful educational innovation have come together. Progress in that school system rests on:

1) fundamental new concepts of education,

2) a receptive community,

3) talented teachers who are willing to try unfamiliar ideas and who are at ease in the nonauthoritarian environment of learner-centered learning,

4) a school administration that is applying a systems viewpoint in seeking total quality, mutual understanding, and continuous improvement,

5) a supportive school board,

6) and a "citizen champion" who, without a personal vested interest in the outcome except for a desire to facilitate improvement in education, has helped by inspiring teachers, finding funding, arranging for computers, and, above all, facilitating convergence of political differences in the community.

The Catalina Foothills district did not have its own high school. Students went into the Greater Tucson system. After seeing the impact on several hundred students of the new educational philosophy embedded in the Orange Grove junior high school, parents became reluctant to have children revert to a traditional high school. The District in 1990 voted a $30 million bond issue to

create a high school in the educational pattern that had been pioneered in the junior high school.

In March 1992 a "Systems Thinking in Education Conference" was held in Tucson. Two hundred people attended six plenary sessions and seven sequences of parallel sessions. Enthusiasm was high with reports of systems activity from fourth to twelfth grades.

The Educational Testing Service has established the Systems Thinking and Curriculum Innovation Network Project (STACI) involving about a dozen schools to explore the use of system dynamics in classrooms.[4]

> "The approach consists of three separate but interdependent components: system dynamics, the theoretical perspective; STELLA, a simulation modeling software package; and the Macintosh computer.... The STACI Project is an implementation and research effort that examines the cognitive and curricular impact of using the systems thinking approach in pre-college instruction.... Because it is critical for teachers to be able to seek assistance easily from experts and other teachers, an electronic mail network using AppleLink has been established among the schools... the project focuses on the examination of cognitive and learning outcomes.... the systems approach is being used in courses that reach a range of students. Contrary to initial beliefs, the perspective can be used to facilitate instruction of low- as well as high-ability students.... from initial results, the use of the systems approach for less able learners seems to be yielding promising outcomes." (Mandinach and Cline, 1989)

Some other countries are moving ahead rapidly in using system dynamics as a foundation for an educational system below the college level. The Scandinavian countries are working together. Davidsen[5] describes their guiding philosophy:

> "System dynamics is a method, used in the study of complex, dynamic systems. Its pedagogical qualities are under investigation in several countries.... our final goal is to provide

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[4] Ellen B. Mandinach and Hugh F. Cline, Educational Testing Service, Princeton, NJ 08541, USA.

[5] Pål I. Davidsen, Department of Information Science, University of Bergen, Thormøhlensgt 55, N-5006 Bergen, NORWAY.

> our students with an effective way of thinking about complex, dynamic systems. Thus we want to change their cognitive style. Far beyond establishing a basis of values, attitudes, and factual knowledge, our schools significantly influence the way each one of our students will be thinking.... we encourage our students to become critical users of models and to question assumptions underlying models, used for professional and political purposes. They should gain respect for real life complexity and variety and question simple solutions to complex problems.... In Norwegian and Nordic schools, we have chosen to utilize the conceptual framework offered by system dynamics for our educational purposes... When we have established an understanding of the basic dynamic processes, we are ready to address ourselves to reality. Then we will have to tackle systems of far greater complexity, typically characterized by feedback, delays, nonlinearities, and noise.... (pursuing) causal chains until they close upon each other, leads us to a multi-disciplinary approach.... Academic boundaries no longer constitute the boundaries of our imagination or our investigation. Historic and economic considerations are merged with physics and chemistry in our study of ecological issues." (Davidsen, 1990)

I have received a German book detailing their experimental use of system dynamics and the STELLA software for teaching high school physics (Bethge and Schecker, 1992).[6]

Several schools are making good progress with system dynamaics and learner-centered learning below the level of junior high school students. In the public schools of Ridgewood, New Jersey, Timothy Lucas and Rich Langheim have been focusing on first through fifth grades.

5. The Future

Over the next several decades, an improved kind of education can evolve. The growing frustrations in corporate, economic, social, political, and international organizations demonstrate the need for better understanding. The basis now exists for a far more effective educational process. But a vast amount of work remains to build on the present foundation. Adequate educational materials are yet to

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[6] Horst Schecker, Institute of Physics Education, Department of Physics, University of Bremen, Postbox 330440, D-2800 Bremen 33, GERMANY.

be developed. One book was written especially for high schools (Roberts, et al., 1983). Although not written specifically for pre-college use, other introductory system dynamics books are available (Forrester, 1961; Forrester, 1968; Forrester, 1969; Forrester, 1975; Goodman, 1974; Richardson and Pugh, 1981). Nevertheless, the published material does not yet adequately convey the background, simulation models, related teacher-support materials, and guidance on teaching methods. Much material already exists in places ranging from files at MIT to work of teachers who are pioneering in systems thinking and learner-centered learning. But most existing materials are not now widely accessible.

No network has existed before 1992 for interchanging information among all interested innovators in pre-college education. But that missing link is now being remedied by a new office, the Creative Learning Exchange,[7] established by John R. Bemis, to receive, print, and distribute system dynamics educational materials. That office will maintain communications among schools, encourage training seminars for teachers, advise teachers in preparing new materials for wider dissemination, and assist in maintaining the integrity and practicality of the system dynamics content of emerging curricula.

A group of students in the MIT Undergraduate Research Opportunities Program are working with me to develop educational materials for use in schools. They are working with teachers in the Cambridge Rindge and Latin High School to test materials and acquire experience in the real world of teachers and classrooms. In a current project they are creating a “Road Maps” agenda for self study in systems dynamics as applied to education. The agenda is a guide to using available published material, which will be supplemented by papers written by the students and some selections from more than 4000 memoranda in the files of the MIT System Dynamics Group. The material from this “System Dynamics in Education Project” will be distributed through the Creative Learning Exchange. This project is creating examples of quality systems work to help establish standards for educational programs. It is not the intention to create entire unified courses of

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[7] Ms. Lees Stuntz, Executive Director, Creative Learning Exchange, 1 Keefe Road, Acton, MA 01720, USA, tel: 508-287-0070, fax: 508-287-0080

study, but rather to generate examples that teachers can use in a wide range of educational settings.

Many private individuals are moving ahead to provide financial assistance to the development of systems education, rather than waiting for public political organizations to innovate. Private support can operate with a freedom and a clarity of purpose that is seldom possible with the bureaucratic processes of government and large foundations.

I believe that the immediate goal is to reach a point where at least twenty schools have been unambiguously successful and have achieved self-sustaining momentum. Thus far, many schools are making good progress but are still relying on outside guidance to assist when barriers are encountered. Some are beginning to emerge from such dependence on external assistance, but there are not yet sufficient examples of on-going, independent successes to over-shadow failures that are almost certain to occur. Preliminary results from system dynamics in primary and secondary schools show such promise that too many schools without the ingredients for success may begin, then fail. As a result, systems education might be discredited unless sufficient successes have been demonstrated to sustain the hope and promise of a more effective education.

The politics and processes of moving from a traditional school to a radically different style of education must be better understood. No one yet knows what percentage of present teachers can make the transition from traditional teacher-dominated classrooms to the free-wheeling, research atmosphere of a learner-centered classroom. To some teachers, the transition is threatening. Little is known about how to evaluate students coming out of this different kind of education. Standardized evaluation probably is not desirable or possible in a program that emphasizes individual development and diversity.

Creating a new kind of education will take substantial time. Planning and funding should provide for long-run continuity based on step-by step progress. Funding will be needed for developing materials, retraining teachers, and launching demonstration schools.

A core of system dynamics experts should monitor progress and continually nudge the activities toward higher quality. There are many ways in which erroneous concepts can creep into such an education. If such fallacies go uncorrected, systems education may be perceived as superficial and unsound and lead to negative backlash. Contributions are essential from experienced teachers, who understand the problems and opportunities in class rooms, and can translate ideas into effective teaching materials. "Citizen champions" can serve an important role to draw together teachers, school administrators, school boards, parents, concerned public, and governmental officials. Such influential groups are beginning to coalesce around the combined concepts of system dynamics and learner-centered learning.

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COUNTERINTUITIVE BEHAVIOR OF SOCIAL SYSTEMS[1]

by

JAY W. FORRESTER[2]

ABSTRACT

This paper addresses several social concerns: population trends; quality of urban life; policies for urban growth; and the unexpected, ineffective, or detrimental results often generated by government programs.

Society becomes frustrated as repeated attacks on deficiencies in social systems lead only to worse symptoms. Legislation is debated and passed with great hope, but many programs prove to be ineffective. Results are often far short of expectations. Because dynamic behavior of social systems is not understood, government programs often cause exactly the reverse of desired results.

The field of system dynamics now can explain how such contrary results happen. Fundamental reasons cause people to misjudge behavior of social systems. Orderly processes in creating human judgment and intuition lead people to wrong decisions when faced with complex and highly interacting systems. Until we reach a much better public understanding of social systems, attempts to develop corrective programs for social troubles will continue to be disappointing.

This paper cautions against continuing to depend on the same past approaches that have led to present feelings of frustration. New methods developed over the last 30 years will lead to a better understanding of social systems and thereby to more effective policies for guiding the future.

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[1] This paper was first copyrighted © 1971 by Jay W. Forrester. It is based on testimony for the Subcommittee on Urban Growth of the Committee on Banking and Currency, U.S. House of Representatives, on October 7, 1970. The original text appeared in the January, 1971, issue of the *Technology Review* published by the Alumni Association of the Massachusetts Institute of Technology. All figures are taken from *World Dynamics* by Jay W. Forrester, Pegasus Communications, Waltham MA. Updated March, 1995

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I. A NEW APPROACH TO SOCIAL SYSTEMS

The human mind is not adapted to interpreting how social systems behave. Social systems belong to the class called multi-loop nonlinear feedback systems. In the long history of evolution it has not been necessary until very recent historical times for people to understand complex feedback systems. Evolutionary processes have not given us the mental ability to interpret properly the dynamic behavior of those complex systems in which we are now imbedded.

The social sciences, which should be dealing with the great challenges of society, have instead retreated into small corners of research. Various mistaken practices compound our natural mental shortcomings. Computers are often being used for what computers do poorly and the human mind does well. At the same time the human mind is used for what the human mind does poorly and computers do well. Furthermore, impossible tasks are attempted while achievable and important goals are ignored.

Until recently, no way to estimate the behavior of social systems existed except by contemplation, discussion, argument, and guesswork. As a way out of the present dilemma, I will sketch here an approach that combines the strength of the human mind and the strength of today's computers. The approach grows out of developments over the last 60 years, in which much of the pioneering research occurred at the Massachusetts Institute of Technology. Concepts of feedback system behavior apply sweepingly from physical systems through social systems. Feedback system ideas were first developed and applied to engineering systems. Understanding closed-loop (feedback) systems has now reached practical usefulness in social systems.

I am speaking of the professional field of *system dynamics*. Applications of system dynamics have been made to corporate policy, behavior of diabetes as a medical system, growth and stagnation of urban areas, and to world forces representing the interactions of population, pollution, industrialization, natural resources, and food.

Development of system dynamics began at M.I.T. in 1956. Continuing development of system dynamics has now spread to many countries. The international System Dynamics Society, and its journal the System Dynamics Review, unify work in the profession. Dozens of books and thousands of papers now exist on system dynamics and its applications.[3]

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[3]. A bibliography with over 3000 entries on computer disks is available from the System Dynamics Society, Milne 300, Rockefeller College, University at Albany, SUNY, Albany, NY 1222, USA

II. COMPUTER MODELS OF SOCIAL SYSTEMS

People would never send a space ship to the moon without first testing prototype models and making computer simulations of anticipated trajectories. No company would put a new household appliance or airplane into production without first making laboratory tests. Such models and laboratory tests do not guarantee against failure, but they do identify many weaknesses which can be corrected before they cause full-scale disasters.

Social systems are far more complex and harder to understand than technological systems. Why then do we not use the same approach of making models of social systems and conducting laboratory experiments before adopting new laws and government programs? The customary answer assumes that our knowledge of social systems is not sufficient for constructing useful models.

But what justification can there be for assuming that we do not know enough to construct models of social systems but believe we do know enough to directly redesign social systems by passing laws and starting new programs? I suggest that we now do know enough to make useful models of social systems. Conversely, we do not know enough to design the most effective social policies directly without first going through a model-building experimental phase. Substantial supporting evidence is accumulating that proper use of models of social systems can lead to far better systems, laws, and programs.

Realistic laboratory models of social systems can now be constructed. Such models are simplifications of actual systems, but computer models can be far more comprehensive than the mental models that would otherwise be used.

Before going further, please realize that there is nothing new in the use of models to represent social systems. Each of us uses models constantly. Every person in private life and in business instinctively uses models for decision making. The mental images in one's head about one's surroundings are models. One's head does not contain real families, businesses, cities, governments, or countries. One uses selected concepts and relationships to represent real systems. A mental image is a model. All decisions are taken on the basis of models. All laws are passed on the basis of models. All executive actions are taken on the basis of models. The question is not to use or ignore models. The question is only a choice among alternative models.

Mental models are fuzzy, incomplete, and imprecisely stated. Furthermore, within a single individual, mental models change with time, even during the flow of a single conversation. The human mind assembles a few relationships to fit the context of a discussion. As debate shifts, so do the mental models. Even when only a single topic is being discussed, each participant in a conversation employs a different mental model to interpret the subject. Fundamental assumptions differ but are never brought into the open. Goals are different but left unstated.

It is little wonder that compromise takes so long. And even when consensus is reached, the underlying assumptions may be fallacies that lead to laws and programs that fail. The human mind is not adapted to understanding correctly the consequences implied by a mental model. A mental model may be correct in structure and assumptions but, even so, the human mind--either individually or as a group consensus--is apt to draw the wrong implications for the future.

Inability of the human mind to use its own mental models becomes clear when a computer model is constructed to reproduce the assumptions contained in a person's mental model. The computer model is refined until it fully agrees with the perceptions of a particular person or group. Then, usually, the system that has been described does not act the way the people anticipated. There are internal contradictions in mental models between assumed structure and assumed future consequences. Ordinarily assumptions about structure and internal governing policies are more nearly correct than are the assumptions about implied behavior.

By contrast to mental models, system dynamics simulation models are explicit about assumptions and how they interrelate. Any concept that can be clearly described in words can be incorporated in a computer model. Constructing a computer model forces clarification of ideas. Unclear and hidden assumptions are exposed so they may be examined and debated.

The primary advantage of a computer simulation model over a mental model lies in the way a computer model can reliably determine the future dynamic consequences of how the assumptions within the model interact with one another. There need be no doubt about a digital computer accurately simulating the actions that result from statements about the structure and policies in a model.

In some ways, computer models are strikingly similar to mental models. Computer models are derived from the same sources; they may be discussed in the same terms. But computer models differ from mental models in important ways. Computer models are stated explicitly. The "mathematical" notation used for describing the computer models is unambiguous. Computer simulation language is clearer, simpler, and more precise than spoken languages. Computer instructions have clarity of meaning and simplicity of language syntax. Language of a computer model can be understood by almost anyone, regardless of educational background. Furthermore, any concept that can be clearly stated in ordinary language can be translated into computer-model language.

There are many approaches to computer models. Some are naive. Some are conceptually inconsistent with the nature of actual systems. Some are based on methodologies for obtaining input data that commit the models to omitting major relationships in the psychological and human areas that we all know to be crucial. With so much activity in computer models and with the same terminology having different meanings in the different approaches, the situation is confusing to a casual observer. The key to success is not in having a computer; the important thing is how the computer is used. With respect to models, the key

is not to computerize a model, but, instead, to have a model structure and decision-making policies that properly represent the system under consideration.

I am speaking here of system dynamics models—the kind of computer models that are only now becoming widely used in the social sciences. System dynamics models are not derived statistically from time-series data. Instead, they are statements about system structure and the policies that guide decisions. Models contain the assumptions being made about a system. A model is only as good as the expertise which lies behind its formulation. A good computer model is distinguished from a poor one by the degree to which it captures the essence of a system that it represents. Many other kinds of mathematical models are limited because they will not accept the multiple-feedback-loop and nonlinear nature of real systems.

On the other hand, system dynamics computer models can reflect the behavior of actual systems. System dynamics models show how difficulties with actual social systems arise, and demonstrate why so many efforts to improve social systems have failed. Models can be constructed that are far superior to the intuitive models in people's heads on which national social programs are now based.

System dynamics differs in two important ways from common practice in the social sciences and government. Other approaches assume that the major difficulty in understanding systems lies in shortage of information and data. Once data is collected, people have felt confident in interpreting the implications. I differ on both of these attitudes. The problem is not shortage of data but rather inability to perceive the consequences of information we already possess. The system dynamics approach starts with concepts and information on which people are already acting. Generally, available information about system structure and decision-making policies is sufficient. Available information is assembled into a computer model that can show behavioral consequences of well-known parts of a system. Generally, behavior is different from what people have assumed.

III. COUNTERINTUITIVE NATURE OF SOCIAL SYSTEMS

Our first insights into complex social systems came from corporate work. Time after time we went into corporations that were having severe and well-known difficulties. The difficulties would be obvious, such as falling market share, low profitability, or instability of employment. Such difficulties were known throughout the company and were discussed in the business press.

One can enter a troubled company and discuss what people see as the causes and solutions to their problems. One finds that people perceive reasonably correctly their immediate environments. They know what they are trying to accomplish. They know the crises which will force certain actions. They are sensitive to the power structure of the organization, to traditions, and to their own personal goals and welfare. When interviewing circumstances are conducive to

frank disclosure, people state what they are doing and can give rational reasons for their actions. In a troubled company, people are usually trying in good conscience and to the best of their abilities to help solve the major difficulties. Policies are being followed that they believe will alleviate the difficulties. One can combine the stated policies into a computer model to show the consequences of how the policies interact with one another. In many instances it emerges that the known policies describe a system which actually causes the observed troubles. In other words, the known and intended practices of the organization are sufficient to create the difficulties being experienced. Usually, problems are blamed on outside forces, but a dynamic analysis often shows how internal policies are causing the troubles. In fact, a downward spiral can develop in which the presumed solutions make the difficulties worse and thereby cause greater incentives to redouble the very actions that are the causes of trouble.

The same downward spiral frequently develops in government. Judgment and debate lead to a program that appears to be sound. Commitment increases to the apparent solution. If the presumed solution actually makes matters worse, the process by which degradation happens is not evident. So, when the troubles increase, the efforts are intensified that are actually worsening the situation.

IV. DYNAMICS OF URBAN SYSTEMS

Our first major excursion outside of corporate policy began in February, 1968, when John F. Collins, former mayor of Boston, became Professor of Urban Affairs at M.I.T. He and I discussed my work in system dynamics and his experience with urban difficulties. A close collaboration led to applying to cities the same methods that had been created for understanding corporations. The resulting model structure represented fundamental urban processes. The computer-model structure showed how industry, housing, and people interact with each other as a city grows and decays. The results are described in my book *Urban Dynamics* (Forrester, 1969).

I had not previously been involved with urban behavior, but the story emerging from the urban model was strikingly similar to what we had seen in corporations. Actions believed to alleviate the difficulties of a city can actually make matters worse. We examined four common programs for improving the depressed nature of central cities. One program was creation of jobs by busing the unemployed to suburban jobs or through governmental jobs as employer of last resort. Second was a training program to increase skills of the lowest-income group. Third was financial aid to depressed cities from federal subsidies. Fourth was construction of low-cost housing. All of these were shown to lie between neutral and highly detrimental regardless of the criteria used for judgment. The four programs range from ineffective to harmful judged either by their effect on the economic health of a city or by their long-range effect on the low-income

population. The results both confirm and explain much of what has been happening over the last several decades in cities.

The investigation showed how depressed areas in cities arise from *excess* low-income housing rather than from a commonly presumed housing shortage. The legal and tax structures have combined to give incentives for keeping old buildings in place. As industrial buildings age, employment opportunities decline. As residential buildings age, they are used by lower-income groups who are forced to use them at higher population densities. Therefore, aging buildings cause jobs to decline and population to rise. Housing, at the higher population densities, accommodates more low-income urban population than can find jobs. A social trap is created where excess low-cost housing beckons low-income people inward because of the available housing. Unemployed people continue coming to a city until their numbers sufficiently exceed the available jobs that the standard of living declines far enough to stop further inflow. Income to the area is then too low to maintain all of the housing. Excess housing falls into disrepair and is abandoned. Extreme crowding can exist in those buildings that are occupied, while other buildings become excess and are abandoned because the economy of the area cannot support all of the residential structures. Excess residential buildings threaten an area in two ways—they occupy land so it cannot be used for job-creating buildings, and they attract a population that needs jobs.

Any change, which would otherwise raise the standard of living, only takes off the economic pressure momentarily and causes population to rise enough that the standard of living again falls to the barely tolerable level. A self-regulating system is thereby at work which drives the condition of the depressed area down far enough to stop the inflow of people.

At any time, a near-equilibrium exists affecting population mobility between different areas of a country. To the extent that there is disequilibrium, it means that some area is slightly more attractive than others and population begins to move in the direction of the more attractive area. Movement continues until rising population drives the more attractive area down in attractiveness to again be in equilibrium with its surroundings. Other things being equal, an increase in population of a city crowds housing, overloads job opportunities, causes congestion, increases pollution, encourages crime, and reduces every component of the quality of life.

A powerful dynamic force establishes equilibrium between all areas in total attractiveness. Any proposed social program should take into account the eventual shifts that will occur in the many components of attractiveness. As used here, attractiveness is the composite effect of all factors that cause population movement toward or away from an area. Most areas in a country have nearly equal attractiveness most of the time, with only sufficient disequilibrium in attractiveness to account for the shifts in population. But areas can have the same composite attractiveness with very different mixes in the components of attractiveness. In one area component A could be high and B low, while the

reverse could be true in another area that nevertheless had the same total composite attractiveness. If a program makes some aspect of an area more attractive than its neighbor's, and thereby makes total attractiveness higher momentarily, population of that area rises until other components of attractiveness are driven down far enough to again establish an equilibrium. Efforts to improve some condition of a city will result primarily in increasing population until other conditions deteriorate to reestablish an equilibrium. The overall condition of urban life, for any particular economic class of population, cannot be appreciably better or worse than that of the remainder of the country to and from which people may come. Programs aimed at improving a city can succeed only if they result in eventually raising the average quality of life for the country as a whole.

V. ON RAISING THE QUALITY OF LIFE

There is substantial doubt that urban programs have been contributing to the national quality of life. Concentrating population in urban locations, undermining the cohesiveness of communities, and making government bureaucracy so big that individuals feels powerless, all reduce the quality of life.

Any proposed program should deal with both the quality of life and the factors affecting population. "Raising the quality of life" means releasing stress from crowding, reducing pollution, alleviating hunger, and treating ill health. But these pressures are the influences that control population movement. If one pressure is relaxed, population will then move in until other pressures rise to stop the inflow. To raise one component of quality of life without intentionally creating compensating counter pressures to prevent a rise in population will be self-defeating.

Consider the meaning of interacting attractiveness components as they affect a depressed ghetto area of a city. First, we must understand the way population density is already being controlled. A set of forces exist that determine why population density is not far higher or lower than it is. There are many possible combinations of forces that an urban area can exert. The particular combination will determine the population mix and the economic health of a city. The depressed areas of most American cities are created by a combination of forces in which there is a job shortage and a housing excess. The availability of housing draws the lowest-income group until they so far exceed the economic opportunities of the area that the low standard of living, the frustration, and the crime rate counterbalance the housing availability. Until the pool of excess housing is reduced, little can be done to improve the economic condition of an inner city. A low-cost housing program alone moves exactly in the wrong direction. It draws more low-income people. It makes the area differentially more attractive to the poor who need jobs and less attractive to those who create jobs. In the new population equilibrium that develops, some characteristics of the social system must counterbalance the additional attractiveness created by the low-

cost housing. That counterbalance is a further decline of the economic condition of the area. Unfortunately, as the area becomes more destitute, pressures rise for still more low-cost housing. The consequence is a downward spiral that draws in the low-income population, depresses their economic condition, prevents escape, and reduces hope. All of this is done with the best of intentions.

My paper, "Systems Analysis as a Tool or Urban Planning" (Forrester, 1969), from a symposium in October, 1969, at the National Academy of Engineering, suggests a reversal of present practice by simultaneously reducing the aging housing in decaying cities and allocating land to income-earning opportunities. The land shifted to industry permits the "balance of trade" of an area to be corrected by allowing labor to create and export products to generate income streams with which to buy the necessities of modern life from the outside. The concurrent reduction of excess housing is absolutely essential. It supplies the land for new job-creating structures. Equally important, the resulting housing shortage creates the population-stabilizing pressure that allows economic revival to proceed without being inundated by rising population. Revival of an urban area can be done without driving the present low-income residents out of an area. Revival policies should create upward economic mobility to convert the low-income population to a self-supporting basis.

Many people, at first, believe these revival policies of less low-cost housing and conditions to favor business to create jobs will not be accepted by elected officials or residents of depressed urban areas. However, some of the strongest support has come from within those groups that are closest to the symptoms, who have lived through the failures of the past, and who must endure present conditions until lasting solutions are found.

The country has slipped into short-term policies for managing cities that have become part of the system that is generating even greater troubles. If we were malicious and wanted to create urban slums, trap low-income people in ghetto areas, and increase the number of people on welfare, we could do little better than follow present policies. The trend toward stressing income and sales taxes and away from the real estate tax encourages old buildings to remain in place and block self-renewal. The concessions in the income tax laws to encourage low-income housing do, in the long run, actually increase the total low-income population. Highway expenditures and government loans for suburban housing have made it easier for higher-income groups to abandon urban areas than to revive them. Expanding the areas incorporated into urban government, in an effort to increase revenue base, has been more than offset by lowered administrative efficiency, more citizen frustration, and the accelerated decline that is triggered in the annexed areas. The belief that more money will solve urban problems has taken attention away from correcting the underlying causes and has instead allowed the problems to grow to the limit of available money.

VI. CHARACTERISTICS OF SOCIAL SYSTEMS

Many characteristics of social systems mislead people. Behavior that people do not anticipate appears in corporate and urban systems and in world-wide pressures now enveloping the planet. Three counterintuitive behaviors of social systems are especially dangerous.

First, social systems are inherently insensitive to most policy changes that people choose in an effort to alter the behavior of systems. In fact, social systems draw attention to the very points at which an attempt to intervene will fail. Human intuition develops from exposure to simple systems. In simple systems, the cause of a trouble is close in both time and space to symptoms of the trouble. If one touches a hot stove, the burn occurs here and now; the cause is obvious. However, in complex dynamic systems, causes are often far removed in both time and space from the symptoms. True causes may lie far back in time and arise from an entirely different part of the system from when and where the symptoms occur. However, the complex system can mislead in devious ways by presenting an apparent cause that meets the expectations derived from simple systems. A person will observe what appear to be causes that lie close to the symptoms in both time and space—shortly before in time and close to the symptoms. However, the apparent causes are usually coincident occurrences that, like the trouble symptom itself, are being produced by the feedback-loop dynamics of a larger system. For example, human suffering in cities is accompanied (some think caused) by inadequate housing. As a result, housing is increased and population rises to defeat the effort. More people are trapped in the depressed urban system. As another example, symptoms of excess population are beginning to overshadow all countries. Symptoms appear as urban crowding and social pressure. Rather than face the rising population problem squarely, governments try to relieve the immediate pressures by more policemen, financial aid, busing to suburban schools, and subsidized health facilities. As a consequence, increasing population reduces the quality of life for everyone.

Second, social systems seem to have a few sensitive influence points through which behavior can be changed. These high-influence points are not where most people expect. Furthermore, when a high-influence policy is identified, the chances are great that a person guided by intuition and judgment will alter the system in the wrong direction. For example, in an urban system, housing is a sensitive control point but, if one wishes to make the city a better place for low-income as well as other people, it appears that low-income housing should be reduced rather than increased. Another example lies in the world-wide problem of rising population and the disparity between the standards of living in the developed and the underdeveloped countries. System dynamics models suggest sensitive control points for increasing the world-wide quality of life exist in the rate of generation of capital investment and in food production, but that expansion of industrialization and food output are the counter productive

directions, both should be restrained. The common answer to world distress has been to increase industrialization and food production, but hope for long-term improvements probably lies in reducing emphasis on both. Contrary to intuitive expectations, the opposite of present practice may actually raise the quality of life and contribute to stabilizing population.

Third, social systems exhibit a conflict between short-term and long-term consequences of a policy change. A policy that produces improvement in the short run is usually one that degrades a system in the long run. Likewise, policies that produce long-run improvement may initially depress behavior of a system. This is especially treacherous. The short run is more visible and more compelling. Short-run pressures speak loudly for immediate attention. However, sequences of actions all aimed at short-run improvement can eventually burden a system with long-run depressants so severe that even heroic short-run measures no longer suffice. Many problems being faced today are the cumulative result of short-run measures taken in prior decades.

VII. A GLOBAL PERSPECTIVE

After discussing social organizations in corporations and cities, the following discussion turns to an even larger world-wide system.

In July, 1970, we held at MIT a two-week international conference on world dynamics. A meeting was organized for the Club of Rome, a private group of about 100 individuals drawn from many countries who had joined together to attempt a better understanding of world problems. Their concern lay in the interactions of population, resources, industrialization, pollution, and world-wide disparities of standard of living. The July agenda included the theory and behavior of complex systems and talks on specific social systems ranging through corporations, commodity markets, biological systems, drug addiction, and growth and decline of cities. A dynamics model, especially prepared for the conference, showed interactions among world population, industrialization depletion of natural resources, agriculture, and pollution. A detailed discussion of the world model appears in *World Dynamics* (Forrester, 1971). That model was refined in the "Project on the Predicament of Mankind" sponsored by the Club of Rome at MIT (Donella Meadows, et al, 1972; Dennis Meadows, et al, 1973 and 1974).

The model of world interactions showed different alternative futures depending on whether social policies are adopted to limit population growth while a high standard of living is still possible or whether the future is ignored until population is suppressed by pollution, crowding, disease, water and resource shortage, social strife, hunger. Malthus dealt only with the latter, but it is possible for civilization to encounter other controlling pressures before a food shortage occurs.

It is certain that resource shortage, pollution, crowding, disease, food failure, war, or some other equally powerful force will limit population and

industrialization if persuasion and psychological factors do not. Exponential growth cannot continue forever. At present population growth rates, there would remain only one square yard per person in less than 400 years. Our greatest challenge is to guide the transition from growth to equilibrium. There are many possible mechanisms for limiting growth. That current growth rates of population and industrialization will stop is inevitable. Unless we choose favorable processes to limit growth, the social and environmental systems by their internal processes will choose for us. The natural mechanisms for terminating exponential growth appear the least desirable. Unless the world understands and begins to act soon, civilization will be overwhelmed by forces we have created but can no longer control.

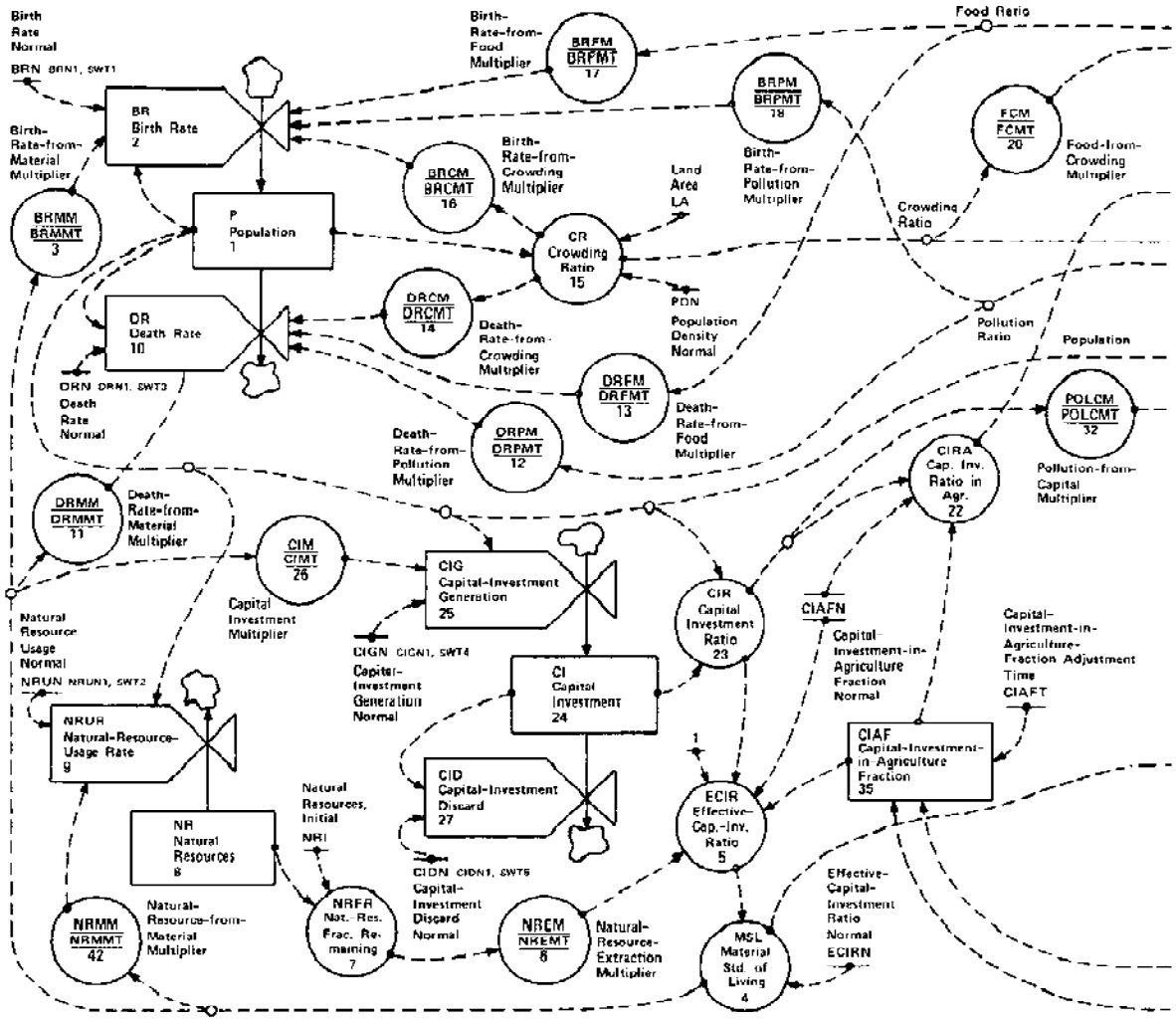
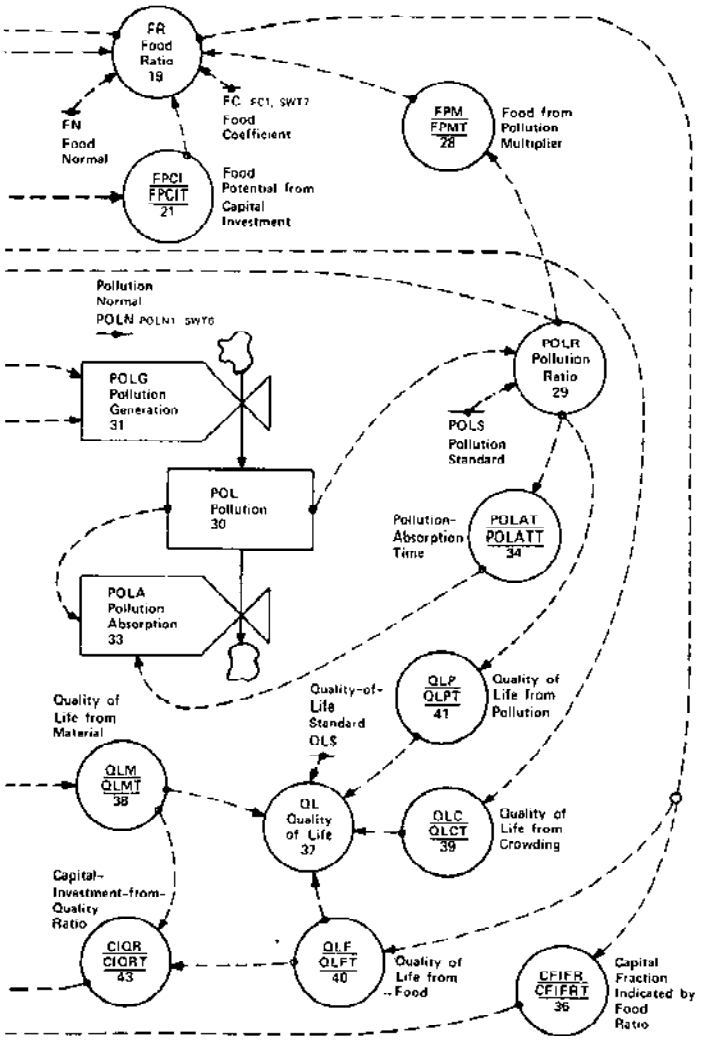


Fig. 1 This world model is a beginning basis for analyzing the effect of changing population and economic growth over the next 50 years. The model includes interrelationships of population, capital investment, natural resources, pollution, and agriculture.

Figure 1 (from Forrester, 1971) interrelates the mutual effects of population, capital investment, natural resources, pollution, and agriculture as the five system "levels" shown by the rectangles. Each level is caused to change by the rates of flow in and out, such as the birth rate and death rate that increase and decrease population. The dotted lines, through intermediate concepts shown at the circles, control the rates of flow. For example, death rate at Symbol 10 depends on population P and the "normal" lifetime as stated by death rate normal (DRN). But death rate also depends on conditions in other parts of the system. From Circle 12



comes the influence of pollution that here assumes death rate will double if pollution becomes 20 times as severe as in 1970; and, progressively, that death rate would increase by a factor of 10 if pollution became 60 times as much. Likewise from Circle 13 the effect of food per capita increases death rate as food becomes less available. The detailed definition of the model states how each rate of flow is assumed to depend on the levels of population, natural resources, capital investment, capital devoted to agriculture, and pollution.

Individually the assumptions in the models are plausible, create little disagreement, and reflect common discussions and assertions about the world system. Each assumption is explicit and can be subjected to scrutiny. From one viewpoint, the system of Figure 1 is very simplified. It focuses on a few major factors and omits most of the substructure of world social and economic activity. But from another viewpoint, Figure 1 is comprehensive and complex. The system is far more complete and the theory described by the accompanying computer model is much more explicit than the mental models that are being used as a basis for world and governmental planning. The model incorporates many nonlinear relationships. The world system shown here exhibits provocative and even frightening possibilities.

Using a system dynamics model, a computer can show how a system, as described for each of its parts, would behave. Given a set of beginning conditions, the computer can calculate and plot the results that unfold through time. The world today is entering a condition in which pressures are rising simultaneously from every one of the influences that can suppress growth—depleted resources (especially water and energy), pollution, crowding,

and insufficient food. It is still unclear which counter force from the environment will dominate if mankind continues along the present path.

Figure 2 shows the mode of behavior of this model system if industrialization is suppressed by falling natural resources. The model system starts with estimates of conditions in 1900. Adjustments were made so that the generated paths pass through the estimated conditions of 1970.

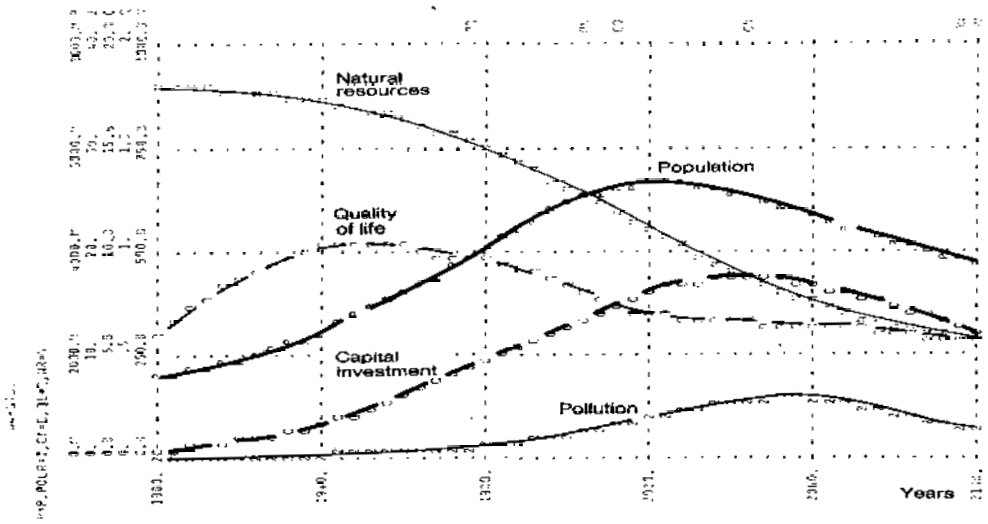


Fig. 2. Basic world model behavior showing the mode in which industrialization and population are suppressed by falling natural resources.

In Figure 2, the quality of life peaks in the 1950s and by 2020 has fallen far enough to halt further rise in population. Declining resources and the consequent fall in capital investment exert further pressure to gradually reduce world population.

VIII. ATTRACTIVE POLICIES CAN CREATE DISASTERS

We may not be fortunate enough to run gradually out of natural resources. Science and technology may very well find ways to use the more plentiful metals and atomic energy so that resource depletion does not intervene. If so, the way then remains open for another growth-resisting pressure to arise. Figure 3 shows what happens if the resource shortage is avoided. Here the only change from Figure 2 is in usage rate of natural resources after year 1970. In Figure 3, resources are used after 1970 at a rate 75 per cent less than assumed in Figure 2. In other words, the standard of living is sustained with a lower drain on the expendable and irreplaceable resources. But the picture is even less attractive! By not running out of resources, population and capital investment are allowed to rise until a pollution crisis is created. Pollution then acts directly to reduce birth rate, increase death rate, and depress food production. Population, which according to

this simple model peaks at the year 2030, has fallen to one-sixth of the peak population within an interval of 20 years—a world-wide catastrophe of a magnitude never before experienced. Should it occur, one can speculate on which sectors of the world population will suffer most. It is quite possible that the more industrialized countries (which are the ones that would have caused such a disaster) would be the least able to survive such a disruption to environment and food supply. Industrialized countries might take the brunt of the collapse.

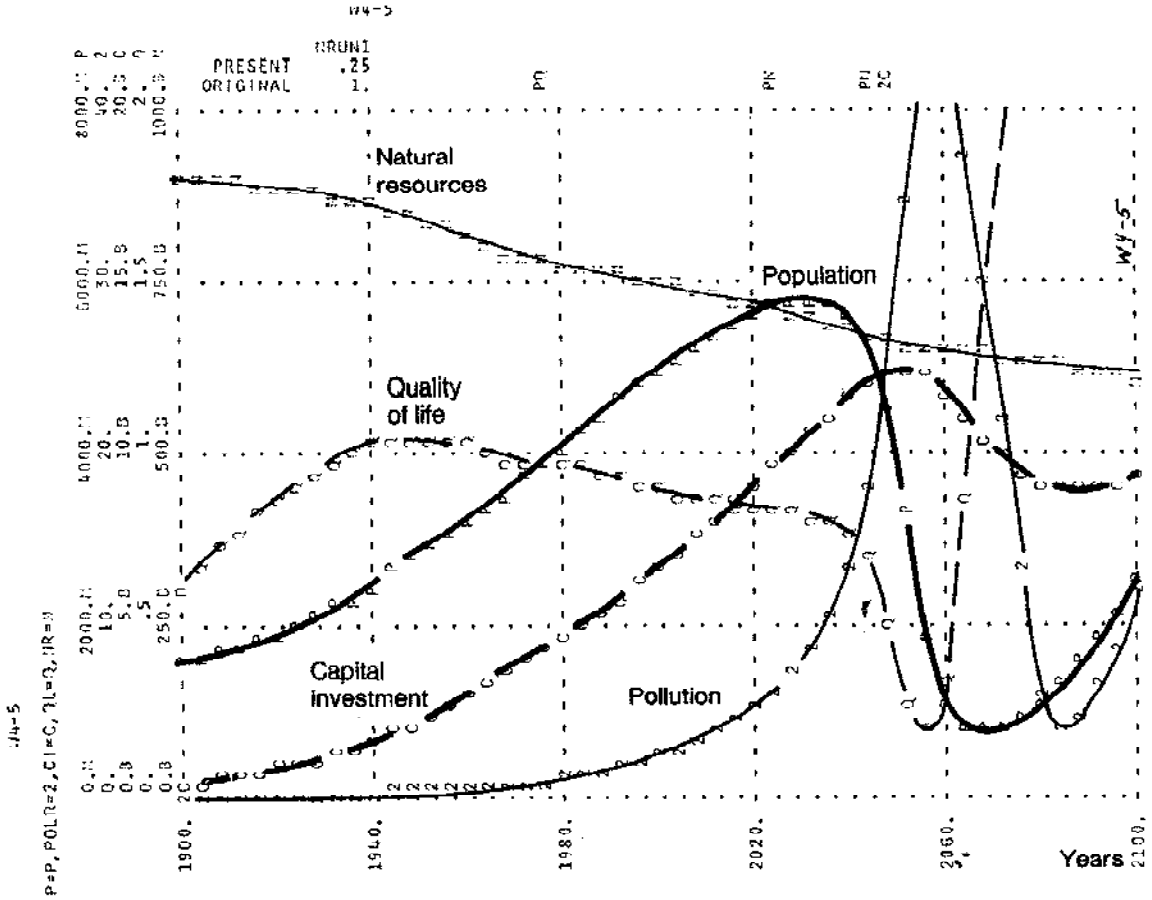


Fig. 3. A pollution crisis is precipitated by lower usage of natural resources. In 1970, natural resource usage is reduced 75 per cent by more effective technology without affecting material standard of living.

Figure 3 shows how a technological success (reducing our dependence on natural resources) can merely save us from one fate only to fall victim to something worse (a pollution catastrophe). Throughout the world an undercurrent of doubt is developing about technology as a savior from social and environmental ills. There is a basis for such doubt. The source of doubt lies not in technology itself but in management of the entire technological-human-political-economic-natural complex.

Figure 3 is a dramatic example of the general process discussed earlier wherein a program aimed at one trouble system results in creating a new set of troubles in some other part of the system. Here, success in alleviating a natural resource shortage throws the system over into a pollution crisis that stops and reverses population expansion. The way in which a solution to one trouble creates a new problem has defeated many past governmental programs and will continue to do so until more effort is devoted to understanding the dynamic behavior of social systems.

Suppose in the basic world system of Figures 1 and 2 we ask how to sustain the quality of life which is beginning to decline after 1950. One attempt, which is popular, might be to increase the rate of industrialization by raising the rate of capital investment. A dynamics model can shed light on such hypothetical questions in a few minutes and at negligible cost. Figure 4 shows what happens if the "normal" rate of capital accumulation is increased by 20 per cent in 1970. Again, the pollution crisis appears. This time the cause is not more efficient use of natural resources but an upsurge of industrialization that overtaxes the environment before resource depletion has a chance to depress industrialization. Again, an "obvious" desirable policy has caused troubles worse than the ones that were originally being corrected.

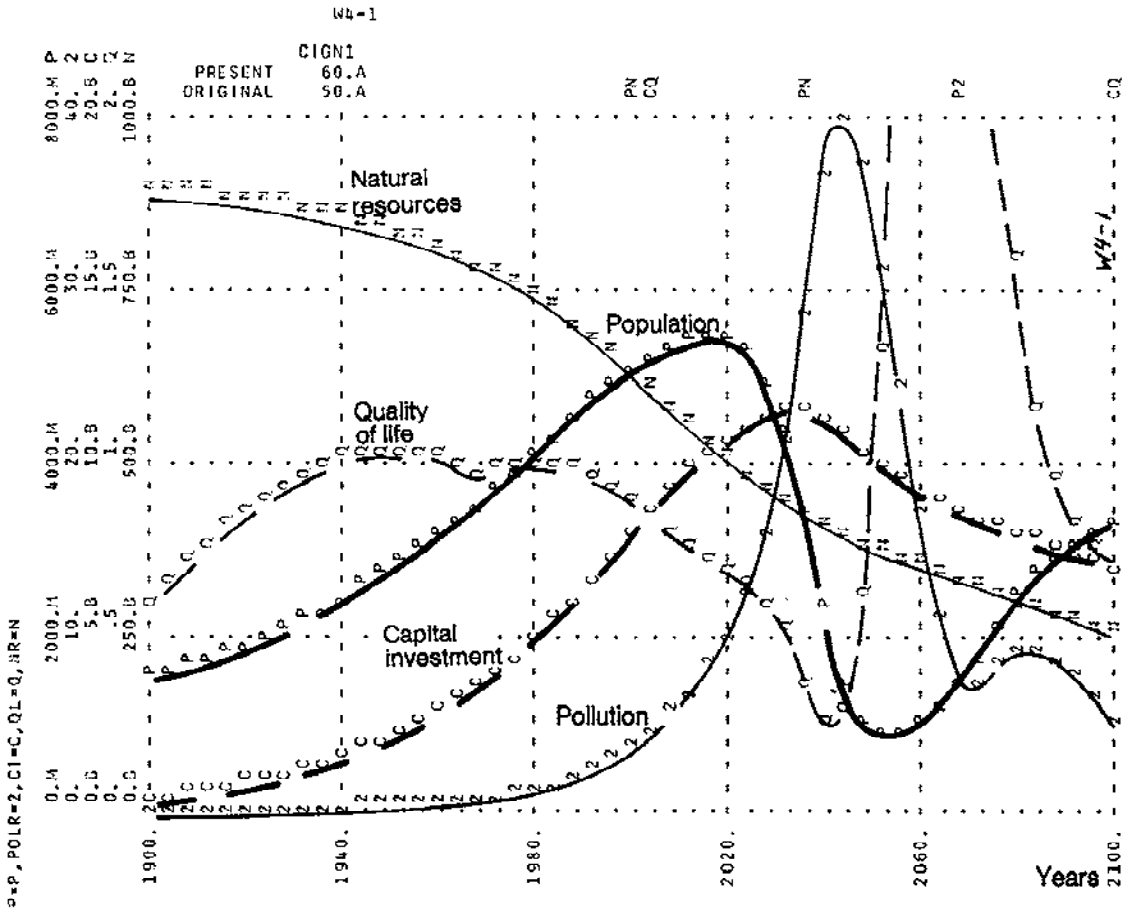


Fig. 4. In 1970, the rate of capital accumulation is increased 20 per cent in an effort to reverse the decline in quality of life. The pollution crisis occurs before natural resources are depleted.

This triggering of a pollution crisis is important, not only for its own message, but because it demonstrates how an apparently desirable change in a social system can have unexpected and even disastrous results.

Figure 4 should make us cautious about rushing into programs on the basis of short-term humanitarian impulses. The eventual result can be anti-humanitarian. Emotionally inspired efforts often fall into one of three traps set for us by the nature of social systems: (1) The programs are apt to address symptoms rather than causes and attempt to operate through points in the system that have little leverage for change; (2) the characteristic of systems whereby a policy change has the opposite effect in the short run from the effect in the long run can eventually cause deepening difficulties after a sequence of short-term actions; and (3) the effect of a program can be along an entirely different direction than was originally expected, so that suppressing one symptom only causes trouble to burst forth at another point.

IX. COMBINATIONS OF "OBVIOUS" POLICIES

Figure 5 retains the 20 per cent additional capital investment rate after 1970 from Figure 4 and in addition explores birth rate reduction as a hope of avoiding crisis. Here the normal birth rate has been cut in half in 1970. (Changes in normal rates refer to coefficients which have the specified effect if all other things remain the same. But other things in the system change and also exert their effects on the actual rates.) The result shows interesting behavior. Quality of life surges upward for 30 years for the reasons that are customarily expected. Although not shown in the figure, food-per-capita grows, material standard of living rises, and crowding does not become as great. But the more affluent world population continues to use natural resources and to accumulate capital plant at about the same rate as in Figure 4. Load on the environment is more closely related to industrialization than to population so the pollution crisis occurs at about the same time as in Figure 4.

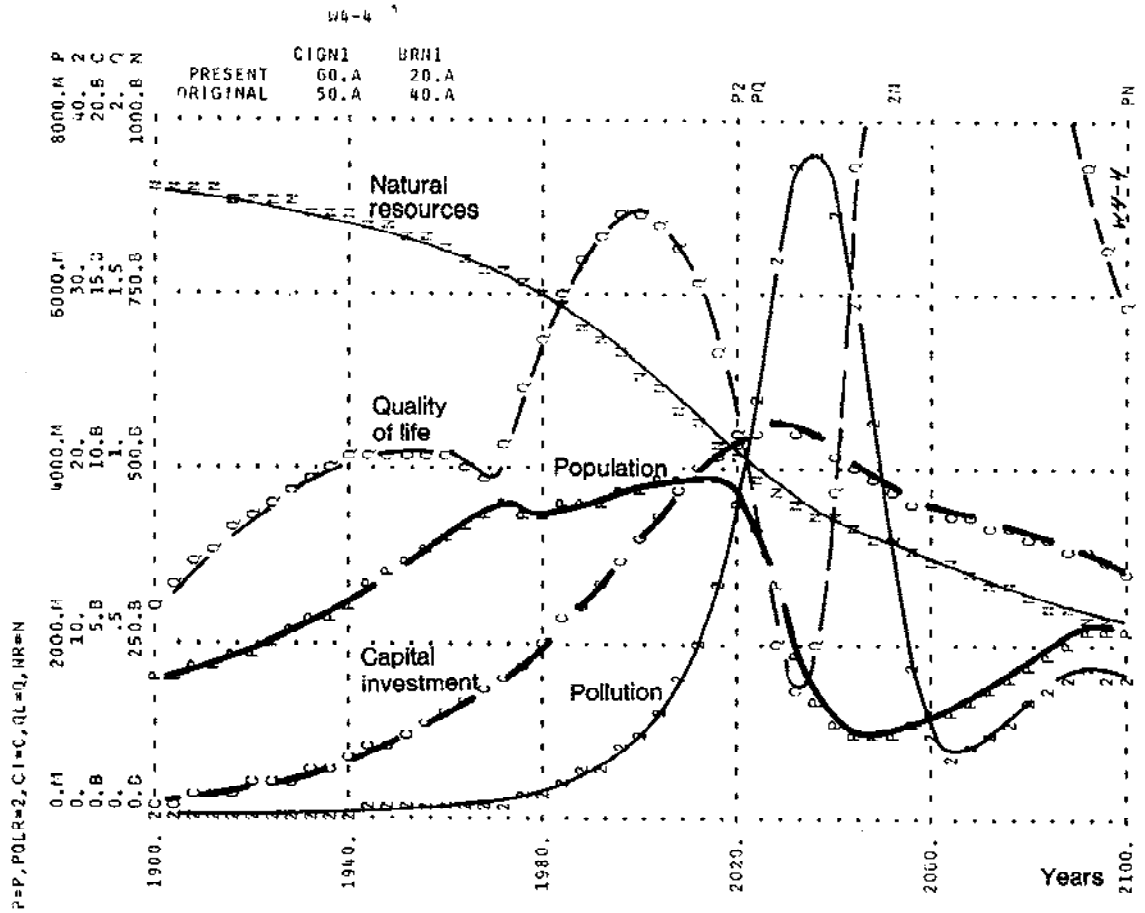


Fig. 5. In 1970 the 20 per cent increase in capital accumulation of Figure 4 is retained and "normal" birth rate is reduced 50 per cent. Capital investment continues to grow until a pollution crisis develops. After an initial decline, population is again pushed up by the rapid rise in quality of life that precedes the collapse.

Figure 5 shows that the 50 per cent reduction in normal birth rate in 1970 was sufficient to start a decline in total population. But the rising quality of life and the reduction of pressures act to start the population curve upward again. Serious questions are raised about the effectiveness of birth control as a means of controlling population when birth rate reduction is the only policy. Secondary consequences of a birth control program will increase influences that raise birth rate and reduce the apparent pressures that require population control. A birth control program which would be effective, all other things being equal, may largely fail because other things will not remain equal. Early success can set in motion forces that eventually defeat the program.

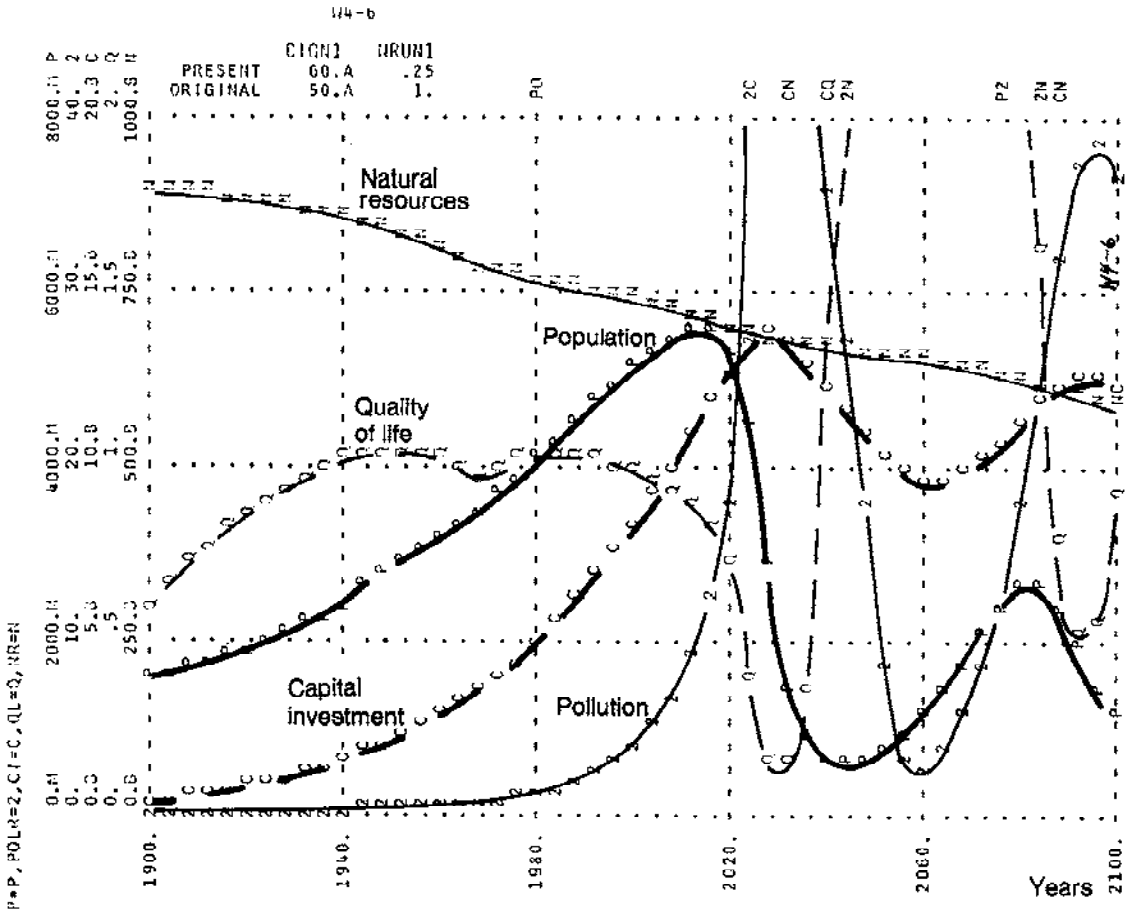


Fig. 6. The 20 per cent increase of capital investment from Figure 4 and the 75 per cent reduction of natural resource usage from Figure 3 are combined.

Figure 6 combines the reduced resource usage rate and increased capital investment rate of Figures 3 and 4. As a result, population collapse occurs slightly sooner and more severely based on the modified system of Figure 6.

Figure 7 examines the result if technology finds ways to reduce pollution generated by industrialization. Pollution rate, other things being the same, is reduced by 50 per cent from that in Figure 6. The result is to postpone the day of reckoning by 20 years and to allow population to grow 25 per cent greater before population collapse occurs. The "solution" of reduced pollution has, in effect, caused more people to suffer the eventual consequences. Figure 7 again reveals the dangers of partial solutions. Actions at one point in a system to relieve one kind of distress produce an unexpected result in some other part of the system. If the interactions are not sufficiently understood, the consequences can be bad as or worse than those that led to the initial action.

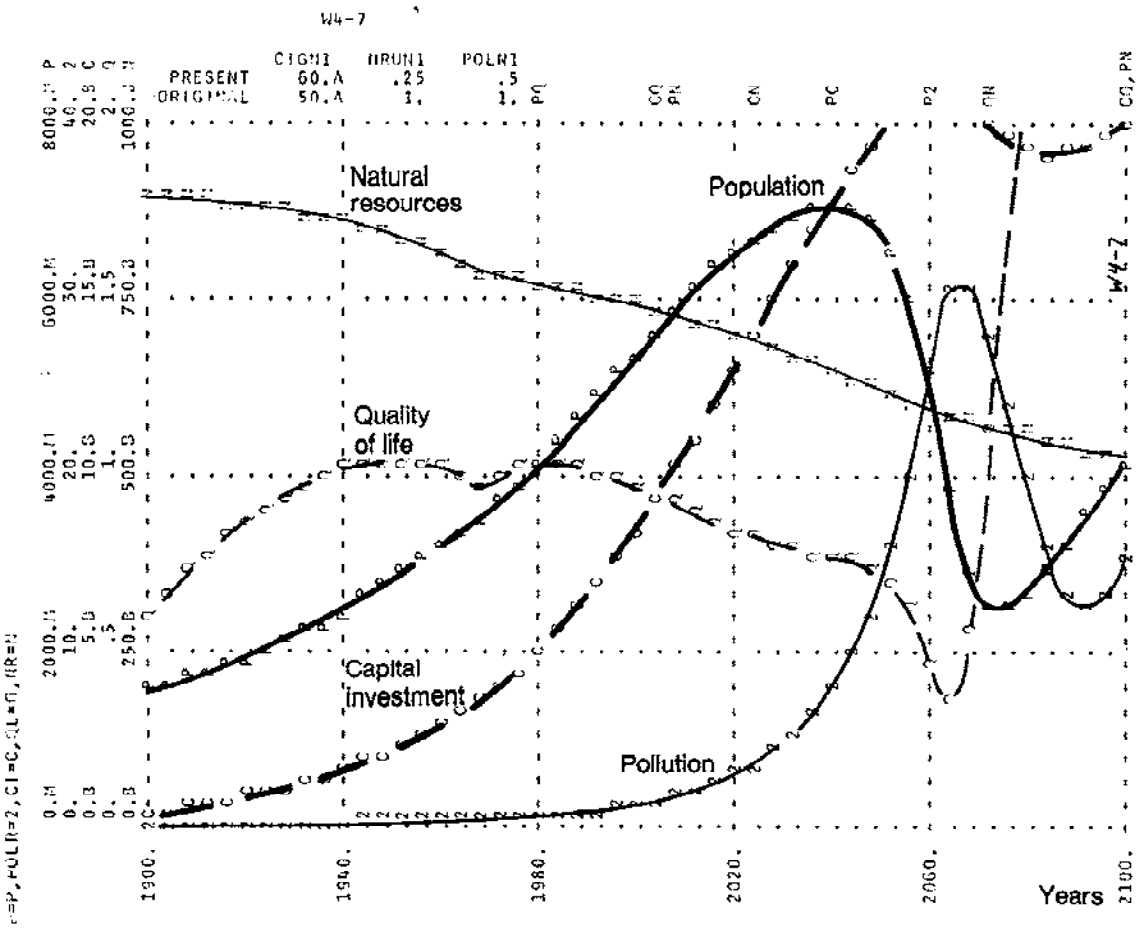


Fig. 7. Increased capital investment rate and reduced natural resource usage from Figure 6 are retained. In addition in 1970 the normal rate of pollution generation is reduced 50 per cent. The effect of pollution control is to allow population to grow 25 per cent further and to delay the pollution crisis by 20 years.

X. AN ALTERNATIVE TO CATASTROPHE

There are no utopias in social systems. There appear to be no sustainable modes of behavior that are free of pressures and stresses. But many modes of behavior are possible and some are more desirable than others. The more attractive behaviors in social systems seem possible only if we act on a good understanding of the dynamic behavior of systems and are willing to endure the self-discipline and short-term pressures that will accompany the route to a desirable future. The world system of Figure 1 can exhibit modes that are more hopeful than the crises of Figure 2 through 7. The more promising modes may require a degree of restraint and dedication to a long-range future that people are not capable of sustaining.

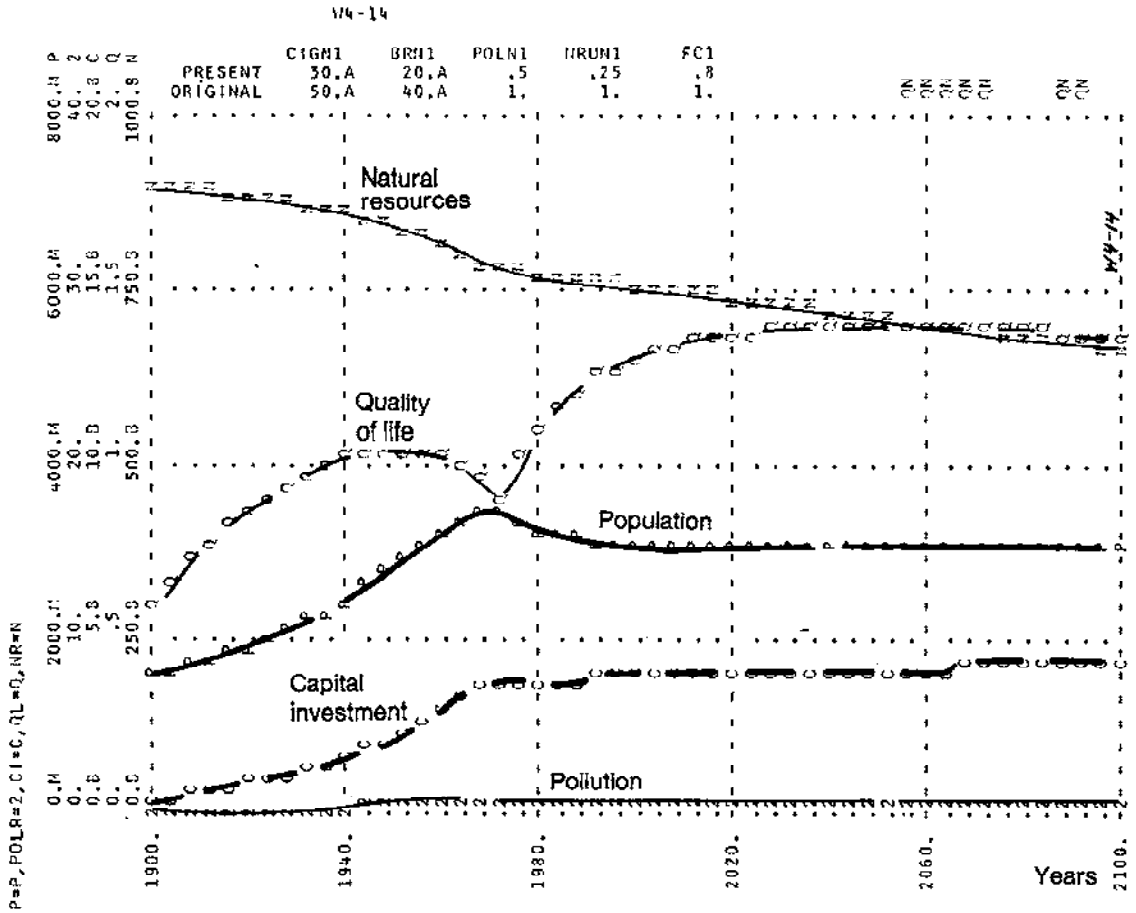


Fig. 8. One set of conditions that establishes a world equilibrium at a high quality of life. In 1970 normal capital investment rate is reduced 40 per cent, normal birth rate is reduced 50 per cent, normal pollution generation is reduced 50 per cent, normal natural resource usage rate is reduced 75 per cent, and normal food production reduced 20 per cent.

Figure 8 shows the world system if several policy changes are adopted together in the year 1970. Population is stabilized. Quality of life rises about 50 per cent. Pollution remains at about the 1970 level. Would such a world be accepted? It implies an end to population and economic growth.

In Figure 8 the normal rate of capital accumulation is reduced 40 per cent from its previous value. The normal birth rate is reduced 50 per cent from its earlier value. The normal pollution generation is reduced 50 per cent from the value before 1970. The normal rate of food production is reduced 20 per cent from its previous value. (These changes in "normal" values are the changes for a specific set of system conditions. Actual system rates continue to be affected by the varying conditions of the system.) However, reduction in investment rate and reduction in emphasis on agriculture are counterintuitive and not likely to be accepted without extensive system studies and years of argument—perhaps more years than are available. The changes in pollution generation and natural resource usage may be easier to understand and to achieve. The severe reduction in world-wide birth rate is the most doubtful. Even if technical and biological methods existed, the improved condition of the world might remove the incentive for sustaining the birthrate reduction.

XI. FUTURE POLICY ISSUES

Changes in the world as a whole bear directly on the future of the United States. American urbanization and industrialization are a major part of the world scene. The United States is setting a pattern that other countries are emulating. However, the example set by the United States is not sustainable. Our foreign policy and our overseas commercial activity are running contrary to changes that should occur in the world system.

The following issues are raised by preliminary studies. Implications for action must be examined more deeply and confirmed by more research into the assumptions about structure and detail of the world system.

(a) Industrialization may be a more fundamentally disturbing force in world ecology than is population. In fact, the population explosion is perhaps best viewed as a result of technology and industrialization. I include medicine and public health improvements as a part of the industrialization that has led to rising population.

(b) Within the next century, the world will be facing a four-pronged dilemma—suppression of modern industrial society by a natural resource shortage, collapse of world population from changes wrought by pollution, population limitation by food shortage, or population control by war, disease, and social stresses caused by physical and psychological crowding.

(c) We may now be living in a "golden age" where, in spite of the world-wide feeling of malaise, the quality of life is, on the average, higher than ever before in history and higher now than the future offers.

(d) Efforts for direct population control may be inherently self-defeating. If population control results, as hoped, in high per capita food supply and material standard of living, these very improvements can generate forces to trigger a resurgence of population growth.

(e) The high standard of living of modern industrial societies results from a production of food and material goods that has been able to outrun rising population. But, as agriculture reaches a space limit, and as both reach a pollution limit, population tends to catch up. Population will then grow until the "quality of life" falls far enough to generate sufficiently large pressures to stabilize population.

(f) There may be no realistic hope for the present underdeveloped countries reaching the standard of living demonstrated by the present industrialized nations. The pollution and natural resource load placed on the world environmental system by each person in an advanced country is probably 10 to 20 times greater than the load now generated by a person in an underdeveloped country. With four times as much population in underdeveloped countries as in the present developed countries, their rising to the economic level of the United States could mean an increase of 40 times the present natural resource and pollution load on the world environment. Noting the destruction that has already occurred on land, in the air, and especially in the oceans, no capability exists for handling such a rise in standard of living for the present total population of the world.

(g) A society with a high level of industrialization may be nonsustainable. Present societies may be self-extinguishing if they exhaust the natural environments on which they depend. Or, if unending substitution for declining natural resources is possible, the international strife over pollution and environmental rights may pull the average world-wide standard of living back to the level of a century ago.

(h) From the long view of a hundred years hence, the present efforts of underdeveloped countries to industrialize along Western patterns may be unwise. They may now be closer to the ultimate equilibrium with the environment than are the industrialized nations. The present underdeveloped countries may be in a better condition for surviving the forthcoming worldwide environmental and economic pressures than are the advanced countries. When one of the several potential forces materializes that is strong enough to cause a collapse in world population, the advanced countries may suffer far more than their share of the decline.

XII. A NEW FRONTIER

It is now possible to take hypotheses about separate parts of a social system, combine them in a computer model, and learn the consequences. The hypotheses may at first be no more correct than the ones we are using in our intuitive thinking. But the process of computer modeling and model testing requires these hypotheses to be stated explicitly. The model comes out of the hazy realm of mental models into unambiguous model statements to which all have access. Assumptions can then be checked against all available information and can be rapidly improved. The great uncertainty with mental models arises from inability to anticipate the consequences of interactions between parts of a system. This uncertainty about future dynamic implications of assumptions in a model is totally eliminated in computer models. Given a stated set of assumptions, the computer traces the resulting consequences without doubt or error. Computer simulation is a powerful procedure for clarifying issues. It is not easy. Results will not be immediate.

We are on the threshold of a great new era in human pioneering. In the past there have been periods characterized by geographical exploration. Other periods have dealt with the formation of national governments. At other times the focus was on the creation of great literature. Most recently we have been through the pioneering frontier of science and technology. But science and technology are now a routine part of life. Science is no longer a frontier. The process of scientific discovery is orderly and organized.

The next frontier for human endeavor is to pioneer a better understanding of environmental, economic, and social systems. The means are available. The task will be no easier than past development of science and technology. For the next 50 years we can expect rapid advance in understanding the complex dynamics of social systems. To do so will require research, development of teaching methods and materials, and creation of appropriate educational programs. The research results of today will find their way into secondary schools just as concepts of basic physics moved from research laboratories to general education over the last century.

What we do today affects our future many decades hence. If we follow intuition and the fallacies embedded in mental models, the trends of the past will continue into deepening difficulty. If we set appropriate research and education programs, which are now possible, we can expect a far sounder basis for future action.

XIII. THE NATION'S ALTERNATIVES

The record to date implies that people accept future growth of United States population as preordained, beyond the purview and influence of legislative action, and as a ground rule which imposes on the nation a task of finding cities in which

a rising future population can live. However, I have described the circular processes in social systems where there is no unidirectional cause and effect. Instead, a ring of actions and consequences close back on themselves. One could say, incompletely, that population will grow and that cities, space, and food must be provided. But one can likewise say, also incompletely, that the provision of cities, space and food will cause population to grow. Population generates pressure for urban growth, but urban pressures help to limit population.

Population grows until stresses rise far enough, which is to say that the quality of life falls far enough, to stop further increase. Everything we do to reduce those pressures causes the population to rise farther and faster and hastens the day when expediencies will no longer suffice. The United States is in the position of a wild animal running from pursuers. We still have some space, natural resources, and agricultural land left within which to maneuver. We can avoid the question of rising population as long as we can flee into this bountiful reservoir that nature provided. But it is obvious that the reservoirs are limited. A wild animal flees until it is cornered, until it has no more space. Then it turns to fight, but it no longer has room to maneuver. The animal is less able to forestall disaster than if it had fought in the open while there was still room to yield and to dodge. The United States is running away from its long-term threats by trying to relieve social pressures as they arise. But if we persist in treating only symptoms and not causes, the result will increase the ultimate threat and reduce our choices for response.

What does this mean? Instead of automatically accepting the need for new towns and the desirability of locating industry in rural areas, we should consider confining our cities. If it were possible to prohibit the encroachment by housing and industry onto even a single additional acre of farm and forest, the resulting social pressure would hasten the day when we would stabilize population. Some European countries are closer to curtailing urban growth than are we.

As I understand it, farmland surrounding Copenhagen cannot be used for either residence or industry until the severest of pressures forces the government to rezone small additional parcels. When land is rezoned, the corresponding rise in land price is fully taxed to remove the incentive for land speculation. The waiting time for an empty apartment in Copenhagen may be years. Such pressures certainly cause the Danes to face the population problem more squarely than do we.

Our greatest challenge now is handling the transition from growth to equilibrium. For a thousand years, tradition has encouraged and rewarded growth. Folklore and success stories praise growth and expansion. But growth is not the path for an unlimited future. Many present stresses in society arise from pressures that accompany the transition from growth into equilibrium. However, the pressures thus far in cities are minor compared to those which are approaching. Population pressures and economic forces in a city that was reaching equilibrium have in the past been able to escape to new land areas.

Escape is becoming less possible. Until now we have had, in effect, an inexhaustible supply of farm land and food-growing potential, but now we are reaching a critical point where, all at the same time, population is overrunning productive land, agricultural land is almost fully employed for the first time, the rise in population is putting more demand on food supplies, and urbanization is pushing agriculture out of the fertile areas into marginal lands. For the first time demand is rising into a condition where supply will begin to fall while need increases. The crossover from plenty to shortage can occur abruptly.

The fiscal and monetary policies of a country form a complex dynamic system of the kind I have been discussing. It is clear that the United States has no established policies to guide interactions between government, growth, unemployment, and inflation. The need to develop long-term policies becomes ever more urgent as the country moves for the first time from a history of growth into the turbulent pressures accompanying the transition from growth to one of the many possible kinds of equilibrium. We need to choose and work toward a desirable kind of equilibrium before we arrive at a point where the system imposes its own choice of regrettable consequences.

In a hierarchy of systems, a conflict exists between goals of a subsystem and welfare of the broader system. The conflict is seen in an urban system. The goal of a city is to expand and to try to raise its quality of life. But growth policies increase population, industrialization, pollution, and demands on food supply. The broader social systems of a country and the world require that goals of urban areas be curtailed and that pressures from such curtailment become high enough to keep urban areas and population within bounds that are satisfactory to the larger system of which cities are a part. If this nation continues to pursue traditional urban goals, the result will deepen distress of the country as a whole and eventually deepen the crisis in cities themselves. We may be at a point where higher pressures in the present are necessary if insurmountable pressures are to be avoided in the future.

I have given a glimpse of the nature of multi-loop feedback systems, a class to which social system belong. I have shown how these complex systems mislead people because intuition has been formed by experienced from simple systems from which we expect behavior very different from that actually possessed by complex systems. The United States is still pursuing programs that will be even more frustrating and futile than many of the past.

But there is hope. It is now possible to gain a better understanding of dynamic behavior in social systems. Progress will be slow. There are many cross-currents in the social sciences which will cause confusion and delay. The system dynamics approach that I have been describing is very different from the emphasis on data gathering and statistical analysis that occupies much of social research. But there have been breakthroughs in several areas. If we proceed expeditiously but thoughtfully, there is a basis for optimism.

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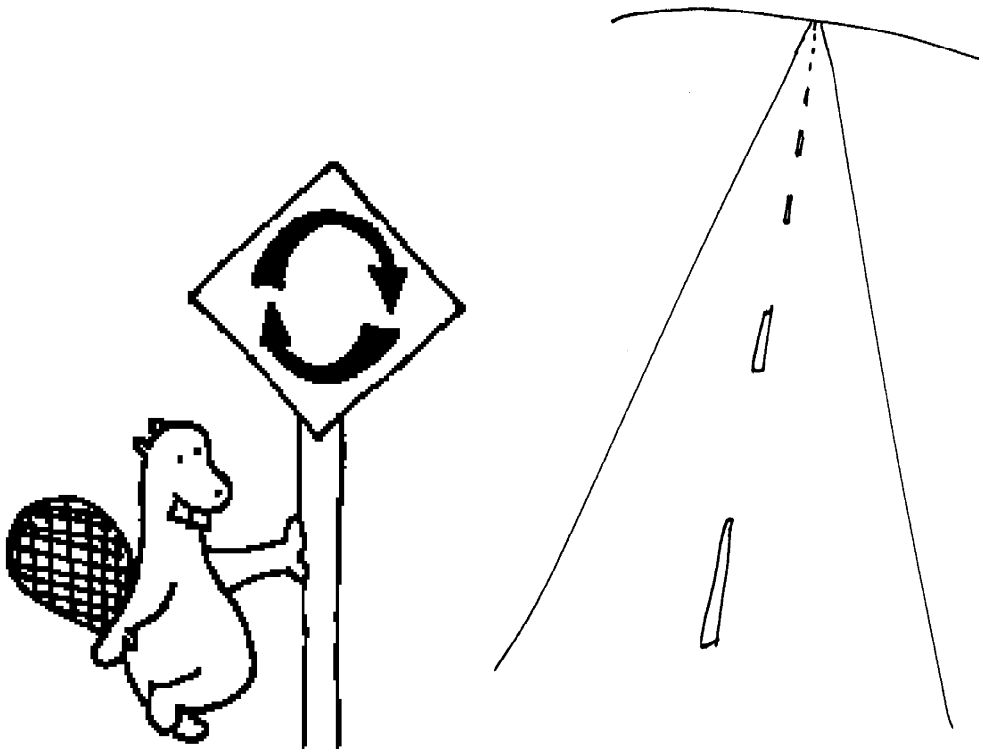
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Road Maps 2

A Guide to Learning System Dynamics



System Dynamics in Education Project

Road Maps 2

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

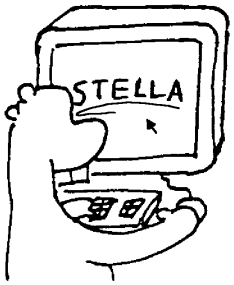
April 23, 1993
Revised June 21, 2000

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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Two!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps Two is the second in the series of chapters in Road Maps. In Road Maps Two, you will begin to model simple systems using STELLA II™ (Systems Thinking Educational Learning Laboratory with Animation) software. STELLA is an easy-to-use software package that allows the user to create and run models on the computer.

Road Maps Two continues to expand on the concept of feedback loops introduced in Road Maps One. It also introduces graphical integration and the concepts of levels and rates, as well as the relationship between feedback loops and levels and rates.

Topics Covered in Road Maps Two

Introduction to Computer Modeling

- *The First Step* (D-4694)
  by Leslie A. Martin
- *Beginner Modeling Exercises* (D-4347-6)
  by Leslie A. Martin

Feedback

- *Introduction to Feedback* (D-4691)
  by Leslie A. Martin

Graphical Integration Exercises

- *Graphical Integration Exercises Part 1: Exogenous Rates* (D-4547-1)
  by Alice Oh

Levels and Rates

- *Introduction to Computer Simulation*, Chapter 13 Levels and Rates
  by Nancy Roberts et al.

Things You'll Need for Road Maps Two

Modeling Software

In order to complete Road Maps Two and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class

processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

Books

You will need chapters from the following book for Road Maps Two. Selected material from this book will also be used in later Road Maps, and is available for download in electronic format from the SDEP server.

Roberts, Nancy, 1983. *Introduction to Computer Simulation.* Portland, Oregon: Pegasus Communications, 562 pp.

How to Use Road Maps Two

Road Maps Two explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let’s get started!

Introduction To Computer Modeling

The first two readings in Road Maps Two will get you started with the basics of modeling in STELLA. You don’t need any experience using STELLA or a computer before. We’ll help you one step at a time, from turning on the computer to actually creating a working model in STELLA.

If you would like to use another modeling software as you study Road Maps, please take some time now to get acquainted with your software. A beginner's tutorial for Vensim PLE[1] is included at the end of Road Maps Two. You are also encouraged to read through the Vensim PLE documentation that is available on the Ventana website. The documentation guides the reader through a series of interesting tutorials, providing an overview of system dynamics and a thorough introduction to the Vensim software. After you have worked through the tutorials or experimented with your software, please proceed with Road Maps by reading *The First Step*, converting between STELLA and your modeling software as you go along.

- *The First Step*[2]

by Leslie A. Martin

The First Step goes over the basics of system dynamics and introduces key terms and concepts. The tutorial will guide you through creating your first models in STELLA and will help you with using the software itself.

Please read *The First Step* now.

After reading *The First Step*...

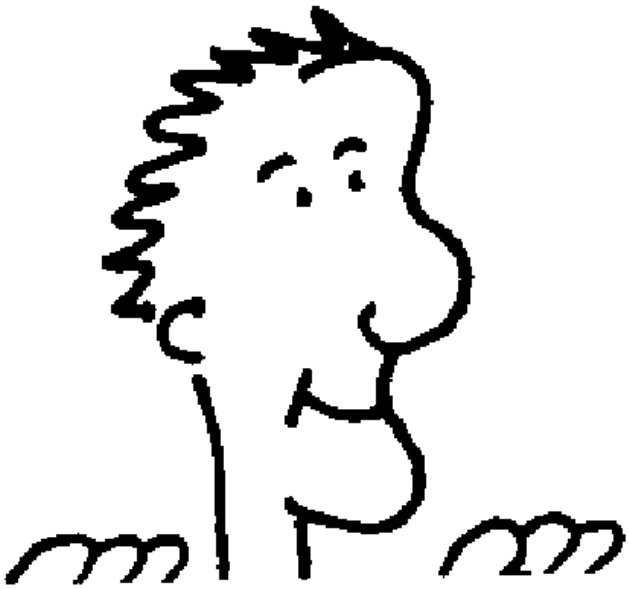
The First Step covered a great deal of important material. It helped you to construct, analyze, and test two one-level multiple-rate systems. It also explained the concepts of stocks and flows and the fundamental role they play in building models of systems.

Before you continue, it is important to note that while *The First Step* uses the terms **stocks** and **flows**, the terms **levels** and **rates** can also be used. 'Stocks' and 'levels' are used interchangeably in system dynamics, as are 'flows' and 'rates'. Also, the two pairs of terms are always used in conjunction with each other. In other words, you should use 'stock' and 'flow' together, or use 'level' and 'rate' together, and not mix the terms, as in 'stock' and 'rate'. Speaking of levels and rates...

---

[1] Nelson Repenning, 1997. *Formulating Models of Simple Systems using VENSIM PLE* (D-4697), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August, 20 pp.

[2] Leslie A. Martin, 1997. *The First Step* (D-4694), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 24, 59 pp. Based on *The First Three Hours* (D-4230-4) by Matthew C. Halbower.



System Principle #2:

Levels and Rates are fundamental to loop substructure.

Look at the final population model in Figures 34 on pages 51 in *The First Step*. Note that the level ("**Population"**) and rates (**"births"** and **"deaths"**) are the primary components of structure. Levels and rates are fundamental to system structure.

System Principle #2 is important to understand because all system dynamics models use levels and rates as loop substructure. The distinction must be made between this principle and System Principle #1, which states that feedback loops are the building blocks of systems. Just as feedback loops are the building blocks of systems, levels and rates are the building blocks of feedback loops.

In order to develop more familiarity with the STELLA software, please take some time now to look over the STELLA tutorial that came included with the software: *Getting Started with STELLA II: A Hands-On Experience*.

The reading uses a predator-prey system to incorporate instructions for building a one-level model, entering the equations, and running the model.[3] The process of building and running the model should be quite similar to what you have just done in *The First Step*. *Getting Started with STELLA II,* however, will also

[3] The iThink tutorial, on the other hand, will guide you through a management flight simulator. The iThink software is the corporate version of the STELLA software.

introduce several unique features of the software, such as using and creating high-level maps and sub-models.

In addition to the tutorial, High Performance Systems also provides several folders of systems modeled with the STELLA II software. A wide range of models are also included with Vensim PLE. We encourage you to look at the models to see some of the many applications of system dynamics.

The next paper will help you develop your modeling skills.

- *Beginner Modeling Exercises* [4]

by Leslie A. Martin

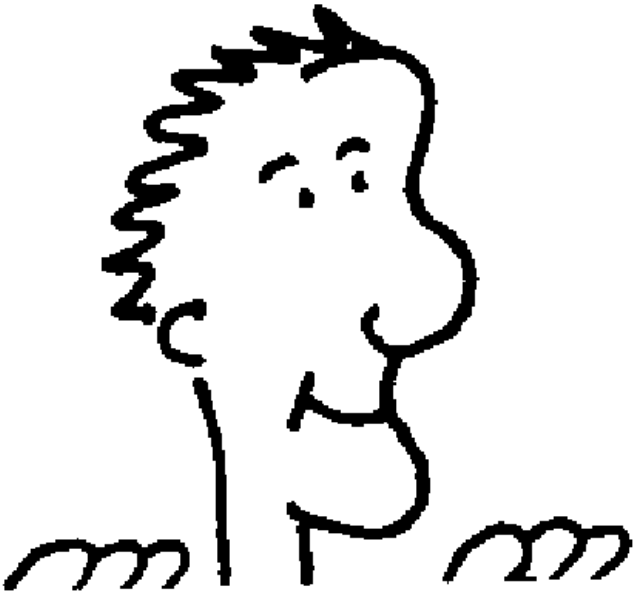
Beginner Modeling Exercises will give you a chance to test yourself on what you've learned so far in Road Maps One and Two. The modeling exercises ask you to identify stocks and flows, distinguish between inflows and outflows, set up stock-and-flow diagrams, and translate short scenarios into equations for the stocks and flows.

Please read and complete *Beginner Modeling Exercises* now.

After completing *Beginner Modeling Exercises...*

The modeling exercises are a good review of what was introduced in Road Maps One and Two to this point. It is important to be able to do all these exercises and, if necessary, to review some of the material before continuing to work through Road Maps.

[4] Leslie A. Martin, 1997. *Beginner Modeling Exercises* (D-4347-6), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 5, 42 pp.



System Principle #3:

Levels and Rates are not distinguished by units of measure.

Units do not determine whether a variable is a level or a rate. In the *Beginner Modeling Exercises*, "velocity" appears in two different systems. In the example in which Randy trains for the Boston marathon, "velocity" is a rate that increases the distance he travels. In the example of a Ferrari accelerating at a green light, "velocity" is a level, the accumulation of the sports car's acceleration. In both cases, "velocity" is in units of distance per time.

System Principle #3 is important in modeling because it tells us not to assume that a particular component of a system is a level or a rate just from looking at its units of measure. Can you identify any other real-life components that can be classified as either a level or a rate?

The next paper will use simple system dynamics models to develop your understanding of feedback processes.

- Introduction to Feedback [5]

by Leslie A. Martin

[5] Leslie A. Martin, 1997. *Beginner Modeling Exercises* (D-4347-6), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 5, 42 pp.

Introduction to Feedback will teach you how to trace causal connections. You will learn how to differentiate between positive and negative feedback loops.

Please read and complete *Introduction to Feedback* now.

After completing *Introduction to Feedback...*

You should now be able to distinguish between positive and negative feedback loops. You have seen several simple feedback structures that you should start to recognize in other people's models and later incorporate into your own models.

Graphical Integration Exercises

Computer programs such as STELLA II are a very powerful tool in simulating systems. However, we must also be able to estimate the system behavior without using a computer. The next reading introduces the method of graphical integration.

- Graphical Integration Exercises Part 1: Exogenous Rates[6]

by Alice Oh

This is the first in a series of papers on graphical integration. It explains the methods of graphical integration for systems with constant and step function rates. You will be asked to do several exercises; you can check your answers in the appendix.

Please read *Graphical Integration Exercises Part 1* now.

After completing *Graphical Integration Exercises Part 1...*

The method of graphical integration is very useful in estimating the behavior of many systems. This paper covered systems with constant positive and negative rates as well as step function rates. It is necessary to understand this method fully before moving to the next paper in this series.

---

[6] Alice Oh, 1995. *Graphical Integration Exercises Part 1: Exogenous Rates* (D-4547-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, December 1, 19 pp.

Levels and Rates

- *Introduction to Computer Simulation*,[7] Chapter 13 Levels and Rates
by Nancy Roberts et al.

Chapter 13 of *Introduction to Computer Simulation,* "Levels and Rates," explains stock-and-flow diagrams as a progression from causal-loop diagrams. The equations that correspond to the variables in stock-and-flow diagrams will also be explained. The equations in Chapter 13 are written in the DYNAMO simulation language. DYNAMO is the equation-based computer simulation language that was used in system dynamics between 1963 and 1986, so we will provide help with understanding DYNAMO equations and converting DYNAMO equations to STELLA.

Please begin Chapter 13 of *Introduction to Computer Simulation* now.
The following tips may help you:

Read Chapter 13 from page 223 up to and including the top of page 229. Do all of the exercises and read through all of the examples.

Modeling Figure 13.11 on STELLA

Read "Simulation, Structure, and Behavior" from page 229 up to and including the first paragraph on page 231.

- Open a new STELLA file.
- Place a stock (▭) in the middle of your screen and rename it "Population."
- Place a flow (⧖) to the left of the "Population" stock and have it flowing into it. Rename the flow "births." Your model on the screen should now look like Figure 13.11.

Defining Figure 13.11 in STELLA

Read from the second full paragraph on page 231 up to Exercise 5 on page 234. Example 2 calls for simulation of the model you are building. The book

[7] Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. *Introduction to Computer Simulation: A System Dynamics Approach*. Portland, Oregon: Productivity Press. 562 pp.

derives and uses DYNAMO equations to do this. Follow the directions below to convert the DYNAMO equations to STELLA.

Look at the DYNAMO equations in the middle of page 233.
- To convert "births" = 100 into a STELLA equation, toggle the globe in the upper-left-hand corner ( ) so that it becomes . Question marks should appear in the stock and flow.
- Double-click on "births" and the dialog box will appear on your screen. Type "100" as the equation for "births". (Units of "births" are in people per year.)
- Click on the *Document* button. In the document box, type "100 babies are born every year in the kingdom of Xanadu." Click OK.
- Double-click on the "Population" stock and type "5510" (the initial population of Xanadu).
- Click on *Document*. Type "The initial population of Xanadu is 5510 people." Click OK.

To view the equations for the model you've just created, click on the arrow ( ) above the icon. Then select *Equation Prefs* under the Edit menu. Once the *Equation Prefs* dialog box appears on your screen, click on *Show Documentation*. Click OK. The following equations will appear:

Population(t) = Population(t - dt) + births * dt
INIT Population = 5510 {people}
DOCUMENT: The initial population of the kingdom of Xanadu is 5510 people.
births = 100 {people/year}
DOCUMENT: 100 babies are born every year in the kingdom of Xanadu.

You will notice that you typed in the information for everything except the first equation for "Population". This is because every stock equation is automatically formulated by STELLA. Notice the similarities between the equation that STELLA uses and the DYNAMO equation for this stock.

- Click on the arrow ( ) to get the model back.

Read and do Exercise 5 on page 234 as it is written in the book.

Simulating Figure 13.11 in STELLA

Read and do Example III on pages 234-238 as it is written in the book,

including Exercises 6 and 7. The hand calculation that Example III requires can be done much more easily by using the STELLA model you've just created. The directions for creating and running a table in STELLA are given below:

- Create a table by clicking on the table icon () and placing it on the workspace. Double-click on the table to make the table dialog box appear. Rename the table "Xanadu Table."
- The table now needs to be defined. Double-click on "Population" and on "births" in the Allowable list (or click on each variable, then click the  icon). "Population" and "births" should now be in the Selected list.
- Click on *Every Dt* to de-select it. "1" should now be highlighted as the Report Interval. Click OK.
- Select *Time Specs* in the Run menu. When the dialog box appears, select "Years." Set the Range values from 2020 to 2040. Click OK.
- To run the table, select *Run* from the Run menu.
- Once the run is complete, note that the numbers in the table agree with Table 13.1 in the book.
- To close the table, click on the box in the upper-left-hand corner.

Exercise 6 asks you to graph the population of Xanadu through the year 2040. Because you have already constructed the model, generating a graph with STELLA is very easy. The time specs you entered for the table are also valid for the graph of the model.

- Create a graph by clicking the graph icon () and placing it onto the workspace. Double-click on the icon.
- Double-click on the graph pad itself to open it. When the dialog box opens, name the graph "Xanadu Graph." Double-click on "Population" and on "births" from the Allowable list to select them to be graphed. Click OK.

When STELLA graphs the variables in a model, each variable has a different scale. Also, the scales do not begin with zero unless the variable itself begins with a zero value.

- To see this, select *Run* from the Run menu.
- When the graphing is complete, double-click on the graph to open the dialog box again. Click on "Population" under the Selected list and click on the double-headed

arrow to the right of "Population" ( ). Set "4000" as the min. and "8000" as the max. for the range of "Population".
- Repeat this process for "births". Use the numbers "0" to "200." Now click Set, then click OK. The graph should now be re-scaled.
- Close the graph.

Modeling and Defining Figure 13.13 in STELLA

Read through Example IV. In this example, a model is created to simulate world population growth. You can build this model in STELLA the same way you built the model for the population of Xanadu. The only difference in the two models is that in the world population model, the birth rate is not constant, but dependent on the population.

- Model the flow diagram in Figure 13.13 on page 238. Use the connector ( ) to connect "Population" to "net births". (The position of the connector can be changed by clicking on and moving the circle at the beginning of the arrow).
- Now you need to enter the equations for your model. Toggle the globe ( ) to and enter the initial world population into the model. Do not forget to document "Population".
- To complete the world population model, study the DYNAMO equations on page 239. Notice that a new variable, "GROWTH FRACTION" (GF), is introduced. Good modeling practice is to give every variable in a system a name and an icon (i.e., stock, flow, converter). Click once on the converter icon ( ) and name it "GROWTH FRACTION". Connect "GROWTH FRACTION" to "net births". Define "GROWTH FRACTION" as "0.02." Do not forget to document "GROWTH FRACTION".
- The only variable left to define is "net births". Double-click on "net births" to open up the dialog box. If the document window is open, click on *Hide Document*. Enter "Population * GROWTH FRACTION" as the equation. Document "net births".
- Compare the STELLA equations to the DYNAMO equations as you did previously.

Simulating Figure 13.13 in STELLA

- Create and define a table ( ) and a graph ( ) for this model as done before. Do not forget to:

Type in a title for each.
Select variables for each.
Change the report interval for the table.
Change the scales for the graph.
Enter the correct time specs for both simulations.
- Compare the table you created to Table 13.2 on page 241.

After reading Chapter 13 of *Introduction to Computer Simulation...*

Chapter 13 gave you more experience with using and creating stock and flow diagrams on STELLA, and explained how to enter the corresponding equations for models in STELLA. It also showed how to recognize and understand the equations for the stocks and flows in DYNAMO. Chapter 13 also illustrates System Principle #4:



System Principle #4:
Levels are accumulations (integrations).

In Example I on pages 227-228 of *Introduction to Computer Simulation*, levels are emphasized as quantities that accumulate over time. Note that the levels **children** and **adults** are accumulations of the rates **births** and **maturing**. Levels accumulate the results of rates (actions) in the system.

Finishing Off Road Maps Two

Road Maps Two explained how to use STELLA to create some simple models of systems on the computer. The ability to use a computer to model a real-world system is a skill that will become vital as you learn more about systems and system behavior. Graphical integration has been introduced as a method to estimate system behavior. Road Maps Two continued to explain how to identify feedback loops and classify the loops in terms of positive or negative feedback. Road Maps

Two also introduced three more principles of systems: a) that levels and rates are fundamental to loop substructure; b) that levels and rates are not distinguished by units of measure; and c) that levels are accumulations.

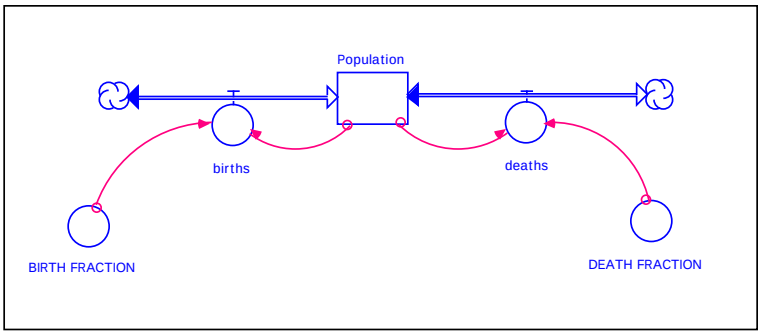
Road Maps Three will continue to have you practice modeling on the computer and creating stock-and-flow diagrams. You will also begin to explore common behaviors of systems in greater detail.

Key Terms and Concepts:

Causal Loop
Connector
Converter
Exogenous Variable
Endogenous Variable
Feedback Loop
Graphical Integration
Level/Stock
Negative Feedback
Positive Feedback
Rate/Flow



The First Step



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Leslie A. Martin
July 24, 1997

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1. ABSTRACT[1]

The current primary and secondary school educational system is poorly serving students in the United States. The result has been a great outcry to improve America's educational system. Unfortunately, many plans aimed at improving education are misguided efforts calling for more of what is already not working, rather than seeking fundamentally new and more effective approaches to education. System dynamics and learner-centered learning are alternative approaches to the status quo of strict factual education that dominates America's primary and secondary schools.

The following tutorial is meant to serve as a hands-on introduction to system dynamics and learner-centered learning for educators and others interested in learning the basics of system dynamics through computer modeling. The tutorial sets forth some of the principles of system dynamics and learner-centered learning by guiding the reader through two simple population models. The reader starts off with a blank screen and develops the models from scratch, learning how to use modeling software, and gaining the confidence to begin to model independently. The authors hope that the skills gained from this tutorial will serve as a basis for continued learning and the development of an educational system based on the principles of system dynamics and learner-centered learning.

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[1] This tutorial is based on the following introductions to STELLA: Matthew C. Halbower, 1991. *The First Three Hours* (D-4230-4), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 19, 53pp, a STELLA tutorial which uses examples from physics, and High Performance Systems, 1990. *STELLA® II User's Guide*, Hanover NH, 218pp.

2. SYSTEM DYNAMICS

System dynamics can provide a common language for mathematics, biology, ecology, physics, history, and literature.

System dynamics is an academic discipline created in the 1960s by Dr. Jay W. Forrester of the Massachusetts Institute of Technology. System dynamics was originally rooted in the management and engineering sciences but has gradually developed into a tool useful in the analysis of social, economic, physical, chemical, biological, and ecological systems.

In the field of system dynamics, a **system** is defined as a collection of elements that continually interact over time to form a unified whole. The underlying relationships and connections between the components of a system is called the **structure** of the system. One familiar example of a system is an ecosystem. The structure of an ecosystem is defined by the interactions between animal populations, birth and death rates, quantities of food, and other variables specific to a particular ecosystem. The structure of the ecosystem includes the variables important in influencing the system.

The term **dynamics** refers to change over time. If something is dynamic, it is constantly changing. A dynamic system is therefore a system in which the variables interact to stimulate changes over time. **System dynamics** is a methodology used to understand how systems change over time. The way in which the elements or variables composing a system vary over time is referred to as the **behavior** of the system. In the ecosystem example, the behavior is described by the dynamics of population growth and decline. The behavior is due to the influences of food supply, predators, and environment, which are all elements of the system.

One feature that is common to all systems is that a system's structure determines the system's behavior. System dynamics links the behavior of a system to its underlying structure. System dynamics can be used to analyze how the structure of a physical, biological, or literary system can lead to the behavior that the system exhibits. By defining the structure of an ecosystem, it is possible to use system dynamics analysis to trace out the behavior over time of the ecosystem based upon its structure.

The diagram in Figure 1 shows how the underlying structure of a system determines the system's behavior. The upward-pointing arrow on the left symbolizes the relationship. The downward-pointing arrow on the right indicates the deeper understanding that is gained from analyzing a system structure. Full understanding can only come when one dives beneath the behavior to understand the structure causing the

behavior.

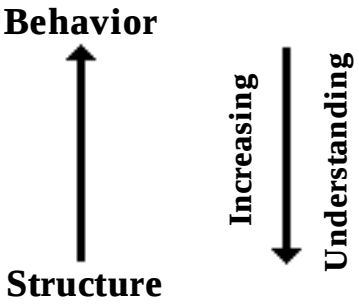


Figure 1: The link between structure and behavior

System dynamics can also be used to analyze how structural changes in one part of a system might affect the behavior of the system as a whole. Perturbing a system allows one to test how the system will respond under varying sets of conditions. Once again referring to an ecosystem, one can test the impact of a drought on the ecosystem or analyze the impact of the elimination of a particular animal species on the behavior of the entire system.

In addition to relating system structure to system behavior and providing students with a tool for testing the sensitivity of a system to structural changes, system dynamics requires a person to partake in the rigorous process of modeling system structure. Modeling a system structure forces a student to consider details typically glossed over within a mental model.

In a book examining the historical development of the earth's ecosystem, J.E. Lovelock has the following to say about system analysis:

> "Think about a temperature controlled oven. Is it the supply of power that keeps it at the right temperature? Is it the thermostat, or the switch that the thermostat controls? Or is it the goal that we established when we turned the dial to the required cooking temperature? Even with this very primitive control system, little or no insight into its mode of action or performance can come from analysis, by separating its component parts and considering each in turn, which is the essence of thinking logically in terms of cause and effect. The key to understanding systems is that, like life itself, they are always more than merely the assembly of constituent parts. They can only be considered and understood as operating systems... whereby the behavior of the system is analyzed in terms of its underlying structure."[2]

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[2] Lovelock, J.E., 1979. "Gaia: A New Look at Planet Earth." Oxford University Press, p.52.

Systems dynamics provides a common communication tool connecting many academic disciplines. System dynamics forces people to think critically about problems because of the process they must go through to develop and analyze system structure. Most importantly, with system dynamics, one can make the mental link between the structure of a system and the behavior that the system produces.

3. COMPUTER SIMULATION

Tell me and I will forget.
Show me and I may remember.
Involve me and I will understand.

One of the best ways to learn is to participate in a project. Educators can stand in front of students all day long and lecture on how to hit a tennis ball, change the oil in a car, or run a corporation. Once a student is in the position where it is necessary to complete one of these tasks, however, the student often cannot. The reason that a student cannot complete the task is because the student has created a mental model of the system, based on lecturing or reading, that does not fit reality. A **mental model** is one's mental perception or representation of system interactions and the behavior those interactions can produce. Due to an incomplete or incorrect mental model, a student cannot apply the principles taught in lectures to tasks in life.

How can educators improve students' mental models of systems? How can students be taught to achieve a greater understanding of a problem or phenomenon and the structure of the system which produces the problem or phenomenon? Just as toddlers learn not to touch a hot stove by the direct feedback signal they receive from touching one, educational systems should incorporate direct feedback between the student and the subject being taught.

System dynamics offers a source of direct and immediate feedback for students to test assumptions about their mental models of reality through the use of computer simulation. **Computer simulation** is the imitation of system behavior through numerical calculations performed by a computer on a system dynamics model. A **system dynamics model** is the representation of the structure of a system. Once a system dynamics model is constructed and the initial conditions are specified, a computer can simulate the behavior of the different model variables over time.

A good model attempts to imitate some aspect of real life. Real life does not allow one to go back in time and change the way things are. Simulation, however, gives

students the power to change system structure and analyze the behavior of the system under many different conditions.

With simulation, students do not have to fly to a Brazilian rain forest and record careful observations for many years to experience how that ecosystem reacts to changes over time. Students can repeatedly simulate the Brazilian ecosystem at home or in the classroom based on varying sets of assumptions about the ecosystem. The idea that one can simulate the experiences of real life is a very powerful concept. Just as jet pilots train on aircraft simulators, so can ecology students gain equivalent experience by training on different ecosystem simulators.

Computer simulation is not only useful for modeling systems that are difficult for students to observe in real life, but it is also more powerful in influencing the learning process when combined with real experimentation. An ideal learning environment would include discussion of a topic, student-directed research, laboratory experimentation, model building and exploration, and computer simulation to verify the link between model behavior and experimental observations. The overall goal is to teach students critical thinking skills and a methodology for dealing with complex problems that they can use later in life as managers, company presidents, journalists, generals, pilots, and engineers. The process of modeling is a continuing companion to the improvement of judgment and human decision making.

4. STELLA®

Systems Thinking Educational Learning Laboratory with Animation

STELLA[3] is a computer simulation program which provides a framework and an easy-to-understand graphical interface for observing the quantitative interaction of variables within a system. The graphical interface can be used to describe and analyze very complex physical, chemical, biological, and social systems. Model builders and users, however, are not overburdened with complexity because all STELLA models are made up of only four building blocks, pictured in Figure 2:

[3]STELLA is not the only modeling software available. Vensim, Powersim, and DYNAMO are three other software packages that modelers frequently use. All references to STELLA in this paper also apply to iThink, the corporate version of the same software.

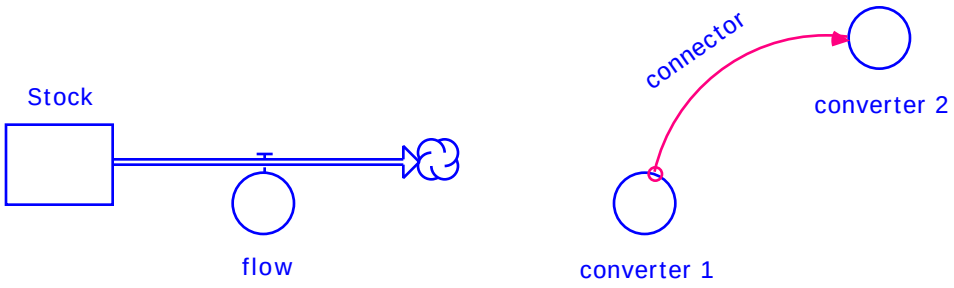


Figure 2: Representations of a stock, flow, converter, and connector

Stock—A stock is a generic symbol for anything that accumulates or drains. For example, water accumulates in your bathtub. At any point in time, the amount of water in the bathtub reflects the accumulation of what has flowed in from the faucet, minus what has flowed out down the drain. The amount of water in the bathtub is the stock of water.

Flow—A flow is the rate of change of a stock. In the bathtub example, the flows are the water coming into the bathtub through the faucet and the water leaving the bathtub through the drain.

Converter—A converter is used to take input data and manipulate or convert that input into some output signal. In the bathtub example, if you were to turn the valve that controls the water flow in your bathtub, the converter would take as an input your action on the valve and convert that signal into an output reflecting the flow of water.

Connector—A connector is an arrow that allows information to pass between converters and converters, stocks and converters, stocks and flows, and converters and flows. In Figure 2 above, the connector from converter 1 to converter 2 means that converter 2 is a function of converter 1; in other words, converter 1 affects converter 2.

The table in Figure 3 provides examples of variables that might be classified as stocks and flows:

Inflows	Stocks	Outflows
births	Population	deaths
planting	Fir Trees	cutting down
eating	Food in a Stomach	digesting
increase	Self Esteem	decrease
hiring	Employees	firing
learning	Knowledge	forgetting
producing	Inventory	shipping
borrowing	Debt	repaying
recovery	Health	decline
build up	Pressure	dissipation
construction	Buildings	demolition
inflow	Water in a Bathtub	outflow

Figure 3: Table of stock and flow examples

5. MACINTOSH BASICS

We will now begin with a short tutorial on some of the basic features of the Macintosh computer.[4] Readers already familiar with the workings of a Macintosh computer may skip ahead to section 6: "The Population Model."

Readers unfamiliar with the Macintosh computer, or computers in general, should not be discouraged. The Macintosh is powerful because it is easy to use. With the Macintosh, one learns through exploration and experimentation. The Macintosh uses picture icons to represent the programs or files that are available on the computer. Typed commands are often unnecessary because manipulation of the programs can be accomplished through the use of the mouse and picture icons. The **mouse** is the small white device connected to the keyboard of the computer. The mouse allows the user to manipulate or control an arrow on the monitor of the Macintosh. By simply learning to

[4] This tutorial was intended for the Macintosh. The STELLA software for the PC, however, is almost identical to the software for the Macintosh. PC users should substitute the *control* key for commands requiring the open-apple (⌘) key.

point and click, anyone can use a Macintosh.

The first thing to do is to turn on the computer by depressing the power switch. It will take a few moments for the computer to start up. The screen that appears is called the **desktop**. The icons that look like file folders are true to their image and simply contain files. File folders allow Macintosh users to personalize their files and store them in the file folders of their liking. To see the contents of a file folder, simply use the mouse to position the arrow over the folder, and click twice on the mouse, quickly, without pausing between the clicks. Or, using the Macintosh lingo, “double-click.” Install your copy of STELLA onto the Macintosh hard drive by following the instructions in the manual enclosed with the software. Specifically, first create a folder and name it STELLA II. Then drag the contents of both diskettes (which came included in the STELLA package) onto the newly created folder. Double-click on the STELLA II folder you just created and again on the STELLA II icon to start the program.

The different versions of STELLA and iThink are not completely compatible with each other. This guide is written for STELLA II v3.0.6 and iThink v3.0.6. If you have a different version of STELLA or iThink, the commands in this guide will be slightly different from the commands necessary for your version, but the interface and commands are similar enough among versions that you will be able to follow along.

6. THE POPULATION MODEL

We are now going to build a population model using the STELLA software.

6.1 Part 1: Constant Flows

After you double-click on the STELLA II icon to start the program, you should see the following screen:

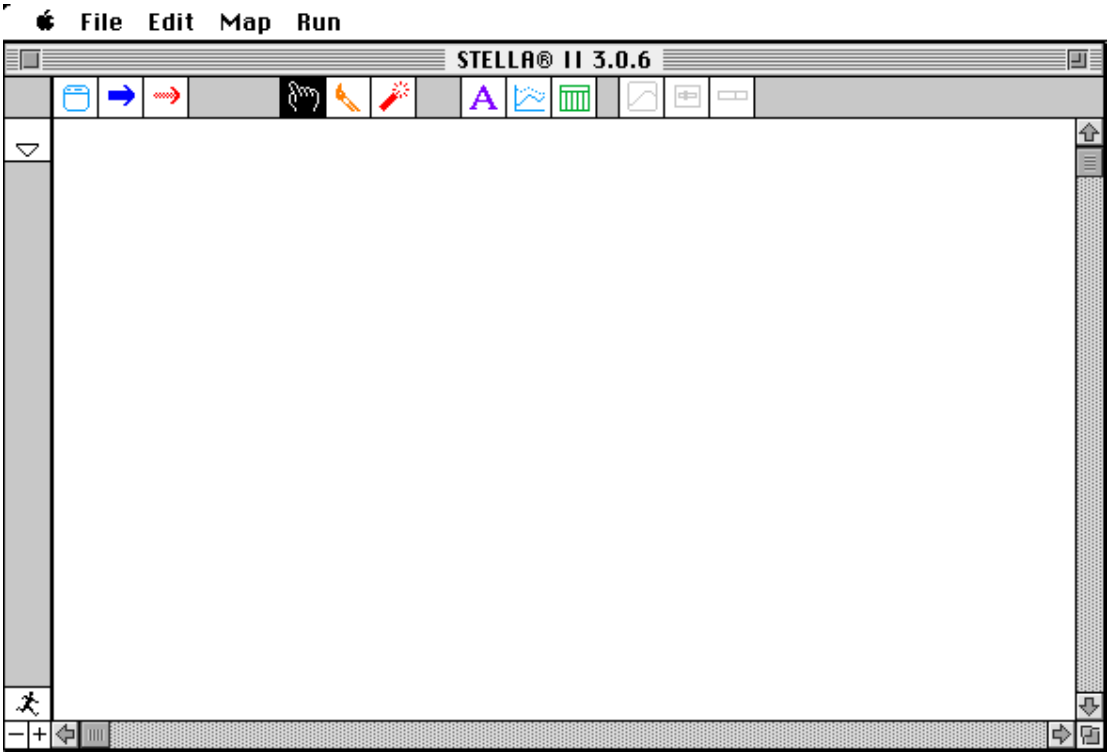


Figure 4: The initial screen

STELLA consists of three major layers. The layer that you see in Figure 4 when you begin STELLA is for high-level mapping, or an elegant presentation of the finished model. The second layer is the model construction layer. The third layer contains a list of the mathematical equations used in the model. The essence of modeling takes place in the second layer, the model construction layer. All the exercises in RoadMaps will take place in the model construction layer.

Let us go to the model construction layer now.

· **Click on the downwards arrow ▽ in the top left corner of the page.**

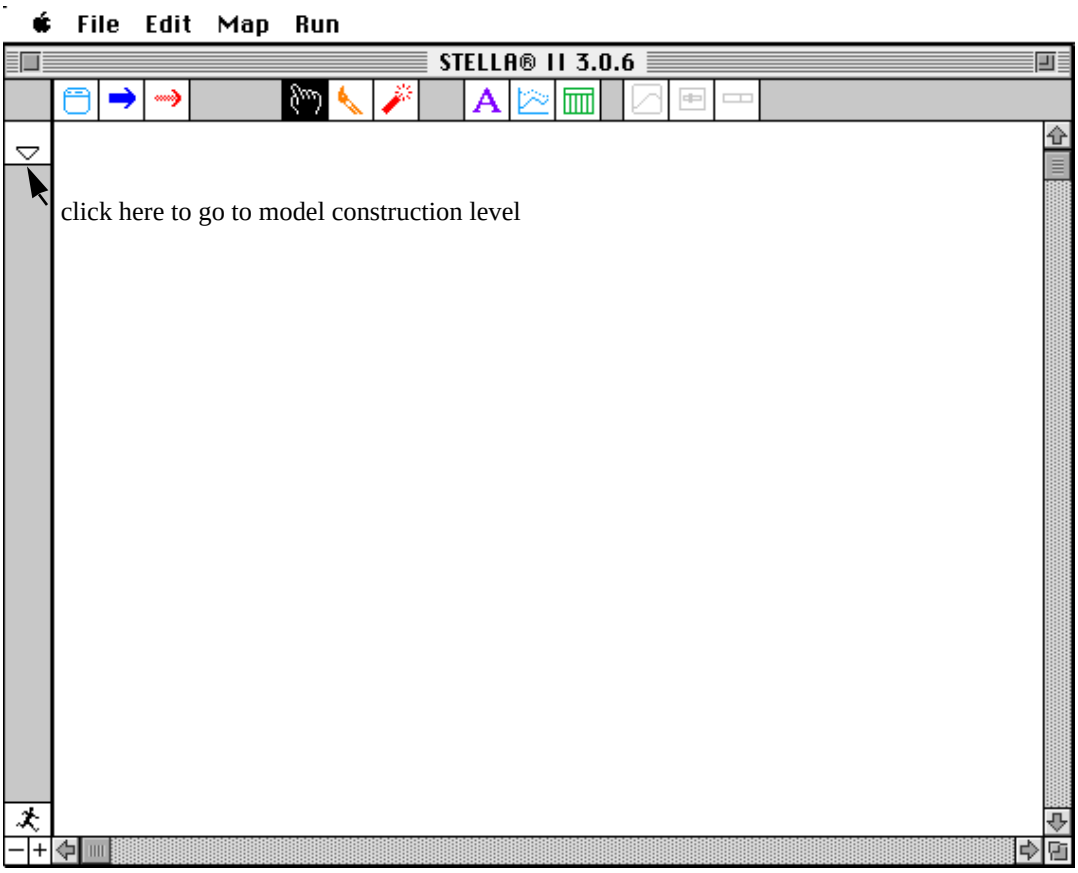


Figure 5: How to go to the model construction level

You are now looking at the diagram window shown in Figure 6. The diagram window is where we will lay out the logic of a simple population system as a diagram. The large white section filling the interior of the diagram window is the “paper” upon which you will construct your model. Although STELLA starts you out with only one 8 1/2’’ by 11’’ piece, you have an essentially “infinite” supply which extends beyond the visible frame.

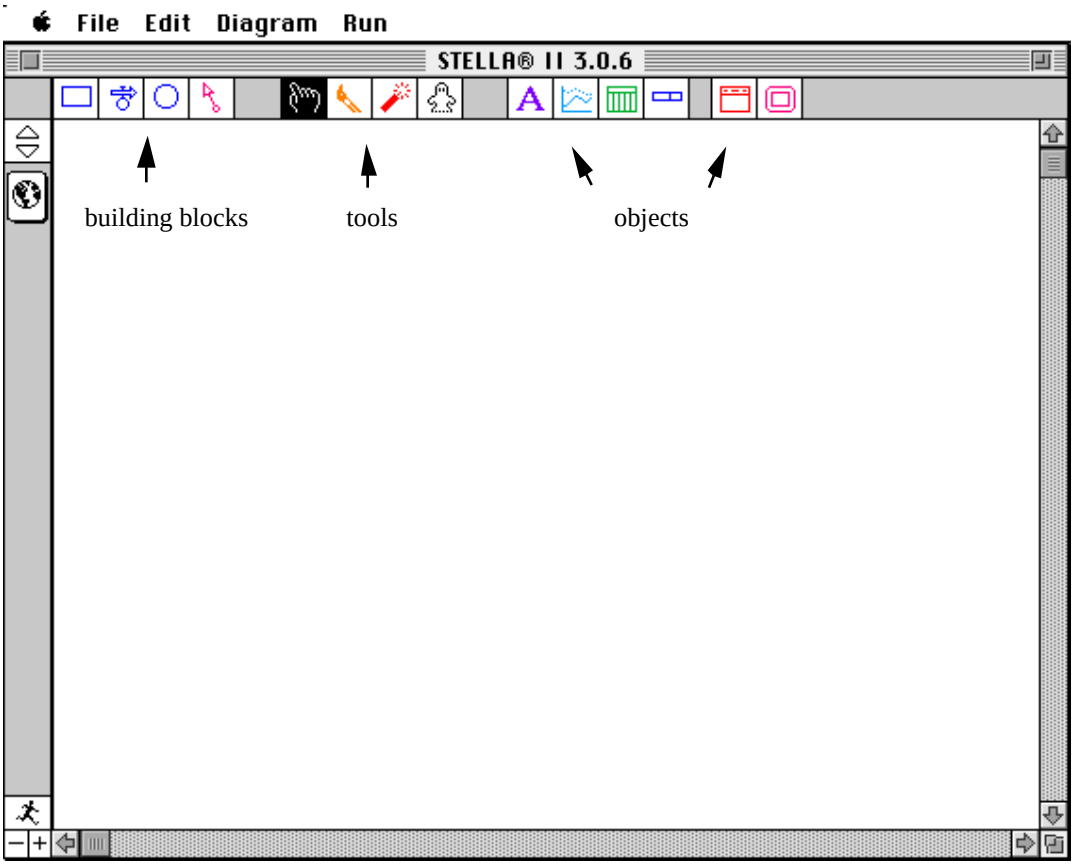


Figure 6: The diagram window

Running across the top of the screen are four building blocks. Following a separation, there are four tools, followed by another separation and six objects. The building blocks are the *only* icons that you will need to construct diagrams with STELLA. The tools and objects will enable you to position, define, replicate, and detonate the building blocks in your diagram.

To represent population, we will use the building block called the **stock**. The stock is used to represent anything that accumulates and/or depletes—be it physical, or non-physical. Population certainly accumulates, as does the cash in your savings account or water in a bathtub. Likewise momentum, anger, commitment, and frustration can also accumulate. Stocks should be used to represent any of these quantities. To create a stock:

- **Select the stock icon** (the first building block, the rectangle ▭) **by clicking once on it with the mouse. Release your click.**
- **Slide the cursor inside the picture border to the upper center.**

- **Click once to deposit it.**

Your diagram should look like the one in Figure 7.

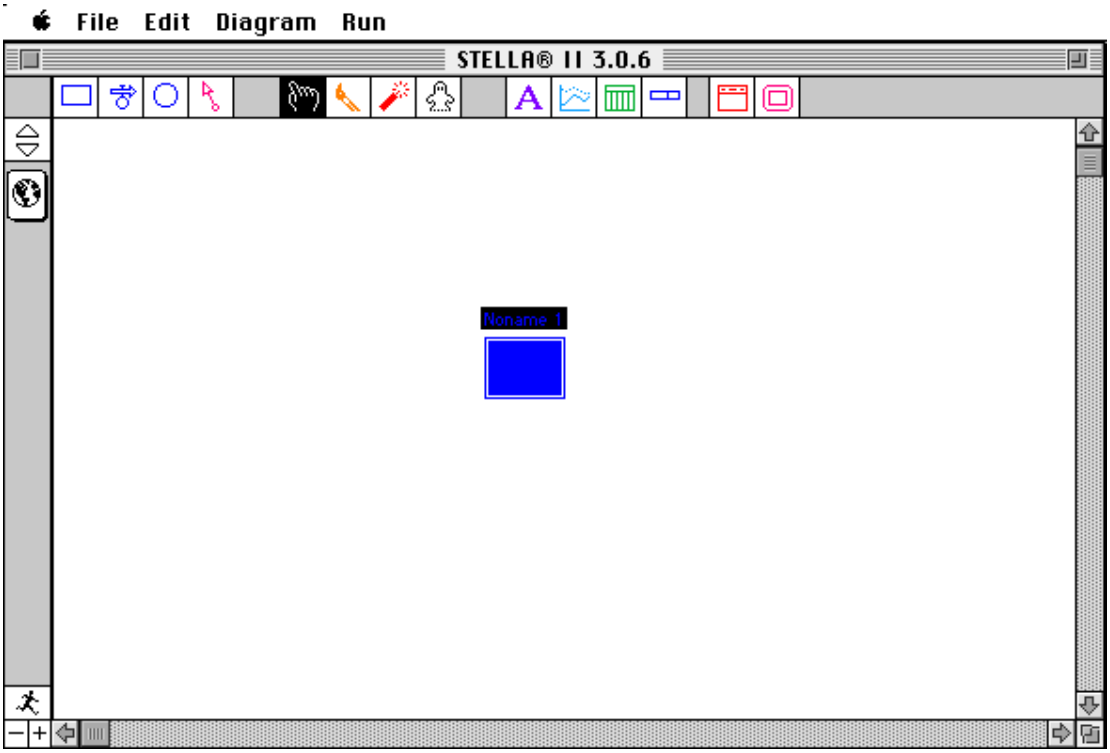


Figure 7: Placing the stock

The deposited stock, is labeled “Noname 1.” (The label is difficult to read because it is highlighted). Each building block that you deposit onto the screen is given a placeholder name (“Noname 1,” “Noname 2,” etc.) You can change any placeholder name by clicking once on the element to select it, and then typing a name.

Let’s name our stock “Population.” Because the stock already is selected (it is highlighted), all we need to do is type a name.

· **With the stock selected, type “Population.”** (If you make a mistake, backspace and correct it just as you would if you were using a word processing program.)

The size of the population at any point in time is equal to the number of people who have flowed into the stock (through births, immigration, or other inflow processes) up to that time, minus the number that have flowed out (via deaths, emigration, etc.) up to that time. Our “Population,” however, has no way to change size because it has no inflows or outflows. Let us add an inflow.

- **Select the flow icon** (located just to the right of the stock,) **by clicking once on it. Be sure to release your click after you select the icon.**
- **Move the cursor onto the page. Position it about two inches to the left of "Population."**
- **Click-and-hold the mouse button. Then, drag the cursor to the right until it makes contact with the left wall of the stock** (the stock should "color" on contact)**. Release your click.**

As you position the flow, your diagram should look like the diagram in Figure 8.

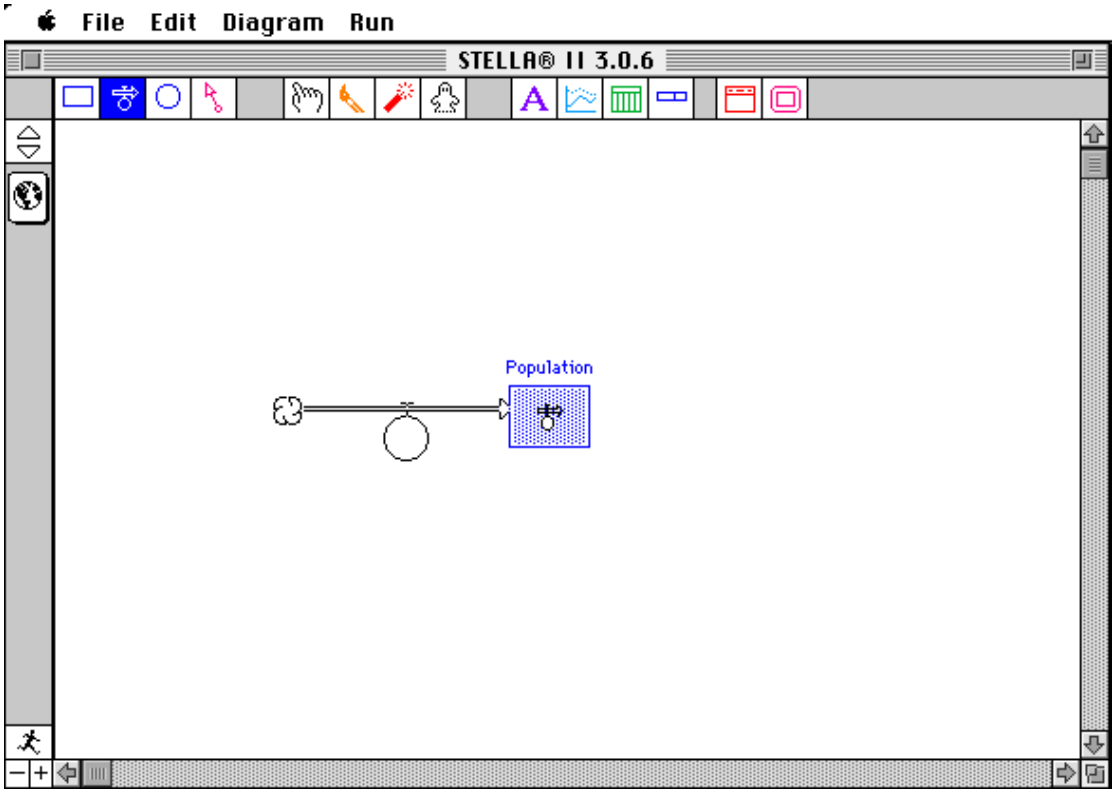


Figure 8: Positioning the flow; the stock "colors"

If you release your click before the flow makes contact with the stock, the flow will appear with **clouds** at both ends of the pipe. But do not worry; you have two options. You can slide the stock onto the cloud until the cloud colors. Or, you can just "blow up" the flow and try again. Let's blow up the flow, just to see how it works (even if you succeeded in correctly creating your flow).

- **Click on the third tool, the *dynamite*. Release your click.**

- **Position the fuse of the dynamite in the center of the circle part of the flow element. Click-and-hold.**

Notice that the circle part of the flow colors. The coloring warns you that the flow is about to become history! If you do not want to detonate the flow, simply slide the dynamite's fuse off the flow and outside the picture border. If you do want to blow up the flow, release your click. Let us do it.

- **Release your click.**

Now you are free to re-create the flow. Follow the steps outlined above. After you have done so...

· **While the flow is selected, name it "births."**

You screen should look like Figure 9.

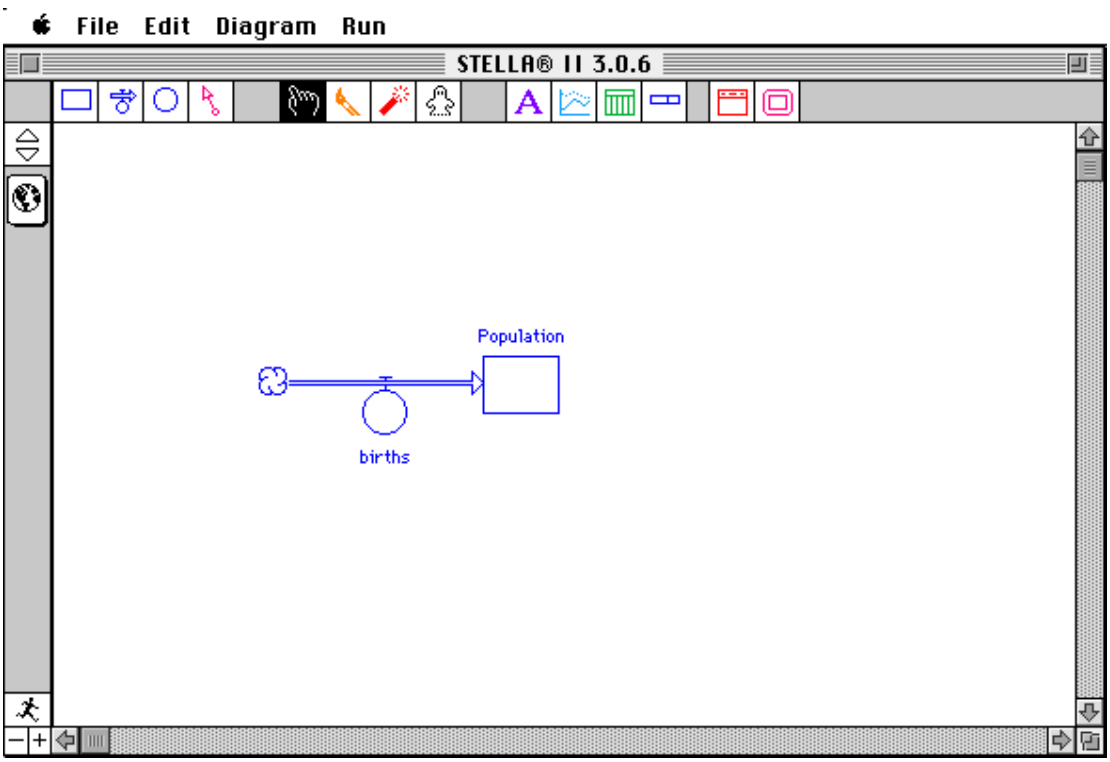


Figure 9: Stock and flow

STELLA II's flow icon requires some explanation. The icon consists of a hollow pipe with an arrow at one end and a cloud on the other. It also has a "T," which pierces the

pipe, with a circle hanging off of the end of the “T.”

The pipes are conduits for carrying flows of things into and out of the stocks. Flows are regulated by the little spigots attached to the top of each pipe (symbolized by the “T”). The circle, attached to the bottom of the spigot, is the receptacle for specifying the logic that will regulate the position of the spigot, and hence, the volume of the flow. The circle and spigot together can be thought of as the flow regulatory assembly. This assembly appears automatically whenever you create a flow. From this point on, we will refer to the whole structure as a flow.

We call the circle a **converter** because it most commonly is used to *convert* things that go into it, into inputs which then can go into something else. Depending on the signal generated by the converter, the spigot can be opened wide to admit a large volume of flow, it can be shut tight so that nothing flows, or it can be anywhere in between those two extremes. Together, the circle and spigot control the flow rate. As you will soon see, not all converters are attached to pipes.

But what about the clouds? All flows must come from somewhere. And, they also must go somewhere. Sometimes, you will care about where they go; other times you will not. Indeed, as you will discover, knowing when it is appropriate to “care” is one of the key skills that you must develop in order to become an effective modeler. The cloud is the symbol that STELLA II uses to indicate that you do not care about where whatever is flowing comes from, or where it goes. If you *do* care, you would replace the cloud with a stock. Then, an inflow to one stock would be an outflow from another. And conversely, an outflow from one stock would be an inflow to another. Clouds thus define the **boundaries** of your model. They define what is relevant to consider, and what is not.

In this model, we will not worry about where the flow of births comes from (i.e., from the stock of fertilized eggs). So, we will let the clouds stand where they are.

Now back to our model. We are ready to define the relationships in our model algebraically. In STELLA, there are two ways to view a model: in mapping mode and in modeling mode. Right now we are in mapping mode, as seen by the little globe in the top left hand corner of the page. Let’s go into modeling mode.

- **Click on the globe in the top left corner of the page.**

The globe should change to the algebraic symbol X^2. Now your diagram should look like the diagram in Figure 10.

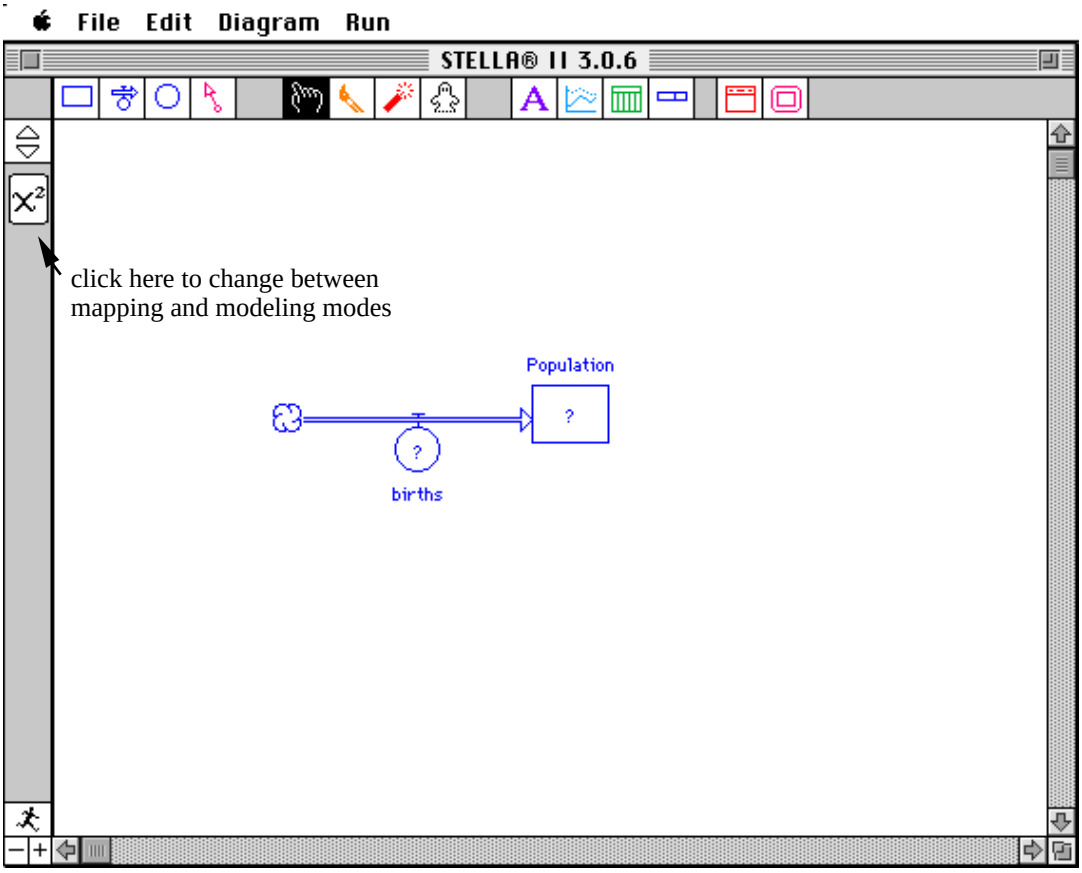


Figure 10: Modeling mode

Notice that there is a "?" inside both the stock and the flow. A "?" means that the building block has not yet been defined by a mathematical equation.

Before we define the variables in our model, we need a scenario to model. We will start with the simplest one. Small-Town, USA, currently has 5,000 inhabitants. Every year, for the last few years, approximately 150 babies are born in Small-Town. Our task is to estimate what will happen to the population of Small-Town in the next few years.

How do we go about modeling this scenario? Well, first let's define the flow of "births."

- **Double-click on "births."**

You will see the diagram box shown in Figure 11.

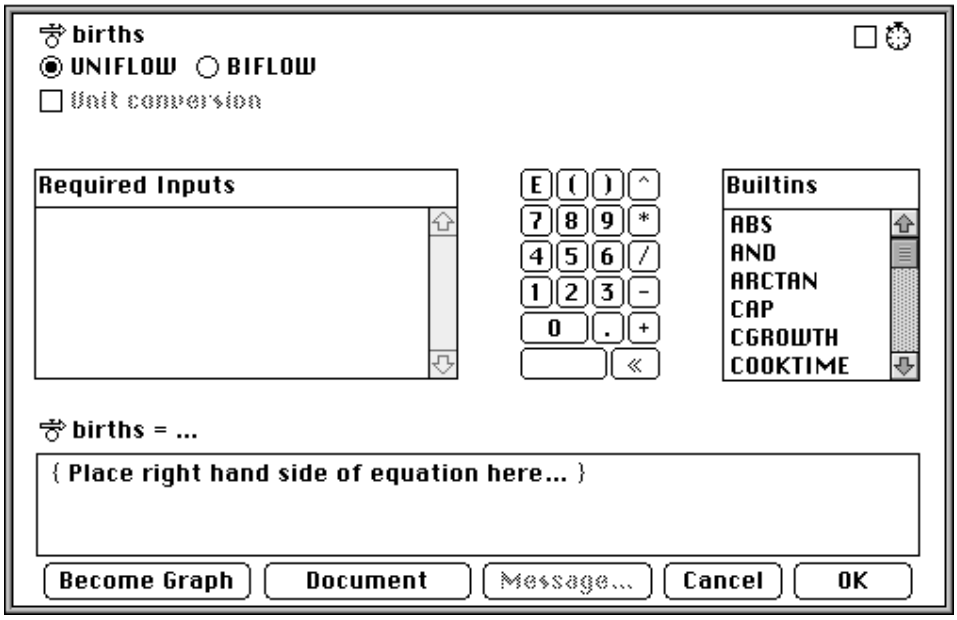


Figure 11: Inside the flow

In the center of the dialog box from left to right, we first have a *Required Inputs* list. This list contains inputs that you *must* use in your equation. For now this box is empty. Next, you see the calculator. You can use it to click in numbers, or arithmetic operators, in forming your equations. Of course, you can always use the keyboard, too. To the right of the calculator is a *Built-ins* list. The list contains both simple and complicated functions that you can click into your equations. For the time being, you will not need to worry about using the *Built-ins.*

At the top, below the name of the flow, is the option between making the flow a "uniflow" or a "biflow." When a flow is defined as a uniflow, it can flow in only one direction; it is either an inflow or an outflow. Biflows can flow in either direction. Note that in Stella II 3.0.6 the uniflow is the default. Change the flow to a biflow.

- **Select *BIFLOW*.**

Even though births always flow in one direction, it is good modeling practice to *always* model with biflows. The equation that you give the flow should guarantee that the flow of "births" never becomes negative, reversing the direction of the flow.

The last box in the dialog box contains the equation for the flow. In our scenario, Small-Town, USA has an average birth rate of 150 babies per year. So let us enter the average birth rate into the equation for "births."

- **Replace {Place right hand side of equation here...} by clicking or typing in "150." Do not include the quotes.**
- **Click on the *Document* button.**

What pops up is a field into which you can type text to document your equations. A few well-chosen words can make it much easier for colleagues or students who did *not* participate in the model-building process to follow your assumptions. It is also a great place for students to explain their logic as to "how this works," making the instructor's evaluation of the thinking behind the model an easier task. Let's put into the words the logic governing the flow of "births":

- **Type, "average number of births per year in the town"**

We also need to make the units of our flow explicit.

- **Type, "UNITS: people/year"**
- **Click once on the *Hide Document* button**

The document will disappear and an asterisk (*) will appear inside the Document button as an indicator that text has been entered.

- **Click on OK.**

Notice that now that "births" has been defined, the "?" has disappeared from within the flow. "Population" still has a "?" and still needs to be defined.

- **Double-click on "Population."**

You will see the diagram box shown in Figure 12.

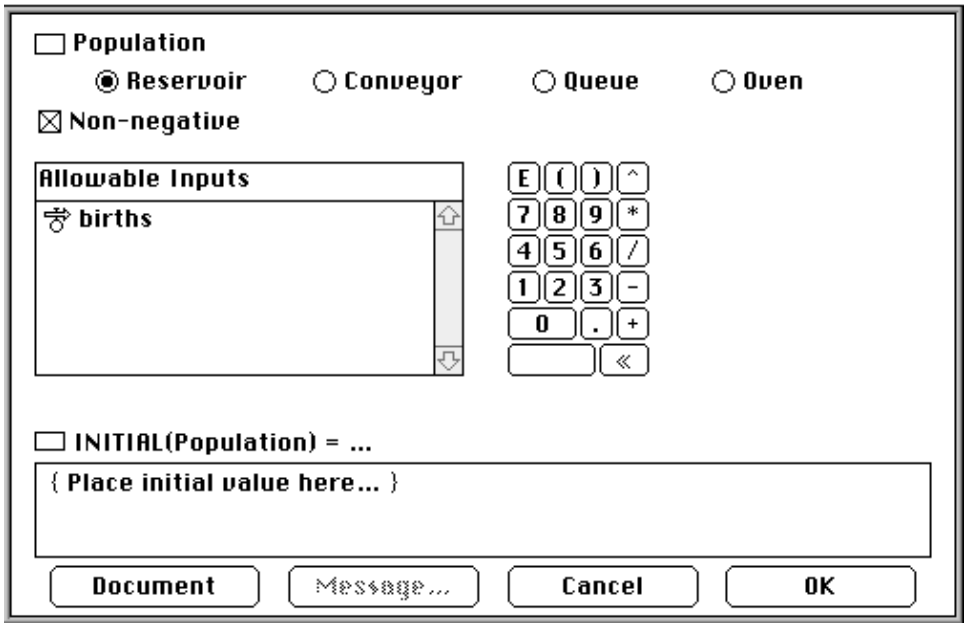


Figure 12: Inside the stock

No, your eyes are not playing tricks on you. The dialog box for stocks is indeed very different from the dialog box for flows. Instead of a *Required Inputs* list, you will see an *Allowable Inputs* list, and you will notice that there are no built-in functions. At the top of the window is a list of possible forms the stock can take (reservoir, conveyor, queue, oven). The last three forms are variations on the first one, the reservoir. Road Maps will exclusively use reservoirs.

The next item in the box is a *non-negative* option. The non-negative option forces the stock to remain either positive or zero, no matter what is trying to flow out of it. Notice that the default is for the stock to be non-negative (there is an "X" in the box.) Let us change the default now.

- **Click to remove the non-negative selection** (by clicking in the small box to the left of *Non-negative*).

Even though it is unrealistic for a human population to ever be negative, it is good modeling practice to let all stocks become negative. A good modeler writes the equations for the inflows and outflows of a stock in a way that ensures that a stock that is always positive in the real world will always be positive in the model.

Notice how the box in which you will define the stock does not ask for an equation but requires an initial value. Flows are defined by equations, but stocks are defined with

initial values. Stocks can only be changed by inflows or outflows. Think of the stock as a bathtub. You want to know every minute, on the minute, how much water is in the tub. You can control the system by turning on the flow of water and pulling open the drain. You observe how much water is flowing in and how much is flowing out. You can then calculate how much the level of water is changing, for example, at what rate the tub is filling up or draining. But to know exactly how much water is in the tub at any point in time, you need to know how full the tub was to begin. So to define a stock all we need to do is give it an initial value. Any variable in your model is "allowable" in defining an initial value for the stock. In our model, we will initialize "Population" with a number. Which one? Well, we know that initially Small-Town, USA, has a population of 5,000 people.

- **Replace {Place initial value here...} by clicking or typing "5000." Do not use any commas.**

Again, we need to document our definition.

- **Click on *Document*.**
- **Type "population of the town"**
- **Type "UNITS: people."**
- **Click the button *Hide Document.***
- **Click OK.**

Now that all of the "?"s have been extinguished, we are ready to run our first simulation. In order to see the results of our simulation, we need to set up a graph.

- **Select the graph icon** (the second object, the little graph ) **by clicking once on it. Release your click.**
- **Slide the cursor inside the picture border to the top right side.**
- **Click once to deposit the icon.**

A graph pad will pop up in a separate window. Your screen should look like the screen in Figure 13. Notice how the graph window is in the foreground and the diagram page has faded into the background. To get back to the diagram page you will need to click anywhere on the surface of the page.

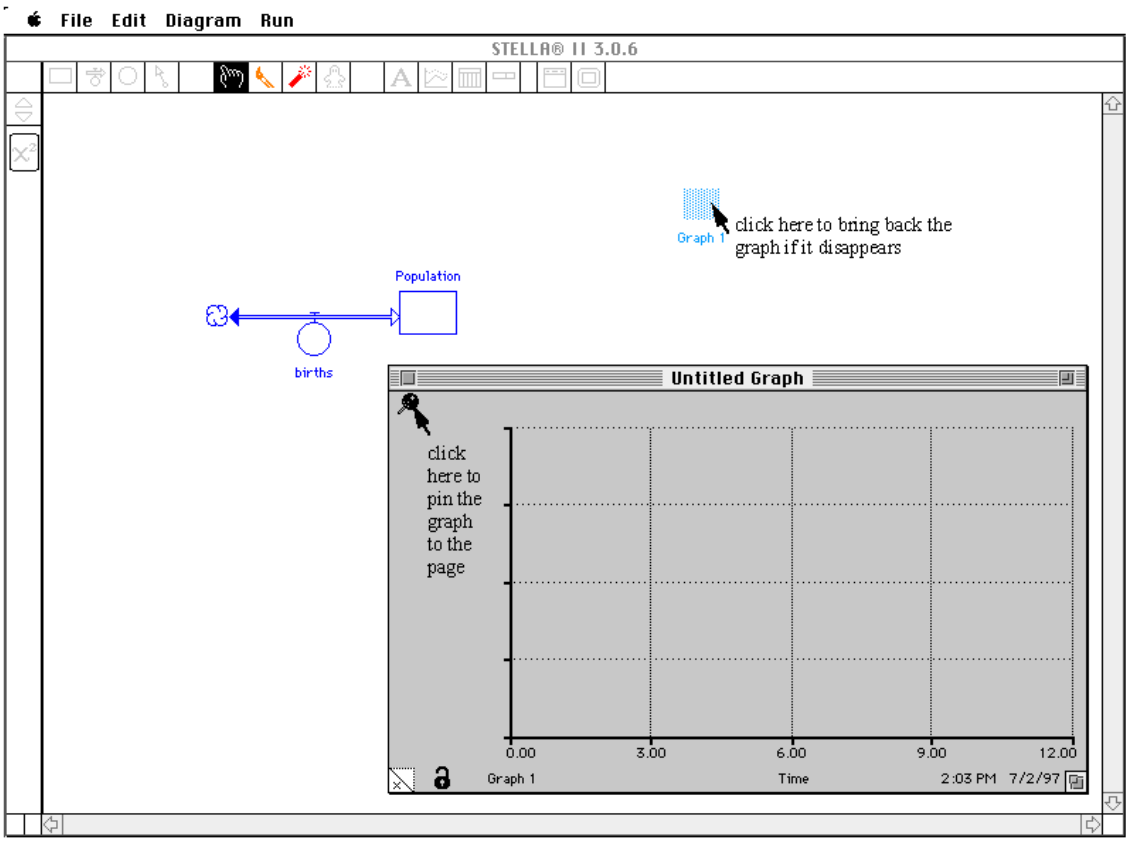


Figure 13: Adding a graph

You can move the graph window anywhere you would like by clicking and dragging the window. You can "pin" the graph to the diagram page by clicking on the pin in the top left corner of the graph. If you pin the graph, the graph will attach to the diagram window and always stay on top of the page. If you click on the diagram window without first pinning the graph to the page, the graph will disappear behind the diagram page. To retrieve it, simply double click on the Graph 1 icon that you placed on the screen. For the rest of the First Three Hours, I will keep the graph off to the side without pinning it down. Now let us set up the graph.

- **Double-click on the surface of the graph pad** (not the graph icon)**.**

The window that will pop up is similar to the window in Figure 14.

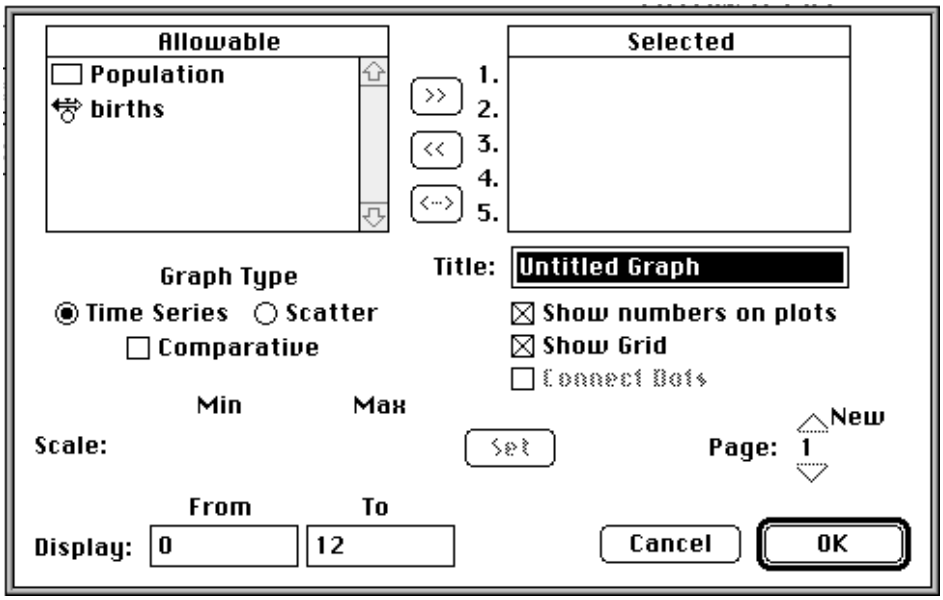


Figure 14: Setting up the graph

The box on the left lists all of the variables in the model. The box to the right contains all the variables that you have selected to graph. You can easily move variables onto and off of the graph by moving them from the list of *Allowable* variables to *Selected* variables and back. Let's graph both "Population" and "births."

· **Double-click on "Population."**

Notice how Population is now displayed in the *Selected* box and shadowed in the *Allowable* box.

- **Click once on "births" to highlight it.**
- **Click on the double arrow pointing into the *Selected* box.**

You can select variables in either way. If you want to remove a variable from the *Selected* box highlight it and click on the double arrow that points into the *Allowable* box.

The final step in setting up the graph will be to name it.

- **In the title box, replace the words "Untitled Graph" with "Small-Town, USA."**
- **Click OK** to close the graph window.

We are now ready to run the model. Click on the diagram page. The graph window will disappear. We need to bring up the run controller.

· **Click on the little runner on the bottom left corner of the screen.**

A run controller window, just like the window in Figure 15, will pop up. You can position the run controller window anywhere on your page by clicking on it, dragging it to its new location and then releasing.



Figure 15: Run controller window

The run controller looks like—and works like—the controls on a cassette player. The first button is the *play* button that starts the simulation. The second button is the *pause* button that you can use to pause a simulation after it is started. The third button is the *stop* button that will stop the run, even if the computer has not finished simulating. We will look at the *Specs* button later.

Notice that the same simulation commands can also be found by pulling down the run menu on the top of the screen.

Now let us bring the graph pad into view.

- **Double-click on the graph icon.**

Now, to run our first model.

- **Click on the *play* ▶ button.**

The computer will plot the graph that you can see in Figure 16.

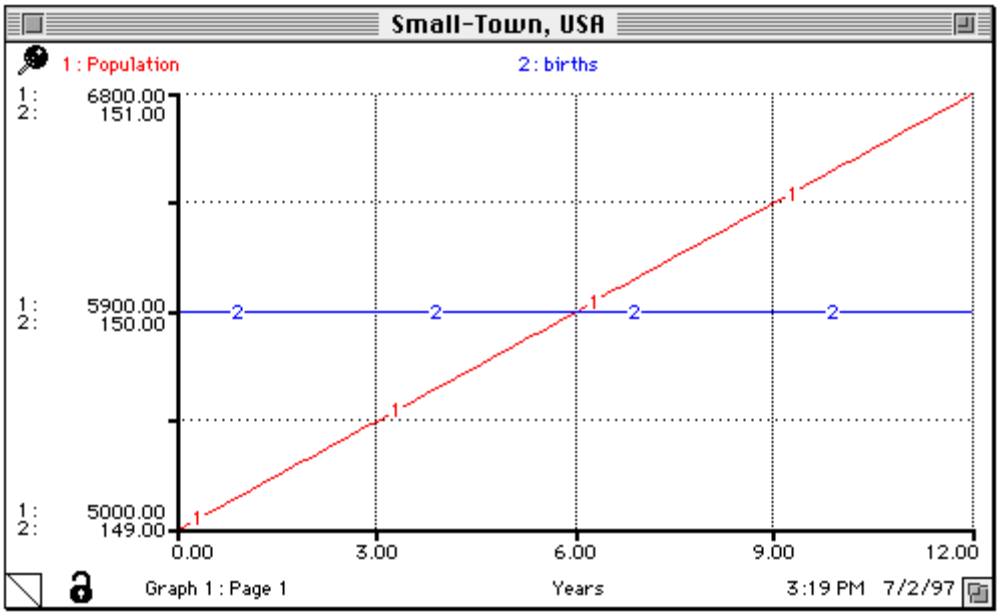


Figure 16: The population grows from 5,000 to 6,800 people in 12 years

What does the graph tell us? At the top of the graph "Population," the first variable we selected, is associated with the number 1. The "births" flow is associated with the number 2. So, on the graph the curve representing the behavior of "Population" over time is labeled with the number 1, while the curve representing "births" is labeled with a 2. Because "births" is a constant 150 people per year, the value for "births" does not change and its plot over time is a straight horizontal line. "Population," however, increases steadily over time. Over a period of twelve years, the population of Small-Town supposedly increases from 5,000 to 6,800 people! The behavior our model produced—a population increase of 1,800 people in 12 years—does not seem very realistic. We will need to refine our model.

What very important flow did we leave out of the model? Why, deaths of course. We will need to add an outflow to the stock of people.

- **Select the flow icon by clicking once on it. Be sure to release your click after you select the icon.**
- **Move the cursor onto the page. Position it in the center of the stock "Population."**
- **Click-and-hold the mouse button. Then, drag the cursor to the right about two inches. Release your click.**

· **Name the outflow "deaths."**

Your diagram should look like the diagram in Figure 17.

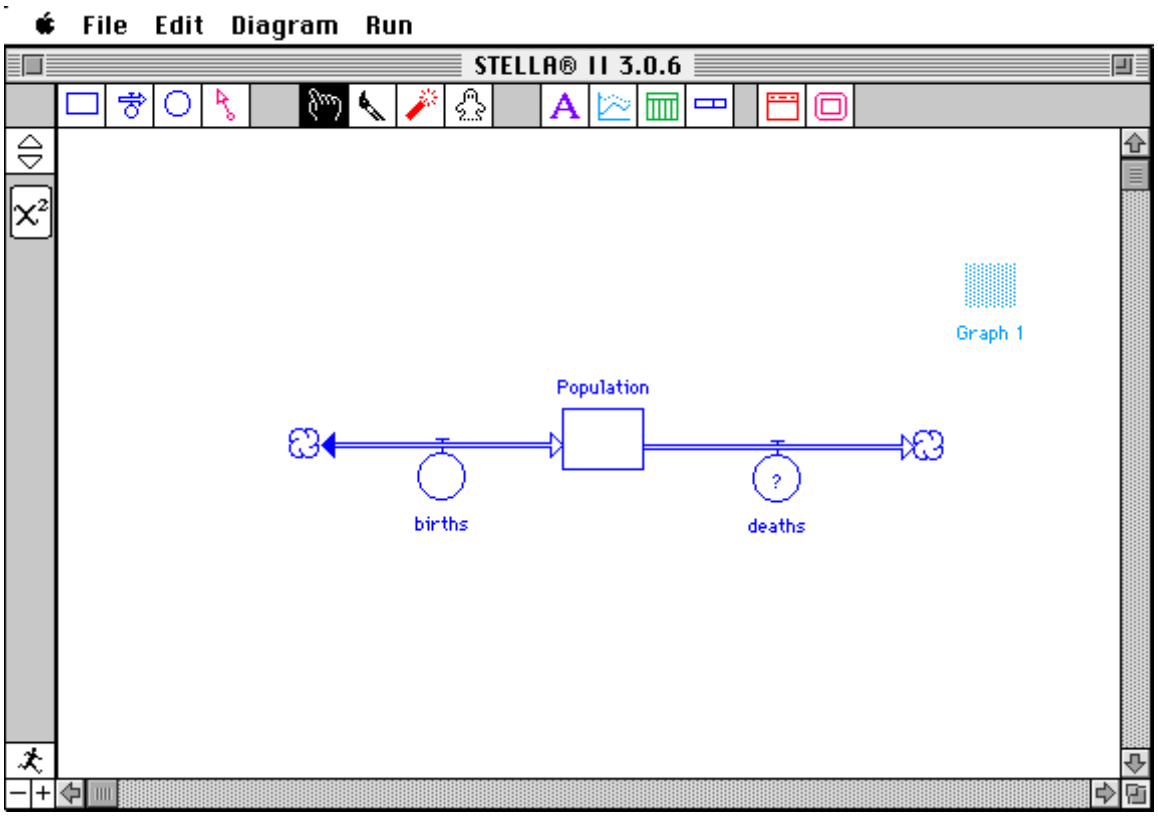


Figure 17: Stock with an outflow

Now we need to look up some information about Small-Town, USA. We call up the mayor's office to find out that, indeed, 150 new babies are born every year in Small-Town but approximately 75 people (mainly the elderly) also die every year.

We can now enter an equation for the outflow.

· **Double-click on "deaths."**

The first thing we want to do is change the uniflow to a biflow.

· **Select *BIFLOW*.**

Now we can enter the information we just gathered into the equation for "deaths."

· **In the equation box, type "75."**

Again we need to include documentation.

· **Click on the *Document* button.**

- **Type “average number of people who die each year in the town”**
- **Type “UNITS: people/year”**
- **Click on the *Hide Document* button.**

Your window should look like the window in Figure 18.

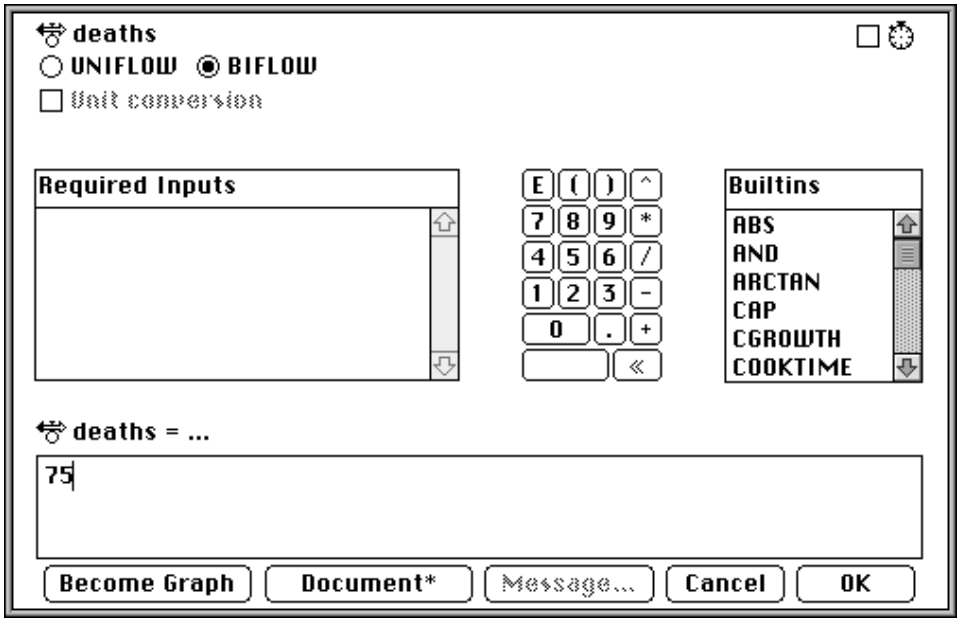


Figure 18: What the “deaths” flow window should look like

- **Click on OK.**

Now we need to add “deaths” to our graph pad.

- **Double-click on the graph icon to bring the graph pad into view.**
- **Double-click on the graph pad.**
- **Double-click on “deaths” in the *Allowable* inputs list.**

“Deaths” now figures in the *Selected* inputs list.

- **Click on OK.**

We are ready to simulate a second time. If the run controller window is not on the page, click on the little runner again to bring back the window.

- **Click on the *play* button.**

Watch as the computer plots the graph reproduced in Figure 19.

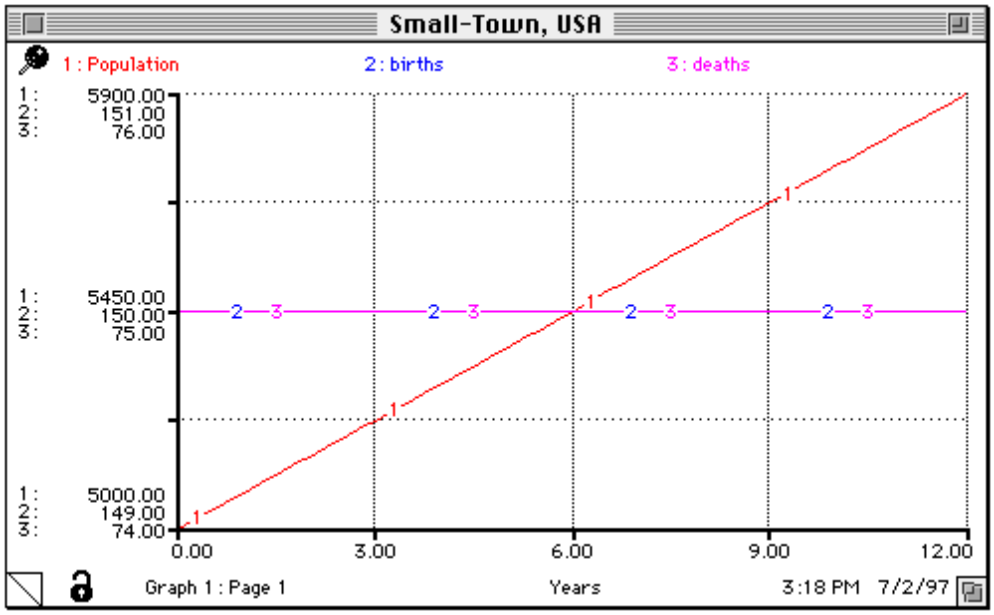


Figure 19: Graph of the population with a deaths flow

Now, with our improved model, the population of Small-Town grows in twelve years from 5,000 people to 5,900 people—definitely a more reasonable value.

Notice how STELLA plots all three variables with different scales. One way to make graphs easier to interpret is to graph a set of variables on the same scale. We can plot "births" and "deaths" on the same scale.

· **Double-click on the graph pad.**

You will see the already familiar graph-setting window.

- **Click once on "births" in the Selected variables box to highlight it.**
- **Hold down the *shift* key on the keyboard.**
- **Click once on "deaths." Release the *shift* key.**

Now both the variables "births" and "deaths" will be highlighted. Another way to highlight a series of variables is to click-and-hold on the first variable and then drag the pointer down to the last variable before releasing.

- **Click once on the double arrow ⇕ to the right of any of the highlighted variables.**

The arrows will change from ↕ to ↕. On the graph, the range of the curves is now limited by a floor and a ceiling. Your window will now look like the window in Figure 20.

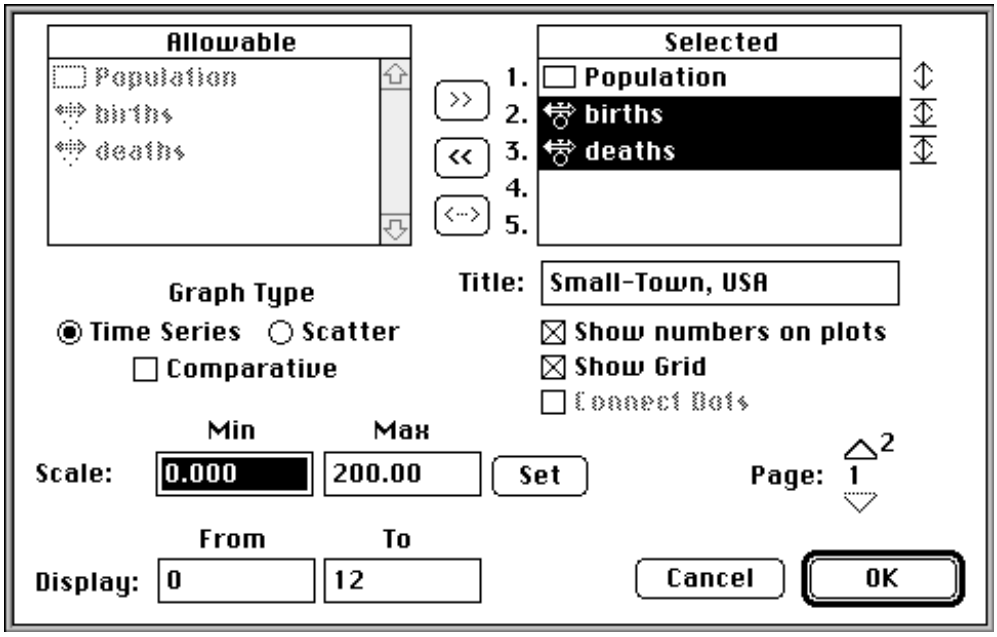


Figure 20: Setting the scale

Notice that now STELLA gives you the option of setting the scale of your variables Let us set the minimum value, the floor, to be zero.

· **Type or click "0" into the *Min* box.**

We want the maximum value, the ceiling, to be larger than the maximum value of the largest of the scaled variables. Now "births" is the larger variable with a flow of 150 people per year, so the upper limit of the scale should be greater than or equal to 150. Take, as a round number, 200.

· **Type or click "200" into the *Max* box. Do not use any commas.**

• **Click the *Set* button.**

• **Click OK.**

The graph automatically adopts the new scale. It should now look like the graph displayed in Figure 21.

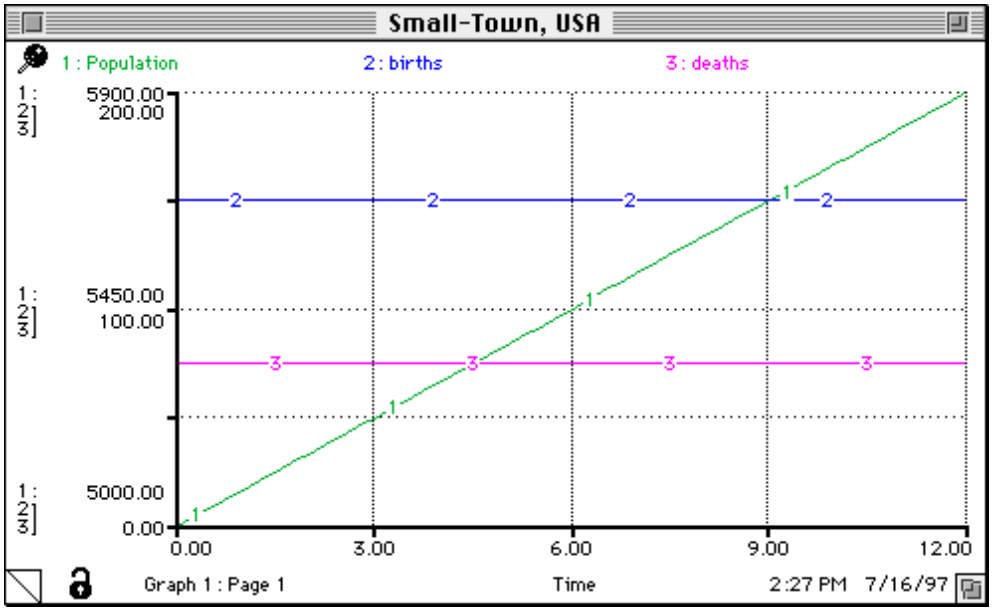


Figure 21: Small-Town, the flows on the same scale

Why does the population of Small-Town increase? Well, you can see on the graph that the line labeled 2—"births"— is higher, or greater, than the line labeled 3—"deaths." Every year there are 150 new births and 75 additional deaths; so the overall population increases by 75 people per year. Likewise, if the rate of water flowing into a bathtub is greater than the rate of water flowing out of the tub, the water level will increase and increase. If I accumulate debt faster than I can pay it off, then my debt will grow. And similarly, if I bake brownies faster than my little brother can eat them, then the number of brownies on my plate will grow.

Now let us try modeling another scenario: Dilapidated-Village, USA. Dilapidated-Village, also has a population today of 5000 inhabitants. In Dilapidated-Village, however, the daycare centers are empty, yet the cemeteries are full. Only an average of 50 babies are born each year.. Yet, on average, 125 people die every year in Dilapidated-Village. What do you think will happen to the population of Dilapidated-Village?

How should we change the model to represent this new scenario? The initial "Population" can remain the same, but we need to change the rates of "births" and "deaths."

- **Double-click on "births."**
- **In the Equation box, type "50." Click OK.**

- **Double-click on “deaths.”**
- **In the Equation box, type “125.” Click OK.**

Before we run the model again, let us save our old graph of Small-Town, USA.

- **Click on the lock in the lower left corner of the graph pad.**

When the lock closes, , the graph is locked, so it will not change when you run the model again.

Now we need to add a new sheet to the graph pad.

- **Double-click on the graph pad.**

The graph window will pop up again. We want add a new page to the graph pad.

- **Click on the upwards-pointing arrow next to the word *new*** (beneath the selection box, above the page number)**.**

You are now on page 2 of the graph window. You can return to page 1 by clicking on the downwards-pointing arrow right beneath the page number.

- **Title the graph “Dilapidated-Village, USA”**

Again select all the variables that we want to graph.

- **Double-click on “Population.”**
- **Double-click on “births.”**
- **Double-click on “deaths.”**

Because we will be comparing the two graphs, let us set the scale on the graph of Dilapidated-Village to be the same as the graph of Small-Town.

- **Click-and-hold on “births” in the *Selected* inputs box.**
- **Drag the pointer down to “deaths.” Release.**
- **Click once on the double arrow to the right of any of the highlighted variables.**
- **Type or click “0” into the *Min* box.**
- **Type or click “200” into the *Max* box.**
- **Click the *Set* button.**

Your window should look like the window in Figure 22.

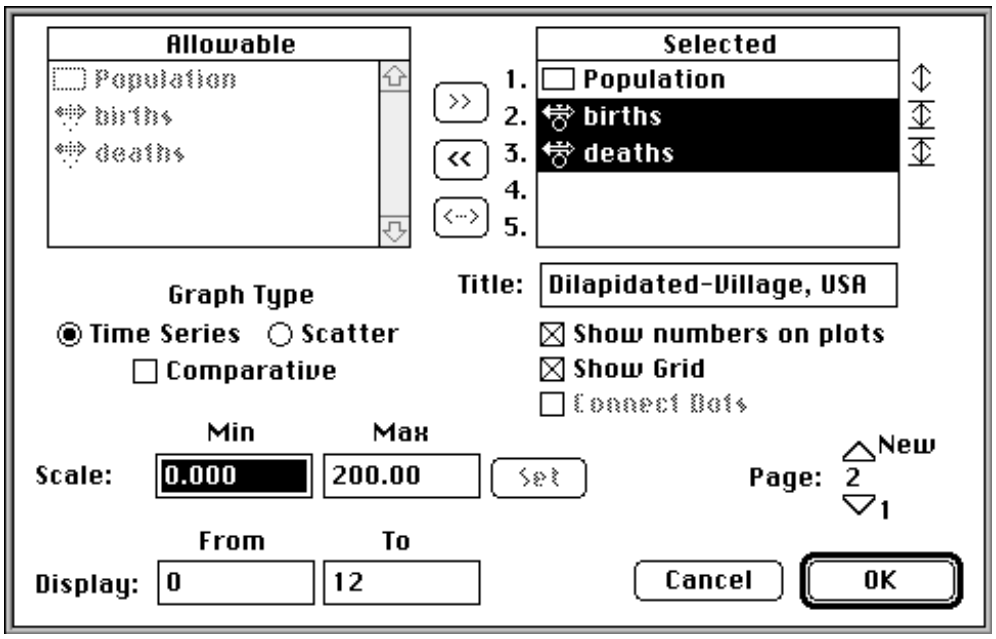


Figure 22: Preparing a second graph

- **Click OK**

If the run controller window is not on the page, click on the little runner again to bring back the window.

· **Click on the *play* button.**

As the simulation runs, you will see the graph reproduced in Figure 23.

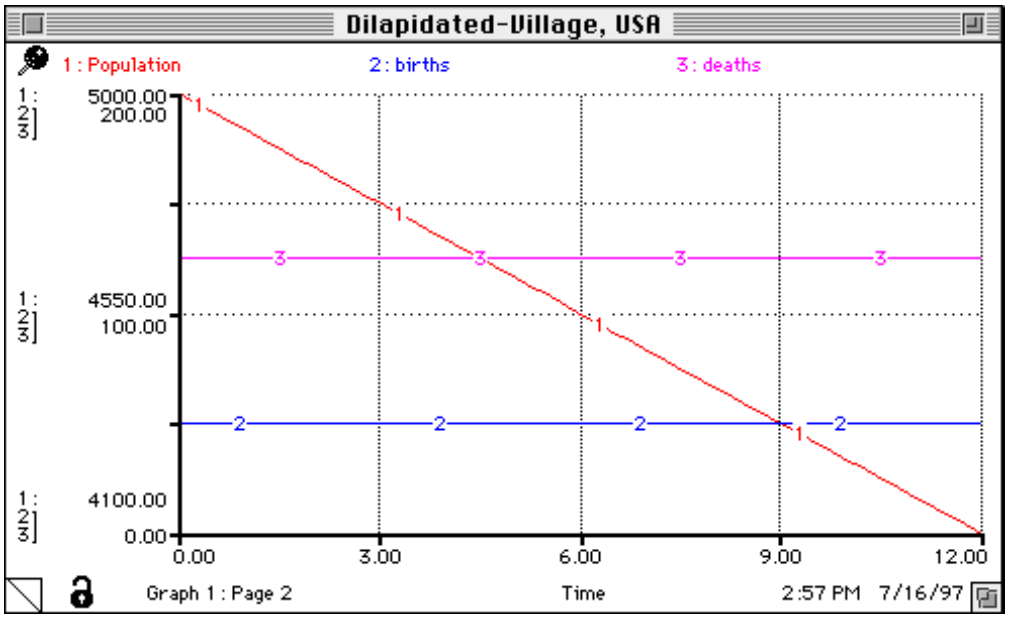


Figure 23: The population of Dilapidated-Village decreases over time

How does Figure 23 differ from Figure 21? How is Small-Town different from Dilapidated-Village? You will notice that page 2 is folded-over at the bottom left-hand corner of the graph pad. To flip back and forth between the graph of Small-Town and the graph of Dilapidated-Village, click on the graph page protruding from behind the fold.

Compare the two graphs. In Small-Town, we noticed that the flow of births is greater than the flow of deaths. Correspondingly, the population increases over time. On the other hand, in Dilapidated-Village, the flow of deaths is greater than the flow of births. On the graph, variable 3, "deaths," is always higher than variable 2, "births." Therefore the population decreases over time.

Right now, you might want to take a short break. If you want to save your model and graphs, you can hold down the open-apple key (with the symbols ⌘) and press "s." Or click-and-hold the *File* option on the menu bar, move the pointer down to the option *Save*, and release. The first time you save your model, you will be prompted to specify a name for the model. Replace the heading *untitled* with any name you like, preferably a name that will help you recognize your model without having to open it. Note that you do not have to save your model in the default directory but can change directories by clicking and holding the name of the directory at the top of the window and then sliding the pointer down to the upper-level directory of your choice. When you come back from your break, we will improve our model and begin to explore the concept of feedback.

6.2 Part 2: Feedback

Welcome back. Now, what do you think would happen if we ran our models over a longer period of time? Instead of running the models for 12 years, let us run them for 100 years. Can you predict what will happen to the populations of the two towns?

To change the time scale, we need to change the run specifications. Bring up the run controller if it is not already on the screen (by clicking on the little runner in the lower left corner of your screen).

- **Click-and-hold the *Specs* button.**
- **Drag the pointer down to the option *Time Specs...*** as done in Figure 24. **Release.**

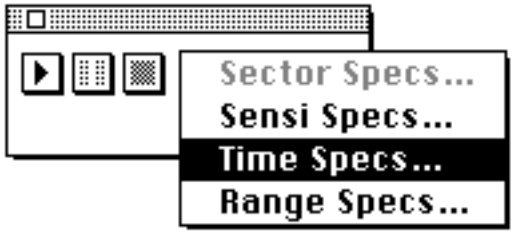


Figure 24: Using the run controller to change the time specifications

The window that pops up has five categories: length of simulation, integration method, unit of time, run mode, and interaction mode. You only need to pay attention to the length of simulation and unit of time categories. We can go ahead and change the time horizon.

- **In the length of simulation category, change the number in the *TO:* box from "12.00" to "100."**

Now we should specify the units of time. Someone who looks at your graph might not know whether the behavior of the system you modeled is being graphed over a period of 100 months, 100 years, or 100 decades. In the documentation, we specified that the flows of "births" and "deaths" were in units of people per year. So we should explicitly show in our graphs that we are simulating "Population" over a period of 100 years.

- **In the unit of time category, click on the circle in front of the selection *Years*.**

Your window should now look like the window in Figure 25.

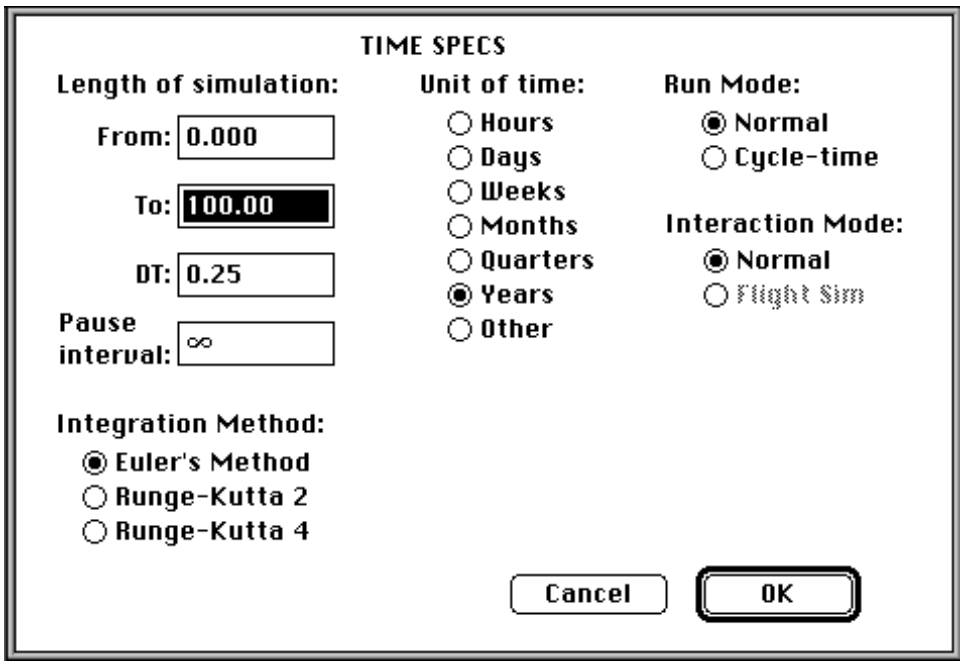


Figure 25: Changing the time horizon

- **Click OK.**

Now let's start with Small-Town, USA. First we need to change the equations for the "births" and "deaths" flows.

- **Click on "births."**
- **Enter "150" into the equation box. Click OK.**
- **Click on "deaths."**
- **Enter "75" into the equation box. Click OK.**

Now we need to set up the graph pad. Bring the graph into view (by double-clicking on the icon) if it is hidden.

- **Flip to the graph titled "Small-Town, USA"** (by clicking on the page protruding behind the fold in the low left corner of the graph pad)**.**
- **Unlock the graph.**
- **Double-click on the graph pad.**

Because we know that "births" and "deaths" will stay constant, we do not need to see them this time.

- **Click-and-hold on "births" in the *Selected* inputs box.**

- **Drag the pointer down to also highlight "deaths." Release.**
- **Click on the double arrow << to send the two flows back to the *Allowable* inputs box.**

"Population" is now the only variable left in the *Selected* inputs box. We should now de-set the scale for "Population," because we do not know what maximum value "Population" will take in our new run.

- **Click once on "Population" to highlight it.**
- **Click twice on the arrow ↕ to the right of "Population."**

The arrow should once again look like ↕.

- **Click on the *De-Set* button. Click OK.**

Now, we are ready to run the model. Another way of running a model is to hold down the open-apple key (with the symbols ⌘) and press "r." Let's do it now.

- **Hold down the open-apple key and press the "r" key.**

You should see the graph displayed in Figure 26.

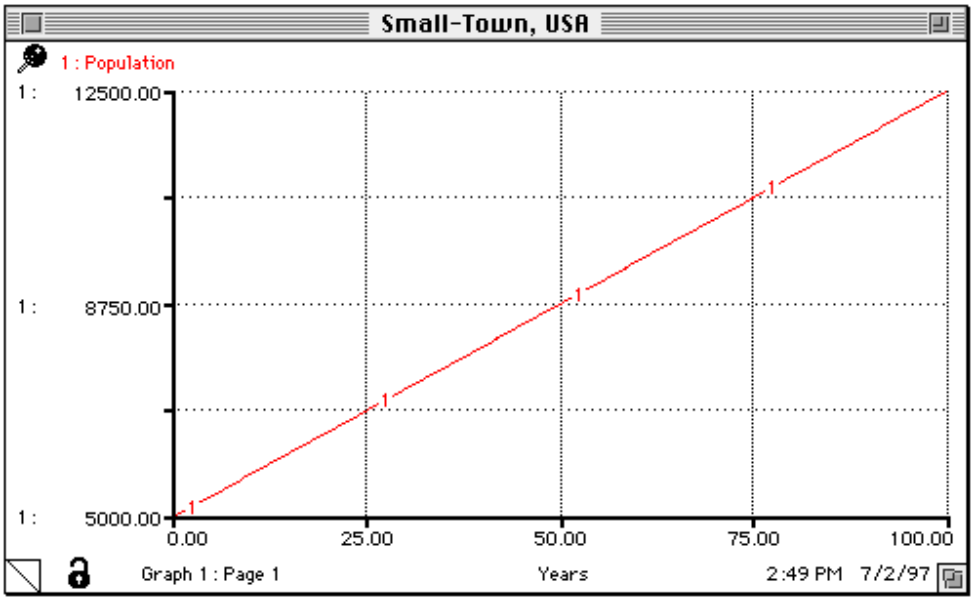


Figure 26: Small-Town over a period of 100 years

Now lock the graph for Small-Town by clicking once on the lock and repeat the same procedure for Dilapidated-Village.

- **Click on "births."**

Enter 50 into the equation box. Click OK.

- **Click on "deaths."**

Enter 125 into the equation box. Click OK.

Now we need to set up the second graph.

- **Flip to the graph titled "Dilapidated-Village, USA"** (by clicking on the page protruding behind the fold in the low left corner of the graph pad)**.**
- **Unlock the graph.**
- **Double-click on the graph pad.**

Again let us remove the constant "births" and "deaths" flows.

- **Click-and-hold on "births" in the *Selected* inputs box.**
- **Drag the pointer down to also highlight "deaths." Release.**
- **Click on the double arrow << to send the two flows back to the *Allowable* inputs box.**

We still need to de-set the scale for "Population:"

- **Click once on "Population" to highlight it.**
- **Click twice on the arrow ↧ to the right of "Population."**

The arrow should once again look like ↕.

- **Click on the *De-Set* button. Click OK.**

Now, we are ready to run the model. Bring the run controller into view (by clicking on the little runner in the bottom left corner of the screen).

- **Click on the *play* button.**

You will see the graph displayed in Figure 27.

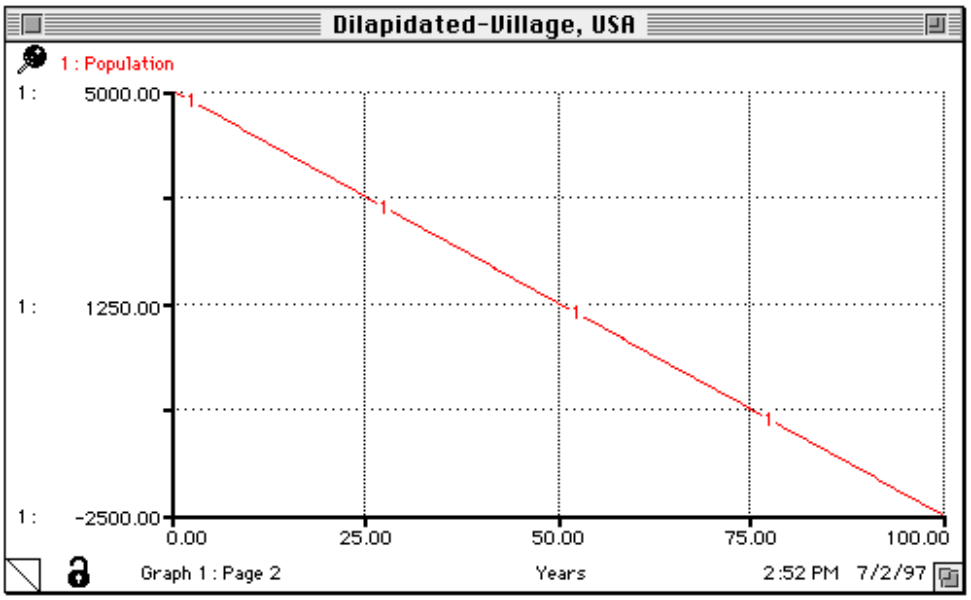


Figure 27: The population of Dilapidated-Village over a period of 100 years

Wait! What happened?! Supposedly, in 100 years, the population of Dilapidated-Village will be negative 2,500 people! Why does the model generate such behavior? Well, each year 50 new babies are born in Dilapidated-Village. Meanwhile, 125 people die. Therefore each year the total population of the town is increased by 50 people and decreased by 125 people. The net effect is for the total population to be 75 people smaller each year. After 65 years the population is down to (5000 people initially minus 75 people a year times 65 years) 125 people. By year 66 there are only 50 people left in Dilapidated-Village. During the following year, if again 50 people are born and 125 die, then the population should equal negative 25 people. Can you see the error in our logic?

The error lies in the fact that in the 67th year there are only 50 people left in Dilapidated-Village, yet, according to our formulation, those 50 people give birth to 50 babies. Even less realistic is the assumption that, of those 50 people, 125 people will die!

Why did the model give us such unrealistic behavior? Because we oversimplified our representation of the “births” and “deaths” flows. There is no way that the flow of deaths can be greater than the actual population. In real life, the death rate of a population depends on the current size of that population. So does the birth rate.

Initially, we made the assumption that the birth and death rates of Small-Town and Dilapidated-Village would be constant. We could make this assumption because we

were running the model over a relatively short time period: 12 years. In 12 years, babies grow to be teenagers, but do not yet contribute new babies to the population. In 12 years population levels only slightly rise or fall. However, when we decided to look at the population of our two towns over a period of 100 years, we could no longer assume that the population would stay pretty much the same and that the birth and deaths rates would remain constant. Over a period of 100 years, a population can change drastically—and the birth and death rates of that population would change accordingly.

What really determines a birth rate? Let us say that the number of births per year is some fraction of the existing population. So, for any given year, the number of births will *depend* upon the size of the population that year. We can show such a dependence with a **connector** (the fourth building block).

- **Select the connector . Release your click.**
- **Place the cursor inside the "Population" stock.**
- **Click-and-hold. Drag the cursor out of the stock until it makes contact with the circle portion of the births flow** (the flow will color upon contact)**. Release your click.**

Your diagram should look like the diagram in Figure 24.

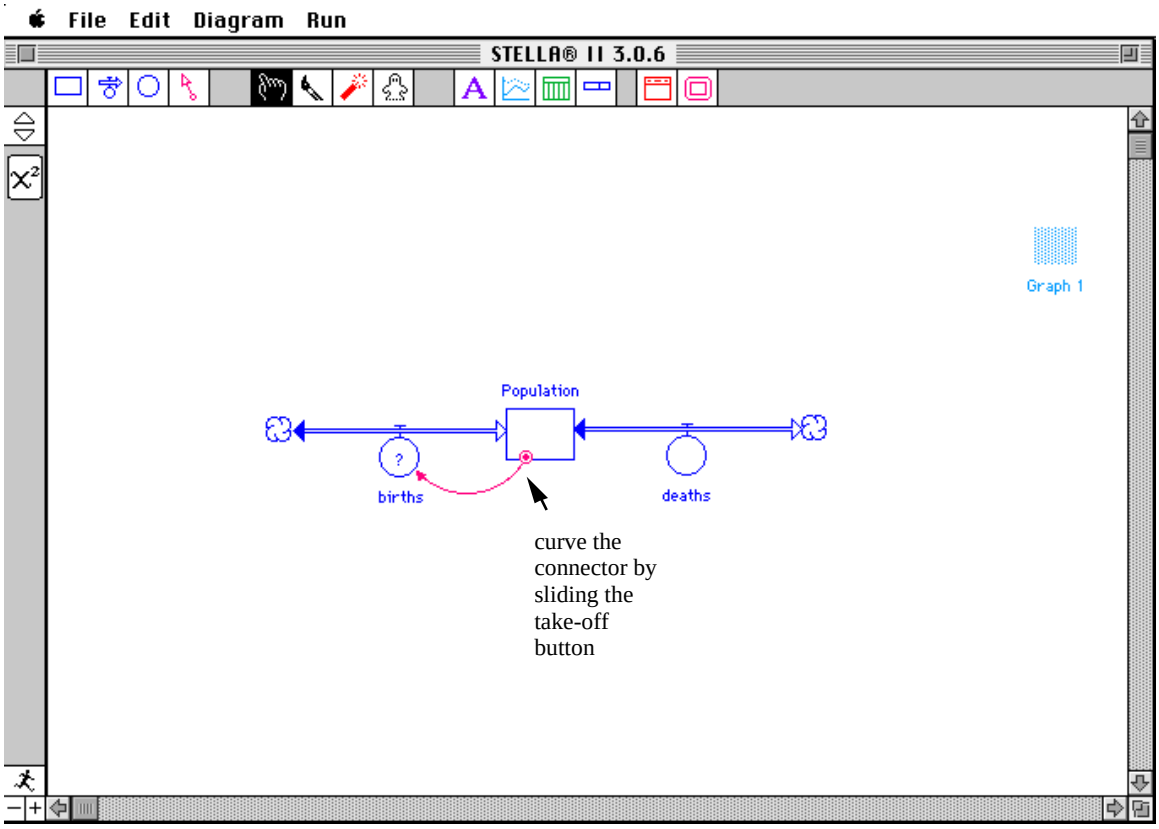


Figure 28: Adding a connector

You have now shown that a relationship exists between “Population” and “births.” If you are picky about the arc traced by your connector, you can bend and straighten it any way you like. To do so, click-and-hold on the little “take-off” button located on the edge of the building block (here the stock) from which the connector takes off. You will then be able to slide the button around the edge. As you do, the arc of the connector will change. Try it.

Notice that now “births” contains a “?” again. By drawing a connector between “Population” and “births,” you are saying that the equation for “births” must contain the variable “Population.” The current equation, 150 people/year, is no longer valid. With the “?” STELLA is reminding us that we need to redefine our flow.

We said that the birth rate would be a fraction of the existing population. Any old fraction? No. The fraction would represent the fertility of the population: how often the population reproduces. Small-Town, USA had an initial population of 5000 people. Initially, they also had a birth rate of 150 people/year. To calculate the birth fraction for

Small-Town, we divide the average birth rate by the average population to obtain the birth rate per person living in Small-Town. Therefore the birth fraction for Small-Town is be 150 people/year divided by 5000 people which equals 0.03 or 3% per year. Therefore, every year, the population of Small-Town increases by 3%.

What do you think the birth fraction is for Dilapidated-Village? Well, initially the birth rate was 50 people/year. The initial population was also 5000 people. So the birth fraction is 50 people/year divided by 5000 people which equals 0.01 or 1% per year.

We can put our birth fraction into our model by using a converter. Let us place a converter in our model now.

- **Select the converter ○. Release your click.**
- **Position the converter below and to the left of "births."**
- **Click to place it on the page.**

Your screen will look like the screen in Figure 29.

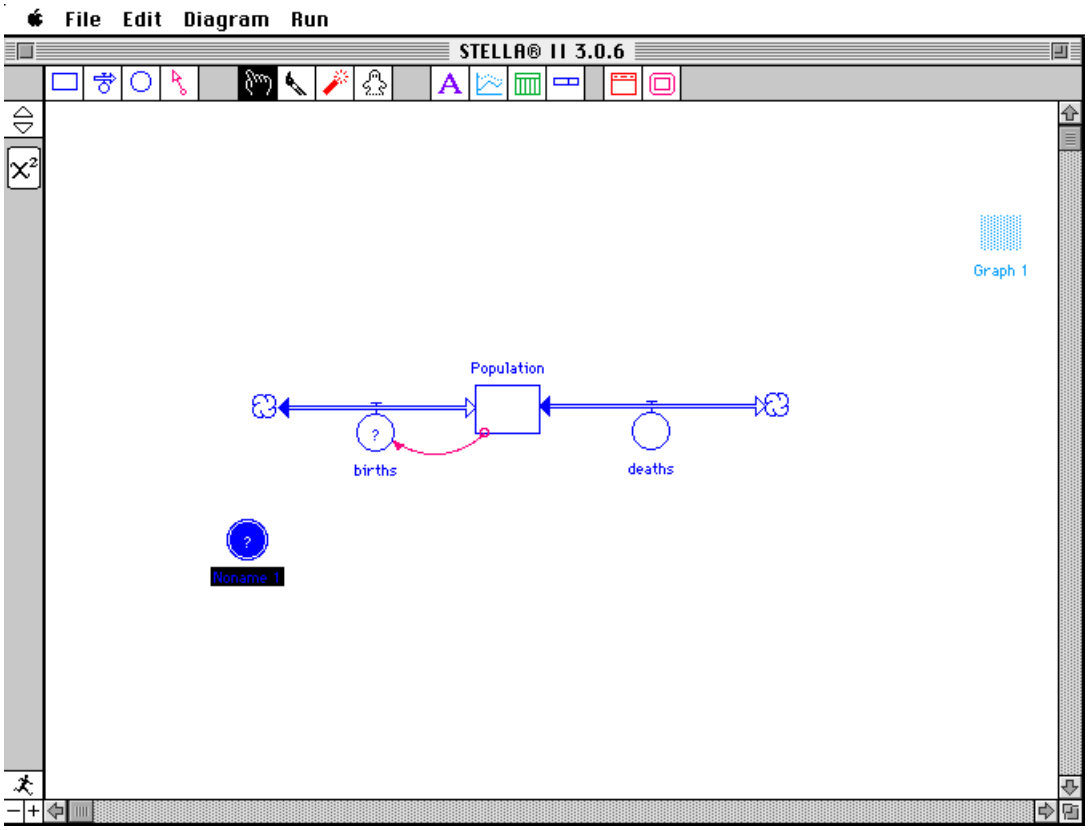


Figure 29: Adding a converter

Now we should name the converter.

- **Label the converter "BIRTH FRACTION."**

We know that the flow of "births" depends on the "BIRTH FRACTION," so we need a connector linking the "BIRTH FRACTION" to "births." Let's make the link.

- **Select the connector . Release your click.**
- **Place the cursor inside the "BIRTH FRACTION" converter.**
- **Click-and-hold. Drag the cursor out of the converter until it makes contact with the circle portion of the "births" flow** (the flow will color upon contact)**. Release your click.**

You diagram will look like the diagram in Figure 30.

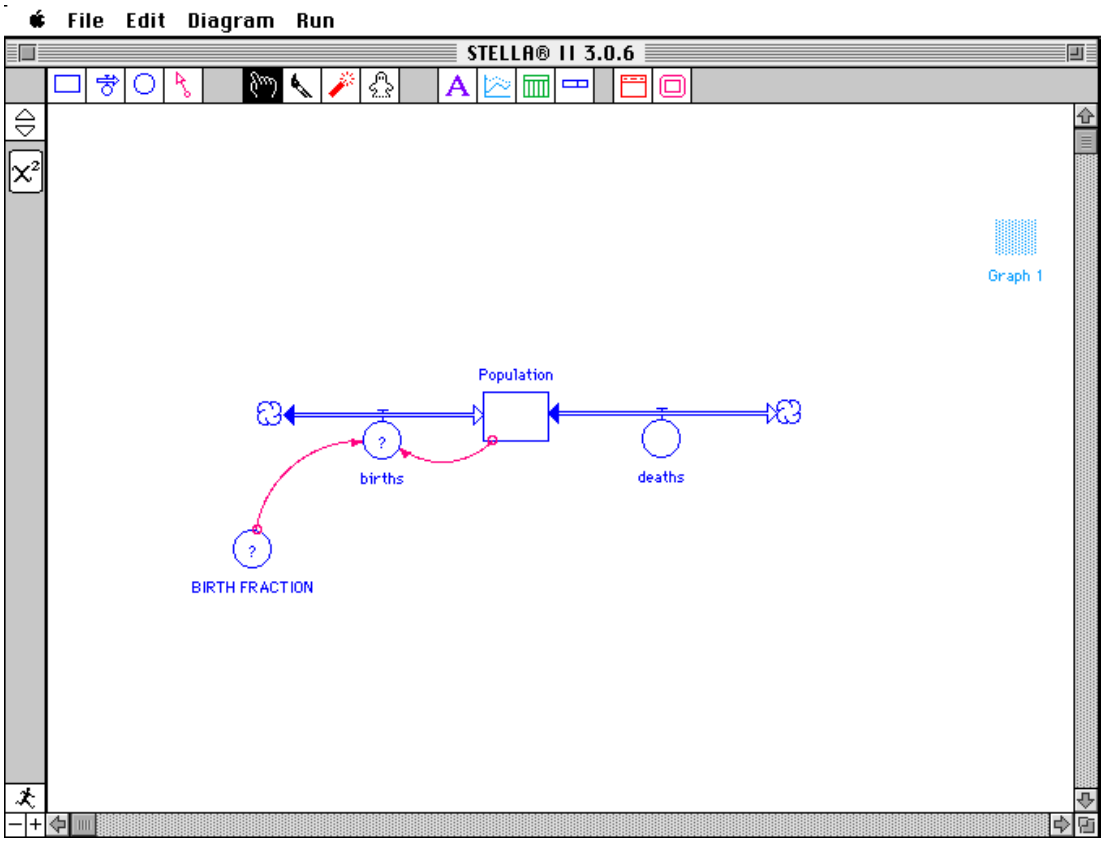


Figure 30: Population model with a birth fraction

Now let's define the "BIRTH FRACTION."

- **Double-click on "BIRTH FRACTION."**

The window that defines converters looks exactly like the window used to define flows.

We will first model our first scenario: Small-Town. We calculated earlier that the birth fraction for Small-Town is 3%, or 0.03 per year.

- **Type "0.03" in the equation box.**
- **Click on the *Document* button.**
- **Type "The birth fraction for the town is calculated by dividing the average birth rate by the average population. The birth fraction represents the fertility of the population."**
- **Type "UNITS: (people/year)/person which simplifies to 1/year"**
- **Click on the *Hide Document* button. Click OK.**

We are finally ready to redefine the "births" flow.

- **Double-click on "births."**

You will see the window replicated in Figure 31.

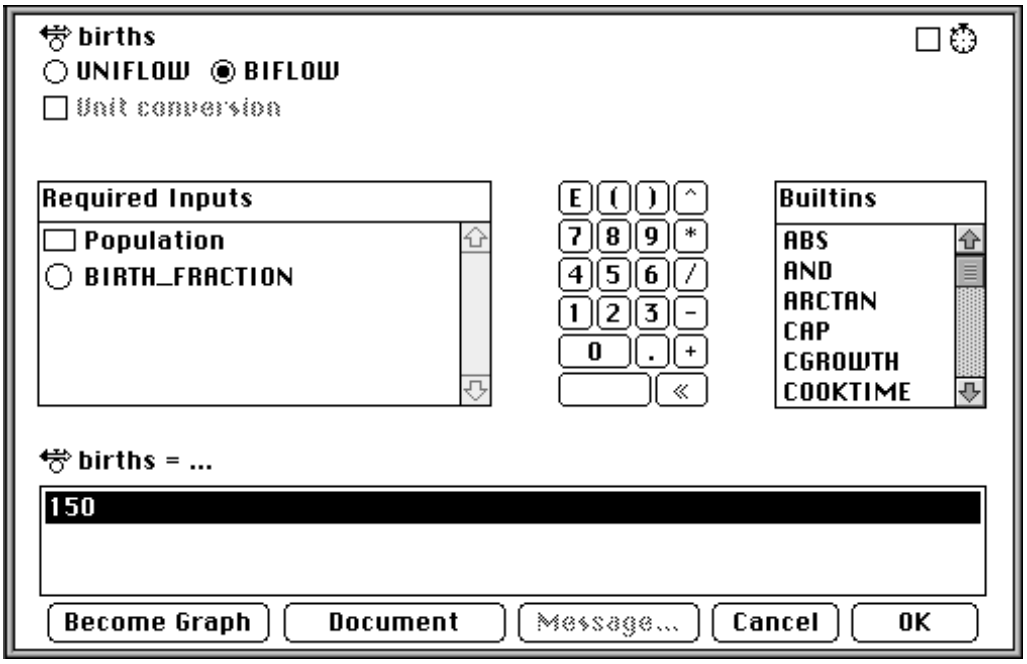


Figure 31: Redefining the flow

As you can see, the Required Inputs list now contains both "Population" and "BIRTH FRACTION." Remember that "Population" and "BIRTH FRACTION" are *required* because we drew connectors from "Population" and "BIRTH FRACTION" to "births," meaning that "Population" and "BIRTH FRACTION" are *inputs* to "births." In other words, "births" *depend* on both "Population" and "BIRTH FRACTION."

- **Click on "Population" in the *Required Inputs* box.** It will appear in the Equation box.
- **Click or type the asterix "*" the symbol for multiplication.**
- **Click on "BIRTH_FRACTION" in the *Required Inputs* box.**

Your equation will now say: births = Population*BIRTH_FRACTION. We need to modify our documentation.

- **Click on the *Document* button.**
- **Replace the old documentation with "Births depend on the current population and a birth fraction that represents the fertility of the population."**
- **Click OK.**

We now have to make some very similar changes to the "deaths" flow. The death rate is not really constant. "Deaths" also depend on the current population and a death fraction. The death fraction represents the mortality of the population. To calculate the death fraction, we need to divide the average death rate by the average number of people in the population. In Small-Town, the initial death rate was 75 people per year. The population was initially 5000 people. So the death fraction is 75 people per year divided by 5000 people, which equals 0.015, or 1.5%, per year. Can you calculate the death fraction for Dilapidated-Village? In Dilapidated-Village the initial death rate was 125 people/year and the initial population was 5000 people. So the death fraction is 125 people/year divided by 5000 people which equals 0.025, or 2.5%, per year. Let us incorporate the death fraction into the model.

- **Select the converter. Release your click.**
- **Position the converter below and to the right of "deaths."**
- **Click to place it on the page.**
- **Label the converter "DEATH FRACTION."**

Now we need to add the connectors.

- **Select the connector. Release your click.**
- **Place the cursor inside the "DEATH FRACTION" converter.**
- **Click-and-hold. Drag the cursor out of the converter until it makes contact with the circle portion of the "deaths" flow** (the latter will color upon contact)**. Release**

your click.

Now you will learn a trick in STELLA to avoid having to select an object twice in a row.

- **Press down on the *alt option* key on your keyboard.**

You will notice that the icon you are holding is once again a connector.

- **Still holding down the *alt option* key, click once in the stock of "Population."**
- **Click-and-hold. Drag the cursor out of the converter until it makes contact with the circle portion of the "deaths" flow** (the latter will color upon contact)**. Release your click.**

Your screen should look like the screen in Figure 32.

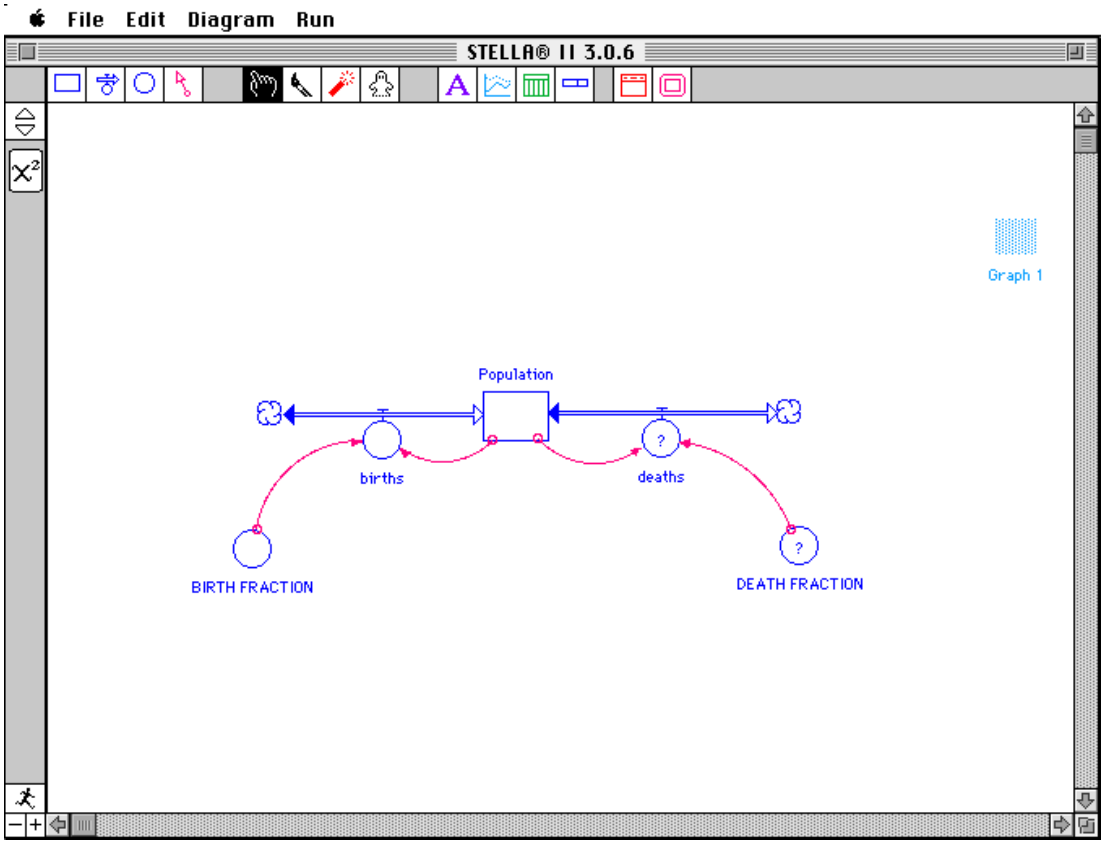


Figure 32: Incorporating a death fraction

Now let's define the "DEATH FRACTION."

- **Double-click on "DEATH FRACTION."**

We calculated earlier that the birth fraction for Small-Town is 1.5%, or 0.015 per year.

- **Type "0.015" in the equation box.**

- **Click on the *Document* button.**
- **Type "The death fraction for the town is calculated by dividing the average death rate by the average population. The death fraction represents the mortality of the population."**
- **Type "UNITS: (people/year)/person which simplifies to 1/year"**
- **Click on the "Hide Document" button. Click OK.**

We are finally ready to redefine the "deaths" flow.

- **Double-click on "deaths."**

The *Required Inputs* list now contains both "Population" and "DEATH FRACTION."

- **Click on "Population" in the *Required Inputs* box.** It will appear in the Equation box.
- **Click or type the asterix "*" the symbol for multiplication.**
- **Click on "DEATH_FRACTION" in the *Required Inputs* box.**

Your equation will now say: deaths = Population*DEATH_FRACTION. To modify the documentation:

- **Click on the *Document* button.**
- **Replace the old documentation with "Deaths depend on the current population and a death fraction that represents the mortality of that population."**
- **Click OK.**

Now, for the sake of clarity, we will open a new graph pad for the next simulations.

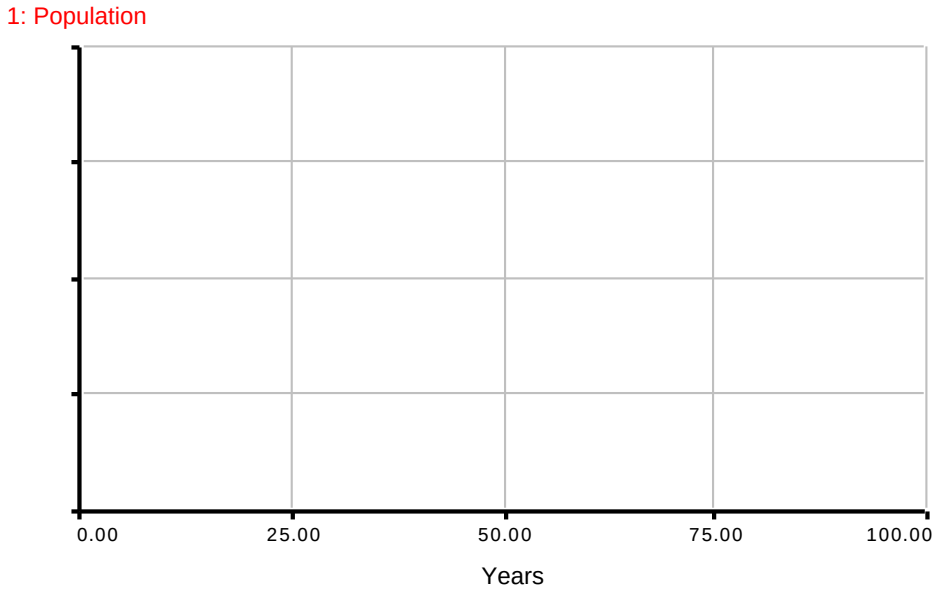
- **Select the graph icon** (the second object, the little graph) **by clicking once on it. Release your click.**
- **Slide the cursor anywhere inside the picture border.**
- **Click once to deposit the graph icon.**

For our first run, we will only look at the behavior of the population of Small-Town over time.

- **Double-click on the graph pad.**
- **Double-click on "Population" in the *Allowable Inputs* list.**
- **Click OK.**

Now, make sure that both of our earlier graphs with constant flows are locked. (You can check that the padlock in the lower left corner of the graphs is closed.)

Finally, before we run the model, we should get some idea of the behavior that we expect to see. We can then see if our expectations are confirmed. In the graph below, sketch what you think might happen to the population of Small-Town.



Now we are ready to run the model.

- **Hold down the open-apple key (with the symbols ⌘) and press the "r" key.**

You should see the graph reproduced in Figure 33.

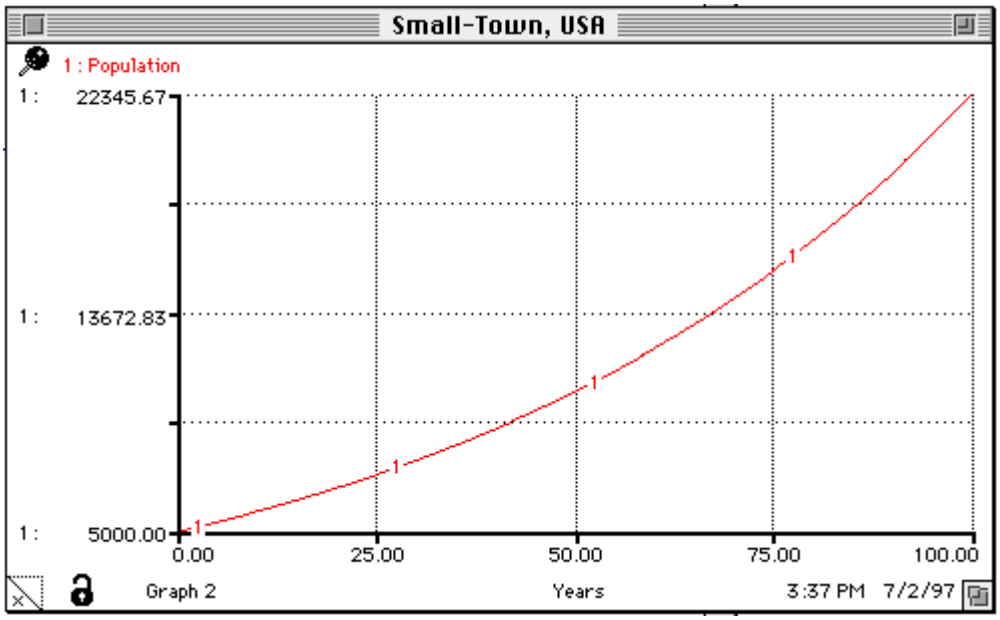


Figure 33: The behavior of the population of Small-Town

The population of Small-Town skyrockets to over 22,000 people! Can you explain why?

The key to understanding the behavior of the "Population" of Small-Town lies in understanding the principles of feedback. Feedback occurs when an output of a system is fed back into the system as an input. What does this mean in English? Take as an example one of the two feedback loops we just built into our population model. Follow the arrows in Figure 34. The rate of "births" determines the "Population." At the same time, "Population" determines "births." The more people there are, the more births there are, which leads to there being even more people. The compounding process is called positive, reinforcing, feedback. Positive feedback generates the exponential growth observed in the population of Small-Town.

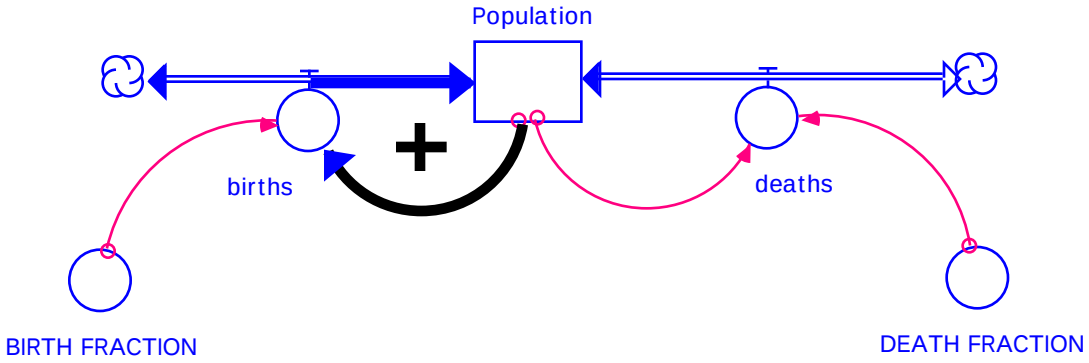


Figure 34: Positive feedback drives the population of Small-Town

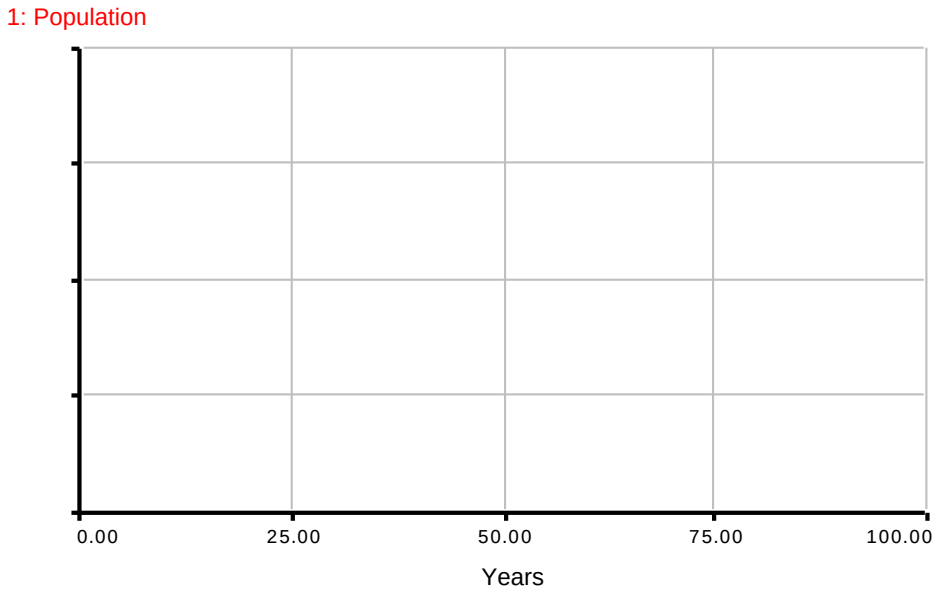
Now let us also graph the behavior of Dilapidated-Village. First, lock the graph of Small-Town. Then...

- **Double-click on the graph pad.**
- **Select a new graph.**
- **Label the new graph "Dilapidated-Village, USA"**
- **Double-click on "Population" in the *Allowable* inputs box. Click OK.**

Now to change the model:

- **Double-click on "BIRTH FRACTION."**
- **Type "0.01" into the equation box. Click OK.**
- **Double-click on "DEATH FRACTION."**
- **Type "0.025" into the equation box. Click OK.**

Before you run the model, sketch what you think will happen in the graph below.



Now, see how closely your prediction mirrors the simulation. Another way to run a model is to select the run option from the menu bar. The menu bar is the row along the top of your screen with the options File, Edit, Diagram, and Run.

- **Click-and-hold on *Run*.**
- **Drag the pointer down to the first selection: Run. Release.**

You will see the graph displayed in Figure 34.

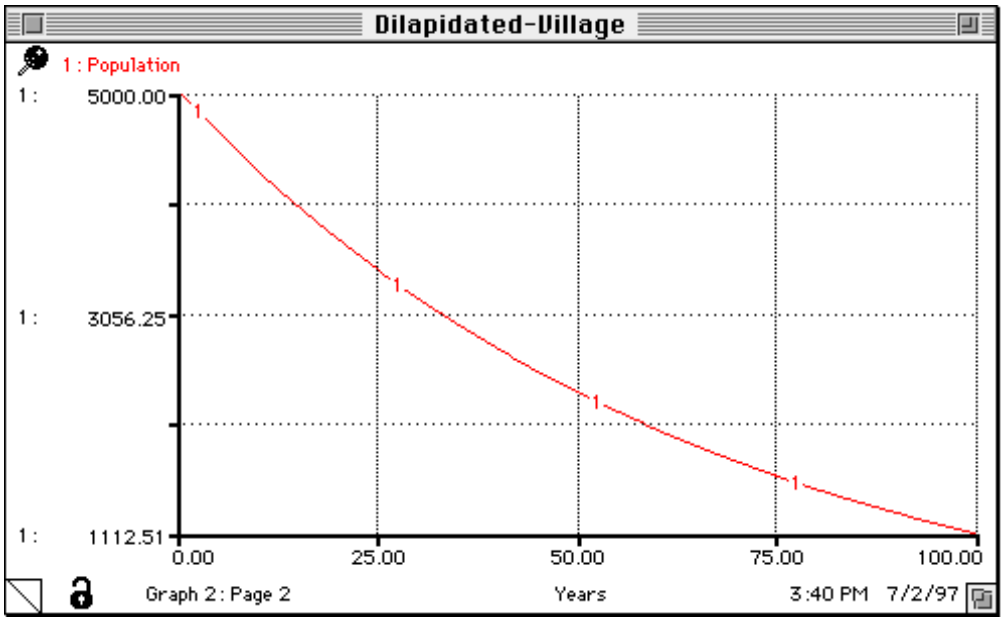


Figure 35: The behavior of the population of Dilapidated-Village

The population of Dilapidated-Village experiences exponential decay. Small-Town was characterized by a high birth rate. Dilapidated-Village on the other hand had a heavy death rate. The population of Small-Town was driven by a positive feedback loop. In contrast, Dilapidated-Village is driven by a negative, balancing, loop. Follow the loop in Figure 36. The greater the population is, the higher the death rate is, which reduces the population.

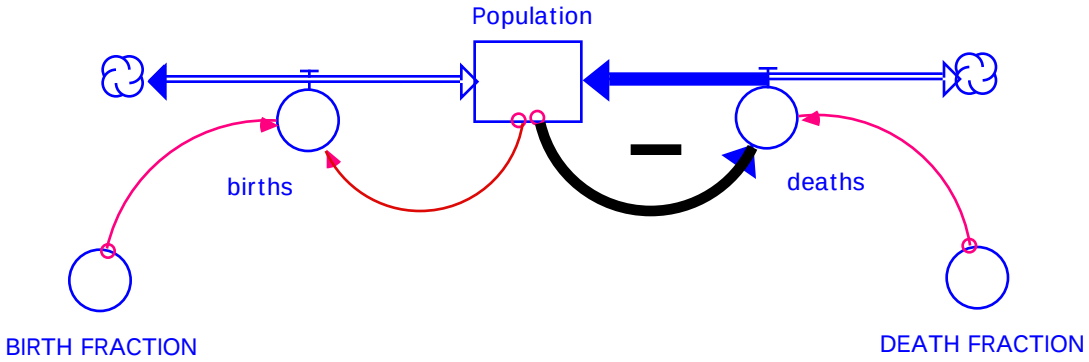


Figure 36: Negative feedback drives the population of Dilapidated-Village

Similarly, the smaller the population is, the less the death rate is, so while the population still decreases, it decreases by less. As the population gets smaller and smaller, the

number of deaths also gets smaller and smaller. So "Population," instead of becoming negative, asymptotically approaches zero.

Do not worry if terms like exponential or asymptotic are unfamiliar. The First Three Hours is meant to serve only as an introduction to system dynamics modeling. As you continue working through Road Maps, the spiral-learning approach will visit and revisit these new concepts.

We are now ready to move on to the debriefing. Let us save the model.

· **Hold down the open-apple key (with the symbols ⌘) and press "s."**

Once the model is saved, we can quit STELLA.

· **Hold down the open-apple key (with the symbols ⌘) and press "q."**

You can also select *File* the tool bar and drag your pointer down to the option *Quit*.

7. DEBRIEFING

You have just completed a great deal of material. You have built a population model from scratch. In the process, you have experimented with the STELLA software. You have learned to use all of the basic STELLA commands. You have also been quickly introduced to several of the key concepts in system dynamics: the use of stocks and flows, model boundaries, time horizons, and feedback. All of these concepts will be taken, one by one, and explored in depth in Road Maps.

At this point you may see some potential in system dynamics modeling applied to the classroom. Most educators, however, ask how they are supposed to incorporate system dynamics and learner-centered learning projects into an already overloaded curriculum. Consider what a science teacher has to say about his experience with introducing system dynamics.

> "Our classrooms have undergone an amazing transformation. Not only are we covering more material than just the required curriculum, but we are covering it faster (we will be through with the year's curriculum this week and will have to add more material to our curriculum for the remaining 5 weeks), and the students are learning more useful material than ever before."[5]

Courtesy of Frank Draper. Used with permission.

[5] Frank Draper. Orange Grove Middle School. Letter to Jay Forrester, May 2, 1989, p.2.

Consider also the following extract from the evaluation by two science teachers:

> "We utilize a teaching style that transfers the focus of the classroom from the teacher to the student. Students shift from being passive consumers of information that need only be remembered until the next exam, to being active participants in the acquisition and utilization of knowledge. In this environment they become intrinsically motivated to extend the information they have learned. Assigning two or three students to a computer allows them to teach themselves and each other as they work through science projects. An important benefit for teachers is that the learning environment is exciting and rejuvenating for both teachers and students. When students have a good time while learning, classroom discipline problems drop and teachers have more time to spend with slower students. In an open-ended survey, 70 percent of the students in our classes called the systems projects the "best part" of the school year."[6]

By integrating system dynamics and learner-centered learning into current curricula, educators create a learning environment which will serve to anchor facts and details through structural and behavioral relationships. Students might even have a great deal of fun using system dynamics to explore the world through computer simulation.

Included on the next few pages is a list of ideas for other models that could be relevant to your classroom. The following models, used by teachers in their classrooms, are available from the Creative Learning Exchange. By the time you finish Road Maps, you should be able to build similar models on your own.

Pre-college System Dynamics Curricula

Ozone Depletion Model - A model illustrating the chemical reactions involved between CFCs and ozone in the upper atmosphere.

$$Cl + O_3 \rightarrow ClO + O_2$$

$$ClO + O \rightarrow Cl + O_2$$

This model could then be coupled to models of CFC production and ultraviolet radiation. Further expansion could explore increased radiation effects on earth in terms of increased melanoma and crop loss.

Convection Model - A model of solar energy's effect on the temperature of air and the convection currents which arise from these temperature imbalances.

---

[6] Frank Draper and Mark Swanson, 1990. "Learner-Directed Systems Education: A Successful Exampl*e* ." System Dynamics Review, Vol. 6, No. 2, pp. 209-213.

Boiling Water Model - A model illustrating the heat of vaporization as well as the difference between temperature and heat can be produced and studied.

Heat Diffusion - A model of heat diffusion between two regions of different temperatures could be applied to the example of a home during a New England winter. With the addition of a furnace control mechanism, the model could be expanded to illustrate feedback between the desired temperature on the thermostat and the actual temperature. A cost accounting system could then be added for the purpose of allowing students to design their own furnace control mechanism (feedback mechanism) which would operate with minimal costs and maximum convenience.

Nuclear Chain Reaction - A nuclear chain reaction model could show the behavior and structure of a chain reaction as well as keep track of energy released. Additionally, the structure associated with a chain reaction is very generic. This type of generic structure means that a model of a chain reaction could be changed to a model of a disease epidemic by simply changing variable names.

Radioactive Decay - Model of radioactive half-life with a linkage to energy of radiation given off.

Greenhouse Effect - Like many of the other models, a greenhouse effect model can be as simple or complex as you desire. Students might start off with the chemistry and physics behind the greenhouse effect. With the initial concepts mastered, structure might be added so that students could become the policy makers in a greenhouse effect simulation. They would have to learn the science behind the greenhouse effect as well as balance the competing interests of industry and the environment. There might be subsystems representing alternative fuels, the automobile industry, or any other party involved in the system.

Kaibab Plateau Ecosystem - The Kaibab Plateau ecosystem is a region of approximately 730,000 thousand acres north of the Grand Canyon. In the early 1900s a bounty was awarded to hunters who killed the predators in the region. Once the predator population disappeared, the deer population began to multiply to such a large extent (from 4,000 to 100,000) that the deer destroyed their own food supply which ended up nearly destroying the herd. An excellent model of these ecosystem dynamics already exists. Further, rather than simply studying the dynamics, students can act as the

managers of the Kaibab Plateau Wildlife Preserve. This allows them to experiment with policies which will bring the ecosystem into balance.

Electrical, Physical Generic Structure - The idea here would be to show the similarity in structure (generic structure) which is shared by an electrical and physical system.

Insulin Production and Blood Sugar Levels - A quantitative model examining the feedback processes between blood sugar level and insulin production is being produced. Emphasis will be placed on examining the system changes which occur for victims of diabetes. This model can be further coupled to a model of digestion. It could then be expanded to other bodily systems until a model of the human body is produced.

Chemical Reactions
Regulation of Body Temperature
Temperature and Rainfall Regulation
Projectile Motion Model
Motion Down an Inclined Plane
Pressure, Temperature, Volume Interactions
Model of Lightning and Potential Difference
Lynx and Hares Ecosystem
Energy Efficiency in the food chain
Stomate in a Leaf
Photosynthesis

Social Science:

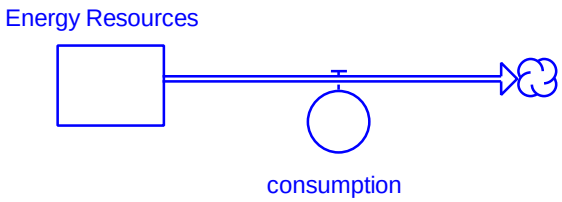
Urban Growth Model - One of the original applications of system dynamics was an analysis of the causes of urban growth and decay which examined the effects of a number of urban renewal programs, including low-income housing, job training, and new enterprise production.[7] Since that time, system dynamics has been applied to the relationships between population, pollution, natural resources, and economic growth.

---

[7] Forrester, Jay. Urban Dynamics. Portland, OR: Productivity Press, 1969.
Alfeld, Louis and Graham, Alan. Introduction to Urban Dynamics, Portland, OR: Productivity Press, 1976
Mass, Nathaniel. Readings in Urban Dynamics Volume 1, Portland, OR: Productivity Press, 1974.
Schroeder, Walter; Sweeney, Robert and Alfeld, Louis. Readings in Urban Dynamics Volume 2, Portland, OR: Productivity Press, 1975.

Pollution causes and Environmental awareness
Case study of Tragedy of Sahel in Sahara Desert
Traffic Jams
AIDS Epidemic
Finite Natural Resources
Causes of the American Revolution

Beginner Modeling Exercises



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Leslie A. Martin
September 5, 1997
Vensim Examples added October 2001

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1. ABSTRACT

The goal of this paper is to teach the reader how to distinguish between stocks and flows. A stock is an accumulation that is changed over time by inflows and outflows. The reader will gain intuition about stocks and flows through an extensive list of different examples and will practice modeling simple systems with constant flows.

2. INTRODUCTION

What is the difference between a stock and a flow? Stocks are accumulations. Stocks hold the current state of the system: what you would see if you were to take a snapshot of the system. If you take a picture of a bathtub, you can easily see the level of the water. Water accumulates in a bathtub. The accumulated volume of water is a stock. Stocks fully describe the condition of the system at any point in time. Stocks, furthermore, do not change instantaneously: they change gradually over a period of time.

Flows do the changing. The faucet pours water into the bathtub and the drain sucks water out. Flows increase or decrease stocks not just once, but every unit of time. The entire time that the faucet is turned on and the drain unplugged, water will flow in and out. All systems that change through time can be represented by using only stocks and flows.

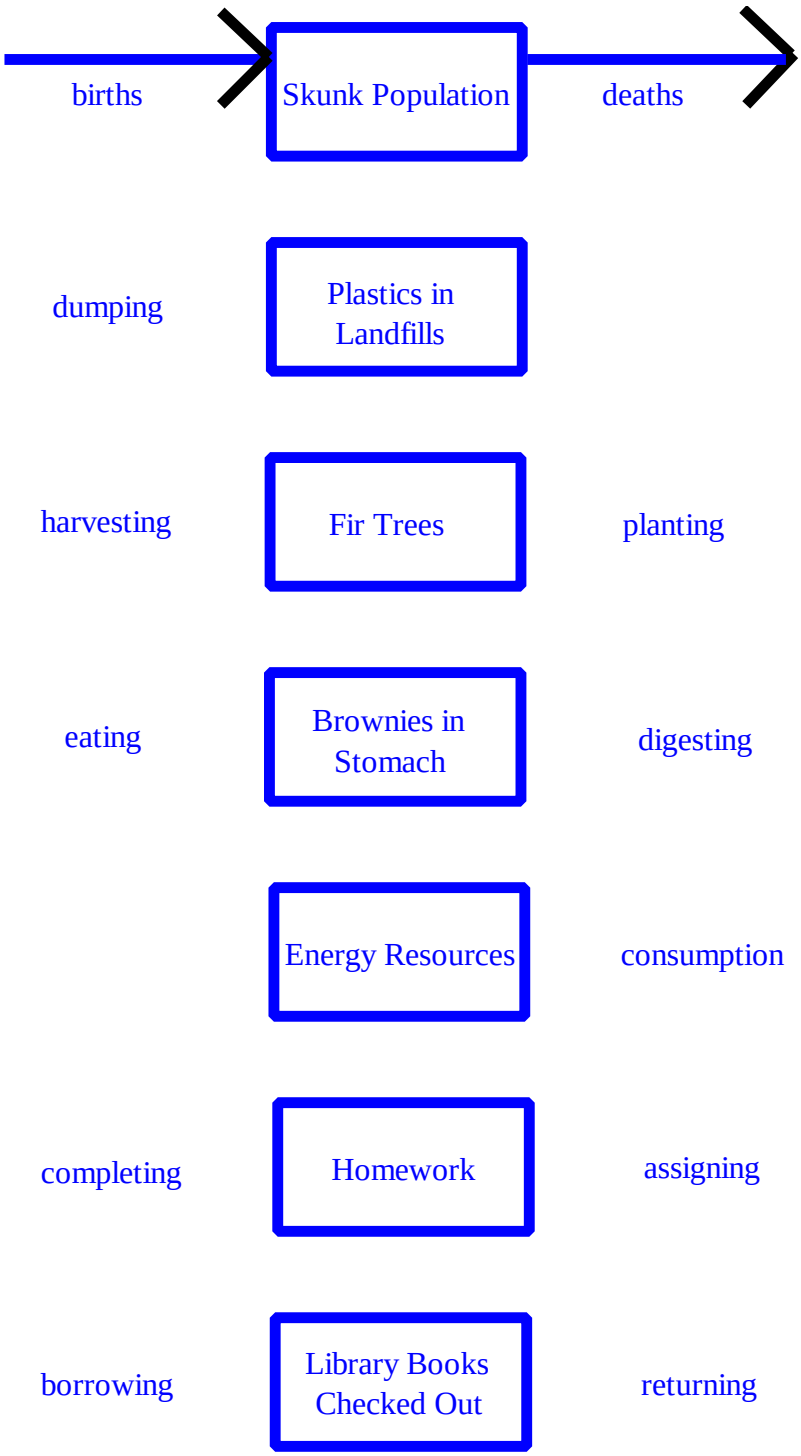
3. STOCKS AND FLOWS

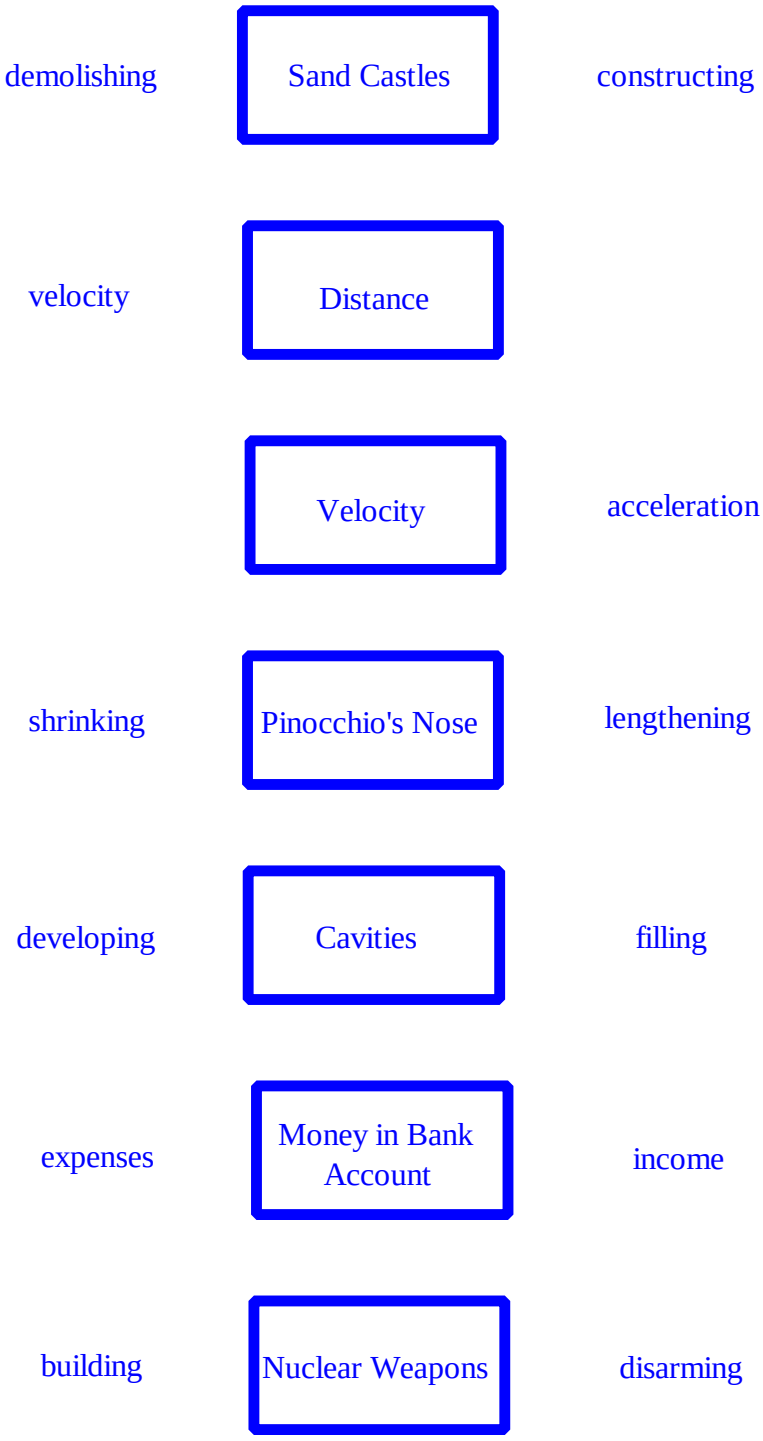
Below are fourteen rows of variables. For each row, identify which variable is a stock and which are the flows that change the stock. Draw a box around the stock. The first row has already been done as an example. The population of skunks is a stock. The size of the skunk population changes with a number of births each year and a number of deaths each year.

births	deaths	skunk population
dumping	plastics in landfills	
harvesting	fir trees	planting
brownies in stomach	eating	digesting
consumption	energy resources	
completing	assigning	homework
returning	borrowing	library books checked out

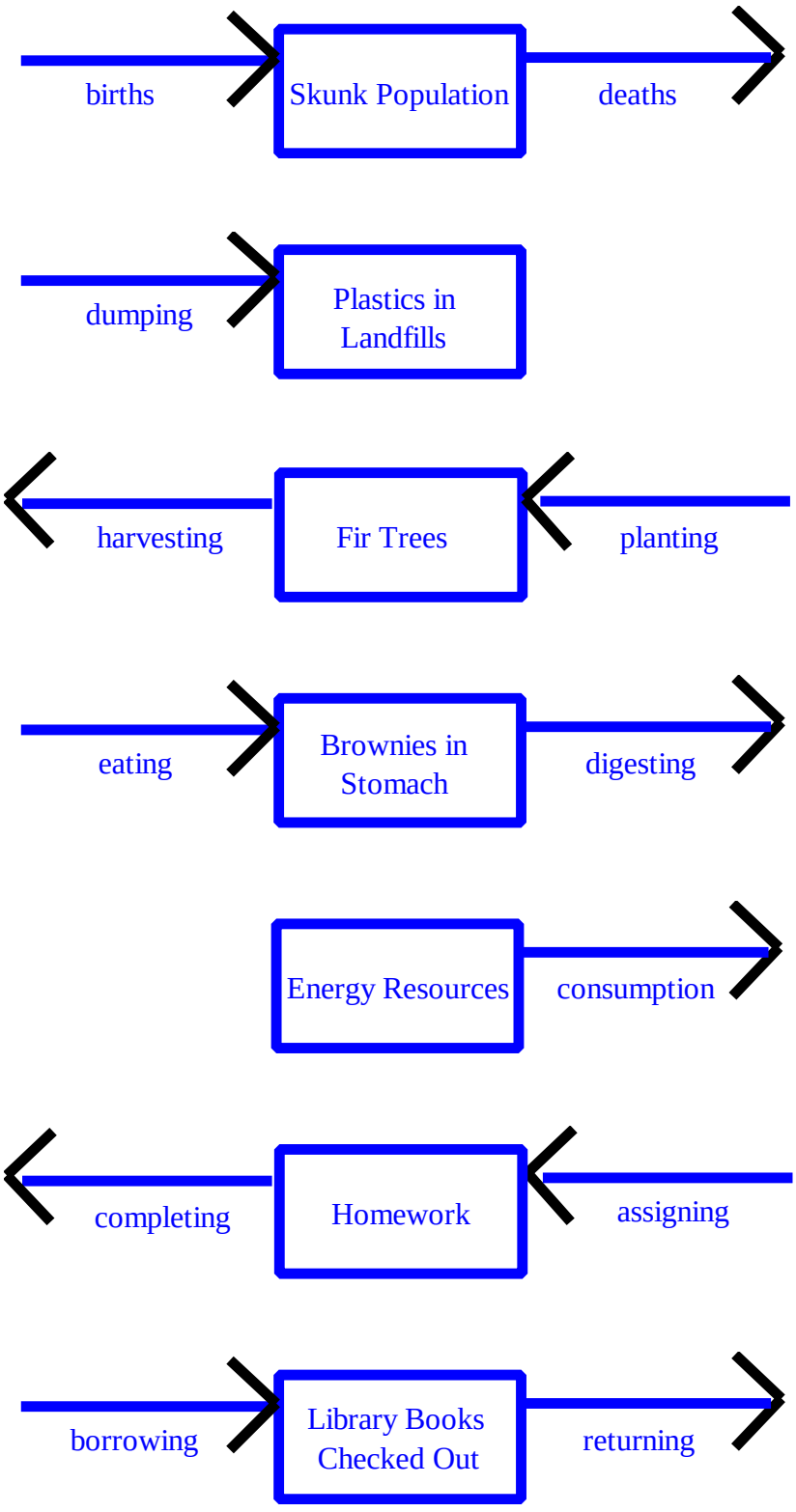
velocity	distance	
velocity	acceleration	
sand castles	demolishing	constructing
shrinking	Pinocchio’s nose	lengthening
cavities	developing	filling
expenses	income	money in bank account
building	nuclear weapons	disarming

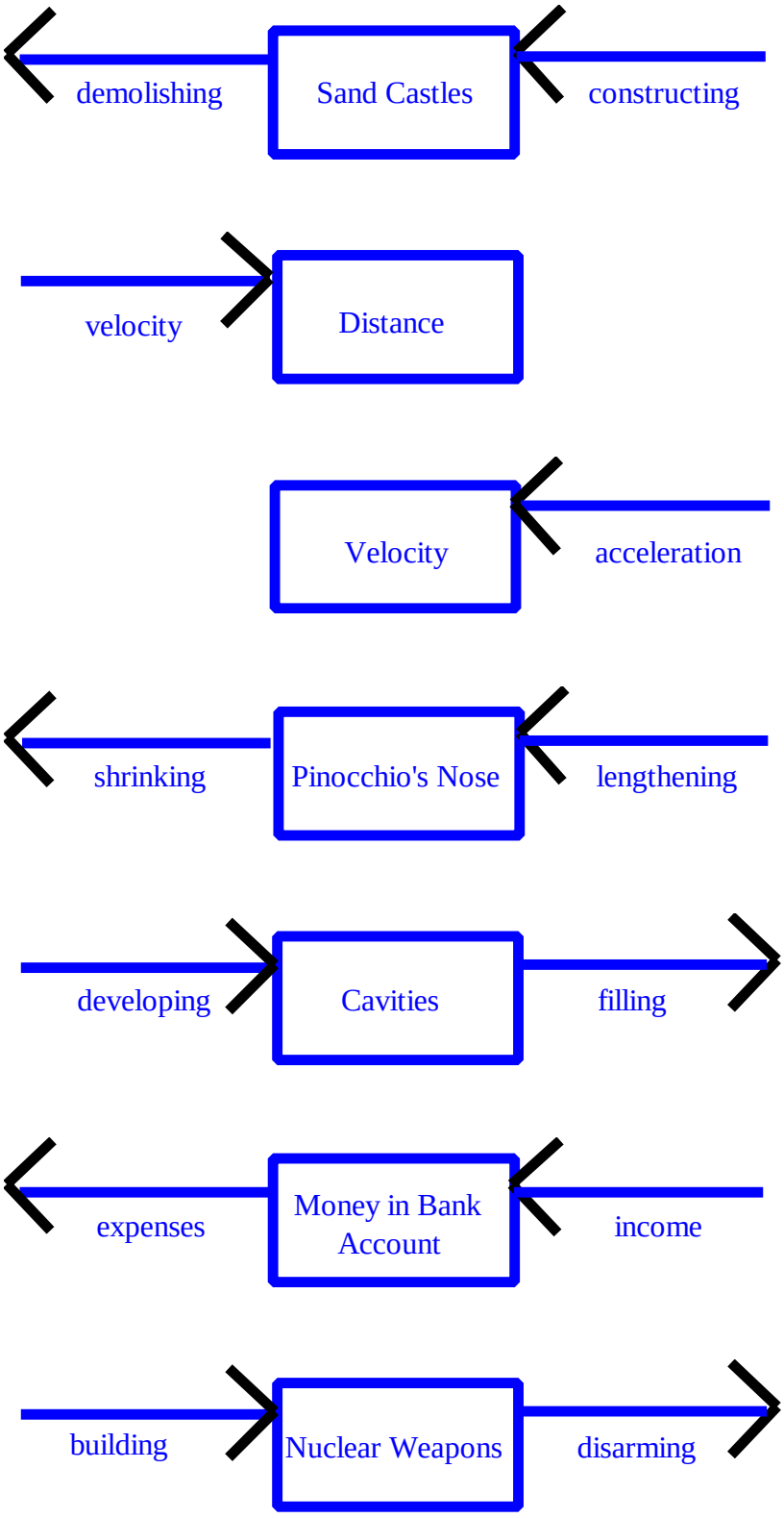
The solution is pictured below. The stocks are in the center, boxed, and the flows are on the outside. Now determine which flows are inflows and which are outflows by drawing arrows into or out of the stocks. The first row has been done as an example. The skunk population is *increased* by births and *decreased* by deaths.




demolishing
Sand Castles
constructing
velocity
Distance
Velocity
acceleration
shrinking
Pinocchio's Nose
lengthening
developing
Cavities
filling
expenses
Money in Bank Account
income
building
Nuclear Weapons
disarming

The solution is depicted below:




demolishing
Sand Castles
constructing
velocity
Distance
Velocity
acceleration
shrinking
Pinocchio's Nose
lengthening
developing
Cavities
filling
expenses
Money in Bank Account
income
building
Nuclear Weapons
disarming

4. MODELING WITH STELLA

4.1 Skunks



Scenario: Five hundred skunks live in the wooded grassy area near the intersection of two interstate expressways. Every year 100 baby skunks are born. Life on a highway takes its toll, though, and every year 120 skunks die.

Question: How many skunks will live near the highway in 10 years?

? Open a new STELLA file.

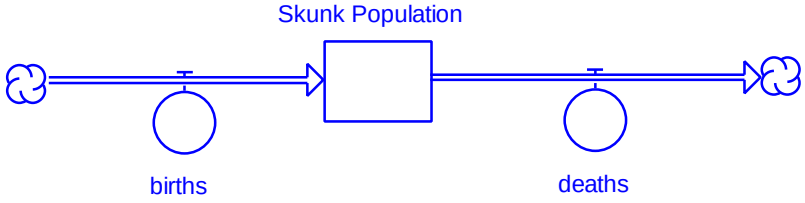
? Go down to the model construction level by clicking on the down arrow ( ).

? Place a stock ( ) in the middle of your screen and name it "Skunk Population."

? Place a flow ( ) to the left of the "Skunk Population" stock and drag the flow into the stock. Name the flow "births."

? Place a flow in the "Skunk Population" stock and drag the flow to the right. Name the flow "deaths."

Your model will look like the model below:



? To go to the equations mode, toggle the globe in the upper-left-hand corner ( ) so that it becomes x^2. Question marks should appear in the stock and flow.

? Double-click on "births" and the dialog box will appear on your screen. Type "100" as the equation for "births."

? Click on the *Document* button. In the document box, type "100 baby skunks are born every year."

? On the next line type "UNITS: skunks/year." Click OK.

? Double-click on "deaths" and the dialog box will appear on your screen. Type "120" as the equation for "deaths."

? Click on the *Document* button. In the document box, type "120 skunks die every year."

? On the next line type "UNITS: skunks/year." Click OK.

? Double-click on the "Population" stock and the dialog box will appear on your screen. Type "500" as the initial skunk population.

? Click on Document. Type "The initial skunk population is 500 skunks."

? On the next line type "UNITS: skunks." Click OK.

To view the equations for the model you have just created, click on the down arrow ( ) above the x^2 icon. Then select Equation Prefs under the Edit menu. Once the Equation Prefs dialog box appears on your screen, click on Show Documentation. Click OK. The following equations will appear:

□ **Skunk_Population**(t) = Skunk_Population(t - dt) + (births - deaths) * dt

INIT Skunk_Population = 500

DOCUMENT: The initial skunk population is 500 skunks.

UNITS: skunks

☞ **births** = 100

DOCUMENT: 100 baby skunks are born every year.

UNITS: skunks/year

☞**deaths** = 120

DOCUMENT: 120 skunks die every year.

UNITS: skunks/year

You will notice that you typed in the information for everything except the first equation for “Population.” Every stock equation is automatically formulated by STELLA when you diagram the flows, so all you need to do is define an initial value.

? Click on the up arrow (⌓) to return to the model construction level.

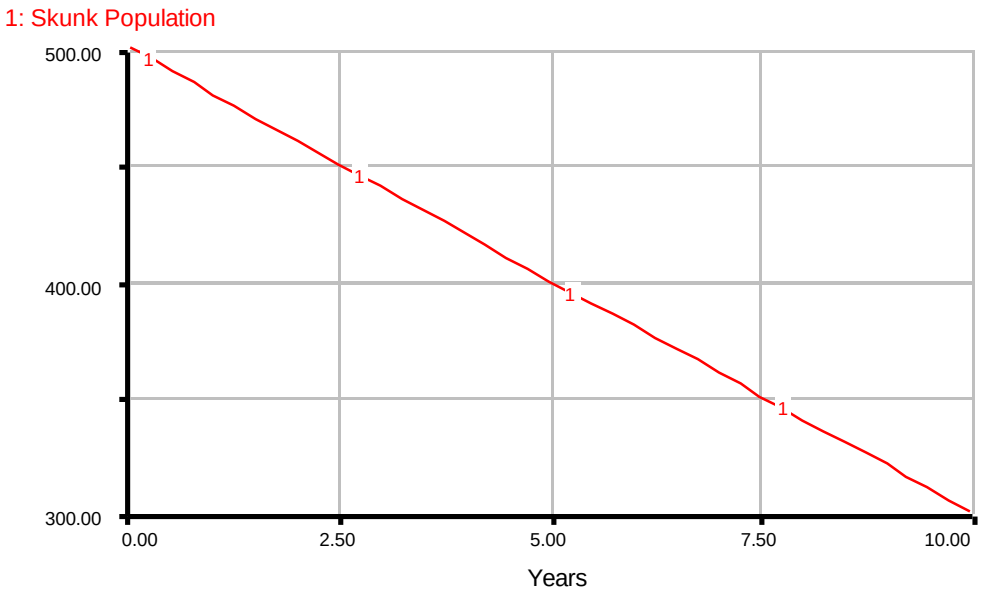
? Create a graph by clicking the graph icon () and placing it onto the workspace. Double-click on the icon.

? Double-click on the graph pad itself to open it. When the dialog box opens, name the graph "Skunk Population." Double-click on “Population” from the Allowable Inputs list to select it to be graphed. Click OK.

? Select Time Specs in the Run menu. When the dialog box appears, select “Years.” Set the Range values from 0 to 10. Click OK.

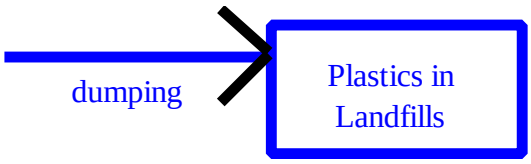
? To run the graph, either select Run from the Run menu, hold down both open-apple (⌘) and “R,” or click on the little runner to bring up the run controller window and click on the play button.

You will see the graph replicated below:



One quick glance at the graph will give you the answer to our question: Ten years from now, 300 skunks will populate the grassy area beyond the interstate.

4.2 Landfills

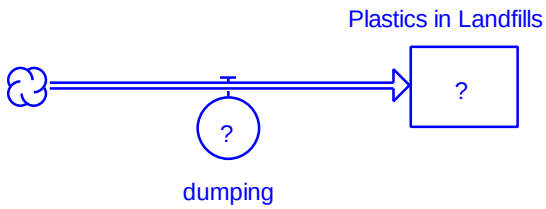


Scenario: The city of Boise, Idaho is building a new landfill. The city council wants to know how large the landfill will be in twenty years so that it can plan ahead and allocate enough space for all of the trash that will be dumped into the landfill. The trash in the landfill can be separated into two categories: the trash that will quickly decompose, like yard leaves, and the trash that will take a long time to decompose, like plastics. The city council predicts that, over the next twenty years, the citizens of Boise will be dumping approximately five thousand gallons of plastics into the landfill every day.

Question: How many gallons of plastics will the Boise landfill contain in twenty years time?

? Dynamite the skunk population model by positioning the dynamite icon ( ) over the stock and flows.
? Place a stock ( ) in the middle of your screen and rename it "Plastics in Landfills."
? Place a flow ( ) to the left of the "Plastics in Landfills" stock and drag the flow into the stock. Rename the flow "dumping."

The plastics will not significantly decompose over a period of 20 years, so you do not need to add an outflow to the stock of "Plastics in Landfills." Your model will look like the model below:



The citizens of Boise dump 5,000 gallons of plastics a day into the landfill. Because there are on average 365 days in a year, the citizens of Boise dump 5,000 * 365 gallons of plastics each year.

? Double-click on "dumping" and the dialog box will appear on your screen. Type "5000 * 365" as the equation for "dumping."

? Click on the *Document* button. In the document box, type "Citizens in Boise will dump 5,000 gallons of plastics into the landfill every day for the 365 days that make up a year."

? On the next line type "UNITS: gallons of plastics/year." Click OK.

? Double-click on the "Plastics in Landfills" stock and the dialog box will appear on your screen. Type "0" as the initial skunk population.

? Click on Document. Type "Initially the landfill is empty."

? On the next line type "UNITS: gallons of plastics." Click OK.

To view the equations for the model you have just created, click on the down arrow (▽) above the x^2 icon. The following equations will appear:

□ **Plastics_in_Landfills**(t) = Plastics_in_Landfills(t - dt) + dumping * dt

INIT Plastics_in_Landfills = 0

DOCUMENT: Initially the landfill is empty

UNITS: gallons of plastics

☞ **dumping** = 5000 * 365

DOCUMENT: Citizens in Boise will dump 5,000 gallons of plastics into the landfill every day for the 365 days that make up a year.

UNITS: gallons of plastics/year

? Click on the up arrow (⌓) to return to the model construction level.

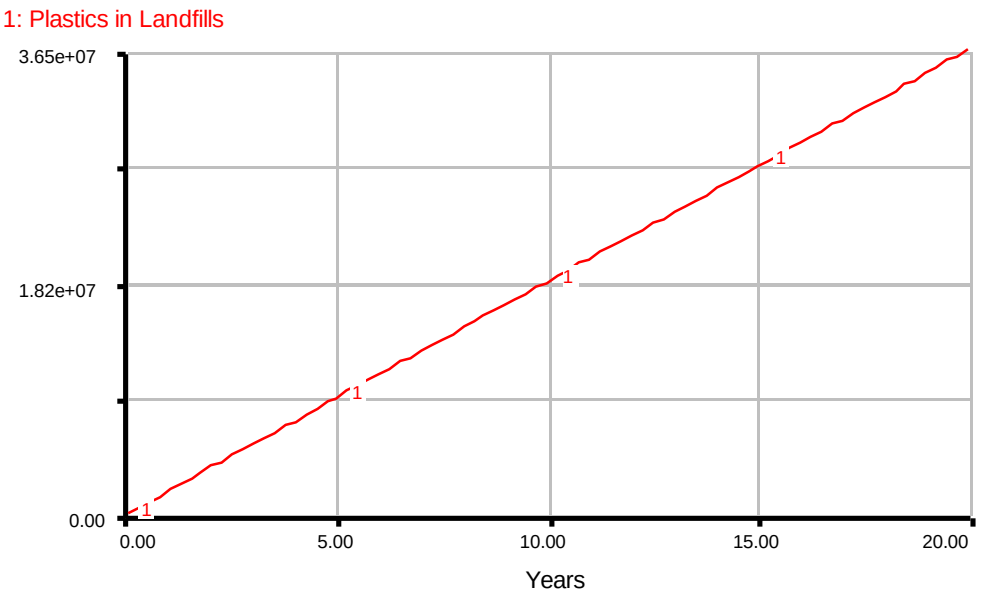
? Double-click on the graph icon (📈) if the graph pad is no longer in view. Notice that the graph was erased when you used the dynamite to blow up the skunk population model.

? Double-click on the graph pad itself to open it. When the dialog box opens, name the graph "Landfills." Double-click on “Plastics in Landfills” from the Allowable Inputs list to select it to be graphed. Click OK.

? Select Time Specs in the Run menu. “Years” should already be selected. Set the Range values from 0 to 20. Click OK.

? To run the graph, either select Run from the Run menu, hold down both open-apple (⌘) and “R,” or click on the little runner to bring up the run controller window and click on the play button.

You will see the graph displayed below:



From the graph you can answer our question by translating 3.65e+07 from scientific notation (by multiplying 3.65 by 10^7): in 20 years, the landfill outside of Boise will contain 36 and a half million gallons of plastics.

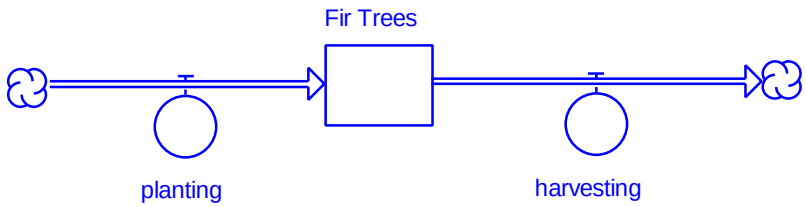
4.3 Fir Trees



Scenario: Today, approximately five million fir trees stand tall in Hardwood Forest. A lumber company has been cutting down, harvesting, approximately one hundred thousand trees every year. An environmental group, worried that the forest will be entirely destroyed, has been working hard to plant as many new fir trees as possible. They have been able to plant approximately five thousand trees every year.

Question: How many fir trees will there be in Hardwood Forest in thirty years?

Answer: 2 million 150 thousand fir trees



Fir_Trees(t) = Fir_Trees(t - dt) + (planting - harvesting) * dt

INIT Fir_Trees = 5e6

DOCUMENT: The initial number of fir trees is 5 million (or 5,000,000 or 5e6)

UNITS: fir trees

planting = 5000

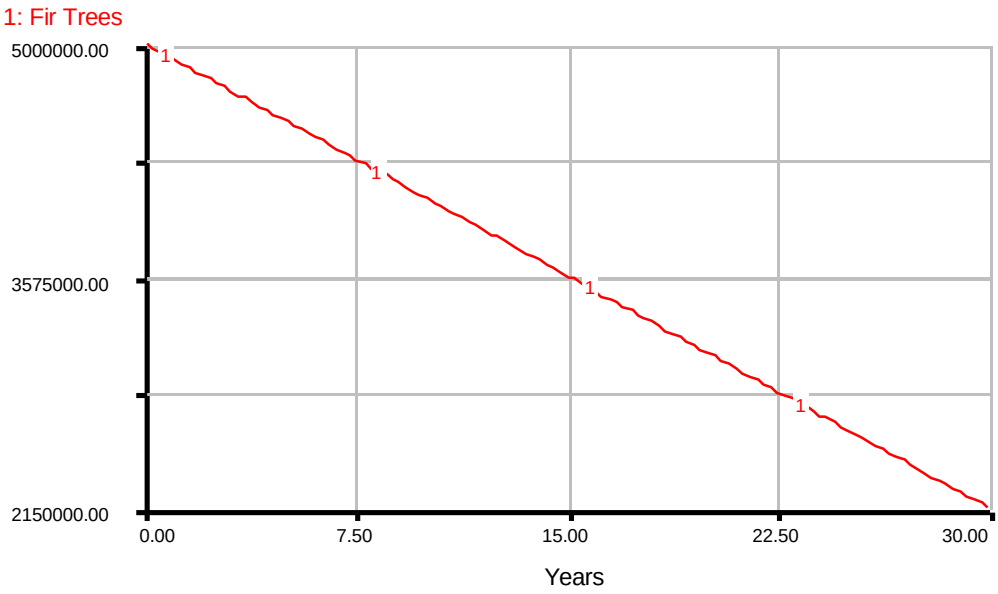
DOCUMENT: an environmental group plants 5000 trees every year

UNITS: fir trees/year

harvesting = 1e5

DOCUMENT: a lumber company harvests 100,000 trees every year (or 1e5)

UNITS: fir trees/year


1: Fir Trees
5000000.00
3575000.00
2150000.00
0.00
7.50
15.00
22.50
30.00
Years
1
1
1
1

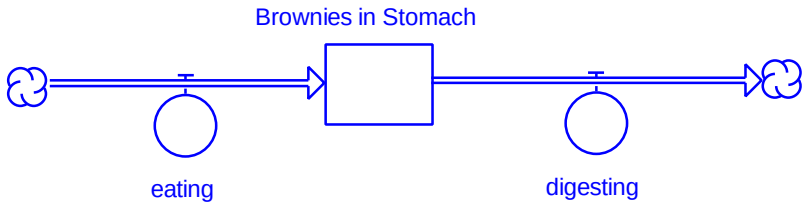
4.4 Brownies



Scenario: It's Saturday and Mathilda is working alone at home on a group project. Mathilda's best friend feels guilty that she's not contributing to the team effort, so she bakes Mathilda an enormous plate of brownies that she brings over with many words of encouragement. Mathilda nibbles on the brownies as she works. She eats a brownie every hour. As Mathilda works on the group project, her stomach works on digesting the brownies. Mathilda digests a brownie every two hours.

Question: Eight hours later, when Mathilda finishes her work on the group project, how many brownies does she have in her stomach?

Answer: 4 brownies



▭ **Brownies_in_Stomach**(t) = Brownies_in_Stomach (t - dt) + (eating - digesting) * dt

INIT Brownies_in_Stomach = 0

DOCUMENT: Initially Mathilda's stomach is empty.

UNITS: brownies

eating = 1

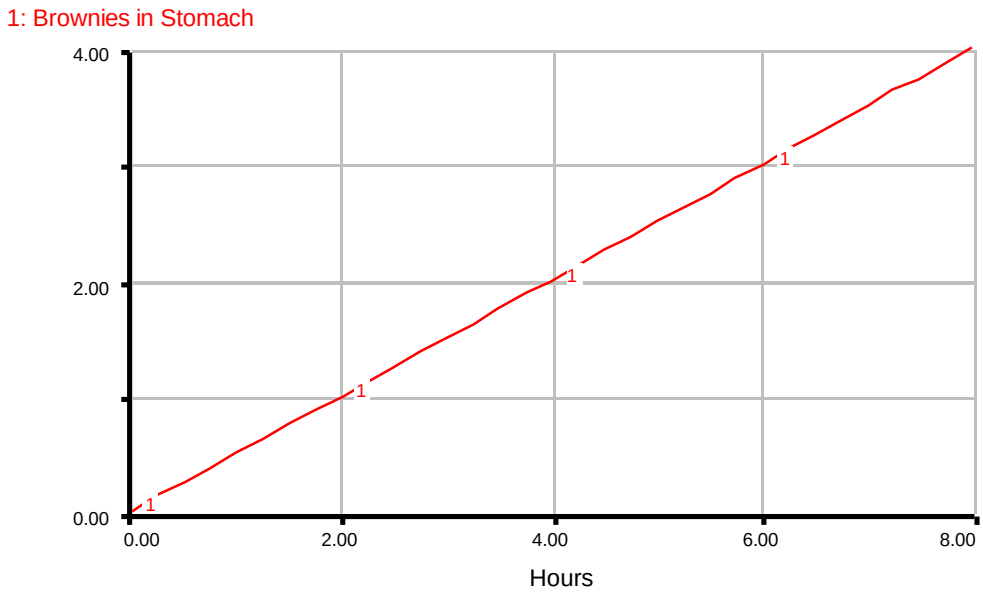
DOCUMENT: Mathilda eats a brownie every hour.

UNITS: brownies/hour

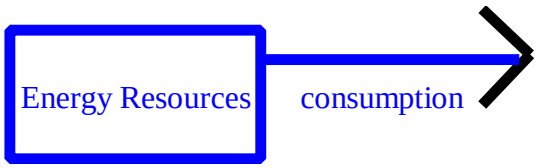
digesting = 1/2

DOCUMENT: Mathilda digests 1 brownie every 2 hours. She therefore digests a half a brownie every hour.

UNITS: brownies/hour


1: Brownies in Stomach
4.00
2.00
0.00
0.00
2.00
4.00
6.00
8.00
Hours
1
1
1
1

4.5 Energy Resources

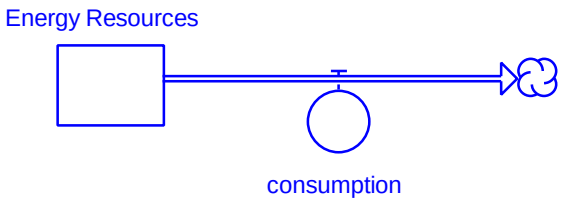


Scenario: In 1972 the world's known reserves of chromium were about 775 million metric tons, of which about 1.85 million metric tons are milled annually at present.[1] Make the temporary assumption that the world population is not growing and industrializing, increasing the demand for chromium exponentially, but instead is at some (unrealistic) equilibrium so that the demand for chromium is constant.

Question: At the current rate of consumption, approximately how long will the known reserves last? (Hint: Try running the model several times, increasing or decreasing the time scale, until you find the numbers of years after which the chromium reserves drop to zero.)

[1] Donella H. Meadows, Dennis L. Meadows, Jorgen Randers, and William W. Behrens III, 1972. The Limits to Growth. New York, NY; Universe Books, p. 61.

Answer: 420 years



Energy_Resources(t) = Energy_Resources (t - dt) - consumption * dt

INIT Energy_Resources = 7.75e8

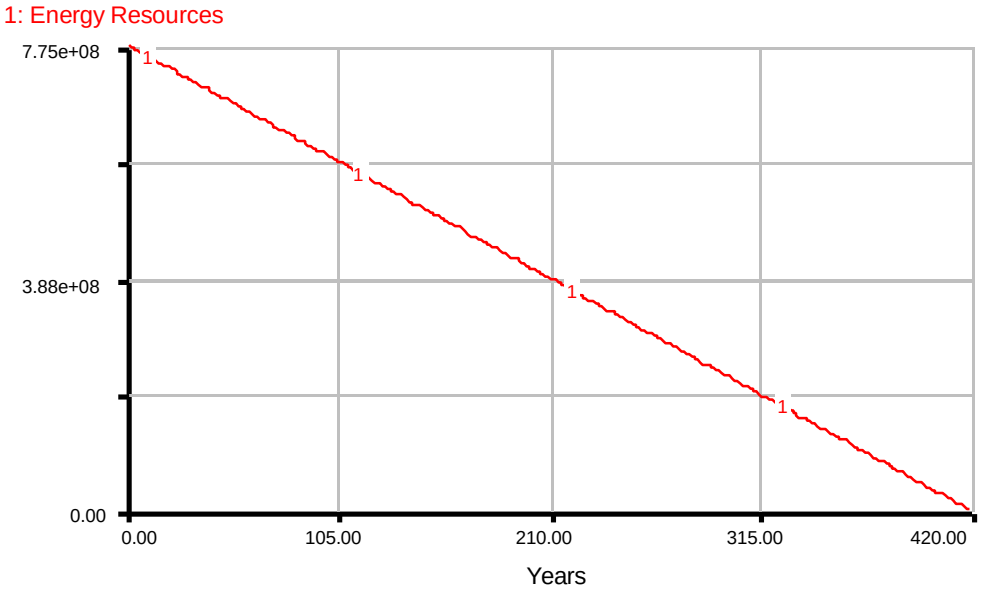
DOCUMENT: In 1972 the world's known reserves of chromium were about 775 million metric tons.

UNITS: metric tons

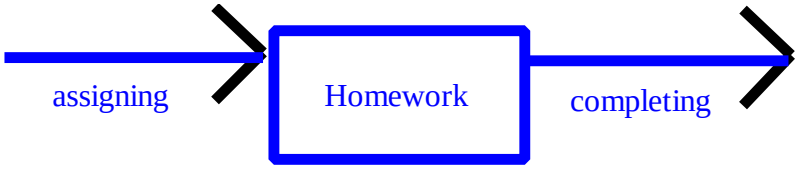
consumption = 1.85e6

DOCUMENT: 1.85 million metric tons are milled annually at present.

UNITS: metric tons/hour



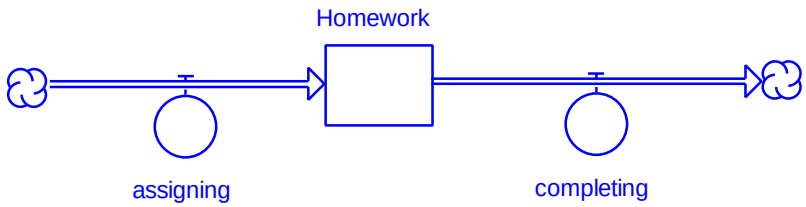
4.6 Homework



Scenario: Mathilda, the ever-studious student, diligently does her homework. Corky, on the other hand, is a slacker. He lets his work build up. Every week he receives seven new assignments. Over the course of the week he completes one or two of the assignments. (On average, he completes one and a half). The semester is twelve weeks long.

Question: How many assignments does Corky have to do at the end of the semester, right before his final exams?

Answer: 66 assignments



▭ **Homework** (t) = Homework (t - dt) + (assigning - completing) * dt

INIT Homework = 0

DOCUMENT: Corky begins the year on a clean slate.

UNITS: assignments

assigning = 7

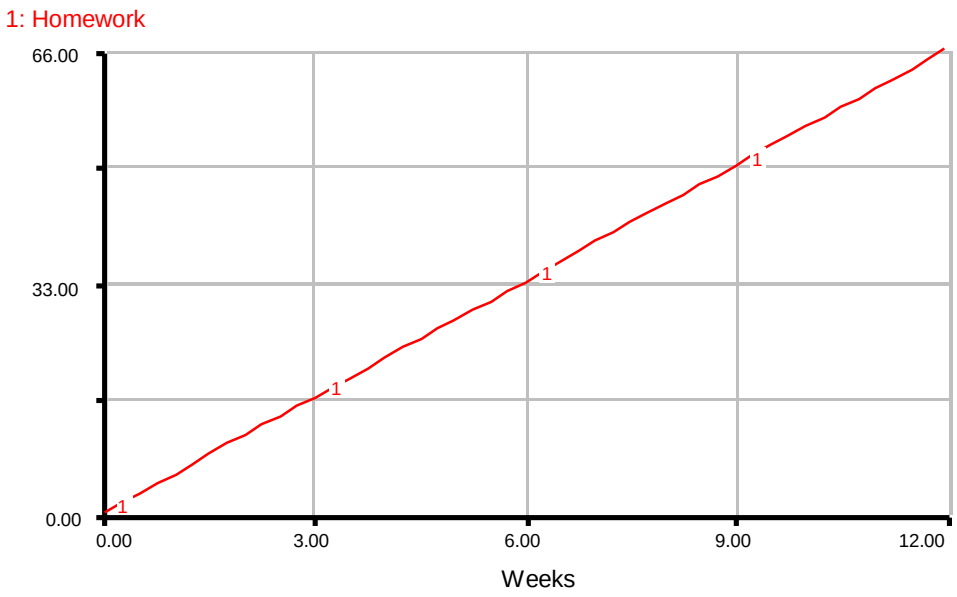
DOCUMENT: Every week Corky receives 7 new assignments.

UNITS: assignments/week

completing = 1.5

DOCUMENT: Over the course of the week Corky completes one or two of his assignments, so on average he completes one and a half assignments.

UNITS: assignments/week


1: Homework
66.00
33.00
0.00
0.00
3.00
6.00
9.00
12.00
Weeks
1
1
1
1

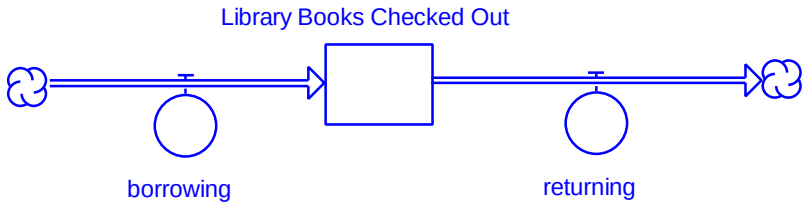
4.7 Library Books



Scenario: Laughton has a pile of five library books on the floor next to his desk. He loves to read. Every week Laughton returns four of the books that he has read. He also checks out four new books.

Question: How large is Laughton's pile of library books after eight weeks?

Answer: 5 books



▭ **Library_Books_Checked_Out** (t) = Library_Books_Checked_Out (t - dt) + (borrowing - returning) * dt

INIT Library_Books_Checked_Out = 5

DOCUMENT: Laughton has a pile of 5 library books on the floor next to his desk.

UNITS: books

borrowing = 4

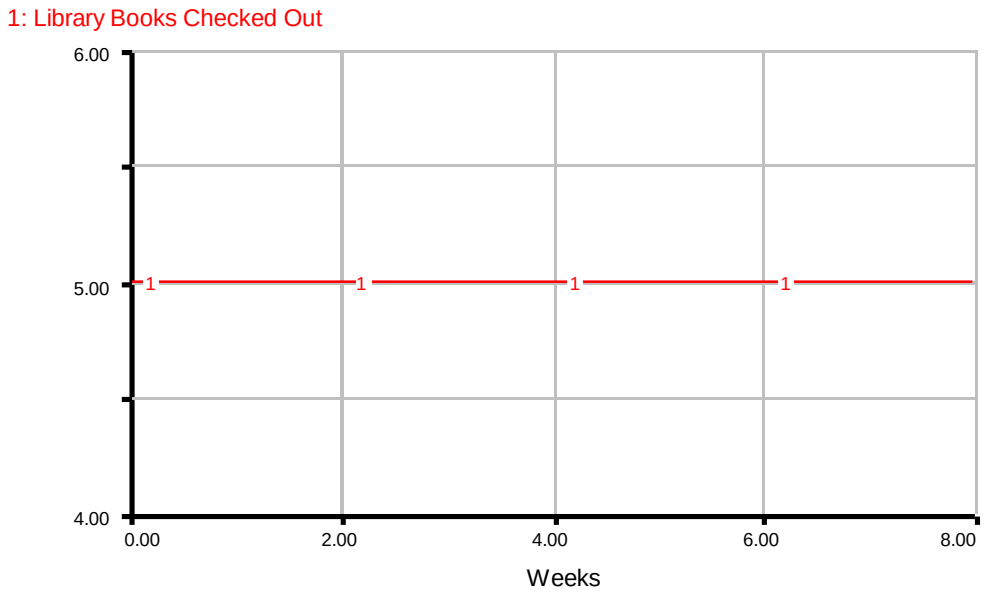
DOCUMENT: Every week Laughton checks out 4 new books.

UNITS: books/week

returning = 4

DOCUMENT: Laughton returns 4 library books every week.

UNITS: books/week


1: Library Books Checked Out
6.00
5.00
4.00
1
1
1
1
0.00
2.00
4.00
6.00
8.00
Weeks

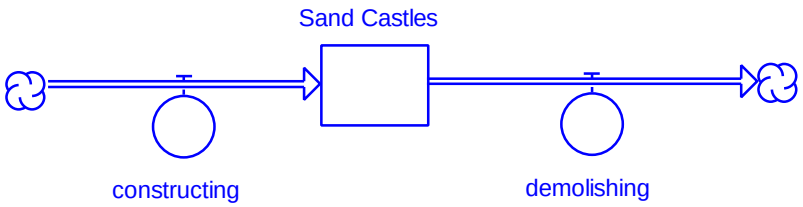
4.8 Sand Castles



Scenario: A beach club in St. Tropez is organized a sand castle contest. At 10 AM, eighty children gathered on the beach to make their sand castles. Unfortunately, at that time the tide was on the rise. Each child was able to build a new sand castle every hour. The incoming tide and the advancing waves demolished fifty sand castles every hour.

Question: How many sand castles were left on the beach at 6 PM? (Hint: you can either run your simulation from 10:00 to 18:00 or from 0 to 8 hours after the beginning of the contest)

Answer: 240 sand castles



▭ **Sand_Castles** (t) = Sand_Castles (t - dt) + (constructing - demolishing) * dt

INIT Sand_Castles = 0

DOCUMENT: When the children arrive the sandy beach is barren.

UNITS: castles

constructing = 80

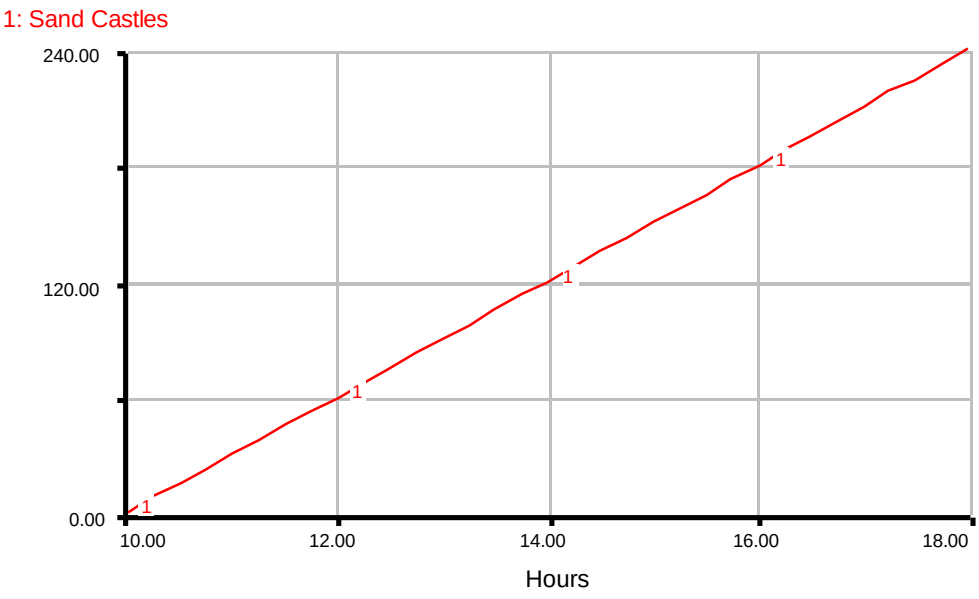
DOCUMENT: Each of the 80 children builds a new sand castle every hour.

UNITS: castles/hour

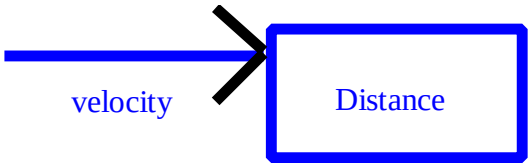
demolishing = 50

DOCUMENT: The advancing waves demolished 50 sand castles every hour.

UNITS: castles/hour


1: Sand Castles
240.00
120.00
0.00
10.00
12.00
14.00
16.00
18.00
Hours
1
1
1
1

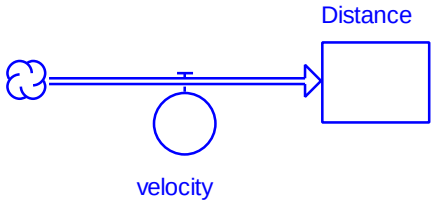
4.9 Distance



Scenario: Randy is training to run in the Boston Marathon. If he paces himself, he can run eight minute miles. Randy likes to run in the morning, before breakfast. He wakes up at 7 AM, and eats breakfast at 8 AM.

Question: How many miles can Randy run before breakfast? (Hint: you can run the simulation from 7 to 8 hours or for 60 minutes. It does not matter which units you choose, but you *must* be consistent and use *either* minutes *or* hours throughout.)

Answer: 7.5 miles



▭ **Distance**(t) = Distance (t - dt) + velocity * dt

INIT Distance = 0

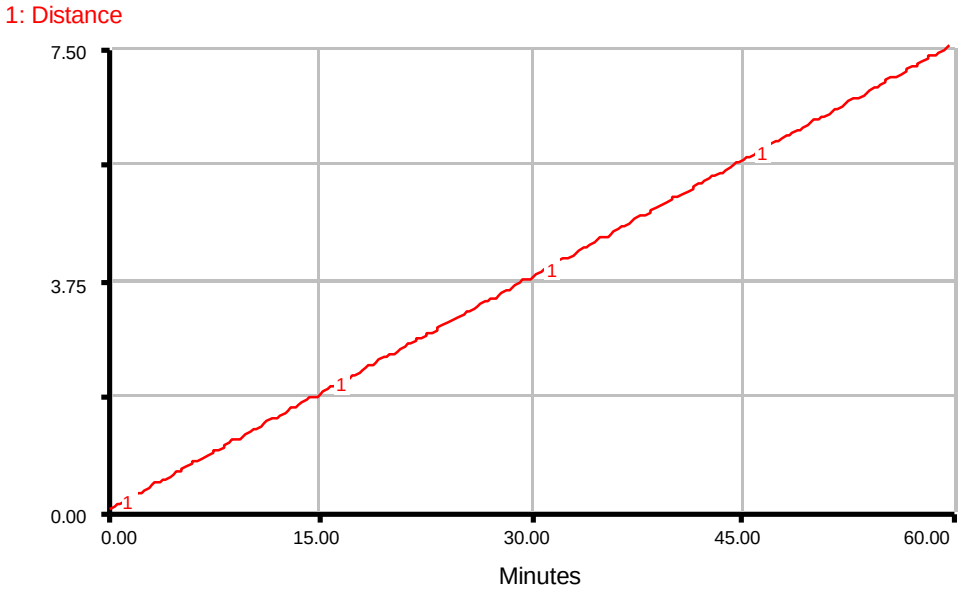
DOCUMENT: At 7 AM Randy has not run any miles yet.

UNITS: miles

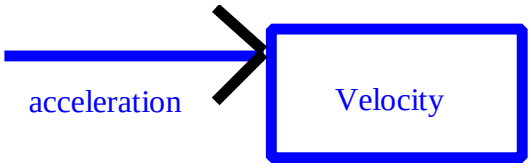
velocity = 1 / 8

DOCUMENT: Because Randy can run 1 mile in 8 minutes, he can run 1/8 of a mile in one minute.

UNITS: miles/minute



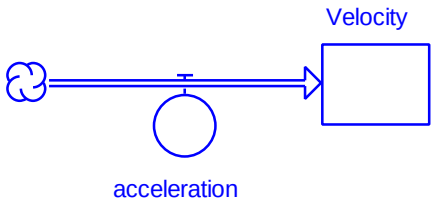
4.10 Velocity



Scenario: A Ferrari is paused at a red light. The light turns green. The driver slams down the accelerator and the sports car springs forward. He keeps his foot tight on the accelerator. The car accelerates at five miles per hour per second.

Question: How fast will the Ferrari be cruising sixteen seconds later?

Answer: 80 miles per hour



▭ **Velocity**(t) = Velocity (t - dt) + acceleration * dt

INIT Distance = 0

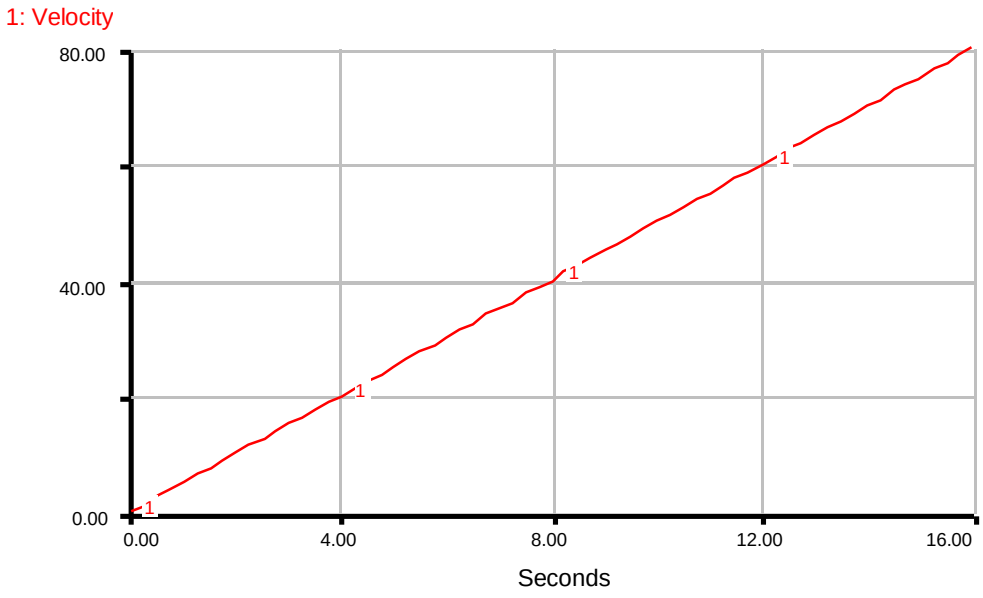
DOCUMENT: At the red light the car is stopped, so its velocity is 0.

UNITS: miles per hour

acceleration = 5

DOCUMENT: The car accelerates at 5 miles per hour/second.

UNITS: miles per hour /second



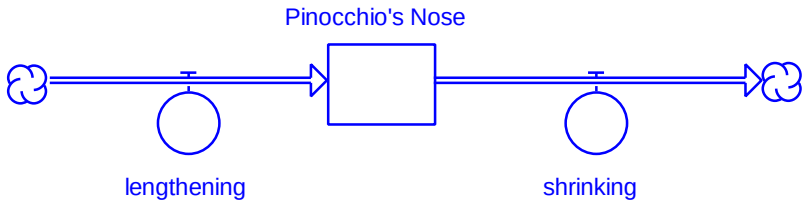
4.11 Pinocchio



Scenario: When Pinocchio lies his nose lengthens by one centimeter. Each time he does a good dead, his nose shrinks five centimeters. Every day, Pinocchio tells an average of 8 lies and performs, on average, one good deed.

Question: If Monday morning Pinocchio's nose is 4 centimeters long, how long will his nose be on Saturday night?

Answer: 22 centimeters



□ **Pinocchio's_Nose** (t) = Pinocchio's_Nose (t - dt) + (lengthening - shrinking) * dt

INIT Pinocchio's_Nose = 4

DOCUMENT: On Monday morning Pinocchio's nose is 4 centimeters long.

UNITS: centimeters

lengthening = 8

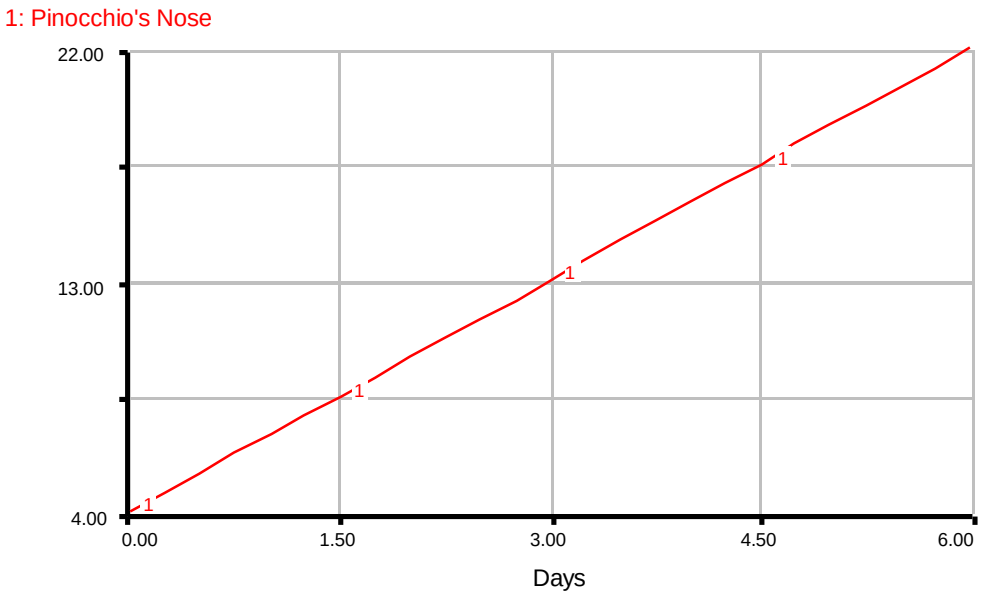
DOCUMENT: Pinocchio tells 8 lies each day and his nose lengthens 1 centimeter for each lie.

UNITS: centimeters/day

shrinking = 5

DOCUMENT: Pinocchio performs 1 good deed each day and his nose shrinks 5 centimeters for that good deed.

UNITS: centimeters/day


1: Pinocchio's Nose
22.00
13.00
4.00
0.00
1.50
3.00
4.50
6.00
Days
1
1
1
1

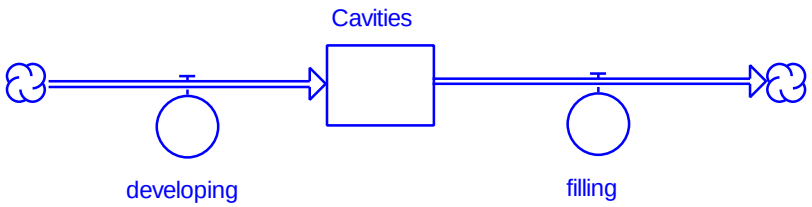
4.12 Cavities



Scenario: I develop a full-blown cavity every two years. I do not go to see my dentist very often; I get a cavity filled every three years. Because I wait so long the process is often quite painful.

Question: I currently don't have any cavities. How many will I have in 15 years?

Answer: 2 and a half cavities



□ **Cavities** (t) = Cavities (t - dt) + (developing - filling) * dt

INIT Cavities = 0

DOCUMENT: I currently don't have any cavities.

UNITS: cavities

developing = 1/2

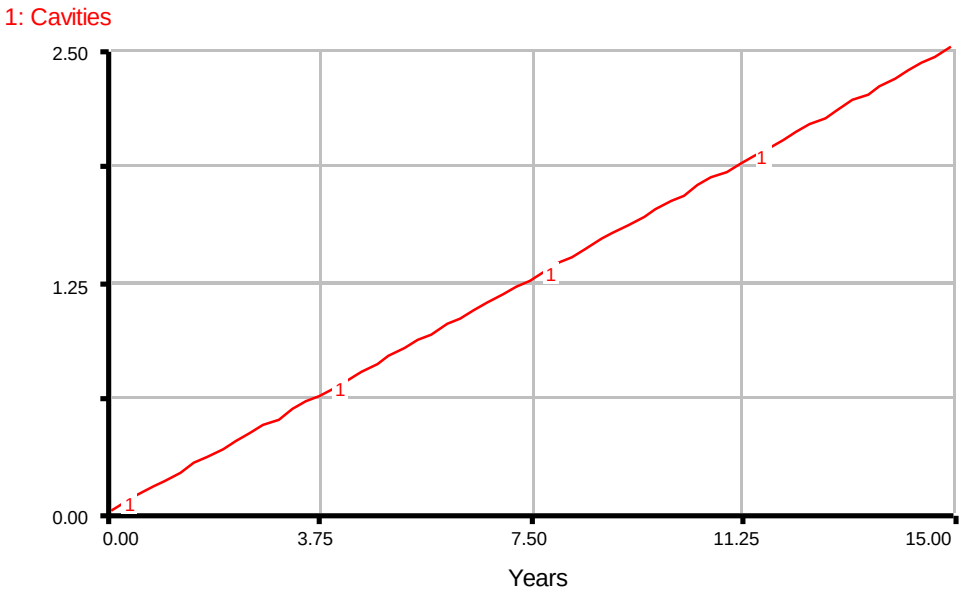
DOCUMENT: I develop a full-blown cavity every two years.

UNITS: cavities/year

filling = 1/3

DOCUMENT: I get a cavity filled every three years.

UNITS: cavities/year



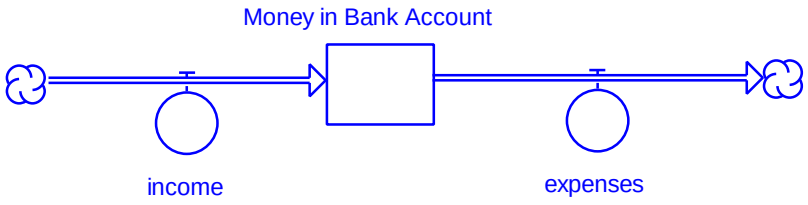
4.13 A Bank Account



Scenario: Stephanie has $678.53 in her bank account. Every week she earns $120. Each week $23.70 are deducted from her paycheck for social security, Medicare, local, state, and federal taxes. She spends $7.75 every week to go out for a movie and approximately $60 every three weeks on clothes.

Question: How much money will Stephanie have in her bank account in four months (sixteen weeks from now)?

Answer: 1775.33 dollars



□ **Money_in_Bank_Account** (t) = Money_in_Bank_Account (t - dt) + (income - expenses) * dt

INIT Money_in_Bank_Account = 678.53

DOCUMENT: Stephanie has $678.53 in her bank account.

UNITS: dollars

income = 120 - 23.7

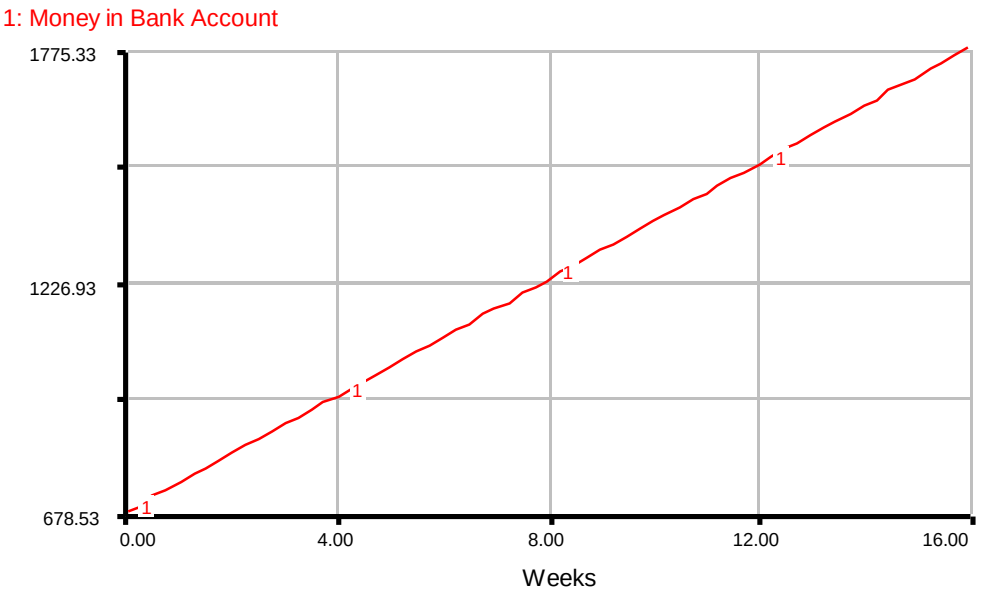
DOCUMENT: Every week she gets a paycheck for $120 minus $23.70 in taxes.

UNITS: dollars/week

expenses = 7.75 + 60 / 3

DOCUMENT: She spends $7.75 every week on a movie and $60 every three weeks on clothes.

UNITS: dollars/week


1: Money in Bank Account
1775.33
1226.93
678.53
0.00
4.00
8.00
12.00
16.00
Weeks
1
1
1
1

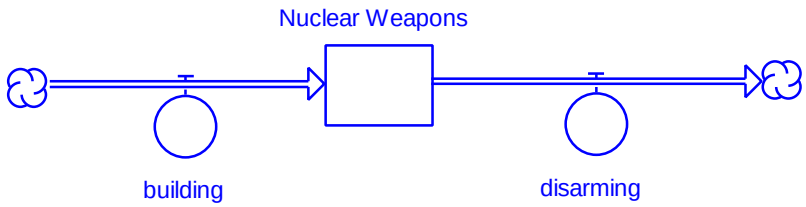
4.14 Nuclear Weapons



Scenario: In 1990 the fictional country of Euromerica had an arsenal of approximately ten thousand nuclear weapons. Every year, scientists and engineers design and manufacture five hundred new nuclear weapons. Because of an international pressure in favor of disarmament, Euromerica disarms approximately six hundred of its older nuclear weapons each year.

Question: If Euromerica continues at its current rates of building and disarming, how many nuclear weapons will the nuclear arsenal contain in the year 2010?

Answer: 8000 nuclear weapons



□**Nuclear_Weapons**(t) = Nuclear_Weapons(t - dt) + (building - disarming) * dt

INIT Nuclear_Weapons = 10000

DOCUMENT: In 1990 Euromerica has 10,000 nuclear weapons.

UNITS: weapons

building = 500

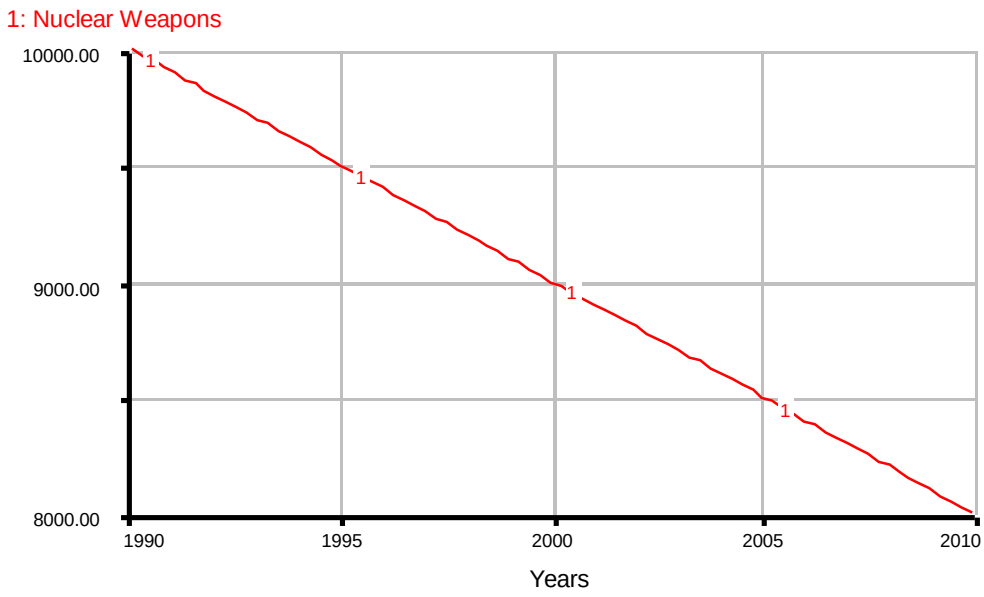
DOCUMENT: Every year Euromerica builds 500 new nuclear weapons.

UNITS: weapons/year

disarming = 600

DOCUMENT: Every year Euromerica disarms 600 nuclear weapons.

UNITS: weapons/year


1: Nuclear Weapons
10000.00
9000.00
8000.00
1990
1995
2000
2005
2010
Years

Vensim Examples:
Beginner Modeling Exercises

By Lei Lei and Nathaniel Choge
October 2001

Section 4.1: Skunks

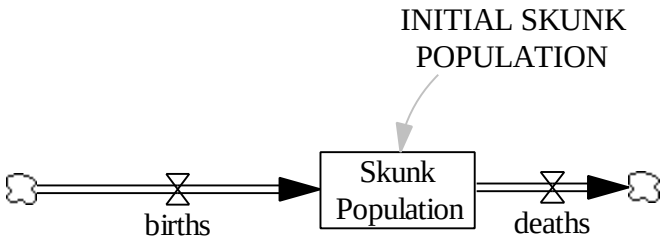


Figure 1: Vensim Equivalent of Skunks model

Documentation for Skunks Model:

(1) births = 100

Units: skunks/Year

100 baby skunks are born every year.

(2) deaths = 120

Units: skunks/Year

120 skunks die every year.

(3) FINAL TIME = 10

Units: Year

The final time for the simulation.

(4) INITIAL SKUNK POPULATION = 500

Units: skunks

The initial skunk population is 500 skunks.

(5) INITIAL TIME = 0

Units: Year

The initial time for the simulation.

(6) SAVEPER = TIME STEP

Units: Year

The frequency with which output is stored.

(7) Skunk Population= INTEG (births-deaths, INITIAL SKUNK POPULATION)

Units: skunks

The skunk population.

(8) TIME STEP = 0.0625

Units: Year

The time step for the simulation.

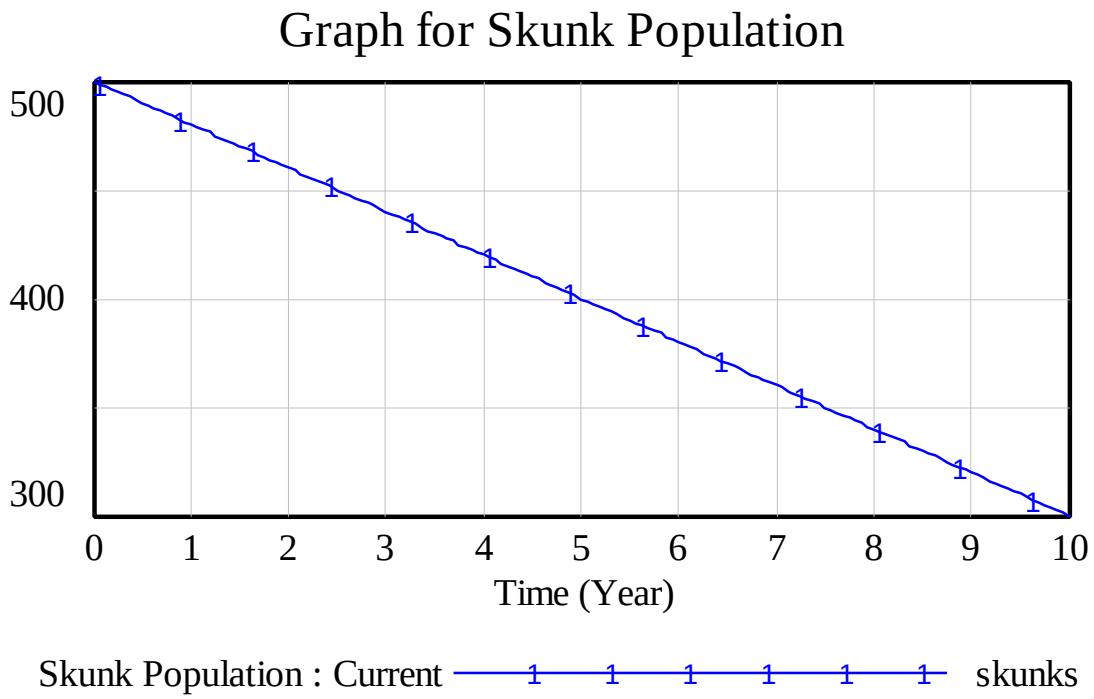


Figure 2: Vensim Equivalent of Simulation for Number of Skunks

Section 4.2: Landfills

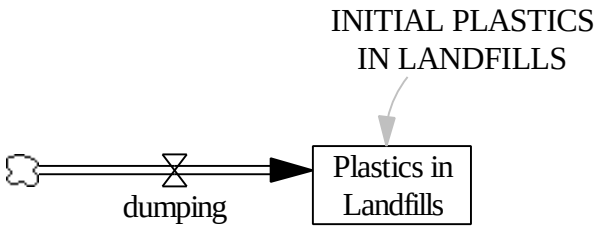


Figure 3: Vensim Equivalent of Landfills

Documentation for Landfills Model:

(1) dumping = 5000*365

Units: gallons of plastics/Year

Citizens in Boise will dump 5,000 gallons of plastics into the landfill for the 365 days that make up the year.

(2) FINAL TIME = 20

Units: Year

The final time for the simulation.

(3) INITIAL PLASTICS IN LANDFILLS = 0

Units: plastics

Initially the landfill is empty.

(4) INITIAL TIME = 0

Units: Year

The initial time for the simulation.

(5) Plastics in Landfills= INTEG (dumping, INITIAL PLASTICS IN LANDFILLS)

Units: gallons of plastics

Gallons of plastics in the landfill.

(6) SAVEPER = TIME STEP

Units: Year

The frequency with which output is stored.

(7) TIME STEP = 0.0625

Units: Year

The time step for the simulation.

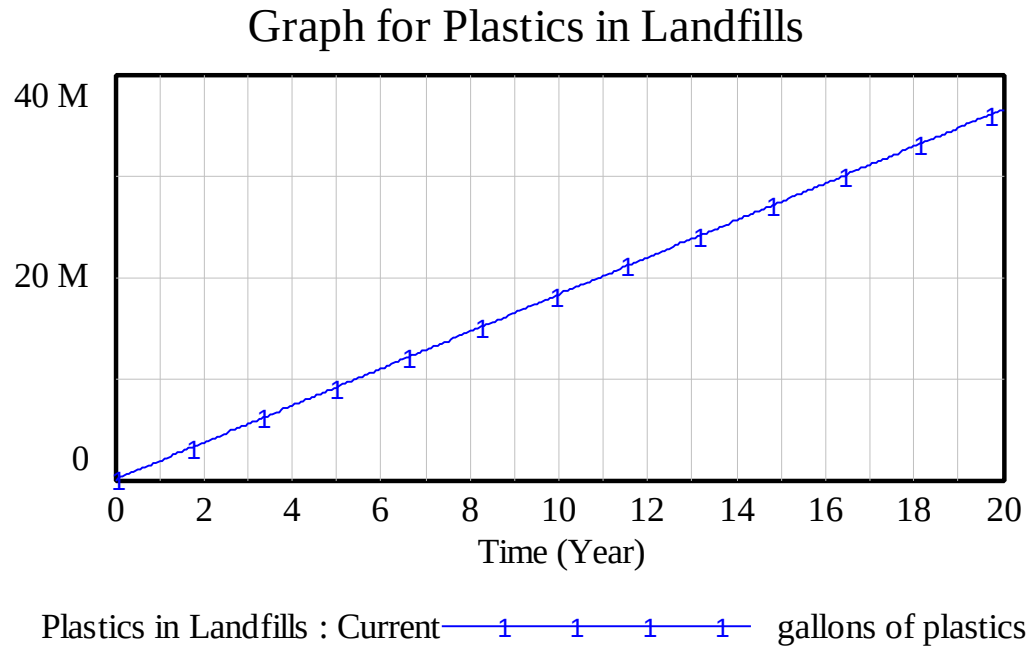


Figure 4: Vensim Equivalent of Simulation for plastics in landfills

Section 4.3: Fir Trees

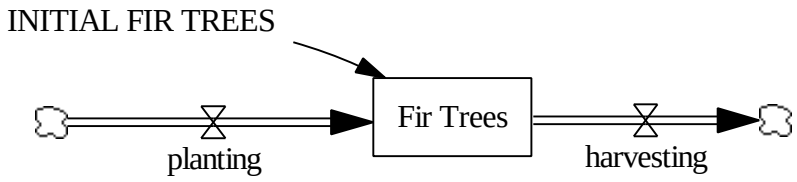


Figure 5: Vensim Equivalent of Fir Trees

Documentation for Fir Trees model:

(1) FINAL TIME = 30
Units: Year
The final time for the simulation.

(2) Fir Trees = INTEG (+planting-harvesting, INITIAL FIR TREES)
Units: fir trees

(3) harvesting = 100000
Units: fir trees/Year
A lumber company harvests 100,000 trees every year.

(4) INITIAL FIR TREES = 5e+006
Units: fir trees

(5) INITIAL TIME = 0
Units: Year
The initial time for the simulation.

(7) planting = 5000
Units: fir trees/Year
An environmental group plants 5000 trees every year.

(8) SAVEPER = TIME STEP
Units: Year
The frequency with which output is stored.

(9) TIME STEP = 0.0625
Units: Year
The time step for the simulation.

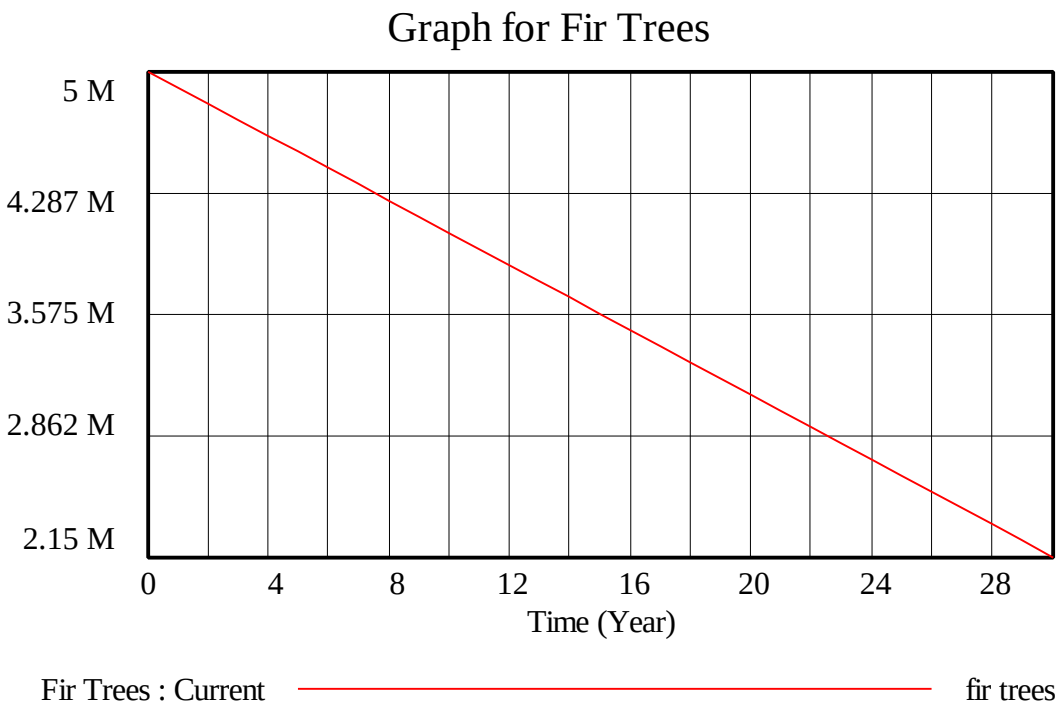


Figure 6: Vensim Equivalent of Simulation for Number of Fir Trees

Section 4.4: Brownies

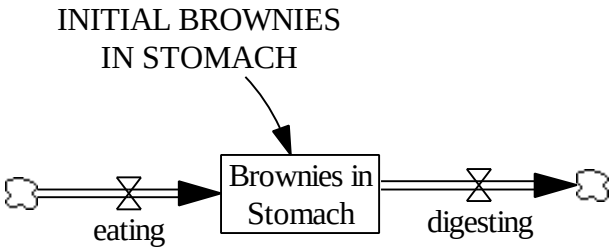


Figure 7: Vensim Equivalent of Brownies Model

Documentation for Brownies Model:

(1) Brownies in Stomach = INTEG (+eating-digesting, INITIAL BROWNIES IN STOMACH)

Units: brownies

Initially Mathilda's stomach is empty.

(2) digesting = 0.5

Units: brownies/hour

Mathilda digests 1 brownie every 2 hours. She therefore digests a half a brownies every hour.

(3) eating = 1

Units: brownies/hour

Mathilda eats a brownie every hour.

(4) FINAL TIME = 100

Units: Month

The final time for the simulation.

(5) INITIAL BROWNIES IN STOMACH = 0

Units: brownies

(6) INITIAL TIME = 0

Units: Month

The initial time for the simulation.

(7) SAVEPER = 1

Units: Month

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Month

The time step for the simulation.

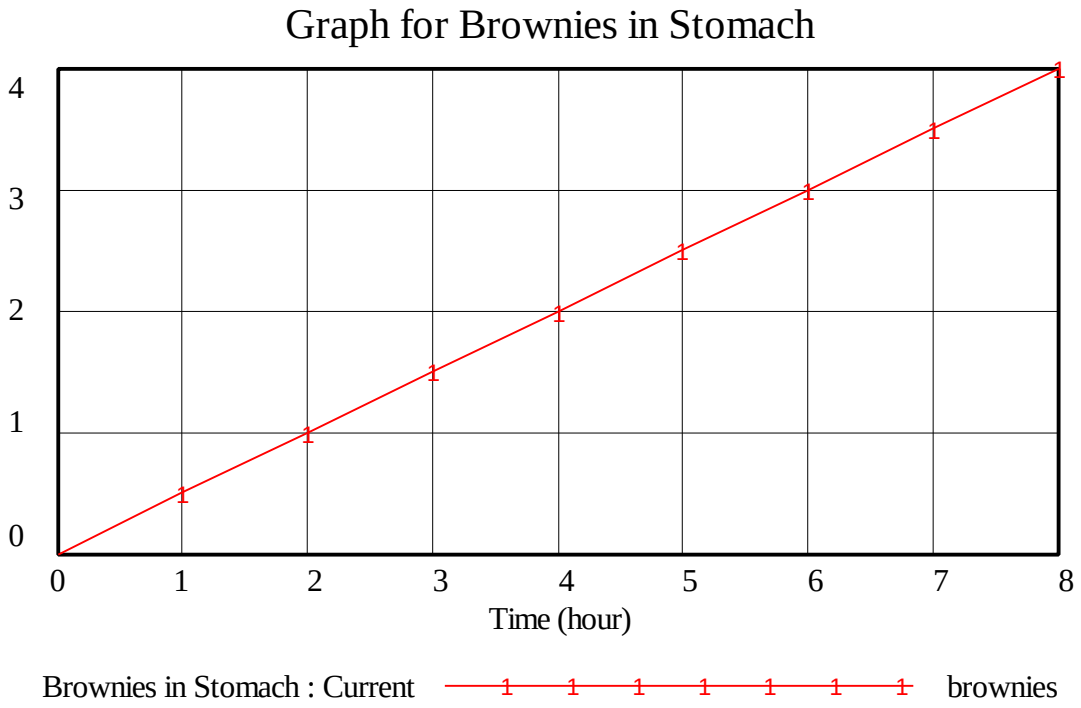


Figure 8: Vensim Equivalent of Simulation for Number of Brownies

Section 4.5: Energy Resources

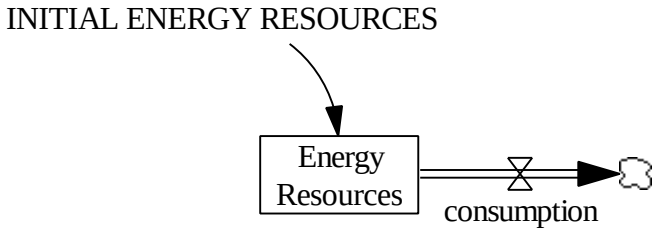


Figure 9: Vensim Equivalent of Energy Resources Model

Documentation for Energy Resources Model:

(1) consumption = 1.85e+006

Units: metric tons/hour

1.85 million metric tons are milled annually at present.

(2) Energy Resources = INTEG (-consumption, INITIAL ENERGY RESOURCES)

Units: metric tons

In 1972 the world's known reserves of chromium were about 775 million metric tons.

(3) FINAL TIME = 420

Units: Year

The final time for the simulation.

(4) INITIAL ENERGY RESOURCES = 7.75e+008

Units: metric tons

(5) INITIAL TIME = 0

Units: Year

The initial time for the simulation.

(6) SAVEPER = TIME STEP

Units: Year

The frequency with which output is stored.

(7) TIME STEP = 0.0625

Units: Year

The time step for the simulation.

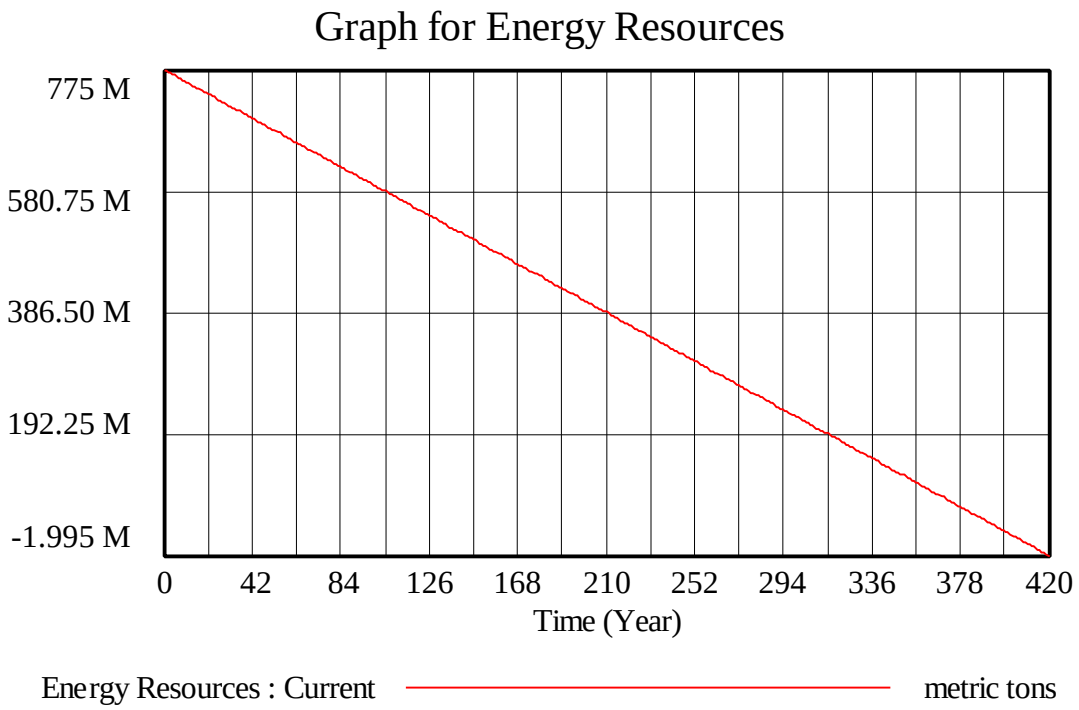


Figure 10: Vensim Equivalent of Simulation of Amount of Energy Resources

Section 4.6: Homework

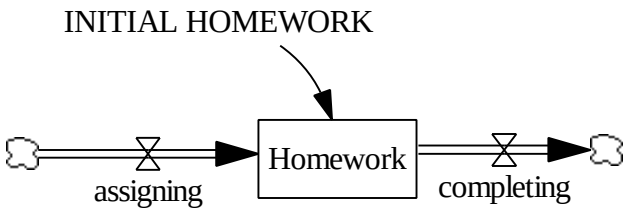


Figure 11: Vensim Equivalent of Homework Model

Documentation for Homework Model:

(1) assigning = 7

Units: assignments/Week

Every week Corky receives 7 new assignments.

(2) completing = 1.5

Units: assignments/Week

Over the course of the week Corky completes one or two of his assignments, so on average he completes one and a half assignments.

(3) FINAL TIME = 12

Units: Week

The final time for the simulation.

(4) Homework = INTEG (assigning-completing, INITIAL HOMEWORK)

Units: assignments

Corky begins the year on a clean slate.

(5) INITIAL HOMEWORK = 0

Units: assignments

(6) INITIAL TIME = 0

Units: Week

The initial time for the simulation.

(7) SAVEPER = TIME STEP

Units: Week

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Week

The time step for the simulation.

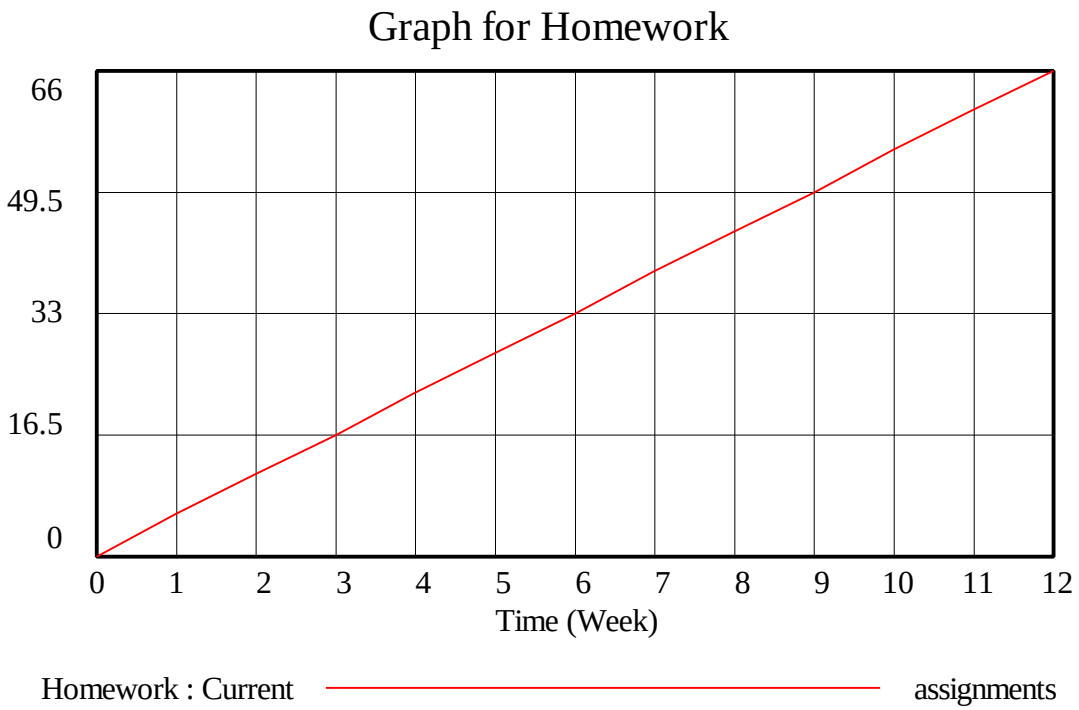


Figure 12: Vensim Equivalent of Simulation for Number Homework

Section 4.7: Library Books

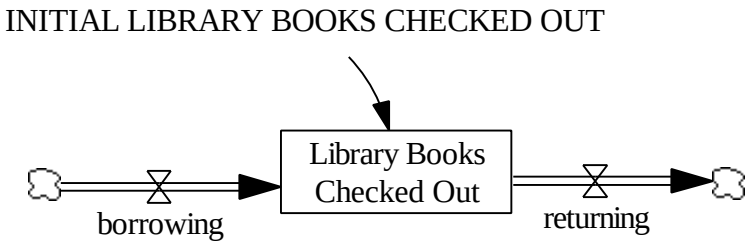


Figure 13: Vensim Equivalent of Library Books Model

Documentation for Library Books Model:

(1) borrowing = 4
Units: books/Week
Every week Laughton checks out 4 new books.

(2) FINAL TIME = 8
Units: Week
The final time for the simulation.

(3) INITIAL TIME = 0
Units: Week
The initial time for the simulation.

(4) INITIAL LIBRARY BOOKS CHECKED OUT = 5
Units: books

(5) Library Books Checked Out = INTEG (borrowing-returning, INITIAL LIBRARY BOOKS CHECKED OUT)
Units: books
Laughton has a pile of 5 library books on the floor next to his desk.

(6) returning = 4
Units: books/Week
Laughton returns 4 library books every week.

(7) SAVEPER = TIME STEP
Units: Week
The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Week

The time step for the simulation.

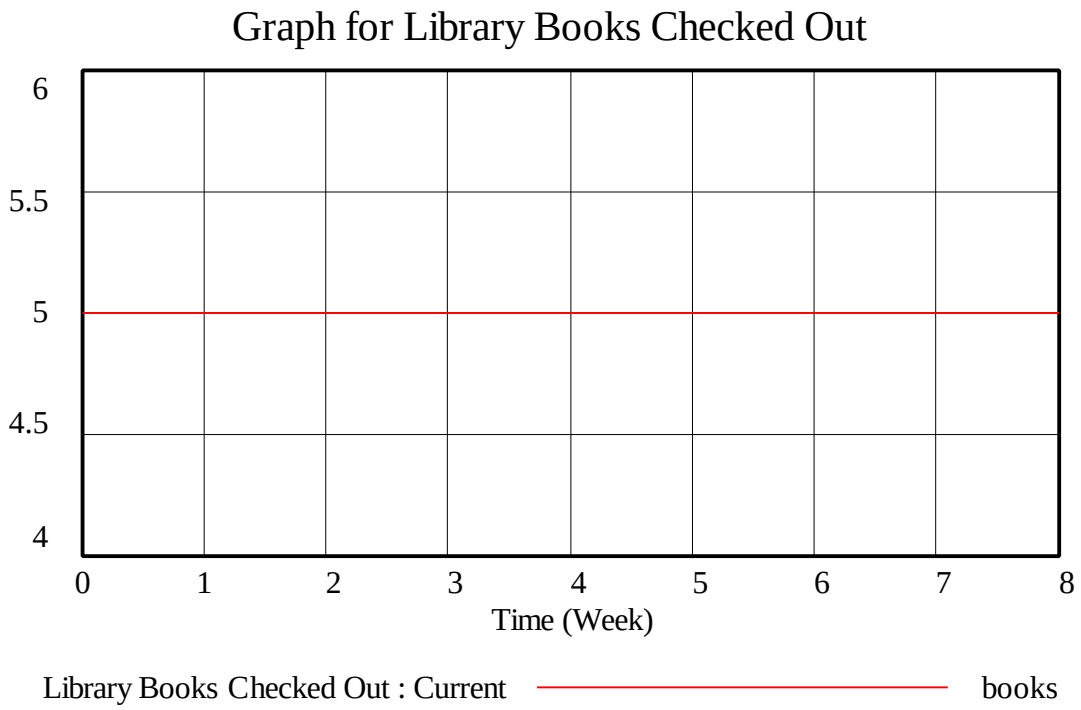


Figure 14: Vensim Equivalent of Simulation for Number of Library Books

Section 4.8: Sand Castle

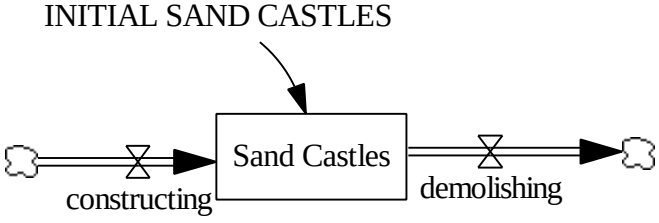


Figure 15: Vensim Equivalent of Sand Castle Model

Documentation for Sand Castle Model:

(1) constructing = 80
Units: castles/Hour
Each of the 80 children builds a new sand castle every hour.

(2) demolishing = 50
Units: castles/Hour
The advancing waves demolished 50 sand castles every hour.

(3) FINAL TIME = 18
Units: Hour
The final time for the simulation.

(4) INITIAL SAND CASTLES = 0
Units: castles

(5) INITIAL TIME = 0
Units: Hour
The initial time for the simulation.

(6) Sand Castles = INTEG (constructing-demolishing, INITIAL SAND CASTLES)
Units: castles
When the children arrive the sandy beach is barren.

(7) SAVEPER = TIME STEP
Units: Hour
The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Hour

The time step for the simulation.

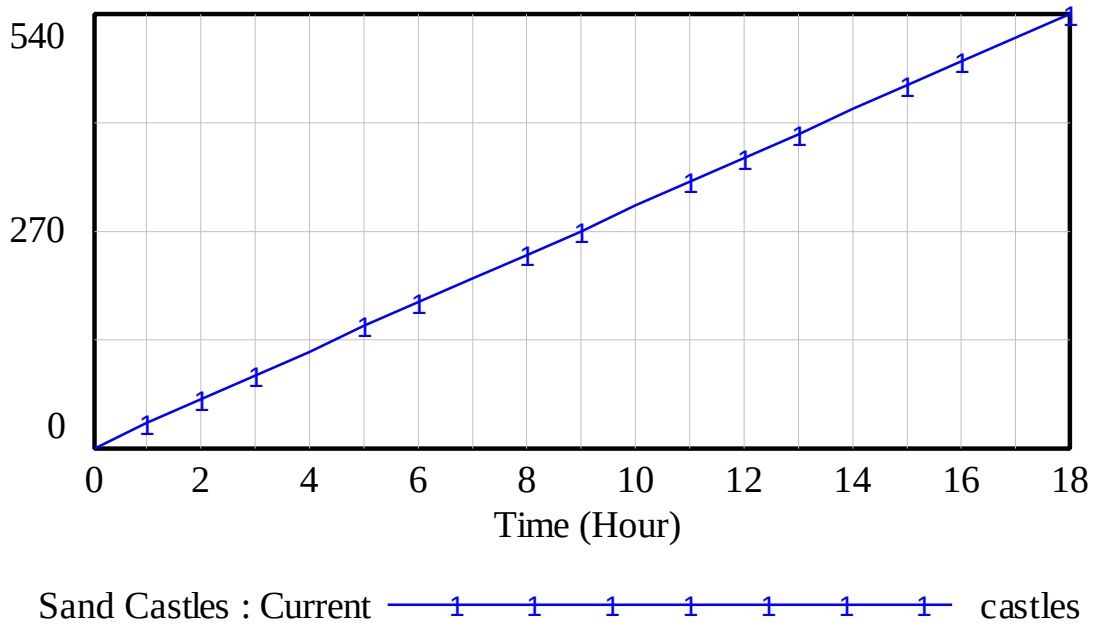


Figure 16: Vensim Equivalent of Simulation of Number of Sand Castles

Section 4.9: Distance

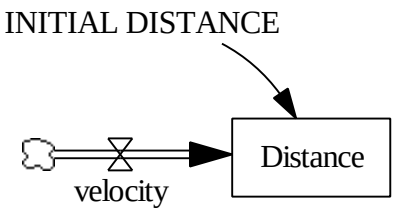


Figure 17: Distance Model

Documentation for Distance Model:

(1) Distance = INTEG (velocity, INITIAL DISTANCE)

Units: miles

At 7AM Randy has not run any miles yet.

(2) FINAL TIME = 60

Units: Minute

The final time for the simulation.

(3) INITIAL DISTANCE = 0

Units: minute

(4) INITIAL TIME = 0

Units: Minute

The initial time for the simulation.

(5) SAVEPER = TIME STEP

Units: Minute

The frequency with which output is stored.

(6) TIME STEP = 0.0625

Units: Minute

The time step for the simulation.

(7) velocity = 0.125

Units: miles/Minute

Because Randy can run 1 mile in 8 minutes, he can run 0.125 of a mile in one minute.

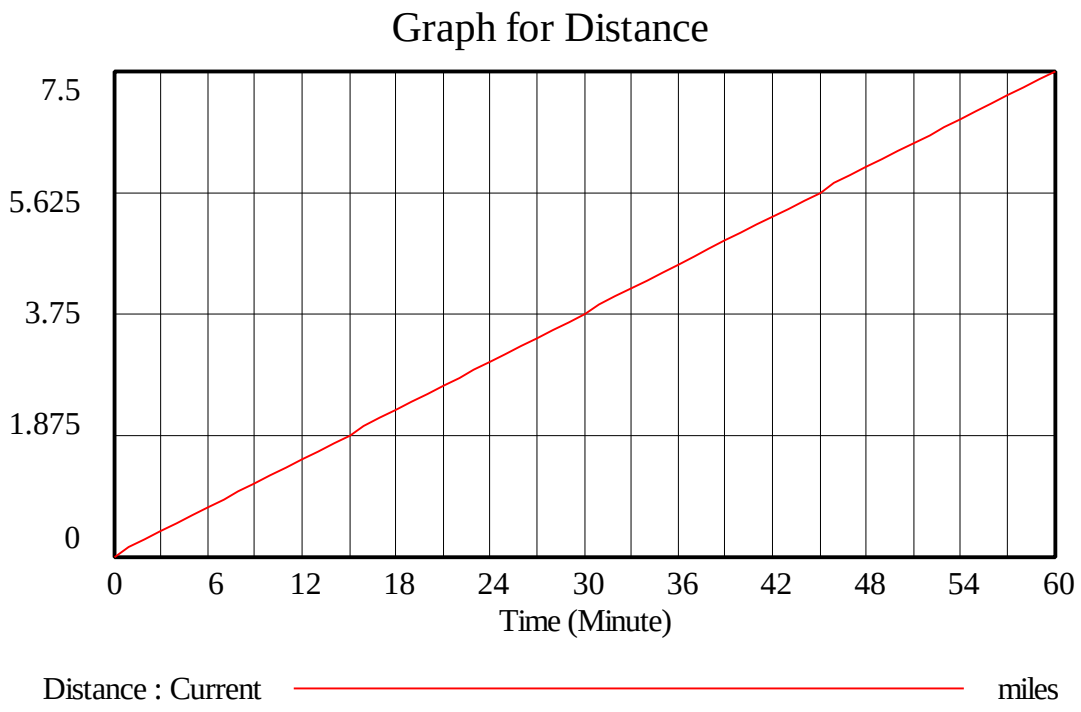


Figure 18: Vensim Equivalent of Simulation for Distance

Section 4.10: Velocity

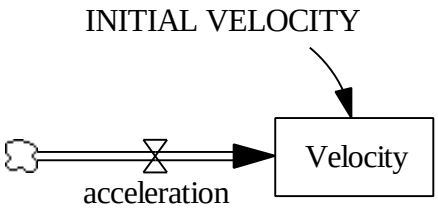


Figure 19: Vensim Equivalent of Velocity Model

Documentation for Velocity Model:

(1) acceleration = 5

Units: miles/hour/Second

The car accelerates at 5 miles per hour/second.

(2) FINAL TIME = 16

Units: Second

The final time for the simulation.

(3) INITIAL TIME = 0

Units: Second

The initial time for the simulation.

(4) INITIAL VELOCITY = 0

Units: miles/hour

(5) SAVEPER = TIME STEP

Units: Second

The frequency with which output is stored.

(6) TIME STEP = 0.0625

Units: Second

The time step for the simulation.

(7) Velocity = INTEG (acceleration, INITIAL VELOCITY)

Units: miles/hour

At the red light the car is stopped, so its velocity is 0.

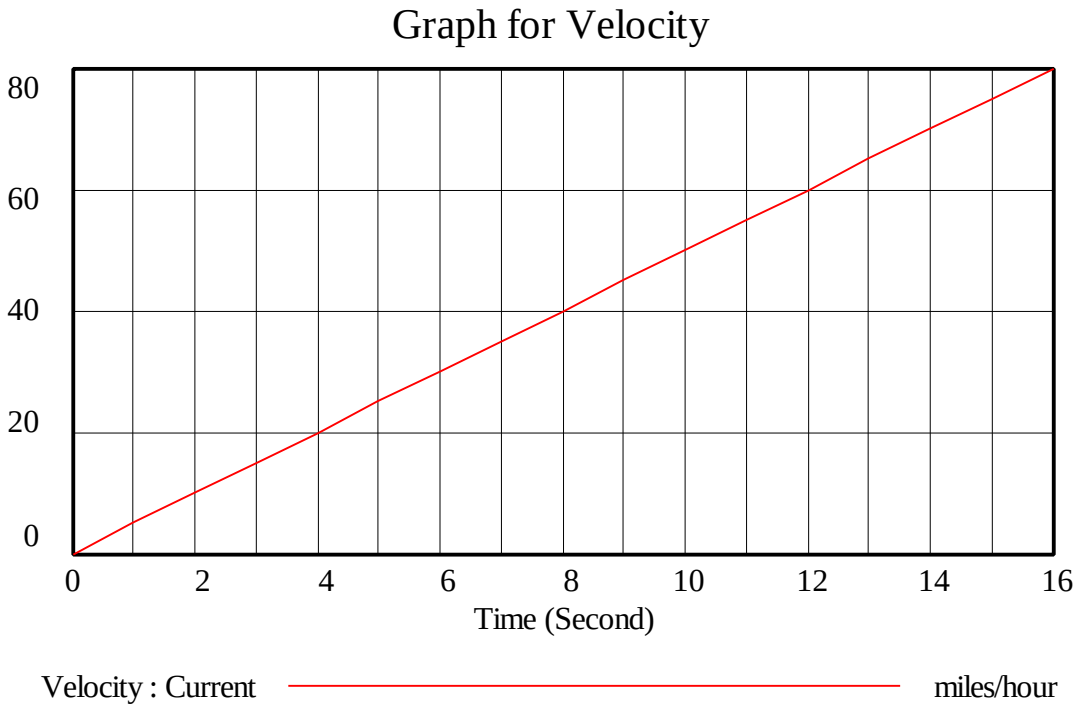


Figure 20: Vensim Equivalent of Simulation for Velocity

Section 4.11: Pinocchio's Nose

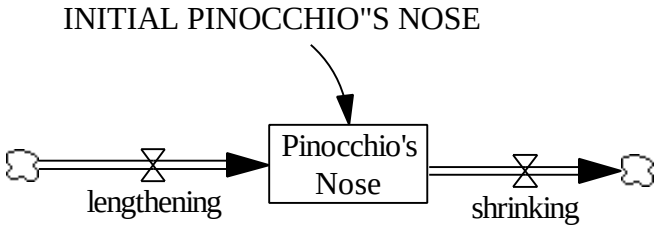


Figure 21: Vensim Equivalent of Pinocchio's Nose Model

Documentation for Pinocchio's Nose Model:

(1) FINAL TIME = 6

Units: Day

The final time for the simulation.

(2) INITIAL PINOCCHIO’S NOSE = 4

Units: centimeters

(3) INITIAL TIME = 0

Units: Day

The initial time for the simulation.

(4) lengthening = 8

Units: centimeters/Day

Pinocchio tells 8 lies each day and his nose lengthens 1 centimeter for each lie.

(5) Pinocchio's Nose = INTEG (lengthening-shrinking, INITIAL PINOCCHIO’S NOSE)

Units: centimeters

On Monday morning Pinocchio's nose is 4 centimeters long.

(6) SAVEPER = TIME STEP

Units: Day

The frequency with which output is stored.

(7) shrinking = 5

Units: centimeters/Day

Pinocchio performs 1 good deed each day and his nose shrinks 5 centimeters for that good deed.

(8) TIME STEP = 0.0625

Units: Day

The time step for the simulation.

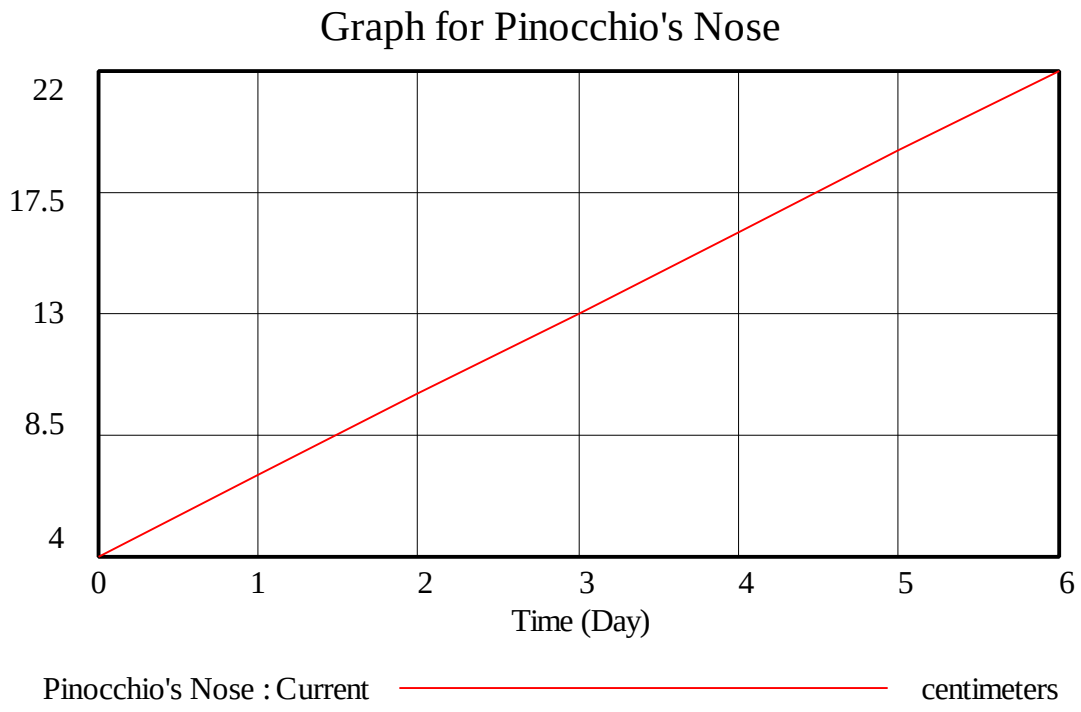


Figure 22: Vensim Equivalent of Simulation for Length of Pinocchio's Nose

Section 4.12: Cavities

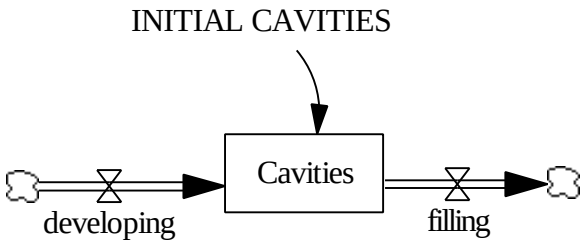


Figure 23: Vensim Equivalent of Cavities Model

Documentation for Cavities Model:

(1) Cavities = INTEG (developing-filling, INITIAL CAVITIES)
Units: cavities
I currently don't have any cavities.

(2) developing = 0.5

Units: cavities/Year

I develop a full-blown cavity every two years.

(3) filling = 0.333333

Units: cavities/Year

I get a cavity filled every three years.

(4) FINAL TIME = 15

Units: Year

The final time for the simulation.

(5) INITIAL CAVITIES = 0

Units: cavities

(6) INITIAL TIME = 0

Units: Year

The initial time for the simulation.

(7) SAVEPER = TIME STEP

Units: Year

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Year

The time step for the simulation.

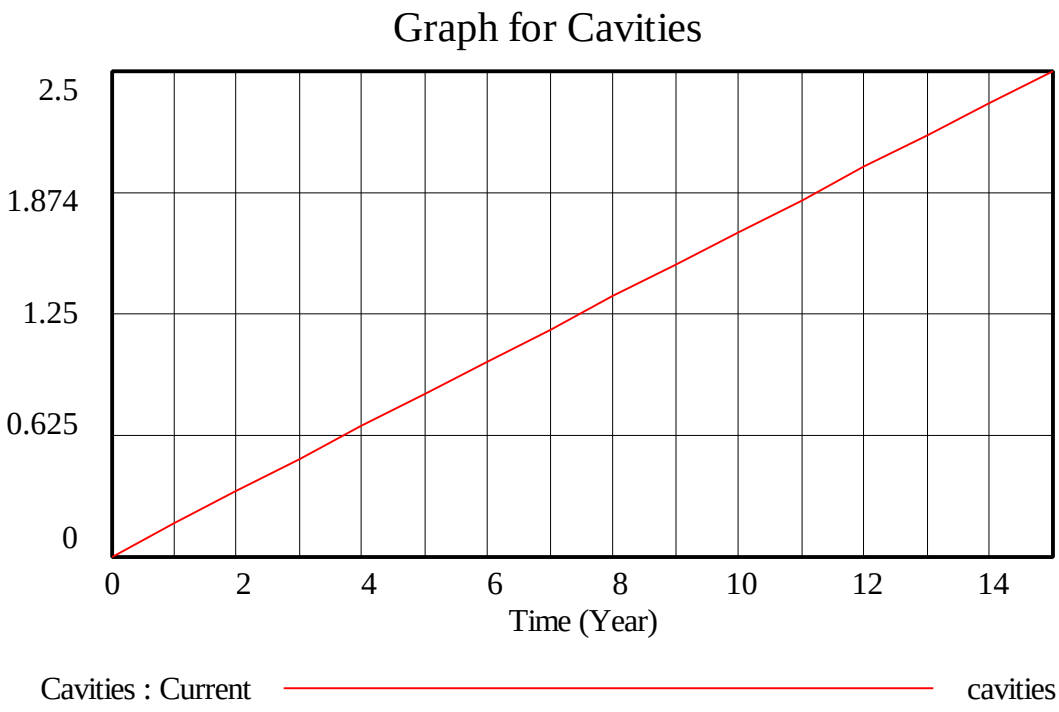


Figure 24: Vensim Equivalent of Simulation for Number of Cavities

Section 4.13: Money in Bank Account

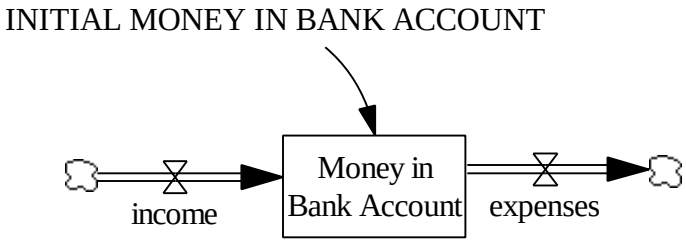


Figure 25: Vensim Equivalent of Money in Bank Account Model

Documentation for Money in Bank Account Model:

(1) expenses = 7.75 +60/3

Units: dollars/Week

She spends $.75 every week on a movie and $60 every three weeks on clothes.

(2) FINAL TIME = 16

Units: Week

The final time for the simulation.

(3) income = 120-23.7

Units: dollars/Week

Every week she gets a paycheck for $120 minus $23.70 in taxes.

(4) INITIAL MONEY IN BANK ACCOUNT = 678.53

Units: dollars

(5) INITIAL TIME = 0

Units: Week

The initial time for the simulation.

(6) Money in Bank Account = INTEG (+income-expenses, INITIAL MONEY IN BANK ACCOUNT)

Units: dollars

Stephanie has $678.53 in her bank account.

(7) SAVEPER = TIME STEP

Units: Week

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Week

The time step for the simulation.

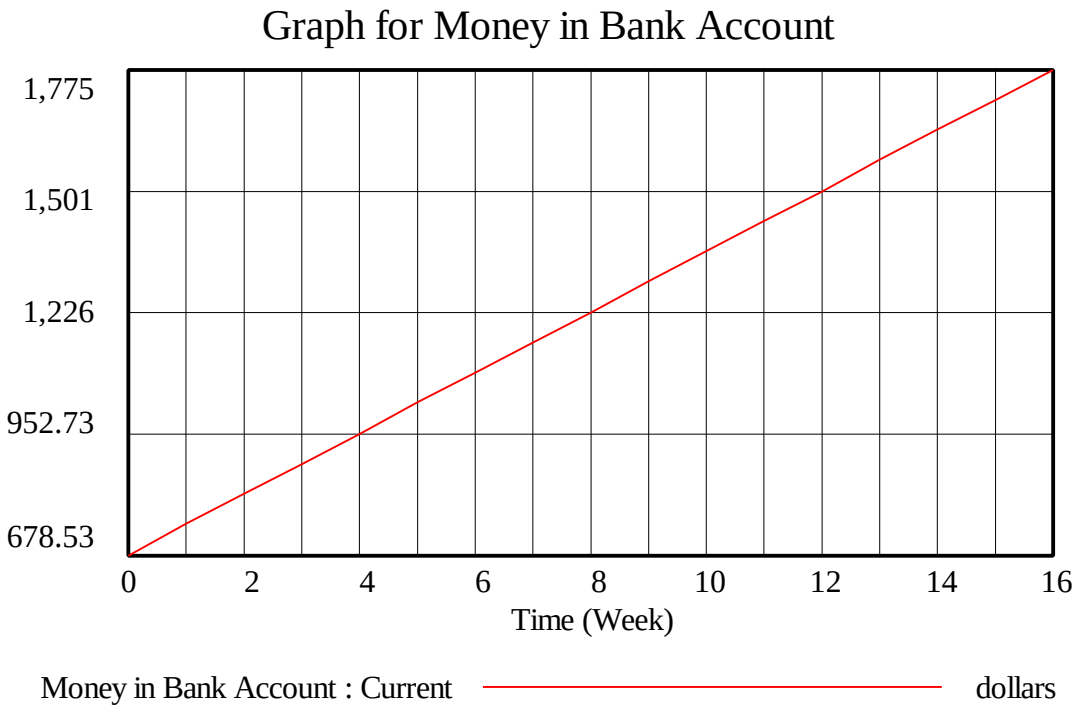


Figure 26: Vensim Equivalent of Simulation for Amount of Money in Bank Account

Section 4.14: Nuclear Weapons

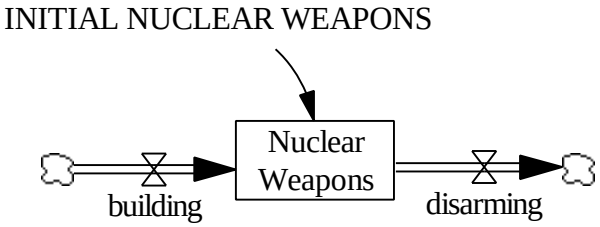


Figure 27: Vensim Equivalent of Nuclear Weapons Model

Documentation for Nuclear Weapons Model:

(1) building = 500

Units: weapons/Year

Every year Euroamerica builds 500 new nuclear weapons.

(2) disarming = 600

Units: weapons/Year

Every year Euroamerica disarms 600 nuclear weapons.

(3) FINAL TIME = 2010

Units: Year

The final time for the simulation.

(4) INITIAL NUCLEAR WEAPONS = 10000

Units: weapons

(5) INITIAL TIME = 1990

Units: Year

The initial time for the simulation.

(6) Nuclear Weapons = INTEG (building-disarming, INITIAL NUCLEAR WEAPONS)

Units: weapons

In 1990 Euroamerica has 10,000 nuclear weapons.

(7) SAVEPER = TIME STEP

Units: Year

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Year

The time step for the simulation.

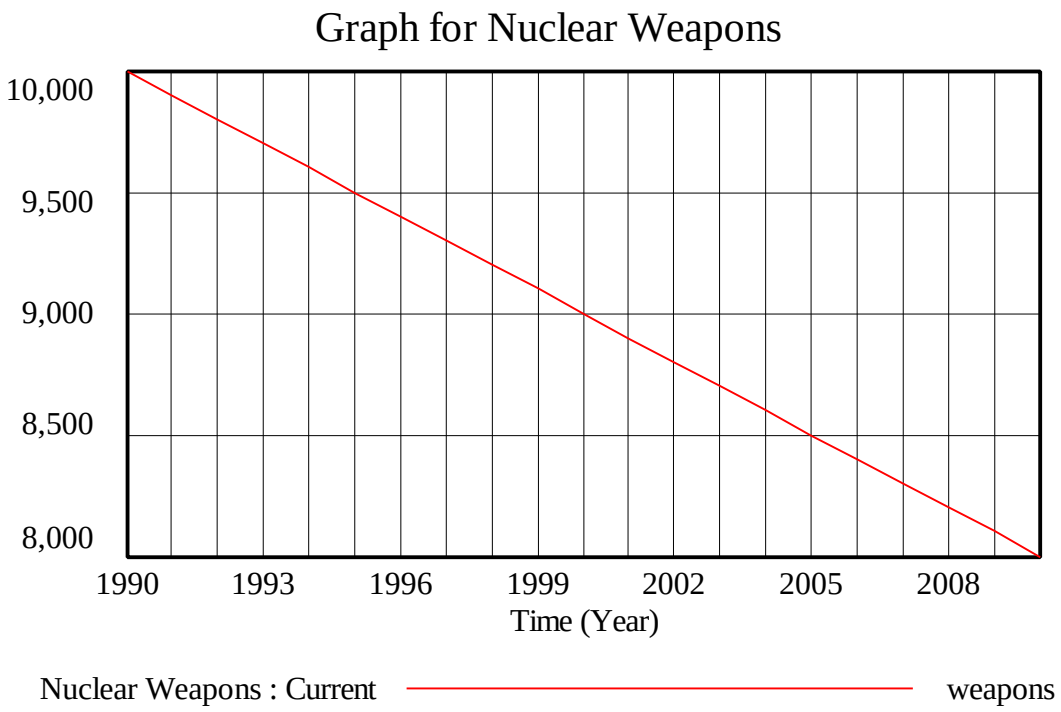
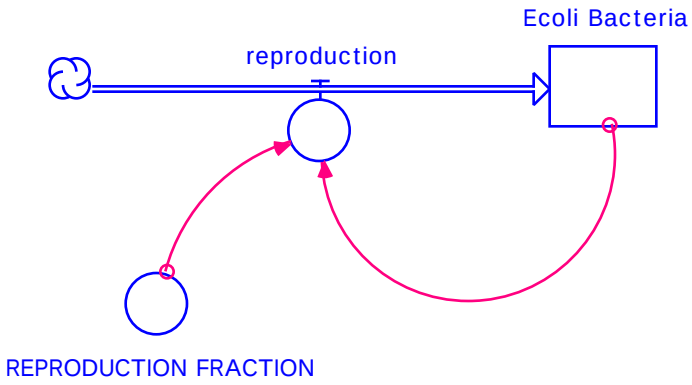


Figure 28: Vensim Equivalents of Simulation for Number of Nuclear Weapons

An Introduction to Feedback



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Leslie A. Martin
October 24, 1997

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1. ABSTRACT

This paper will develop the concept of feedback. Feedback is the process through which a signal travels through a chain of causal relations to reaffect itself. Feedback can be divided into two categories: positive and negative. Feedback is *positive* if an increase in a variable, after a delay, leads to a further increase in the same variable. Positive feedback is found in compounding, reinforcing, or amplifying systems that produce exponential behavior. On the other hand, feedback is *negative* if an increase in a variable eventually leads to a decrease in the variable. Negative feedback drives balancing or stabilizing systems that produce asymptotic or oscillatory behavior. This paper will reinforce the general theory of feedback with many different examples in the form of exercises. The exercises will involve the reader in making the distinction between positive and negative feedback.

2. WHAT IS FEEDBACK?

"Feedback is a process whereby an initial cause ripples through a chain of causation ultimately to reaffect itself."[1] The easiest way to explain the concept of feedback is with an example:

> "As a simple example, consider an initial disturbance in the temperature of a room caused by a sudden cold spell. This drop in temperature might in turn cause various types of activities. For example, persons in the room might put on sweaters or move to a warmer part of the house. Also, the thermostat might turn on the furnace. The activity of the furnace might in turn cause a number of things to happen. Furnace activity might cause the level of fuel oil in the storage tank to go down, which in turn might cause the future purchase of more fuel oil. Furnace activity might also cause wear and tear on the burner unit, which might cause further repairs to be made. However, none of these causal chains feeds back to influence room temperature. The important effect of furnace activity, for our purposes of analyzing the control of room temperature, is the heating up of the radiators in the room, which eventually causes the room temperature to rise."[2]

The thermostat example provides instances of both open-loop and closed-loop systems. An open-loop system is a simple chain of causality: the room temperature drops so people put on sweaters. Notice that the act of wearing a sweater does not at all affect the temperature in the room. A closed-loop system, on the other hand, is a circular chain of causality that "feeds back" to itself: the room temperature drops and activates the thermostat, which turns on the furnace, which heats up the radiators, which raises the room temperature. A change in room temperature automatically causes another change in room temperature. In the thermostat example, the effect of the two changes is balancing: a drop in room temperature causes a rise in room temperature. If a thermostat is working correctly, the temperature in the room should never deviate very far from the ideal setting.

Closed-loop systems are most commonly called **feedback systems**. In a feedback system, each variable is—at the same time—cause and effect. Room temperature is *effected by* the activity of the radiators and *affects* the thermostat, which affects the furnace and the future activity of the radiators. In a feedback system, a change in the environment (for example, a change in room temperature) leads to a decision (made by the thermostat) that results in an action (turning on the furnace) that affects the

---

[1] Nancy Roberts, et al, 1983. Introduction to Computer Simulation. Reading, MA: Addison-Wesley, p. 16.
[2] ibid.

environment (the radiators raise the room temperature) and thereby influences future decisions (the thermostat can shut off once the desired room temperature has been restored).[3]

Feedback can be categorized as either positive or negative. Nothing is especially great about "positive" feedback and nothing is especially depressing about "negative" feedback. The designations "positive" and "negative" indicate whether changes in the feedback system move in the same direction to produce compounding, reinforcing behavior, or move in opposite directions to produce balancing, stabilizing behavior.

3. POSITIVE FEEDBACK

Positive feedback drives growth and change. Figure 1 demonstrates an example of positive feedback. A biology research assistant is growing bacteria in a shaking flask. Reproduction increases the number of *E. coli* bacteria. The higher the reproduction rate, the more bacteria in the flask. What determines the reproduction rate? *E. coli* bacteria reproduce by dividing and multiplying. The reproduction rate depends directly on how many bacteria are already in the flask.

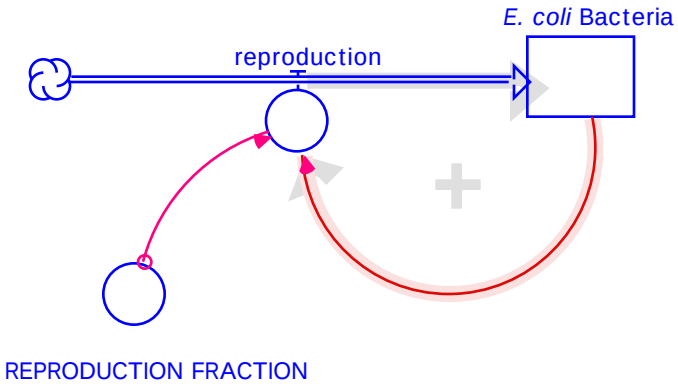


Figure 1: Positive feedback: *E. coli* bacteria

The shaded arrows in Figure 1 are not information links or material links but *causal* links. The reproduction flow of bacteria each hour into the stock of accumulated bacteria is a *material* link. The connector which ties the reproduction rate to the current number of bacteria is called an *information* link because it carries the information about the current state of the system (the value of the stock) to the mechanisms that drive the system to change (the flow). The material and information links help determine the

[3] Jay W. Forrester, 1961. Industrial Dynamics. Portland, OR: Productivity Press, p. 14.

behavior of the system over time. The shaded arrows, on the other hand, do not drive the system but simply describe the flow of causality. The number of E. coli bacteria in the flask influences the reproduction rate which, in turn, influences the number of E. coli bacteria. You can trace through the feedback loop by following the causal arrows. Increasing the number of *E. coli* bacteria will in turn increase the reproduction rate, which compounds the initial increase of the number of *E. coli* bacteria in the shaking flask.

What do you think happens to the number of bacteria over time? Assume the shaking flask holds initially 100 bacteria cells. It takes approximately a half an hour for a population of *E. coli* to double. Assume that the bacteria will have enough room and nourishment to grow unhindered. How many bacteria will the research assistant find in her shaking flask four hours from now? Sketch your guess below:

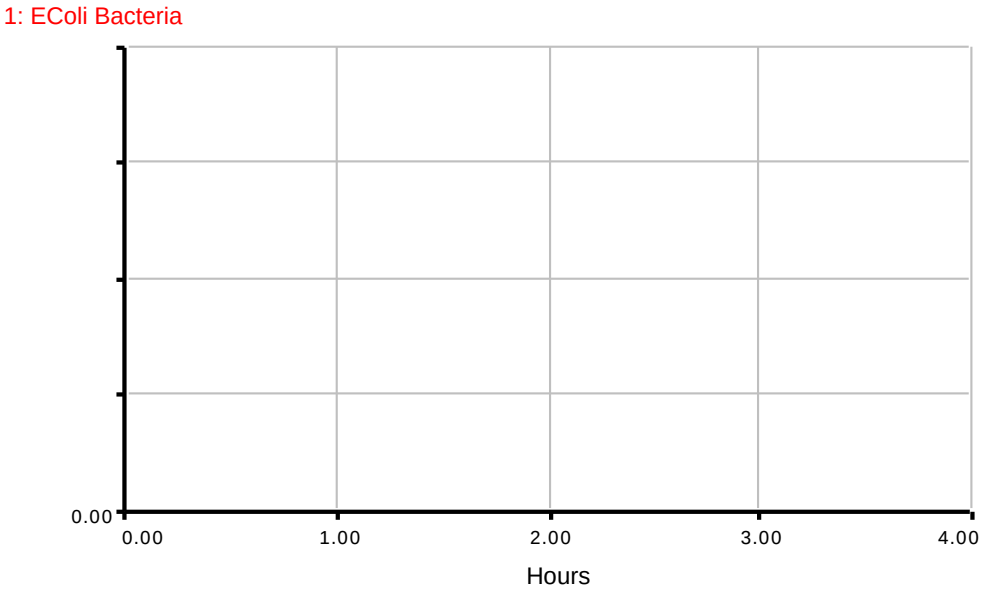


Figure 2 shows the behavior of the *E. coli* bacteria when the model in Figure 1 is run in STELLA. In four hours the population grows from 100 to 25,600 bacteria!

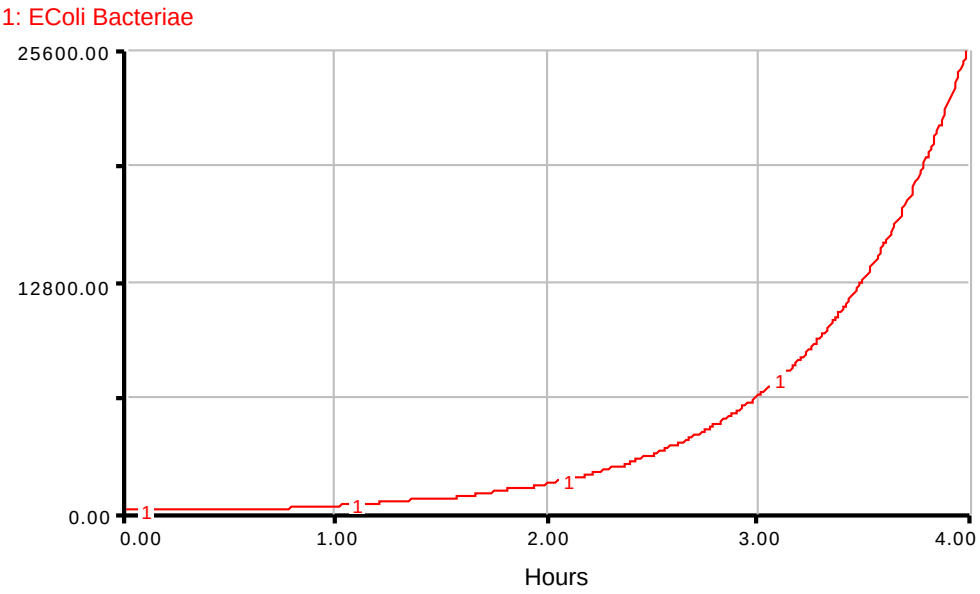


Figure 2: Exponential growth

The behavior depicted in Figure 2 is called **exponential growth**. Exponential growth is compounding and reinforcing. Another positive feedback system that exhibits exponential growth is represented in the model in Figure 3.

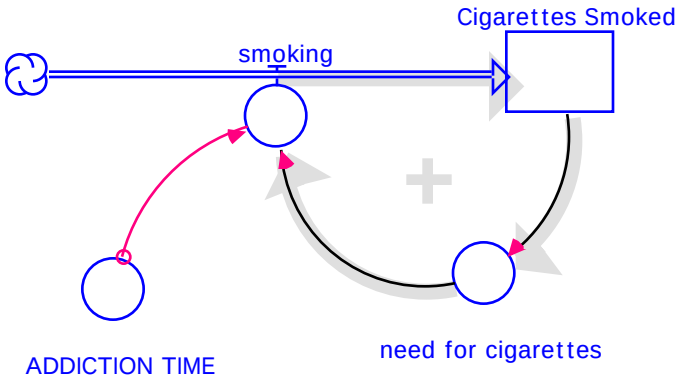


Figure 3: Positive feedback: addiction

The more my uncle smokes, the more addicted he becomes to the nicotine in his cigarettes. After smoking a few cigarettes a long time ago, my uncle began to develop a need for cigarettes. The need caused him to smoke even more, which produced an even

stronger need to smoke. The reinforcing behavior in the addiction process is characteristic of positive feedback.

4. NEGATIVE FEEDBACK

Negative feedback negates change and stabilizes systems. In positive feedback, an increase in a variable eventually leads to a further increase in the same variable. In negative feedback, on the other hand, an increase in a variable eventually leads to a decrease in that variable. The thermostat example contains a negative feedback system: an increase in room temperature eventually causes a decrease in room temperature. A child who learns to ride a bicycle is also using a negative feedback process: when the bike sways too much to the left, the child corrects by swaying to the right, and then when the bicycle sways too much to the right, he compensates again by swaying to the left. Negative feedback loops are balancing and stabilizing.

One example of a negative feedback system is the gradual decay of radioactive nuclei. Every year, a fraction of the total number of radioactive nuclei decays.

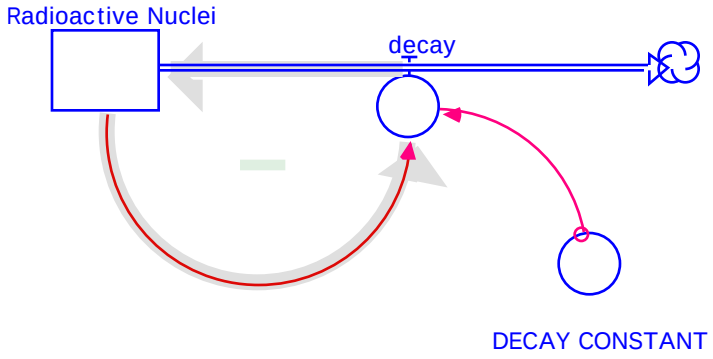
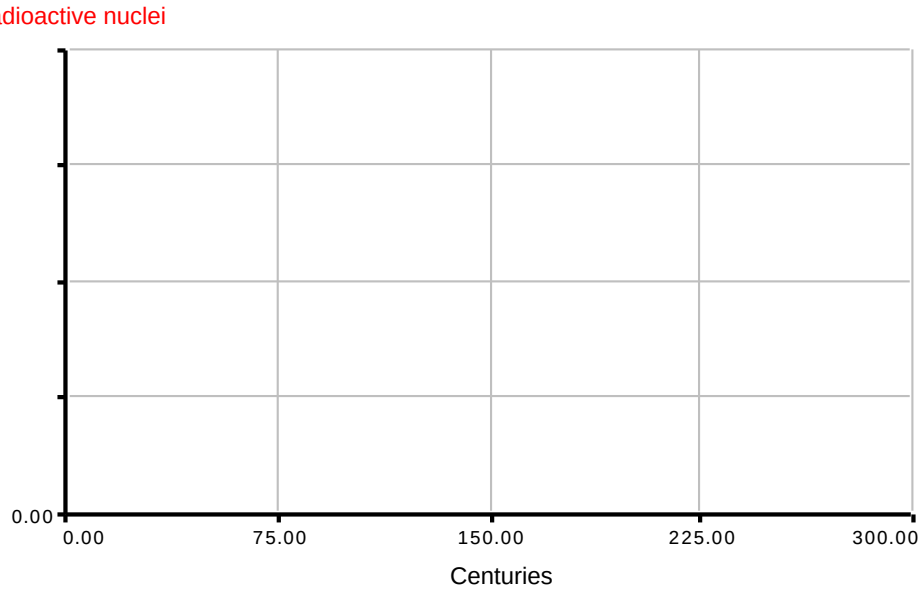


Figure 4: Negative feedback: radioactive nuclei

Figure 4 depicts a radioactive decay structure. Be careful to distinguish between the material flow out of the stock and the causal arrow into the stock. The decay rate constantly draws nuclei *out* of the stock of accumulated nuclei. So the decay rate is affecting the stock by causing it to decrease. Trace through the feedback loop, starting with the stock. At first the number of radioactive nuclei is large. Because so many nuclei are present, the decay rate is large. The decay reduces the number of nuclei. Loop through a second time. Now a smaller number of nuclei remain, but the number is still significant. The decay rate is still high, but not as high as before. The number of nuclei

is further reduced, but not as drastically. Loop through a several more times. Nuclei continue to decay, so the nuclei become fewer and fewer. As the nuclei decay, the decay rate becomes weaker and weaker, so the decaying process gets slower and slower.

Given that carbon-14 has a half-life of 5230 years, what do you think that the behavior of 1000 radioactive carbon-14 nuclei would be over a period of 300 centuries?



The behavior of the radioactive nuclei, modeled in STELLA, is replicated in Figure 5. The behavior in Figure 5 is called **exponential decay** or **asymptotic decline**.

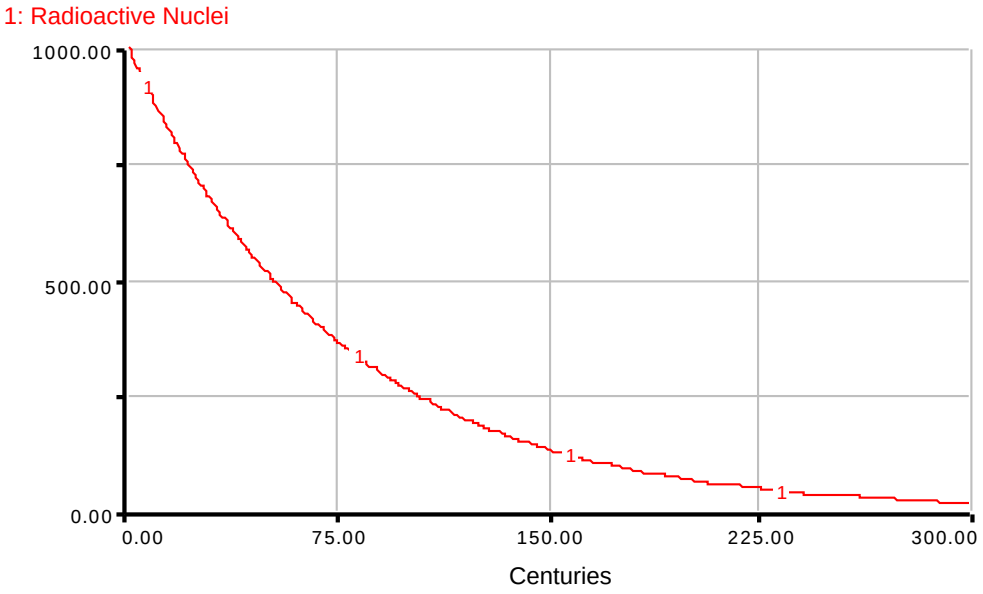


Figure 5: Asymptotic decline

My dieting is another example of a negative feedback system. The model representing how I diet is shown in Figure 6. The more I am overweight, the more I diet, which makes me less overweight. As my weight approaches my desired weight, I feel less pressured to diet.

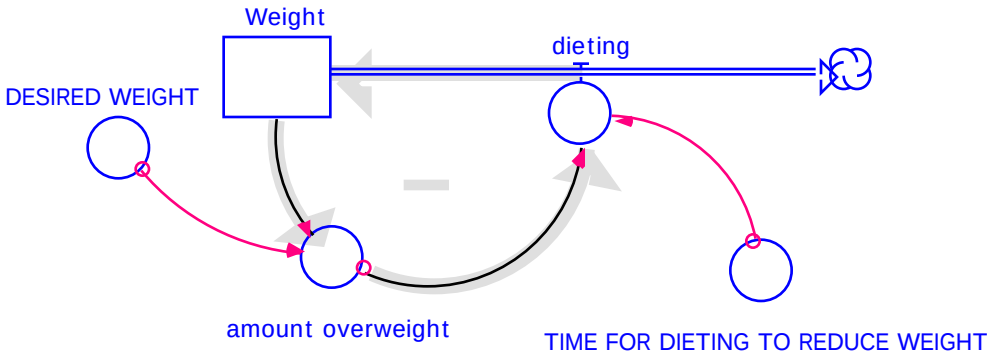


Figure 6: Negative feedback: dieting

As an example, today I weigh 200 pounds. I would really love to weigh 150 pounds, so I begin a strict diet. Figure 7 demonstrates how my weight decreases over a

period of 24 weeks, or 6 months. My weight drops, sharply at first and then more and more slowly, as it approaches the 150 pounds that I consider to be ideal.

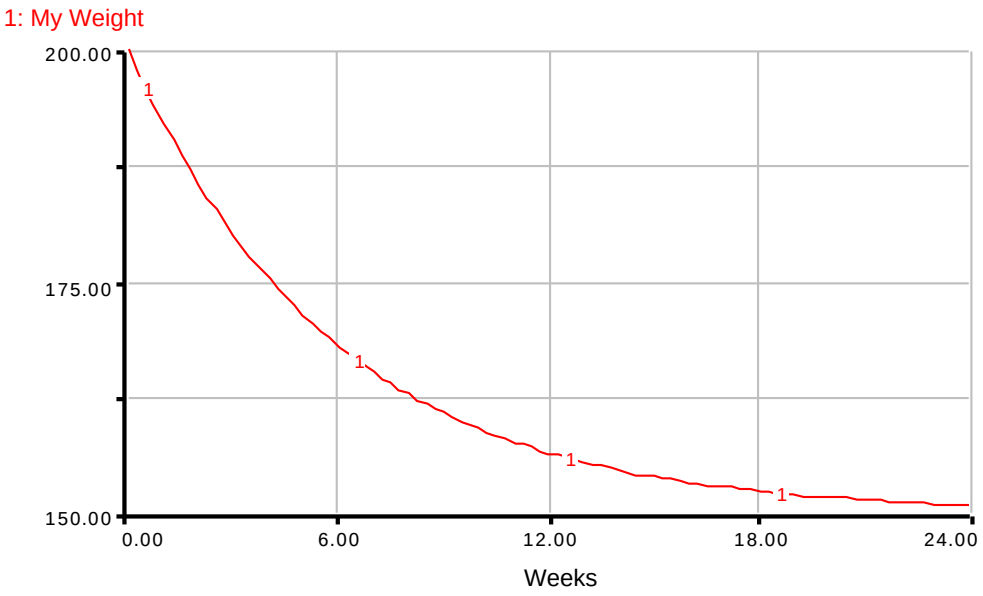


Figure 7: Asymptotic decline

Notice the similarities between the behavior of my weight and the behavior of the radioactive nuclei. Negative feedback exhibits goal-seeking behavior. I diet in order for my weight to reach a goal of 150 pounds. Radioactive nuclei decay towards a goal of zero; they decay until none remain.

In negative feedback, the stock in question does not need to *fall* towards its goal. The stock can also *climb* towards the goal. Take dieting as an example. I may be overweight, but my younger brother is underweight. He wants to weigh 150 pounds but currently weighs a puny 100 pounds. So whereas I begin to diet, he starts to eat heartily. Figure 8 shows how both weights change over the same 24-week period.

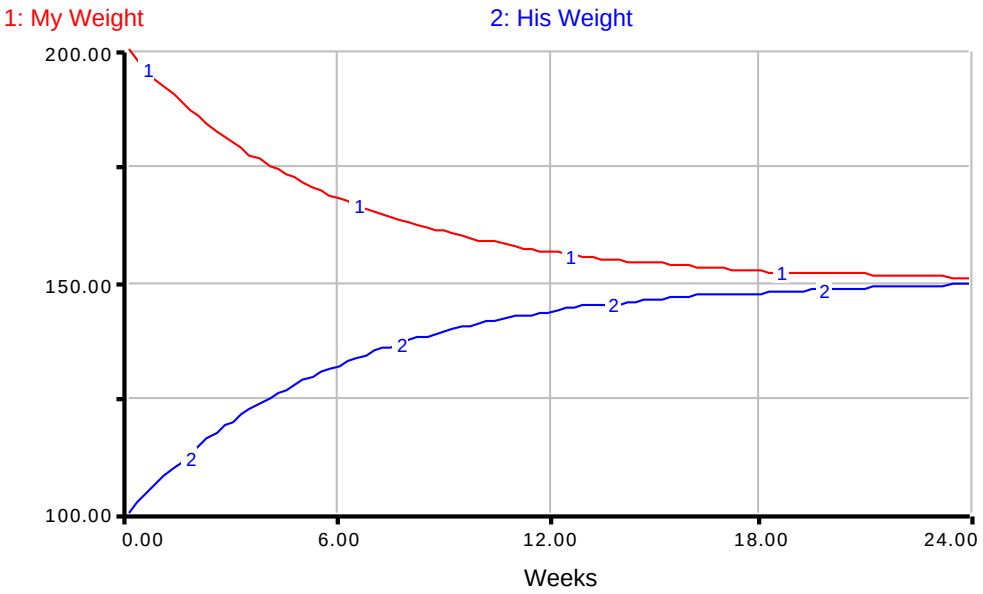


Figure 8: Asymptotic behavior

The behavior of both of the stocks in Figure 8 is asymptotic. The curves tend towards a goal, the desired weight. Negative feedback causes goal-seeking behavior. In simple systems like the ones presented in this paper, goal-seeking behavior will be asymptotic; in complex systems, goal-seeking behavior may be oscillatory. Positive feedback, in contrast, causes exponential, amplifying behavior. The two types of feedback, positive and negative, combine to create all of the behavior observed in complex systems.

5. EXERCISES

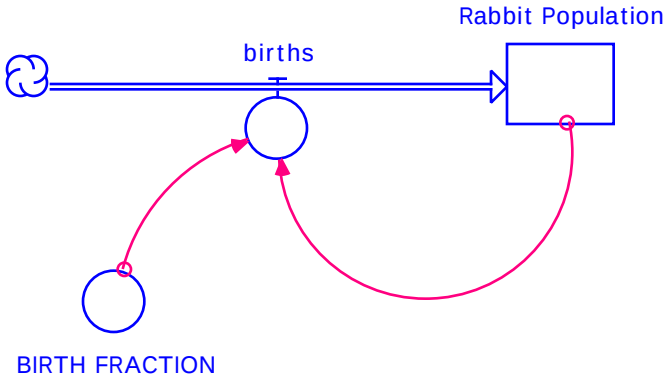
The goal of these exercises is to learn to differentiate between positive and negative feedback. For each exercise, first trace through the causal links by identifying whether each variable causes an increase or a decrease in the next. Circle the correct choice. Then determine whether the feedback loop is positive or negative: is a change in the stock reinforced or balanced? Circle the correct choice. Then trace through the feedback loop on the stock and flow diagram with your pen and label the loop as positive or negative. When you are done, flip to the next page to check your answers.

5.1 Rabbit Population

· An increase in the rabbit population *increases* *decreases* the birth rate.

· An increase in the birth rate *compounds* *counterbalances* the initial increase in the rabbit population.

· The loop, therefore, is a *positive* *negative* feedback loop.

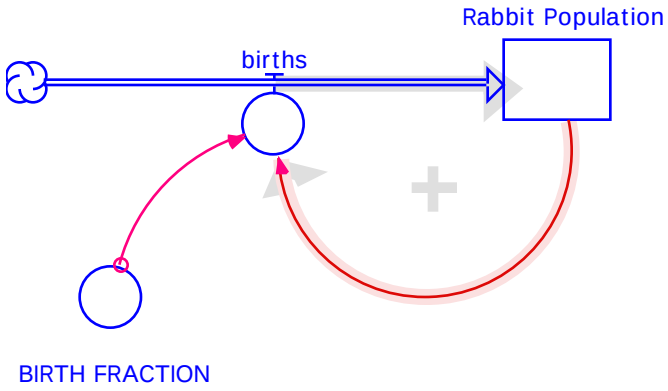


births = Population * BIRTH FRACTION

UNITS: people/year

Rabbit Population: Answers

- An increase in the rabbit population *increases* the birth rate.
- An increase in the birth rate *compounds* the initial increase in the rabbit population.
- The loop, therefore, is a *positive* feedback loop.



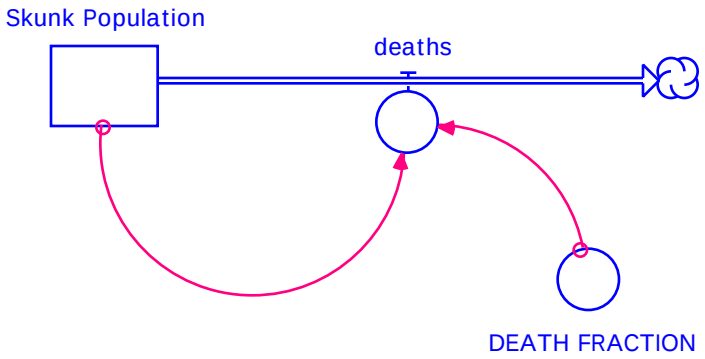
The larger the population, the greater the number of births. The more the birth rate increases, the faster the population increases.

5.2 Skunk Population

· An increase in the skunk population *increases* *decreases* the death rate.

· An increase in the death rate *compounds* *counterbalances* the initial increase in the skunk population.

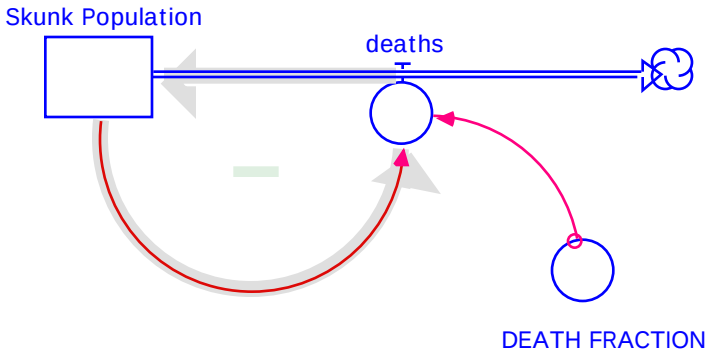
· The loop, therefore, is a *positive* *negative* feedback loop.



deaths = Skunk Population * DEATH FRACTION
UNITS: skunks/year

Skunk Population: Answers

- An increase in the skunk population *increases* the death rate.
- An increase in the death rate *counterbalances* the initial increase in the skunk population.
- The loop, therefore, is a *negative* feedback loop.



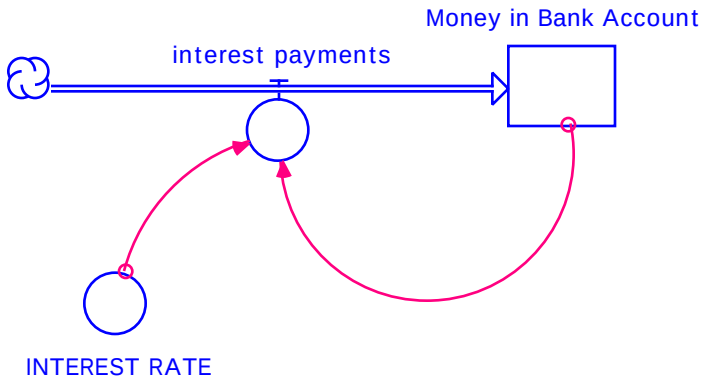
The number of deaths each year is a certain fraction of the skunk population. The death fraction represents the mortality of the population. A large skunk population has a high death rate, which sharply reduces the population. As the population decreases, the death rate becomes less drastic, causing the population to continue to decrease, but not as severely. Eventually, all the skunks die.

5.3 Bank Account

· An increase in the amount of money in a bank account *increases* *decreases* the interest payments.

· An increase in the interest payments *compounds* *counterbalances* the initial increase in the amount of money in the bank account.

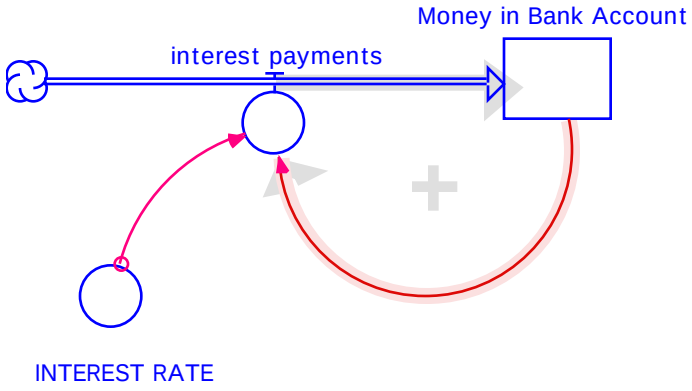
· The loop, therefore, is a *positive* *negative* feedback loop.



interest payments = Money in Bank Account * INTEREST RATE
UNITS: dollars/year

Bank Account: Answers

- An increase in the amount of money in a bank account *increases* the interest payments.
- An increase in the interest payments *compounds* the initial increase in the amount of money in the bank account.
- The loop, therefore, is a *positive* feedback loop.



Interest payments are calculated by multiplying the amount of money in a bank account by a given interest rate. The more money is in the account, the higher the interest payments, which adds even more money into the account. The rich get richer.[4]

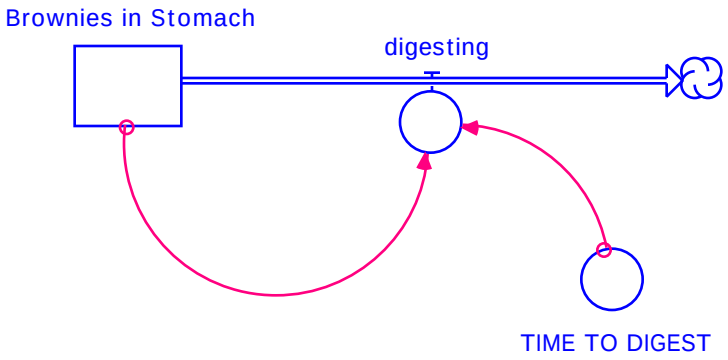
[4] Careful: Note that an interest *rate* is not actually a rate in the system dynamics sense of the word! An interest rate is a certain percentage per year. For example, the money in my bank account might be subject to 5% interest each year. A *rate* that changes the balance of my bank account is the flow of money into the account, my interest payments. If initially my bank account has $1,000 and if my account is subject to 5% interest, compounded annually, then a *rate* of $50 in interest payments flow into my account the first year.

5.4 Brownies

· An increase in the number of brownies in your stomach *increases* *decreases* your digestion rate.

· An increase in your digestion rate *compounds* *counterbalances* the initial increase in the number of brownies in your stomach.

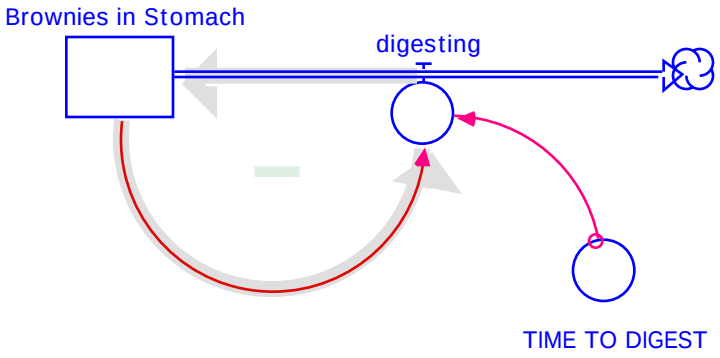
· The loop, therefore, is a *positive* *negative* feedback loop.



digesting = Brownies in Stomach / TIME TO DIGEST
UNITS: brownies/hour

Brownies: Answers

- An increase in the number of brownies in your stomach *increases* your digestion rate.
- An increase in your digestion rate *counterbalances* the initial increase in the number of brownies in your stomach.
- The loop, therefore, is a *negative* feedback loop.



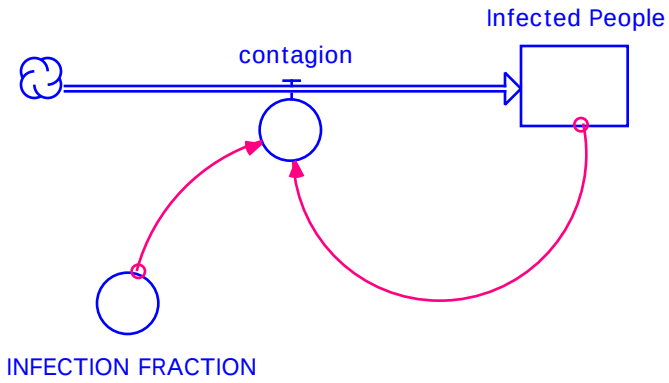
Food takes time to digest. The more brownies you have in your stomach, the more brownies you digest, which increases the rate at which brownies leave your stomach. As the number of brownies in your stomach decreases, the digesting rate slows down until all of the brownies have been digested.

5.5 Infection

· An increase in the number of infected people *increases* *decreases* the contagion rate.

· An increase in the contagion rate *compounds* *counterbalances* the initial increase in the number of infected people.

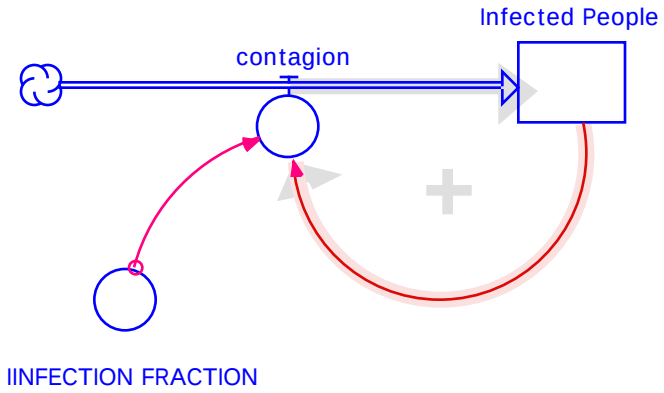
· The loop, therefore, is a *positive* *negative* feedback loop.



contagion = Infected People * INFECTION FRACTION
UNITS: people/day

Infection: Answers

- An increase in the number of infected people *increases* the contagion rate.
- An increase in the contagion rate *compounds* the initial increase in the number of infected people.
- The loop, therefore, is a *positive* feedback loop.



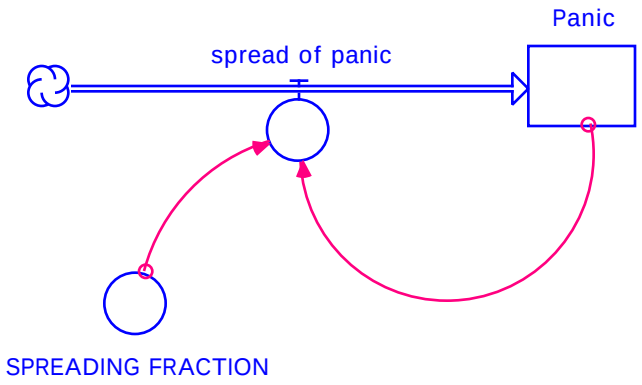
When many students in a classroom come to school with the flu, the remaining students have a higher chance of becoming sick. As the number of infected people increases, so does the rate of contagion. In turn, more and more people will be infected with the illness.

5.6 Panic

· An increase in the amount of panic *increases* *decreases* the rate at which panic spreads.

· An increase in the rate at which panic spreads *compounds* *counterbalances* the initial increase in the amount of panic.

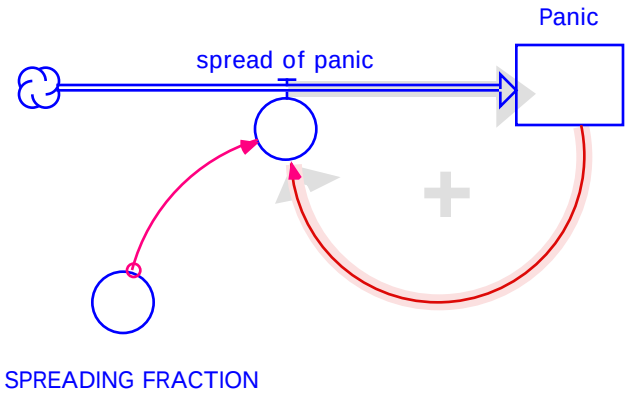
· The loop, therefore, is a *positive* *negative* feedback loop.



increase in panic = Panic * SPREADING FRACTION
UNITS: panic/minute

Panic: Answers

- An increase in the amount of panic *increases* the rate at which panic spreads.
- An increase in the rate at which panic spreads *compounds* the initial increase in the amount of panic.
- The loop, therefore, is a *positive* feedback loop.



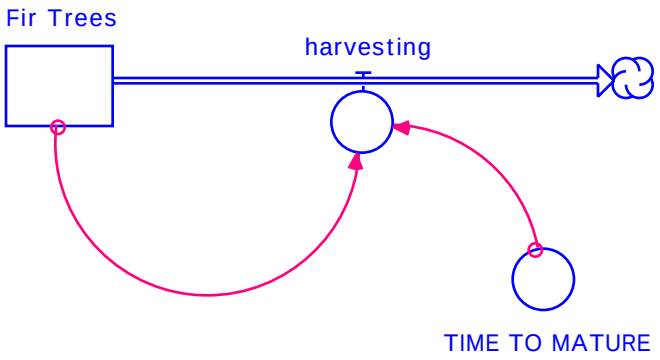
Someone shouts “fire!” in a theater. As people panic, the spread of panic increases. Across the theater hall, more and more people try to rush out. As the spread of panic increases, the total amount of panic skyrockets.

5.7 Fir Trees

· An increase in the number of fir trees *increases* *decreases* the harvesting rate.

· An increase in the harvesting rate *compounds* *counterbalances* the initial increase in the number of fir trees.

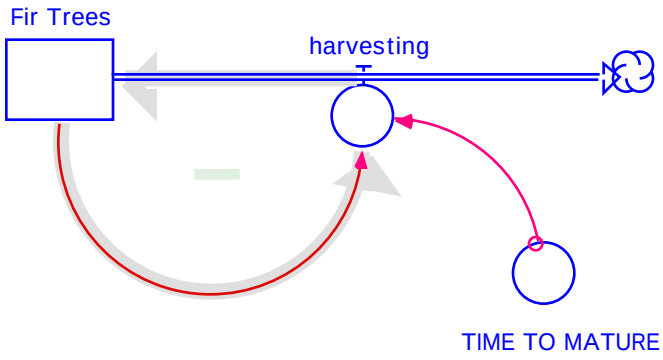
· The loop, therefore, is a *positive* *negative* feedback loop.



harvesting = Fir Trees / TIME TO MATURE
UNITS: trees/year

Fir Trees: Answers

- An increase in the number of fir trees *increases* the harvesting rate.
- An increase in the harvesting rate *counterbalance*s the initial increase in the number of fir trees.
- The loop, therefore, is a *negative* feedback loop.



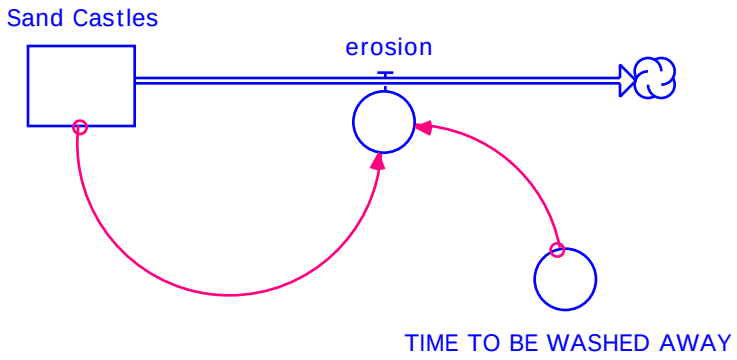
Timberjacks chop down great, towering trees; they harvest the mature trees in the forest. The more fir trees in the forest, the more trees that the timberjacks can harvest. As the number of fir trees in the forest decreases, the timberjacks can harvest fewer and fewer trees. Eventually, the trees will all get chopped down.

5.8 Sand Castles

· An increase in the number of sand castles *increases* *decreases* the erosion rate.

· An increase in the erosion rate *compounds* *counterbalances* the initial increase in the number of sand castles.

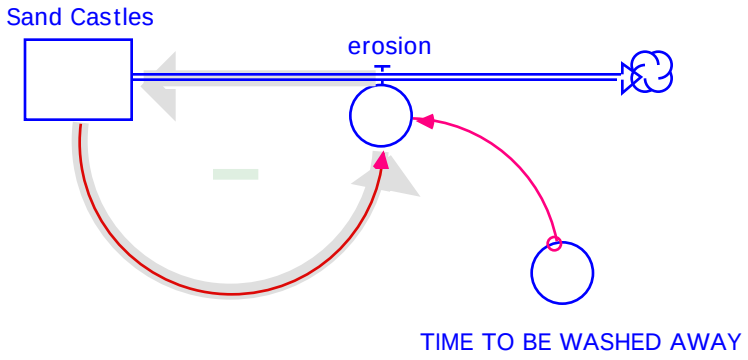
· The loop, therefore, is a *positive* *negative* feedback loop.



erosion = Sand Castles / TIME TO BE WASHED AWAY
UNITS: castles/hour

Sand Castles: Answers

- An increase in the number of sand castles *increases* the erosion rate.
- An increase in the erosion rate *counterbalances* the initial increase in the number of sand castles.
- The loop, therefore, is a *negative* feedback loop.



As the tide rises, waves wash away sand castles. The more sand castles are on the beach, the more castles the waves erode. As there are fewer sand castles left, the erosion rate becomes less dramatic until finally the last surviving castle loses its last walls.

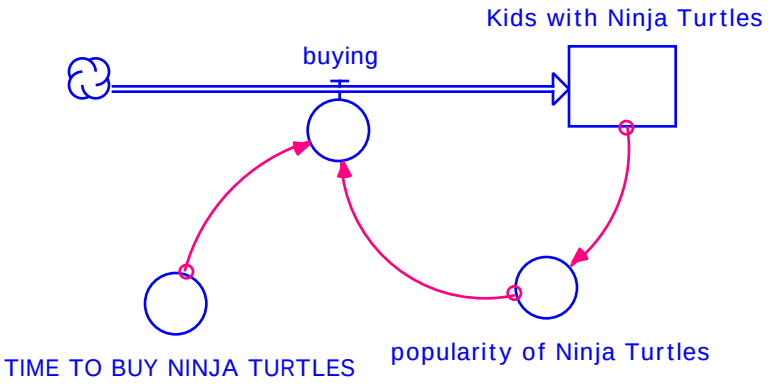
5.9 Kids with Ninja Turtles

· An increase in the number of kids with Ninja Turtles *increases* *decreases* the popularity of Ninja Turtles.

· An increase in the popularity of Ninja Turtles *increases* *decreases* the rate at which kids buy Ninja Turtles.

· An increase in the buying rate *compounds* *counterbalances* the initial increase in the number of kids with Ninja Turtles.

· The loop, therefore, is a *positive* *negative* feedback loop.

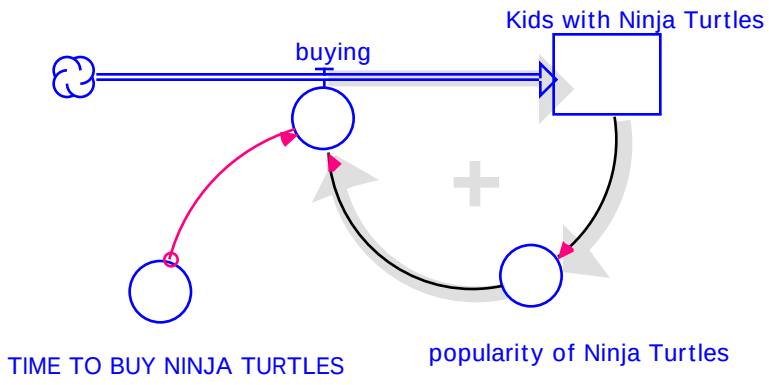


buying = Kids with Ninja Turtles / TIME TO BUY NINJA TURTLES

UNITS: kids/month

Kids with Ninja Turtles: Answers

- An increase in the number of kids with Ninja Turtles *increases* the popularity of Ninja Turtles.
- An increase in the popularity of Ninja Turtles *increases* the rate at which kids buy Ninja Turtles.
- An increase in the buying rate *compounds* the initial increase in the number of kids with Ninja Turtles.
- The loop, therefore, is a *positive* feedback loop.



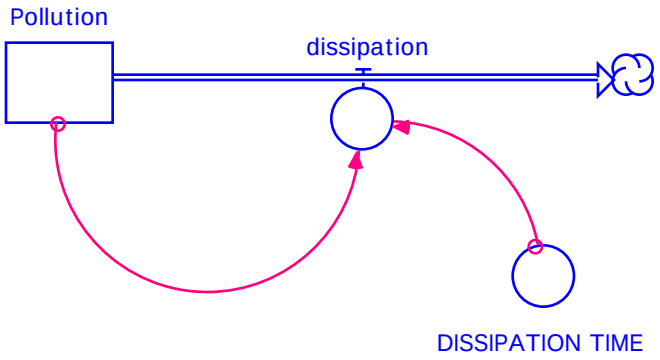
The spread of a fad is a reinforcing process. As more and more kids have Ninja Turtles, the toys become more and more popular; children (and adults) want to have the same toys as their friends. As the popularity of the toys increases, more and more people buy the Ninja Turtles.

5.10 Pollution

· An increase in the amount of pollution *increases* *decreases* the dissipation rate.

· An increase in the dissipation rate *compounds* *counterbalances* the initial increase in the amount of pollution.

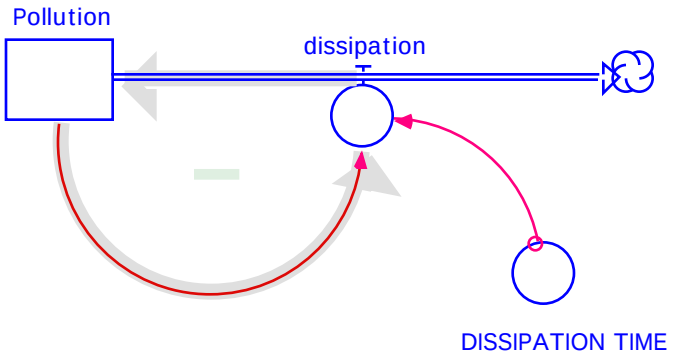
· The loop, therefore, is a *positive* *negative* feedback loop.



dissipation = Pollution / DISSIPATION TIME
UNITS: pollution/year

Pollution: Answers

- An increase in the amount of pollution *increases* the dissipation rate.
- An increase in the dissipation rate *counterbalances* the initial increase in the amount of pollution.
- The loop, therefore, is a *negative* feedback loop.



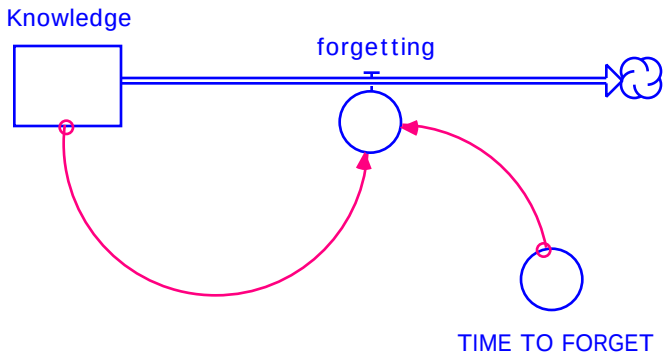
Pollution takes a long time to dissipate. The more pollution, the higher the dissipation rate, which, after a long delay, will reduce the overall level of pollution by a greater amount. As the amount of pollution decreases, the dissipation rate also decreases, until all the pollution dissipates away.

5.11 Knowledge

· An increase in the extent of your knowledge *increases* *decreases* the rate at which you forget.

· An increase in the rate at which you forget *compounds* *counterbalances* the initial increase in the extent of your knowledge.

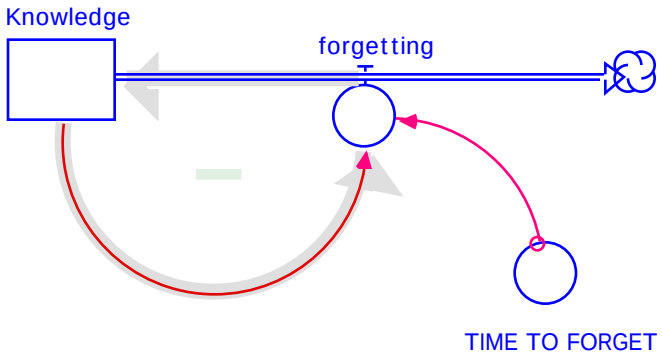
· The loop, therefore, is a *positive* *negative* feedback loop.



forgetting = Knowledge / TIME TO FORGET
UNITS: facts/year

Knowledge: Answers

- An increase in the extent of your knowledge *increases* the rate at which you forget.
- An increase in the rate at which you forget *counterbalances* the initial increase in the extent of your knowledge.
- The loop, therefore, is a *negative* feedback loop.



Sherlock Holmes used to choose not to remember certain facts because he knew that the more facts you know, the more you forget. As the number of facts you know decreases, the rate at which you forget also decreases. You may easily forget the phone numbers of the first 500 listings in the phone book that you memorized to impress your friends. On the other hand, a few select facts, like your phone number or the date of your birth, are more difficult to forget and remain with you for a longer time.

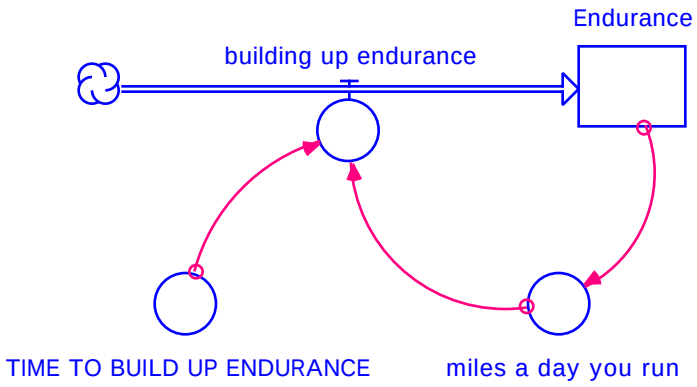
5.12 Endurance

· An increase in your endurance *increases* *decreases* the number of miles a day you run.

· An increase in the number of miles a day you run *increases* *decreases* the rate at which you build up your endurance.

· An increase in the rate at which you build up your endurance *compounds* *counterbalances* the initial increase in your endurance.

· The loop, therefore, is a *positive* *negative* feedback loop.

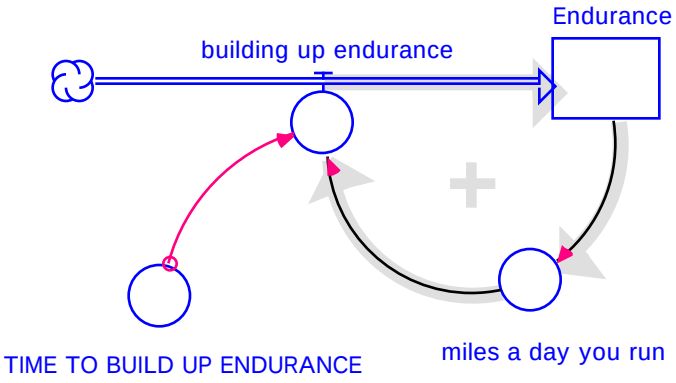


building up endurance = endurance / TIME TO BUILD UP ENDURANCE

UNITS: miles per day /week

Endurance: Answers

- An increase in your endurance *increases* the number of miles a day you can run.
- An increase in the number of miles a day you can run *increases* the rate at which you build up your endurance.
- An increase in the rate at which you build up your endurance *compounds* the initial increase in your endurance.
- The loop, therefore, is a *positive* feedback loop.



The more endurance you have, the more miles you can run every morning. The more you run, the faster you build up your endurance. Your ability to run improves with practice.

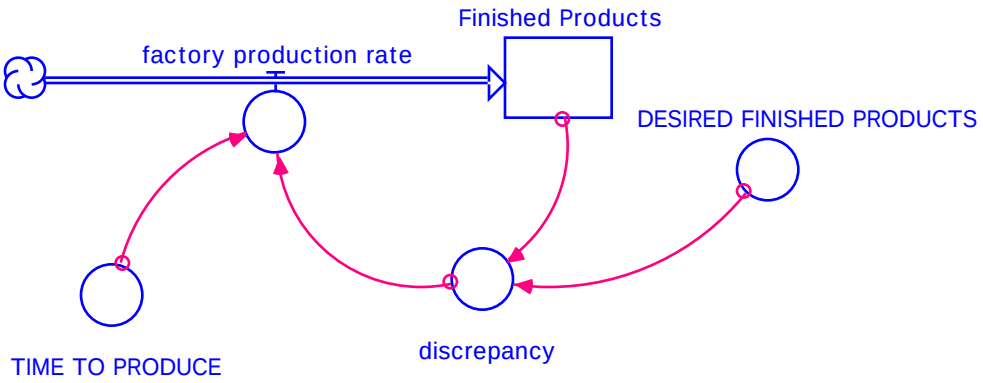
5.13 Finished Products

· An increase in the number of finished products *increases* *decreases* the discrepancy between the desired and actual number of finished products.[5]

· A decrease in the discrepancy *increases* *decreases* the factory production rate.

· A decrease in the factory production rate *compounds* *counterbalances* the initial increase in the accumulation of finished products.

· The loop, therefore, is a *positive* *negative* feedback loop.



factory production rate = discrepancy / TIME TO PRODUCE

UNITS: widgets/year

discrepancy = DESIRED FINISHED PRODUCTS - Finished Products

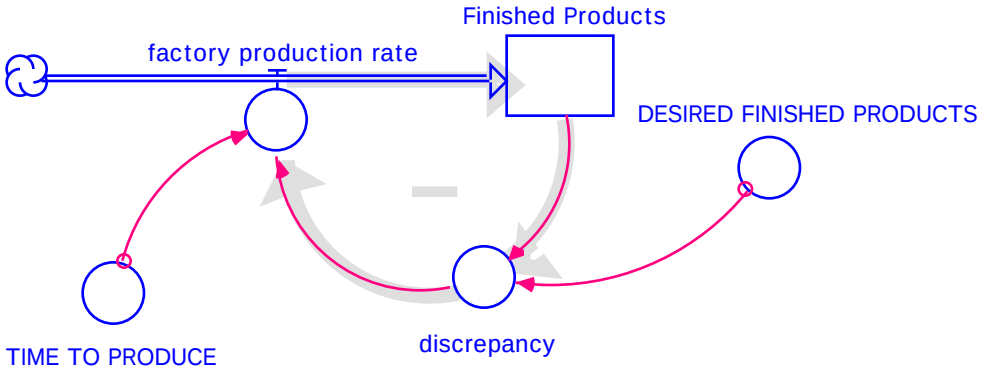
UNITS: widgets

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[5] Assume that initially the factory has fewer "Finished Products" than "DESIRED FINISHED PRODUCTS."

Finished Products: Answers

- An increase in the number of finished products *decreases* the discrepancy between the desired and actual number of finished products.
- A decrease in the discrepancy *decreases* the factory production rate.
- A decrease in the factory production rate *counterbalances* the initial increase in the accumulation of finished products.
- The loop, therefore, is a *negative* feedback loop.



The managers of a factory decide how many finished products they want to produce. They then compare the actual number of finished products already in their warehouses to the desired number of finished products and work to eliminate any discrepancy between what they have and what they want. At first, the factory runs full strength because it has many products to make. As it gets closer and closer to finishing the job, the discrepancy decreases and production slackens until finally the desired number of finished products has been produced.

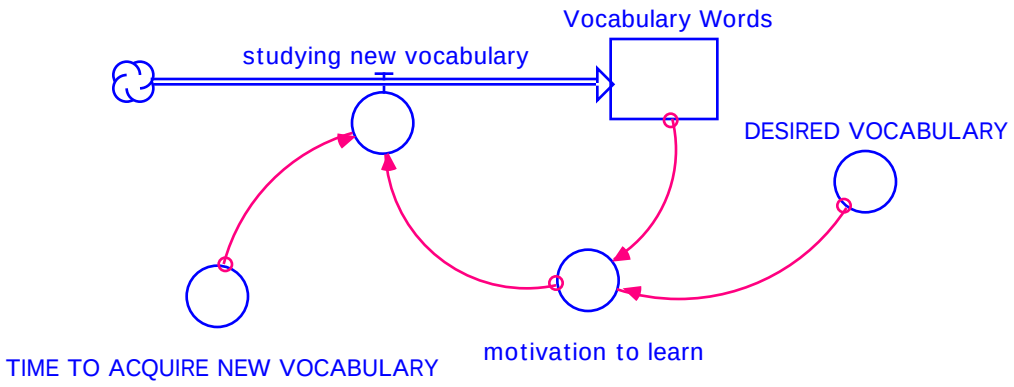
5.14 Vocabulary Words

· An increase in the number of vocabulary words that you know, compared to the number of vocabulary words you want to know, *increases* *decreases* your motivation to learn new words.[6]

· A decrease in your motivation to learn new words *increases* *decreases* the rate at which you study new vocabulary.

· A decrease in the rate at which you study new vocabulary *compounds* *counterbalances* the initial increase in the number of vocabulary words that you know.

· The loop, therefore, is a *positive* *negative* feedback loop.

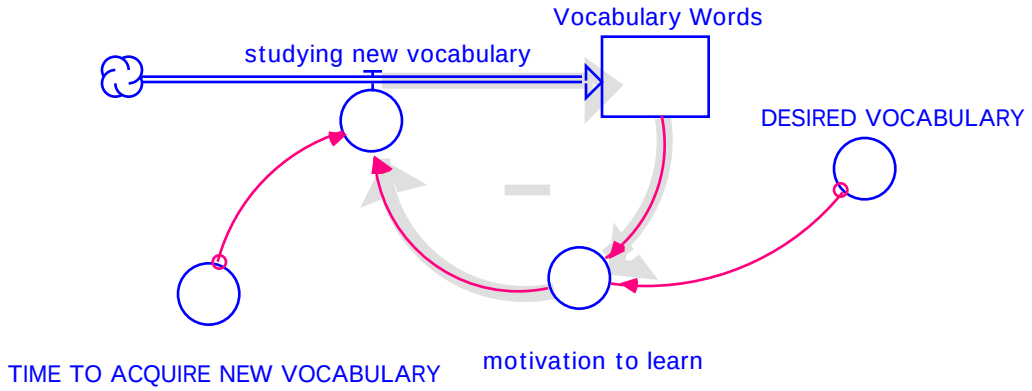


studying new vocabulary = motivation to learn / TIME TO ACQUIRE NEW VOCABULARY
UNITS: words/week

[6] Assume that initially your "DESIRED VOCABULARY" is larger than the number of "Vocabulary Words" that you know.

Vocabulary Words: Answers

- An increase in the number of vocabulary words that you know, compared to the number of vocabulary words you want to know *decreases* your motivation to learn new words.
- A decrease in your motivation to learn new words *decreases* the rate at which you study new vocabulary.
- A decrease in the rate at which you study new vocabulary *counterbalances* the initial increase in the number of vocabulary words that you know.
- The loop, therefore, is a *negative* feedback loop.



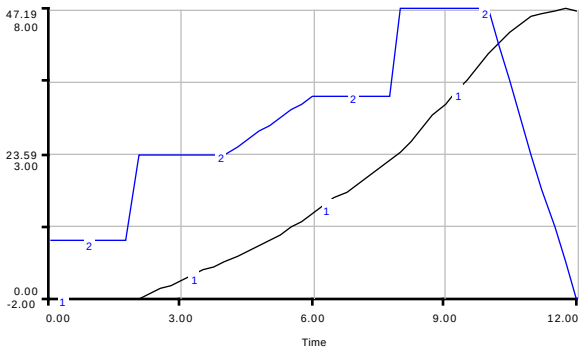
I am getting ready for a trip to France. I bought a beginner's language book that contains 1000 of the most important French words that I will need to know. I want to learn all of them. At first I study really hard and learn many new words because I am very motivated. As I learn more and more words, however, I start to feel satisfied with the words that I do know, my motivation to learn decreases, and it is only slowly and grudgingly that I commit to memory the last words.

6. SUMMARY

Feedback is the process through which a signal travels through a chain of causal relations to reaffect itself. Feedback can be divided into two categories: positive and negative. Feedback is *positive* if an increase in a variable leads to a further increase in the variable. Positive feedback is a reinforcing, compounding, or amplifying process. The reproduction of *E. coli* bacteria and the behavior of the development of a nicotine addiction are examples of positive feedback. Positive feedback systems produce exponential behavior. On the other hand, feedback is *negative* if an increase in a variable eventually leads to a decrease in the variable. Examples of negative feedback include the decay of radioactive nuclei or the process of dieting to reach an ideal weight. Negative feedback drives balancing, stabilizing systems that produce asymptotic or oscillatory behavior. The two types of feedback, positive and negative, combine to create all of the behavior observed in complex systems.

Graphical Integration Exercises

Part One: Exogenous Rates



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
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December 1, 1995

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1. Abstract

This is a first paper in the graphical integrations exercises series. The series is intended to teach its readers the basic principles and applications of graphical integration. This paper discusses graphical integration methods for systems with exogenous rates.

2. Introduction

Let us consider a simple system of a bathtub with a constant inflow through a faucet but no drainage of water. Can we predict the behavior of this system? We know that if we turn on the faucet to fill a tub, go outside for five minutes and come back to find the tub one-fourth full, then it will take ten minutes to fill up halfway, fifteen minutes to be three-fourths full, and in a total of twenty minutes from the start of the inflow, the tub will be completely full of water.

What we just did was graphical integration without explicitly drawing graphs. We were able to do this because the system in question was very simple. However, many systems, in real-life situations are much more complex, and it becomes difficult to predict how they will behave. Enough practice, however, will make it easier to use graphical integration to understand the behavior of many systems.

Although we have access to sophisticated computer programs that simulate many complex systems, it is important that we can intuitively predict and understand what we see on the graph after running a simulation. This series of graphical integration papers will enable us to do this. In this first paper of the series, we will explore the most basic case, that of exogenous rates.

An exogenous rate to a system is a rate variable that cannot be affected by that system. In other words, its value does not depend on any level of the system. Although many of the systems we see in the real world are affected by endogenous rates (the opposite of exogenous rates), learning the behavior of simple systems will set the ground for more complicated systems to come later in the series.

3. Systems with One Exogenous Rate

In this section, we will consider a system with a single exogenous rate. We will look at the system's behavior when the rate is constant and when the rate varies with time.

3.1 Constant Rate

If we turn on the faucet and leave it as it is, the rate of water flow into the tub is constant. The behavior of this system can easily be predicted.

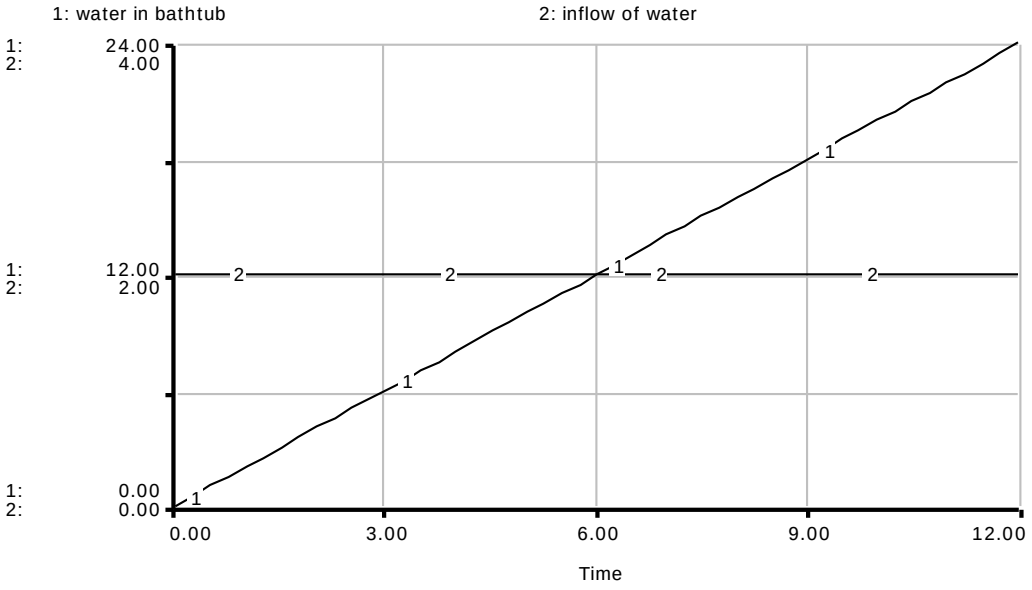


Figure 1: Water Inflow and Bathtub Level
This is an illustration of a bathtub filling with water. If the inflow is constant and positive, the water level increases linearly.

Figure 1 is a graphical analysis of what we predicted in the introduction. Notice that *the slope of the level, measured in amount of water per unit time, is equal to the value of the constant rate, also measured in amount of water per unit time.* In the above example, the water level rises to 12 units in 6 units of time, so the slope of the level is equal to 2 units of water per unit time, which is the value of the inflow.

What if there is no inflow but only an outflow? An example would be a full bathtub being drained at a constant rate. In that case, the value of the inflow is 0 and the value of

the outflow is a positive number. The net constant flow, defined as *net flow = inflow - outflow,* is *net flow* = 0 - (positive number) = (negative number), thus the slope of the level is also negative. How would the system behave in response to a negative net flow? This is illustrated in Figure 2.

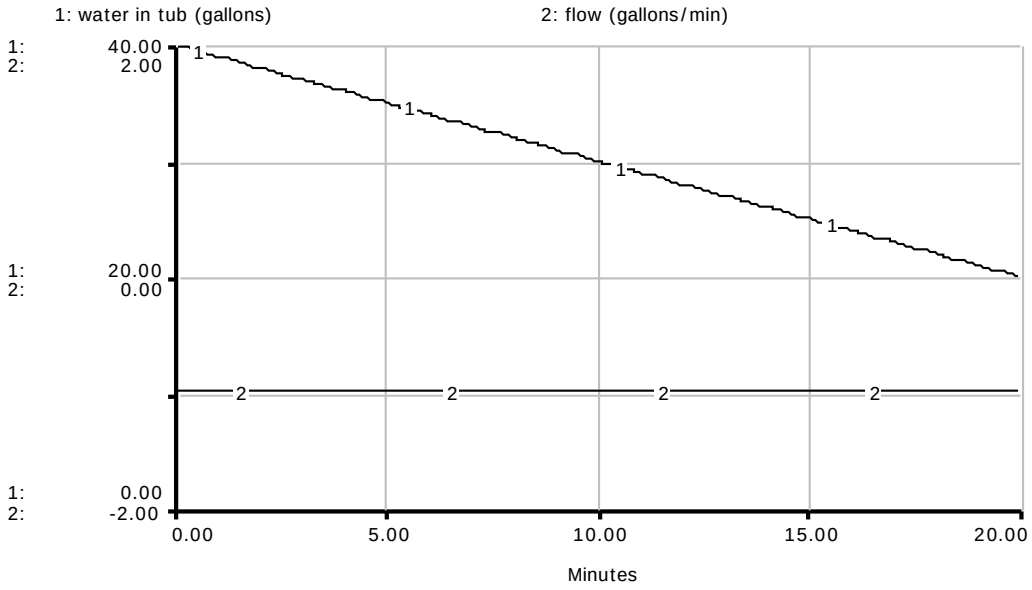


Figure 2: Water Outflow and Amount of Water in Tub
The inflow of water is equal to 0 gallon/min, and the outflow of water is equal to 1 gallon/min. Consequently, the net flow of the system is -1 gallon/min, so the slope of the level is –1 gallon/min.

As you may have expected, with a negative net constant flow, the water level *drops* at a constant rate, in this case at a rate of 1 gallon per minute.

3.2 Step Function

Now, let us look at what happens when we delay turning on the faucet.

What you will see on the next page in Figure 3 is called a *step function*. As the name suggests, a step function starts off at a constant value, in this case 0, and steps either up or down to another constant value. In this case, we step up to +3 at *Time* = 5. A function may have several steps, and we will see examples of that later in this paper.

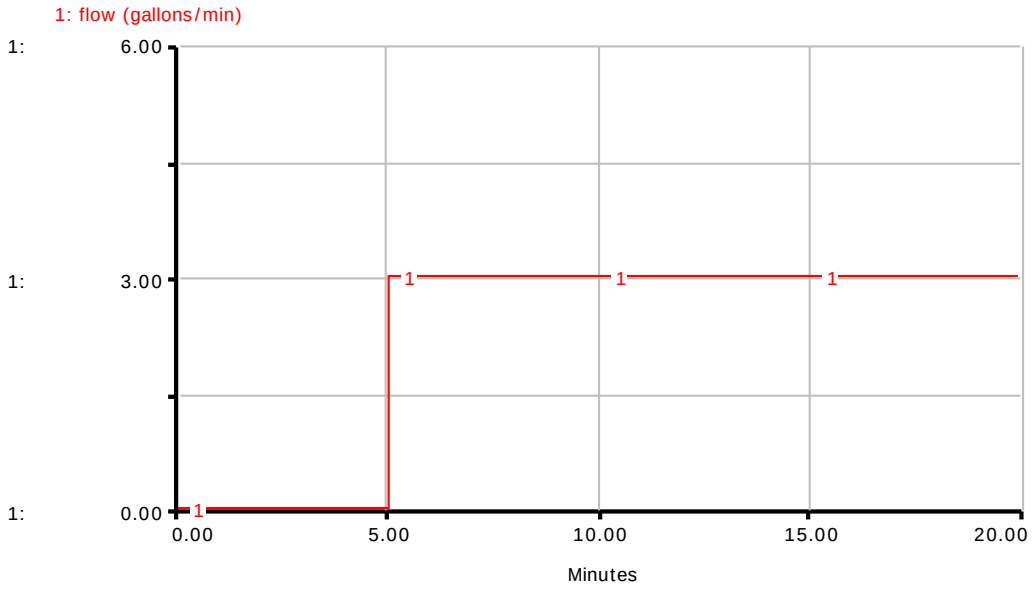


Figure 3: Step Function
Instead of starting the flow at time 0, we start the flow after waiting five minutes.

The result is illustrated in Figure 4 below. Until *Time = 5 minutes*, the stock stays at zero, and when the flow increases to 3, the stock accumulates at the rate of 3 gallons per minute.

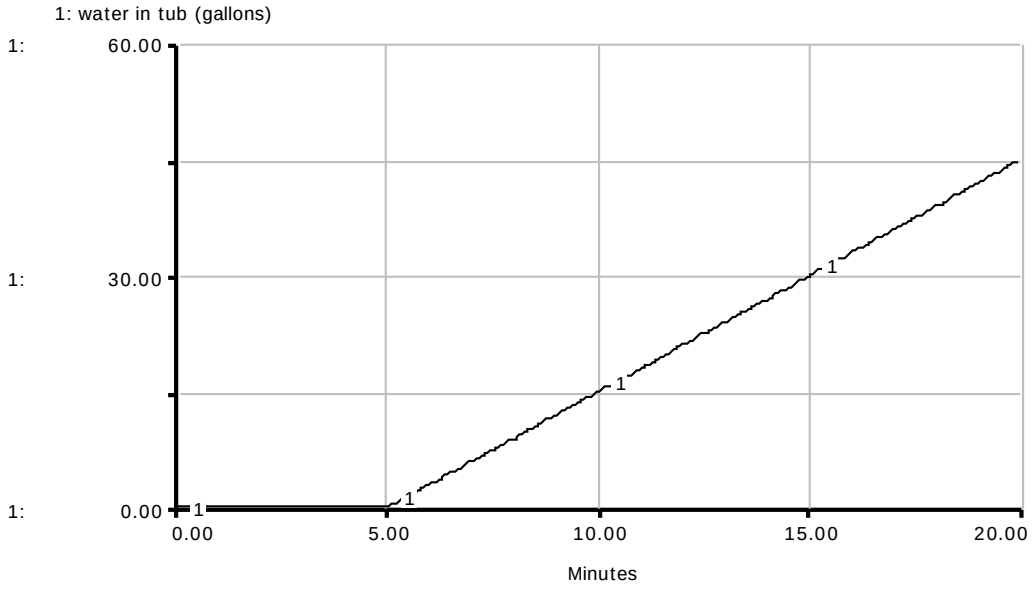


Figure 4: Graph of Stock Resulting from Step Function
Stock remains at zero until the flow increases from 0 to 3 at *Time* = 5.

Here is another example. This one has two steps, one step up at *Time* = 5 and another at *Time* = 15.

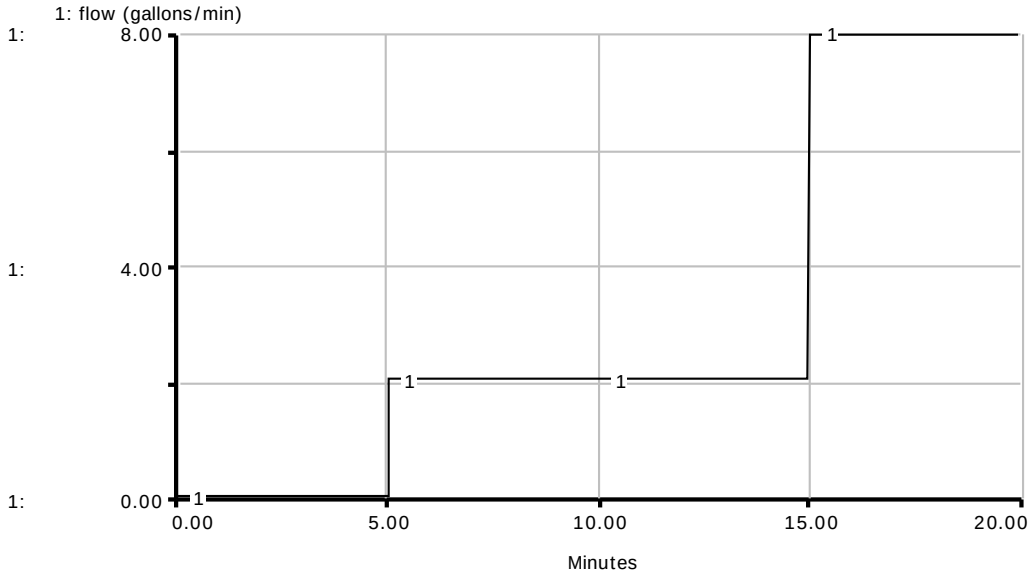


Figure 5: A Step Function Rate

Graphically integrating the rate in Figure 5, we get the following behavior of the stock.

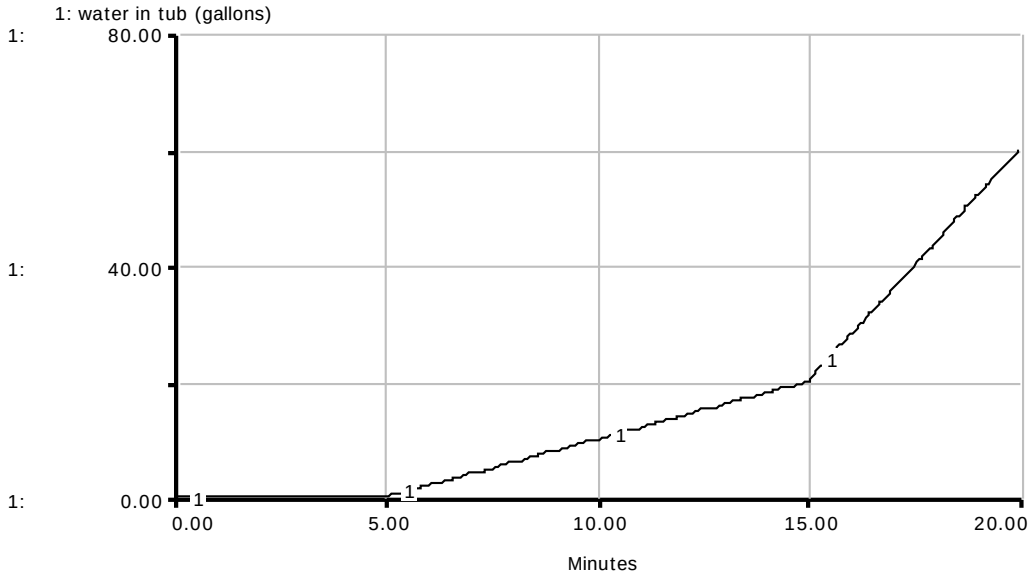


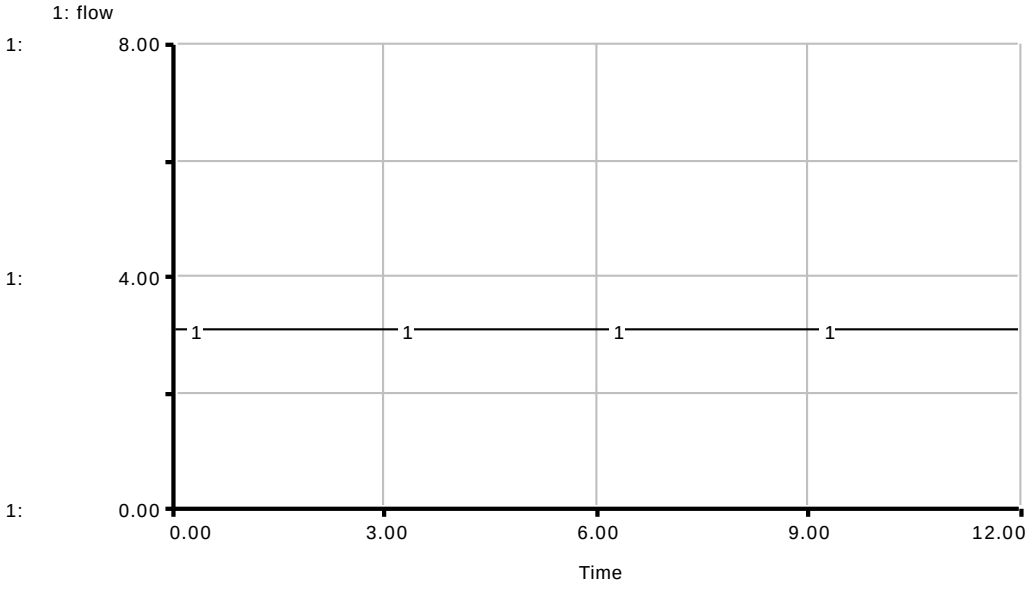
Figure 6: Value of Stock Changing as the Flow Changes in Figure 5
From *Time* = 0 to *Time* = 3, the value of the flow is 0, so the slope of the stock is also 0. From *Time* = 5 to *Time* = 15, the flow is 2 gallons/min, so the slope of the stock is also 2 gallons/min. From *Time* = 15 to *Time* = 20, the flow is 8 gallons/min, so the slope of the stock is 8 gallons/min.

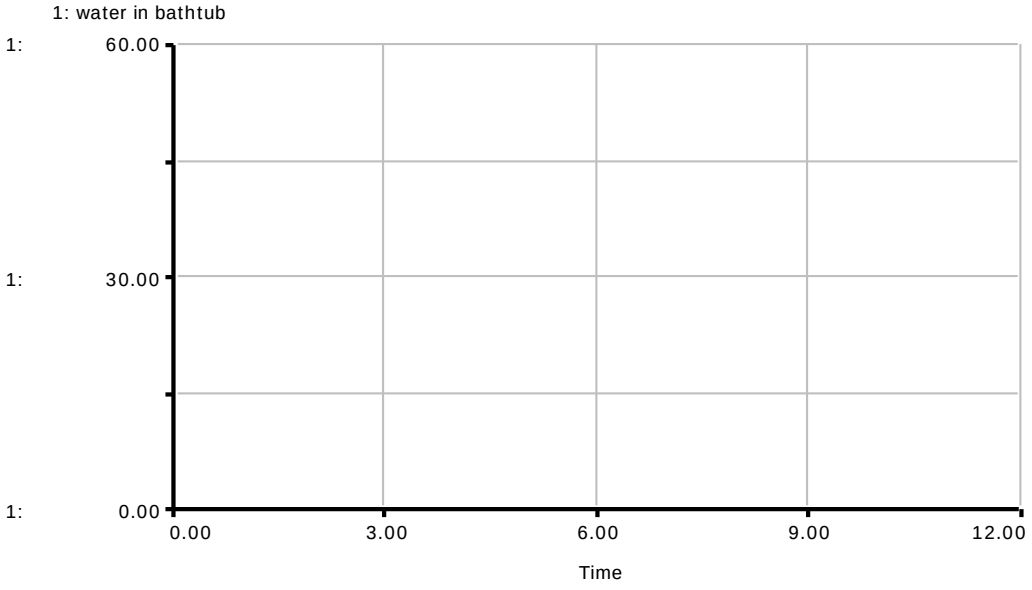
You may be wondering how one can accurately measure the slope of the stock in the above example. The values on the y-axis are not the simple numbers used in the previous examples, and the values of the flow are somewhat difficult to accurately guess. For instance, the first step in Figure 5 could have gone up to 1.9 or 2.1 instead of 2, and the second step could have been up to 7.8 or 7.9. Sometimes you do not know the exact values of flows and stocks. What is important in graphical integration is that you understand the general idea. In Figure 5, you can clearly see that the flow starts out at 0, goes up to some value, and then goes up to a bigger value. From that, you should be able to predict that the stock remains unchanged until the first step, then goes up with a constant slope, and then goes up with a steeper slope.

3.3 Exercises

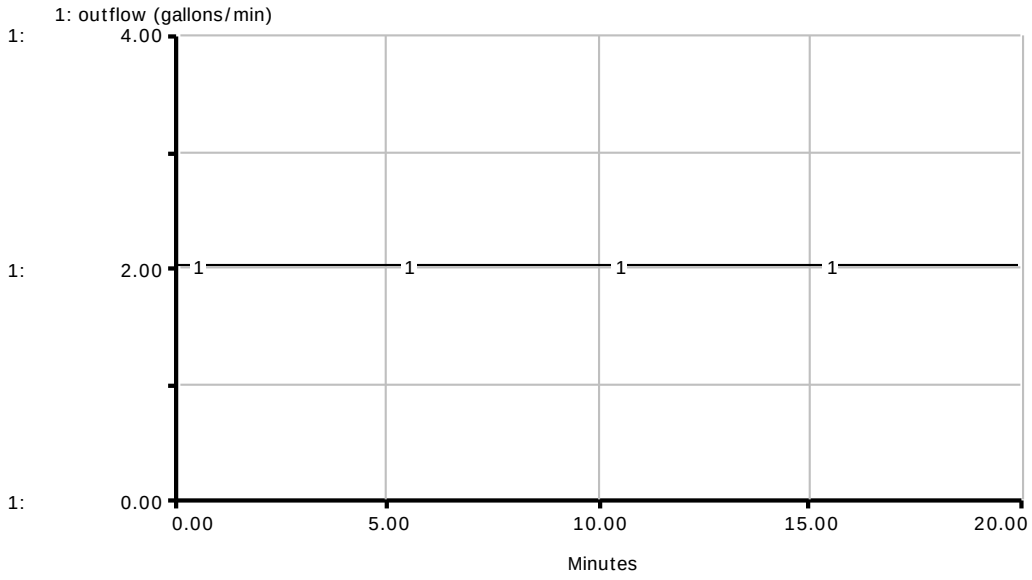
Now, let's try some exercises.

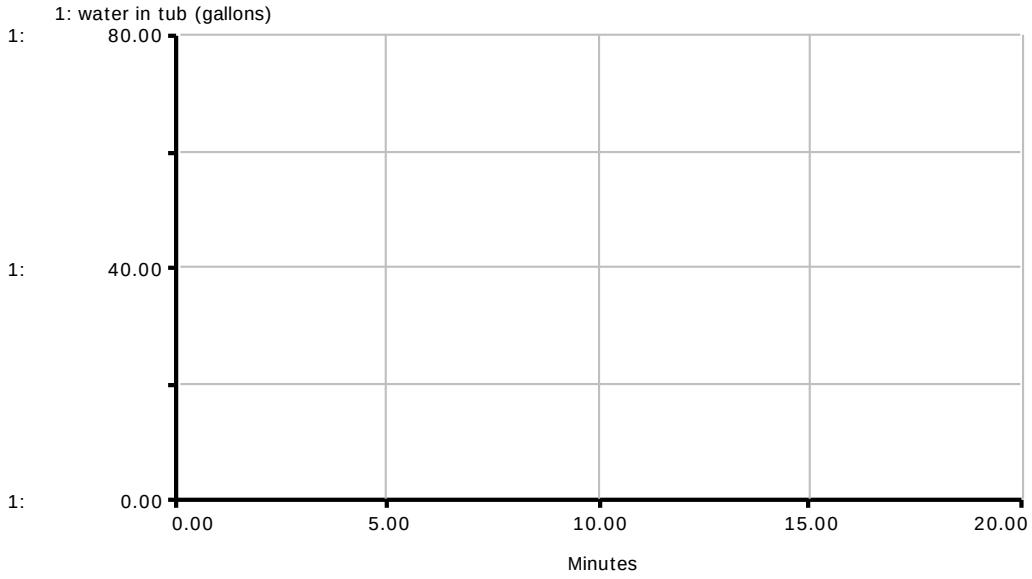
1. In this first exercise, the rate is +3. Using the blank graph pad below, graph the behavior of the system, assuming the initial value of the level is 0.



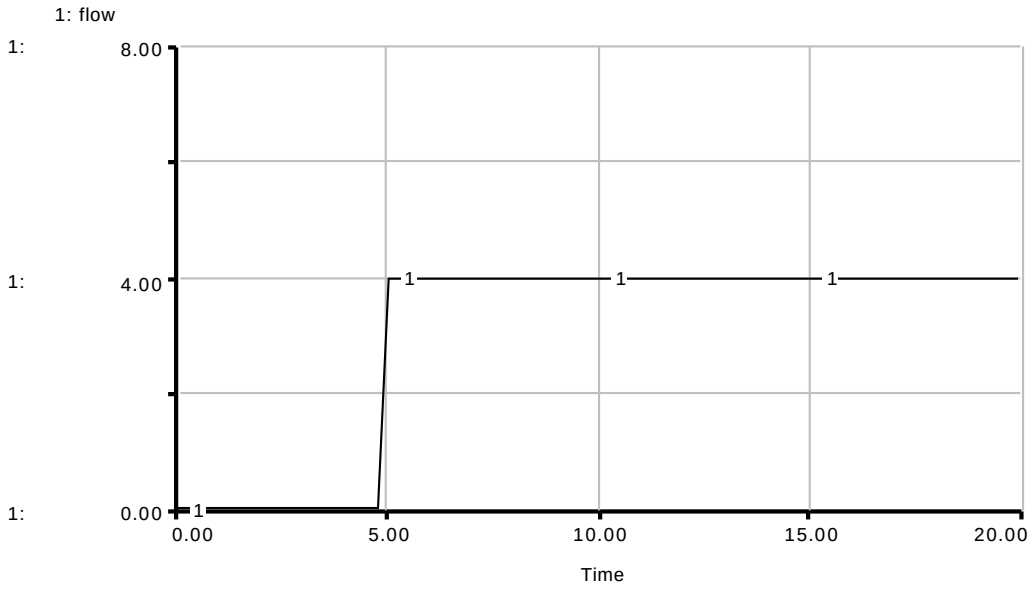


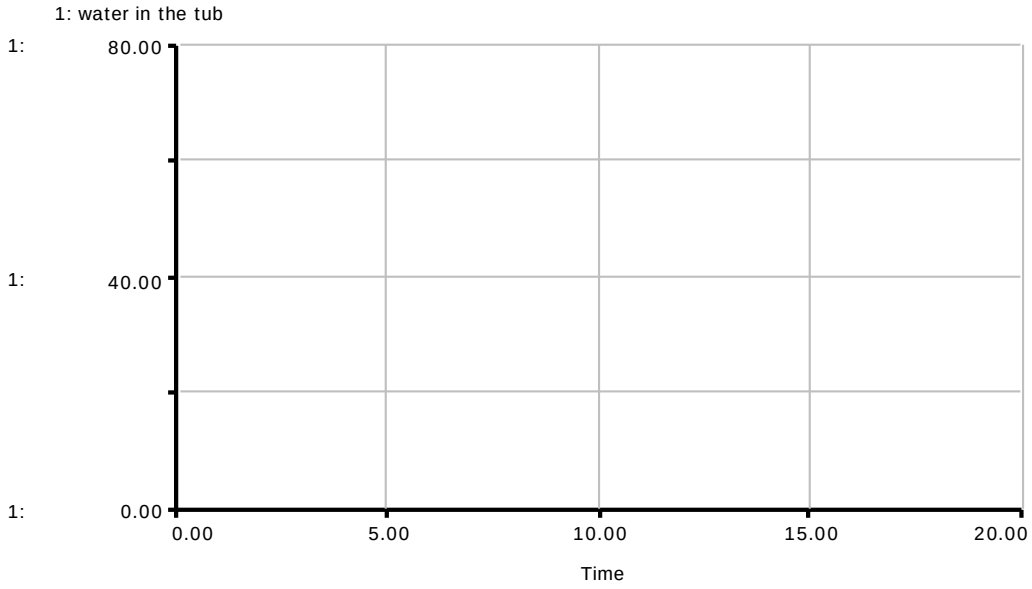
2. For this second exercise, assume there is no inflow and outflow = 2. Graph the behavior of the system below, assuming the initial value of the level is 40.



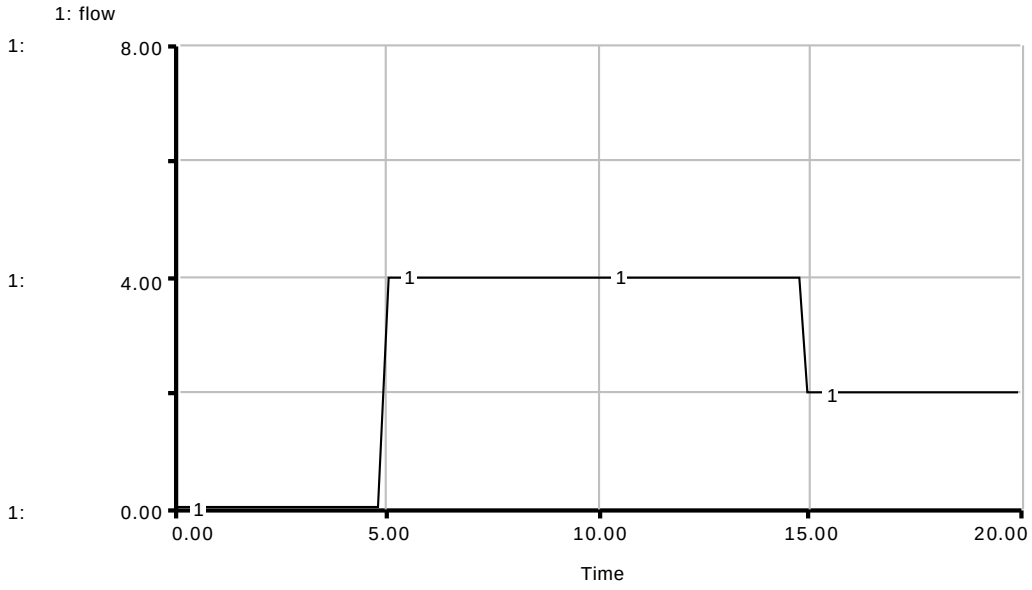


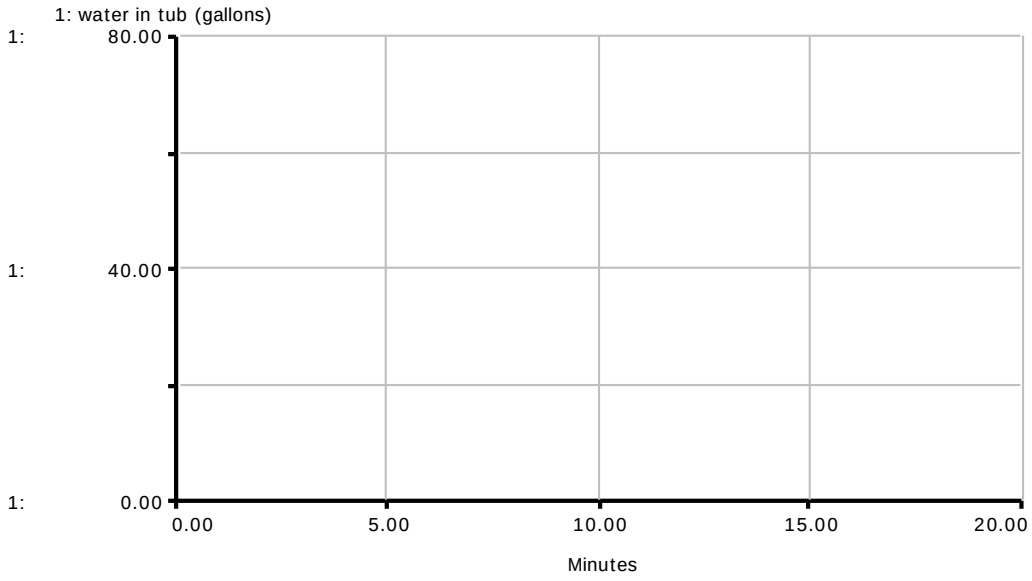
3. Now, let's try a step function. The flow in this exercise remains 0 until a step up to 4 at *Time* = 5. Assume the initial value of the stock is 0, and graph the predicted behavior below.



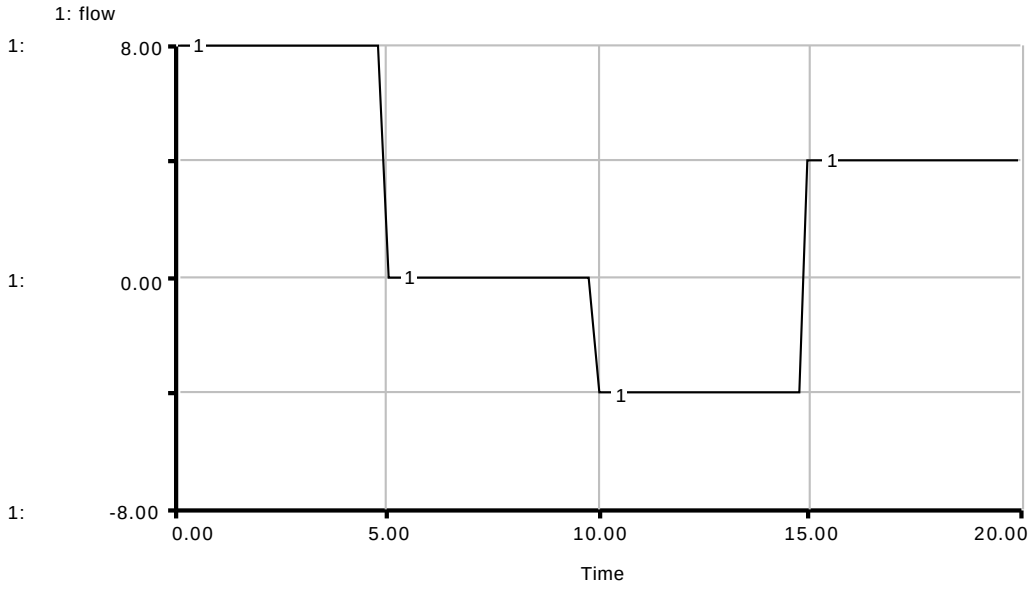


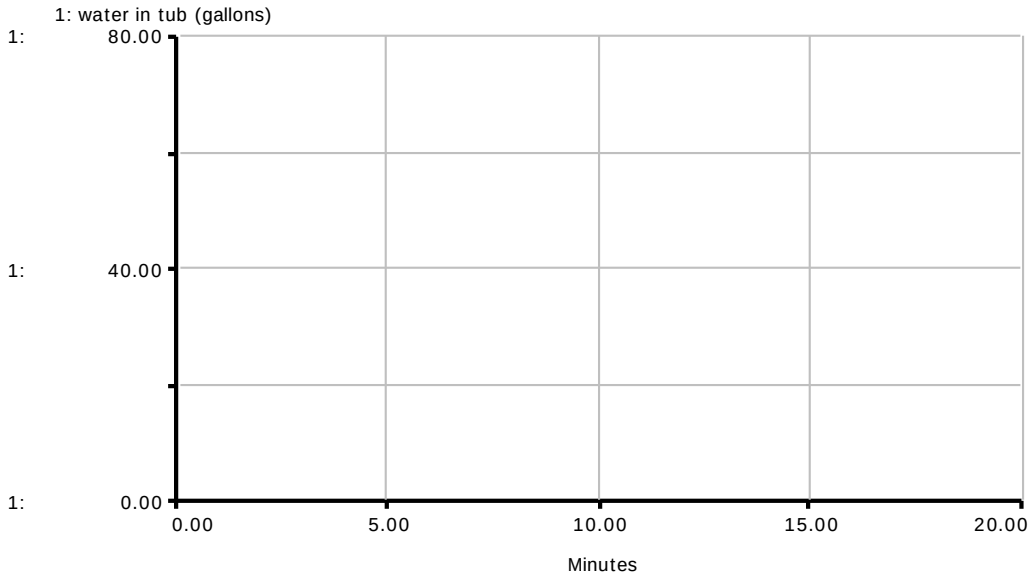
4. Let us try a more complex problem. We start with *flow* = 0, then step up to 4 at *Time*=5, then step down to 2 at *Time*=15. Assuming the initial value of the stock is 0, graph the result below.





5. Here is another one. It might look complicated, but if you break it up into small pieces, it is not difficult. The initial value of the net flow is +8, then it steps down to 0 at *Time* = 5, then down again to -4 at *Time* = 10, then up to +4 at *Time* = 15. Don't forget that the value of the net flow at a given time is equal to the slope of the stock.





The answers to the above exercises are in the Appendix.

4. Conclusion

So far we've looked at exogenous rates and how they affect the behavior of the stock. You should now feel comfortable using graphical integration method to predict the results of constant positive and negative rates as well as step function rates.

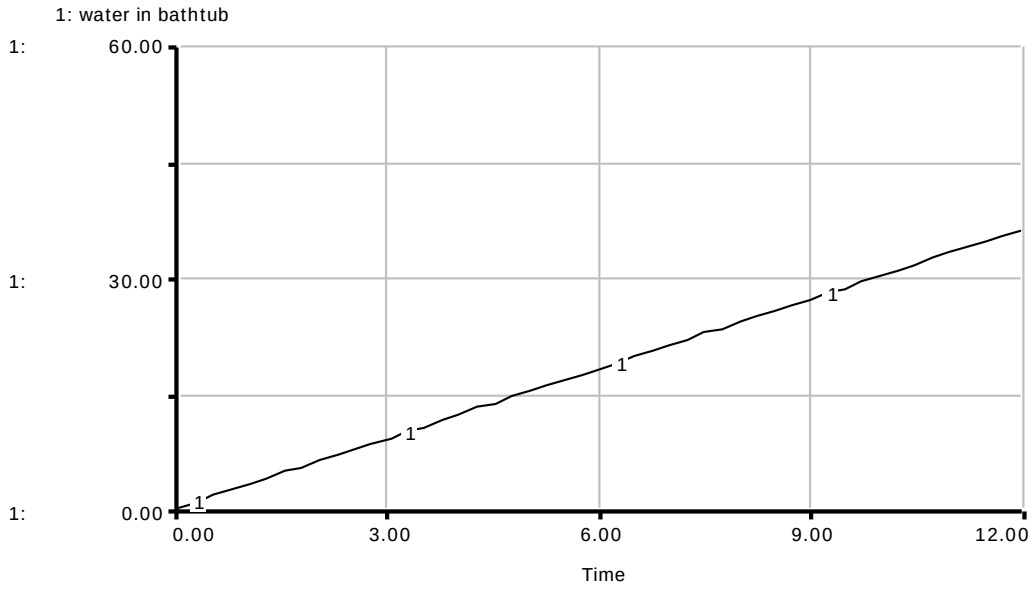
In the next paper in this series, you will go through similar steps, but with ramp functions and feedback systems. If you do not feel you have built a strong foundation of graphically integrating constant and step function rates, please go back and review this paper before going on to the next step.

Good luck!

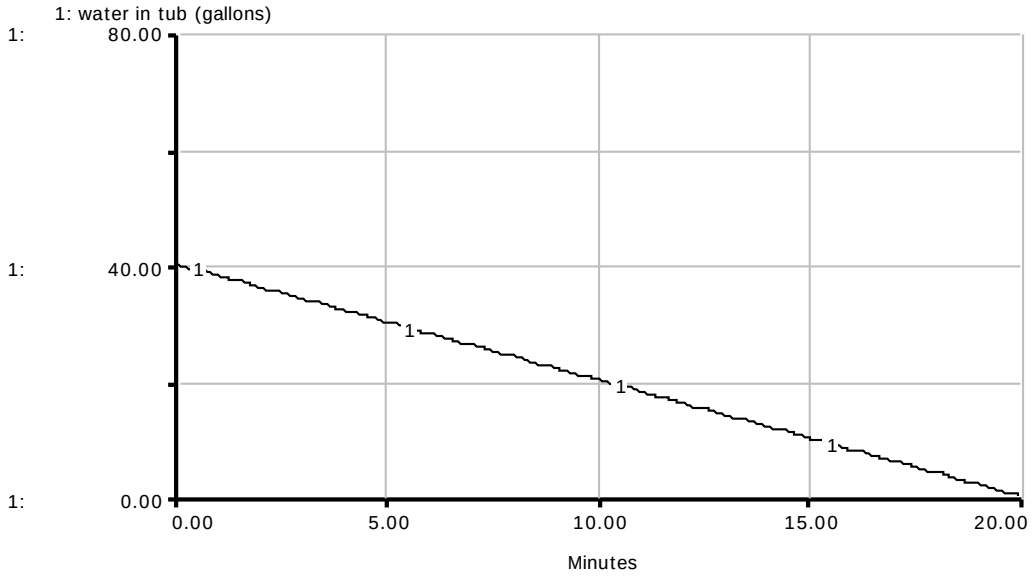
5. Appendix

Here are the answers to exercises.

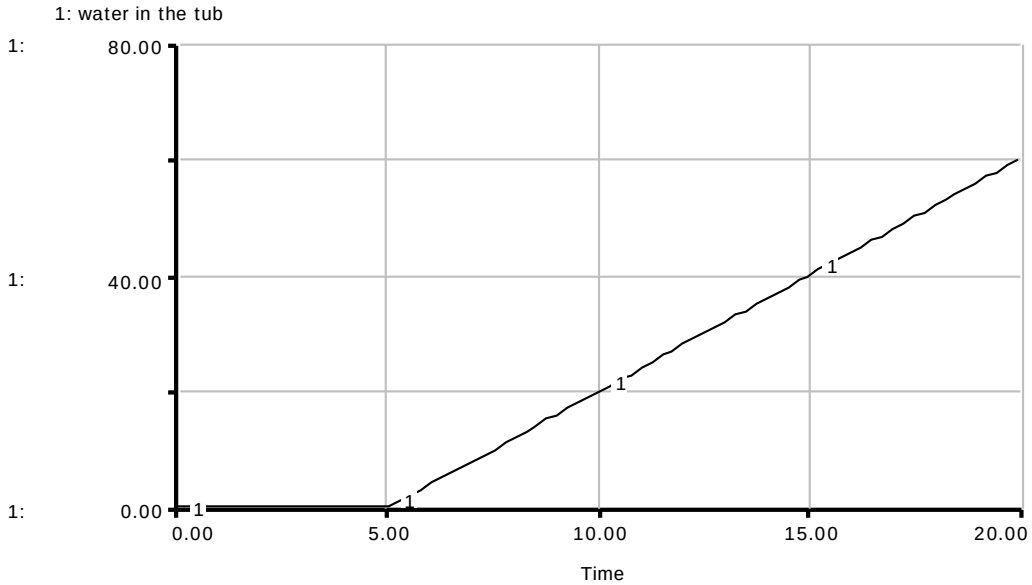
1. This first one is just a straight line with a slope of +3. Notice that the value of the constant rate was also +3.



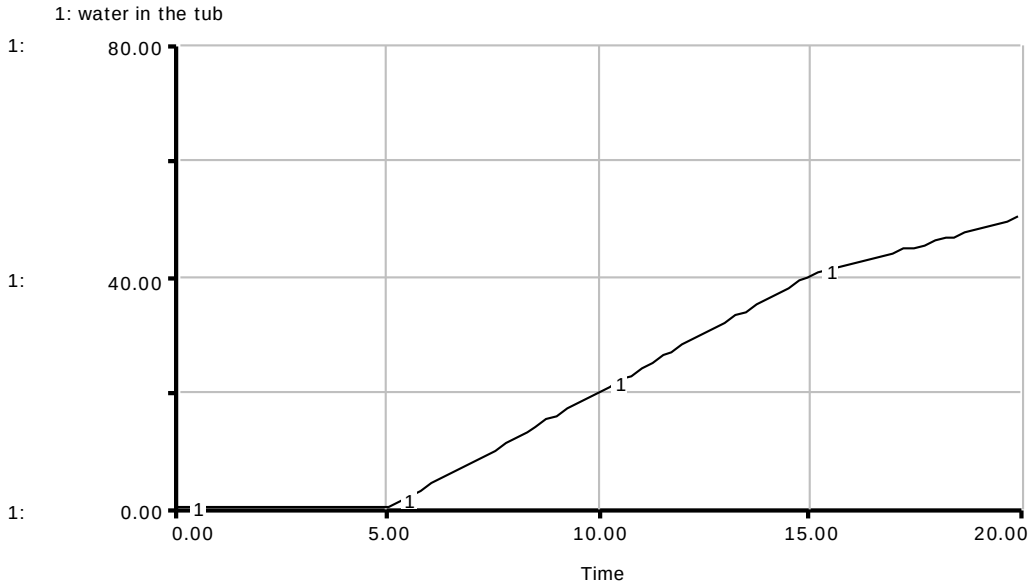
2. This is similar to exercise 1, but the slope is -2. We see that the tub is being drained.



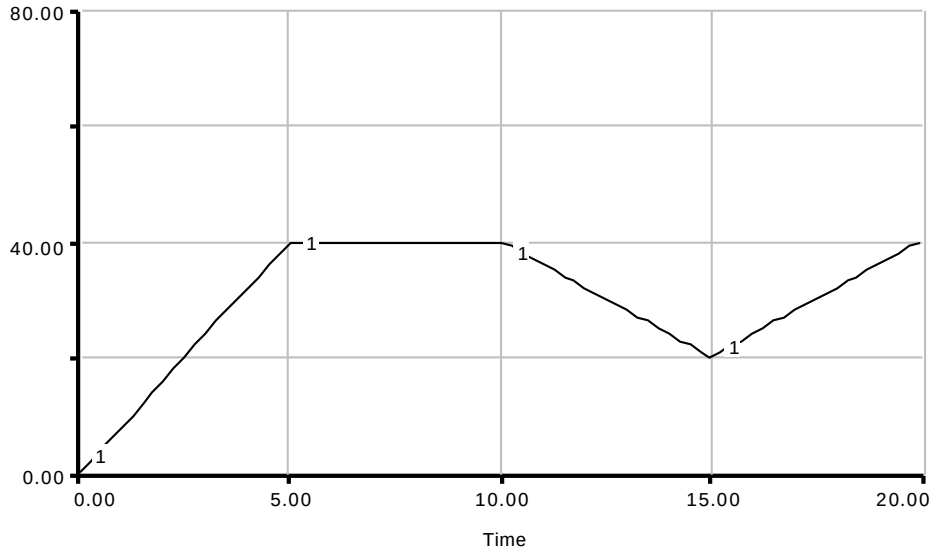
3. There is a delay in turning on the faucet. The water level remains at 0 until the value of the rate steps up to 4 at *Time* = 5.



4. This is the same as exercise 3 until *Time* = 15, at which point the slope changes to 2.



5. The initial rate is +8, so the value of the stock at *Time* = 5 is 8 units of water / unit time x 5 units of time = 40 units of water. The value of the net flow from *Time* = 5 to *Time* = 10 is 0, so the stock remains at 40. Then the slope drops down to -4, and then back up to +4.



D-4697-2

Formulating Models of Simple Systems

using

Vensim PLE

version 3.0B

Professor Nelson Repenning
System Dynamics Group
MIT Sloan School of Management
Cambridge, MA O2142

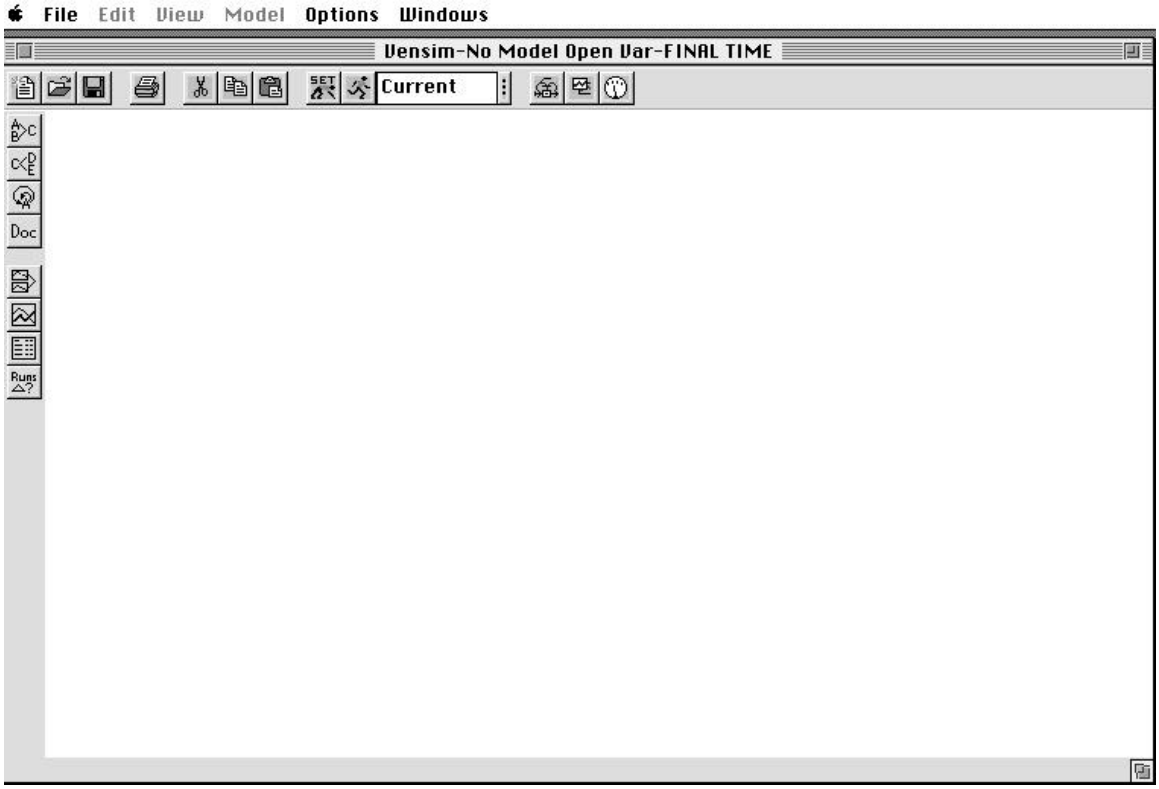
Edited by Laura Black, Farzana S. Mohamed, and students in the System Dynamics in Education Project, April 1998.

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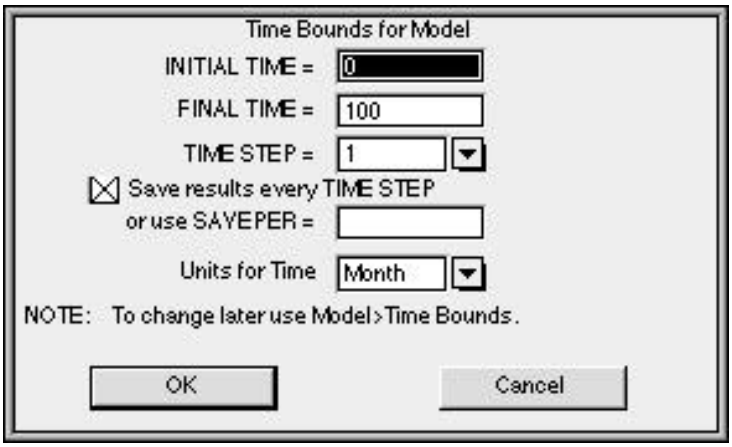
I. Introduction and Getting Started

The purpose of this tutorial is to help you develop some familiarity with building and analyzing system dynamics models using the Vensim PLE software. In order to become familiar with Vensim PLE, you are going to build a simple model of the federal deficit.

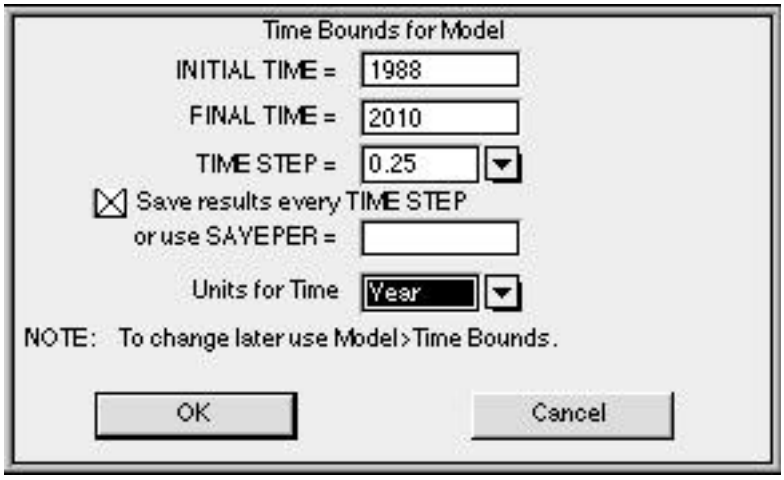
To begin you need to get Vensim PLE ready for modeling. This tutorial makes use of the Macintosh version on Vensim PLE; the IBM-Compatible version should work similarly, but some of the screens may look different. When you first open Vensim PLE on your computer, the screen should look like this:



To start working on a new model go to the **File** menu and select **New Model**. Vensim PLE will return the following dialog box:



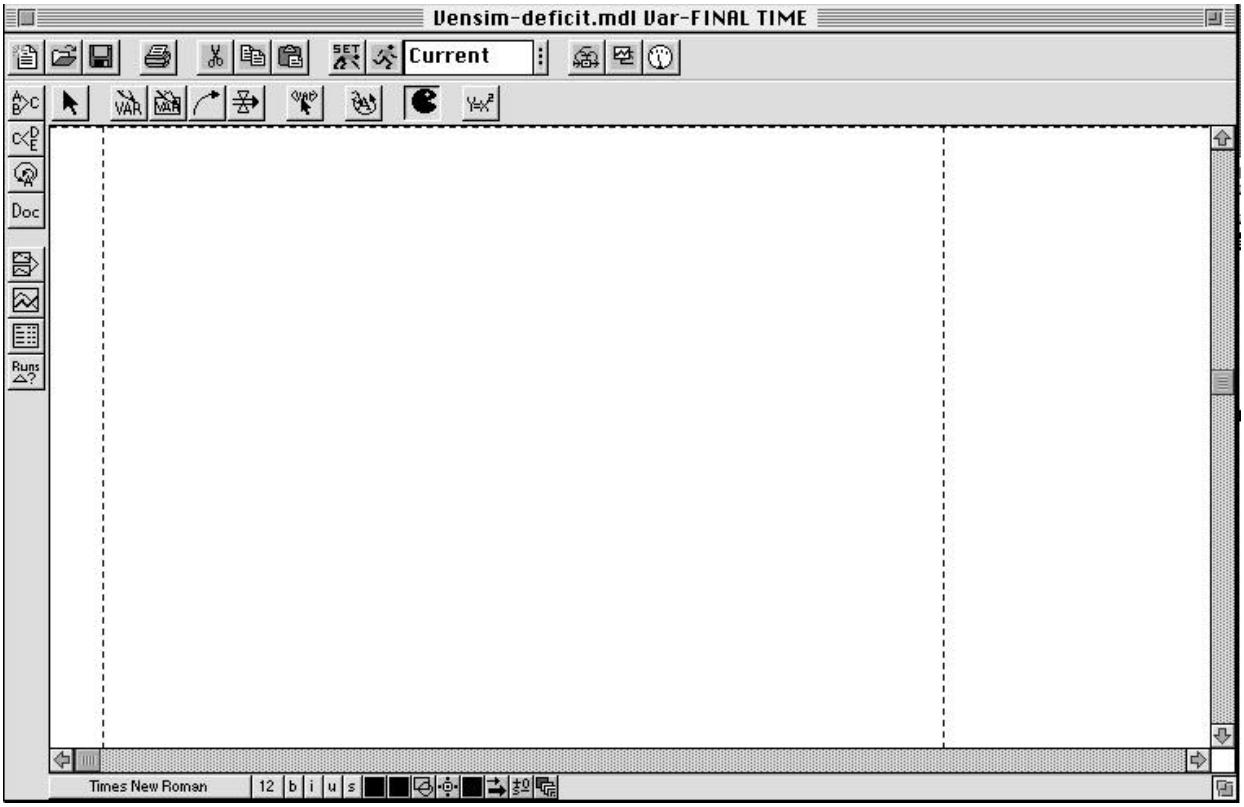
To begin your effort you must choose the time horizon of your model (when your simulation will start and finish), the appropriate time step (how accurately you wish to simulate your model), and the units of time. Start your model of the deficit in 1988 (enter 1988 in the **INITIAL TIME** box) and simulate it through the year 2010. Select a **time step** of 0.25 years. Finally, change the **units of time** from **Month** to **Year**. Your dialog box should now look like this:



Click on **OK** or hit return. To give your model a name, choose the **Save As...** command from the **File** menu and enter the desired name in the text field and click on OK. (Vensim PLE should automatically supply the **.mdl** extension. If you are working with a different version of Vensim and see a *Show all of type* option on the right side of the dialog box, make sure that the **.mdl Fmt Models** extension is selected. This allows Vensim PLE to save the model in a format that can be used by both Macintosh and IBM-compatible computers.)[□]

[□] Vensim saves every simulation run and custom graph you produce as a separate file. It supplies a **.vdf** extension for simulation runs. These files cannot be opened from outside the Vensim application; they can be opened from inside Vensim through the **Datasets / Simulate Model...** and **Control / Custom Graphs** dialog boxes.

Your screen should now look like this:



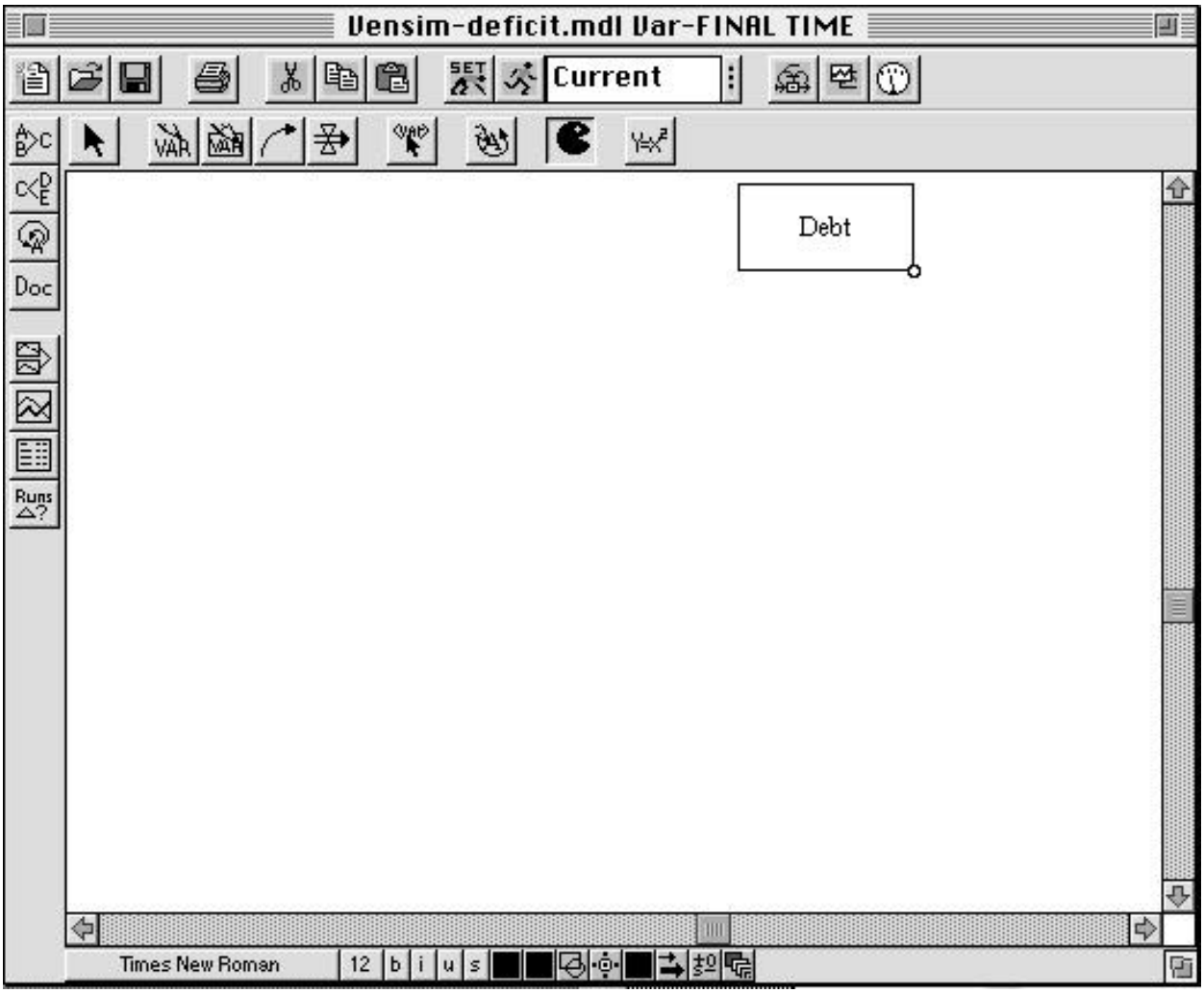
You are ready to start building your model.

II. Developing the Stock, Flow, and Feedback Structure

The Vensim PLE software is designed using the metaphor of a "work bench." The large blank area in the middle of the screen is your work area, where you actually develop and analyze your model. The different buttons on the border of the work area represent the different "tools" available as you work on your model. The upper toolbar consists of the Title Bar, a Menu, a Main Toolbar, and Sketch Tools. The Main Toolbar comprises two sets of tools: file operation tools that control standard file functions—opening, closing and saving files, printing, cutting, copying, and pasting—as well as simulation and graphing tools that will allow you to set up and run simulations, and set up display graphs. The sketch tools allow you to build in model components. The tools on the Status Bar (the bottom of the window) allow you to change the formatting of the diagram. The Analysis Tools on the left on the window are tools that you will use to analyze your model to understand its behavior. You will become familiar with many of these tools as you build the deficit model.

To begin, add a stock representing the outstanding federal debt to your model. Click on the  button (the one with the box in it) on the sketch tools bar and then click in the right center of the screen. You use this tool whenever you want to add a stock (also known as a level) variable to your model. Vensim PLE then returns an empty text box and a blinking cursor. Type the word **Debt** and then hit the return key.

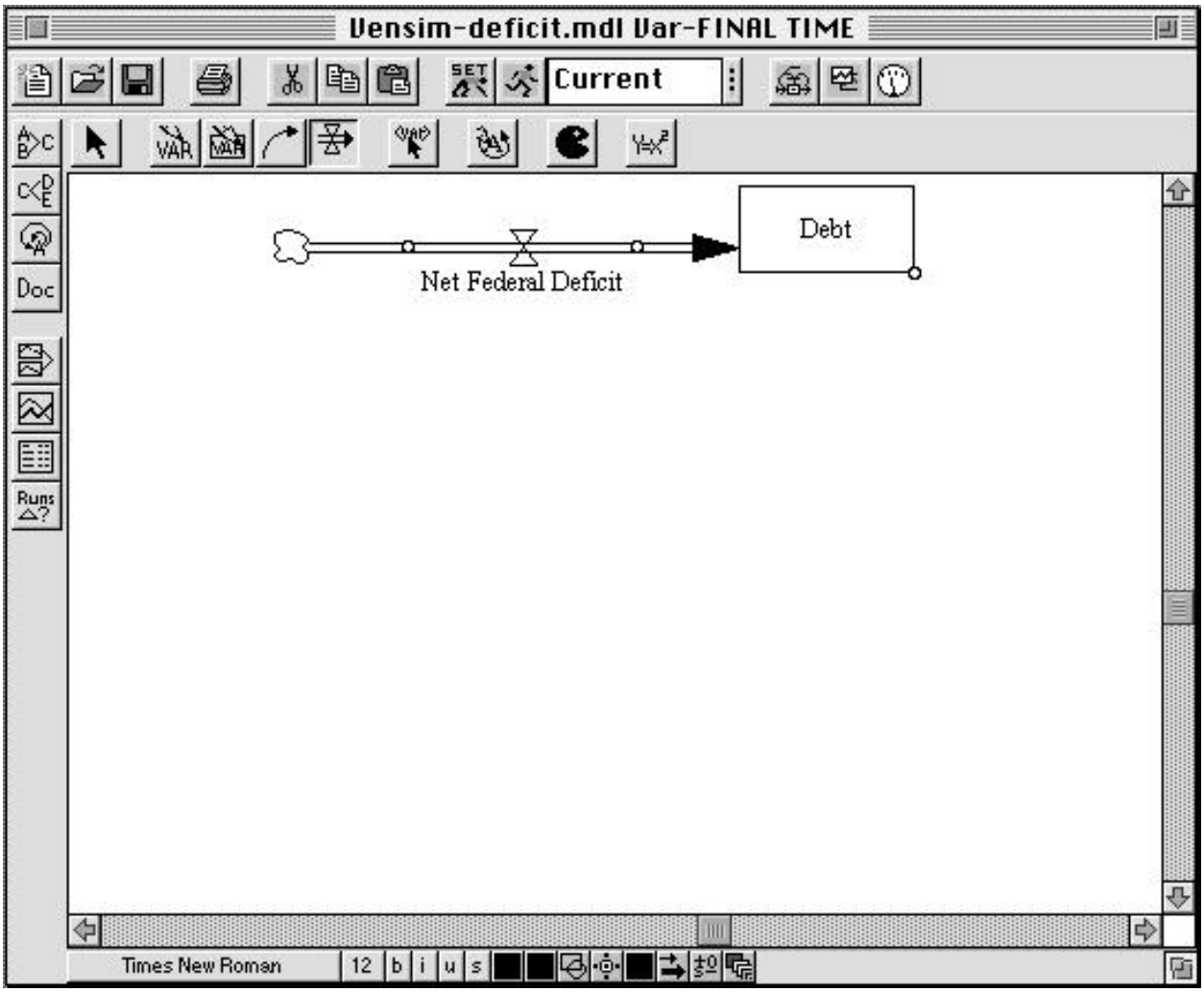
Your screen should now look like this:



You have just created the first variable in your model, the stock of money that constitutes the federal debt.

Now, add the inflow to the stock of **Debt**. Click on the  button on the sketch tools bar. Now, click and release once to the left of the **Debt** stock, move the cursor so that it sits *inside* the **Debt** stock, (the stock should "blacken" if you do this correctly) and click and release again. Vensim PLE then gives you an empty text box and a blinking cursor. Type **Net Federal Deficit** and hit the return key.

Your screen should now look like this:



Note: The ⟲ icon (which is supposed to resemble a cloud) represents the boundary of your model. In this case the "cloud" on the left side of the flow signifies that you do not, at the moment, care about where the deficit comes from — you are not keeping track of the stock that is being drained by the deficit flow. You do care, however, where the deficit goes: hence, you are accumulating the deficit in the **Debt** stock. If your deficit flow has "clouds" on both ends, then you have not hooked the flow to the stock correctly. To fix this problem, click on the tool and then click on the flow valve. This action will remove the flow from the model and let you start over again.

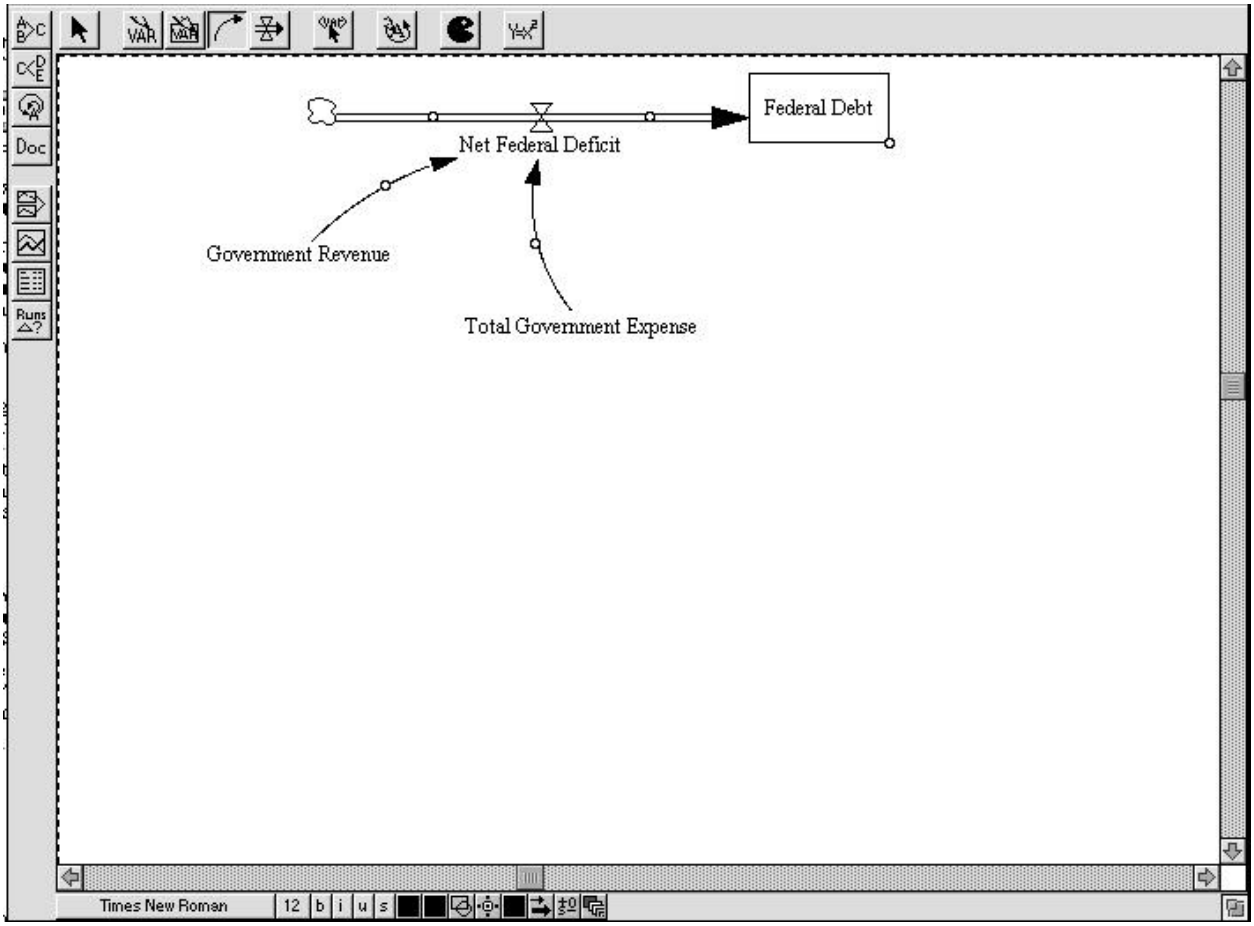
You have now created the flow, **Net Federal Deficit**, which increases the stock of **Debt**. At this point you may you wish to change the name of the stock variable from **Debt** to **Federal Debt**. Click on the VAR button (the one without the box in it) on the sketch tool bar and then click on the **Debt** stock. Vensim PLE gives you a text box with **Debt** already written. You can now edit the text in any way you choose. Click in front of the **D,** add the word **Federal**, and hit the return key.

Now you need to create the variables needed to determine the **Net Federal Deficit**. Assume the **Net Federal Deficit** is determined by two variables, **Government Revenues** and **Total Government Expenses**. Click on the  button again and add these two variables to your diagram (click and release on the work area, type the variable name, hit return, and then click and release again, on a different part of the work area, to add the next variable). Unlike the previous two variables — **Federal Debt**, a stock, and **Net Federal Deficit**, a flow — the new variables are not attached to a valve or a box. These variables are called *auxiliary variables*.

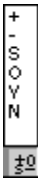
To show pictorially that the **Net Federal Deficit** is determined by **Government Revenues** and **Total Government Expenses**, we connect them with causal arrows. First click on the  button to select the causal arrow tool. Now, click and release on the variable **Government Revenue** and then click and release again on **Net Federal Deficit**. Do the same for **Total Government Expenses**. Make sure your causal arrows actually end on the words **Net Federal Deficit**. They should *not* be attached to the cloud, the stock, or directly to the valve.

You can delete arrows using the  tool.

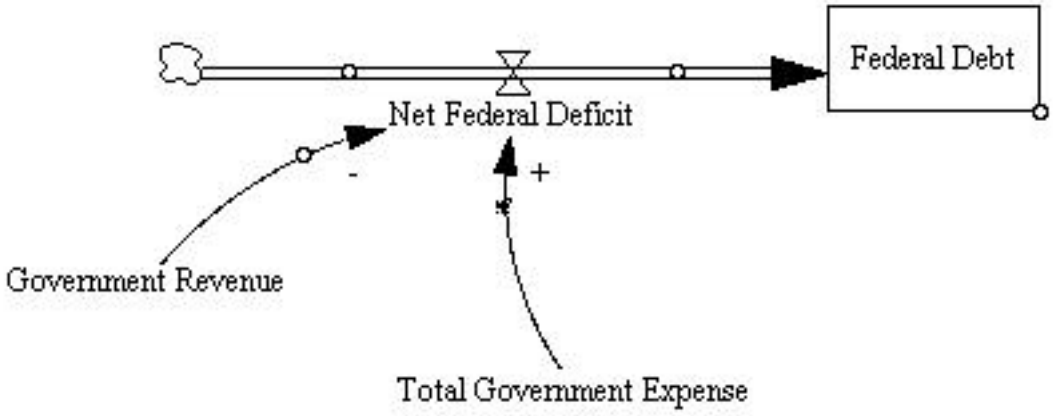
Clicking on the  button allows you to select the variables you have created and move them to different places on the screen. To move variables, place the arrow cursor over the variable you wish to move, hold down the mouse button, move the variable to the desired place, and then release the mouse button. You can also select the "handles" of the causal arrows (the small circles in the middle of the arrow) and change the curvature of the arrow. Arrange your variables and arrows so that your diagram looks approximately like this:



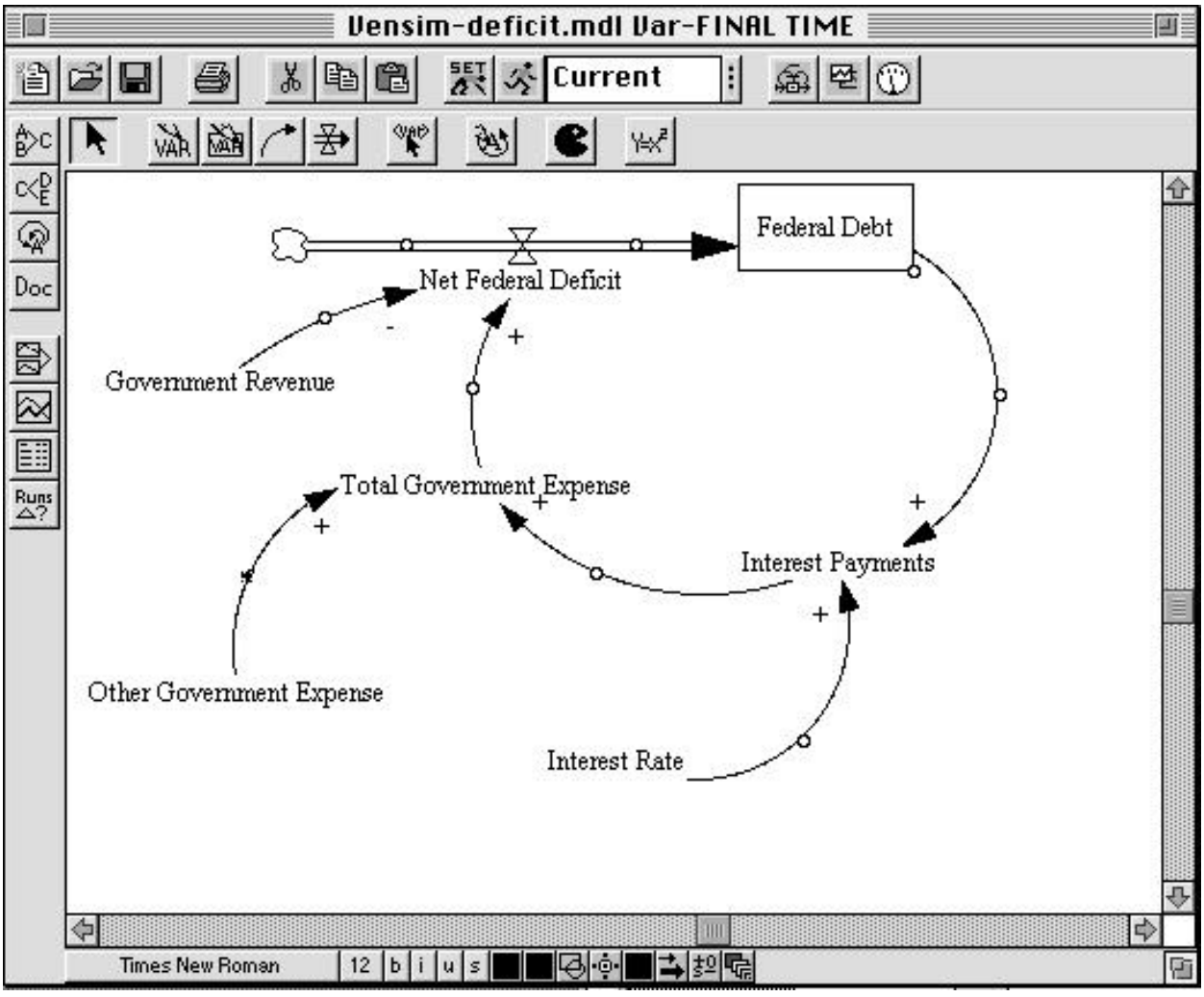
Now, you may want to update your diagram by labeling the arrows to show that **Government Revenue** and **Total Government Expense** affect the **Net Federal Deficit** in different ways. Specifically, an *increase* in revenue causes the deficit to *decrease*, while an *increase* in expenses causes the deficit to *increase.* To do this, first click on the  button. Then select the "handle" of the arrow you wish to label by clicking and releasing on the small circle in the middle of the arrow (the handle darkens when selected). Now, with the handle selected, click and release the  button on the bottom horizontal toolbar. You then see a pop-up menu that looks like this:



Click and release on the desired label, and it will show up in the diagram. Label your two causal arrows so your diagram looks like this:

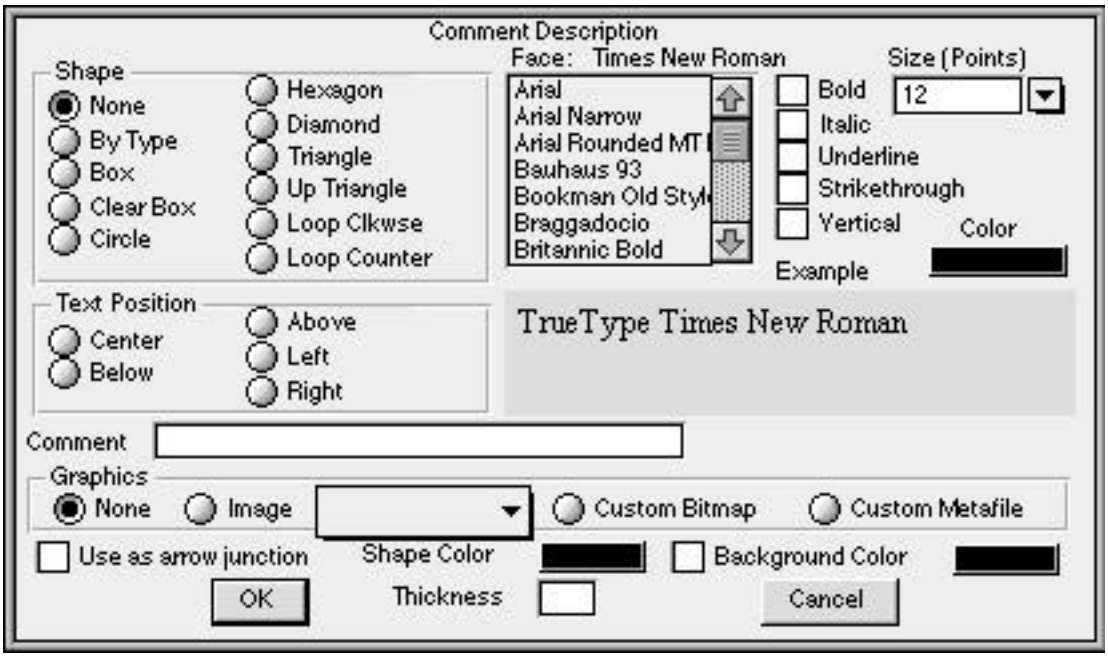


Now, using the same steps discussed above, complete the stock, flow and feedback so your diagram looks like this:

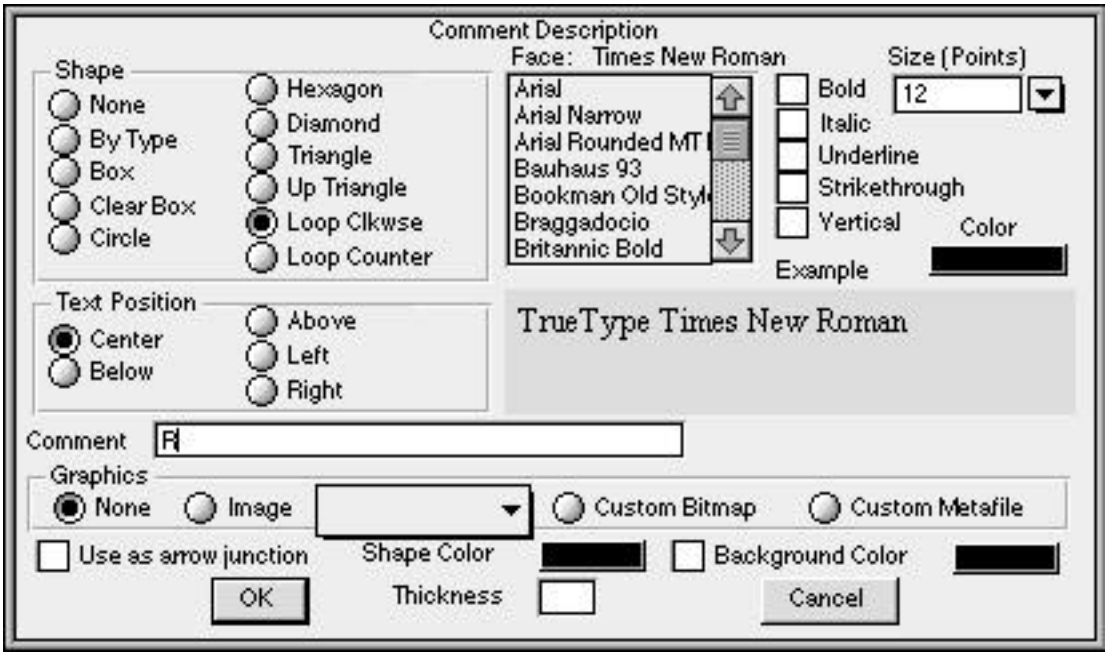


You may want to slide the handle of each arrow close to its arrowhead, so each label is clearly associated with its causal arrow.

Finally, you may wish to label the positive feedback loop you have just created. Click on the  button and then click in the center of the feedback loop. You can use this tool to create comments that, while having no structural use, can greatly help someone else to understand your model diagram. After clicking in the center of the loop, you should see the following dialog box:

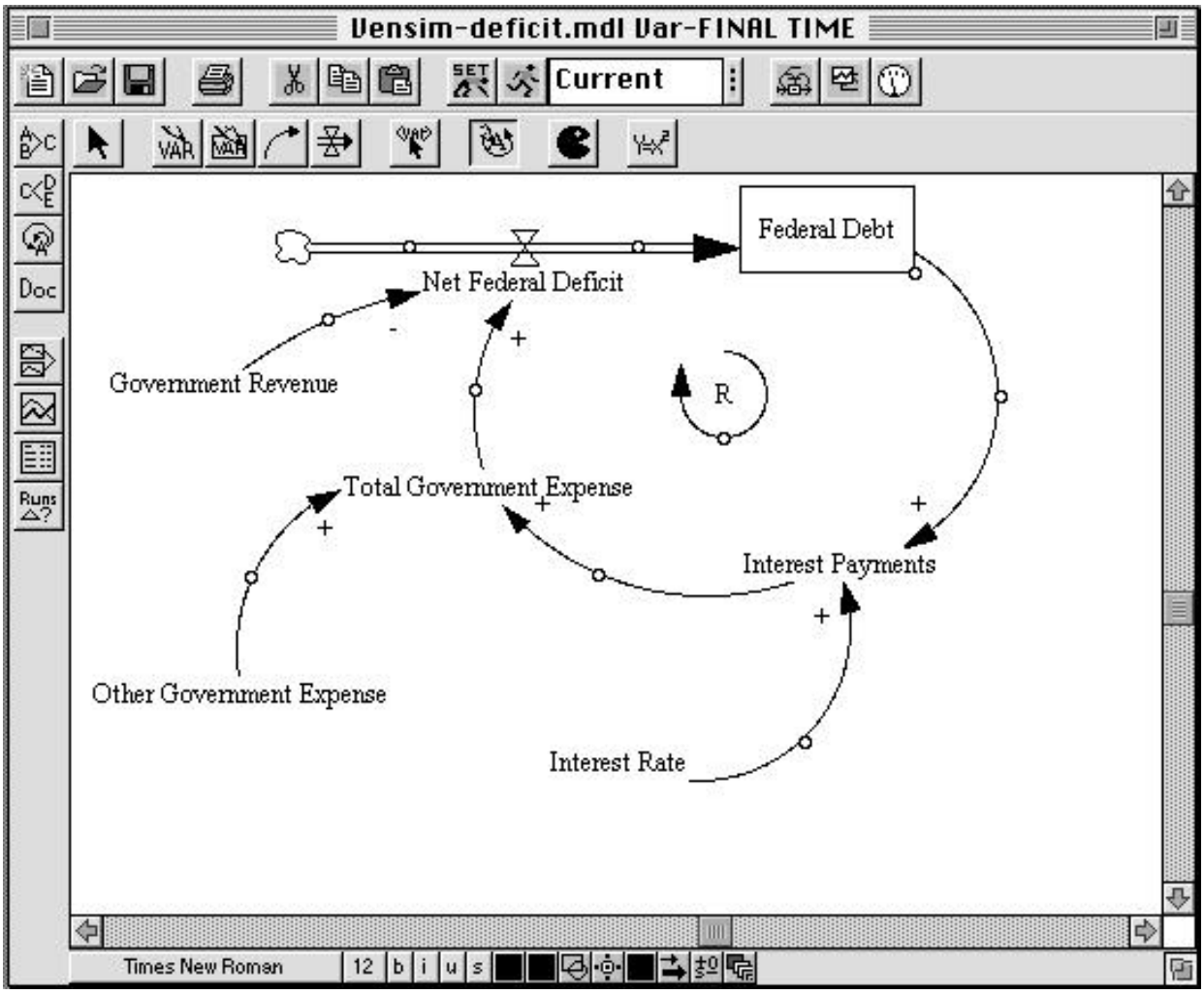


Click on the **Loop Clkwse** button in the **Shape** box; click on **Center** in the **Text Position** box; and type **R**, for **reinforcing**, in the **Comment** box. You may also type **+** or **P** to denote a positive feedback, also known as a reinforcing, loop. Your screen should now look like this:



Click on the **OK** button or hit return.

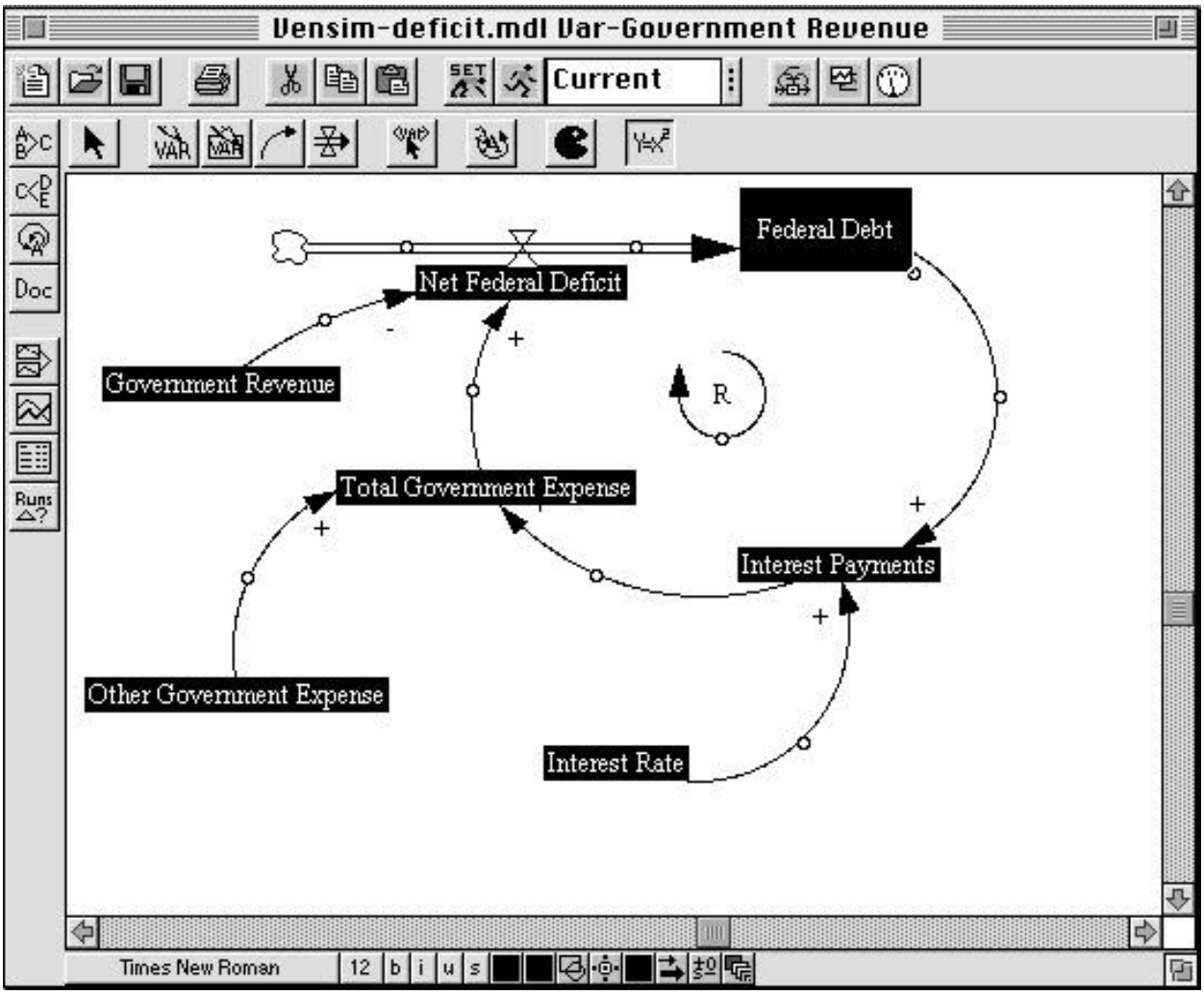
Your screen should now appear as:



III. Specifying Equations for Your Model

Now that you have developed a complete stock, flow, and feedback representation of the deficit, you need to write equations for each of the variables. Equation formulation is a critical step in the process of model building and is a key part of the process of developing a rigorous understanding of the problem at hand.

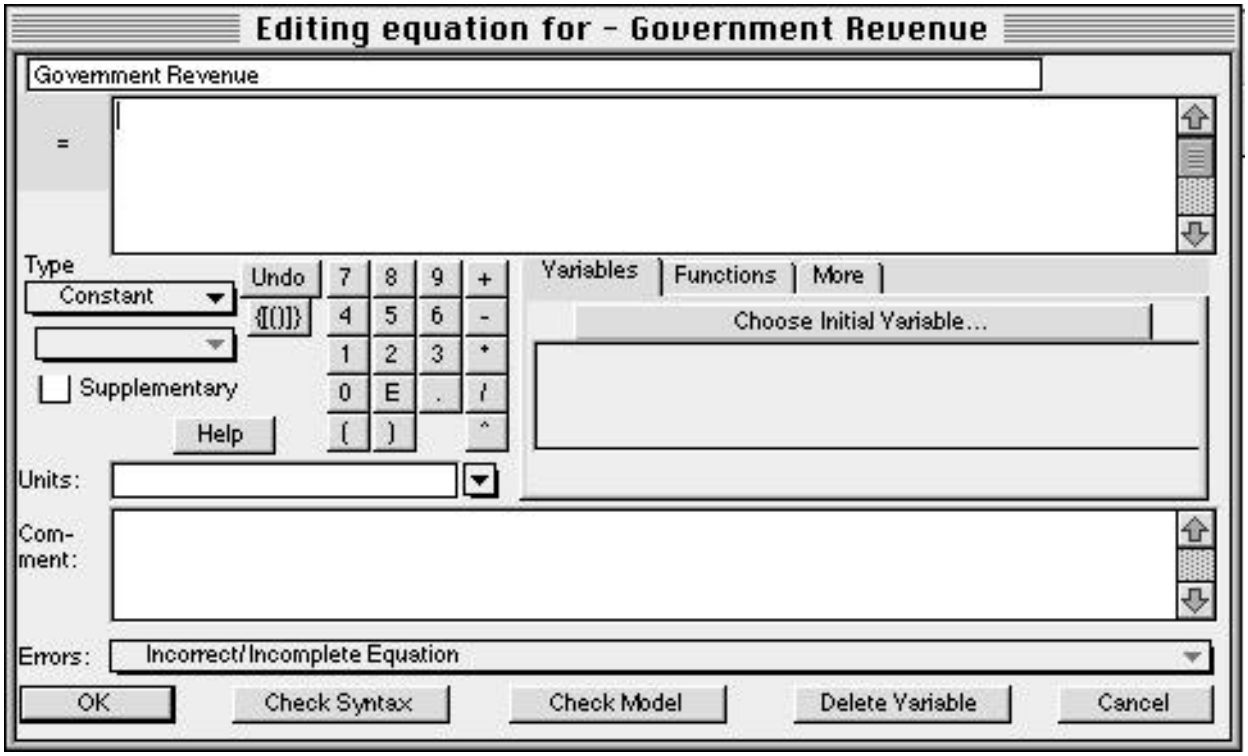
To begin writing equations, click on the  button on the sketch tool bar. The variables in your diagram become highlighted.



A highlighted variable indicates that the equation for that variable is incomplete.

Variables in system dynamics models are classified as either exogenous or endogenous. Exogenous variables are those that are not part of a feedback loop, while endogenous variables are members of at least one feedback loop. Your deficit model has three exogenous variables—**Government Revenue**, **Other Government Expenses**, and the **Interest Rate**—and four endogenous variables—**Interest Payments**, **Total Government Expense**, **Net Federal Deficit**, and the **Federal Debt**.

Start by writing the equations for the exogenous variables. To begin, click on the highlighted variable **Government Revenue**. You then see the following dialog box:



Good modeling practice requires that each equation in a model have three elements: the equation itself, specified units of measure, and complete documentation. You enter the equation in the box to the right of the **=** sign. You enter the unit of measure in the text field to the right of the word **Units**. Equation documentation or "comment" is entered in the box to the right of the word **Comment**.

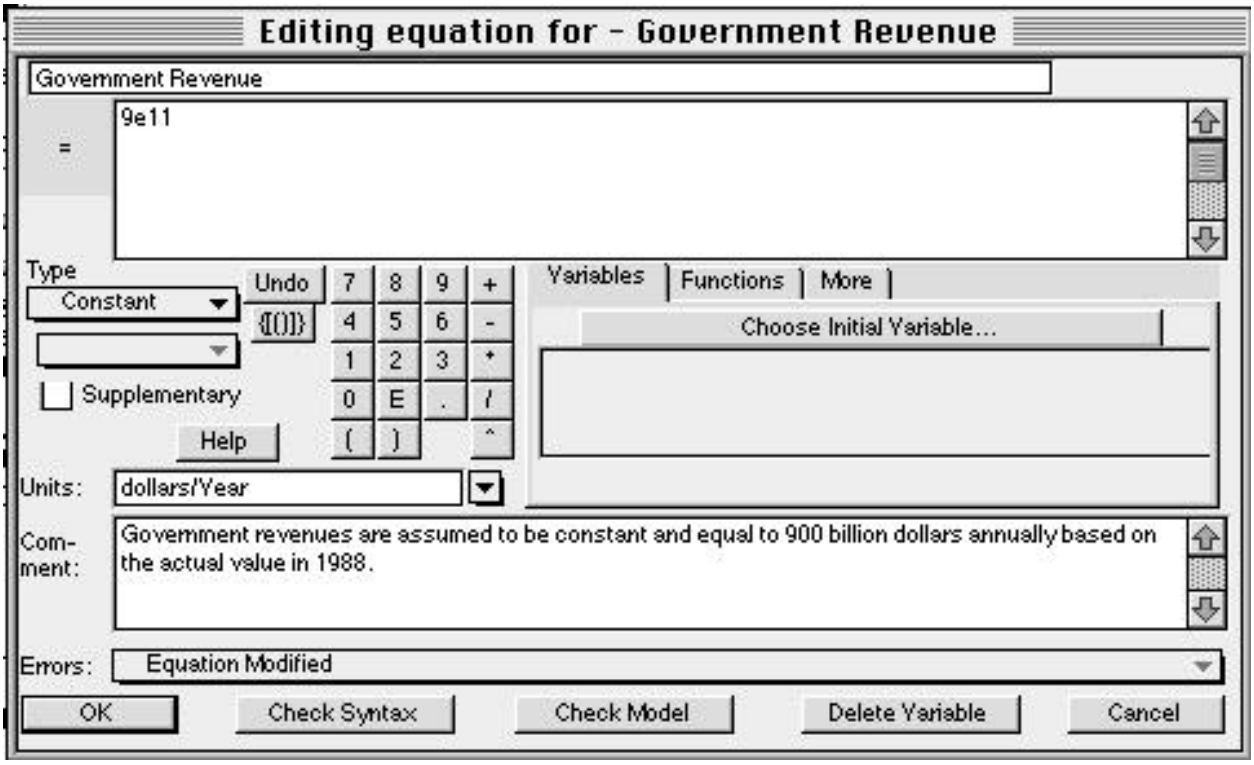
To write an equation for Government Revenue, click in the box to the right of the **=** sign. Assume that government revenue is constant, so that all you need to do is enter the appropriate number for government revenue. In 1988, government revenue was about 900 billion dollars annually, so type 900000000000 in the box. Alternatively, you can write 9e11, which is Vensim PLE shorthand for $9 * 10^{11}$.

Now, fill in the units. Revenue is a flow variable, so the appropriate unit of measure for this equation is dollars/time unit. Because you already chose to run the model in time steps of 1 year, the appropriate unit is dollars/year. Type **dollars/year** in the units field. (The next time you specify the units for a variable in this model, dollars/year will appear in the units pull-down menu. You can click on the arrowhead to the right of the units field to see units already specified for other variables in the model, and then use the mouse to select the units from that list when appropriate.)

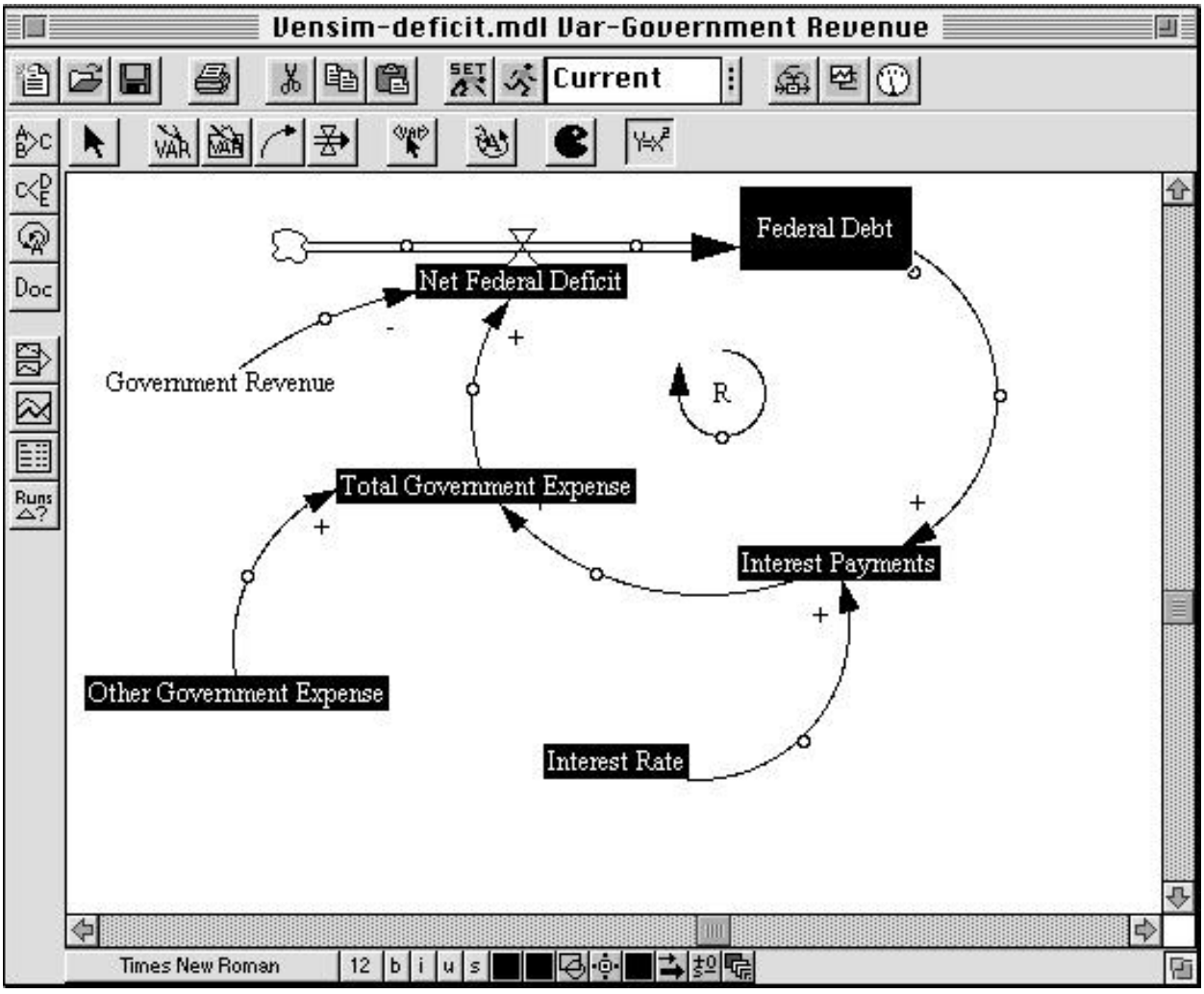
Finally, provide a description of this equation in the comment field. A good comment will be brief, but it will also give the reader the logic behind the equation as well as state the key assumptions. For example, one might write for this equation:

> **Government revenues are assumed to be constant and equal to 900 billion dollars annually based on the actual value in 1988.**

Your dialog box should now look like this:



Click on **OK** or hit return and your diagram will look like this:



Government Revenue is no longer highlighted because you have just specified its equation.

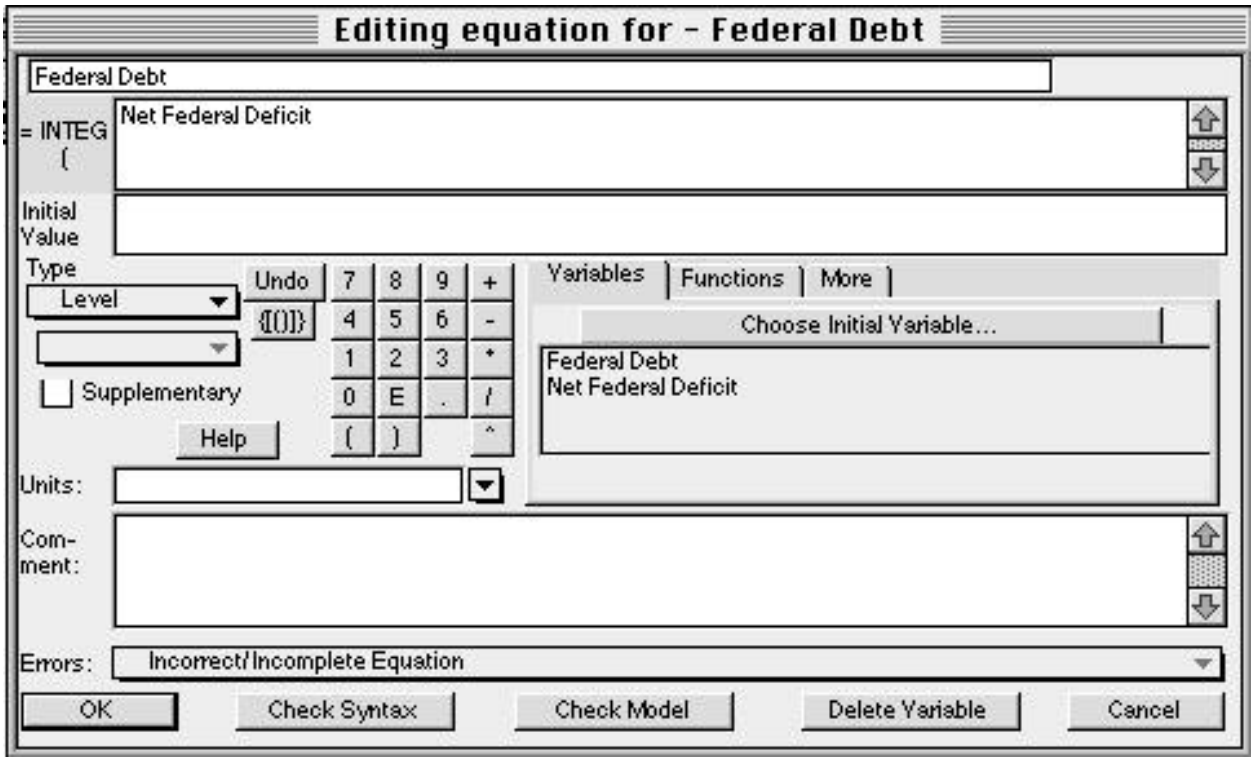
Following the process above, write equations for the two other exogenous variables, **Interest Rate** and **Other Government Expenses**. Use the following information:

- Government expenses, excluding interest on the debt, were approximately 900 billion dollars in 1988.
- The interest rate paid on the national debt in 1988 was around 7%/year.

Now that the equations for the exogenous variables are formulated, turn your attention to the endogenous variables. Writing equations for the stocks and the flows is a little different, so let's do an example of each. First we formulate the equation for the stock, **Federal Debt.**

Again, click on the  button on the sketch tool bar and then click on the stock, **Federal Debt**.

The following dialog box will be displayed:



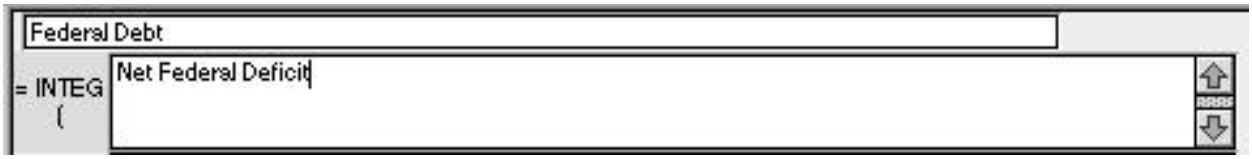
Unlike flows and constants, a stock requires that an additional element be specified in its formulation; after you specify the equation, you need to select an initial or starting value.

You enter the equation for the stock in the box to the right of the word **Integ**. **Integ** stands for "integrate" and simply means that the stock at any moment in time is equal to the sum of all the inflows minus the sum of all the outflows plus the initial value.

When you created the stock, flow, and feedback diagram, you connected the flow **Net Federal Deficit** to the stock **Federal Debt**. Vensim PLE captures this stock-flow dependency by providing a list of the required **Variables** to the stock **Federal Debt** on the right side of the equation dialog box. (The variable we are formulating, **Federal Debt**, itself also appears in the **Variables** box, but we focus on the input **Net Federal Deficit.** In general, you will never want to have the same variable on both the left and right sides of an equation.)

Because the model diagram shows the flow **Net Federal Deficit** feeding into the stock **Federal Debt**, Vensim has anticipated that the flow is an input to the stock equation and has placed the **Net Federal Deficit** variable name in the box to the right of **Integ**. If this is not the case in your version of Vensim PLE, then simply click in the box to the right of the **Integ** and then click on the variable **Net Federal Deficit** in the **Variables** box to write the equation for the change in **Federal Debt**. (Note: If **Net Federal Deficit** is *not* in the **Variables** box, then your model diagram is incorrect and needs to be changed—make sure the flow is attached to the stock).

The **Integ** box should now look like this:



Below the **Integ** box is the **Initial Value** box. Here you enter the initial condition or starting point for the stock. In 1988, the outstanding federal debt was approximately 2.5 trillion dollars, so enter 2500000000000 in the initial value box (alternatively you can write 2.5e12, which is Vensim PLE shorthand for 2.5×10^{12}). The **Initial Value** box should look like this:

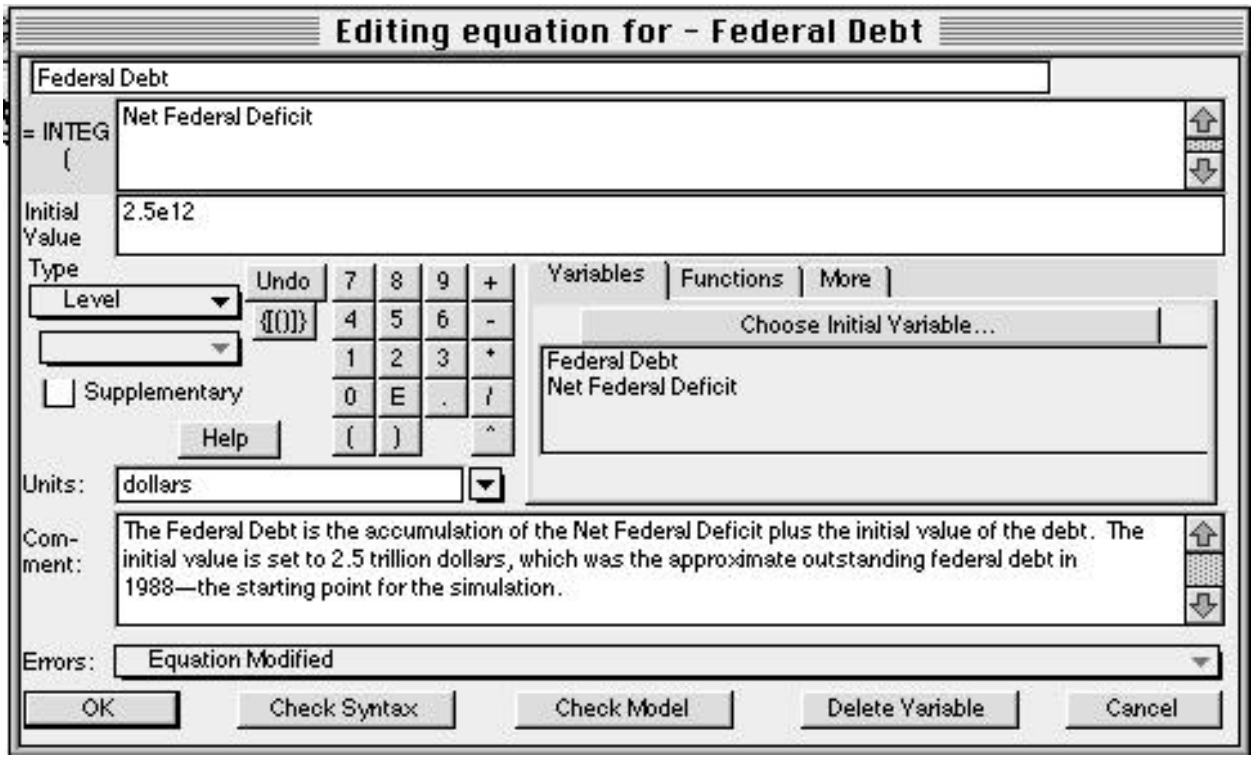


Now the equation specification for the **Federal Debt** stock is complete. Your equation indicates that the federal debt is simply the accumulation of the **Net Federal Deficit** since 1988 added to the initial value.

You still need to specify the unit of measure and document your equation in the comment field. The units should be fairly straightforward. The **Federal Debt** is a stock and its units are dollars. Useful comments briefly explain the structure of the equation and highlight the key assumptions made. A sample comment for **Federal Debt** is:

> **The Federal Debt is the accumulation of the Net Federal Deficit plus the initial value of the debt. The initial value is set to 2.5 trillion dollars, which was the approximate outstanding federal debt in 1988—the starting point for this simulation.**

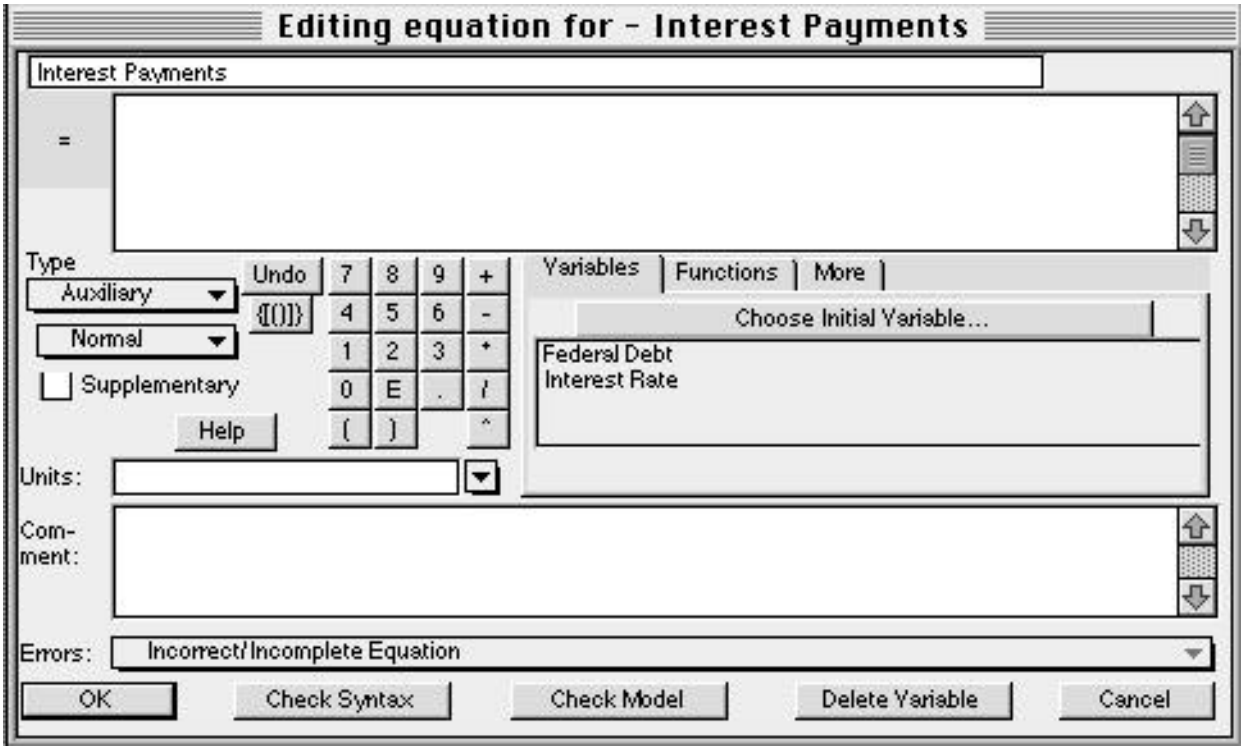
Your dialog box should now look like this:



Click on **OK** or press return.

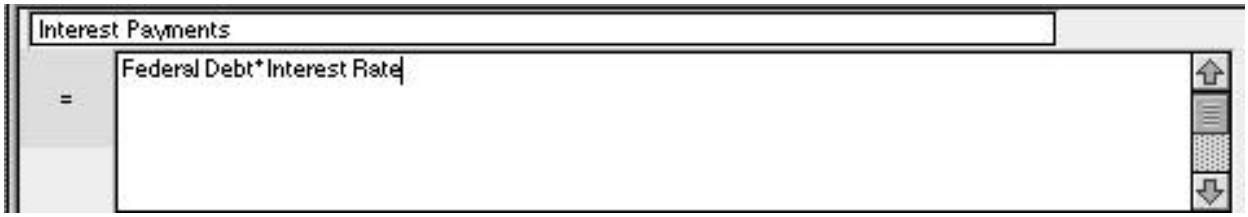
Now you need to specify the equations for the auxiliary variables and the flow.

Using the  tool on the sketch tool bar, click on the **Interest Payments** variable. You should see a dialog box that looks like this:



This box is identical to those used to specify the exogenous variables, but, in this case, there are two other variables in the **Variables** box; you are required to use these variables in the equation. When you developed the stock, flow, and feedback diagram, you drew causal arrows connecting the variable **Federal Debt** and constant **Interest Rate** to the variable **Interest Payments**. Vensim PLE has conveniently recognized this fact and has provided a list of the required inputs to your equation based on the picture you have already created. In fact, if you try to write your equation without using the two required variables, Vensim PLE will give you an error message.

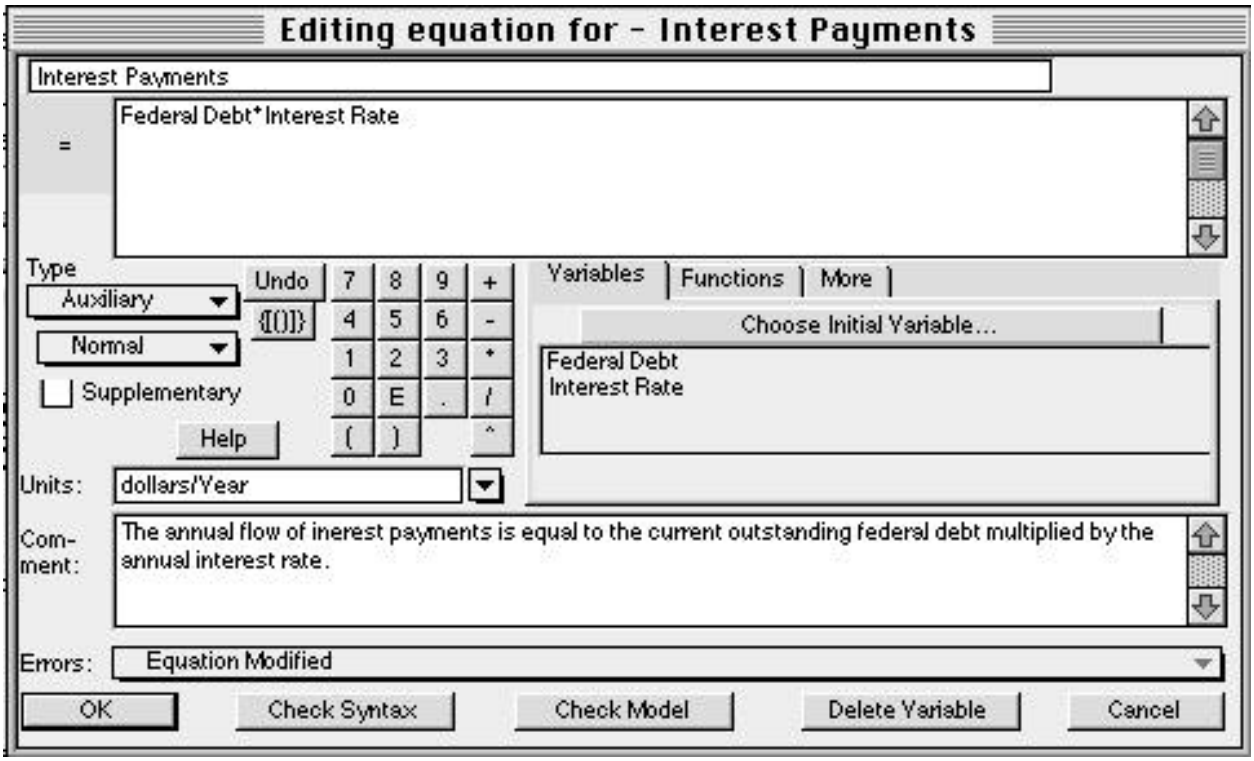
The rate of interest payment is simply equal to the current debt stock multiplied by the interest rate. To enter this equation, first click on the **Federal Debt** variable in the **Variables** box. It now appears in the equation box. Now type * (alternatively you can click on the  button), and then click on the **Interest Rate** variable in the **Variables** box. Your equation box should now look like this:



To complete the equation, you need to specify the units, dollars/year, and document your equation in the comment field. An appropriate comment might look like the following:

> **The annual flow of interest payments is equal to the current outstanding federal debt multiplied by the annual interest rate.**

The dialog box for the variable **Interest Payments** should now look like this:



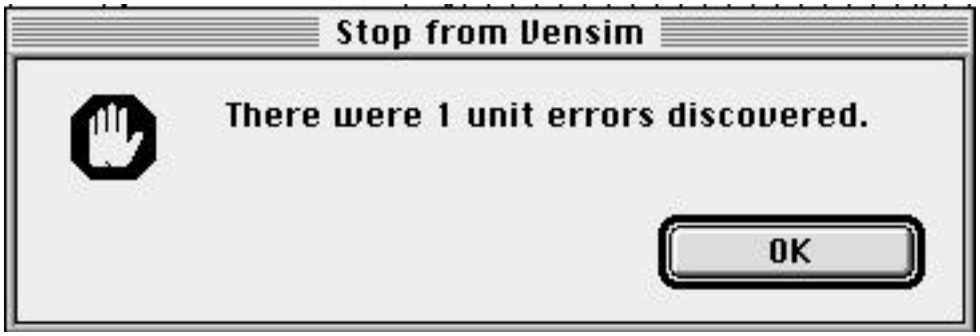
Following a similar process to the one outlined above, you should now be able to complete your model.

IV. Using the Model Structure Analysis Tools

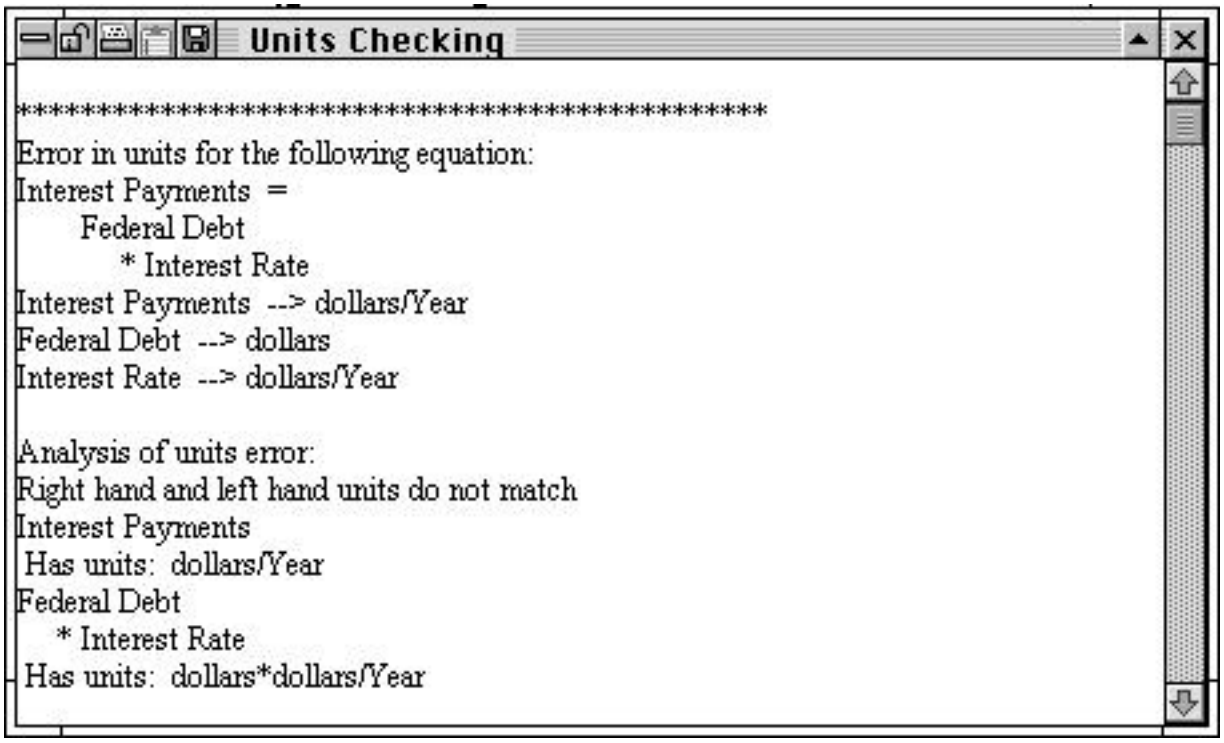
Vensim PLE provides five tools for analyzing and understanding the structure of your model.

By far the most important of these is the unit-checking tool.

An important feature of any system dynamics equation is *dimensional consistency*, which is just a fancy way of saying that the units of measure must be the same on both the left and right sides of the equation. As an example, suppose you had chosen the units of the **Federal Debt** stock to be dollars and the units of the **Interest Rate** to be dollars/year. If so, then pressing the *apple* key and *u* (alternatively, you could select the **Model** menu, then select **Units Check**) simultaneously would yield the following message:



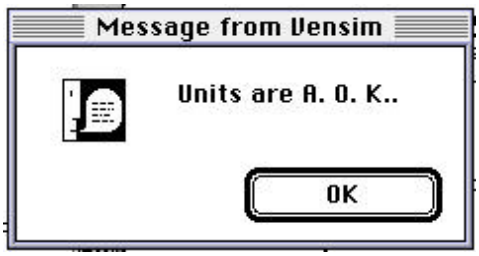
Followed by:



The problem is that, in this example, the equation for **Interest Payments** is not dimensionally consistent: the right and left sides of the equation have different units. The flow **Interest Payments** is measured in dollars per year. The **Federal Debt**, because it is a stock, is measured

in dollars. Multiplying **Federal Debt** by something with units in dollars/year results in a quantity that has units in dollars^2/year—hence the error.

The cause of the problem is that the unit of measure for **Interest Rate** is incorrect. The interest rate is not measured in dollars per year. An easy way to think about this fact is to recognize that the interest rate really has nothing to do with dollars. It could easily apply to any other currency or any other type of measurement unit. In fact, the interest rate has no unit of measure; it is dimensionless. Although it has no unit of measure, it is, nevertheless, time-dependent; an annual interest rate is not equivalent to a monthly one. As a result, the appropriate unit of measure for **Interest Rate** is 1/year. If you enter 1/year into the unit field of the interest rate variable and simultaneously press the *apple* key and u, you should receive the following message:



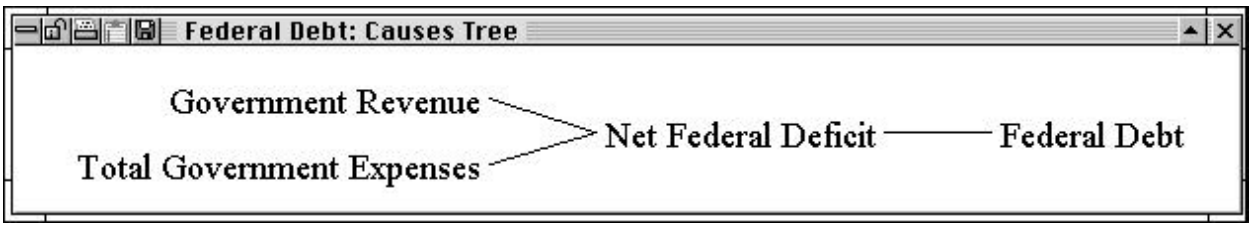
The units in your model now balance.

In this example, the unit-checking tool identified an incorrect assumption in a common mental model of the interest rate. Dimensional consistency is an important feature of system dynamics models, and Vensim PLE's unit-checking feature often helps you to identify serious flaws in both your understanding of the system under consideration and your resulting model formulations. Always make sure that the units in your model balance!

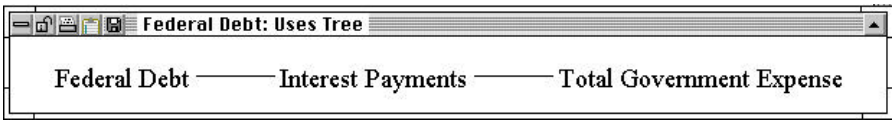
The other analysis tools that Vensim PLE provides can also be useful. The  and the  buttons create "causes" and "uses" trees for a variable. To use these tools, you need to first "select" a variable. To select a variable, first click on the  button and then double-click on the variable you wish to select. You can tell what variable is selected by looking at the top border of the Vensim PLE window. If you select the variable **Federal Debt,** the top border will look like this:



Having selected **Federal Debt**, clicking on the two "causes" and "uses" buttons in sequence gives you:



and



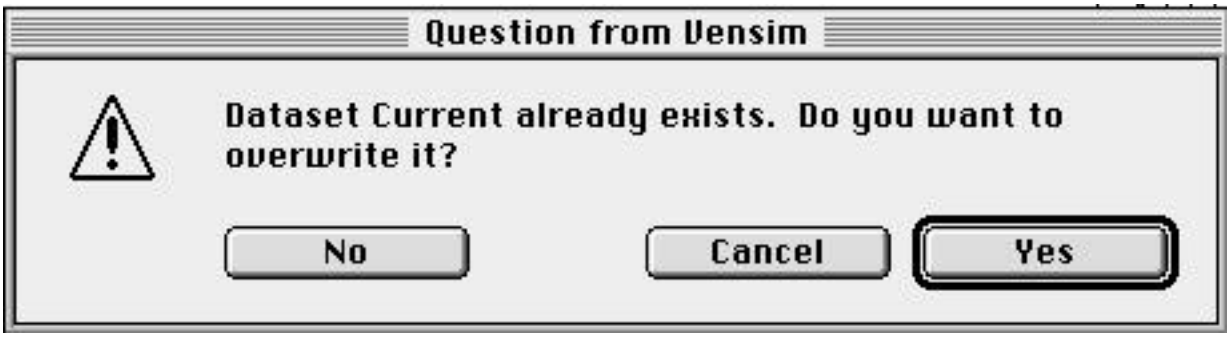
The 

button on the analysis tool bar provides you with a complete listing of the equations in your model. The  tool on the analysis tool bar identifies all the feedback loops of which the selected variable is a member.

V. Simulating Your Model

Vensim PLE also has tools to help you analyze the behavior of your model. Before analyzing the behavior, however, you must actually simulate the model so that you have some behavior to analyze.

To run a simulation, you first need to click on the running man  icon on the top toolbar. Vensim may display the following dialog box.



Clicking on yes will overwrite the "current" dataset displayed in the box to the right of the running man icon. Selecting "No" will allow you to create a different dataset. It is helpful to choose names that suggest some idea of what is being tested rather than simply using name like SIM1, SIM2, etc. Because this run is the base case run for your model, you might choose to call the run BASE.*

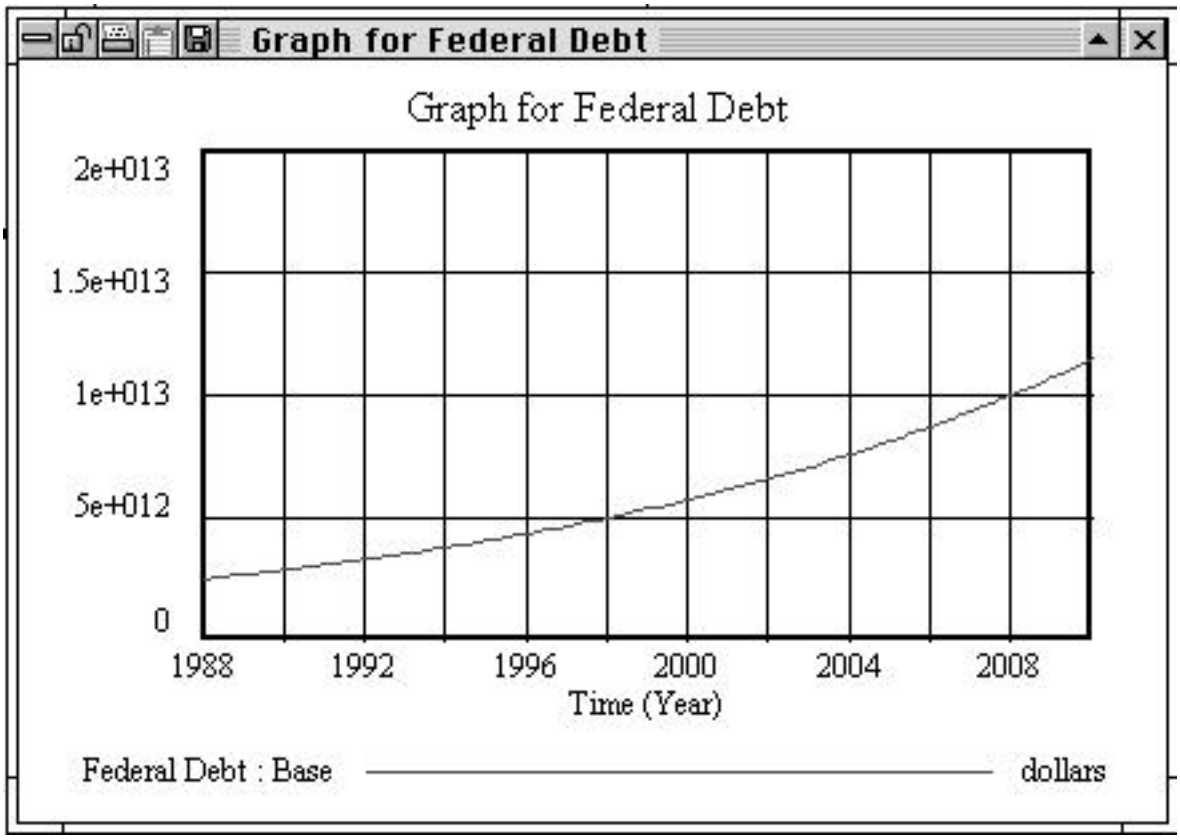
Click on **No**, type in BASE as your new dataset name, click on **Save** or hit return, and your model will start simulating.

Once the simulation run is completed, you can look at the results of your simulation. Vensim PLE provides many tools with which to view simulation output. The most basic, and often the most useful, of these tools is the strip graph. To create a graph of the **Federal Debt**, first click on the  tool and then double-click on the stock to select the variable **Federal Debt**.

To see a strip graph, click on the  button on the analysis tool bar.

* Advanced Tip: Vensim PLE also offers you the choice of two numerical integration methods, Euler and Runge-Kutta 4. Runge-Kutta 4 is a more accurate integration method, but it is also more computationally intensive. Generally it is better to use the Euler method and only change if you believe you are seeing integration error.

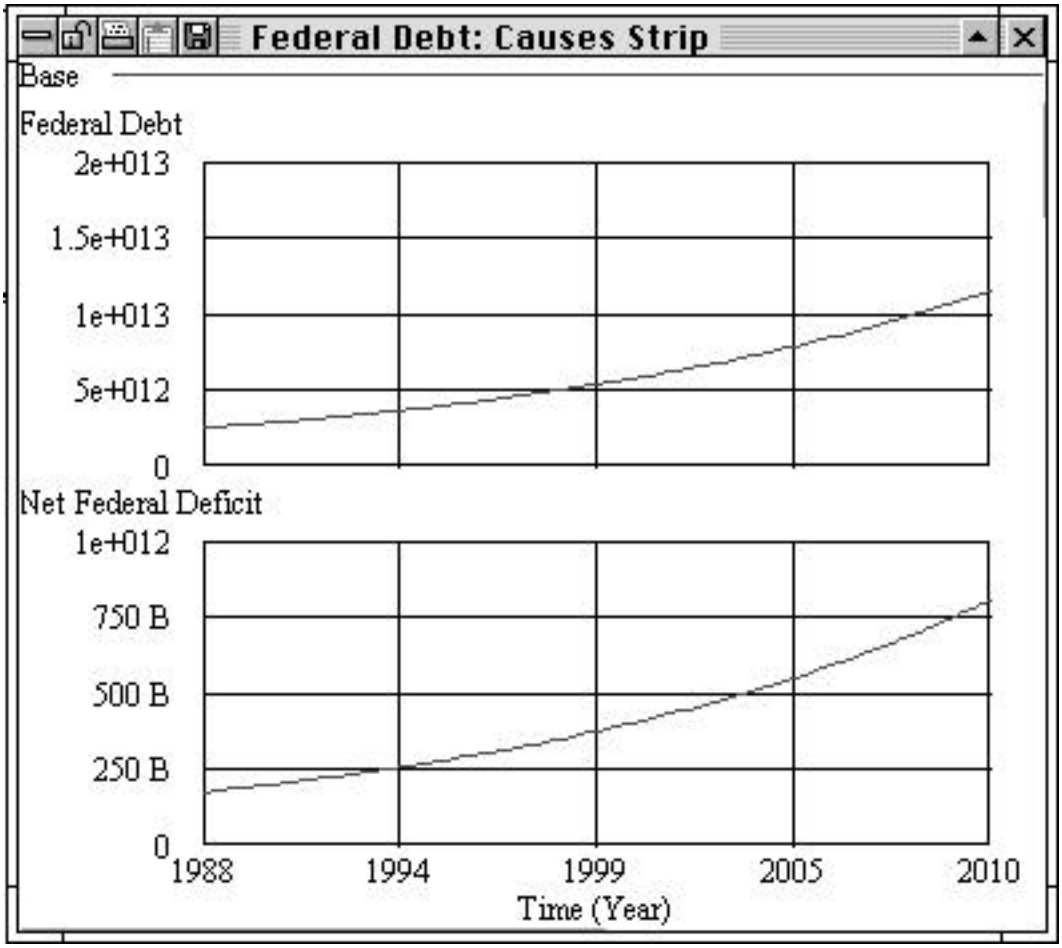
You then see:



By the year 2010, given the current assumptions in the model, the federal debt will grow to more than 10 trillion dollars, four times its value in 1988.

Besides the strip graph, Vensim PLE provides many other ways to examine simulation output.

The  button displays a strip graph of the currently selected variable, along with graphs of all the variables that determine the value of the selected variable (the causes). Clicking on the button gives you:

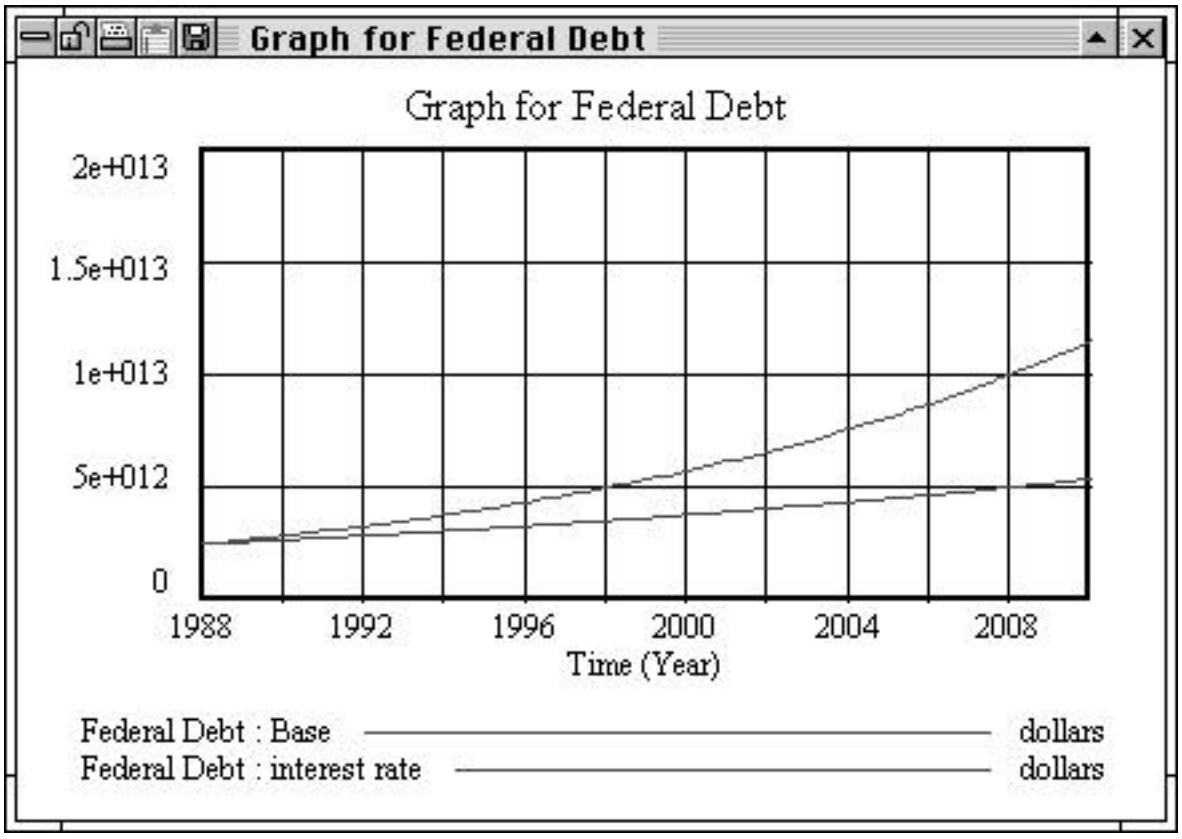


Vensim PLE also can present the output in the form of a table rather than a graph. To see a table of the selected variable simply click on the  button.

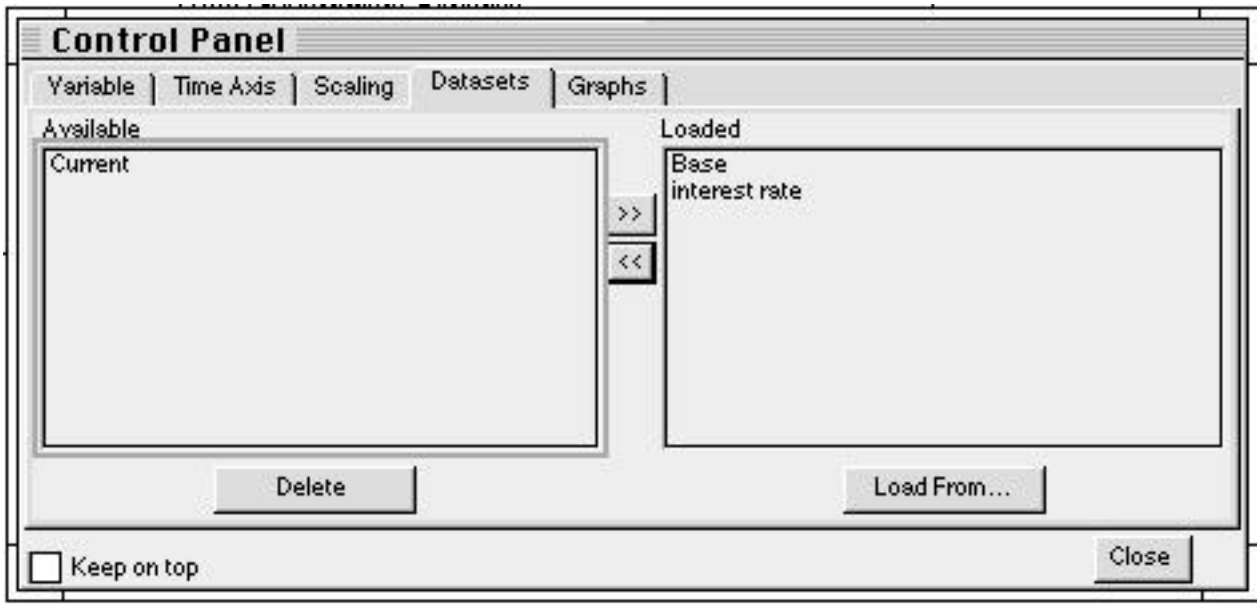
Having analyzed this simulation, you may wish to run additional simulations under different assumptions. For example, what might happen if the prevailing interest rate were 3.5% rather than 7%?

One way to change the parameter is to change the model equation in the original model. With the  button selected, click on **Interest Rate.** A dialog box will appear. In the constant box change the interest rate from 7% to 3.5%. Again, run this new simulation but do not overwrite the simulation named **Base**. Instead, name it **interest rate**.

Your new graph should look like this:



If you do not wish to see the previous run (base) displayed with the new simulation run, click on the  button on the upper toolbar. Your screen should look like this:



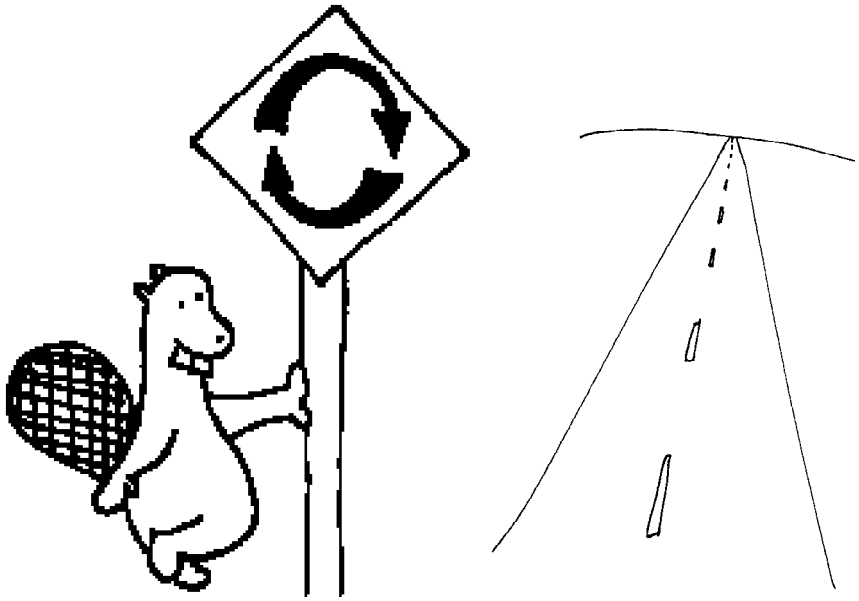
A dialog box appears and shows on the left side the two simulation runs you have created so far. Double-click on the name of the simulation run you wish to *remove* from the graph (or highlight it and click on the **<<** button to remove it from the right side of the dialog box). Close the **Datasets** window and close and re-display the strip graph. Now, only the new simulation run should appear.

You may also wish to run the model for a longer period of time. In this case, select **Time Bounds...** from the **Model** menu. You then see the same dialog box that you saw when you first started to develop your model.

You can extend your simulation by setting a new date for your final time. Run your model out to the year 2075.

Road Maps 3

A Guide to Learning System Dynamics



System Dynamics in Education Project

Road Maps 3

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

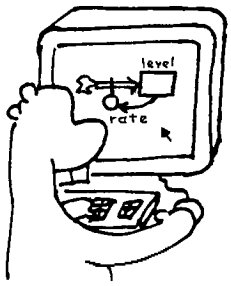
August 21, 1992
Latest Revision March 18, 2001

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Macintosh is a trademark of Apple Computers, Inc.

Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Three!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps Three is the third in the series of chapters in Road Maps. In Road Maps Three you will reinforce the material you learned earlier in Road Maps One and Two while continuing to develop your modeling skills. Specifically, you will learn how to convert causal loop diagrams into flow diagrams and DYNAMO equations into their corresponding STELLA models and equations. Road Maps Three will expand your graphical integration as well as mental simulation skills. The modeling in Road Maps Three begins with positive and negative feedback loops and goes on to combining them to form an S-shaped growth structure. The final section of Road Maps Three is Professor Jay Forrester's book, *Urban Dynamics*. The book describes a system dynamics model used to study urban problems.

Topics Covered in Road Maps Three

Graphical Integration Exercises

- *Graphical Integration Exercises Part 2: Ramp Functions* (D-4571)
by Kevin Agatstein and Lucia Breierova

Beginner Modeling Exercises

- *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback* (D-4487)
by Joseph G. Whelan
- *Beginner Modeling Exercises Section 3: Mental Simulation of Simple Negative Feedback* (D-4536)
by Helen Zhu

Positive and Negative Feedback Loops

- *Study Notes in System Dynamics*, Exercises 4, 5
by Michael R. Goodman

- *Introduction to Computer Simulation*, Chapter 15: Using Simulation to Analyze Simple Positive and Negative Loops
by Nancy Roberts et al.

Practice with More Extensive Computer Modeling

- *Study Notes in System Dynamics, Section 3.10-3.12*
by Michael R. Goodman

Development and Modeling of S-Shaped Growth

- *Study Notes in System Dynamics*, Chapter 4: S-Shaped Growth Structure and Chapter 5: Review of Simple Structures
by Michael R. Goodman

Real-Life Applications of System Dynamics

- *Urban Dynamics*
by Jay W. Forrester

Things You'll Need for Road Maps Three

Modeling Software

In order to complete Road Maps Three and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on

Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

See the Appendix for more information about obtaining Vensim and Powersim.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

Books

You will need the following three books for Road Maps Three. *Study Notes in System Dynamics,* by Michael R. Goodman, and *Introduction to Computer Simulation,* by Nancy Roberts et al, were previously required for Road Maps Two. These books will also be used in later Road Maps. Please note that required chapters for the Roberts text can be downloaded from the SDEP server in electronic format.

Goodman, Michael R, 1974. *Study Notes in System Dynamics*. Portland, Oregon: Productivity Press, 388 pp.

Roberts, Nancy, 1983. *Introduction to Computer Simulation.* Portland, Oregon: Productivity Press, 562 pp.

Forrester, Jay W., 1969. *Urban Dynamics*.

Portland, Oregon: Productivity Press, 285 pp.

If you have any problems in getting these books, contact Productivity Press (see Appendix).

How to Use Road Maps Three

Road Maps Three explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

Graphical Integration Exercises

The first reading in Road Maps Three develops your understanding and skills of graphical integration. Road Maps Two introduced the use of graphical integration to estimate the behavior of systems with constant positive and negative flows, as well as step functions flows. The following paper explores ramp function flows.

- Graphical Integration Exercises Part 2: Ramp Functions [1]

by Kevin Agatstein and Lucia Breierova

A "ramp function" in a flow is a linearly increasing or decreasing flow. The paper describes the behaviors caused by such flows: parabolic growth or ìdownwardsî parabolic growth. To practice the methods of graphical integration, you will be asked to do some exercises. Solutions are provided at the end of the paper.

Please read *Graphical Integration Exercises Part 2* now.

After completing *Graphical Integration Exercises Part 2*...

This paper explained how to use graphical integration to estimate the behavior of systems with linearly increasing and decreasing flows. It showed how to calculate the level of a stock by calculating the area under the flow graph, and how to calculate the slope of the stock from the value of the flow. It is very important to understand these methods before moving on to the next paper in the Graphical Integration series.

Beginner Modeling Exercises

The next two readings continue the series of Beginner Modeling Exercises which was first introduced in Road Maps Two. In Road Maps One, feedback loops were described as the basic structural elements of systems. Positive and negative feedback cause almost all dynamic behavior of systems.

- Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback [2]

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[1] Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part 2: Ramp Functions* (D-4571), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Feb. 7, 25 pp.

by Joseph G. Whelan

The second paper in the Beginner Modeling Exercises series discusses the effects of positive feedback on dynamic systems. It first describes how to recognize a positive feedback loop in a system, and how to use mental simulation to explore such a system. You will then be asked to do several exercises to improve your understanding of systems with simple positive feedback. Solutions to the exercises are provided.

Please read *Beginner Modeling Exercises Section 2* now.

After reading *Beginner Modeling Exercises Section 2...*

This paper explained how to identify a positive feedback loop in a system. A positive feedback loop generates exponential growth. It also showed how to find the doubling time of a stock. It is important to be able to do all the exercises before moving to the next reading.

- Beginner Modeling Exercises Section 3: Mental Simulation of Simple Negative Feedback [3]

by Helen Zhu

The third paper in the Beginner Modeling Exercises series explains how negative feedback affects the behavior of systems. As the previous paper, it first describes how to recognize negative feedback and how to mentally simulate the behavior of systems containing negative feedback loops. To make sure that you understand what you have just learned, you will be asked to complete several exercises. Solutions are again provided.

Please read *Beginner Modeling Exercises Section 3* now.

After reading *Beginner Modeling Exercises Section 3...*

The paper explained why negative feedback creates asymptotic growth, as well as described its goal-gap structure. It showed a method to calculate the half-

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[2] Joseph G. Whelan, 1995. *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback* (D-4487), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 27, 15 pp.

[3] Helen Zhu, 1996. *Beginner Modeling Exercises Section 3: Mental Simulation of Simple Negative Feedback* (D-4536), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, January 28, 23 pp.

time of the stock, and explained how to use this information to estimate the behavior of the system. Understanding the concepts and exercises is very important before starting the next section of Road Maps Three.

Positive and Negative Feedback Loops

The next three sections of Road Maps Three also deal with positive and negative feedback loops. Road Maps One provided the basics of positive and negative feedback loops, Road Maps Two offered beginner modeling exercises which involved positive and negative feedback loops, and we have just explained how to use mental simulation to understand the behavior of systems with simple positive or negative feedback. Now we will go one step further by modeling positive and negative feedback loops on a computer using STELLA.

- *Study Notes in System Dynamics*,[4] Exercise 4

by Michael R. Goodman

The next section of Road Maps Three is Exercise 4 of *Study Notes in System Dynamics*, by Michael R. Goodman. Exercise 4 uses a DYNAMO model which you will need to convert to STELLA. If you have any problems with the conversion, please refer to Road Maps Two which worked with a similar model and provided supplemental information to help with the conversion.

The exercise provides you with a causal loop diagram and the equations for the DYNAMO system to be modeled. Before you are asked to model it on STELLA, you are asked to answer questions intuitively about the system. <u>It is extremely important to think about the questions and write them down before modeling it and finding out the correct answers</u>.

Please read and do Exercise 4 now. The following tips may help you.

Some tips on STELLA: *Multiple Graphs*

There are two different methods for creating multiple graphs in STELLA:

1. Open up the dialog box of the first graph, and click once on the upward pointing arrow next to *new*. Define the graph that you want. When you close the graph

[4] Goodman, Michael R., 1974. Study Notes in System Dynamics, Portland, Oregon: Productivity Press. 800/394-6868. 388 pp.

dialog box, you can turn the pages of the graph by clicking on the small triangle in the lower left-hand corner of the graph.

2. Open a new graph by clicking on the graph icon and by placing it on the screen.

A few notes on DYNAMO: *Converting to STELLA*

The DYNAMO equations given in the solution on page 171 contain DYNAMO commands not previously mentioned in Road Maps. The new commands define the DYNAMO graph for a DYNAMO simulation. The resulting graph in DYNAMO, S4.9, is shown on page 172. For further information on the DYNAMO equations see pages 88-94 "The SPEC statement" in *Introduction to System Dynamics Modeling with DYNAMO* by Richardson and Pugh.

Some DYNAMO terms are clarified below in terms of STELLA software. If you have any further questions consult the STELLA manual.

[**DT = .1**] The model will be simulated every 0.1 year.

The equivalent in STELLA is to enter ".1" in the *dt* box in the *Time Specs* dialog box. Then click once on the time units desired for simulation.

[**Pltper = 1**] A point will appear on the graph every year, and a line will be drawn between all the points to see a curve on the graph.

There is no equivalent for this in STELLA. The closest is the *Report Interval* box in the table dialog box.

[**Length = 50**] The graph will show 50 years of output.

The equivalent to this in STELLA is to enter "0" in the *From* box and "50" in the *To* box in the *Time Specs* dialog box.

[**Plot Pop = P(0,320)/BR=B(0,24)**] This command assigns scales and variables for the graph. The equivalent in STELLA is to change the scales in the *Graph* dialog box for each variable.

After finishing Exercise 4...

This exercises provided more experience with converting DYNAMO equations to STELLA. It also explained the basic characteristics of positive feedback loops. Positive feedback loops are a fundamental building block of system dynamics and of computer modeling. They will be explored again in Road Maps Three, and they will be used throughout the rest of Road Maps.

The model that you just built also illustrates a systems principle:



Systems Principle #5:

Levels are changed only by the Rates.

Look at the model just created in STELLA (same as the model on p. 171 of *Study Notes in System Dynamics*). Note that the level, **population** is influenced only by the rate, **birth rate**. Only rates can change levels.

System Principle #5 is important in modeling because it helps clarify the interactions between the different levels and rates.

- *Study Notes in System Dynamics*, Exercise 5

by Michael R. Goodman

The next reading in Road Maps Three is Exercise 5 of *Study Notes in System Dynamics*. This exercise has the same pattern as Exercise 4; however, Exercise 5 deals with negative feedback loops. The exercise asks for intuitive answers to questions before you simulate your model. Answering these questions before you run the model is extremely important for learning about the feedback loops. Once again, if you need help with the conversion from DYNAMO to STELLA there is some supplemental information included in Road Maps Two that can assist you.

Please read and do Exercise 5 now. The following tip may help you.

A tip for modeling

To model an outflow, begin with the flow *inside* of the stock and drag the flow two inches *out* of the stock. Then release the mouse.

After finishing Exercise 5...

Exercise 5 gave you even more experience with converting equations and models from DYNAMO to STELLA. It also explained the basic characteristics of a negative feedback loop. Negative feedback loops are another fundamental building block of system dynamics and computer modeling. They will be explored again in Road Maps Three, and they will be used throughout the rest of Road Maps.

- *Introduction to Computer Simulation*,[5] Chapter 15: Using Simulation to Analyze Simple Positive and Negative Loops

by Nancy Roberts et al.

Chapter 15 discusses the fact that both positive and negative feedback loops are the only two types of loops in a system. Everything else is built from positive and negative feedback loops. This fact was mentioned in System Principle #1 which stated that feedback loops are the basic building blocks of systems. Chapter 15 is the last section of Road Maps Three dealing specifically with positive and negative feedback loops. After this chapter, we will move on to more complex structures.

Chapter 15 also uses DYNAMO equations. However, Chapter 15 includes some DYNAMO equations that will be new to you. To help you, we have included some supplemental information.

Please read Chapter 15 and do all of the exercises as the book asks. Refer to the following supplement as necessary.

Tips for "Using Simulation to Analyze Simple Positive and Negative Loops"

Read Chapter 15 from page 261 up to and including Exercise 3 on page 262. Do all of the examples and exercises as the book asks. Example I refers to equations from the same book in Chapter 14, which is not used in Road Maps. The equations, converted to STELLA, follow:

```
YEAST_CELLS(t) = YEAST_CELLS(t-dt) + (BUDDING) * dt
INIT YEAST_CELLS = 10 {CELLS}
INFLOWS:
BUDDING = YEAST_CELLS * BUDDING_FRACTION {CELLS/HOUR}
BUDDING_FRACTION = 0.1 {1/HOUR}
```

[5] Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. *Introduction to Computer Simulation: A System Dynamics Approach*, Portland, Oregon: Productivity Press. 800/394-6868. 562 pp.

dt = 1
simulation length = 30 hours

A tip on STELLA: *Sensi Specs*

STELLA provides an easy way to view the effects of several different values for the same variable on one graph. The three values that you are experimenting with in Exercise 3 are 0.2, 0.1, and 0.05 for BUDDING_FRACTION. To see all of these values on the same graph, the graph needs to be created under *Sensi Specs*.
- First, close all graphs and dialog boxes on the screen if any are open.
- Pull down the *Run* menu, and release the mouse on *Sensi Specs*. A *Sensi Specs* dialog box will open up on the screen.
- Double click on BUDDING_FRACTION in the *Allowable* list.
- Then click once on BUDDING_FRACTION in the *Selected(value)* list.
- Click once on the button next to *ad hoc*. Type "0.2." Click once on *Set* and type "0.1" Click once on *Set*, and type "0.05." Click once on *Set*.
- Click once on *Define: Graph* button in the lower left-hand corner. The cursor will be a graph icon. Click once on an empty space to place the graph. The graph and graph dialog box will open up with a new graph ready to be defined, and *Comparative* selected.
- Define the graph with only one variable. Double click on YEAST_CELLS.
- Then title the graph and click once on OK.
- Choose S-Run from the Run menu.

Please read and do Exercise 4 on the bottom of page 262 through the top of page 264. Then read through Exercise 5 on page 263. In Exercise 5 you will modify the model from Exercise 4 by adding an outflow from the stock of BANK BALANCE. The addition of an outflow will make the model a two-flow model. It will be the first two-flow model that you model by yourself in Road Maps.
- To add the new outflow, WITHDRAWALS, place the flow inside the stock of BANK BALANCE. Then drag outward away from the inflow of INTEREST. Putting the flows on opposite sides of the stock makes the model easier to read.
- Define and document the outflow, WITHDRAWALS.
- View the equations for the STELLA model, and notice the change to the BANK BALANCE equation.

Finish part b of Exercise 5.

If you want to see WITHDRAWAL graphed remember to add WITHDRAWAL to the *Selected* list in the graph dialog box. Part c and d of Exercise 5 can be easily done using *Sensi Specs* in STELLA as done above. (See A tip on STELLA: *Sensi Specs*)

Please read "Starting a Model in Equilibrium," and do Exercise 6 on page 263.

If you have difficulty with Exercise 6 try running the model for various balances, interest rates and withdrawal rates.

Read the section "Examining a System's Response to Disturbances—Part I" on page 264. Then read through Exercise 7 on page 265.

A tip on STELLA: *Step Functions*

This section describes how to include instantaneous changes which occur during the simulation of a model by using the step function. The step function works the same in STELLA as it does in DYNAMO.

- To change the WITHDRAWAL equation in your model open the WITHDRAWALS dialog box. Add "+ STEP(25,5)" to your equation so that it reads "50 + STEP(25,5)."
- Graph the results.

Please read Example II from page 265 up to and including Exercise 10 on page 266.

Do all of the examples and exercises as the book asks. Example I refers to equations from a chapter of the same book not used in Road Maps; the equations (converted to STELLA) follow:

```
YEAST_CELLS(t) = YEAST_CELLS(t-dt) + (- YEAST_DEATHS) * dt
INIT YEAST_CELLS = 10 {CELLS}
OUTFLOWS:
YEAST_DEATHS = YEAST_CELLS/AVERAGE_LIFETIME {CELLS/HOUR}
AVERAGE-LIFETIME = 0.1 {1/HOUR}
```

Read through Exercise 11 on page 266. Before doing this exercise remember that good modeling practice requires that the inflow, YEAST BUDDING, and the outflow, YEAST DEATHS, are on opposite sides of the stock, YEAST CELLS.

Do Exercises 12 and 13 on page 266 through the top of page 267. Example III, including Exercise 14 is very important to learn first-hand the behavior of the system. Try to do the experiment.

Read "Modeling Temperature Change" up to Exercise 15 on page 269. Read through Exercise 15 on the bottom of page 269 through the top of page 270. But before modeling the system, read "Auxiliary Variables" from page 270 through 271. "Auxiliary Variables" will help you to formulate a more user-friendly model for Exercise 15. Do Exercise 15.

One more input that is not shown in the flow diagram on page 271 is COOLING CONSTANT, (T). In the STELLA model, COOLING CONSTANT should be a converter connected to DECLINE so that it is not hidden in the model.

Read from "The Goal-Gap Formulation" on page 271 up to Example IV: Pushups and Practice on page 274. Do the exercises and examples as they are written in the book.

Read through Example IV from the bottom of page 274 through the top of page 277. Example IV refers to a part of the book not used in Road Maps. The discussion referred to can be inferred from the causal loop diagram in Figure 15.9 on page 274.
- Create the model on STELLA as described by Exercise 18 on page 276. Your model should be similar to Figure 15.11 on page 276. The difference will be the auxiliary variables visible on your model:

PRACTICE TIME PER PUSHUP = 0.5 {MIN/PUSHUP}

MAINTENANCE = 10 {MIN}

PRACTICE EFFECTIVENESS = 0.2 {PUSHUPS/MO/MIN}

When creating the flow, IMPROVEMENT RATE, click once on the *bi-flow* button in the top left-hand corner of the flow dialog box. Jim can improve as well as worsen.

Do Exercise 18b through 18e using *Sensi Specs* on STELLA.
Read Example V on page 277 up to and including Exercise 20 on the top of page 282.
For Ex. 19 on page 278, remember to name all parameters and give them an icon.

NUMBER OF PUSHUPS = 10 {INITIAL NUMBER OF PUSHUPS}

IMPROVEMENT RATE = (AMOUNT OF PRACTICE-BASE PRACTICE) *
PRACTICE EFFECTIVENESS {PUSHUPS/MO}
BASE PRACTICE = 30 {MIN}
AMOUNT OF PRACTICE = PRACTICE TIME PER PUSHUP * DIFFERENCE {MIN}
DIFFERENCE = GOAL FOR PUSHUPS - NUMBER OF PUSHUPS {PUSHUPS}
GOAL FOR PUSHUPS = 50 {PUSHUPS}
PRACTICE TIME PER PUSHUP = 0.5 {MIN/PUSHUP}
PRACTICE EFFECTIVENESS = 0.2 {PUSHUPS/MO/MIN}

For "Choosing a Value for DT" on pages 279-282, STELLA II v. 2.2.1. unfortunately does not allow a value of DT greater than one, but you should follow along with this section and run the simulations you can. All the models used here are ones you have already created. The information on DT, the solution interval, will be expanded upon in further parts of the Road Maps.

Please read and do Exercise 21 on page 282.

Exercise 21 is your chance to create your own model. You can make this as complicated as you wish. Try varying the structure and complexity to demonstrate the behavior you have seen in previous models such as S-shaped growth, exponential growth and decay.

This is the end of **Tips for "Using Simulation to Analyze Simple Positive and Negative Loops."**

After finishing Chapter 15...

Chapter15 improved your STELLA modeling skills, as well as your skills for conversion from DYNAMO to STELLA. It provided more explanation of positive and negative feedback loops. As we continue in Road Maps you will see that positive and negative feedback loops are the building blocks for all other structures. Being familiar with the characteristics of feedback loops is essential for learning about system dynamics and computer simulation.

The following System Principle is one that we have already presented in Road Maps Two. We are restating the principle, as part of our spiral approach, to emphasize its importance.

System Principle #2:
Levels and Rates are fundamental to loop substructure.

A feedback loop consists of two distinct types of variables, the **levels** (also called stocks or states) and the **rates** (also called flows or actions). These two variables are both necessary to represent the structure in a feedback loop.

As you are reading Chapter 15 of *Introduction to Computer Simulation*, trace through the feedback loops in Figure 15.1, 15.3, and 15.5, which belong to Examples I, II, and III, respectively. Notice that both the levels and the rates are fundamental to the substructure of the feedback loops. The loop dynamics cannot be represented without their inclusion.

Practice with More Extensive Computer Modeling

- *Study Notes in System Dynamics*, Section 3.10-3.12
by Michael R. Goodman

This section introduces more advanced models than you have seen before. It demonstrates how more complicated models are built on positive and negative feedback loops. You will not have to derive the models yourself because the flow diagrams, DYNAMO equations, and run outputs are provided with each example. You will have to convert the DYNAMO equations to STELLA and then run the model for each example.

Please read and do Section 3.10-3.12 now. The following tip may help you.
A tip for STELLA: *Table Graphs*

One new DYNAMO equation, a table equation, is introduced.
The DYNAMO equations, 5 and 5.1 from page 63 are:

PAT.K = TABLE(PATT,POLR,0,80,10)
PATT =.6 / 2.5 / 5 / 8 / 11.5 / 15.5 / 20 / 31 / 50
PAT - POLLUTION ABSORPTION TIME (YEARS)
POLR - POLLUTION RATIO (DIMENSIONLESS)

PAT.K — This is the variable in the model that will represent the table. In the STELLA model the name will be POLLUTION ABSORPTION TIME.

- Open the dialog box for POLLUTION ABSORPTION TIME.

TABLE(.....) — This defines the variable, PAT.K as a table.
In STELLA, the equivalent is to open the dialog box of POLLUTION ABSORPTION TIME and to define it as "A graph of ..." POLLUTION RATIO.

- Select POLLUTION RATIO from the *Required inputs* box.
- Click once on the *Become Graph* button on the bottom of the dialog box. A new dialog box will appear, to define the table.
- Click once on the *To Equation* button on the bottom of the dialog box to view the equation. You will see POLLUTION ABSORPTION TIME = "graph of..." POLLUTION RATIO.
- Click once on the *To Graph* button.

PATT — In STELLA, this name is the same as PAT.
DYNAMO requires two different names for a table, whereas STELLA requires only one.

POLR — This is the variable of which POLLUTION ABSORPTION TIME is a function. In STELLA this means that one value for POLLUTION ABSORPTION TIME corresponds to one value for POLLUTION RATIO.

0 — This value represents the minimum value that POLLUTION RATIO can have.

80 — This value represents the maximum value that POLLUTION RATIO can have.

- Return to the graph dialog box. Notice that the horizontal and vertical side of the graph ranges from 0 to 100 (100 is the default number in STELLA). But, POLLUTION RATIO on the horizontal axis ranges from 0 to 80, as the equation tells us.
- Notice that "100," the maximum range, is highlighted on the horizontal axis for POLLUTION RATIO. To enter the correct number for POLLUTION RATIO, type "80."

10 — This value is only necessary in DYNAMO. It is the length of incrementations on the graph between 0 and 80.

PATT = .6 / 2.5 / 5 / 8 / 11.5 / 15.5 / 20 / 31 / 50 — These nine values represent the values that the table will have. Thus the range for POLLUTION ABSORPTION TIME for the vertical axis is 0.6 to 50.

- Press *tab* on your keyboard to highlight the maximum value for POLLUTION ABSORPTION TIME and type "50." Press *tab* on your keyboard again to highlight the minimum value for POLLUTION ABSORPTION TIME and type "0.6."
- Notice that there are eleven points to be defined on the output table in STELLA, but only 9 points given in *Study Notes* for POLLUTION ABSORPTION TIME.
- Highlight the number 11 in the *Data Points* box. Type "9."
- Highlight the first number, 0.000 under *Output*, by clicking once on it. Type "0.6." Press *return* on your keyboard. Type "2.5." Press *return* on your keyboard, and type "5." Press *return*.... Continue until you have filled in all of the points under *Output*.
- The graph should resemble as much as possible the table in Figure 3-20 on page 61 in *Study Notes in System Dynamics*.
- If it does, click once on *OK*.
- See page 26 in *Getting Started with STELLA II: A Hands-on Experience* for more information on table functions in STELLA.

After finishing Section 3.10-3.12...

Working with the models in Sections 3.10-3.12 gave you more a sense of the system dynamics approach. We now introduce another important system principle.



System Principle #6:

Levels exist in conservative subsystems.

A conserved quantity has the property that it is never created or destroyed (within its system); it is only moved around. Look at Figure 3-12 on page 52 in *Study Notes in System Dynamics*, a simple inventory control system. Notice that the level **Inventory** contains goods that are moved in from the **Factory** and out to

the **Customers**. The goods are neither created nor destroyed within the level. All levels contain conserved quantities.

Development and Modeling of S-shaped Growth

The next two sections of Road Maps Three build on your knowledge of positive and negative feedback loops and guide the development of a model of an S-shaped growth system.

- ***Study Notes in System Dynamics***, Chapter 5

by Michael R. Goodman

Chapter 5 helps you develop a STELLA model of a system generating S-shaped growth. When you are finished, compare the output of your STELLA model to the answer in DYNAMO. The output of your model should match the answer given.

Please read and do the exercise in Chapter 5 now.
After finishing Chapter 5...

Chapter 5 developed the concepts of positive and negative feedback loops, and introduced the basic ideas behind S-shaped growth. It also reviewed the use of table functions in STELLA.

The following principle is another recurrent principle. We are restating System Principle #3 in order to emphasize its importance.

System Principle #3:
Levels and Rates are not distinguished by units of measure.

The units of measure of a variable do not distinguish between a level and a rate. The modeler must recognize the difference between a variable created by integration and one that is a policy statement in the system.

Look at Figure 5-11 of *Study Notes in System Dynamics*. Here, and in all models, the units of a rate are the units of its associated level over time. A good rule of thumb for distinguishing levels from rates is to imagine what would happen if action were halted. Rates will cease when action stops, but levels will continue to exist. In this case, notice that **Industrial Structures** has units of [structures], and **Industrial Construction** has units of [structures/year]. If action were to cease, then construction would cease as well, but the structures would stay (their number simply

would not change). Thus **Industrial Structures** is a level and **Industrial Construction** is a rate.

- Study Notes in System Dynamics, Chapter 4

by Michael R. Goodman

Chapter 4 focuses on the structure and behavior of the different variables in an S-shaped growth system. The chapter explores two systems which exhibit S-shaped growth. One is an epidemic and the second is a pendulum. Although the pendulum example may seem confusing without prior knowledge of the equations that govern a pendulum's movements, work through the model along with the book, and focus on understanding the system through your knowledge of system dynamics.

Please read and do Chapter 4 now.

After finishing Chapter 4...

Chapter 4 described the key characteristics behind S-shaped growth. Because systems producing this behavior are very common, you will be modeling several more S-shaped growth systems in later Road Maps.
Some key characteristics are:

- exponential growth
- goal-seeking behavior
- loop dominance
- dynamic equilibrium

Real-life Applications of System Dynamics

Revised by Lei Lei

- Urban Dynamics [6]

Road Maps 3 concludes with the reading of one of Professor Jay Forrester's classic system dynamics books. It is not necessary to complete the book before continuing on to the next chapter of Road Maps. However, reading *Urban Dynamics* will aid the reader in giving real world relevance to system dynamics. *Urban Dynamics* is an introduction to complex urban systems and their

[6] Forrester, Jay W., 1969. *Urban Dynamics*, Waltham, Massachusetts: Pegasus Communications. 285 pp.

characteristics. The reader should not be discouraged if s/he cannot fully understand the models introduced in the book. The reading is assigned as part of Road Map's spiral-learning approach, where concepts are referred to and expanded upon continually throughout the series.

Urban Dynamics provides a glimpse of the origins of the field of system dynamics before the name "system dynamics" was coined. The book presents excellent examples of using simulation models to understand and explain the counterintuitive nature of complex social systems. *Urban Dynamics* is one of Professor Jay W. Forrester's more controversial books. The publication of *Urban Dynamics* in 1969 marks a unique event in the analysis of social systems.

Although *Urban Dynamics* addresses crises that our cities faced twenty years ago, many of the same problems still exist today. The *Urban Dynamics* model refutes the conventional wisdom that urban problems are caused by such factors as rural-urban migration, dwindling fiscal resources, and suburbanization.

Instead, the book suggests that most urban problems arise from the interactions of processes that occur within the cities themselves. After showing what established policies have actually done to our cities and why, Forrester uses the model to test new policies and programs for the revival of urban cities. The policies presented in the book challenge the status quo. For example, Forrester suggests that demolishing, rather than constructing, low-cost housing will be a better long-term solution to the urban crises. By reading *Urban Dynamics*, the reader will gain a deeper understanding of how high leverage policies can be pushed in the wrong direction.

Forrester discusses how cities are nonlinear systems that contain multiple feedback loops. Thus, fully understanding and managing the complex systems defies simple human reason and intuition. Forrester suggests, however, that by examining urban problems with computer simulation, we can better understand outcomes of various policies. The book uses DYNAMO models of generic urban areas to discuss and test urban policy options presented by the author.

At the time *Urban Dynamics* was published, the book sparked a controversy that related to the nationwide debate over urban crises. The controversy is summarized as follows:

Business Week praised the book in 1969 and said, "*Urban Dynamics* is an unsettling, complex and ground-breaking new book about our cities and the decay that afflicts them."[7] On the other hand, Kain says in *Fortune*, "such models (as

[7] "A daring look at city ills," Business Week, June 14, 1969, pp. 145-146.

used in *Urban Dynamics*) have a great deal of potential for the analysis of urban problems," but also says, "before adequate models become available, many inadequate ones will be put forward. Forrester's model is a conspicuous example."

A study by Jerome Rothenberg published in the *Journal of Urban Economics* in 1974 pointed out that, "the insistence upon treating 'the city' as a self-contained system results in grave difficulties." Jantsch, in *Futures*, best summarizes the controversy as, "*Urban Dynamics* had become the subject of heated debate even prior to publication. It has captured the imagination of politicians and managers in the public domain, and it has been furiously attacked and rejected by social scientists." In 1969 Jantsch said, "It is a safe prediction to state that it [*Urban Dynamics*] will become one of the most provocative and stimulating books of our time." "But," he also said, "*Urban Dynamics* is not the only approach to long-range planning and it is not perfect." Each article above concludes that *Urban Dynamics* is a notable piece of work, and highlights the controversy created by the book.

Urban Dynamics is one of the most important books in the history of system dynamics. The reader should not be frustrated if s/he cannot fully comprehend the book at this stage in the learning process and should focus on increasing understanding of a complex social system. *Urban Dynamics* shows how system dynamics can be applied to real-world problems.

Finishing off Road Maps Three

Road Maps Three explained positive and negative feedback loops, and described how they are the building blocks of all other models in system dynamics. This chapter improved your graphical integration, mental simulation, and STELLA modeling skills, as well as your ability to convert from DYNAMO to STELLA. We have presented two more system principles: a) that levels are changed only by the rates; and b) that levels exist in conservative subsystems.

In Road Maps Four you will find out why some of the models you have built so far look similar. You will further develop your mental simulation skills through more Beginner Modeling Exercises. Road Maps Four also contains the Fish Banks Game and introduces "The Tragedy of the Commons."

Key Terms and Concepts:

Dynamic Equilibrium
Exponential Growth
Goal-seeking Behavior
Graphical Integration
Loop Dominance
S-shaped Growth



Graphical Integration Exercises

Part Two: Ramp Functions

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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by
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March 25, 1996

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1. Abstract

This paper is the second in a series of papers on developing skills in graphical integration. It will study the use of graphical integration to estimate the behavior of simple systems with linearly increasing or decreasing flows. The paper assumes you can estimate the behavior of models with constant flows. You should know how to graphically integrate constant flows as well as step functions. Also, you should be familiar with the terms "slope" and "area," and you should know how to calculate them.[1]

2. Introduction

Let us consider a simple system, that of a bathtub with a constant inflow through a faucet. We know that if we turn on the faucet to fill a tub, go outside for five minutes, and come back to find the tub one-fourth full, then it will take five more minutes to fill up halfway. It will then take five more minutes to be three-fourths full, and a total of twenty minutes for the tub to be completely full of water.

What we just did, essentially, is graphical integration without explicitly drawing graphs. We were able to perform this integration because the system in question was very simple. However, many systems are not as simple as the bathtub system. Thus it becomes difficult to predict how they will behave. Practice in using graphical integration will make it easier to use it to better estimate the behavior of many systems.

Although we have access to sophisticated computer programs that simulate models of great complexity, it is important that we can intuitively estimate and understand what we see on the graph pad before and after running a simulation. Graphical integration helps improve our understanding of the relation between structure and behavior and also to catch errors in the model built. In this second paper of the series, we will explore flows that take on linearly increasing or decreasing values. This linearly increasing or decreasing behavior is often called a "ramp function."

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[1] If you need to develop these skills, please refer to: Alice Oh, 1995. *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Dec. 1, 19pp.

3. Flows as Ramp Functions

The first paper in the series, *Graphical Integration Exercises Part One: Exogenous Rates*, studied graphical integration of constant flows and step functions. We will now turn to explore flows as ramp functions. A flow ramp function is a flow that is increasing or decreasing linearly, and thus it is not constant over time. An example of a linearly increasing flow ramp function is the filling of a bathtub by constantly turning the faucet in the "on" direction. As time goes on, not only does the amount of water in the tub increase, but so does the rate at which water enters the tub from the faucet. Another example of a ramping flow function, only linearly decreasing, is filling the same bathtub with the faucet initially at its highest flow value, and steadily turning the valve to decrease the rate at which water enters the tub. The tub fills, more and more slowly, until the valve is turned all the way and the water ceases to flow into the tub.

For this paper we will not concern ourselves with why the rate increases or decreases linearly. We will simply study how we can estimate the effect of the flow on the behavior of the stock.

3.1 Linearly Increasing Flows Cause Parabolic Growth

In the first paper of the Graphical Integration Exercises series, we noted that *the slope of the stock is equal to the value of the flow.* The same principle applies to all flows irrespective of whether they are constant or variable. Let us first look at an example: an initially empty bathtub with a linearly increasing (also called positively ramping) net flow.

Figure 1 shows the behavior of such a flow.

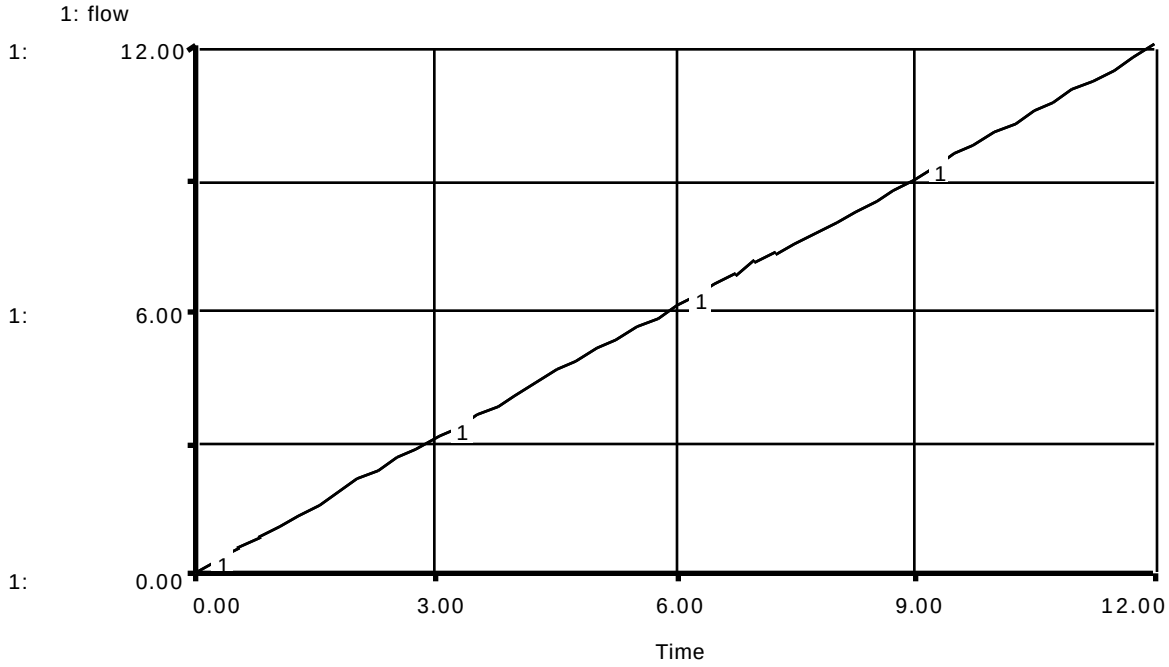


Figure 1: Linearly increasing flow with positive slope.

Figure 2 shows the resulting behavior of the stock.

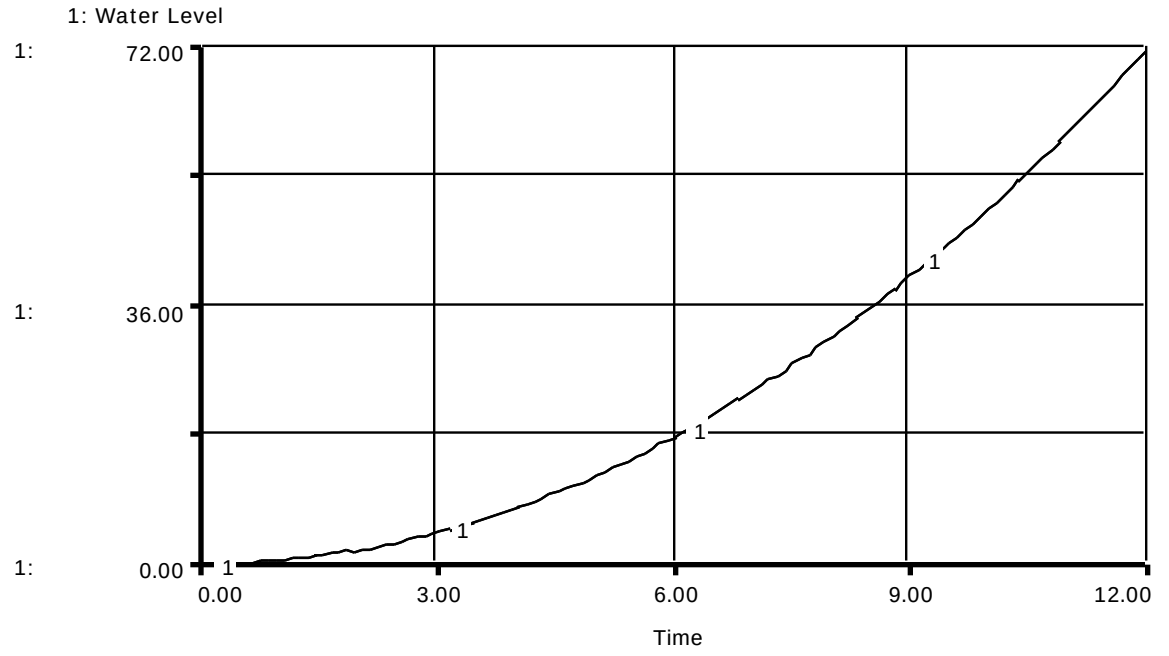


Figure 2: Behavior of stock with a linearly increasing flow.

Linearly increasing flows result in parabolic[2] growth of the stock. The area between the flow graph and the line representing a flow of zero equals the change in the stock over a time period. This is because the area, which is also the product of the flow value and time, is the amount added to the stock. We will explore this concept in greater detail a little later in the paper. You will find in graphical integration that calculating the area is extremely important for determining the behavior of the stock.

The area between the flow graph and the line representing a flow of zero in Figure 1 is the area of a triangle with a length of 12 units of time and a height of 12 units. Thus, the area is:

Total Area = 0.5 * base * height[3]

Total Area = 0.5 * 12 * 12

Total Area = 72

The reason that the change in the value of the stock equals the area between the flow curve and the line representing a flow of zero is that by calculating the area, what you are essentially doing is multiplying the flow by the time. This is the amount added to the stock. Thus, units per time times time are equal to the total number of units. Calculating the area under the flow graph is important if you are to integrate it graphically, because it allows you to easily determine the final value of the stock:

Final Value of Stock = Initial Value of Stock + Total Area

Final Value of Stock = 0 + 72

Final Value of Stock = 72

The parabolic behavior of the stock agrees with the fact that *the slope of the stock at any given time is equal to the value of the flow at that time.* We can calculate the slope of the stock at any point on the above graph by drawing the tangent line and calculating its slope. The tangent line at any given time is the line that touches the curve only at that point in time. Let us look at an example of a tangent line drawn into the curve shown in Figure 2.

---

[2] For those of you mathematically inclined, parabolic growth means that the dependent variable is a quadratic equation: $y = ax^2 + bx + c$. Exponential growth is $y=ae^{bx}$.

[3] Remember, the area of a triangle is 0.5 times base times height.

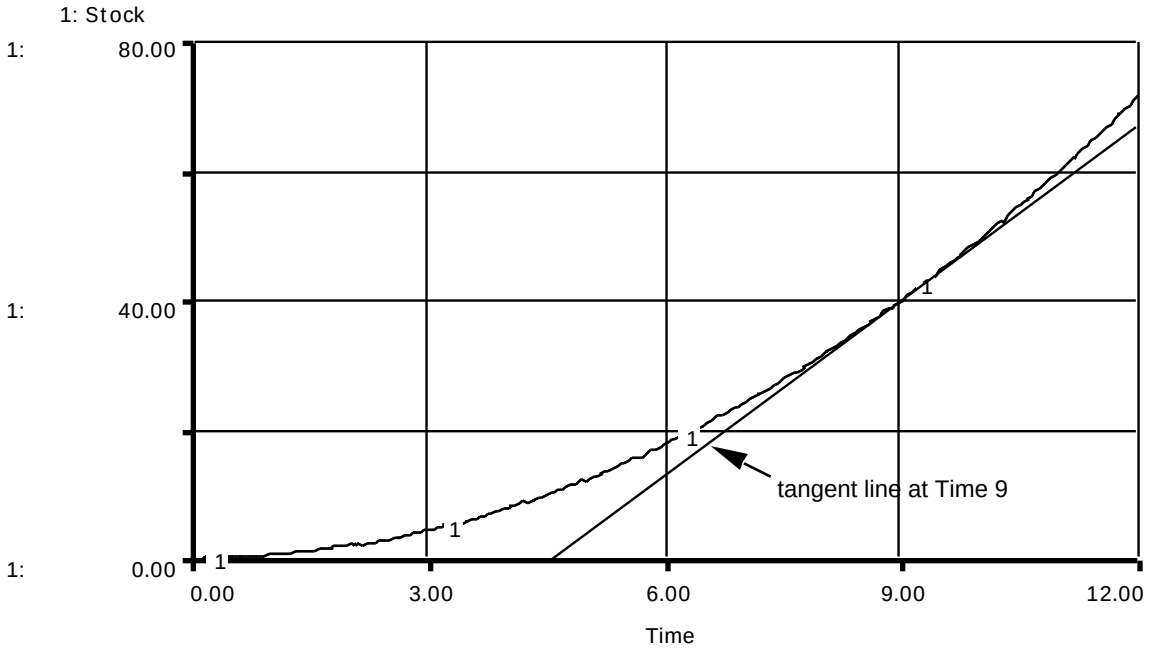


Figure 3: The behavior of an initially empty stock being filled with a linearly increasing flow.

Figure 3 shows the graph of the stock with the tangent line at Time 9. Notice how the slope of the tangent line is approximately (68-0)/(12-4.5) = 9.[4] Looking at Figure 1, we see that the value of the flow at Time 9 is also 9!

Now, let us look at another example. Let us consider again a linearly increasing flow, but with its start delayed until Time 10. We will assume the stock initially to be zero. Figure 4 shows the behavior of this flow.

[4] Slope = difference in y-coordinate/difference in x-coordinate.

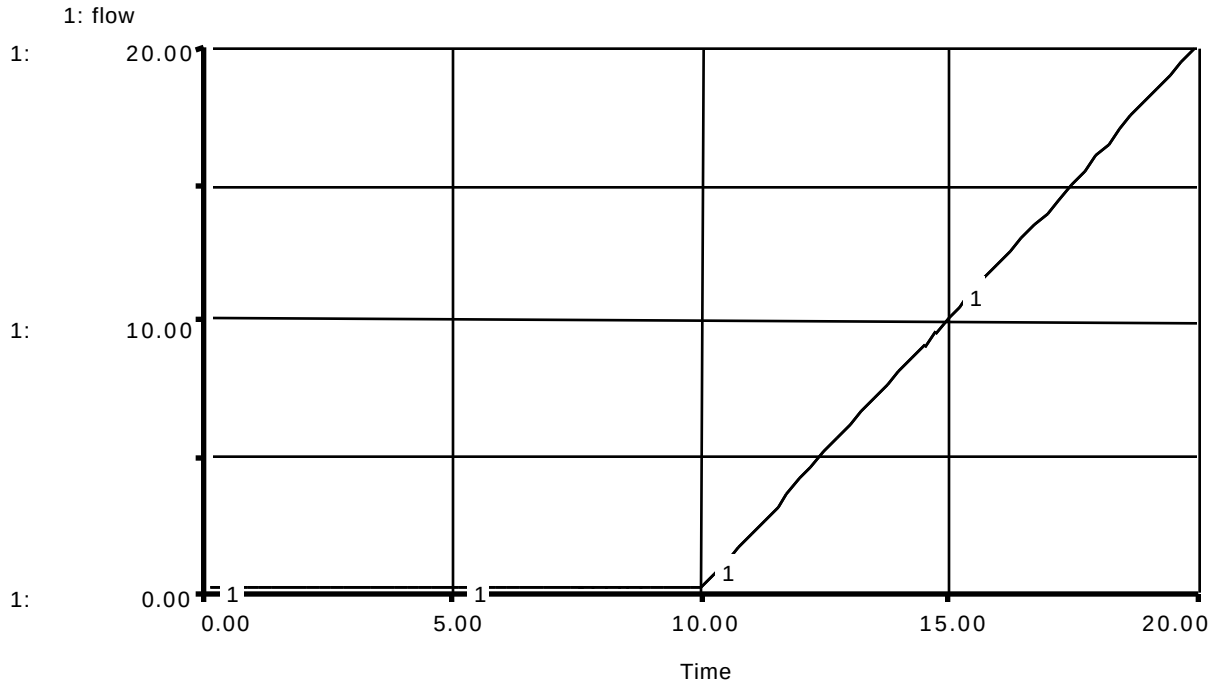


Figure 4: A linearly increasing flow beginning at Time 10.

Figure 5 shows the stock's behavior.

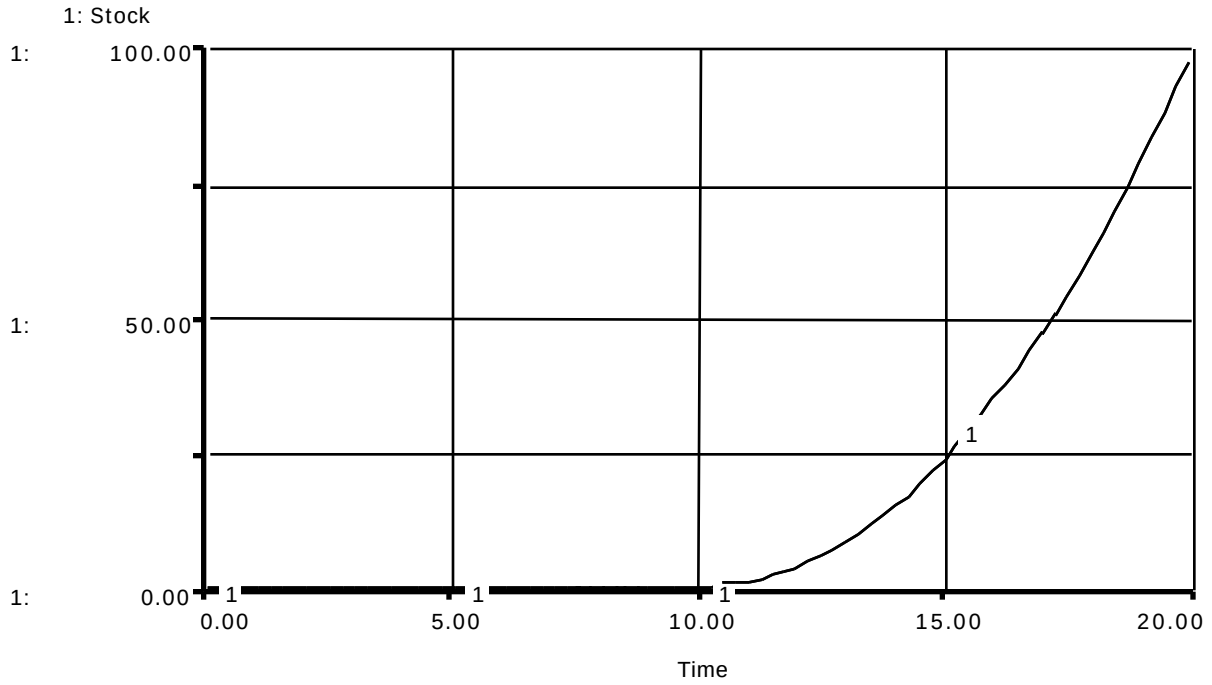


Figure 5: The behavior of the stock resulting from a linearly increasing flow beginning at Time 10.

The stock fills parabolically, beginning at Time 10. Before that time, because the net flow into the stock is zero, we do not add or subtract anything from the stock. At Time 15, the stock is being filled at a rate of 10 units per unit time. At Time 20, the stock is being filled at a net flow of 20 units per unit time. It is the linearly increasing flow that causes the parabolic growth. When we calculate the area under the entire flow graph, we see that this area is 100, which is the change in the value of the stock:

Total Area = Area under the flat line until Time 10 + Area under the triangle
Total Area = 0 + 0.5 * 20 * (20 - 10) = 100

Final Value of Stock = Initial Value of Stock + Total Area
Final Value of Stock = 0 + 100
Final Value of Stock = 100

For practice, try drawing in the tangent line at Time 15. Does the slope of this line equal 10 (which is the value of the flow at Time 15)? (Do not worry if the slope of the line you drew is a little off. Understanding the idea is what is important.)

3.2 Linearly Decreasing Flows Cause "Decreasing" Parabolic Growth

Now that we have looked at linearly increasing flows, let us look at a linearly decreasing flow filling an initially empty stock. Figure 6 shows the behavior of the flow, and Figure 7 shows the resulting behavior of the stock.

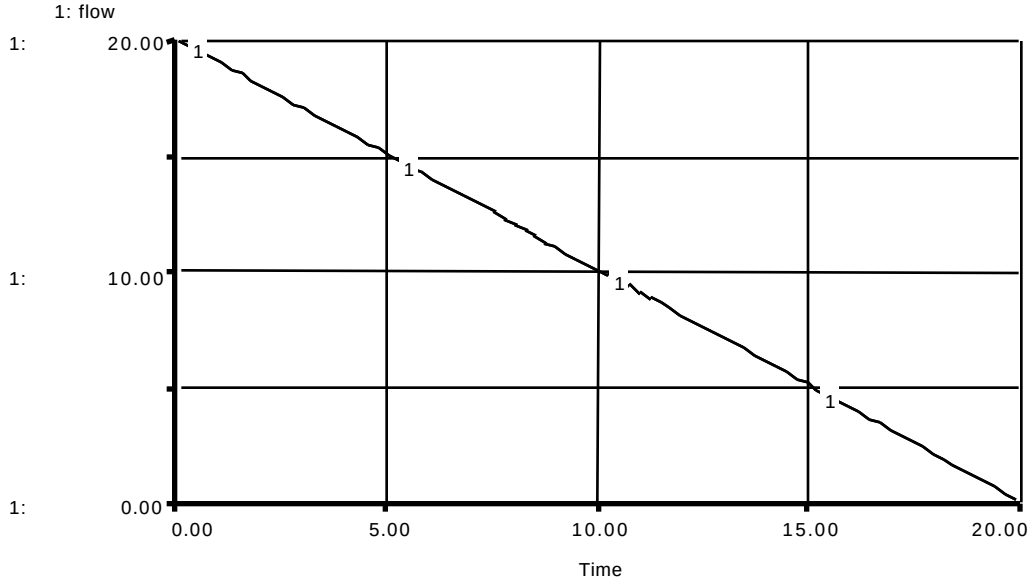


Figure 6: Linearly decreasing flow with negative slope.

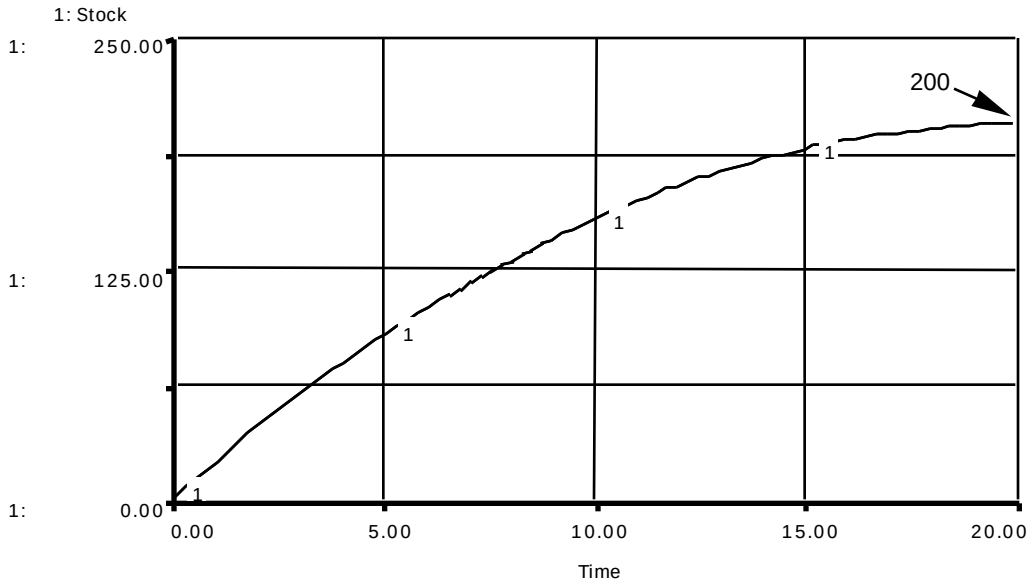


Figure 7: Behavior of Stock with a Linearly Decreasing flow.

As the flow value decreases, the rate of increase of the stock decreases. Notice that even though the flow value is decreasing, the level of the stock is still increasing. **Anytime there is a net positive inflow into a stock, the stock's value will increase.** At Time 0 the stock is filling at a rate of 20 units per unit time. At Time 5 the stock is still being filled, but only at the rate of 15 units per unit time. By Time 20 the filling rate is zero, and the stock remains constant. Thus, unlike systems with a linearly increasing flow that exhibit parabolic behavior, systems with downward sloping flows exhibit decreasing parabolic growth.

The stock is behaving as we expected because the change in the value of the stock is equal to the area under the flow graph.

Area under flow graph = 0.5 * 20 * 20 = 200.

This is equal to the change in the value of the stock:

Final Value of Stock = Initial Value of Stock + Change in the Value of Stock

Final Value of Stock = 0 + 200 = 200.

You can also check the proper behavior of the stock by drawing in tangent lines at various points. When we draw in tangent lines on the stock graph, we see that at the beginning the tangent lines are very steep. By Time 20, when the fill rate is zero, the tangent line is flat (zero slope). To prove this, we can draw two tangent lines into the graph shown in Figure 7.

Figure 8 shows the graph of the stock, the water level of the bathtub we discussed earlier, with tangent lines at Time 5 and Time 20.

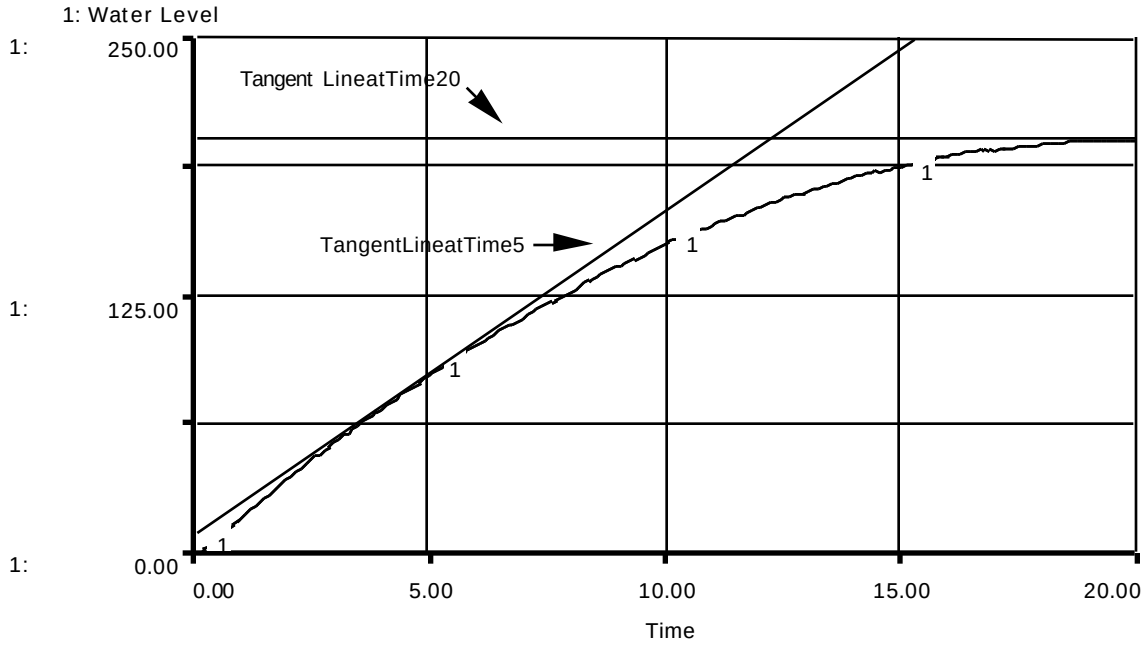


Figure 8: The result of a linearly decreasing flow filling an empty stock with the tangent lines drawn in at Time 5 and Time 20.

Let us look at another example of a linearly decreasing flow. We will again assume the initial value of the stock to be zero. Figure 9 shows the behavior of the flow and Figure 10 shows the resulting behavior of the stock.

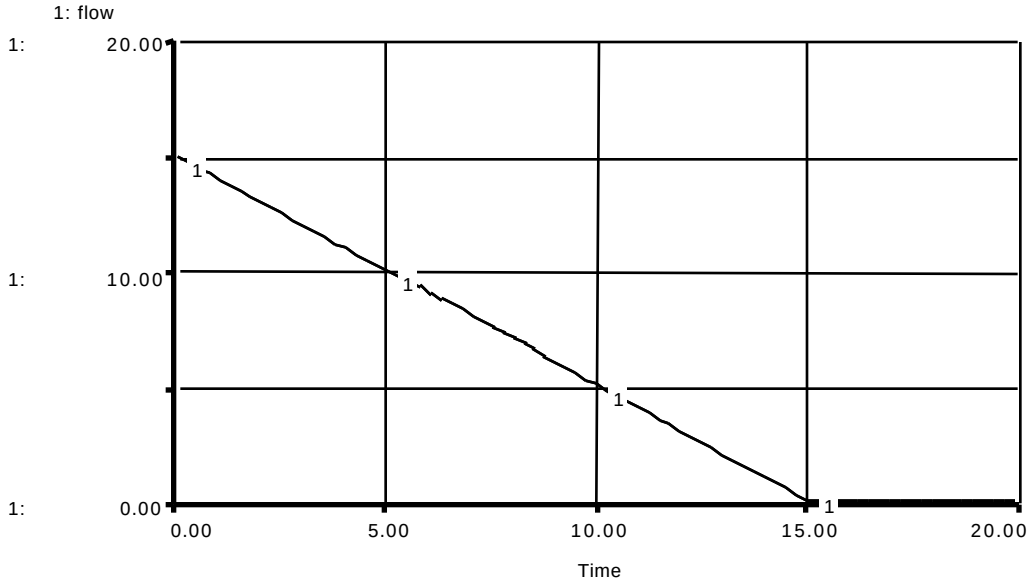


Figure 9: A linearly decreasing flow from Time 0 to Time 15.

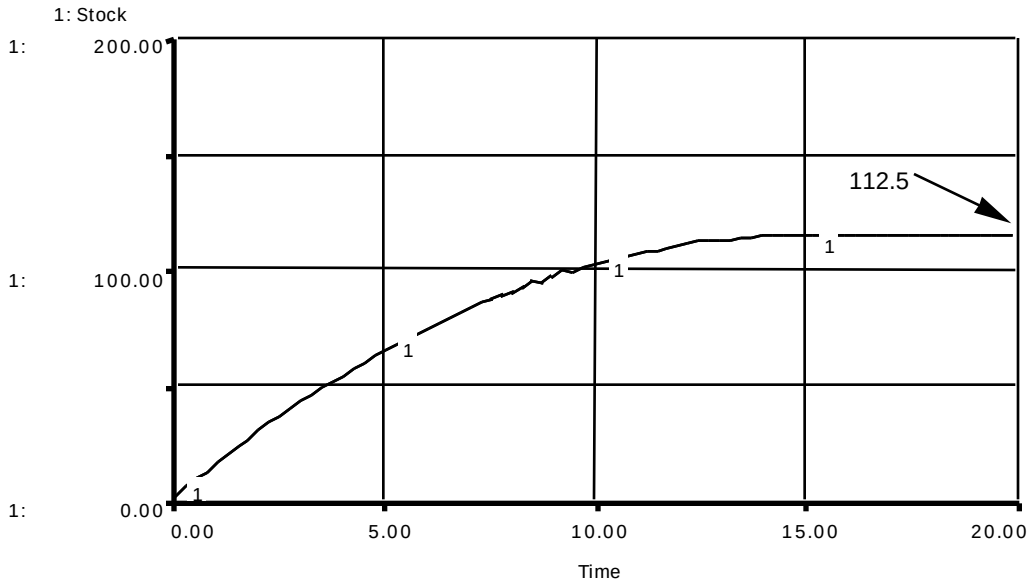


Figure 10: The stock resulting from a linearly decreasing flow from Time 0 to Time 15.

Just as we expected, as shown on Figure 10, the change in the value of the stock is equal to 112.5, which is the area under the flow graph. Also, the slope of the stock starts off steep and reaches flatness by Time 15. The flow into the stock becomes zero at Time 15, after which the level of the stock remains constant. For practice, try drawing the tangent line at Time 5. Does the slope of that line approximately equal 10?

4. Key Ideas

Now that we have seen how stocks behave when the flows are linearly increasing or decreasing, we are ready to try some exercises. Anytime you are performing graphical integration, keep the following rules in mind:

1. **When the net flow is positive, stocks are filled; when the net flow is negative, stocks are emptied.**
2. **The area under the flow graph over a period of time is equal to the change in the value of the stock over that same period of time. This helps us estimate the value of the stock if we know the initial value of the stock and the value of the flow:**
   Final Value of Stock = Initial Value of Stock + Area under flow graph
3. **The value of the flow gives the slope of the stock at that instant of time.**

4. **Constant flows cause the stock to increase linearly with the slope of the stock equal to the value of the flow.**[5]
5. **Linearly increasing flows cause the stock to exhibit parabolic growth.**
6. **Linearly decreasing flows cause the stocks to exhibit "decreasing" parabolic behavior.**
7. **You can often break up complicated graphs into several small, simpler ones.**

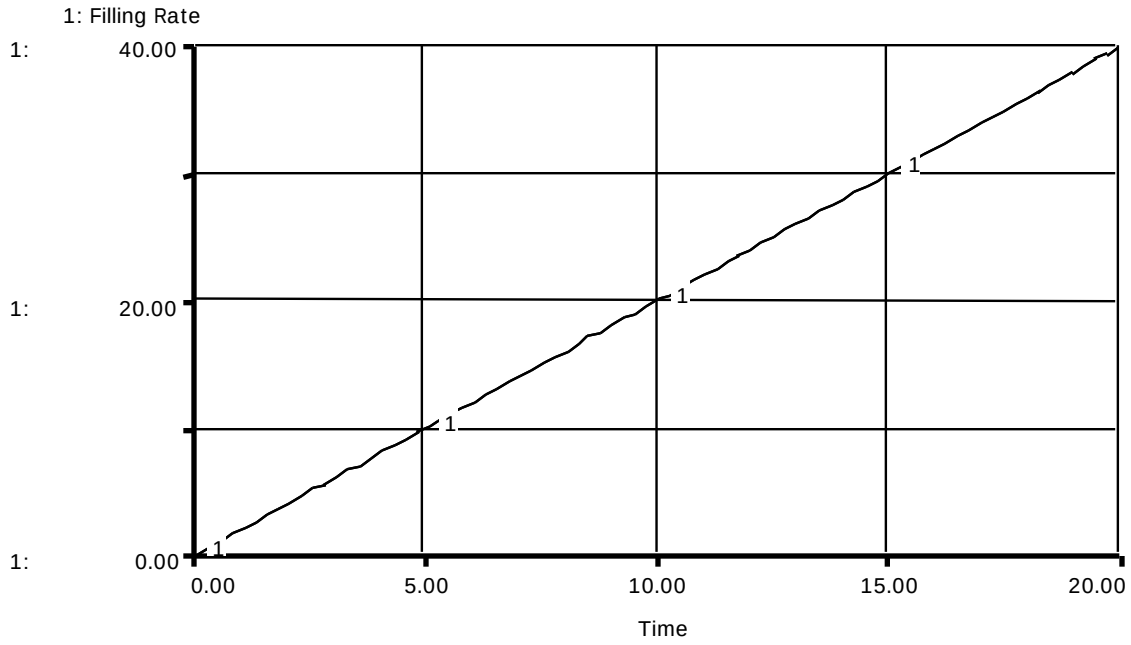
Now, let's try some exercises!!!

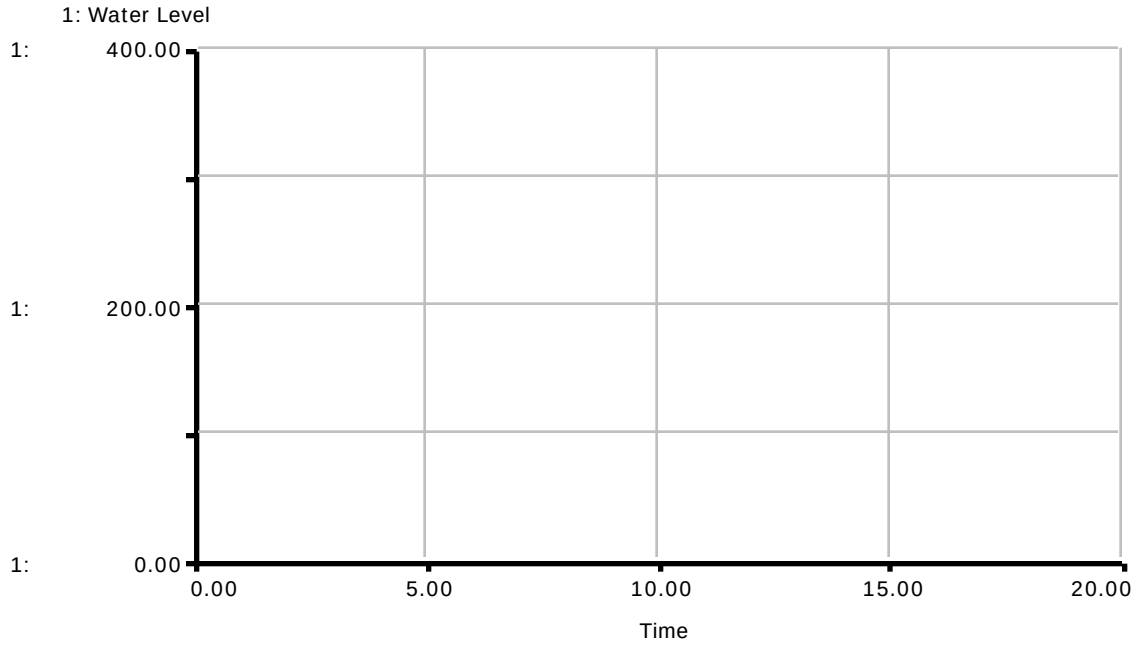
5. Exercises

5.1 Linearly Increasing Flow

In the first exercise, the flow of water filling a bathtub is initially 0 gallons per minute, and it is increasing linearly with a slope of +2 gallons per minute. Using the blank graph pad below, graph the behavior of the system. Assume the initial value of the stock is 0. (Hint: First figure out what type of behavior the stock will display, then calculate the area under the flow graph.)

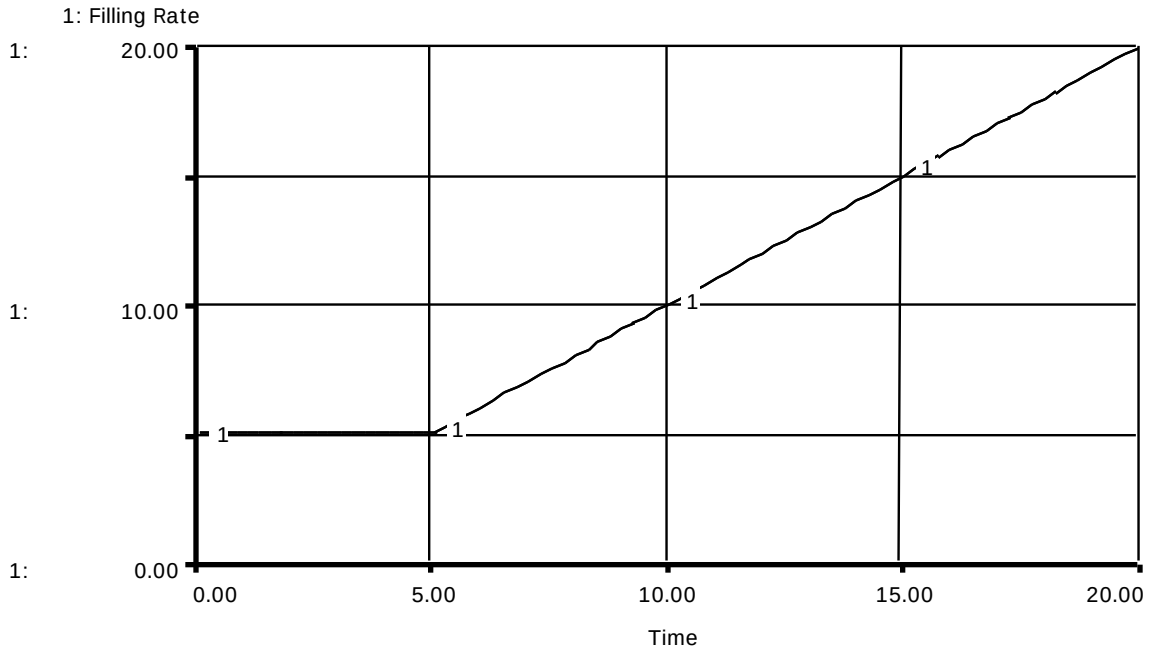
[5] If you are still unclear about how to graphically integrate systems with constant flows, please see: Alice Oh, 1995. *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Dec. 1, 19pp.

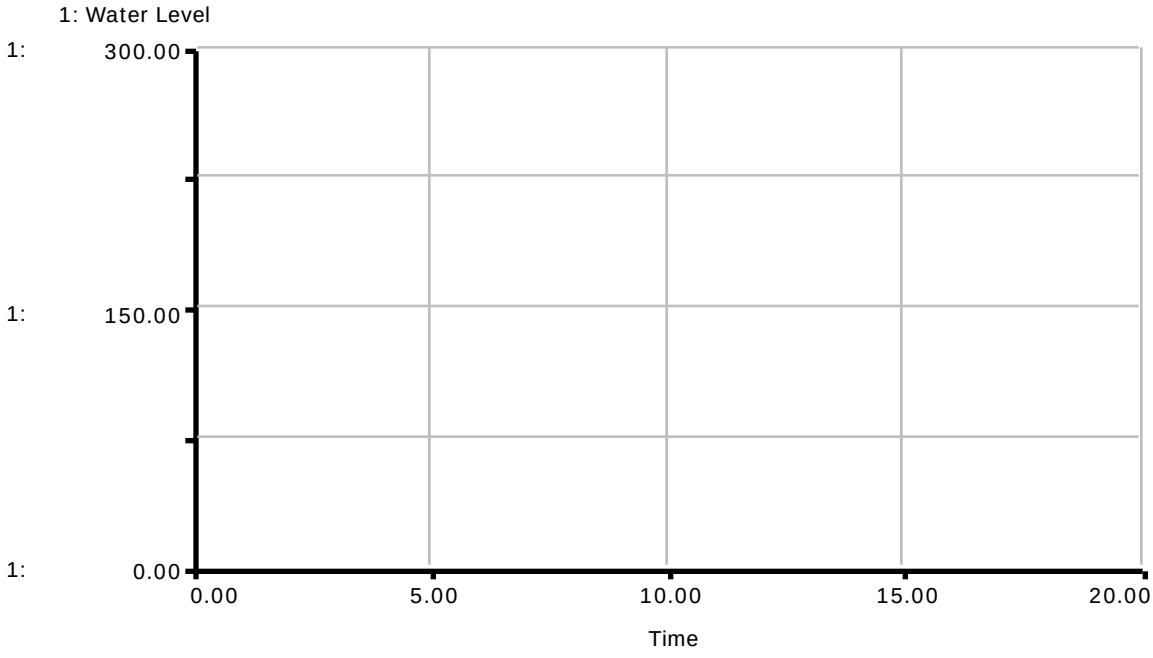




5.2 Constant and Linearly Increasing Flows

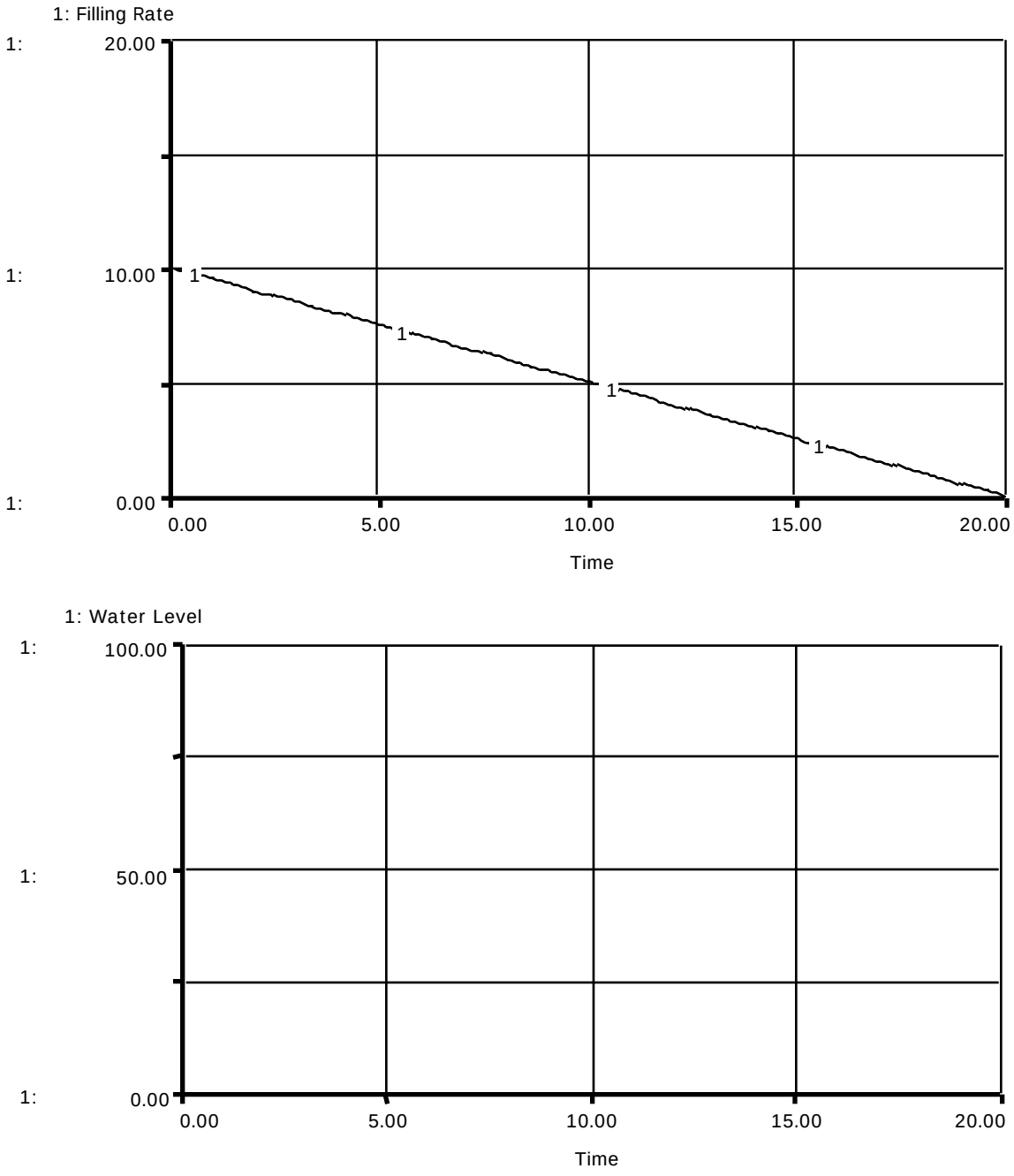
In this case, the rate of filling the bath tub is initially constant at +5 until Time 5. At Time 5, the flow increases linearly with a slope of +1. Assuming the initial value of the stock to be 75, graph its behavior below.





5.3 Linearly Decreasing Flow

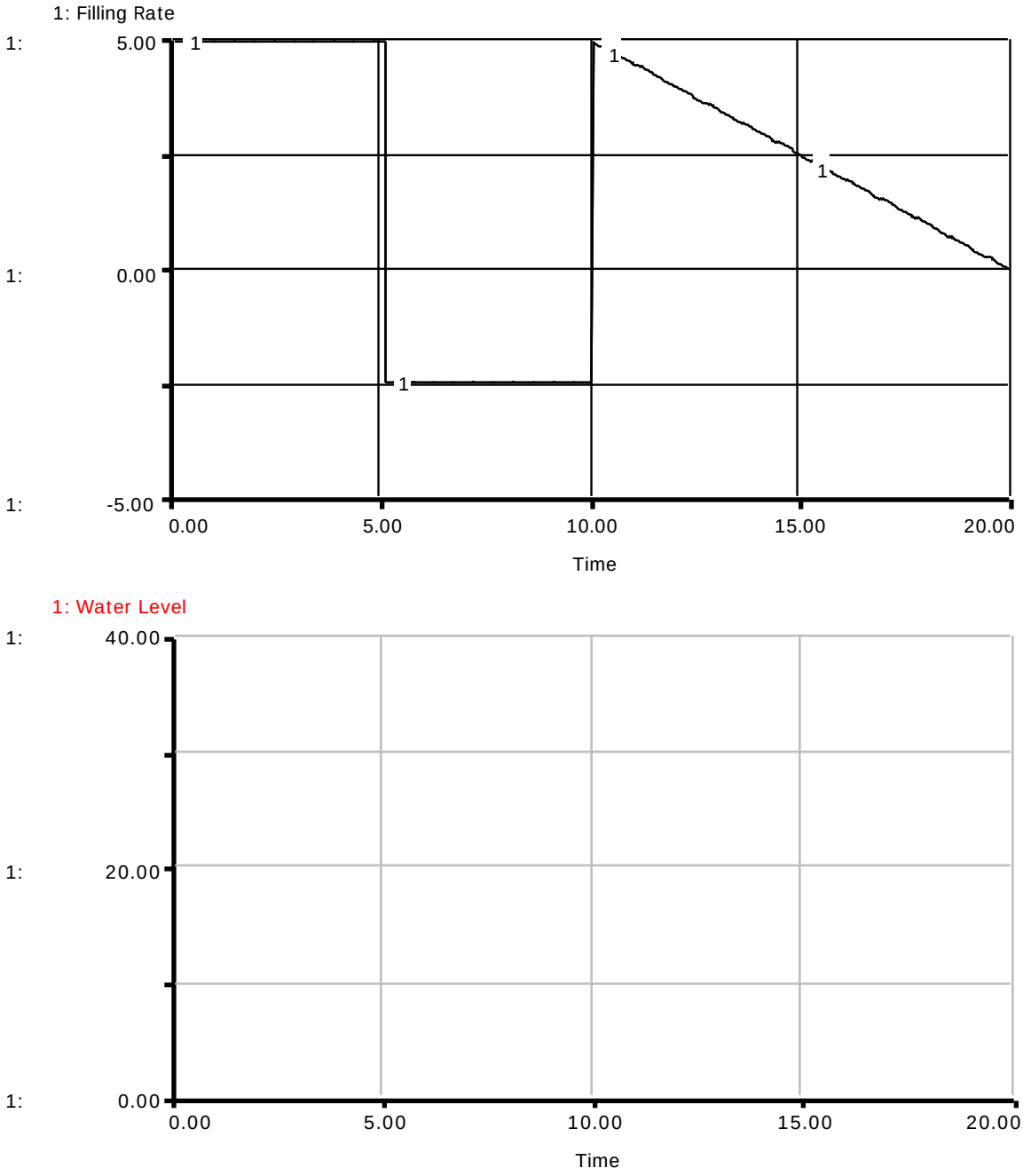
Now, let us try a linearly decreasing flow. The flow in this exercise starts at ten, and decreases to zero at Time 20. The initial value of the stock is zero.



5.4 Constant and Linearly Decreasing Flows

Now let's try a more complex problem. From Time 0 to Time 5, the flow is constant at +5. At Time 5, the flow reverses to –2.5. At Time 10, the flow again turns positive and

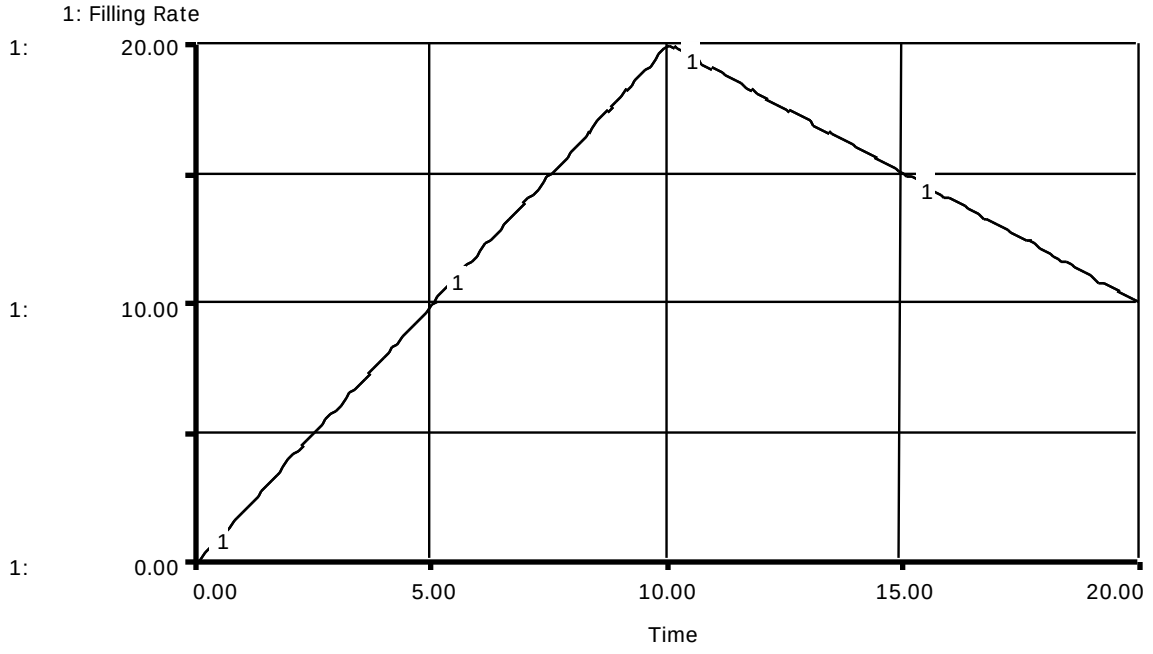
steps up to 5 and linearly decreases to zero by Time 20. Assume the initial value of the stock is zero. (Hint: Break this problem up into three smaller problems.)[6]

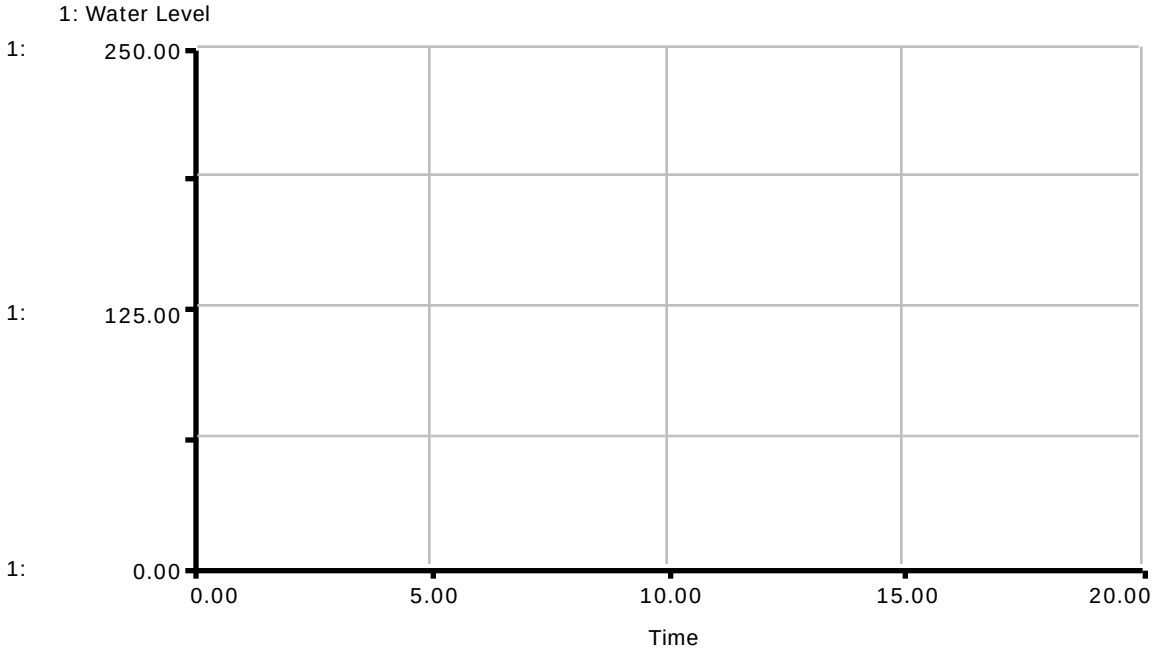


[6] If you feel uncomfortable dealing with negative flows, please review: Alice Oh, 1995. *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Dec. 1, 19pp.

5.5 Linearly Increasing and Decreasing Flows

For our last exercise, let's try both upward and downward sloping flows. From Time 0 to Time 10, the flow is increasing from 0 to 20. From Time 10 to Time 20, the flow decreases from 20 to 10. Assume the initial value of the stock is 0. Graph the behavior of the stock below.





6. Conclusion

In this paper we looked at graphical integration with linearly increasing and decreasing flows. You should now feel comfortable using graphical integration as a method for estimating the behavior of systems with constant flows, step function flows, as well as linearly increasing and decreasing flows.

In the next paper of the Graphical Integration Exercises series, you will learn how to graphically integrate systems with separate inflows and outflows. You will need the skills you've acquired in this paper for the next in the series of papers on graphical integration.

7. Appendix: Answers to Exercises

7.1 Answer to Section 5.1

The water level, which is the stock, is simply a parabolic growth curve. Notice that the area under the flow graph is:

Area under flow graph = 0.5 * 40 * 20

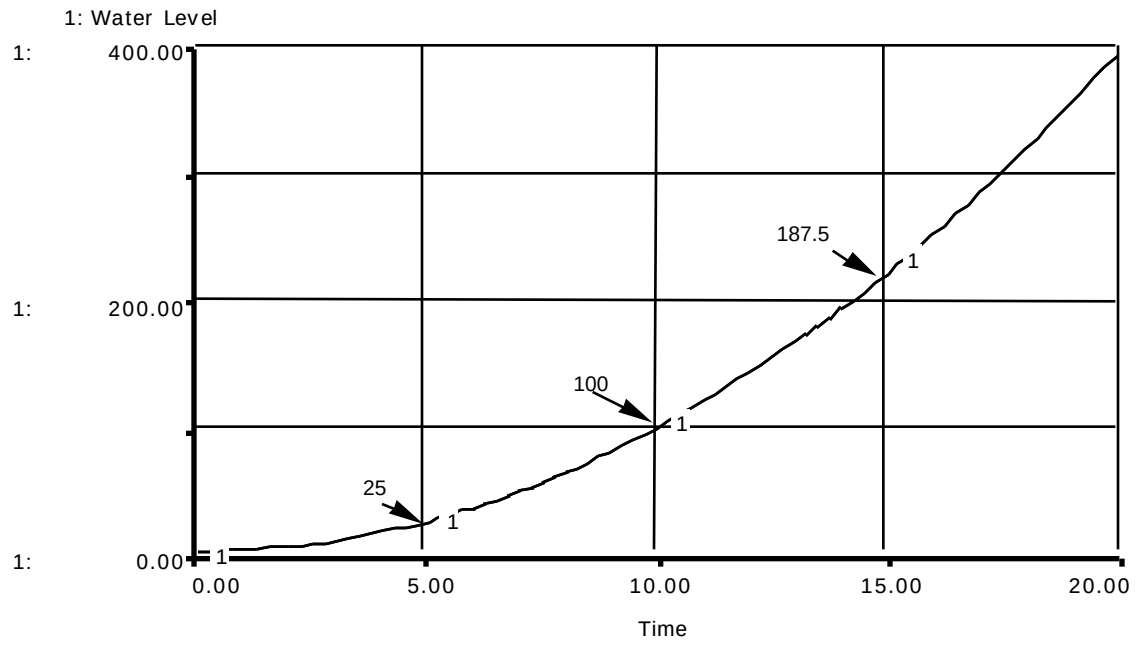
Area under flow graph = 400

This is equal to the change in the value of the stock:

Final Value of Stock = Initial Value of Stock + Change in the Value of Stock

Final Value of Stock = 0 + 400 = 400

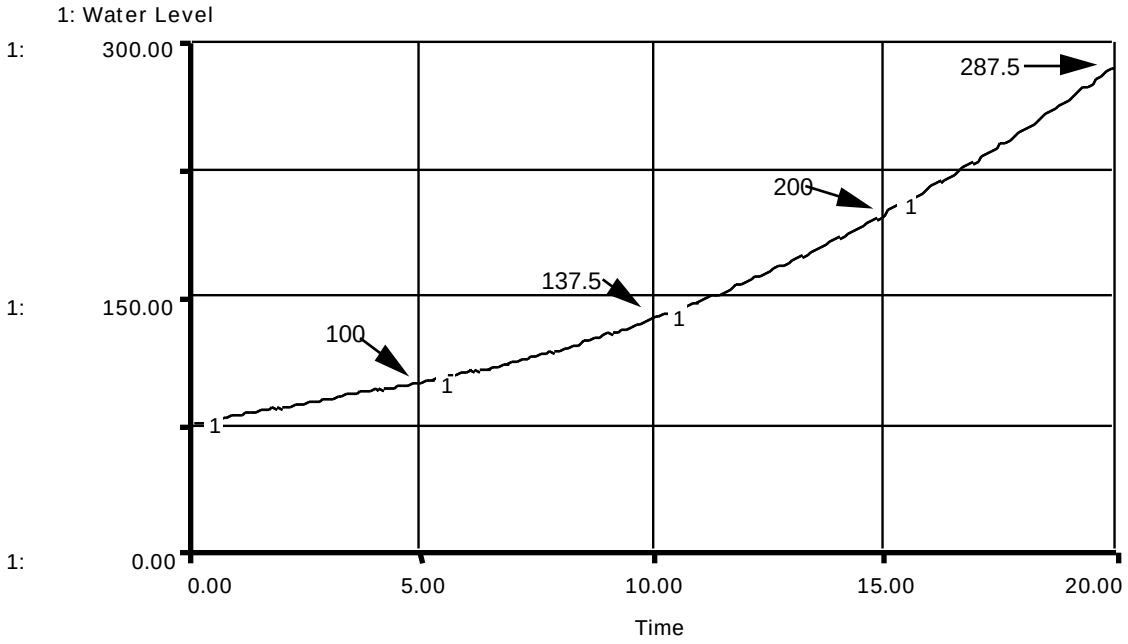
You could have also calculated the partial values of the stock by calculating the areas of the triangles and rectangles below the flow curve after each 5 units of time. Also notice that if you drew in a tangent line at Time 10 its slope would be 20, which is the filling rate at Time 10.



7.2 Answer to Section 5.2

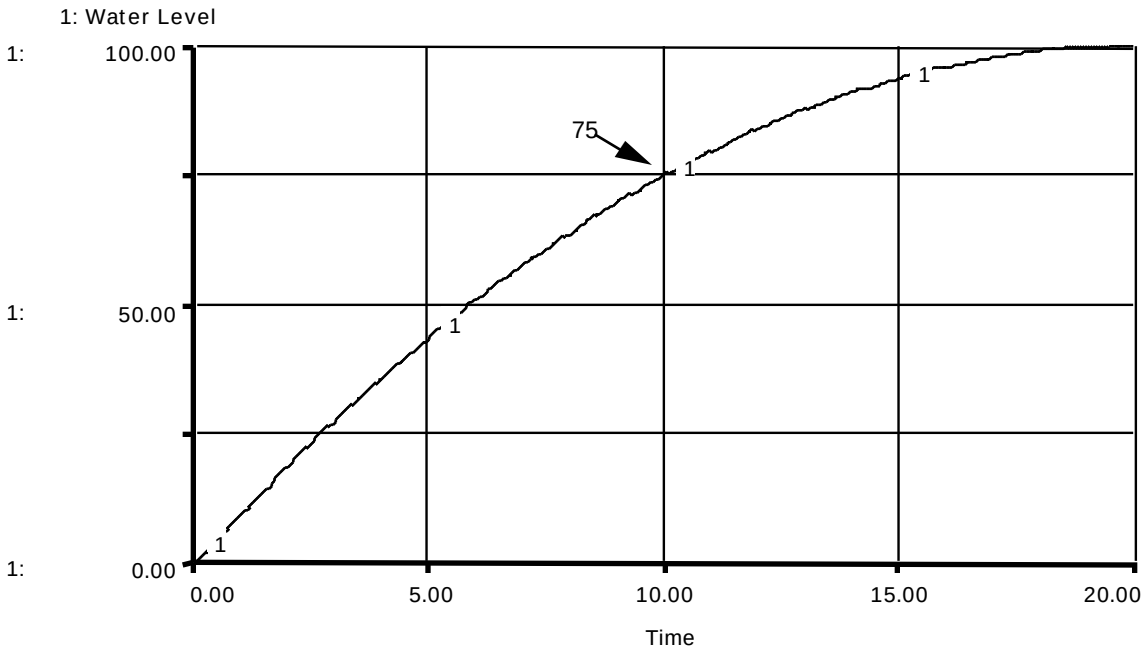
The second problem can be broken into two parts. The first part is from Time 0 to Time 5, when the flow is constant. Notice that the first part of the curve is linear, with a slope of 5, which is the constant flow value. Also notice that the area under the flow graph for this time period is 5 times 5, or 25.[7] This area is the change in the value of the stock. Because the stock was initially 75, at Time 5, the stock's new value is 75 + 25, or 100. For Time 5 to Time 20, the stock undergoes parabolic growth because of the ramping flow. To calculate the change in the level of the stock, we need to calculate the area under the flow graph from Time 5 to Time 20. This is most easily done by breaking the area under the flow graph into a rectangle and a triangle. The rectangle has a length of 15 and a height of 5. Thus its area is 75. The base of the triangle is 15, and its height is 15, so its area is 112.5. Thus the total area under the flow graph from Time 5 to Time 20 is 75 + 112.5 = 187.5. Because the stock, as we determined, was at 100 at Time 5, the value of the stock at Time 20 is 100 + 187.5, or 287.5.

[7] The area of a rectangle is simply its length times its height.



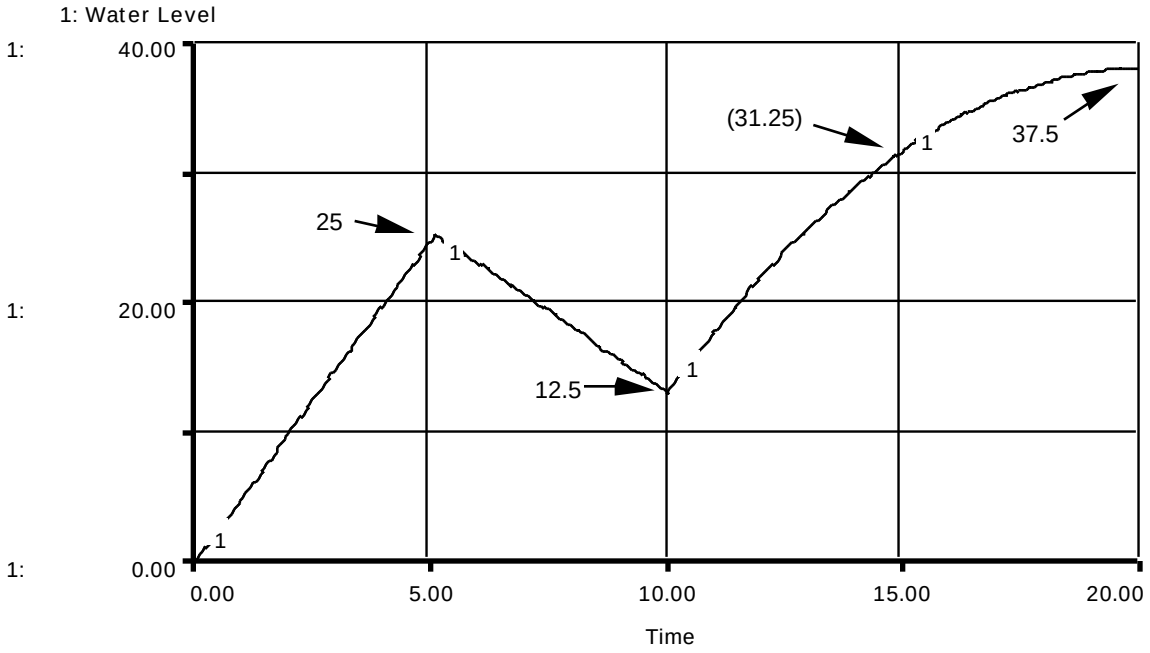
7.3 Answer to Section 5.3

Because the flow is simply a downward sloping line, the stock will undergo a downward parabolic behavior. Initially it is increasing very rapidly. As time continues, the value of increase gets smaller and smaller, equaling zero at Time 20. The area under the flow graph is 0.5 times 20 times 10, which equals 100. Because the stock was initially empty, the final value of the stock is 0 + 100, or 100.



7.4 Answer to Section 5.4

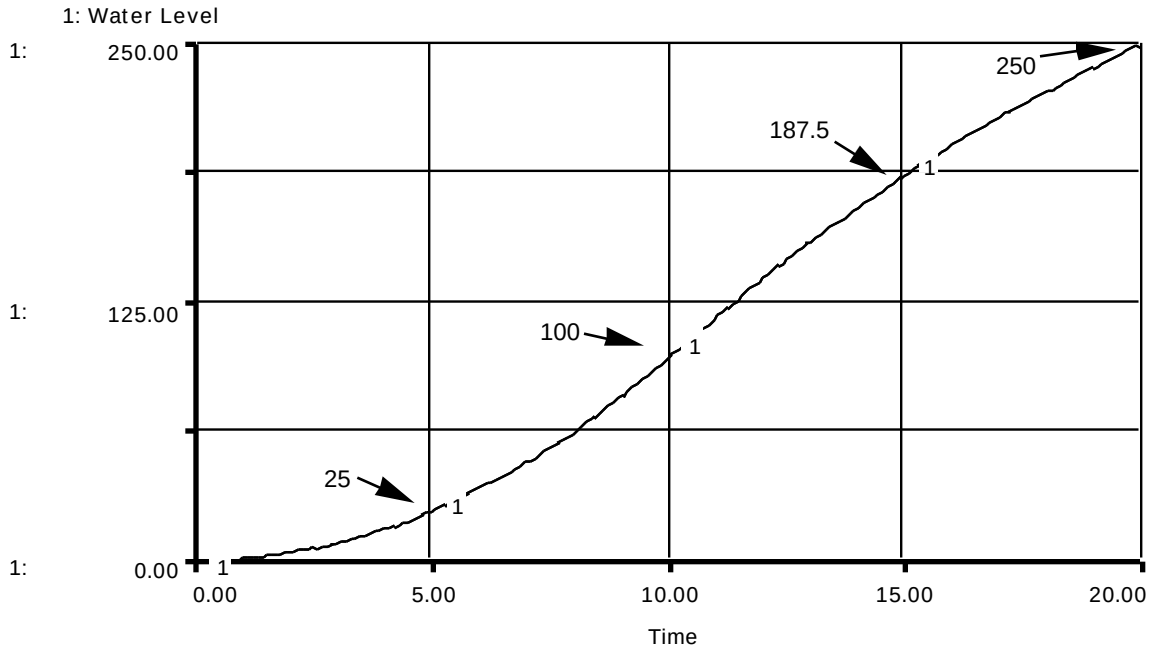
To solve this problem, it is easiest to break it up into three parts: from Time 0 to Time 5, from Time 5 to Time 10, and from Time 10 to Time 20. For the first part, from Time 0 to Time 5, the stock increases linearly because the flow is a constant positive number. To calculate the value of the stock at Time 5, we only need to find the area under the flow graph from Time 0 to Time 5. This area is simply 5 times 5, or 25. Because the initial value of the stock was zero, the value of the stock at Time 5 is 25. For Time 5 to Time 10, the stock decreases linearly because the constant flow is negative. Again, calculating the area between the flow graph and the zero flow line, we see that the stock decreased by 12.5, yielding a value of the stock at Time 10 to be 25 - 12.5, or 12.5. For the third part, the stock displays a downward parabolic behavior, because the flow is ramping downwards. The area under the flow graph is 0.5 times 10 times 5, or 25. Thus, the value of the stock at Time 20 is the value of the stock at Time 10 plus 25. Because the value of the stock at Time 10 was 12.5, the value of the stock at Time 20 is 12.5 + 25 = 37.5.



7.5 Answer to Section 5.5

Again, it is easiest to answer the last problem by separating it into two parts. The first part, where the flow is ramping upwards, displays parabolic growth. The value of the stock at Time 10 is calculated by finding the area under the flow graph, which is 0.5

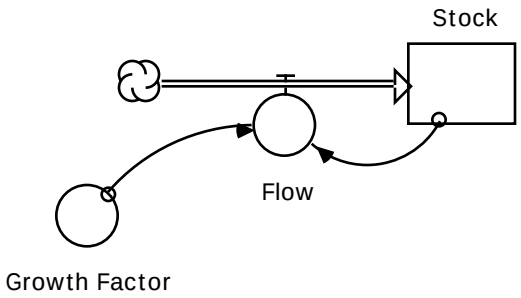
times 10 times 20. This equals 100. The second part, which begins at Time 10, displays a downward parabolic behavior because the flow is sloping downward. The area under the flow graph for this period can be broken up into the area of the rectangle with a length of 10 and a height of 10, plus the area of the triangle with a base of 10 and a height of 10. The sum of these two areas is 100 + 50 = 150. Because the total area under the flow graph is the change in the value of the stock, the value of the stock at Time 20 is 100 (the value of the stock at Time 10) plus 150. Therefore, at Time 20, the value of the stock is 250.



Beginner Modeling Exercises

Section 2

Mental Simulation of Simple Positive Feedback



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Joseph G. Whelan
March 25, 1996

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Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems is the cause of nearly all dynamic behavior. To successfully use system dynamics as a learning tool you must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on your computer[1]. Computer simulation is a very useful tool for exploring systems. However, it is also crucial that you be able to use the other simulation tool of system dynamics: *Mental Simulation.* A strong set of mental simulation skills will enhance your ability to validate, debug and understand dynamic systems and models.

This paper begins with a review of some key concepts of simple feedback systems. A set of exercises is included to help reinforce your understanding of the feedback dynamics in a simple positive feedback loop. Solutions to the exercises are included in the appendix.

[1]There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road maps is geared towards the use of STELLA II which is available from High Performance Systems. (603) 643-9636

Positive Feedback

One of the simplest feedback systems is the positive feedback loop. Positive feedback can be likened to a snowball rolling down a hill. As the ball rolls down the hill, it collects snow. The bigger the snowball gets, the more snow it collects and the faster it grows. Positive feedback occurs when change propagates through a system to produce *more* change in the *same* direction. This is the type of feedback that produces growth.

You see positive feedback every day in the world around you. Everyone knows about the ability of rabbits to multiply at an alarming rate. This is a good example of positive feedback. Every time a new pair of rabbits is born, they contribute to the reproductive capabilities of the total population. As the population grows, so does the rate at which rabbits are born; making the population grow even faster.

Another everyday example of positive feedback is a savings account. Let's say you deposit $10 into a savings account that earns 10% per year (It's just an example ☺) After 1 year, you would have $11 in the bank. In the first year, you earned $1.00 of interest. However, in the second year, your interest earnings are 10% of $11, or $1.10 leaving you with $12.10 after the second year. Each year as your bank account grows, so do the interest payments, causing your bank account to grow even faster. This simple system can be modeled with a STELLA model as shown in Figure 1.

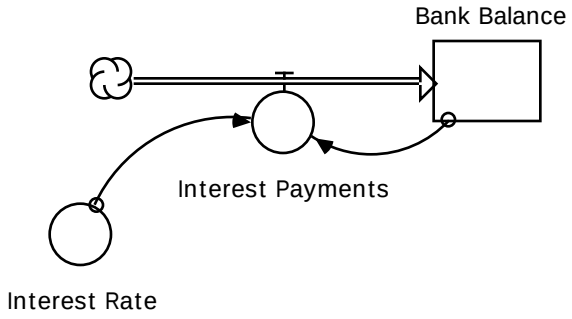


Figure 1: STELLA model of a savings account

The *interest payments* each year are equal to the *bank balance* multiplied by the *interest rate*.

Table 1 below shows the first four years of simulation of the savings account model.

Years	Bank Balance	Interest Payments	Interest Rate
1990	$10.00	1.00	0.10
1991	$11.00	1.10	0.10
1992	$12.10	1.21	0.10
1993	$13.31	1.33	0.10
1994	$14.64	1.46	0.10

Table 1: Model run for savings account

Computing the values of model elements by hand can be a very tedious process and is not the only way to obtain the behavior of a model. If you need a precise result for the behavior of a model, you should build it on the computer and run the simulation. However, often you will only need to know the general behavior pattern of a model, or how the behavior would change if the model were altered. For this, there are many mental simulation tools that will allow you to predict the behavior of a model without ever touching a computer. In this paper, you will learn some of the basics using the savings account model described above.

The first step in predicting the behavior is determining a starting point. Often, you are given the initial value(s) for the stock(s) in a model. If not, you should pick a reasonable value. If you are dealing with a model of a rabbit population you might start with 100 rabbits. If you were dealing with a model of the national debt, you would pick a much larger number, say a few trillion. You can calculate the beginning values for all the flows by using the initial values of the stocks.

In the savings account model, we are given that the initial value for the *bank balance* stock is $10. Since the equation for *interest payments* is:

Interest Payments = Bank Balance Interest Rate

We can calculate that the first value for the flow *interest payments* is $1.00.

We can take the information we have and begin to plot the graph of the model behavior as shown in Figure 2. Recall that the slope of a stock is equal to the net inflow. This means that the initial slope of the *bank balance* stock is equal to the first value of *interest payments*, or 1.

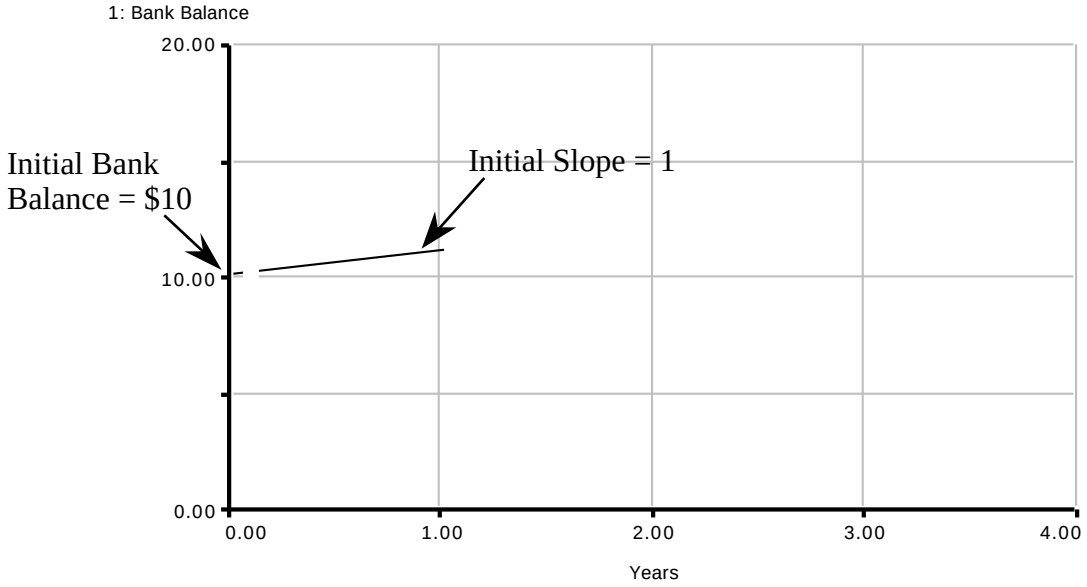


Figure 2: Initial bank balance behavior

Now that we know how the model starts to behave we need to figure out where it goes. We know that the bank balance model is an example of positive feedback, so the stock will continue to increase. But, what would happen if we didn't know that the model exhibited positive feedback? How could you tell? Here's a good test:

Imagine that you came upon a simple system and you wanted to know if it exhibited positive feedback. You know that positive feedback occurs when change propagates throughout a system to produce more change in the same direction. So, let's test the system and see if that happens. Assume that the bank account has $10 in it, so the interest payment for that year is $1. Now pretend that you deposited $5 into the account (a change in the system). What happens? That deposit will cause the interest payments to increase (propagate through the system) from $1 to $1.50. Now, the bank balance will start to increase at a faster rate because the interest payments are larger. So, a change propagated through the system and produced more change in the same direction. A positive feedback loop!

So far, we have established the starting point for the model and the fact that it contains a positive feedback loop. The characteristic behavior of a positive feedback loop is *exponential growth*. Exponential growth of a stock is characterized by the fact that the stock has a constant *doubling time*. This means that if the account balance doubled from $10 to $20 in 5 years it would double from $20 to $40 in the next five years and from $40 to $80 in the five years after that.

The next question to ask is, "How quickly will the growth occur?" The doubling time for a simple positive feedback loop can be approximated as follows:

$$\textbf{Doubling time} = \frac{\textbf{0.7}}{\textbf{interest rate}}$$

The doubling time for this model is:

$$\textbf{Doubling time} = \frac{\textbf{0.7}}{\textbf{.10}} = \textbf{7 years}$$

The graph in Figure 3 shows the exponential growth generated by the savings account model.

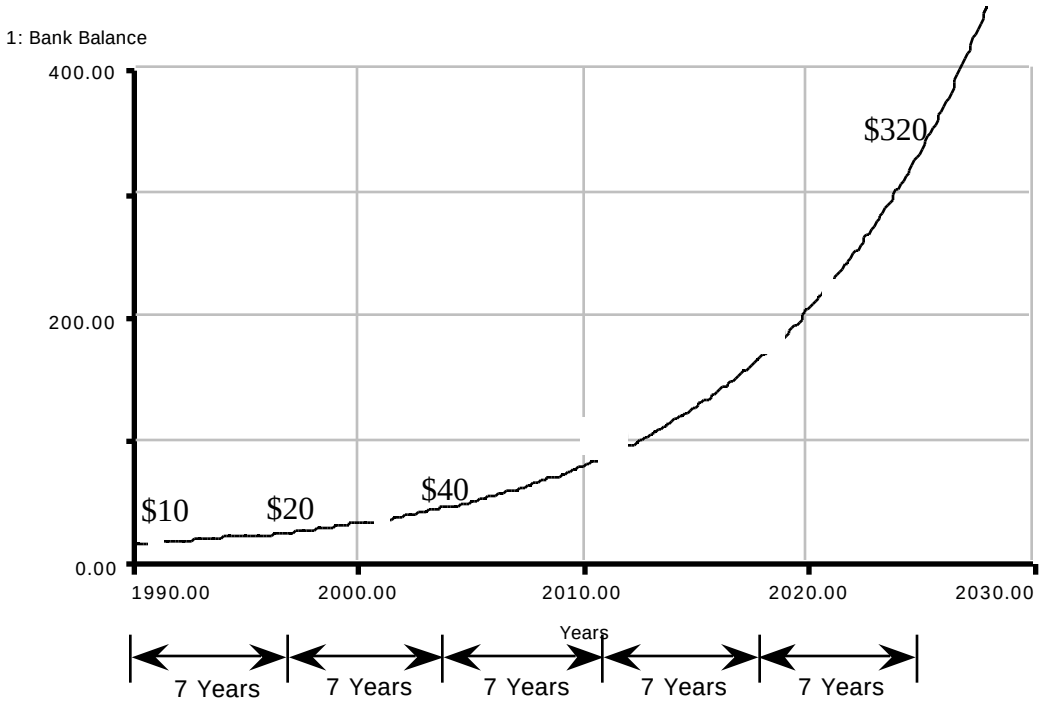


Figure 3: Behavior graph for the Bank Balance Model

The same equation for doubling time can be used for other systems as well. If the rate equation is formulated as follows:

$$\textbf{Inflow} = \textbf{Stock} \cdot \textbf{growth factor}$$

Then, simply substitute the growth factor for the interest rate in the doubling time equation.

There is one more thing about positive feedback that must be covered before going on to the exercises: Positive feedback cannot continue forever. There is *always* a factor or factors in any system that will limit the growth of its elements. In a population system there are limited food and water resources, the snowball will eventually reach the bottom of the hill, and your bank would probably fold sometime before your savings account began rivaling the size of the national debt. The presence of limiting factors in any system containing positive feedback is known as *limits to growth.*

Review:

Below is a brief list of the concepts you have learned so far. If you feel unclear about any of the topics mentioned below, you may wish to refer to the appropriate section of the paper before proceeding to the exploration exercises which follow.

1. A starting point:
   - Initial values of stocks
   - Initial values of flows
2. Where does is go from there:
   - identifying feedback
   - positive feedback
3. How does the stock grow:
   - exponential growth
4. Doubling time

The exercises which follow will help to review the skills you have learned and give you practice in using them. The solutions to the exercises begin on page 12.



Exploration:

#1: All growth processes have some kind of underlying positive feedback. Name some examples of positive feedback in the world around you:

_____

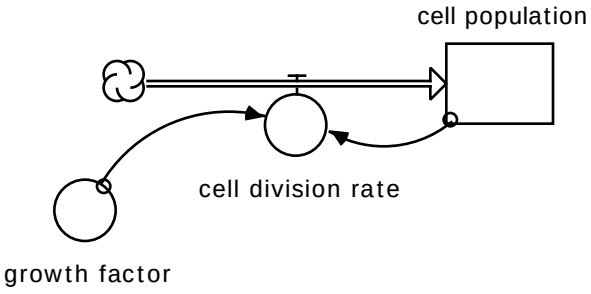
_____

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_____

#2: A friend approaches you with the following model of a yeast cell population. He is new to system dynamics and wants you to check for him if this model will produce positive feedback as he believes it will. Using your knowledge of positive feedback, verify or refute your friend's guess.



Model Equations:

growth factor = 0.2 (UNITS = 1/days)

cell division rate =
cell population * growth factor

Initial cell population = 40 cells

Is your friend's model an example of positive feedback? _____

Why or why not? _____

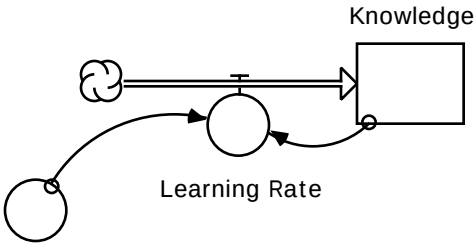
_____

_____

_____

_____

#3: The figure below shows a simple model for the growth of knowledge in a developing civilization. The rate of learning is proportional to the current level of knowledge.



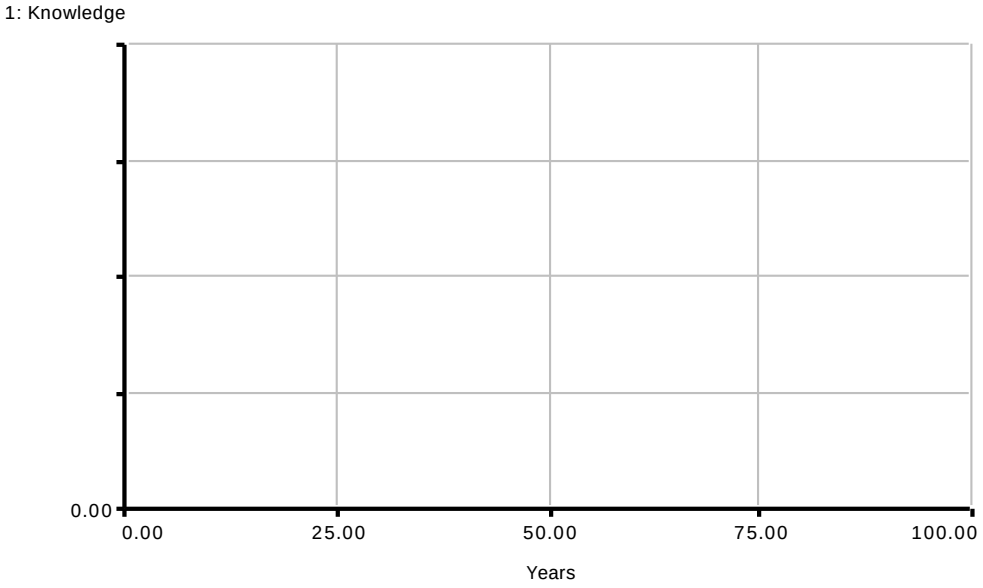
Model Equations

Ability to Learn = .02
Learning Rate =
Knowledge □Ability to Learn
INIT Knowlege = 1

What is the doubling time for the model? (doubling time = 0.7/growth factor) _____

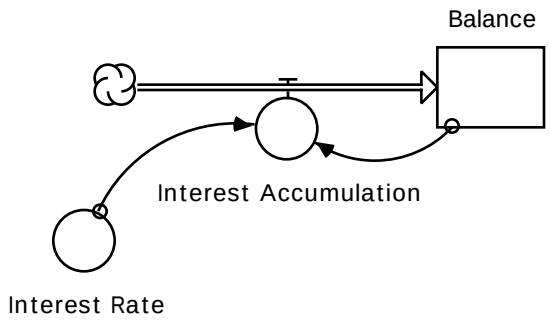
Use the doubling time to find the value of the stock after 100 years. Now, set an appropriate scale on the graph.

On the graph below, draw the behavior of the stock *Knowledge* for 100 years.



#4: The savings account model used in this paper can also be used to model debt. The only difference is that instead of the *balance* being positive, it takes on negative values, representing your debt. The *interest payments* now represent accumulating interest charges and the *interest rate* represents the interest rate you are charged for your debt. (Note that an inflow with a negative value is the same as an outflow.)

We will use this model to explore the accumulation of debt on a credit card. Assume that you owe $100 to a credit card that charges 18% annually. What would happen to the amount that you owed if you didn't make any payments for 10 years. (Not a good idea, but it is possible.)



Verify that this new use of the model is still a positive feedback loop.

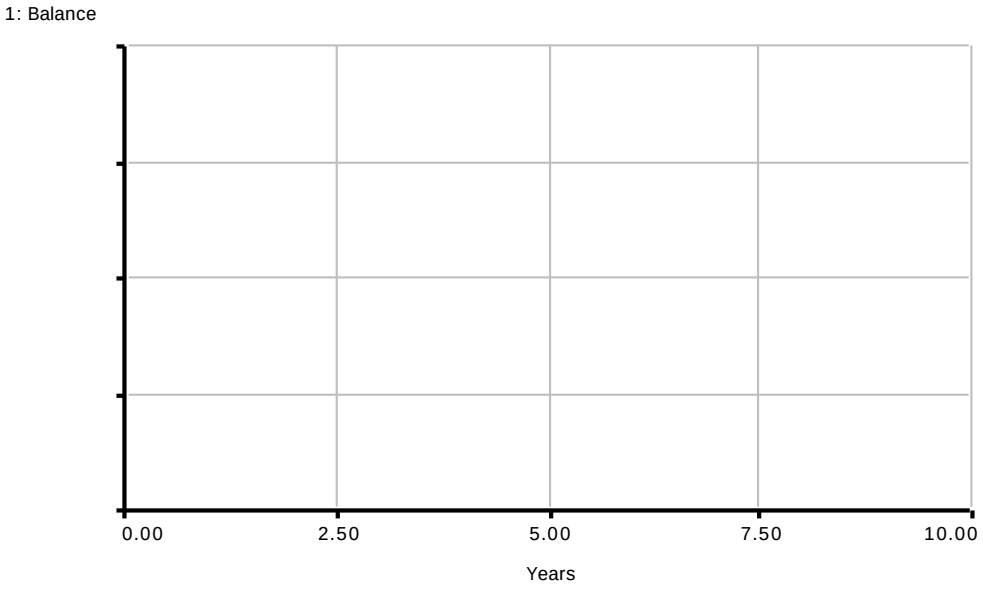
Assume that your owe $100, so the initial value for the *Balance* is -$100. If the interest rate is 18%, what is the first value for *interest accumulation?* _____

Will the *balance* increase or decrease? _____

Will your debt increase or decrease? _____

What is the doubling time for this scenario? _____

Draw on the graph below what you think the behavior of the *balance* will be over the next 10 years.



#5: What would happen if the bank *balance* were started at zero? (i.e., you opened up a savings account, but didn't deposit any money into it.) _____

_____

_____

_____

_____

Solutions:

#1: There are countless examples of positive feedback in the world around us. Here are just a few:

- cell division — As the population grows, so does the number of cell divisions per hour.
- expansion of a population — Similar to the cell population. The larger the population, the greater the number of births per year.
- spread of a rumor — As the number of people that know the rumor increases, more people are able to spread the rumor.
- spread of a disease — This feedback loop works the same way as the spread of a rumor.
- growth of a company — As the size of a company increases, the funding available for expansion also increases allowing it to grow at an increasing rate.
- spread of a forest fire — A forest fire's ability to ignite the land around it increases with the size of the fire.

Recall the definition given for positive feedback: Positive feedback exists in a system when a change propagates through the system to produce more change in the same direction.

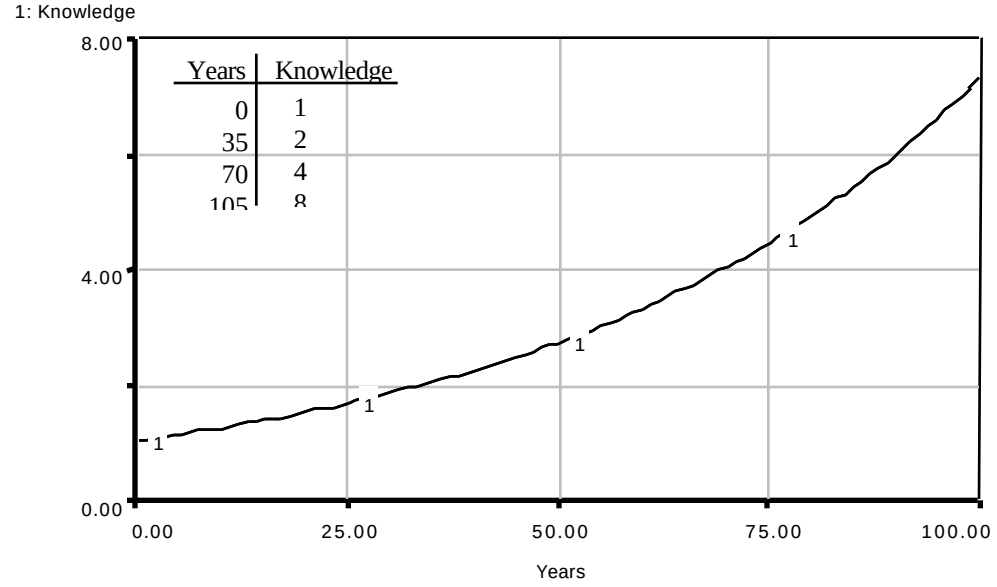
#2: Your friend is correct. This model will produce positive feedback. In fact, this model has exactly the same structure as the bank balance model. The only differences are the names of the model elements and the values of the *growth factor* parameter and the initial value of the *cell population.*

#3: The doubling time is calculated as shown:

$$\textbf{doubling time} = \frac{.7}{\textbf{growth factor}} = \frac{0.7}{.02} = \textbf{35 years}$$

100 years is slightly less than 3 doubling times of growth. So, after 100 years the stock *Knowledge* is approximately equal to 1*2*2*2 = 8. So an appropriate scale to choose is one that goes from 0 to 8. You can use the doubling time to find a few more points on the graph and then draw in the curve by hand.

The behavior of the learning model over a 100 year period is shown below.



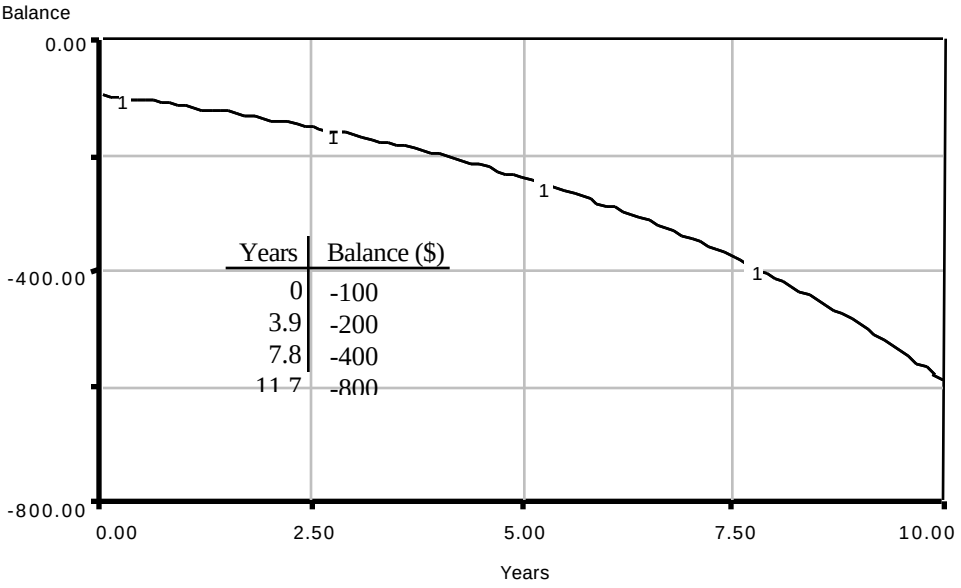
#4: This system contains a positive feedback loop because as the *balance* becomes more negative, so does the rate of *interest accumulation* causing the *balance* to decrease even faster which results in even higher *interest accumulation*.

Assuming you owe $100 and you are paying an *interest rate* of 18%, the first year's worth of *interest accumulation* is $18. After 1 year, the *bank balance* will decrease to -$118, making your debt equal to $118. The balance will continue to decrease exponentially. The doubling time is calculated as follows:

$$\textbf{doubling time} = \frac{.7}{\textbf{growth factor}} = \frac{\mathbf{0.7}}{\mathbf{.18}} = \mathbf{3.9\ years}$$

Since 10 years is between 2 and 3 doubling times, the scale on the graph can be set to -$800: the balance after 3 doubling times. The doubling time can then be used to plot a few more points to complete the curve.

The graph below shows the behavior of the *balance* over a 10 year period.



#6: If the bank balance were to start at zero, this would mean that the value of *interest payments* would also be zero. Since the only flow that affects the *bank balance* stock is equal to zero, the stock will never change and neither will your *bank balance.* This state is called equilibrium. It is a special kind of equilibrium called *unstable equilibrium*. The reason for this name is because a small change in the stock to either increase or decrease it will initiate exponential behavior. This can be likened to a ball balanced on top of a pin. If the ball is exactly centered over the pin, it will not move, but any disturbance at all will cause it to fall all the way off.

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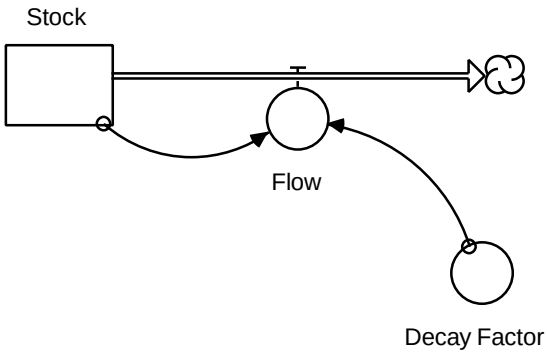
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Beginner Modeling Exercises

Section 3

Mental Simulation of Simple Negative Feedback



Prepared for
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Helen Zhu
Vensim Examples added October 2001

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Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on your computer[1]. Computer simulation is a very useful tool for exploring systems. However, one should be able to use the other simulation tool of system dynamics: *mental simulation.* A strong set of mental simulation skills will enhance ability to validate, debug and understand dynamic systems and models.

This paper deals primarily with negative feedback and begins with a review of some key concepts. A set of exercises at the end will help reinforce understanding of the feedback dynamics in a simple negative feedback loop. Solutions to these exercises are also included. It is assumed that the reader has already studied "Beginner Modeling Exercises: Mental Simulation of Positive Feedback,"[2] and is familiar with fundamental system dynamics terms.

Negative Feedback

Compare negative feedback to letting air out of a balloon. At first, air pressure inside the expanded balloon pushes air out at a high rate, allowing the balloon to deflate. As air escapes, the balloon gets smaller, the air pressure dies down, and the deflation rate drops. This continues until deflation stops completely. Negative feedback occurs when change in a system produces less and less change in the same direction until a goal is reached. In this circumstance, the goal is equal air pressure inside and outside the balloon.

---

[1]There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road Maps is geared towards the use of STELLA II which is available from High Performance Systems (603) 643-9636. Road Maps can be accessed through the internet at http:/sysdyn.mit.edu/.

[2] "Beginner Modeling Exercises: Mental Simulation of Positive Feedback" (D-4487) by Joseph G. Whelan, is part of the Road Maps series.

Systems exhibiting negative feedback are present everywhere, ranging from a population facing extinction to the simple thermostat. In each circumstance, negative feedback exhibits goal-seeking behavior. In other words, the difference between the current state of the system and the desired state causes the system to move towards the desired state. The closer the state of the system is to its goal, the smaller the rate of change, until the system reaches its goal. This goal can differ from system to system. In a population extinction model, the goal is zero — animals continue dying until there are none left. A thermostat, on the other hand, has a goal of a desired room temperature.

Example 1: Rainfall on the sidewalk (Goal = 0)

This section focuses on how to simulate and visualize stock behavior mentally over time for a zero-goal system. The first step explores initial behavior.

The system and its initial behavior

An example of negative feedback is the amount of wet sidewalk from rain. Initially, a 10 square foot block of sidewalk is completely dry. Rain begins to fall at a constant rate. Within the first minute, rain covers 50% of the block surface area, or 5 square feet total. In the next minute, the same amount of rain falls, but some of the raindrops fall on areas that are already wet. The chance of rain falling on a dry spot is one out of two, or 50%. Rainfall covers five square feet in the second minute, but 50% of that is already wet, so only an additional 2.5 square feet becomes wet.

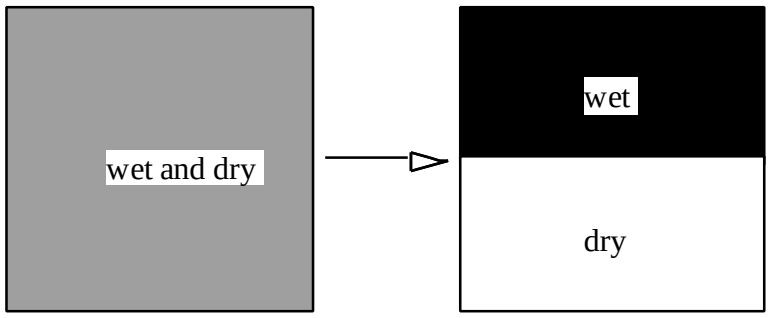


Figure 1: Rain falls on sidewalk, first minute

Although rainfall is evenly distributed in "wet and dry," it quantitatively covers half of the surface area of the block. It is easier to visualize this, for modeling purposes, as its equivalent of one-half wet and one-half dry. Figure 1 above shows both the natural rainfall pattern and its equivalent schematic representation.

Rain continues to fall. Half of it lands on places where the sidewalk is already wet. The other half covers 50% of the remaining dry areas.

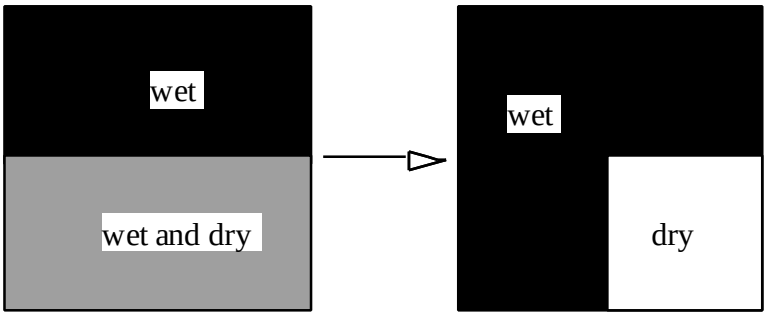


Figure 2: Sidewalk, minute two

Net rainfall is the equivalent of 75% wet, 25% dry. The same process of 50% of the remaining dry surface becoming wet continues, advancing towards covering the entire surface area.

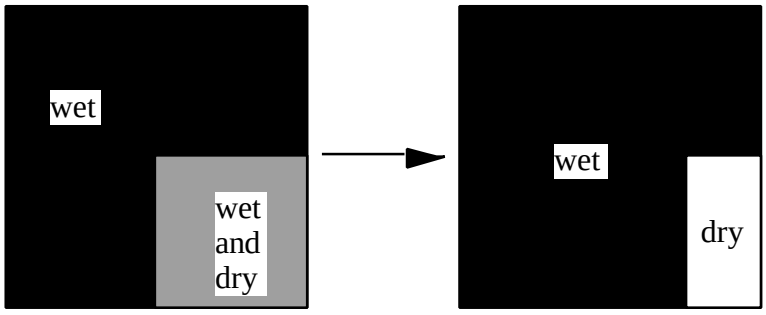


Figure 3: Sidewalk, minute three

Given this description of the rainfall system, we can model it in STELLA as shown in the figure below.

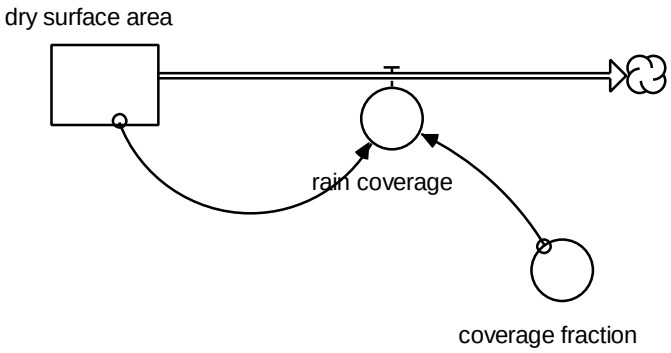


Figure 4: STELLA model of rainfall system

rain coverage = dry surface area * coverage fraction
[square feet/min] [square feet] [/minute]

We know that:

initial *dry surface area* = 10 square feet
coverage fraction = 1/2 /minute

Thus, initial *rain coverage* = 10 * 1/2 = 5 square feet/minute. In other words, as Figure 5 shows, the initial slope of the *dry surface area* is -5 square feet per minute. This negative term indicates that dry *surface area* decreases by *rain coverage* rate.

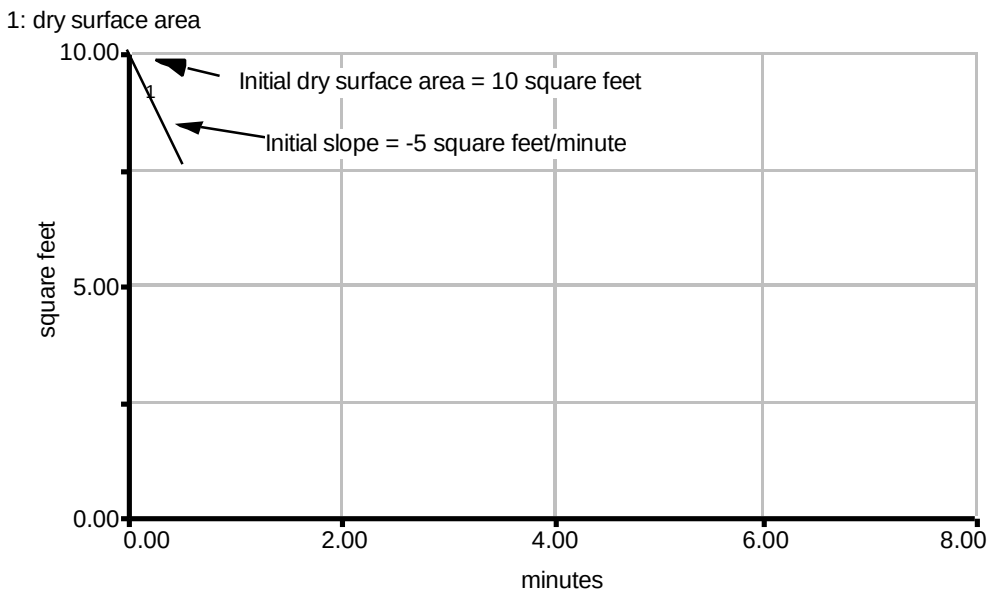


Figure 5: Behavior plot of rainfall system

What happens past the initial point? Eventually rain will cover the entire sidewalk. The system moves toward the goal of zero *dry surface area*. In order to mentally simulate the quantitative behavior of the system over time, one should perform half-life calculations.

Half-life Calculations for rainfall system

Negative feedback systems exhibit asymptotic behavior. Asymptotic decay of a stock has a constant halving time, or half-life, of the stock. Half-life is a measure of the rate of decay. It is the time interval during which distance to the goal decreases by a fraction of one-half. Half-life is the counterpart to the doubling time in positive feedback, and is likewise independent of initial stock value. Approximate the half life by:

Half-life = 0.7 * time constant

where **time constant = 1/ decay fraction**

Is *coverage fraction* a time constant or decay fraction? Both terms refer to a constant value, one being the reciprocal of the other, and vice versa. They differ only in how they are used. In a rate equation, the stock is either multiplied by a decay fraction or

divided by a time constant[3]. In this case, we multiply *coverage fraction* and *dry surface area* to get *rain coverage*, so *coverage fraction* is a decay fraction. The half-life is:

Half-life = 0.7 * (1/(1/2)) = 1.4 minutes

This means that the *dry surface area* halves every 1.4 minutes. Knowing the half-life value, it is easy to plot the curve of the stock by connecting the points representing the value of the stock at each half-life. Figure 6 shows this, in graph form.

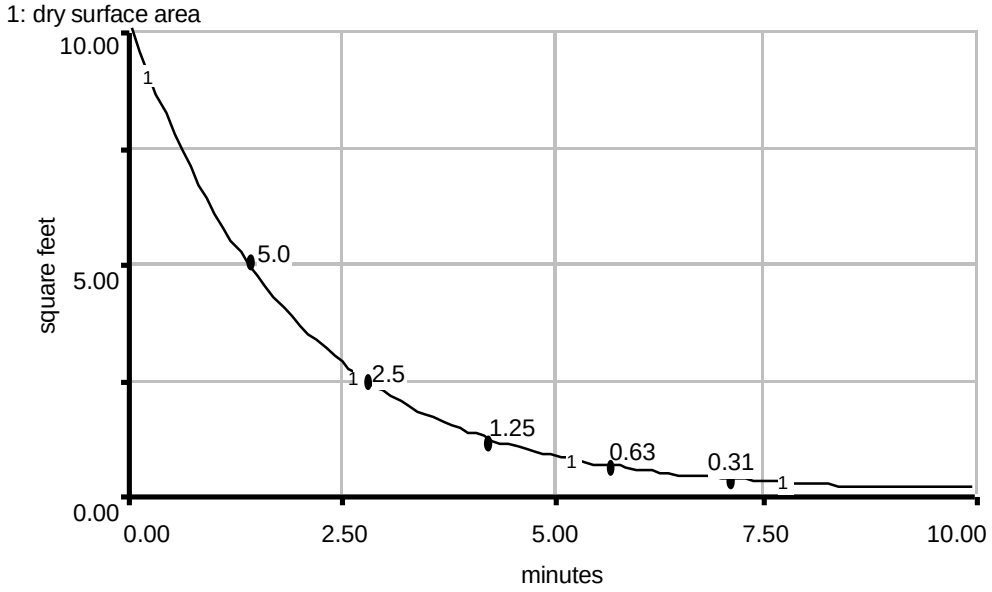


Figure 6: Mental simulation graph of rainfall system

The mental simulation is complete. It is evident from the graph that the system asymptotically declines toward a goal of zero.

Example 2: Chromatography (Goal ? 0)

So far, zero-goal negative feedback systems have been discussed. However, this is a special case and many systems have goals that are not equal to zero. A common experiment in biology classrooms is chromatography, on paper or on silica gel (thin layer chromatography). Chromatography is a method used to separate the component

[3] Notice that multiplying by the time constant is the same as dividing by the decay fraction because: 0.7*time constant=0.7*(1/decay fraction)=0.7/decay fraction

substances in an unknown liquid such as plant pigment or oils. The experimenter places drops of the substance to be tested about 4 cm from the base of the paper. The chromatography paper is placed standing upright in a container of solvent about 3 cm deep (water or dilute acid). As solvent is absorbed and travels upwards on the paper, it carries molecules of the specimen with it. These molecules separate into components according to differing masses and molecule sizes. When the paper is initially placed in the solvent, the dry paper quickly absorbs the solvent, and the solvent rises rapidly. Gradually, as the solvent rises, the forces pulling it up (surface tension, water pressure) and the forces pulling it down (gravity, air pressure) cancel each other. Equilibrium is reached and the paper no longer absorbs any solvent. Figure 7 is an illustration of the chromatography process.

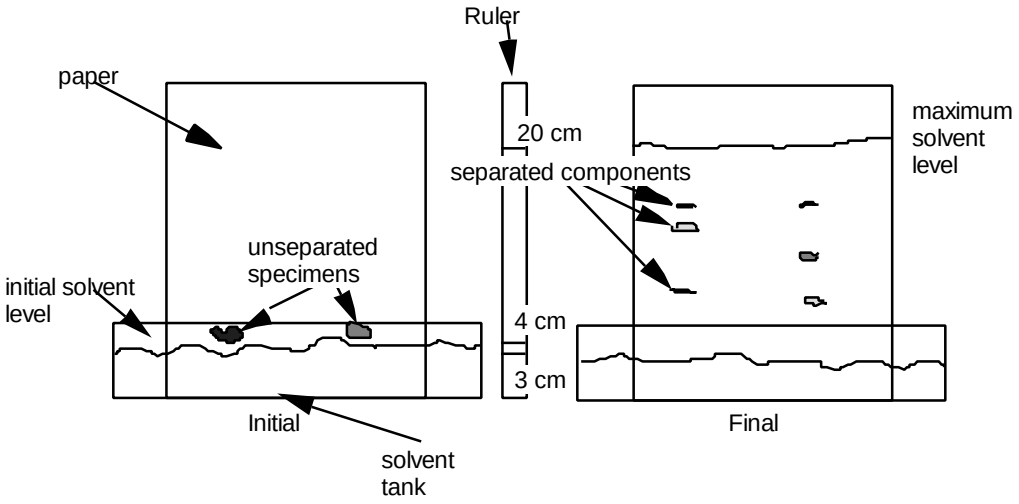


Figure 7: Chromatography: initial and final stages

The chromatography system can be generalized to say that the forces acting on the solvent in the paper are directly dependent on the height of the solvent on the paper. If *current height* (cm) is the level of the solvent mark on the paper at any time, and *max height* (cm) is the highest solvent level attained (through experiment), then the *absorption rate* (cm/minute) is a fraction (*absorption constant*) of the difference between the two heights (or *gap*). In a sample chromatography run, the maximum height (*max height*)

attained by the solvent was about 20 cm, and approximately 1/5 (*absorption constant*) of the *gap* absorbs solvent per minute. Thus the system equations are:

gap = max height - current height
[cm] [cm] [cm]

absorption rate = gap * absorption constant
[cm/minute] [cm] [/minute]

The solvent absorption system can be modeled in STELLA as shown in Figure 8:

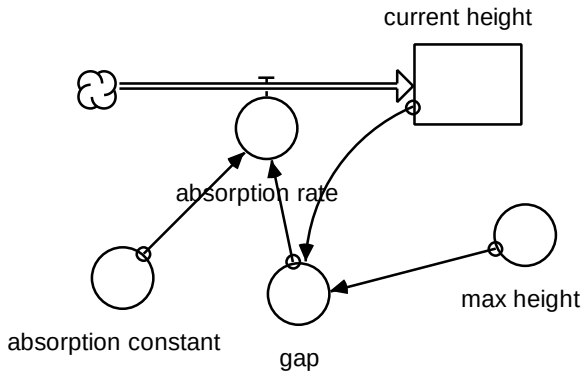


Figure 8: STELLA model of solvent absorption

Try to picture how the system works. The *current height* is initially 3 cm (the height of the initial solvent level), so the *gap* between the *current height* and the *max height* is 17 cm. Multiplying 17 cm by the fraction of 1/5 yields an initial absorption rate of 3.4 cm per minute.

Figure 9 shows the initial slope of the stock behavior to be 3.4 cm per minute:

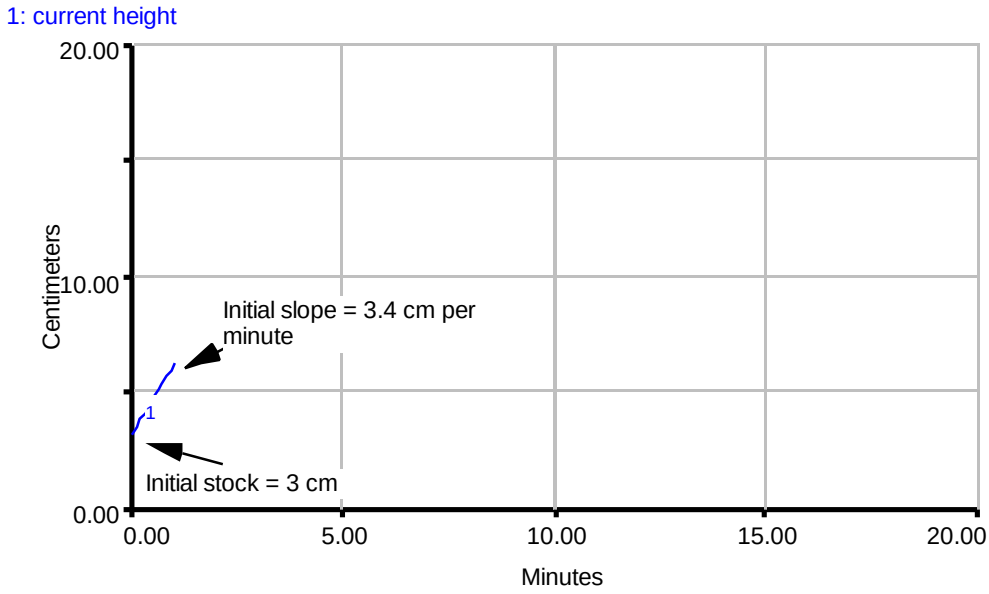


Figure 9: Behavior plot for chromatography system

The chromatography system differs from the rainfall example because the goal is no longer zero, but a set value. There is a gap involved equal to the initial stock value minus the goal, and that is why the system has a goal-gap formulation. The half-life can also be calculated: Half-life = 0.7 * (1 /(1/5)) = 3.5 minutes.

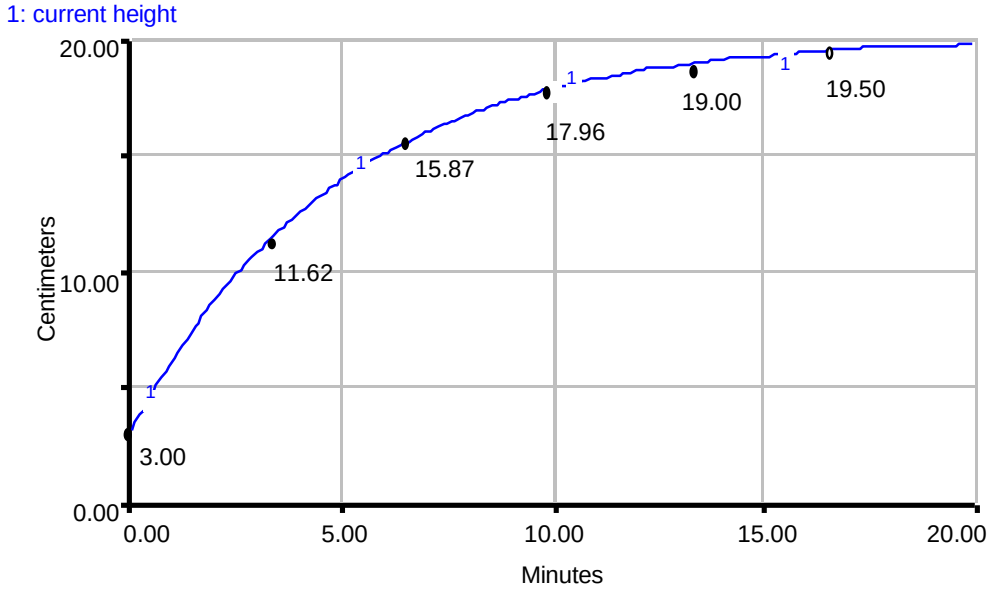


Figure 10: Mental Simulation graph of chromatography model

Review

The exercises in the following section assume a strong understanding of the terms below. If any of these terms is unfamiliar or confusing, please review this paper before attempting to correctly solve the exploration problems.

I. A starting point:
- Initial values of stocks and flows

II. From there on:
- Identifying negative feedback

III. Growth characteristics:
- Asymptotic growth
- Growth to equilibrium
- Above/below goal initial stock value

IV. Goal-gap structure of negative feedback

V. Halving time

The solutions to the following exercises are attached at the end of the paper. If you are having trouble with the exercises, refer to the examples in the paper before looking at the solution.

Exploration

1. Negative feedback is more common in the everyday world than positive feedback. List some examples of negative feedback in the world around you.

_____

_____

_____

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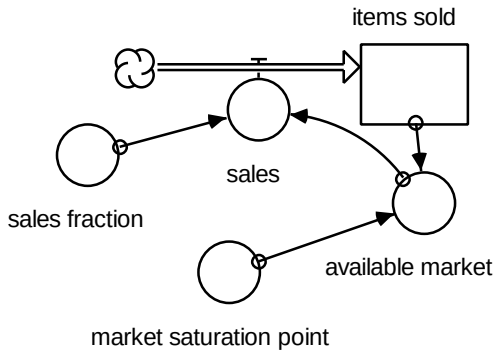
2. Determine whether the following exhibits positive feedback or negative feedback. You may sketch the model structure if necessary.

The federal debt accumulates interest indefinitely.

_____

_____

3. You confront a STELLA model of the sale of a new invention, the widget. Because it is new, no one has this product initially. Assuming that each widget never needs to be replaced, and its reputation does not change so that the sales fraction is constant, the system can be modeled like this:



items_sold(t) = items_sold(t - dt) + (sales) * dt
INIT items_sold = 0 [widgets]
sales = available_market*sales_fraction [widgets/month]
available_market = market_saturation_point - items_sold [widgets]
market_saturation_point = 10000 [widgets]
sales_fraction = 0.2 [/month]

Is this model an example of negative feedback?

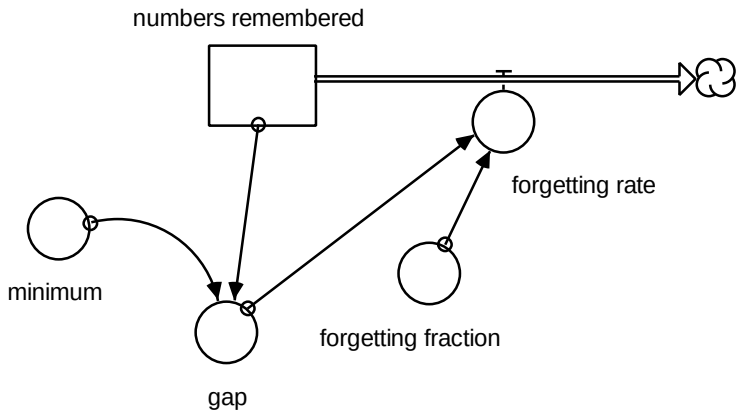
_____

Why or why not?

_____

Can you construct a model that would represent the same system but contains an outflow instead of an inflow? This requires choosing different variables.

4. The figure below shows a simple model for forgetfulness. The more you need to memorize, the more easily you forget. For instance, if you had to memorize 100 phone numbers, it is likely that you will forget about 80 of them, since your brain has to keep track of too many numbers. However, if you only had to memorize 10, you may not forget any.



numbers_remembered(t) = numbers_remembered(t - dt) + (- forgetting_rate) * dt [numbers]
INIT numbers_remembered = 100 [numbers]
forgetting_rate = forgetting_fraction*gap [numbers/day]
forgetting_fraction = 0.8 [/day]

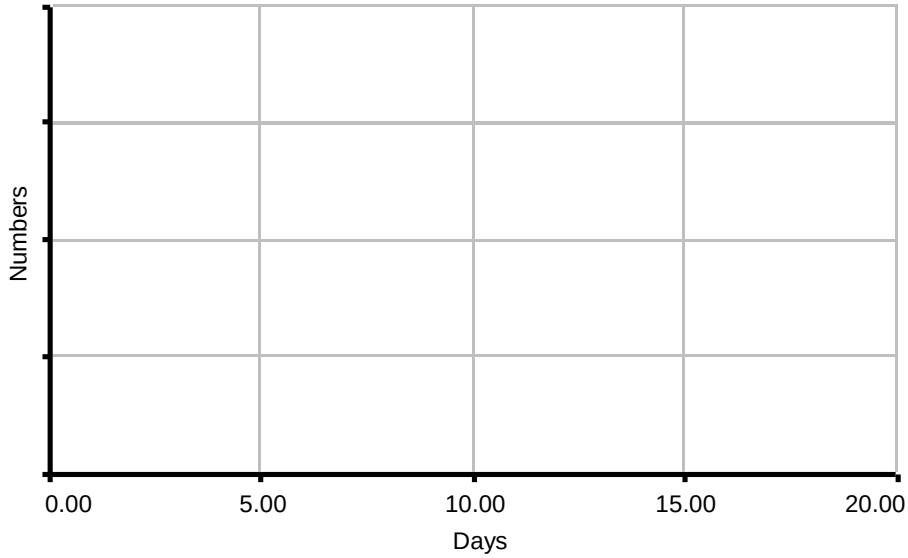
gap = numbers_remembered - minimum [numbers]
minimum = 10 (minimum is maximum number of phone numbers you can remember without forgetting any) [numbers]

What is the halving time for this model? (halving time = 0.7 * time constant)

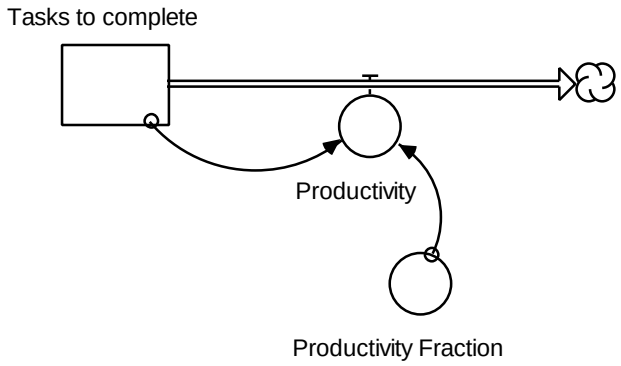
What is the goal in this model?

Use the halving time to approximate how many halving times it takes to reach the goal.

On the graph below, draw the behavior of the stock *Numbers Remembered* for 20 days:



5. Take the following STELLA model of human productivity. When faced with many projects to do, people work faster and harder. But when the demand for hard labor is lower, with fewer tasks to complete, productivity usually drops.



Tasks_to_complete(t) = Tasks_to_complete(t - dt) + (- Productivity) * dt [tasks]

INIT Tasks_to_complete = 100 [tasks]

Productivity = Productivity_Fraction*Tasks_to_complete [tasks/day]

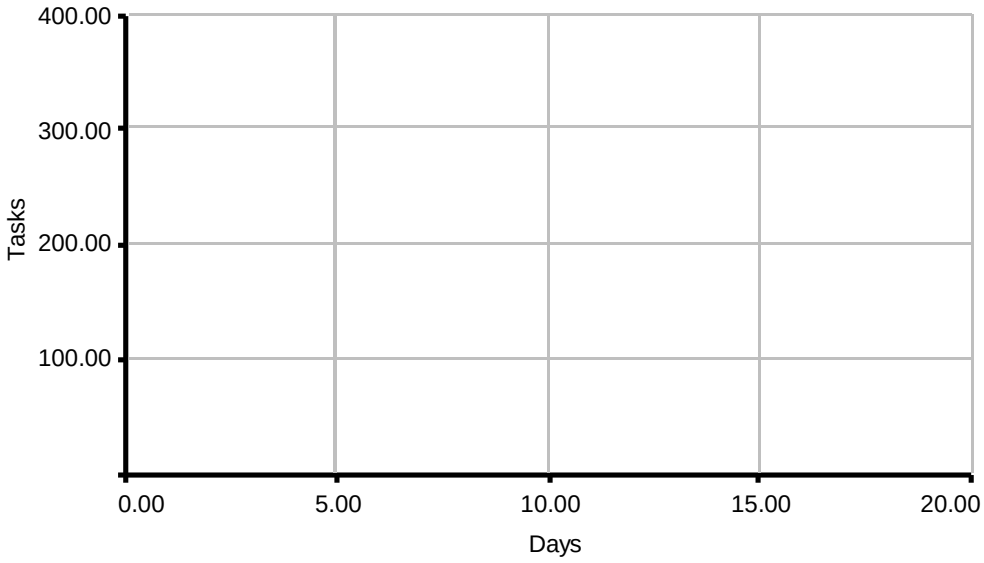
Productivity_Fraction = 0.3 [/day]

What is the goal of this model?

If the initial value of the stock was 200 tasks, what is the half life of this model?

If the initial value was 1000 tasks?

On the graph pad below, draw the behavior of the stock with initial value of 100, 200 and 300 (three lines on one graph) tasks. At what point does each curve reach equilibrium?



Solutions

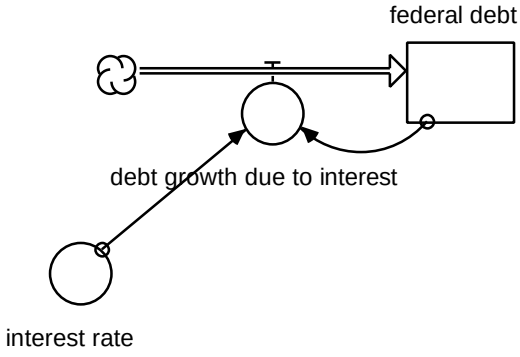
Exercise 1. Several examples of negative feedback that you may be familiar with are:

- running — Initially you go fast, then slow down as you get more and more tired until you reach the point when you're so tired you stop completely.
- music — When you get a new CD, you listen to it maybe ten times the first day, five times the second, and so on until you get sick of it. Eventually you only listen to it once a week or even less often.
- making pals — A college freshman tries to make many friends during orientation, but then starts building a small group of pals to spend time with and is less eager to make more friends. Eventually the individual has "enough" friends, and stops trying to strike up random conversations all the time.
- business — A company lays off a certain percentage of employees as part of cutting back, until it reaches the absolute minimum number needed to function.

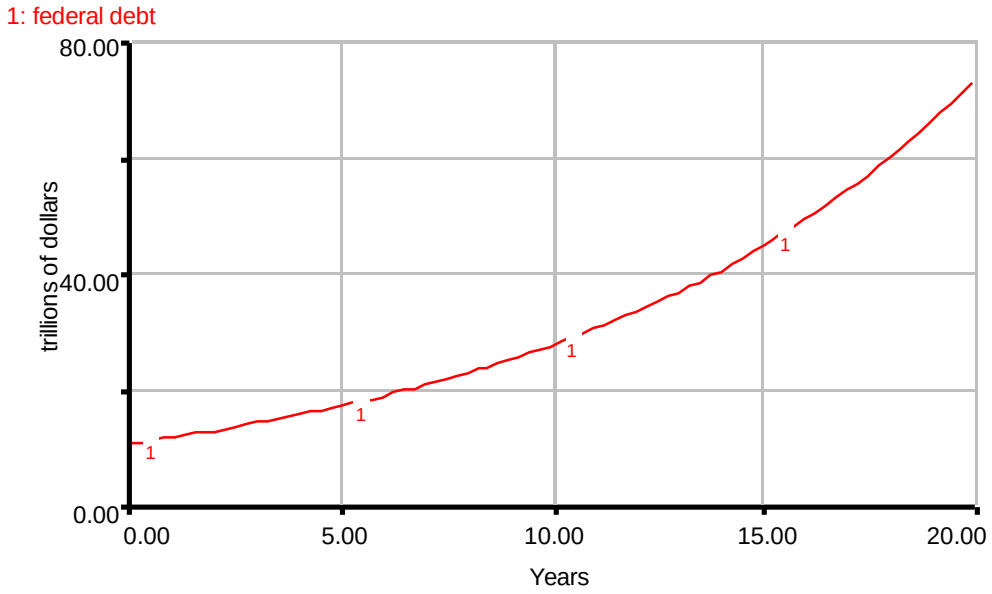
• dress If the temperature is 100 degrees and you're in a huge fur coat with lots of sweaters and accessories, you will immediately throw off some large item to cool yourself down. As you approach comfort level, you gradually take off smaller items like scarves or gloves and make smaller adjustments until you arrive at an ideal temperature.

Exercise 2. The federal debt accumulates interest indefinitely: positive feedback.

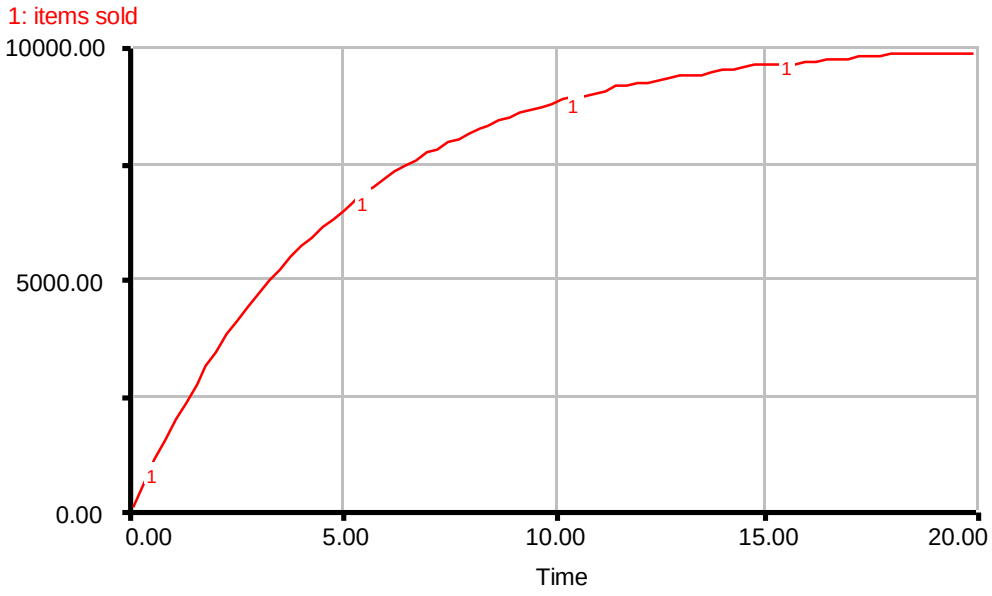
The word “indefinitely” alone should be enough to tell you that this is positive feedback. Interest is always accrued as a certain percentage of the stock. The greater the initial debt, the more interest payments increase, and therefore the larger the debt becomes. Mental simulation shows that this kind of feedback yields exponential growth of the stock, so this is a positive feedback loop. The model is essentially the same as a bank account accumulating interest. It can be modeled as follows:



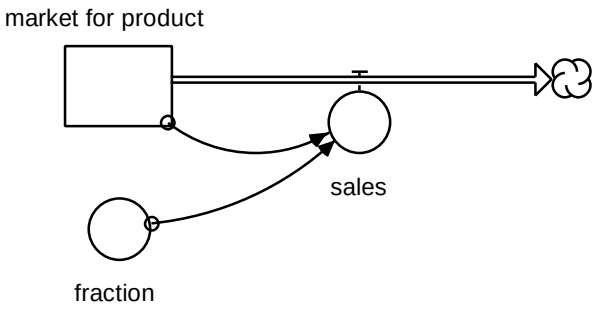
The exhibited exponential behavior, when initial *federal debt* is $10 trillion, with an *interest rate* of 10% per year (0.1), is shown in the following graph of simulation results:



3. Yes, this model is definitely an example of negative feedback. It is easy to identify the goal-gap structure in this model. The goal is the *saturation of market*; the gap is the *available market*. As the *available market* gets smaller and smaller, the *sales* go down. The graph of items sold, therefore, exhibits asymptotic growth approaching the saturation point, in this case 10000 widgets. The following graph is a plot of the stock.

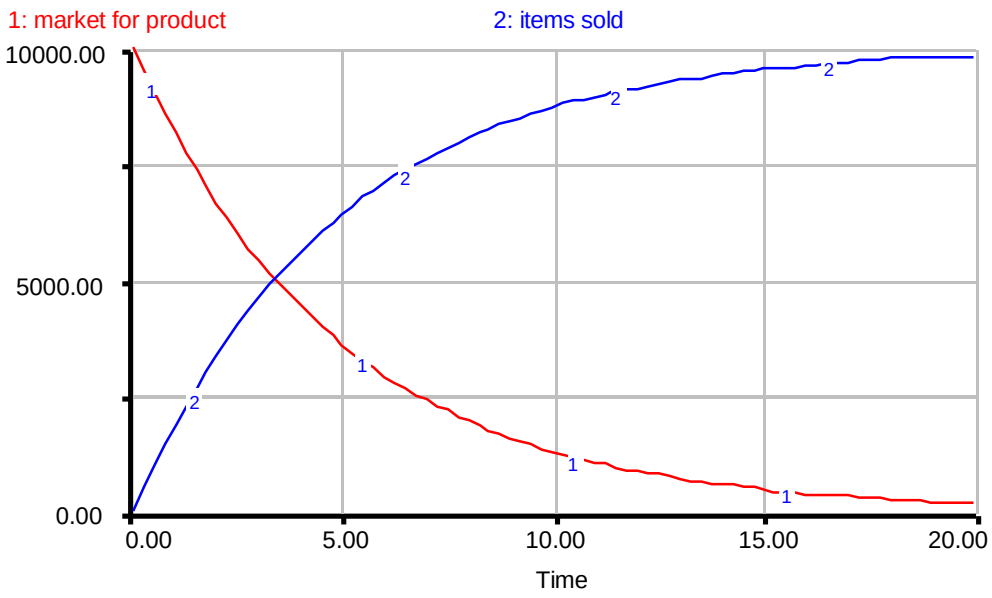


Another way to see this more clearly as a negative feedback loop is to build the model this way:

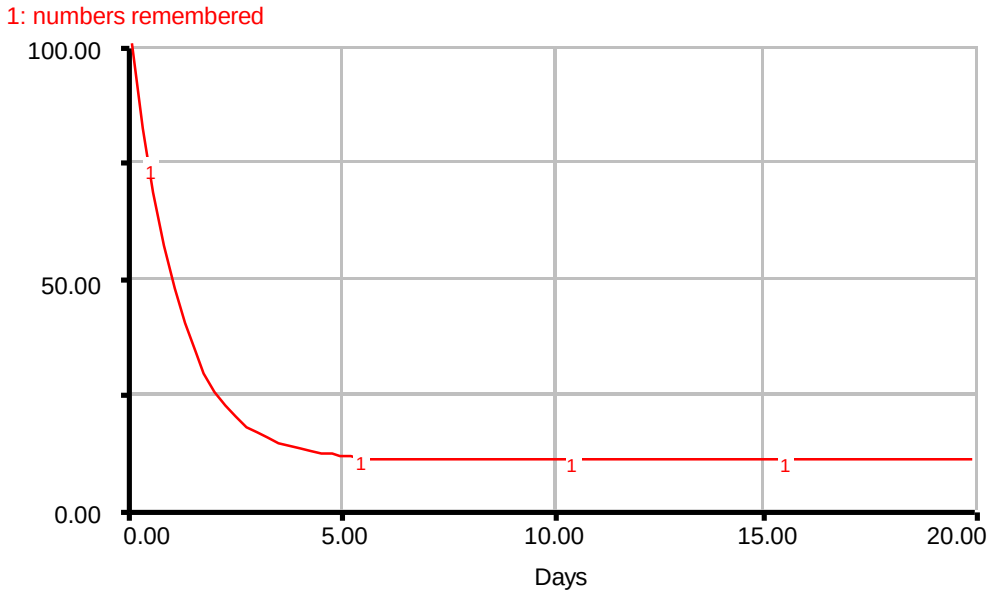


market_for_product(t) = market_for_product(t - dt) + (- sales) * dt [widgets]
INIT market_for_product = 10000 [widgets]
sales = market_for_product*fraction [widgets/month]
fraction = 1/5 [/month]

Notice that the decay fraction remains the same, and the maximum value is still 10000 widgets. Although the graph of the stock in this second case appears to be different, it actually exhibits the same type of behavior. One is asymptotic growth, the other asymptotic decay, and both have the same half-life because their decay fractions are exactly the same.



4. This system contains a negative feedback loop. The fewer phone numbers there are in your brain, the smaller the percentage of those phone numbers you will forget over a certain period of time. Below is a graph showing the asymptotic decline of the stock and rate. The stock approaches its goal of 10 phone numbers.



The halving time calculation is very simple.

Halving time = 0.7 * time constant

Halving time = 0.7 * (1 / decay fraction)

Halving time = 0.7 * (1 / 0.8)

Halving time = 0.7 * 1.25 = 0.875 days

Notice that if you halve the gap, 90, 7 times, the result is less than 1, which we consider a close enough approximation of 0. Thus it takes 7 * 0.875 = 6.125 days for the stock to come to equilibrium.

5. The half life calculation is as follows:

Half life = 0.7 * time constant

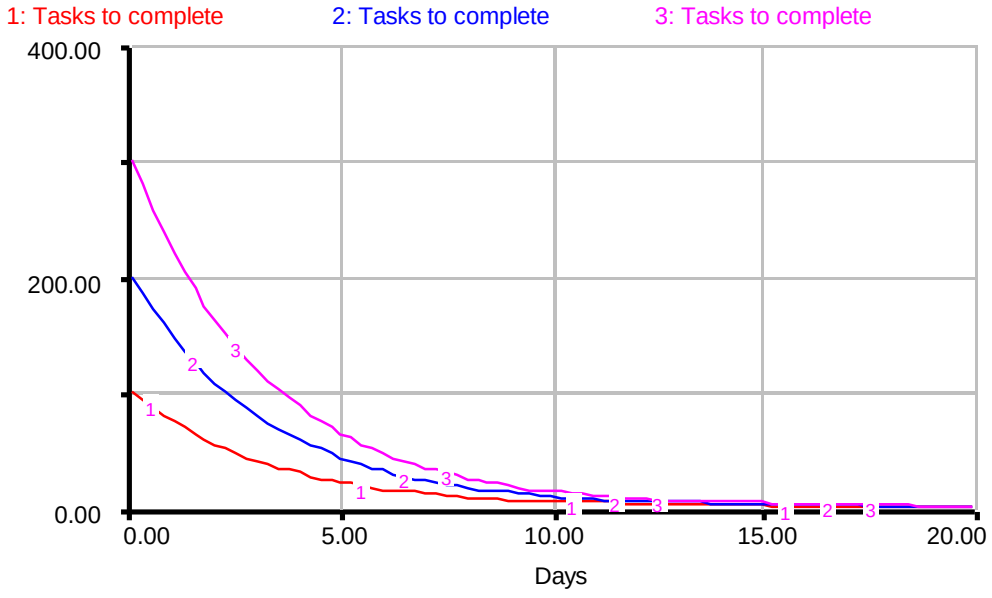
Half life = 0.7 * (1 / decay fraction)

Half life = 0.7 * 3.3

Half life = 2.31 days

Whether the initial value of the stock was 200 or 1000 tasks, the half life of this model would still be the same, because half-life does not depend on the initial value of the stock.

The graph of the three curves looks like this:



Evidently, it is a characteristic of negative feedback that the stock approaches equilibrium at about the same time, no matter what its initial value (this includes initial values above and below the goal). The stock asymptotically grows or decays towards a stable equilibrium.

Bibliography

Forrester, J. W. (1968). Principle of Systems. Cambridge, MA: Productivity Press.

Goodman, M. R. (1974). Study Notes in System Dynamics. Cambridge, MA: Productivity Press.

Roberts, Nancy et al. (1983). Introduction to Computer Simulation. Cambridge, MA: Productivity Press.

Vensim Examples:
Beginner Modeling Exercises: Section 3 Mental Simulation of Simple Negative Feedback

By Lei Lei
October 2001

Example 1: Rainfall on the Sidewalk

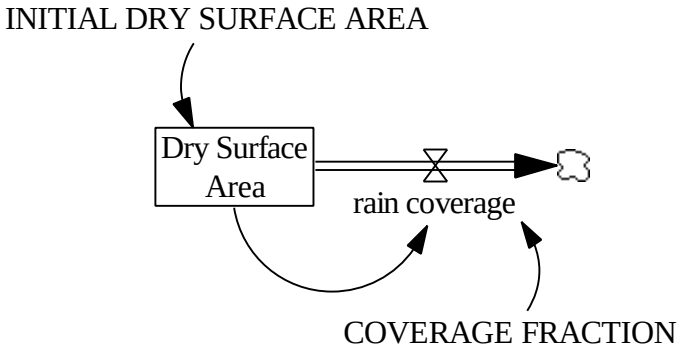


Figure 11: Vensim equivalent of Figure 4. Model of rainfall system

Documentation for Rainfall System model

(1) COVERAGE FRACTION = 0.5

Units: 1/Minute

(2) Dry Surface Area = INTEG (-rain coverage, INITIAL DRY SURFACE AREA)

Units: square feet

(3) FINAL TIME = 10

Units: Minute

The final time for the simulation.

(4) INITIAL DRY SURFACE AREA = 10

Units: square feet

(5) INITIAL TIME = 0

Units: Minute

The initial time for the simulation.

(6) rain coverage = Dry Surface Area * COVERAGE FRACTION

Units: square feet/Minute

(7) SAVEPER = TIME STEP

Units: Minute

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Minute

The time step for the simulation.

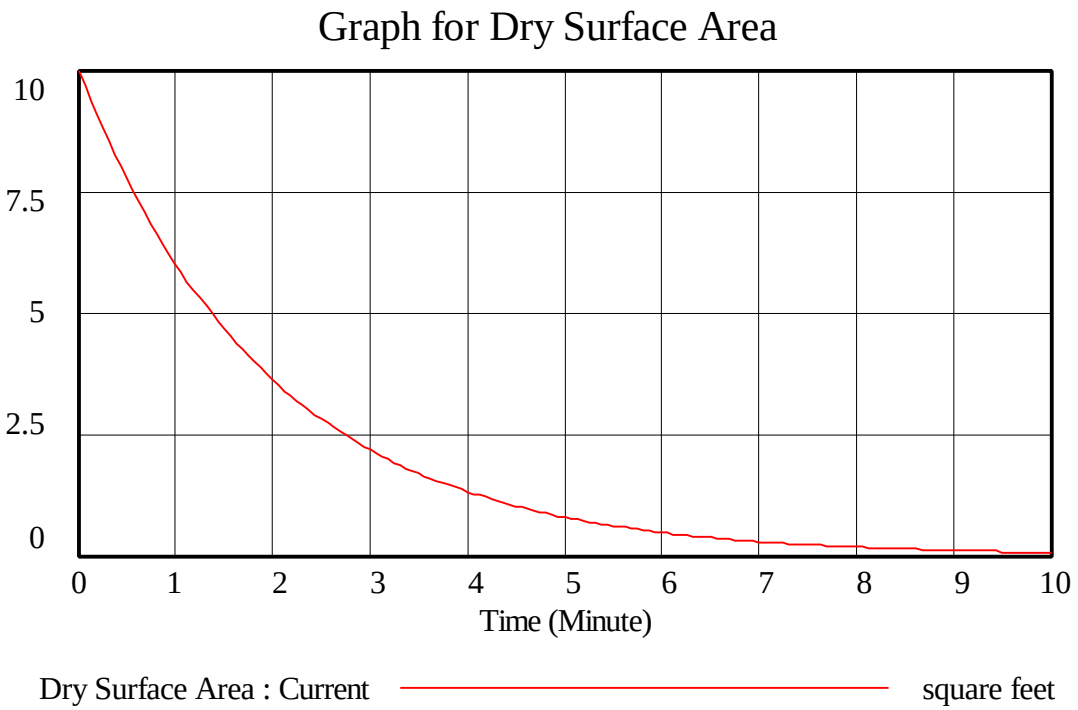


Figure 12: Vensim Equivalent of Figure 6. Mental simulation graph of rainfall system

Example 2: Chromatography

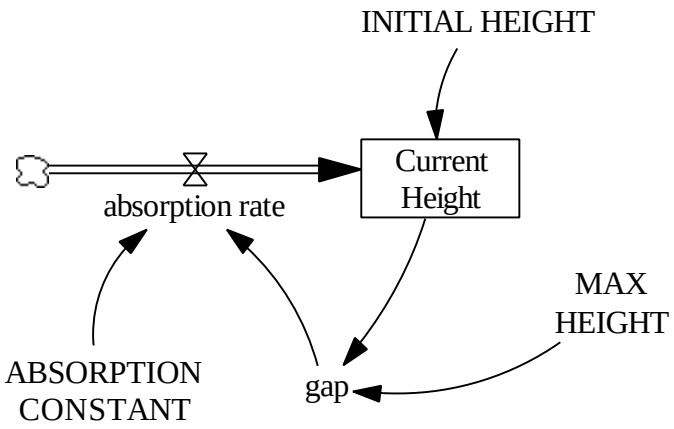


Figure 13: Vensim equivalent of Figure 8. Model of solvent absorption

Documentation for Solvent Absorption model

(01) ABSORPTION CONSTANT = 0.2
Units: 1/Minute

(02) absorption rate = gap * ABSORPTION CONSTANT
Units: cm/Minute

(03) Current Height = INTEG (absorption rate, INITIAL HEIGHT)
Units: cm

(04) FINAL TIME = 20
Units: Minute
The final time for the simulation.

(05) gap = MAX HEIGHT-Current Height
Units: cm

(06) INITIAL HEIGHT = 3
Units: cm

(07) INITIAL TIME = 0
Units: Minute
The initial time for the simulation.

(08) MAX HEIGHT = 20
Units: cm

(09) SAVEPER = TIME STEP

Units: Minute

The frequency with which output is stored.

(10) TIME STEP = 0.0625

Units: Minute

The time step for the simulation.

Exploration 1: Widget

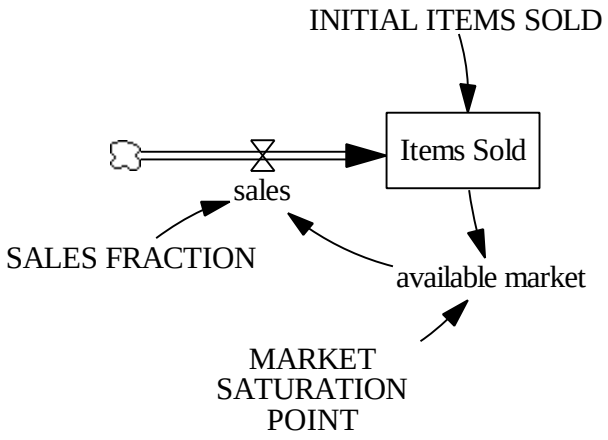


Figure 14: Vensim equivalent of Widget model

Documentation for Widget model

(01) available market = MARKET SATURATION POINT-Items Sold
Units: widgets

(02) FINAL TIME = 20
Units: Month
The final time for the simulation.

(03) INITIAL ITEMS SOLD = 0
Units: widgets

(04) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(05) Items Sold = INTEG (sales, INITIAL ITEMS SOLD)
Units: widgets

(06) MARKET SATURATION POINT = 10000
Units: widgets

(07) sales = available market*SALES FRACTION
Units: widgets/Month

(08) SALES FRACTION = 0.2
Units: 1/Month

(09) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(10) TIME STEP = 0.0625
Units: Month
The time step for the simulation.

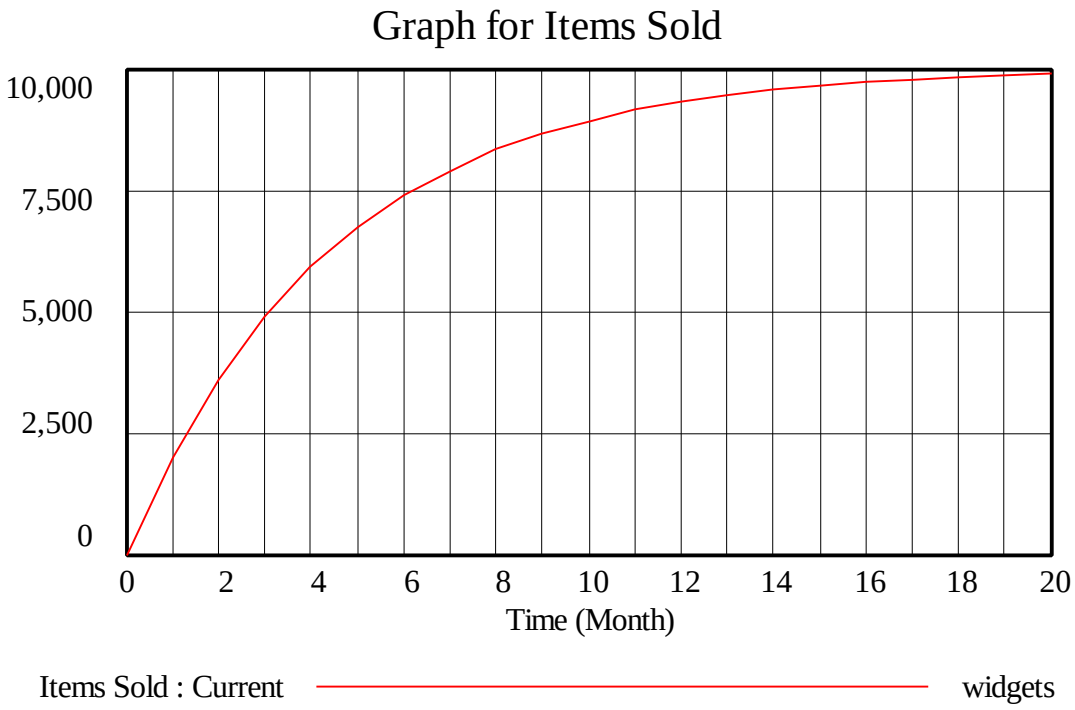


Figure 15: Vensim equivalent of Simulation for widget model

Exploration 2: Forgetting

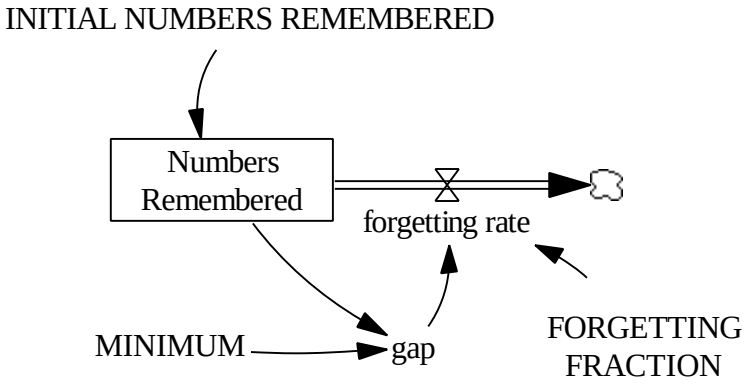


Figure 16: Vensim equivalent of Forgetting model

Documentation for Forgetting model

(01) FINAL TIME = 20
Units: day
The final time for the simulation.

(02) FORGETTING FRACTION = 0.8
Units: 1/day

(03) forgetting rate = FORGETTING FRACTION * gap
Units: numbers/day

(04) gap = Numbers Remembered-MINIMUM
Units: numbers

(05) INITIAL NUMBERS REMEMBERED = 100
Units: numbers

(06) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(07) MINIMUM = 10
Units: numbers
Minimum is maximum number of phone numbers you can remember without forgetting any.

(08) Numbers Remembered = INTEG (-forgetting rate,
INITIAL NUMBERS REMEMBERED)
Units: numbers

(09) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.

(10) TIME STEP = 0.0625
Units: day
The time step for the simulation.

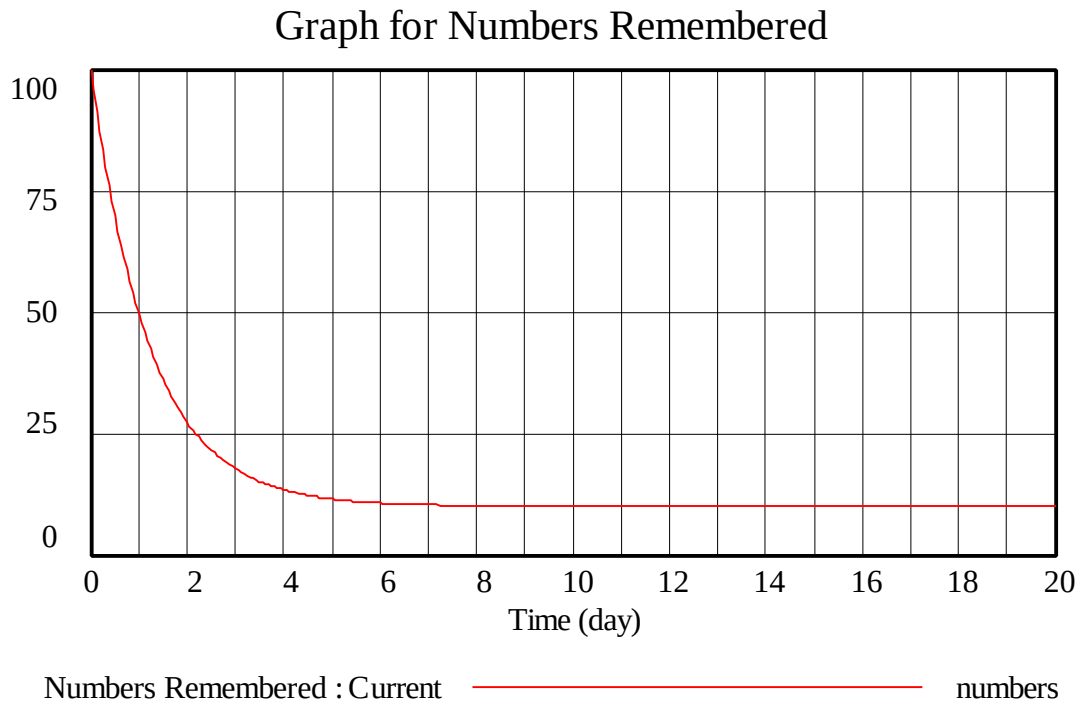


Figure 17: Vensim equivalent of Simulation of Forgetting model

Exploration 3: Productivity

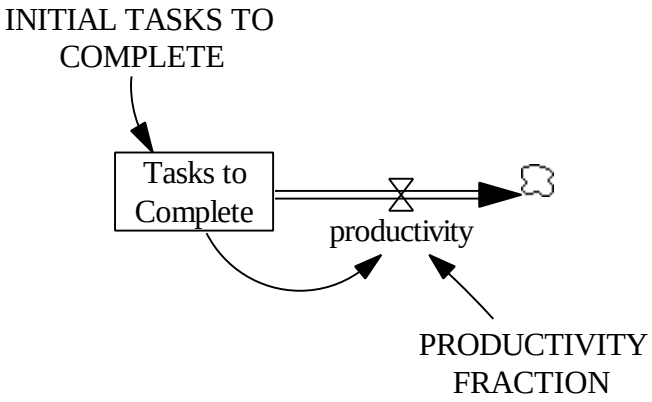


Figure 18: Vensim equivalent of Productivity model

Documentation for Productivity model

(1) FINAL TIME = 20
Units: day
The final time for the simulation.

(2) INITIAL TASKS TO COMPLETE = 300
Units: tasks

(3) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(4) productivity = PRODUCTIVITY FRACTION * Tasks to Complete
Units: tasks/day

(5) PRODUCTIVITY FRACTION = 0.3
Units: 1/day

(6) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.

(7) Tasks to Complete = INTEG (-productivity, INITIAL TASKS TO COMPLETE)
Units: tasks

(8) TIME STEP = 0.0625
Units: day
The time step for the simulation.

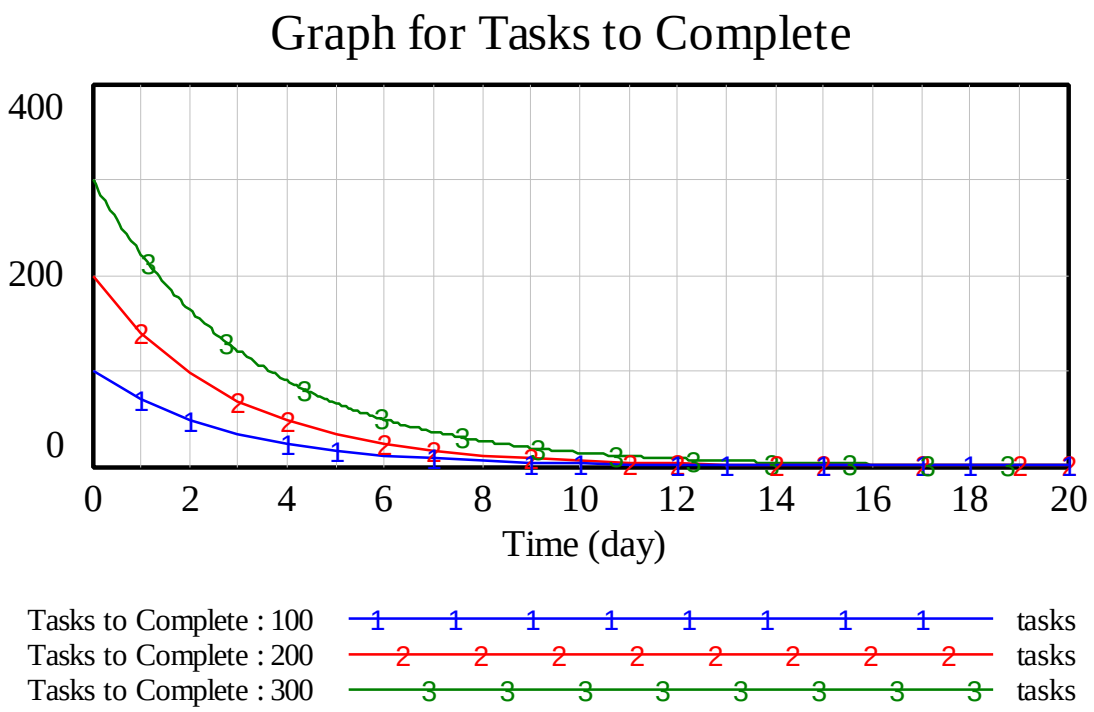


Figure 19: Vensim Equivalent of Simulation of Productivity model

Exploration 4: Federal Debt

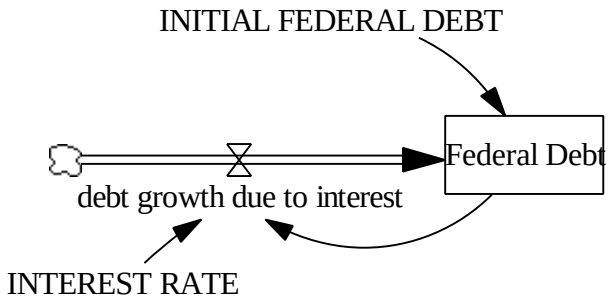


Figure 20: Vensim equivalent of Federal Debt model

Documentation for Federal Debt model

(1) debt growth due to interest = Federal Debt * INTEREST RATE
Units: trillion dollars/Year

(2) Federal Debt = INTEG (debt growth due to interest, INITIAL FEDERAL DEBT)

Units: trillion dollars

(3) FINAL TIME = 20
Units: Year
The final time for the simulation.

(4) INITIAL FEDERAL DEBT = 10
Units: trillion dollars

(5) INITIAL TIME = 0
Units: Year
The initial time for the simulation.

(6) INTEREST RATE = 0.1
Units: 1/Year

(7) SAVEPER = TIME STEP
Units: Year
The frequency with which output is stored.

(8) TIME STEP = 0.0625
Units: Year
The time step for the simulation.

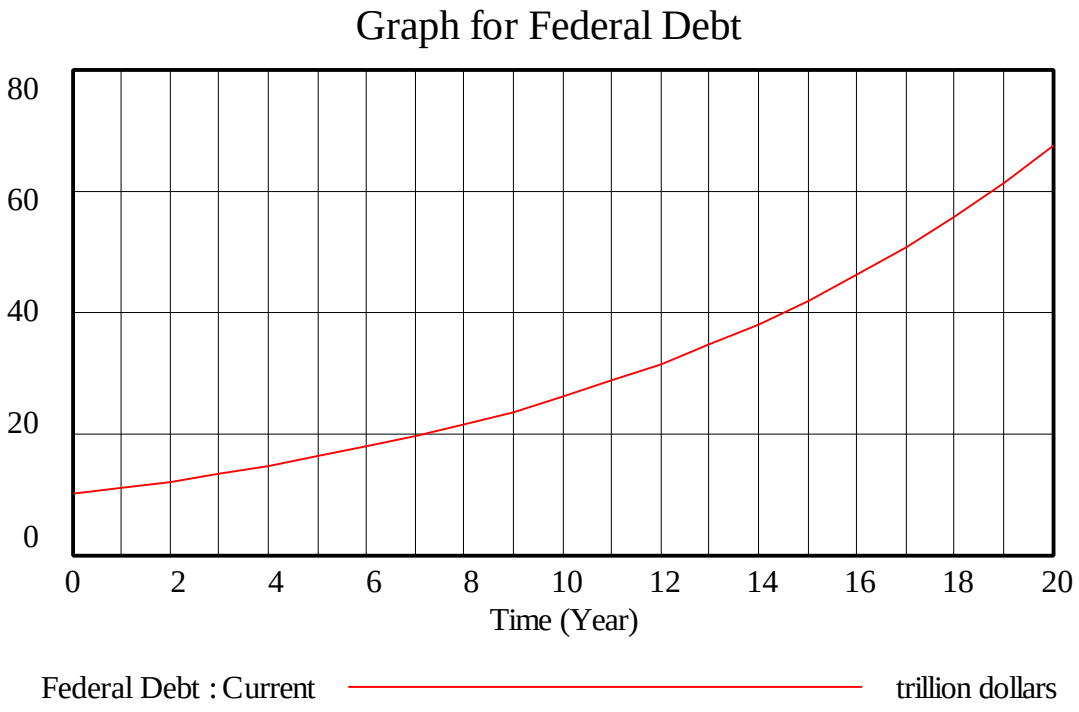


Figure 21: Vensim equivalent of Simulation of Federal Debt model

Exploration 5: Market for Widgets

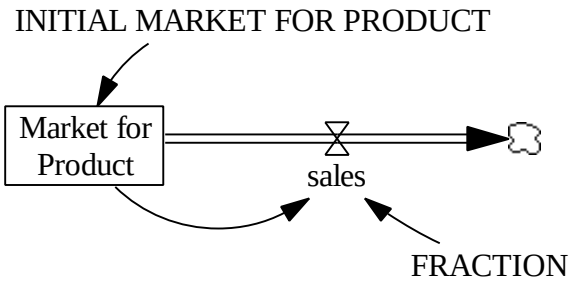


Figure 23: Vensim equivalent of Market for Widgets model

Documentation for Market for Widgets model

(1) FINAL TIME = 20
Units: Month
The final time for the simulation.

(2) FRACTION = 0.2
Units: 1/Month

(3) INITIAL MARKET FOR PRODUCT = 10000
Units: widgets

(4) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(5) Market for Product = INTEG (-sales, INITIAL MARKET FOR PRODUCT)
Units: widgets

(6) sales = Market for Product * FRACTION
Units: widgets/Month

(7) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(8) TIME STEP = 1
Units: Month
The time step for the simulation.

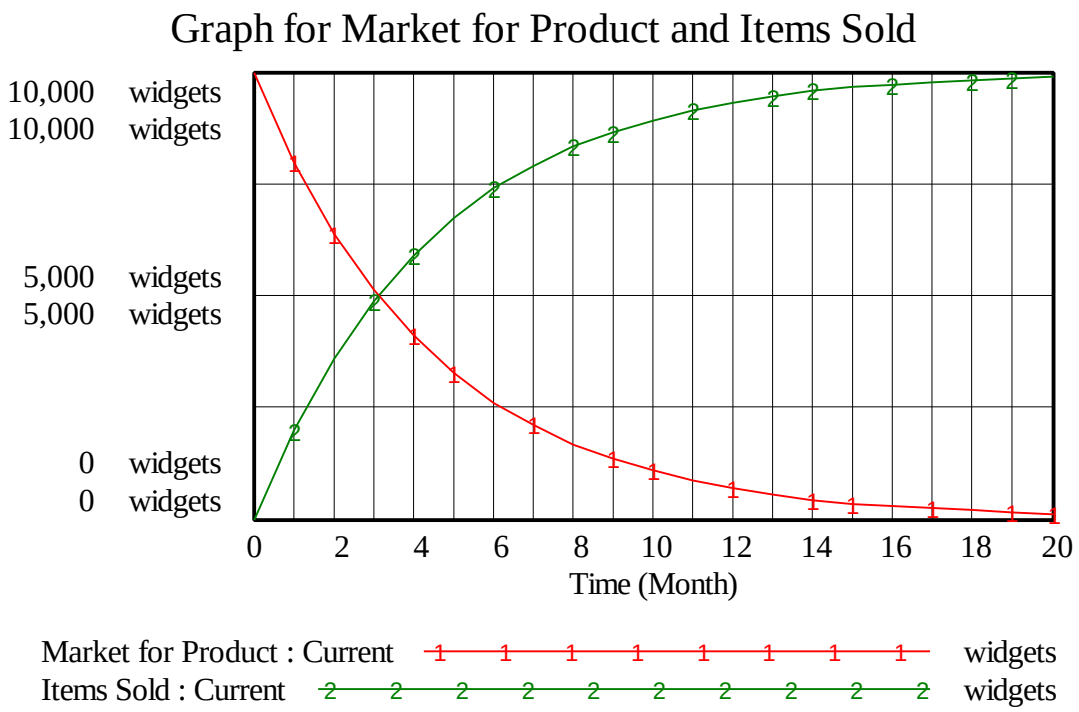
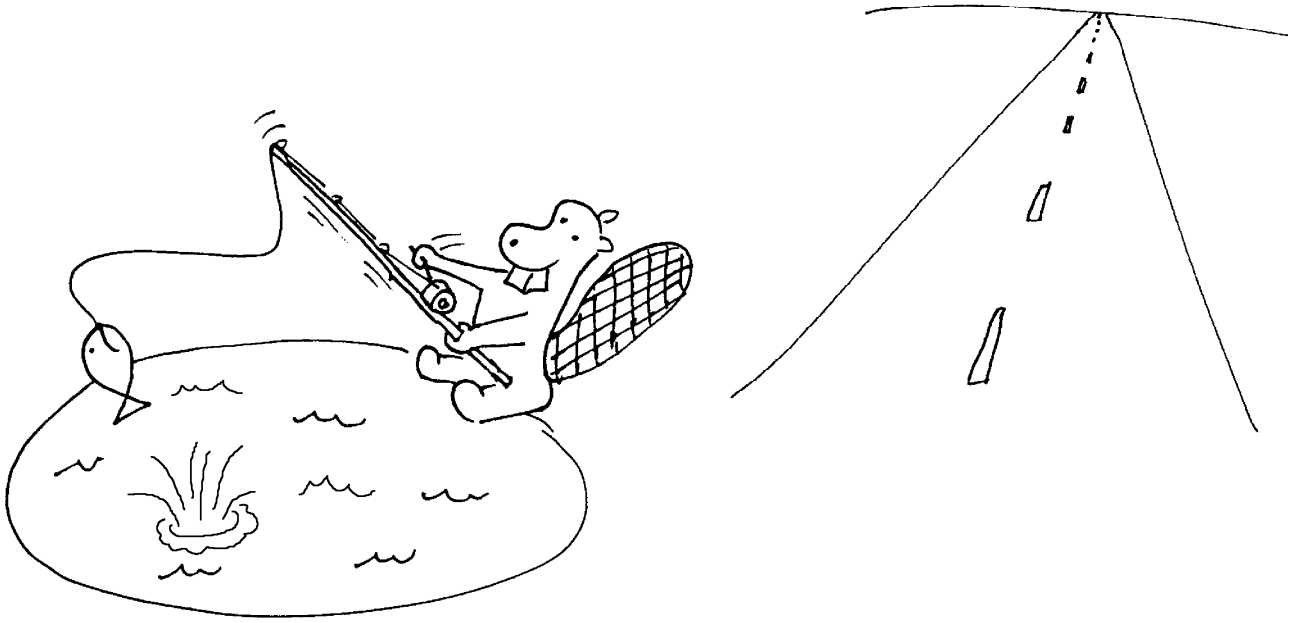


Figure 23: Vensim equivalent of Simulation of Market for Product and Items Sold

Road Maps 4

A Guide to Learning System Dynamics



System Dynamics in Education Project

Road Maps 4

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

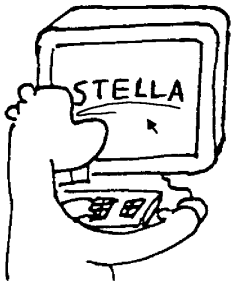
July 18, 1993
Latest Revision March 18, 2001

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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Four!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps Four is the fourth in the series of chapters in Road Maps. The first three chapters of Road Maps gave you a broad introduction to the field of system dynamics and a focus on the building blocks of modeling: positive and negative feedback loops. Road Maps Four will explore and challenge your system dynamics knowledge and understanding.

The first two papers in Road Maps Four introduce the concept of generic structures. You will then do some more beginner modeling exercises. The next paper, *Building the Fish Banks Model and Renewable Resource Depletion*, includes a computer simulation game of a fishing industry based on a system dynamics model. After playing the game, you will create your own model of the fishing industry and try out different policies on the system. At the end of Road Maps Four, we will talk about some of the shortcomings of causal loop diagrams.

Topics Covered in Road Maps Four

Transferability of Structures

- *Generic Structures: First-Order Positive Feedback* (D-4474-1) by Stephanie Albin and Mark Choudhari
- *Generic Structures: First-Order Negative Feedback* (D-4475-1) by Stephanie Albin

Beginner Modeling Exercises

- *Beginner Modeling Exercises Section 4: Mental Simulation: Adding Constant Flows* (D-4546) by Alan Coronado

The Tragedy of the Commons

- *Building the Fish Banks Model and Renewable Resource Depletion* (D-4543) by Joseph G. Whelan and Matthew C. Halbower

Shortcomings of Causal Loop Diagrams

- *Problems with Causal Loop Diagrams* (D-3312)
by George P. Richardson

Things You'll Need for Road Maps Four

Modeling Software

In order to complete Road Maps Four and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: “Vensim Guide (D-4856)” in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

How to Use Road Maps Four

Road Maps Four explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

Transferability of Structures

After going through three sections of Road Maps you might have noticed that some of the models seem to look similar. For example, the model that would be used to describe the filling of a bathtub looks very much like a simple population model. This phenomenon occurs all the time in system dynamics, and is known as the **transferability of structures**. These structures that are common to several systems are called **generic structures**.

***- Generic Structures: First-Order Positive Feedback*[1]**

by Stephanie Albin and Mark Choudhari

The first paper in the series of generic structures introduces the concept of generic structures in systems with first-order positive feedback. After offering and explaining some examples, the paper studies the generic structure and discusses the various types of behavior it can produce. You will then be asked to do several exercises. The solutions are provided at the end of the paper.

Please read *Generic Structures: First-Order Positive Feedback* now.

After reading *Generic Structures: First-Order Positive Feedback...*

This paper described the generic structure of first-order positive feedback systems. In many cases different systems share the same underlying structure. What other systems can you think of that have the same positive feedback loop structure? Transferability of structures is important because it allows you to estimate the behavior of new systems with the same underlying generic structures, based on your knowledge of other systems with which you may already be familiar. It is important to understand all the concepts and to be able to do the exercises before moving to the next paper.

***- Generic Structures: First-Order Negative Feedback*[2]**

by Stephanie Albin

This paper will develop your understanding of generic structures and will present this concept in systems with first-order negative feedback. As in the previous paper, several examples are offered at the beginning, and the paper then explains the generic structure as well as the goal-seeking behaviors it produces. You will again do some exercises; solutions are included at the end of the paper.

Please read *Generic Structures: First-Order Negative Feedback* now.

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[1] Stephanie Albin and Mark Choudhari, 1996. *Generic Structures: First-Order Positive Feedback* (D-4474-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, March 8, 19 pp.

[2] Stephanie Albin, 1996. *Generic Structures: First-Order Negative Feedback* (D-4475-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 22pp.

After reading *Generic Structures: First-Order Negative Feedback...*

This paper discussed the generic structure in first-order negative feedback systems. Again, it explained that different systems often have the same underlying structure. Can you think of any other systems that have the same negative feedback loop structure? The paper emphasized the importance of transferability of structures.

By enabling an efficient transfer of knowledge from one discipline to another, system dynamics serves as the only effective framework for integrating and organizing knowledge from diverse fields. We will return several times in Road Maps to identify and explore generic structures.

Beginner Modeling Exercises

The next reading is the fourth paper in the series of Beginner Modeling Exercises. In Road Maps Three, you learned how to mentally simulate the behavior of simple systems containing either a positive or a negative feedback loop. We will now have a look at how the introduction of a constant flow affects this behavior.

- Beginner Modeling Exercises Section 4: Mental Simulation: Adding Constant Flows[3]

by Alan Coronado

This paper will continue to develop your mental simulation skills, and will apply them to estimate the behavior of systems that also contain a constant flow. Both positive and negative feedback with a constant flow will be explored, and after each section you will be asked to complete some exercises. You can compare your answers to the ones provided at the end of the paper.

Please read *Beginner Modeling Exercises Section 4* now.

After reading *Beginner Modeling Exercises Section 4...*

The addition of a constant flow to a system containing a positive or a negative feedback loop does not change the general pattern of behavior, but changes the

---

[3] Alan Coronado, 1996. *Beginner Modeling Exercises Section 4: Mental Simulation: Adding Constant Flows* (D-4546), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 27 pp.

system goals. This paper explained how to determine the equilibrium stock, the net flow, and the time constant of a system. It is necessary to be able to solve all the exercises before moving to the next paper in the Beginner Modeling Exercises series.

The Tragedy of the Commons

The disappearing tropical rain forests, the dwindling Scandinavian fish supply, and the depletion of fish from the Grand Banks: what do these three have in common? In each case, a shared resource has diminished as a result of individual overuse. In system dynamics this is known as "The Tragedy of the Commons." The next paper examines the system structure that causes this behavior.

- Building the Fish Banks Model and Renewable Resource Depletion[4]

by Joseph G. Whelan and Matthew C. Halbower

This paper leads you through The Fishing Game, a one player computer simulation game about a fishing industry. The game allows you to role-play, making decisions similar to those made by people in the real system. As you will see, situations of this sort can produce unexpected behavior. This game and the model you will create after you finish the game illustrate a common system behavior called "The Tragedy of the Commons."

Computer simulation games allow you to experiment with different policies without taking “risks” in real life. Computer simulation also helps to clear up any uncertainty that you might have about the results of a policy. You simply try the policy in the game and see the results.

Now please read *Building the Fish Banks Model...*

After reading *Building the Fish Banks Model...*

The behavior known as "The Tragedy of the Commons" occurs because of the system's structure. Individuals within the system (in this case fishermen) do not

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4 Joseph G. Whelan and Matthew C. Halbower, 1994. *Building the Fish Banks Model and Renewable Resource Depletion* (D-4448), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July, 55 pp. This paper is accompanied by a Macintosh® computer disk.

see that their goal to maximize their own profits undermines the very source of their livelihood. Do you see any other situations that might have a tragedy of the commons structure?

As discouraging as this situation seems, the model also points out that some policies can prevent the overuse of the resource. Policy analysis is one of the most powerful applications of system dynamics. How do system dynamics models help decision makers (government agencies, regulatory commissions, etc.) in their policy making? This issue will be explored in later chapters of Road Maps.

Note each of the following System Principles that were illustrated in *Building the Fish Banks Model*.



System Principle #7:

Rates depend only on Levels and Constants.

Look at Figure 7 on page 15 in *Building the Fish Banks Model*. Note that the only inputs to the rates **Fish Hatch Rate** and **Fish Death Rate** are the level **FISH** and constants. The two rates do not directly influence each other. Rates depend only on levels and constants.

We now introduce a new principle about decisions within feedback loops.



System Principle #8:

Decisions are always within feedback loops.

Look now at Figure 9 on page 19 of *Building the Fish Banks Model*. The decision made to determine the **Ship Building Rate** is part of two feedback loops (one positive, the other negative). Look again at Figure 7 on page 15. The "decisions" for Fish Hatch Rate and Fish Death Rate also occur within feedback loops.

No matter what the nature of the decision process—human, subconscious, biological, chemical, mechanical, electrical, etc.—it is always imbedded within at least one feedback loop.

It may be difficult to think of a machine or a chemical reaction as making decisions. In System Principle #8 we consider a "decision process" to be any case where a change is made based on information gathered in a system.



System Principle #9:

Every equation must have dimensional equality.

In any equation, every term must be measured in the same dimensions. "One cannot add apples and oranges." Look in *Renewable Resource Depletion* at Figure 11 on p. 16 and the model equations on p. 37. **Yearly Profits** is defined as the difference between **Revenues** and **Costs**. Both **Revenues** and **Costs** must have the same units of measure, in this case $/year. Dimensional inequality between terms indicates a faulty equation formulation.

The following system principle is one that you have seen before. We are revisiting it in our spiral approach to emphasize its importance.

System Principle #2:

Levels and Rates are fundamental to loop substructure.

As you become familiar with the Fish Banks Model, trace through the feedback loops abundant within it. Notice that both the levels and the rates are fundamental substructure of the feedback loops. The loop dynamics cannot be represented without their inclusion. Further, the dynamics can be completely modeled with **only** the level and rate variables. These two variables are both necessary and sufficient to structure in representing a feedback system.

Since causal loop diagrams do not show the substructure described in System Principle #2, they lack much of the clarity of stock-and-flow diagrams. This leads to problems as described in the next section.

Shortcomings of Causal Loop Diagrams

In many previous readings and exercises in Road Maps, we have made use of causal loop diagrams. These diagrams are widely used to illustrate systems and their structure to people who are not familiar with system dynamics. However, as we will soon show you, these diagrams have problems of their own.

***- Problems with Causal Loop Diagrams*[5]**

by George P. Richardson

This paper discusses some of the problems that arise when causal loop diagrams are used to model systems. The basic cause of these problems is that causal loop diagrams are not clear enough to convey structure or behavior, which are the defining characteristics of a system. Although the author redefines positive and negative loops to try to decrease confusion, the best way to avoid confusion is to use stock-and-flow diagrams to model the system directly without needing causal loop diagrams.

Now please read *Problems with Causal Loop Diagrams.*

After you have read *Problems with Causal Loop Diagrams*...

This paper explained why causal loop diagrams are used with caution by system dynamicists. Their structure does not show the difference between a stock and a flow, whereas the stock-and-flow diagram is much less ambiguous. The best way to avoid the confusion is to simply avoid using causal loop diagrams whenever possible. Causal loop diagrams can be used to explain behavior after the model is built, but using these diagrams to build models can be dangerous.

Finishing Off Road Maps Four

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5 George P. Richardson, 1981. *Problems with Causal Loop Diagrams* (D-3312), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, April, 23 pp. Current Address (as of 9/94): The Rockefeller College, SUNY-Albany, 135 Western Avenue, Albany, NY 12222.

Road Maps Four introduced the idea of transferability of structures, and explained that many different systems often exhibit similar types of behavior because of their same underlying structure. This chapter also showed how to mentally simulate the behavior of systems that contain a constant flow in addition to a positive or negative feedback loop. We described the "Tragedy of the Commons," and we explained why it occurs so frequently. Where does this type of behavior occur? Why is it an important phenomenon?

We also explained policy analysis, and the role of computer simulation games in teaching people about systems. What use are games in education? Why should people play these games? Why is policy analysis such a big topic in system dynamics?

In Road Maps Four we also described the limitations of causal loop diagrams, and explained how to avoid future misunderstandings caused by these limitations. Why do these tools break down when explaining systems?

Road Maps Four also presented three new systems principles: a) that rates depend only on levels and constants; b) that decisions are always made within feedback loops; and c) that every equation must have dimensional equality.

In Road Maps Five you will further develop your understanding of S-shaped growth through mental simulation and through studying the generic structures that produce it, and by exploring the spread of an epidemic. More theory about systems and their structure will be introduced as well.

Key Terms and Concepts:

Conserved Flow
Generic Structure
Information Link
Net Rate
Parameter
Policy Analysis
Tragedy of the Commons
Transferability of Structure



Generic Structures:
First-Order Positive Feedback

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1. Introduction

Generic structures are relatively simple structures that recur in many diverse situations. In this paper, for example, the models of a bank account and a deer population are shown to share the same basic structure! Transferability of structure between systems gives the study of generic structures its importance in system dynamics.

Road Maps contains a series of papers on generic structures. In these papers, we will study generic structures to develop our understanding of the relationship between the structure and behavior of a system. Such an understanding should help us refine our intuition about the systems that surround us and allow us to improve our ability to model the behaviors of systems.

We can transfer knowledge about a generic structure in one system to understand the behavior of other systems that contain the same structure. Our knowledge of generic structures and the behaviors they produce is transferable to systems we have never studied before!

It is often the case that the behavior of a system is more obvious than its underlying structure. Systems are then referred to by the common behaviors they produce. However, it is incorrect to assume such systems are capable of exhibiting only their most popular behaviors, and we need to look more closely at the other behaviors possible. In effect, our study of generic structures examines the range of behaviors possible from particular structures. In each case, we seek to understand what in the structure is responsible for the behavior produced.

This paper introduces a simple generic structure of first-order linear positive feedback. We illustrate our study of the positive feedback structure with many examples of systems containing the structure. You will soon begin to recognize the structure in many of the models you see and build. In the exercises at the end of the paper, we provide you with an opportunity to see how you can transfer your knowledge between different systems.

2. Exponential Growth

Exponential growth is produced by a positive feedback loop between the components of a system. The characteristic behavior of exponential or compound growth is shown in Figure 1. Many systems in the world exhibit the exponential behavior of a process feeding upon itself. For example, in an ecological system, the birth of deer increases the deer population, which further increases the number of deer that are born. At your bank, your account balance is increased by the interest you earn on it, and the larger your balance gets, the more interest you earn on it! Another system which can be said to exhibit exponential growth is the knowledge-learning system. Simply stated, the more you know, the faster you learn, and then gain even more knowledge.

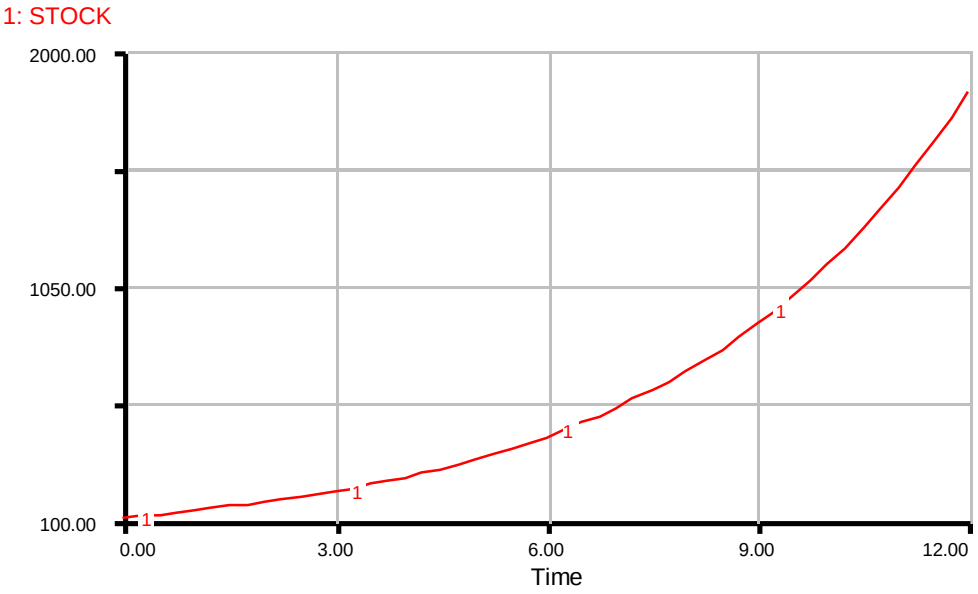


Figure 1 Exponential Growth Curve

These very different systems exhibit the same behavior pattern because the relationship between their components is fundamentally the same. They all contain the first-order linear positive feedback generic structure. Population is related to births in the same way your bank balance is related to the interest it earns and knowledge is related to learning.

Let us begin to explore the nature of this relationship by looking at the structure of our three example systems.

2.1. Example 1: Population-Birth system

Our first example shown in figure 2 is taken from the ecology of a deer population. The **deer population** is the stock, and the **births** of deer is the net inflow to the stock. The amount of deer births is equal to the amount of female deer that reproduce and is calculated as a compounding fraction (called **birth fraction**) of the total deer population. The

births = deer population * birth fraction.

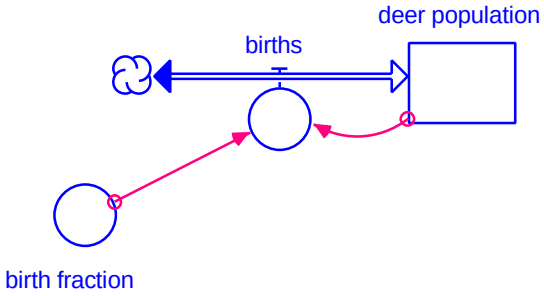


Figure 2 Model of a population-birth system

2.2. Example 2: Bank balance-interest system

Our second example in figure 3 shows the relationship between a bank balance and the interest it earns. The **bank balance** is the stock, and the **interest earned** is the inflow to the stock. The amount of interest earned every year is equal to a compounding fraction (**interest rate**) of the bank balance. The

interest earned = bank balance * interest rate.

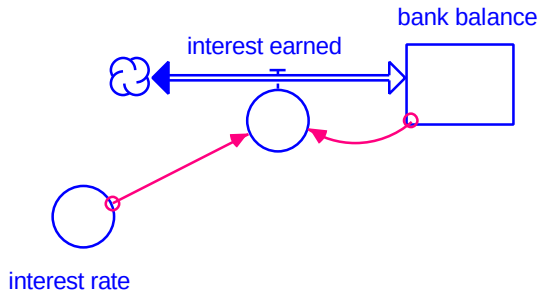


Figure 3 Model of a bank balance system

2.3. Example 3: Knowledge-learning system

This third example is of a more abstract system. Figure 4 shows that the stock of **knowledge** is increased by the net inflow **learning**. The rate of learning is the knowledge spread out over the **time to learn**. Therefore, learning is equal to the knowledge divided by the time to learn. The time to learn is known as the time constant of the system. Basically, the more you know, the faster you learn. The

learning = knowledge/time to learn.

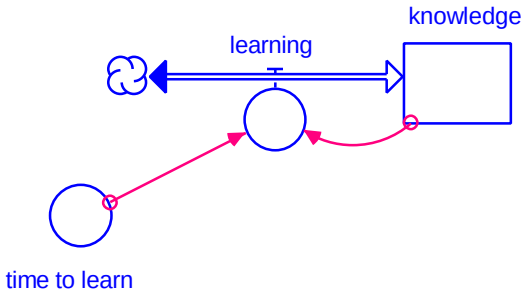


Figure 4 Model of a knowledge-learning system

Note that in the equation of the rate, we divide the stock (knowledge) by time to learn. This is analogous to multiplying by a fraction as seen in examples 1 and 2. The units for time to learn are the units of time like weeks or months. The units of the compounding fraction would be unit/unit/time.

As is evident from Figures 2, 3, and 4, all three of these systems have essentially the same structure.

3. The Generic Structure

We will now study the generic structure and then explore the possible behavior it can produce.

3.1. Model Diagram

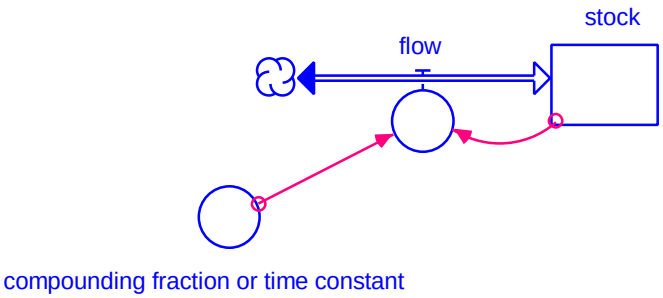


Figure 5 Model of the underlying generic structure

The model diagram of the first-order positive generic structure is shown in figure 5. In the equation of the rate, we multiply the stock by the compounding fraction or divide the stock by the time constant. The time constant is simply the reciprocal of the compounding fraction.

3.2. Model Equations

The equations for the generic structure are

stock(t) = stock(t - dt) + (flow) * dt

DOCUMENT: This is the stock of the system. It corresponds to the deer population, the bank balance, and the stock of knowledge in the examples above.
UNIT: units

INFLOWS:
flow = stock*compounding_fraction
DOCUMENT: The flow is the fraction of the stock that flows into the system per unit time. It corresponds to the births, the interest earned, and the learning in the examples above.
UNIT: units/time

compounding_fraction = *a constant*

DOCUMENT: This is the compounding fraction or growth factor. It determines the inflow to the stock. The compounding fraction corresponds to the birth fraction and the interest rate in the examples above. It is the amount of units added to the stock for every unit already in the stock, every time.
UNIT: units/unit/time

Note: If we had a time constant instead of a compounding fraction the equation for the flow and the time constant would be
INFLOWS:
flow = stock/time constant
UNIT: units/time

time_constant = *a constant*

DOCUMENT: This is the time constant. It is the adjustment time for the stock. It corresponds to the time to learn in the above example. This is the time for each initial unit to compound into a new unit.

UNIT: time

From the comparison of the two possible equations for the rate, we notice that the multiplier in the rate equation is given by

$$\text{multiplier (for the stock) in the rate equation} = \text{compounding fraction} = \frac{1}{\text{time constant}}$$

3.3. Model Behavior

The characteristic feature of exponential growth is its constant doubling time, i.e. the time it takes for the stock to double remains constant. For example in Figure 6, it takes the stock 7 years to double from 100 to 200 and also the same time to double from 800 to 1600!

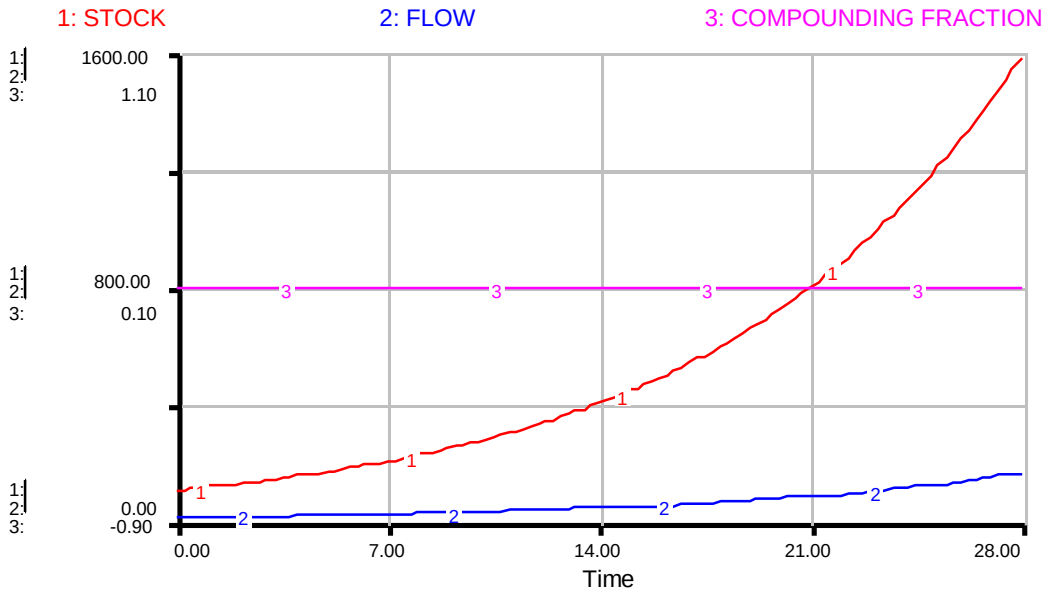


Figure 6 Results of a simulation of the positive feedback generic structure.

To find the doubling time of the stock, we need the time constant of the system. The time constant may either be given to you directly (as the time to learn in Example 3 above), or

if you have a compounding fraction, the time constant is simply it's reciprocal. The time constant is obtained from the compounding fraction by

$$\text{Time constant} = \frac{1}{\text{compounding fraction}}$$

The doubling time for the stock is given by

$$\text{Doubling time} \approx 0.7 \times \text{Time constant}$$ [1]

4. Behaviors produced by the generic structure

To explore the different behaviors possible, let us first experiment by changing the initial value of the stock and keep the value of the compounding fraction constant. The different behaviors for changing values of the initial stock are shown below. The stock is given initial values of -200, -100, 0, 100, and 200 for runs 1 through 5 respectively. The compounding fraction is kept constant at 0.1. The results are shown in Figure 7.

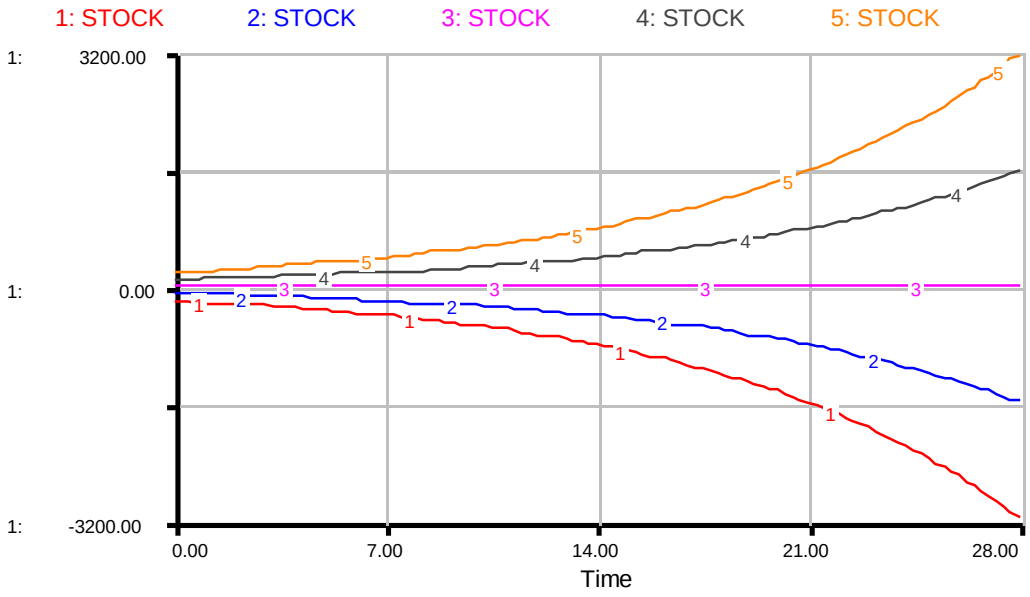


Figure 7 Simulation for different initial values of the stock

The flow is a constant fraction (compounding fraction) of the stock. As the stock increases, the compounding fraction remains the same, but now, the flow is the fraction of a larger stock, and therefore the flow increases with the stock. The slope of the stock at a

[1] ln 2 is approximatly equal to 0.7.

point in time is equal to the net flow into it at that time. Therefore, for each curve, the slope of the stock is increasing or decreasing as the stock is increasing or decreasing.

With a positive value for the compounding fraction, the nature of the behavior is determined by whether the initial value of the stock is positive or negative. For the loop to be a positive feedback loop, we require that the compounding fraction be positive[2].

Therefore, we see that the generic structure of a first-order positive loop can exhibit three types of behavior - positive exponential growth, unstable[3] equilibrium, and negative exponential growth.

Let us now explore what accelerates or retards the exponential growth of a system. We will study the effect of changing the value of the compounding fraction while keeping the initial value of the stock constant. The compounding fraction is given values of 0, 0.1, 0.2, 0.3, and 0.4 for runs 1 through 5 respectively. The initial value of the stock is kept constant at 100. The change in rate of exponential growth is shown in Figure 8.

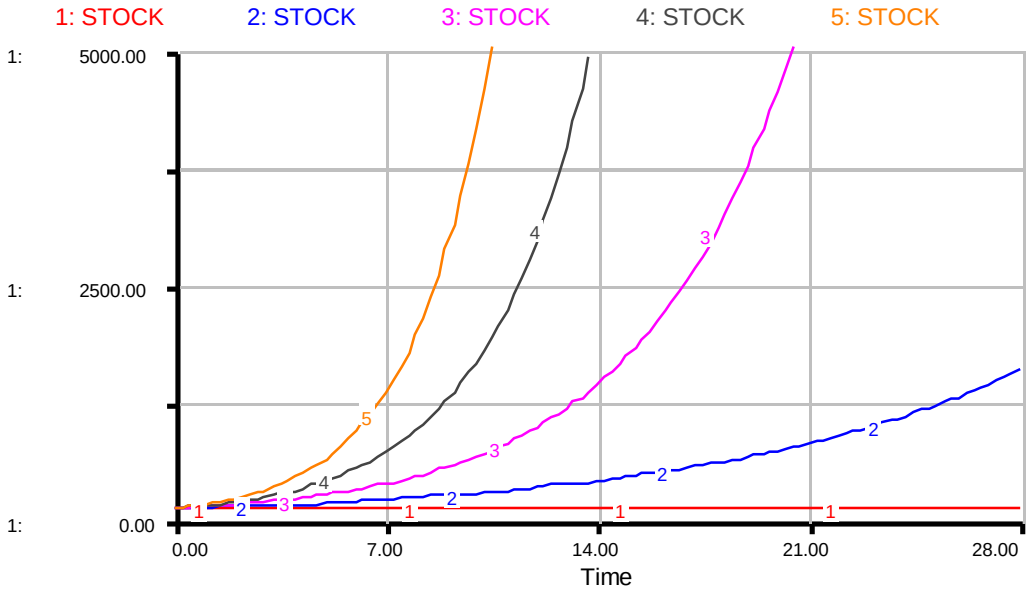


Figure 8 Simulation for different values of the compounding fraction

[2] A positive compounding fraction is required for a positive feedback (reinforcing) loop as it gives the net flow the sign of the stock (either positive or negative). A negative value for the compounding fraction will make the loop a negative feedback or balancing loop.

[3] This equilibrium is called unstable as the slightest deviation of the value of the stock away from zero will destroy the equilibrium and result in exponential growth.

The slope of the stock at a specific point[4] in time is equal to the net flow into it at that time. The flow is a larger fraction of the stock for a larger compounding fraction. Therefore, the slope of the stock is greater for a larger compounding fraction.

The larger the compounding fraction is, the larger the flow and the faster the growth of the stock. A larger compounding fraction accelerates exponential growth.

For a negative initial value of the stock, the effect of the compounding fraction on growth rate is the same, except the growth is in the negative direction.

5. Summary of important characteristics

Structure:

The loop is a positive feedback loop if and only if the stock has a positive sign in the equation for net flow into the stock. A positive sign in the equation for flow gives the flow the same sign as the stock (reinforcing behavior). Therefore, the simplest positive feedback loop requires a positive compounding fraction for the inflow to the stock.

Behavior:

We summarize the behavior of the positive feedback loop in table 1 below. Although positive feedback loops are best known for their exponential growth, they do exhibit other behaviors. Remember: A negative multiplier in the rate does not create a positive feedback loop.

The generic structure of a first-order positive loop can exhibit three types of behavior - positive exponential growth, unstable equilibrium, and negative exponential growth.

For an initial value of the stock and the multiplier in the rate (fraction or time constant), the behavior of the stock is given in italics	**Stock**		
	Negative	**Zero**	**Positive**

[4] The slope of the stock at a point is the slope of the line tangent to the curve at that point.

Compounding Fraction[5]	**Zero**	*Equilibrium*	*Zero*	*Equilibrium*
	Positive	*Negative Exponential growth*	*Zero*	*Positive Exponential growth*

Table 1 Summary of the behavior of a positive feedback loop

Exponential growth requires an initial value of the stock other than zero. Exponential growth has a constant doubling time. The rate at which an exponential growth occurs increases with the value of the compounding fraction.

Look over table 1 and the graphs of the simulation runs till you internalize your knowledge about positive feedback loops. When you feel confident about your understanding of the behavior, you should go on to the exercises in the next section.

6. Using insights gained from the generic structure

We have seen examples of different systems with the same positive feedback loop structure. We studied the underlying generic structure to develop our intuition about positive loops. Now, we apply the insight we gained from the generic structure to understand the behavior of other systems.

To do the exercises below, you need not simulate the models; hand computation should suffice. However, after answering the questions we encourage you to build and experiment with the models.

[5] Zero compounding fraction corresponds to an infinite time constant. This is not a situation you will be confronted with.

6.1. Exercise 1: Software sales

The customer base of a software manufacturer increases with the addition of new customers. Through the word of mouth, a fraction of the present customers encourage other people to become new customers. The model for this simple positive feedback system is shown below.

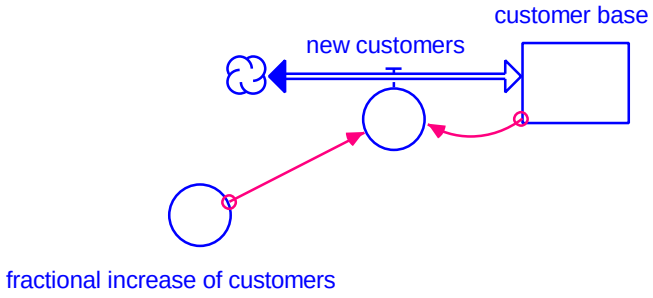


Figure 10 Model for Software sales

There are two software companies, Nanosoft and Picosoft, each of which have a customer base consisting of 10,000 customers, and a fractional increase is 0.1 customers/customer/week (the fraction means that 1 out of 10 customers convinces another person to become a customer each week).

1. What is the time constant and doubling time? Give their units. What are the units of new customers?
2. Approximately how much time does it take for the customer base of Nanosoft to grow to 40,000 customers?
3. If Nanosoft wants to have 80,000 customers in the same amount of time, how could it change the initial value of the stock to achieve this?[6]
4. Picosoft also wants 80,000 customers in the same time but decides to change the fractional increase to achieve this. What change should it make?
5. If Nanosoft has a customer base three times larger than Picosoft, which of the two firms do you think will grow faster? What is the ratio of their customer bases after 14 weeks?

[6] Although changing the initial stock may not be a feasible option in the real system, our purpose here is to understand the effect of different initial values of the stock on its growth.

6.2. Exercise 2: Making Friends

Brenda and Brandon are twins who have just moved into a new town to live with their aunt. Although they are twins, their personalities are quite different. Brenda is very sociable and makes friends easily. She usually makes a new friend, though each friend she already has, every 2 weeks. Brandon, on the other hand, is quite shy; it usually takes him twice as long as Brenda to make a new friend through each of his current friends.

In this new town, Brandon already has 5 friends that he made in previous summer visits. Brenda, however, has never been to this town before and the only 'friend' she has here is her aunt.

Figure 11 is a very simple model of the process by which new friends are made. The model indicates that the rate at which a person makes new friends depends on the amount of friends that a person already has and the time to make a new friend. For example, if Brenda has a lot of friends, she will be introduced to a lot of new people (friends of friends), and if she doesn't take much time to make friends with a new person, then she will make a lot of new friends very quickly.

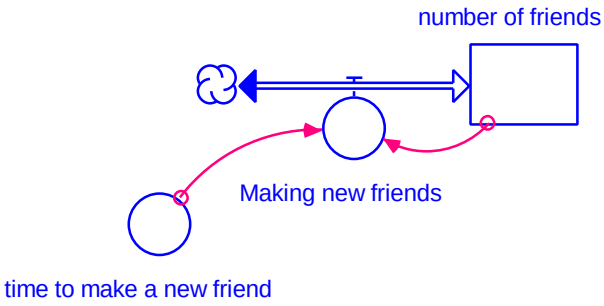


Figure 11 Model for making friends

1. What is the time constant and doubling time for Brenda?
2. What is the time constant and doubling time for Brandon?
3. By the time school starts (9 weeks after moving in) who will have more friends, Brenda or Brandon? You don't need to find exactly how many friends Brenda and Brandon have after 9 weeks; just provide an indication of who has more friends after 9 weeks.

6.3. Exercise 3: Account balance

Brandon decides he has had enough of school, and plans to start his own software business to compete with Nanosoft and Picosoft. We have built a model of his account balance shown in figure 12. A loan is considered as a negative account balance.

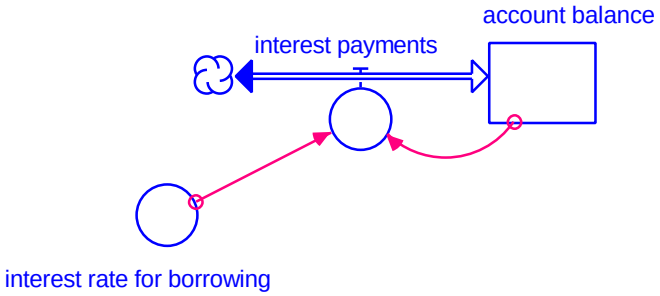


Figure 12 Model for account balance

To buy a computer, he takes out a $2,000 loan from his bank at an interest rate of 5%. On his way home from the bank, he meets Brenda who tells him of another bank that will forgive $1,000 of his loan if he transfers to them. This bank charges an interest rate of 10%. Brandon doesn't understand exponential growth very well and is confused about what he should do. What do you recommend? What will his account balance be after 14 years for each bank?

7. Solutions to Exercises

7.1. Answer to Section 6.1: Software Sales

1. The time constant = $\frac{1}{\text{fractional increase}}$, or 10 weeks.

The doubling time = 0.7 x the time constant, or 7 weeks.

2. For the customer base of 10,000 to grow to 40,000, the initial base doubles twice. Each doubling time is 7 weeks, thus it takes 14 weeks for the firm to reach 40,000 customers.

3. Since the time to reach 80,000 is given as 14 weeks (two doubling times), the initial value for the stock should be 20,000 customers.

4. To reach 80,000 customers, a base of 10,000 doubles three times in the 14 weeks. Working backwards:

1 doubling time = $\frac{14}{3}$ or $4.\bar{6}$ weeks.

1 time constant = $\frac{\text{doubling time}}{0.7} = 6.\bar{6}$ weeks.

Fractional increase = $\frac{1}{\text{time constant}} = \frac{21}{140}$ or .15 customers/cust/week

5. Nanosoft grows faster since it has a larger stock than Picosoft. While the fractional increase of each company is the same, the rate of growth for Nanosoft is larger because the actual number of customers the fractional increase corresponds to is larger. The ratio of customer bases after 14 weeks is still the same, 3 to 1. It may seem impossible for the ratio to remain the same while one firm grows at a faster rate than the other. The key to understand is that Nanosoft does grow faster but also has a larger distance to go to maintain the ratio of 3 to 1.

7.2. Answer to Section 6.2: Making Friends

1. Brenda's time constant is her time to make a new friend, which equals 2 weeks. The doubling time is the time constant x 0.7, or 1.4 weeks.

2. Brandon takes twice as long to make friends. His time to make a new friend is twice that of Brenda's. His time constant is thus 4 weeks. The doubling time is the time constant x 0.7, or 2.8 weeks.

3. The easiest way to answer this question is to use the doubling times and the initial values for the stocks of friends and make a small chart of Brenda and Brandon's number of friends for the summer.

Week	Brenda's Friends	Brandon's Friends
0	1	5
1.4	2	—
2.8	4	10
4.2	8	—
5.6	16	20
7.0	32	—
8.4	64	40

9	more than Brandon	less than Brenda

Because of the nature of the positive generic structure, once Brenda has more friends that Brandon, she will always have more friends that Brandon. We can then infer that in week 9, the end of the summer, Brenda has more friends than Brandon.

7.3. Answer to Section 6.3: Account Balance

Brandon's bank's interest rate is 0.05. The time constant is equal to 20, and the doubling time of the debt is equal to 14 years. The other bank's interest rate is 0.1 and has a time constant of 10 years. The doubling time of a debt is 7 years.

Years	Debt in Brandon's Bank	Debt in Other Bank
0	2000	1000
14	4000	4000
28	8000	16000
42	16000	64000
56	32000	256000

This chart clearly illustrates the power of exponential growth. The bank which Brandon should invest in depends on when he plans on paying back his loan. If he plans to pay back in the first 14 years, then the other bank would save him money. If it will take Brandon over 14 years to repay the loan, the bank he already borrowed from is his best bet.

8. Appendix - Model Documentation

8.1. Documentation for section 2.1: Population-birth system

deer population(t) = deer population(t - dt) + (births) * dt
INIT deer population = 100
DOCUMENT: This is the number of deer present in the system.
UNIT: deer

INFLOWS:

births = deer population * birth fraction

DOCUMENT:This is the number of deer born every year.

UNIT: deer/year

birth fraction = .3

DOCUMENT: This is the number of deer born per deer every year.

UNIT: deer/deer/year

8.2. Documentation for section 2.2: Bank balance-interest system

bank balance(t) = bank balance(t - dt) + (interest earned) * dt

INIT bank balance = 100

DOCUMENT: This is the amount of money in a bank account

UNIT: dollars

INFLOWS:

interest earned = bank balance * interest rate

DOCUMENT: This is the amount of interest earned per year on the money in the account.

UNIT: dollars/year

interest rate = .025

DOCUMENT: This is the number of dollars earned per dollar in 1 year.

UNIT: dollars/dollar/year

8.3. Documentation for section 2.3: Knowledge-learning system

knowledge(t) = knowledge(t - dt) + (learning) * dt

INIT knowledge = 100

DOCUMENT: This is the amount a person knows, measured in facts about a subject.
UNIT: facts

INFLOWS:
learning = knowledge /time to learn
DOCUMENT: This is the rate at which new facts are learned per day.
UNIT: facts/day

time to learn = 3
DOCUMENT: This is the time constant of the system. It takes an average of 3 days for each fact to assist in the learning of a new fact.
UNIT: day

Vensim Examples:
Generic Structures: First-Order Linear Positive Feedback

By Aaron Diamond

October 2001

2.1 Example 1: Population-Birth system

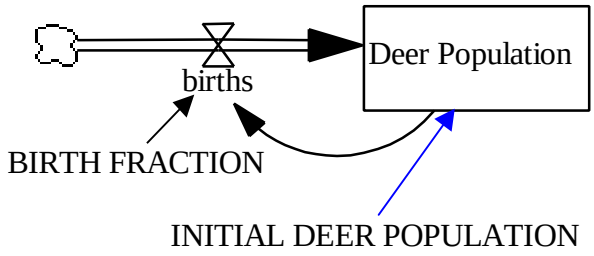


Figure 13: Vensim Equivalent of Figure 2: Model of a population-birth system

Documentation for population-birth model

(1) BIRTH FRACTION=0.3
Units: deer/deer/year
This is the number of deer born per deer every year.

(2) births=Deer Population*BIRTH FRACTION
Units: deer/year
The flow is the number of deer born every year.

(3) Deer Population= INTEG (births, INITIAL DEER POPULATION)
Units: deer
This is the number of deer present in the system.

(4) INITIAL DEER POPULATION=100
Units: deer

(5) FINAL TIME = 28

Units: year
The final time for the simulation.

(6) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(7) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(8) TIME STEP = 0.0625
Units: year
The time step for the simulation.

2.2 Example 2: Bank balance-interest system

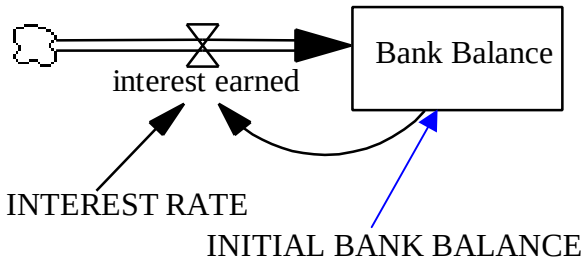


Figure 14: Vensim Equivalent of Figure 3: Model of a bank balance system

Documentation for bank balance model

(1) Bank Balance= INTEG (interest earned, INITIAL BANK BALANCE)
Units: dollars
This is the amount of money in a bank account.

(2) FINAL TIME = 28
Units: year
The final time for the simulation.

(3) INITIAL BANK BALANCE=100
Units: dollars

(4) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(5) interest earned=Bank Balance*INTEREST RATE
Units: dollars/year
The flow is the amount of interest earned per year on the money in the account.

(6) INTEREST RATE=0.025
Units: dollars/dollars/year
This is the number of dollars earned per dollar in 1 year.

(7) SAVEPER =TIME STEP
Units: year
The frequency with which output is stored.

(8) TIME STEP = 0.0625
Units: year
The time step for the simulation.

2.3 Example 3: Knowledge-learning system

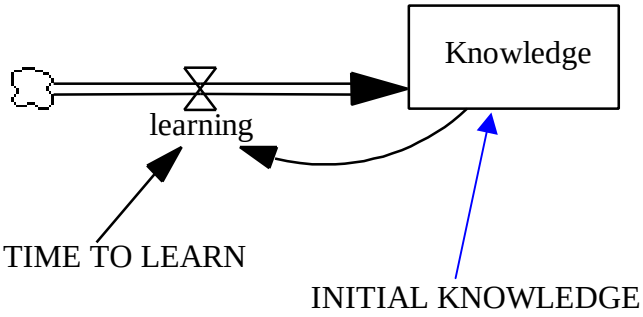


Figure 15: Vensim Equivalent of Figure 4: Model of a knowledge-learning system

Documentation for knowledge-learning model

(1) FINAL TIME = 28
Units: day
The final time for the simulation.

(2) INITIAL KNOWLEDGE=100
Units: facts

(3) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(4) Knowledge= INTEG (learning, INITIAL KNOWLEDGE)

Units: facts
This is the amount a person knows, measured in facts about a subject.

(5) learning=Knowledge/TIME TO LEARN
Units: facts/day
This is the rate at which new facts are learned per day.

(6) SAVEPER =TIME STEP
Units: day
The frequency with which output is stored.

(7) TIME STEP = 0.0625
Units: day
The time step for the simulation.

(8) TIME TO LEARN=3
Units: day
This is the time constant of the system. It takes an average of 3 days for each fact to assist in the learning of a new fact.

3.1. Model Diagram

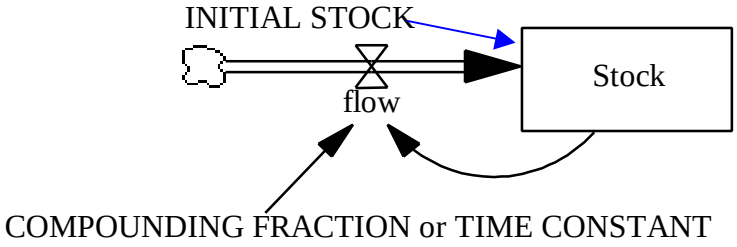


Figure 16: Vensim Equivalent of Figure 5: Model of the underlying generic structure

Documentation for generic structure model

(1) COMPOUNDING FRACTION or TIME CONSTANT=a constant
Units: units/units/time for COMPOUNDING FRACTION,
Units: time for TIME CONSTANT
This is the compounding fraction or growth factor. It determines the inflow to the stock. The compounding fraction corresponds to the birth fraction and the interest rate in the examples above.
It is the amount of units added to the stock for every unit already in the stock, every time.

(2) FINAL TIME =28
Units: Month
The final time for the simulation.

(3) flow=Stock*COMPOUNDING FRACTION, or Stock/TIME CONSTANT
Units: units/time
The flow is the fraction of the stock that flows into the system per unit time. It corresponds to the births, the interest earned, and the learning in the examples above.

(4) INITIAL STOCK=a constant
Units: units

(5) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(6) SAVEPER =TIME STEP
Units: Month
The frequency with which output is stored.

(7) Stock= INTEG (flow, INITIAL STOCK)
Units: units
This is the stock of the system. It corresponds to the deer population, the bank balance, and the stock of knowledge in the examples above.

(8) TIME STEP = .0625
Units: Month
The time step for the simulation.

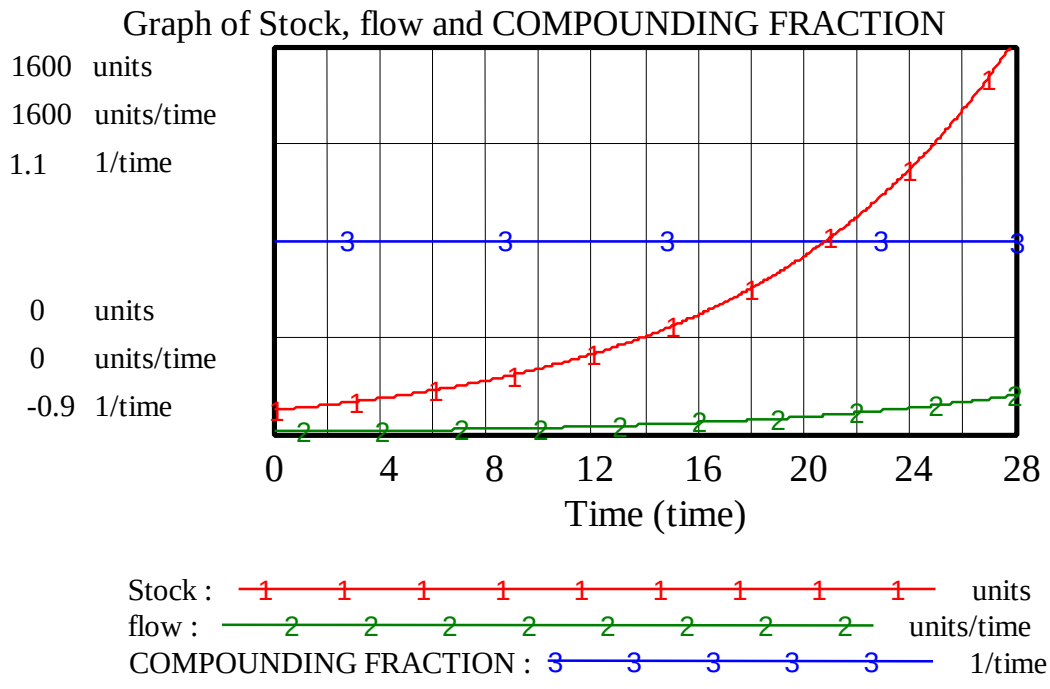


Figure 17: Vensim equivalent of Figure 6: Results of a simulation of the positive feedback generic structure.

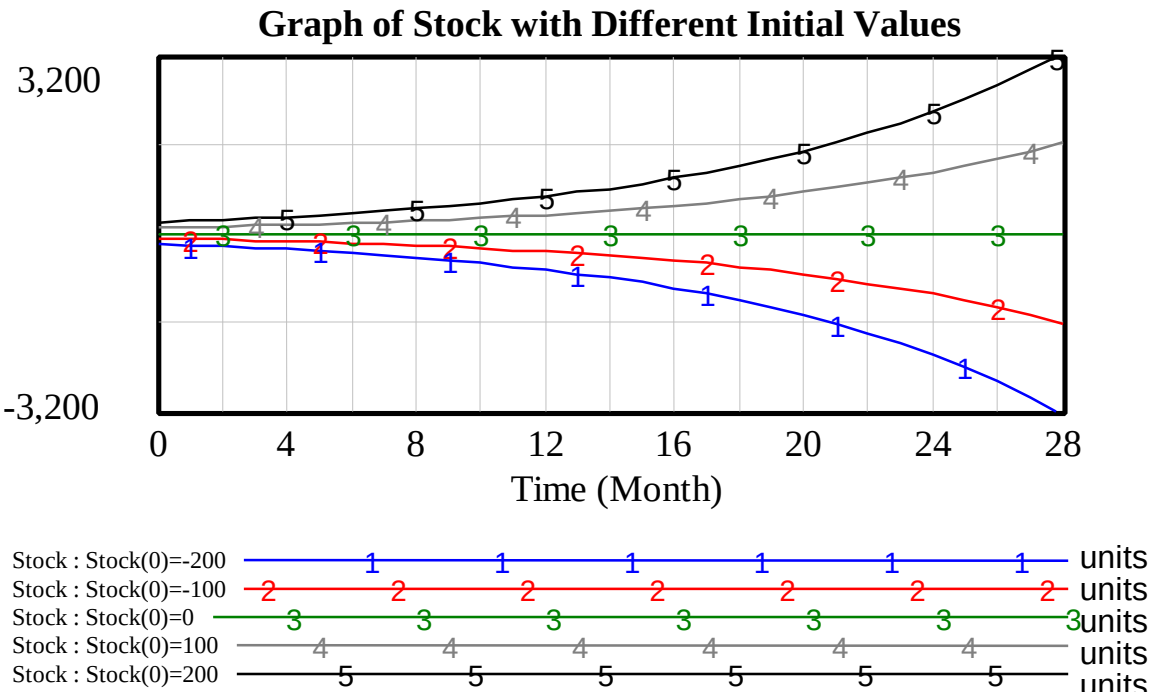


Figure 18: Vensim Equivalent of Figure 7: Simulation for different initial values of the Stock

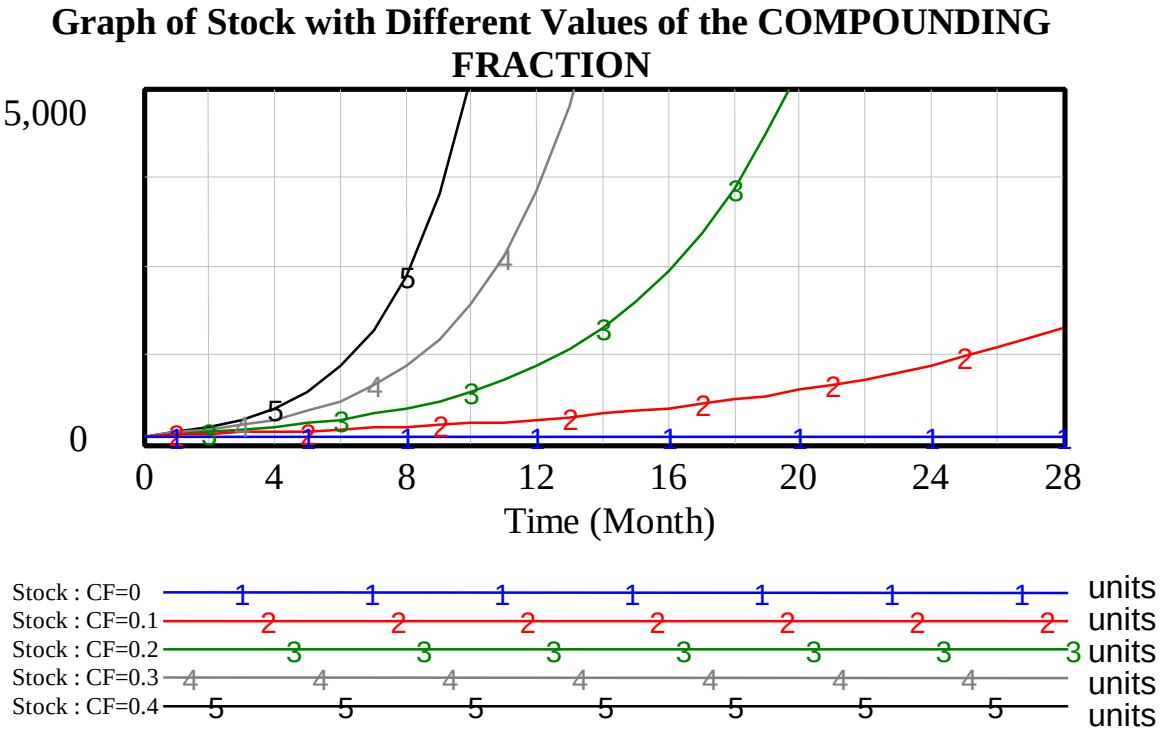


Figure 19: Vensim Equivalent of Figure 8: Simulation for different values of the COMPOUNDING FRACTION

Generic Structures:
First-Order Negative Feedback

Produced for the
System Dynamics in Education Project
MIT System Dynamics Group

Under the Supervision of
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1. Introduction

Generic structures are relatively simple structures that recur in many diverse situations. In this paper, for example, the models of radioactive decay and a population death system are shown to share the same basic structure! The transferability of structure between systems gives the study of generic structures its importance in system dynamics.

Road Maps contains a series of papers on generic structures. These papers use generic structures to develop an understanding of the relationship between the structure and behavior of a system. Such an understanding should help refine intuition about the systems that surround us and facilitate improvement of our ability to model behaviors exhibited by systems.

The knowledge about a generic structure in one system is transferable to understand the behavior of other systems that contain the same structure. Knowledge of generic structures and the behaviors they produce is transferable to systems never studied before!

Behavior of a system is often more obvious than it's underlying structure. It is common practice to refer to systems by the behaviors they produce. However, it is incorrect to assume such systems are capable of exhibiting only their most popular behaviors, and a need for a closer look at the other possible behaviors is present. A study of generic structures examines the range of behaviors possible from particular structures. In each case, one seeks to understand what in the structure causes the behavior produced.

This paper introduces a simple generic structure of first-order linear negative feedback. Many examples of systems containing the basic generic structure illustrates this study. Soon the structure will become recognizable in many new and different models. The exercises at the end of the paper provide an opportunity to transfer your knowledge between different systems.

2. Exponential Decay

Exponential decay is one of the behaviors commonly exhibited by a negative feedback loop. Figure 1 contains a typical exponential decay curve. An important characteristic of exponential decay is its asymptotic behavior.[1] The asymptote that the level approaches is the "goal" of the level. The goal is equal to zero in Figure 1. Another important characteristic of exponential decay is the curve's constant halving time. The halving time is the time it takes for the stock to reduce by one-half. The following simple formula approximates the halving time of a level using the time constant.

The **halving time = 0.7 * time constant**.

Remember, the time constant is the time in which the initial slope reaches the goal. Figure 1 illustrates this concept. The time constant is 3 because the line tangent to the initial stock value crosses the time axis at 3 time units.

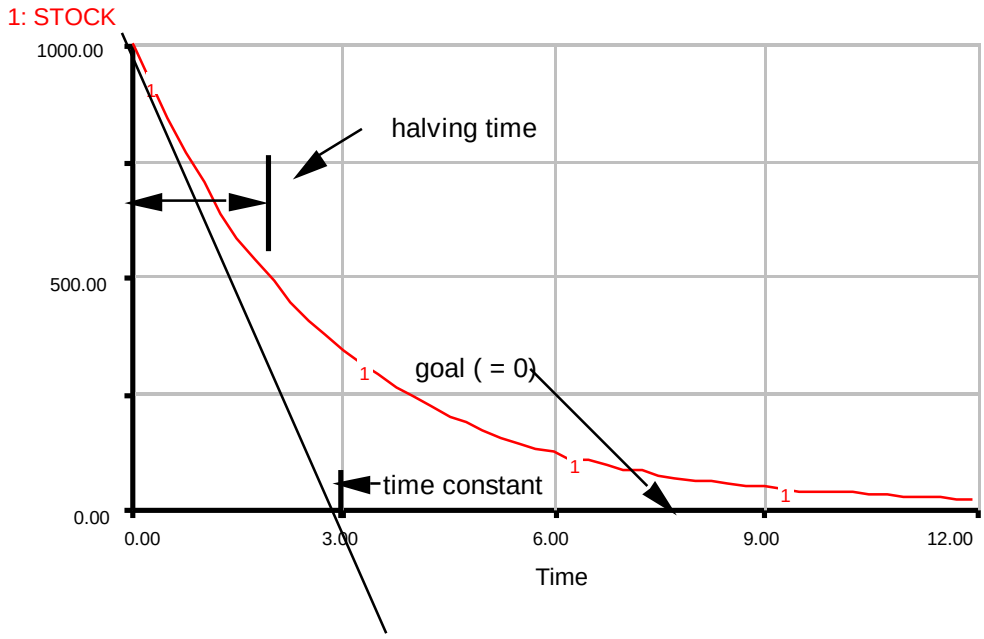


Figure 1. Exponential Decay Curve

[1] A curve exhibiting asymptotic behavior gradually approaches a specific value (the asymptote) over time. The slope of the curve gets closer and closer to the slope of the line.

A simple negative feedback loop consists of a stock, an outflow from the stock, and a goal. The outflow is proportional to the difference between the stock and the goal. One can also refer to negative feedback loops as self-correcting or goal-seeking loops.

For example, a radioactive element possesses a negative feedback structure and the resulting exponential decay. Another system with exponential decay arises from a population with no births or other inflows, and a death rate proportional to the population. Both of these systems share a common structure and exhibit similar behaviors. An additional system that exhibits exponential decay is a company that is downsizing it's work-force to reach an explicit goal. These three systems share the basic negative feedback structure with a goal (in some cases the goal is zero), and exhibit the characteristic behavior of exponential decay.

These three systems mentioned above will now be explored in more detail.

2.1 Example 1: Radioactive Decay

Figure 2 shows radioactive decay. The stock represents an amount of a **radioactive compound** and the natural **rate of decay** of the compound into other compounds is the outflow. The **decay fraction** is the fraction of the initial radioactive compound that decays each time period and is characteristic of the specific compound. Assuming this is a closed system and there is no further addition of the radioactive compound, eventually the entire initial amount will transform into a stable compound. The implicit goal of the level for the system is zero.

The **rate of decay** = **radioactive compound** * **decay fraction**.

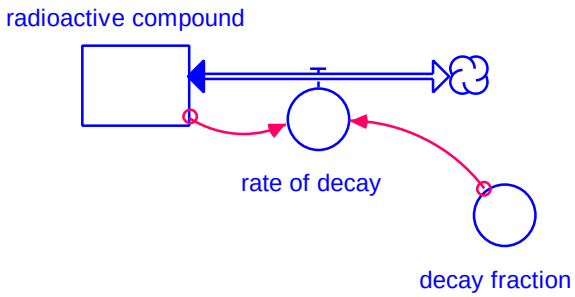


Figure 2. Radioactive Decay

2.2 Example 2: Population-Death system

Figure 3 models the dynamics of a population of mules. The **mule population** is the stock and the **death rate** is the outflow. The **death fraction** is the fraction of mules that die each year. The implicit goal of the mule system is zero. With no births in the system, the mules will eventually die out.

The **death rate** = **mule population** * **death fraction**.

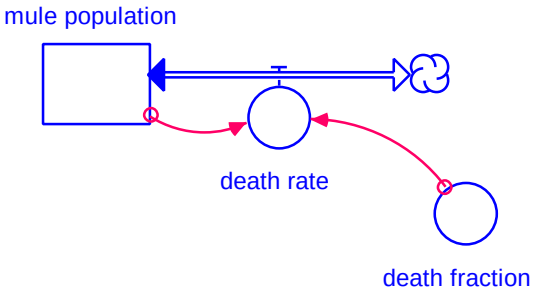


Figure 3. Population-Death system

Examples 1 and 2 both share the same underlying structure of a stock with an outflow proportional to the stock. Both have an implicit goal of zero, which drives the exponential decay.

2.3 Example 3: Company Downsizing System

The third example in Figure 4 shows a negative feedback structure with the **number of employees** in a company as the stock and the **firing rate** as the outflow. Instead of having an implicit goal of zero as in examples 1 and 2, the company's goal is the **desired number of employees.** The **distance to goal** is the difference between the level and the goal, and is simply the number of employees that the firm must reduce by to reach the goal. The

distance to goal = number of employees – desired number of employees.

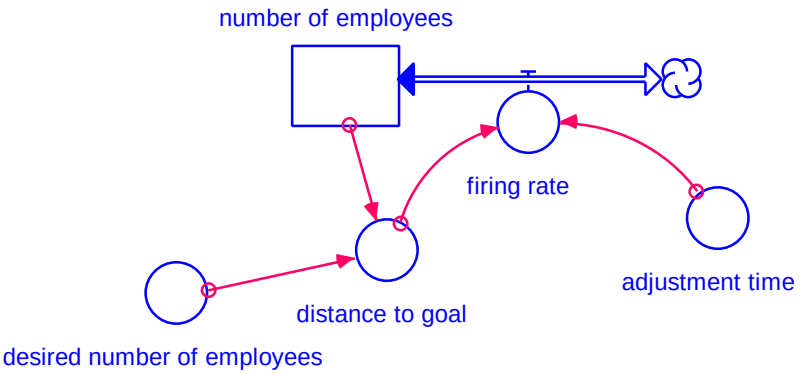


Figure 4. Company Downsizing System

The **firing rate** decreases the stock of employees. The **firing rate** is the number of employees fired per unit time, which is the same as the **distance to goal** spread out over the **adjustment time** Therefore, the **firing rate** is equal to the **distance to goal** divided by the **adjustment time**.

Firing rate = **distance to goal/adjustment time**.

In the equation of the rate, there is division by the **adjustment time,** which is the time constant of the system. Dividing by the adjustment time is analogous to multiplying by a draining fraction as seen in examples 1 and 2. In the equation of the rate, multiplying the **distance to goal** by a draining fraction is identical to dividing by the time constant. The time constant is simply the reciprocal of the draining fraction.

Example 3 has the same basic structure as examples 1 and 2. It is possible to model examples 1 and 2 in this way. They are just specific cases of the generic structure where the goal equals zero. If the desired number of employees equals zero, then the distance to the goal would simply equal the number of employees. The new rate equation becomes:

firing rate = distance to goal/adjustment time
firing rate = (number of employees – desired number of employees) / adjustment time
firing rate = (number of employees – 0)/adjustment time
firing rate = **number of employees/adjustment time**

This last equation uses the simpler structure of examples 1 and 2.

3. The Generic Structure

Now examine the generic structure of negative feedback systems used in all 3 examples. First, a model of the generic structure will be presented. All 3 examples can be modeled using the general form of the generic structure. Then, the model equations for both cases — using a draining fraction or using a time constant — will be examined. Lastly, the characteristic behaviors of the model will be discussed.

3.1 Model Diagram

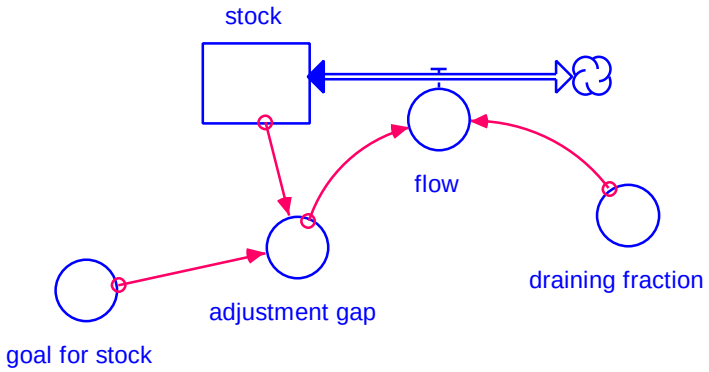


Figure 5. Generic Model

Figure 5 shows the generic structure of a negative feedback loop. This model of the generic structure can model examples 1, 2, and 3 as well as similar systems.

3.2 Model Equations

The equations for the generic structure are

stock(t) = stock(t – dt) + (– flow) * dt
DOCUMENT: This is the stock of the system. It corresponds to the amount of radioactive compound, the mule population, or the number of employees of a firm in the examples above respectively.
UNIT: units

OUTFLOWS:

flow = adjustment gap * draining fraction
DOCUMENT: The draining fraction and the value of the gap set the outflow to the stock. It corresponds to the decay rate, death rate, and firing rate in the above examples.
UNIT: units/time

adjustment gap = stock – goal for stock
DOCUMENT: The adjustment gap is the difference between the stock and the goal for the stock. In the radioactive and population examples it is equal to the difference between the stock and the implicit goal of zero, or simply the value of the stock. The adjustment gap also corresponds to the distance to goal in the employee example.
UNIT: units

draining fraction = *a constant*
DOCUMENT: The draining fraction is the fraction of the gap (equal to the stock when the goal is zero) that is closed each time period. The draining fraction corresponds to the decay fraction and the death fraction in the examples above.
UNIT: 1/time

goal for stock = *a constant*
DOCUMENT: This is the goal for the stock. The goal equals zero in the radioactive compound and population systems, and corresponds to the desired number of employees in example 3.
UNIT: units

Note: If we had a time constant instead of a draining fraction, the equation for the flow and the time constant would be

flow = adjustment gap/time constant
UNIT: units/time

t**ime constant** = *a constant*

DOCUMENT: This is the time constant. It represents the adjustment time for the stock.
UNIT: time

From the comparison of the two possible equations for the rate, we notice that the multiplier in the rate equation is given by[2]

$$\text{multiplier in the rate equation} = \text{draining fraction} = \frac{1}{\text{time constant}}$$

[2]For a more in depth explanation of time constants see the paper Beginner Modeling Exercises Section 3: Mental Simulations of Negative Feedback by Helen Zhu. (D-4536)

3.3 Model Behavior

The characteristic feature of exponential decay is its constant halving time which is the time for the gap to be cut in half. Because the draining fraction is constant at 0.2, the halving time remains constant for the entire simulation of the model. In Figure 6, for example, the initial stock is 100 and the goal is 50, making the gap equal to 50. It takes about 3.5 years for the gap to be cut in half to 25 (stock = 75) and another 3.5 years for the gap to be cut in half again to 12.5 (stock = 62.5)!

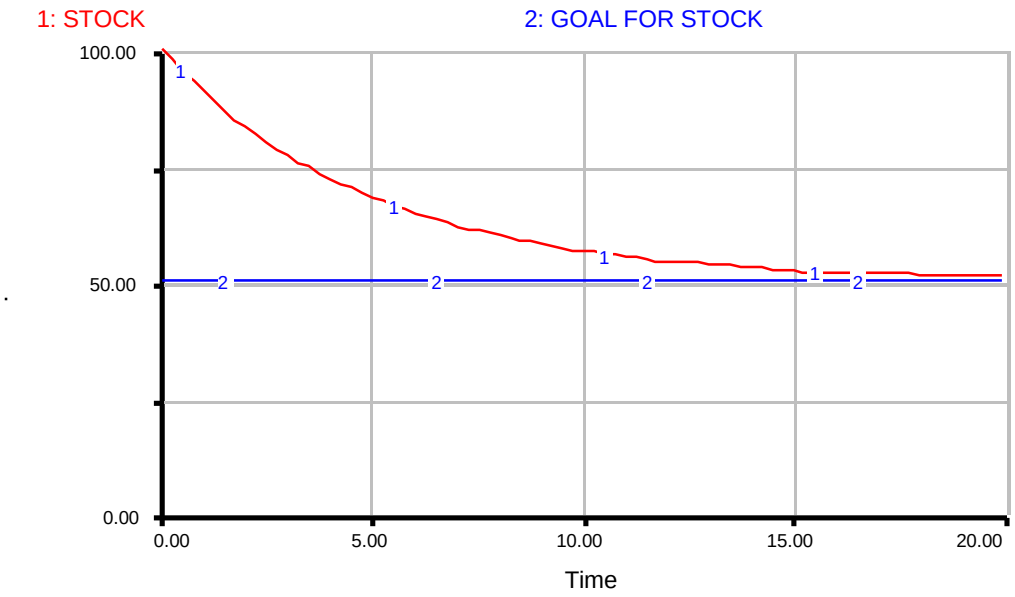


Figure 6. Results of Simulation

4. Behaviors produced by the generic structure

The behavior produced by this structure will vary, depending on the values of three parameters: the initial value of the stock, the goal, and the draining fraction.

4.1 Changing the initial value of the stock

This section explores the effect of varying the initial stock value. In our generic model, the stock has initial values of -8000, -4000, 0, 4000, and 8000 for runs 1 through 5 respectively. The goal is constant and zero, the draining fraction at 0.2.

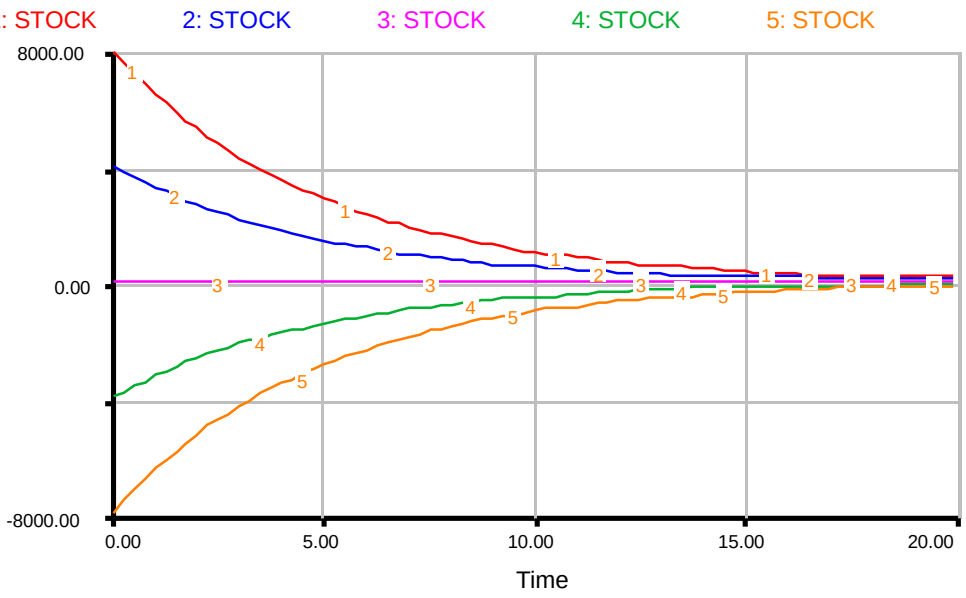


Figure 7. Simulation for different initial values of the stock

Figure 7 shows that the stock will always asymptotically approach its goal (in this case equal to zero) whatever its initial value is. In 5 halving times, the stocks are all about 97% of the way to their goal of zero.

We can clearly see that a negative feedback loop creates goal-seeking behavior. The stock will always asymptotically try to reach the goal to be in a state of stable equilibrium. The generic structure of a first-order negative feedback loop can exhibit three types of behavior: asymptotic (exponential) decay to equilibrium, steady state in equilibrium, and asymptotic growth to equilibrium.

4.2 Changing the value of the draining fraction

Changing the value of the draining fraction accelerates or retards the exponential decay of a system. To study this effect we will vary the draining fraction while keeping the initial value of the stock constant at 8000 and the goal set to zero. The draining fraction has values of 0, 0.1, 0.2, 0.3, and 0.4 for runs 1 through 5 respectively. The change of behavior of the stock due to varying the draining fraction is shown in Figure 8 below.

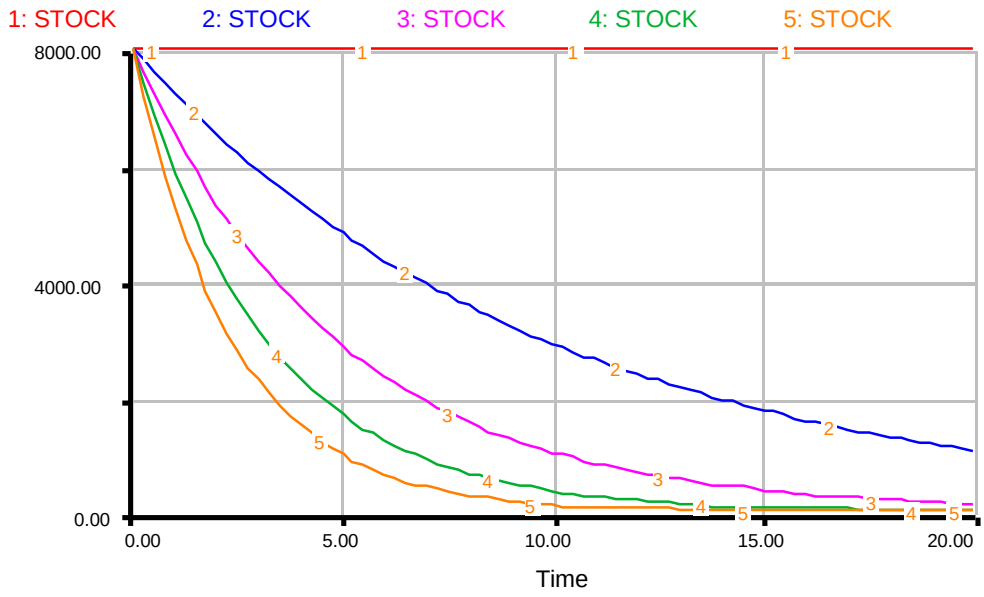


Figure 8. Simulation for different values of the draining fraction

Notice that when the draining fraction is equal to zero, the original level of the stock does not change. This is because the flow is equal to the draining fraction times the level. Zero multiplied by the stock creates a flow of zero. Also, the greater the value of the draining fraction, the faster the stock approaches equilibrium. A larger draining fraction accelerates the exponential decay. An examination of the halving time equation for different values of the time constant[3] can quantitatively verify that acceleration. A draining fraction of 0.1 corresponds to a halving time of 7 time units. A draining fraction of 0.4 corresponds to a halving time of only 1.75 time units. (Note: For a negative initial value of the stock, the effect of the draining fraction on the asymptotic growth rate to the goal is similar. In that case, the stock grows asymptotically. The larger the draining fraction, the faster the stock grows.)

[3] Halving time = 0.7 * time constant = 0.7/draining fraction. For a more in-depth explanation see Zhu, Helen. Beginner Modeling Exercises 3: Mental Simulations of Negative Feedback (D-4536). January 1996, MIT System Dynamics in Education Project

4.3 Changing the value of the goal

Changing the goal for the stock also creates interesting behaviors. We are again keeping the initial stock at 8000 and the draining fraction equal to 0.2. Values of the goal are 8000, 6000, 4000, 2000, and 0 for runs 1 through 5. Figure 9 shows the simulation for different values of the goal. Though the stock never actually reaches the goal, after about 5 halving times the stock is 97% of the way to the goal. All the different runs have the same halving times and traverse this percentage of the gap at the same time.

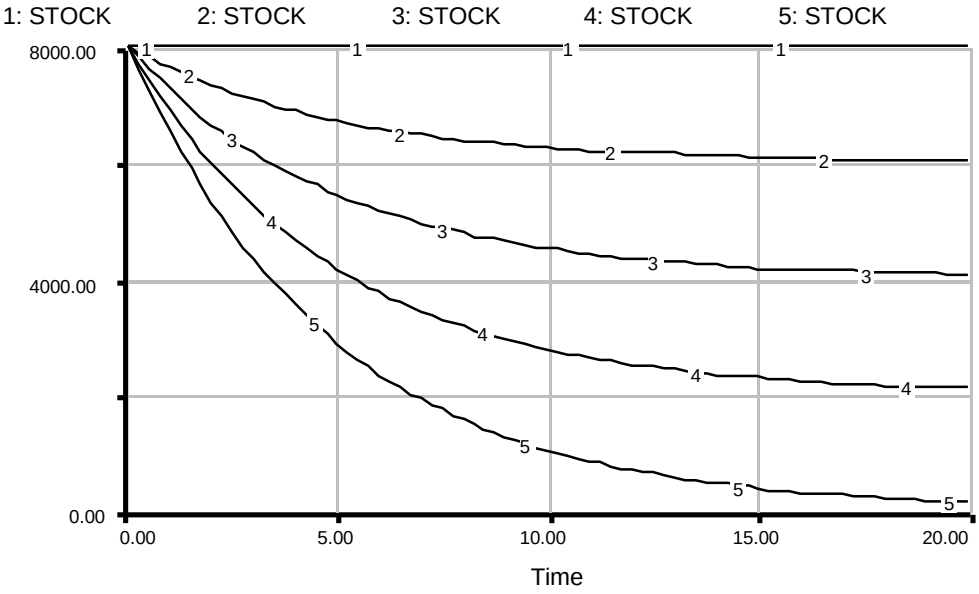


Figure 9. Simulation for different values of the goal for the stock

5. Summary of important characteristics

Structure:

The loop is a negative feedback loop if the flow is specified as an outflow in the model and the draining fraction is positive (or if the draining fraction is negative and the flow is specified as an inflow in the model).[4]

Behavior:

Figure 10 contains a summary of the behaviors that the negative feedback loop can exhibit. Although negative feedback loops are best known for their exponential decay, they do exhibit other behaviors.

The generic structure of a first-order negative loop can exhibit three types of behaviors: asymptotic decay to equilibrium, constant at equilibrium, and asymptotic growth to equilibrium. All three types of behavior are goal-seeking.

The behavior of the stock is in italics for each combination of parameter values. (assume goal = 0)		**Stock**		
		Negative	**Zero**	**Positive**
Draining Fraction	**Zero**	*equilibrium at neg. value*	*equilibrium at zero*	*equilibrium at pos. value*
	Positive	*asymptotic growth to equilibrium*	*equilibrium at zero*	*asymptotic decay to equilibrium*

Figure 10. Summary of the behavior of a negative feedback loop

[4] Since STELLA specifies the outflow as negative in the equations of the stock, we will assume the draining fraction must be positive. Therefore, the simplest negative feedback loop requires a positive draining fraction for the outflow of the stock.

To sum up, asymptotic growth and decay require an initial value of the stock other than zero and have a constant halving time. The rate at which the decay occurs increases with the value of the draining fraction. If either the stock or the draining fraction is zero, the outflow is also zero and equilibrium exists.

Look over Figure 10 and all the graphs of simulation included in the paper. When you feel confident about your understanding of the behavior, you should go on to the exercises in the next section.

6. Using Insights gained from the generic structure

We have seen many examples of different systems, with both implicit and explicit goals with the same basic structure. Now, we apply the insight we gained from understanding the generic structure and its behavior to understand the behavior of other systems.

To do the exercises, you need not simulate the models; hand computation should suffice. However, after answering the questions, we encourage you to build and experiment with the models.

6.1 Exercise 1: Radioactive Decay

In section 2.1 we introduced the simple structure of radioactive decay. Carbon-14 decays into the stable element Nitrogen with a half-life of 5700 years.

1. Using the above information, draw the stock and flow diagram of the decay of Carbon-14.
2. What is the value of the time constant? the draining fraction?

6.2 Exercise 2: Package Deliveries

Below is a simple negative feedback system.

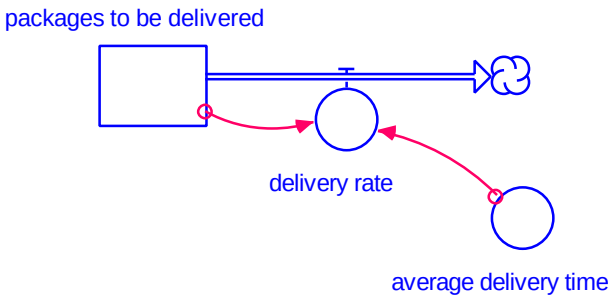


Figure 11. Package Deliveries

Jay runs a delivery service. His average delivery time is 2 days. A computer company, Nanosoft, just sent him 500 packages **to deliver** immediately. The more packages Jay has to deliver, the more pressure he is under to deliver them.

1. What is the time constant and halving time? Give their units. What are the units of the delivery rate?
2. What is the goal of the stock of this system?
3. How much time will it take for 50% of the packages to be delivered? 75% of the packages?

6.3 Exercise 3: Company Downsizing

Nanosoft has been losing money due to the competition of rival Picosoft. Managers at Nanosoft decided that by cutting the number of employees from 20,000 to 12,000 Nanosoft could save money and still maintain production levels. The total time allotted for the downsizing is 7 years. (Since it takes 5 halving times to reach 97% of the goal—close enough to consider the downsizing complete—the halving time is 1.4 years long and the time constant is 2 years.)

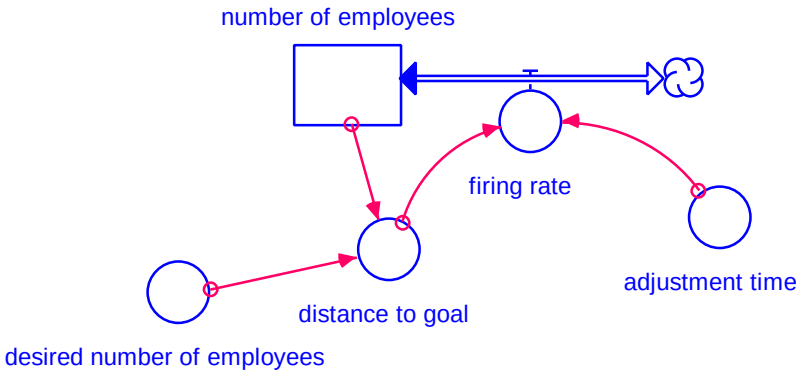


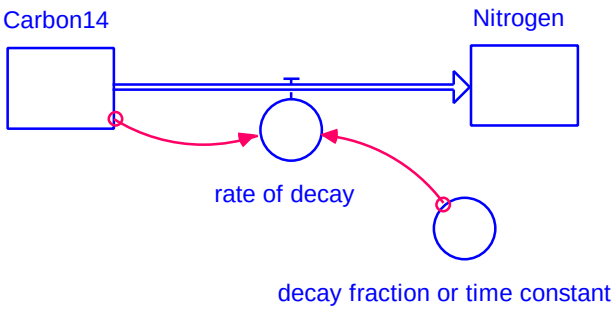
Figure 12. Company Downsizing

1. What is the draining fraction and the adjustment time? What is the halving time? Give their units.
2. In 3 years, what is the approximate number of employees working at Nanosoft?
3. Nanosoft decided they want exactly 16,000 employees working at the firm in 4 years. How can Nanosoft get this result by changing the adjustment time while keeping the desired number of employees at 12,000?

7 Solutions to exercises

7.1 Solution to 6.1: Radioactive Decay

1.



We have a set amount of Carbon-14 that does not grow,thus our stock has no inflows and one outflow for decay. Carbon-14 decays into Nitrogen, so the outflow from the Carbon stock is the inflow to the Nitrogen stock.

2. The halving time of Carbon 14 is 5700 years.
The time constant ? halving time/ 0.7 ? 5700/0.7 ? 8142.86 years
The draining fraction = 1/time constant = 1/8142.86 ? 0.000123

(NOTE to advanced modelers: The reason we can model the system in such a way, is because the unit of the stock is atoms. The number of atoms in the system is conserved. **Fifteen** atoms of Carbon 14 will decay into 15 atoms of Nitrogen. Had we done the problem with the units of the stock being grams, the system would not be valid. The mass

would not be conserved since the Carbon-14 is losing a beta particle (mass) to the atmosphere which is outside of the system as we modeled it).

7.2 Solution to 6.2: Package Deliveries Solution

1. The time constant = 2 days. The halving time ? 0.7 x time constant ? (2)(0.7) ? 1.4 days

The units for the delivery rate are packages/day.

2. The goal for the system is implicitly zero.

3. For 50% (1 halving time) of the packages to be delivered, Jay takes 1.4 days. For 75% (2 halving times) of the packages to be delivered, Jay takes 2.8 days.

7.3 Solution to 6.3: Company Downsizing

1. The time constant = 2 years, which is the adjustment time. The draining fraction = 1/time constant = 0.5.

The halving time ? 0.7 x time constant = (0.7)(2) = 1.4 years.

2. In 3 years about 2 halving times go by. In two halving times, about 75% of the initial gap is filled. 75% of 8000 is 6000, therefore (20,000 - 6,000) 14,000 employees are still working at Nanosoft.

3. In 4 years, Nanosoft wants 4,000 workers fired. This is 50% of the gap and should take 1 halving time. If 1 halving time = 4 years, the adjustment time ? halving time/ 0.7 ? 4/0.7 = 5.7 years. Nanosoft should increase the adjustment time to 5.7 years.

8 Appendix

Model Documentation for Sections 2.1 to 2.3

Section 2.1: Radioactive Decay Model Documentation

radioactive compound(t) = radioactive compound(t – dt) + (– rate of decay) * dt

INIT radioactive compound = *a constant*

DOCUMENT: This is the amount of radioactive compound present.

UNIT: atoms

OUTFLOWS:

rate of decay = radioactive compound * decay fraction

DOCUMENT: This is the amount of radioactive compound that turns into a stable compound per year.

UNIT: atoms/year

decay fraction = *a constant*

DOCUMENT: This is the draining fraction of the negative feedback system.

UNIT: 1/time

Section 2.2: Population-Death System Model Documentation

mule population(t) = mule population(t – dt) + (– death rate) * dt

INIT mule_population = *a constant*

DOCUMENT: This is the number of live mules in the system

UNIT: mules

OUTFLOWS:

death rate = mule population * death fraction

DOCUMENT: This is the number of mules that die per year.

UNIT: mule/year

death fraction = *a constant*

DOCUMENT: This is the draining fraction of the negative feedback system.

UNIT: 1/years

Section 2.3: Company Downsizing Model Documentation

number of employees(t) = number of employees(t – dt) + (– firing rate) * dt
INIT number of employees = *a constant*
DOCUMENT: This is the number of employee who work for the company.
UNIT: employee

OUTFLOWS:
firing rate = distance to goal/adjustment time
DOCUMENT: This is the number of employees who are fired per week.
UNIT: employee/week

adjustment time = *a constant*
DOCUMENT: This is the time constant of the system.
UNIT: week

desired number of employees = *a constant*
DOCUMENT: This is the goal for the negative feedback system.
UNIT: employee

distance to goal = number of employees – desired number of employees
DOCUMENT: This is the adjustment gap for the negative feedback system.
UNIT: employee

Vensim Examples: Generic Structures: First-Order Negative Feedback

By Aaron Diamond
October 2001

2.1 Example 1: Radioactive Decay

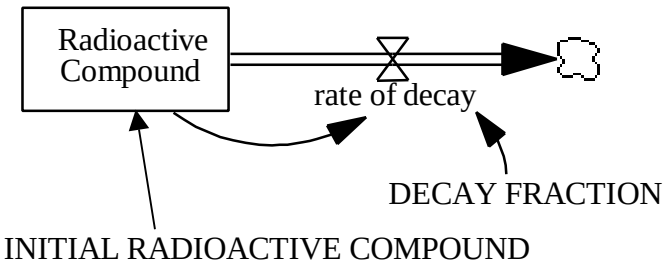


Figure 13: Vensim Equivalent of Figure 2: Radioactive Decay

Documentation for Radioactive Decay model

(1) DECAY FRACTION=a constant
Units: 1/year
This is the draining fraction of the negative feedback system.

(2) FINAL TIME = 100
Units: Year
The final time for the simulation.

(3) INITIAL RADIOACTIVE COMPOUND=a constant
Units: atoms

(4) INITIAL TIME = 0
Units: Year
The initial time for the simulation.

(5) Radioactive Compound= INTEG (-rate of decay, INITIAL RADIOACTIVE COMPOUND)
Units: atoms
This is the amount of radioactive compound present.

(6) rate of decay=Radioactive Compound*DECAY FRACTION
Units: atoms/year
This is the amount of radioactive compound that turns into a stable compound per year.

(7) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(8) TIME STEP = .0625
Units: Year
The time step for the simulation.

2.2 Example 2: Population-Death system

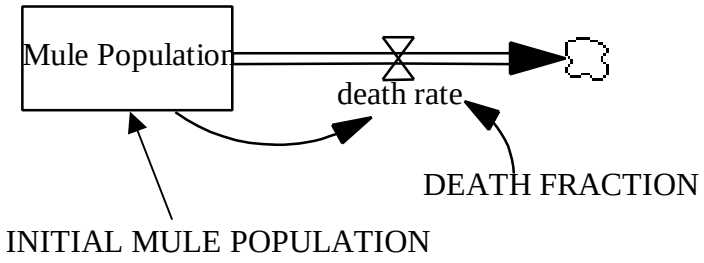


Figure 14: Vensim Equivalent of Figure 3: Population-Death system

Documentation for Population-Death model

(1) DEATH FRACTION=a constant
Units: 1/year
This is the draining fraction of the negative feedback system.

(2) death rate=Mule Population*DEATH FRACTION
Units: mules/year
This is the number of mules that die per year.

(3) FINAL TIME = 100
Units: year
The final time for the simulation.

(4) INITIAL MULE POPULATION
Units: mules

(5) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(6) Mule Population= INTEG (-death rate, INITIAL MULE POPULATION)
Units: mules
This is the number of live mules in the system.

(7) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(8) TIME STEP = .0625
Units: year

The time step for the simulation.

2.3 Example 3: Company Downsizing System

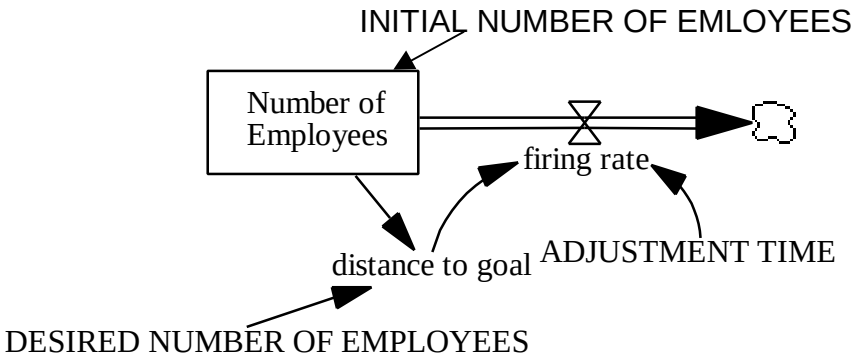


Figure 15: Vensim Equivalent of Figure 4: Company Downsizing System

Documentation for Company Downsizing model

(01) ADJUSTMENT TIME=a constant
Units: week
This is the draining fraction of the negative feedback system.

(02) DESIRED NUMBER OF EMPLOYEES=a constant
Units: employee
This is the goal for the negative feedback system.

(03) distance to goal=Number of Employees-DESIRED NUMBER OF EMPLOYEES
Units: employee
This is the adjustment gap for the negative feedback system.

(04) FINAL TIME = 20
Units: week
The final time for the simulation.

(05) firing rate=distance to goal/ADJUSTMENT TIME
Units: employee/week
This is the number of employees who are fired per week.

(06) INITIAL NUMBER OF EMPLOYEES=a constant
Units: employee

(07) INITIAL TIME = 0
Units: week

The initial time for the simulation.

(08) Number of Employees= INTEG (-firing rate, INITIAL NUMBER OF EMPLOYEES)
Units: employee
This is the number of employees who work for the company.

(09) SAVEPER = TIME STEP
Units: week
The frequency with which output is stored.

(10) TIME STEP = 0.0625
Units: week
The time step for the simulation.

3.1 Model Diagram

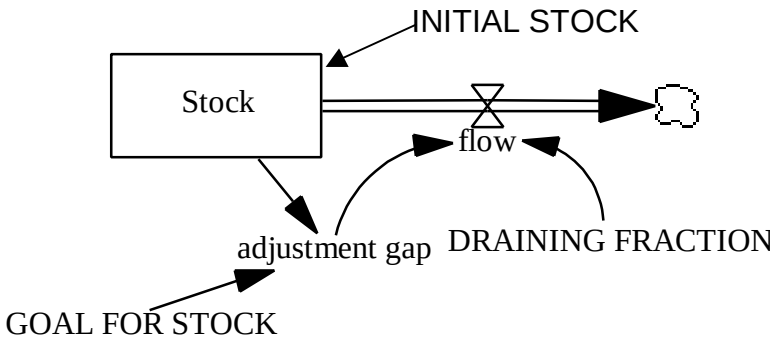


Figure 16: Vensim Equivalent of Figure 5: Generic Model

Documentation for the Generic Model

(01) adjustment gap=Stock-GOAL FOR STOCK
Units: units
The adjustment gap is the difference between the stock and the goal for the stock. In the radioactive and population examples it is equal to the difference between the stock and the implicit goal of zero, or simply the value of the stock. The adjustment gap also corresponds to the distance to goal in the employee example.

(02) DRAINING FRACTION=a constant
Units: 1/time
The draining fraction is the fraction of the gap (equal to the stock when the goal is zero) that is closed each time period. The draining fraction corresponds to the decay fraction and the death fraction in the examples above.

(03) FINAL TIME = 100
Units: time
The final time for the simulation.

(04) flow=adjustment gap*DRAINING FRACTION
Units: units/time
The draining fraction and the value fo the gap set the outflow to the stock. It corresponds to the decay rate, death rate, and firing rate in the above examples.

(05) GOAL FOR STOCK=a constant
Units: units
This is the goal for the stock. The goal equals zero in the radioactive compound and population systems, and corresponds to the desired number of employees in example 3.

(06) INITIAL STOCK= a constant
Units: units

(07) INITIAL TIME = 0
Units: time
The initial time for the simulation.

(08) SAVEPER = TIME STEP
Units: time
The frequency with which output is stored.

(09) Stock= INTEG (-flow, INITIAL STOCK)
Units: units
This is the stock of the system. It corresponds to the amount of radioactive compound, the mule population, or the number of employees of a firm in the examples above respectively.

(10) TIME STEP = 0.0625
Units: time
The time step for the simulation.

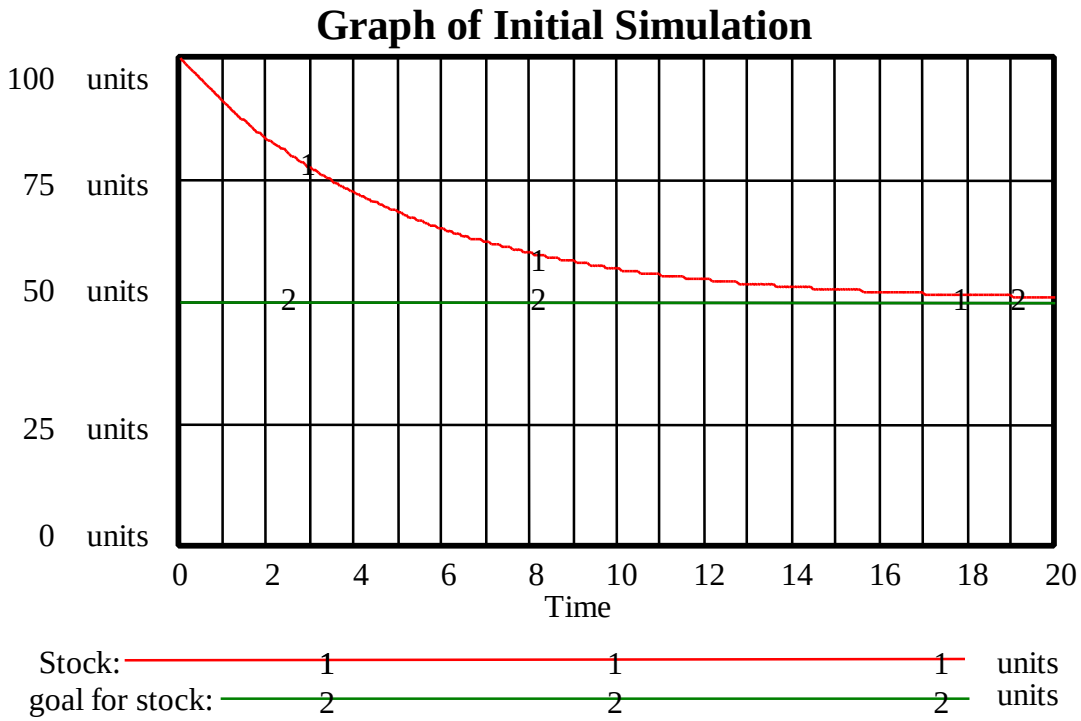


Figure 17: Vensim Equivalent of Figure 6: Results of Simulation

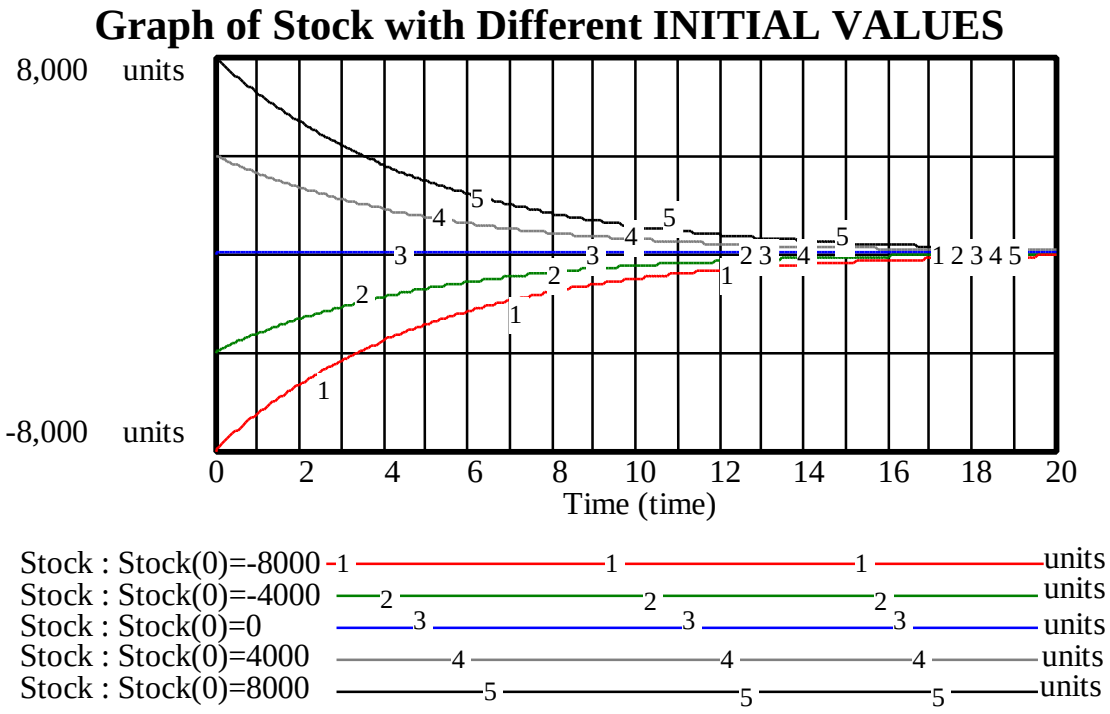


Figure18: Vensim Equivalent of Figure 7: Simulation for different INITIAL VALUES of the Stock

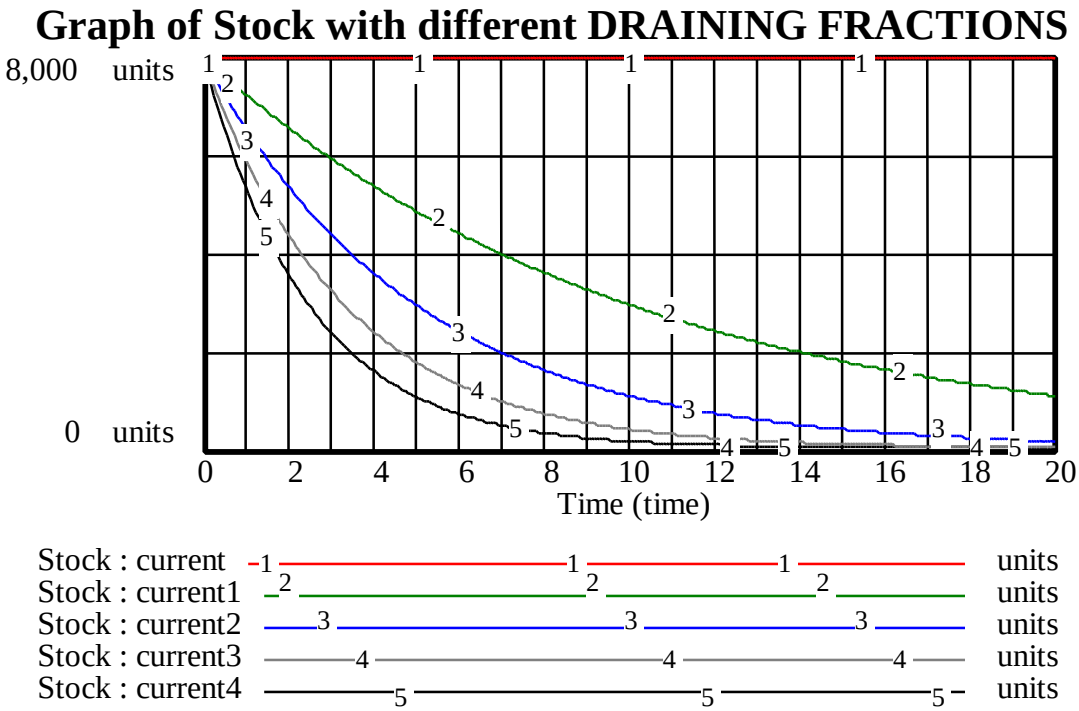


Figure 19: Vensim Equivalent of Figure 8: Simulation for different values of the DRAINING FRACTION

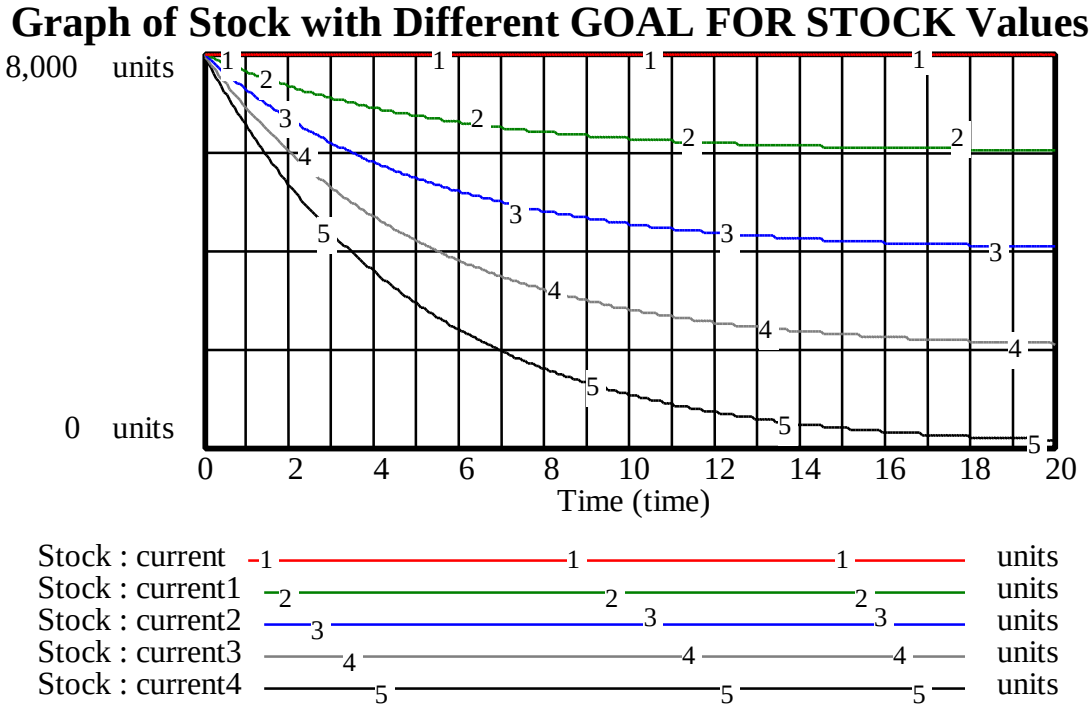
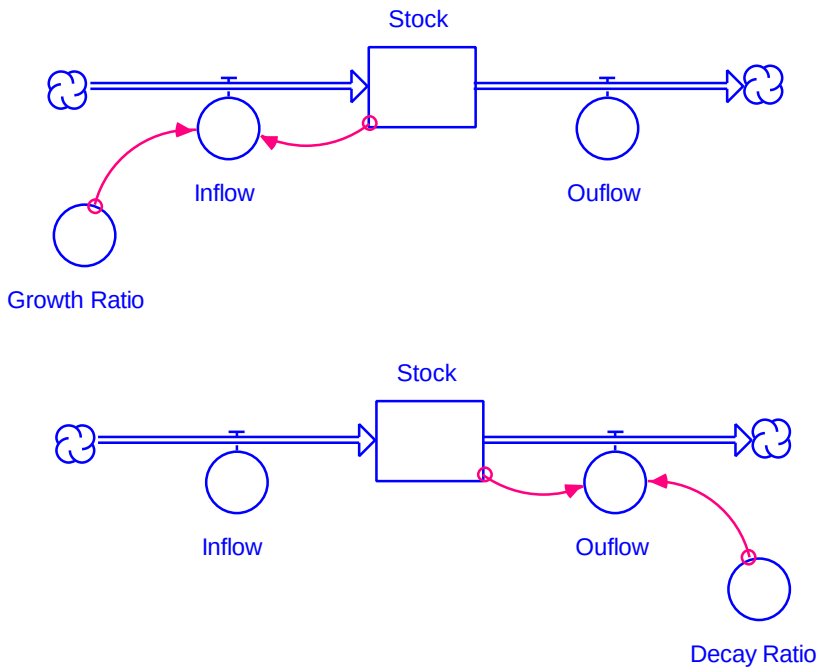


Figure 20: Vensim Equivalent of Figure 9: Simulation for different values of the GOAL FOR STOCK

Beginner Modeling Exercises

Section 4

Mental Simulation: Adding Constant Flows



Prepared for
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Alan E. Coronado, 1996
Vensim Examples added October 2001

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Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool, one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with computer simulation software.[1] Computer simulation is a very useful tool for exploring systems. However, one should be able to use the other simulation tool of system dynamics: *mental simulation.* A strong set of mental simulation skills will enhance ability to validate, debug and understand dynamic systems and models.

This paper provides guidelines to mentally simulate first-order (single-stock) feedback systems containing constant flows into or out of the stock. These guidelines are presented as the following three steps of mental simulation: first, *calculate equilibrium*; then, *determine the behavior mode* that the system will exhibit; and finally, *sketch the expected behavior*. Application of these three steps will first be demonstrated on a positive feedback system containing an outflow and then on a negative feedback system with an inflow. Practice exercises will follow the examples. In order to obtain a better understanding, the reader is encouraged to make a serious attempt at solving the exercises before looking up the answers in the back. It is assumed that the reader has experience mentally simulating simple positive and negative feedback systems.[2]

1. Positive Feedback with Constant Outflow

The purpose of this section is to demonstrate application of the following insights to the mental simulation of positive feedback systems containing constant outflows:

? Adding constant flows to a positive feedback system shifts equilibrium away from zero.

---

[1]There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road Maps is geared towards the use of STELLA II which is available from High Performance Systems (603) 643-9636. Road Maps can be accessed through the internet at http:/sysdyn.mit.edu/.

[2] For practice exercises consult the "Beginner Modeling Exercises: Mental Simulation" papers on "Positive Feedback" (D-4487) by Jospeh Whelan and "Negative Feedback" (D-4536) by Helen Zhu.

? Constant flows do not change the characteristics of exponential growth generated by positive feedback. Thus even in the presence of steady flows, the doubling time can be used to estimate system behavior.

The model to be simulated is built by a scientist interested in breeding a population of fruit flies in order to assure a steady supply for use in experiments. The model contains a stock, representing the fruit fly population, which is subject to two flows. The inflow corresponds to reproductive growth. Since fruit flies reproduce rapidly, adding about half a population every day, the scientist estimates a reproduction ratio of 50% per day. The outflow corresponds to a constant rate of removal of specimen from the stock. In this example the scientist desires a steady supply of about 50 fruit flies per day. In order to determine the right amount of fruit flies needed to begin breeding them, the scientist builds the stock-and-flow model in Figure 1.

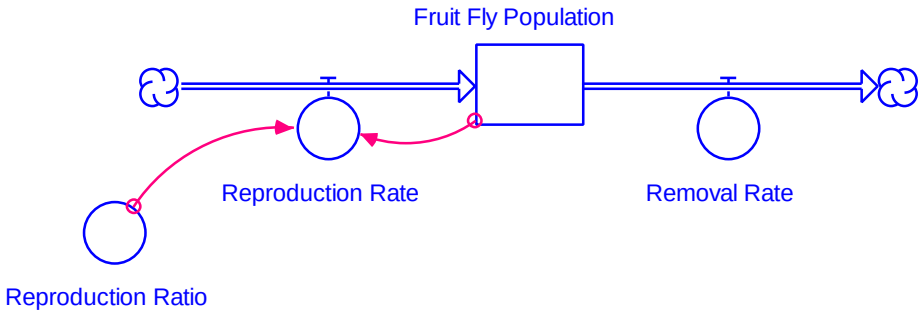


Figure 1: Fruit fly population model

The quantity of fruit flies the scientist needs should allow for the population to remain stable, with a balance struck between the removal and reproduction of fruit flies. Thus the scientist will simulate the system behavior in equilibrium. This will be performed in three steps:

1. *Calculate equilibrium.*

The constant outflow shifts the equilibrium of the positive feedback system away from zero. The equilibrium stock for a first-order system can be obtained by equating the sum of flows into the stock to the sum of flows out of the stock. Thus for the fruit fly system, equilibrium is found by solving the following equation:

Reproduction Rate = Removal Rate,

or:

Fruit Fly Population * Reproduction Ratio = Removal Rate.

Solving this equation we obtain the equilibrium fruit fly population:

Fruit Fly Population = Removal Rate / Reproduction Ratio

= 50 / 0.5 = 100 fruit flies.

2. *Determine the behavior mode.*

First-order positive feedback systems tend to exhibit either exponential growth away from equilibrium, negative exponential change towards equilibrium, or equilibrium. Since we are simulating the behavior of the stock at stability, 100 fruit flies, this last behavior mode is the one we are looking for.

3. Sketch the expected behavior.

Since the system is in equilibrium, the graph will be a horizontal line at 100 fruit flies as shown in Figure 2. Figure 2 also presents the equilibrium graph of the population for the case where there is no constant outflow. From comparing the two graphs it can be concluded that adding the constant outflow shifts the equilibrium of this positive feedback system away from zero.

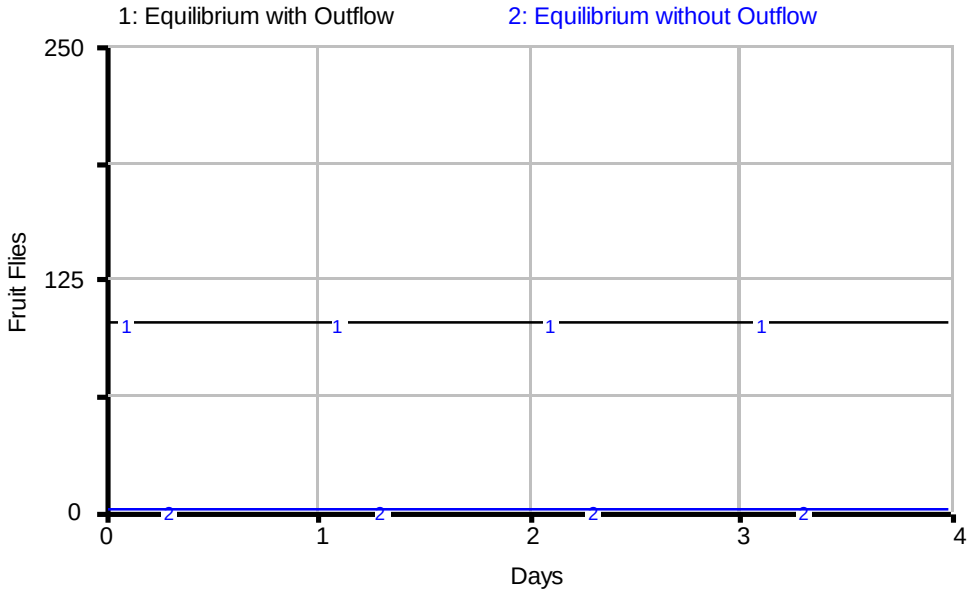


Figure 2: Change in equilibrium as a result of outflow

Since the mental simulation indicates that the population will be stable at 100 fruit flies, the scientist decides to order that amount. However, the lab supplies company mistakenly sends 120 fruit flies instead. The scientist quickly predicts the population behavior in three steps:

1. Calculate equilibrium.

The scientist remembers that the population is at equilibrium when there are 100 fruit flies.

2. Determine the behavior mode.

When there are 120 fruit flies the population clearly is not in equilibrium. Instead, there are *more* fruit flies than at equilibrium. Thus the behavior mode is *exponential growth* away from equilibrium.

3. *Sketch the behavior.*

Since the constant outflow does not change the exponential behavior generated by the positive feedback loop, doubling time can be used to estimate behavior. The doubling time is approximated by:

Doubling Time = 0.7 / Reproduction Ratio = 0.7 / 0.5 = 1.4 days.

Does this mean that the stock of 120 fruit flies doubles to 240 in just 1.4 days? If this assertion is true, then the system will behave exactly as if there were no outflow! Thus, it is obvious that the 120 fruit flies will not grow to 240 in 1.4 days.

From this last observation it might seem as if the doubling time does not describe the rate at which 120 fruit flies reproduce. This observation is misleading because the doubling time is being applied to the wrong stock. Clearly, the exponential growth generated by positive feedback does not describe the behavior of the 100 fruit flies that are being removed at the same rate that they reproduce (remember that the population is at equilibrium when there are 100 specimens).

Instead, doubling time refers only to exponential growth. Only the additional 20 fruit flies that are not subject to removal grow exponentially, unhindered by the constant outflow. Thus, the behavior of the 120 fruit flies can be predicted by dividing the

population up into two groups: the 100 fruit flies at equilibrium and the 20 fruit flies subject to positive feedback.

The key to sketching behavior is graphing the two behaviors separately and then adding them up to produce the behavior of the population as a whole.[3] First, the graph of the 20 fruit flies that are subject to pure positive feedback is graphed as shown in Figure 3. The doubling time of 1.4 days allows for a quick sketch.

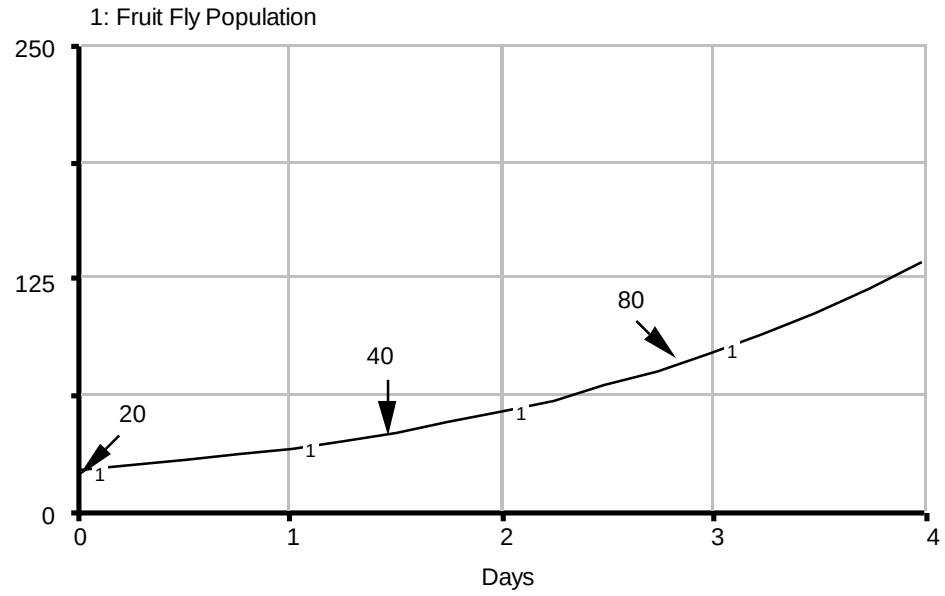


Figure 3: Exponential growth of twenty additional fruit flies

The predicted behavior for the system as a whole is obtained by adding the equilibrium graph, obtained in Figure 2, to the graph in Figure 3. Since the new equilibrium is represented by a horizontal line at 100 fruit flies, adding these behavior modes is tantamount to shifting the exponential curve up by the amount of the new equilibrium. Figure 4 shows the final behavior estimate for the system. The exponential growth generated by the system with the outflow is compared to that without the outflow. The previous and new equilibriums are also compared.

[3] Mathematically, this procedure of adding behavior modes to produce the total system behavior is called "superposition." Superposition is only possible for linear systems, such as those being used in this paper.

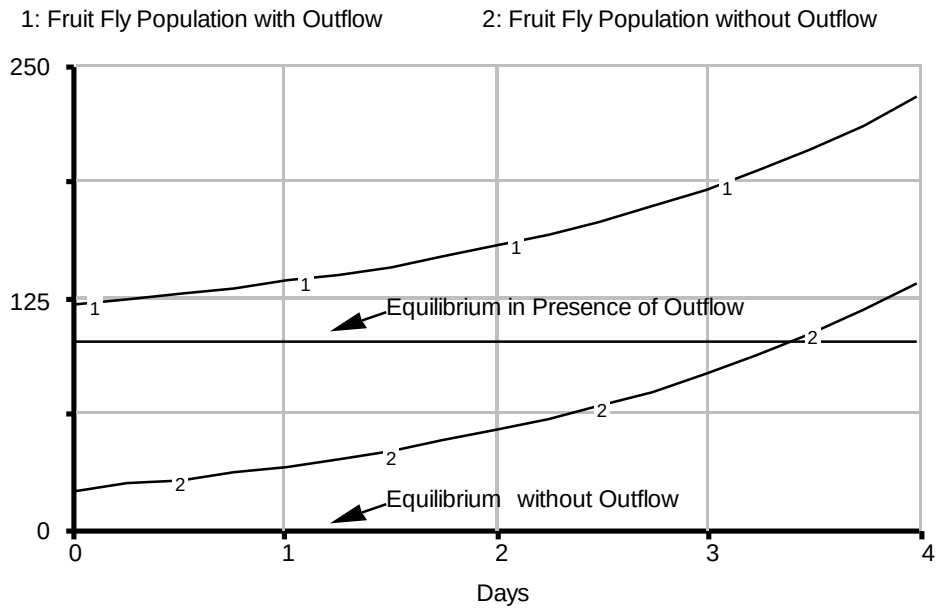


Figure 4: Mental simulation graph of fruit fly population behavior

In Figure 4 we notice that addition of the constant flow did not change the exponential behavior generated by the positive feedback. As a result, sketching positive feedback system with a constant outflow is simple. Just add the two behavior modes: exponential growth generated by positive feedback, and the new equilibrium resulting from addition of the constant outflow. The first is estimated using the doubling time and the second is calculated from the equilibrium relation. The behavior of the system as a whole is found by adding up these two behaviors. This operation amounts to shifting the exponential growth upwards, so that it starts from the new equilibrium.

2. Exercise 1: Nobel Prize Fund

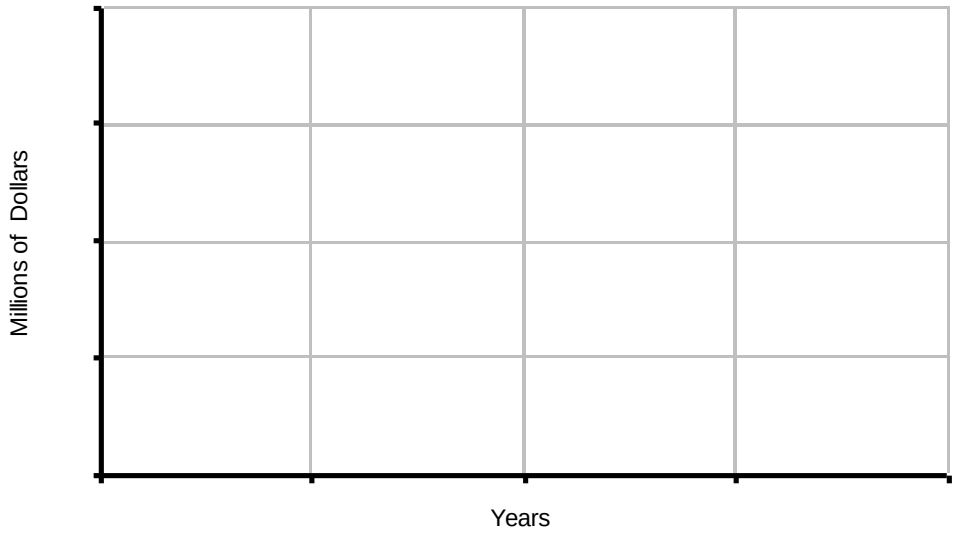
Every year the Nobel Prize Foundation distributes approximately a total of $6,000,000 in cash prizes to those who, during the preceding year, have conferred the greatest benefit on mankind in one of the following areas: Chemistry, Literature, Medicine, Physics, Economics, and Peace. These prizes are financed through interest accumulated on a bank account.

A. Draw a stock-and-flow model that describes the behavior of the Nobel Prize Fund. Treat the prizes in different categories as separate outflows from the bank account.

B. Draw a model that describes the behavior of the Nobel Prize Fund, this time treating the prizes as one big prize, i.e. as a single aggregated flow.

C. The Nobel Prize Fund earns enough interest to offset the cash lost as a result of the awards given. Assuming the interest rate is 10%, what is the minimum balance of the Nobel Prize Fund?

D. Sketch the account behavior assuming the Fund contains $30,000,000 at a time zero. Accuracy is not necessary; a drawing describing the basic behavior of the account is sufficient. For simplicity, treat the accumulation of interest and the withdrawal of cash prizes, as smooth, continuous functions, i.e. that they occur evenly throughout the year.



E. Suppose the Nobel Prize Fund is actually greater than the minimum needed for it to remain steady. This assumption is reasonable as it is unrealistic to expect the account to be exactly, to the last cent, equal to the minimum amount needed to not deplete. Now, suppose the Nobel Prize Foundation members have decided that they have enough money to fund a Nobel Prize "for those who have conferred the greatest benefit on mankind" in the field of System Dynamics. Supposing the Fund contains $60,500,000, how much can the System Dynamics Nobel Prize distribute in cash, assuming money is not taken from the other prizes to fund this new prize?

_____

_____

_____

3. Negative Feedback with Constant Inflow

This section will guide the reader through the mental simulation of a negative feedback system containing a constant inflow. The following insights will prove useful to the mental simulation process:

? Adding constant flows to a negative feedback system shifts equilibrium.

? Constant flows do not change the characteristics of exponential decay produced by negative feedback. As a result, halving time remains a useful mental simulation tool.

The negative feedback system to be simulated is a draining sink that contains an added inflow produced by a leaking faucet. The rate of draining is proportional to the volume of water in the sink. For this specific sink the proportionality constant, or draining fraction, is about 0.1/s. Water flows in at a rate of 30 cm^3/s. The model for the system is depicted in Figure 5.

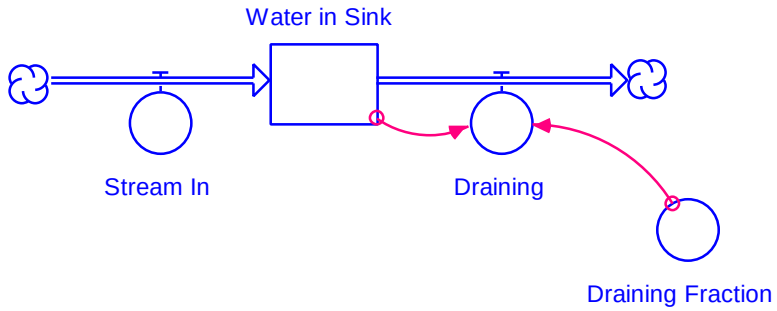


Figure 5: Model for draining sink with constant inflow

Now let us mentally simulate the behavior of the system when it is in equilibrium:

1. *Calculate equilibrium.*

In the absence of an inflow, the system is in equilibrium when the sink is empty. Adding a steady, exogenous flow shifts the equilibrium volume. To find out by how much, the equilibrium condition for first-order systems is used. In other words, the sum of inflows into the stock is equated to the sum of outflows. The inflow is simply a constant stream in. The outflow is given by the product of the volume by the draining fraction. Equating these terms we obtain:

Stream In = Volume * Draining Fraction.

Solving this equation we obtain the equilibrium volume of water:

Volume = Stream In / Draining Fraction = 30 / 0.1 = 300 cm^3.

2. *Determine the behavior mode.*

In first-order negative feedback systems the stock tends to approach equilibrium asymptotically, either from above or from below. Besides asymptotic behavior, the stock can exhibit equilibrium. For this simulation we are attempting to estimate the behavior of the system when the stock is at 300 cm^3, which represents equilibrium.

3. *Sketch the expected behavior mode.*

Since the system is in equilibrium, the graph will be a horizontal line with volume equal to 300 cm^3, as shown in Figure 6. The result of adding the inflow to the negative feedback system has been to shift equilibrium from 0 cm^3 to 300 cm^3.

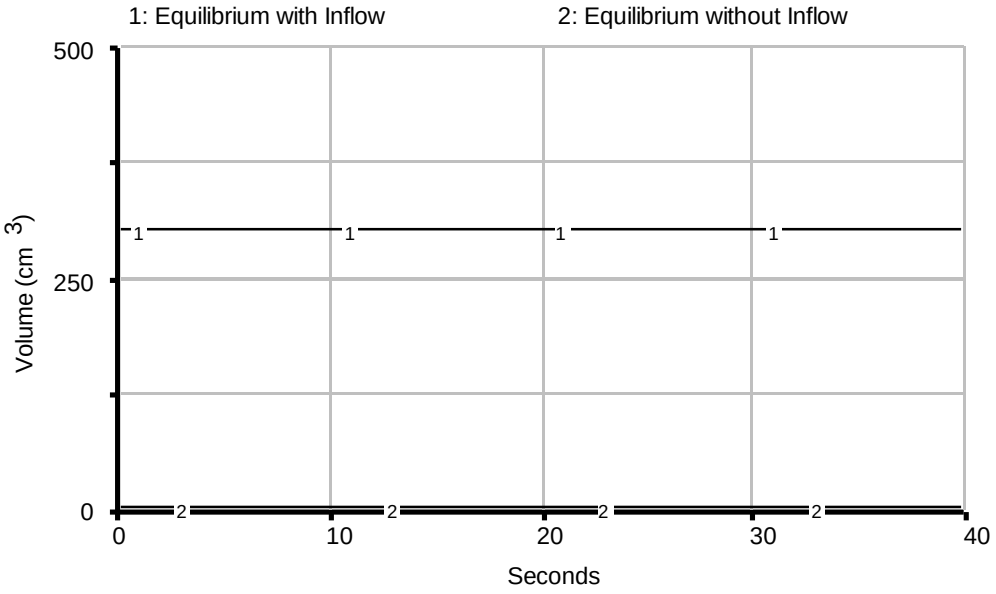


Figure 6: Equilibrium resulting from addition of inflow

Now let us simulate the sink system for the case when the sink contains 500 cm^3 of water at the beginning of the simulation.

1. *Calculate equilibrium.*

From the previous simulation the equilibrium volume is known to be 300 cm^3.

2. *Determine the behavior mode.*

For this simulation, the initial volume of water, 500 cm^3, is *greater* than the equilibrium value. Thus the system approaches equilibrium from *above*.

3. *Sketch the behavior.*

The behavior of the system as a whole can be decomposed into two separate parts that can be graphed separately. From the 500 cm^3 of water present at the beginning of the simulation, 300 cm^3 of it are in equilibrium. The remaining 200 cm^3 are subject to draining. The graph of the equilibrium component was obtained in the previous simulation. Now we shall proceed to sketch the behavior of the volume subject to draining. Subsequently, the behavior modes will be added to obtain the behavior for the system as a whole.

Draining of the 200 cm^3 of water can be sketched quickly using the half-life, which is approximated by:

Half-Life = 0.7 / Draining Fraction = 0.7 / 0.1 = 7 seconds.

Having obtained the half-life, a quick sketch, resembling Figure 7, can be obtained for the 200 cm^3 of water subject to draining.

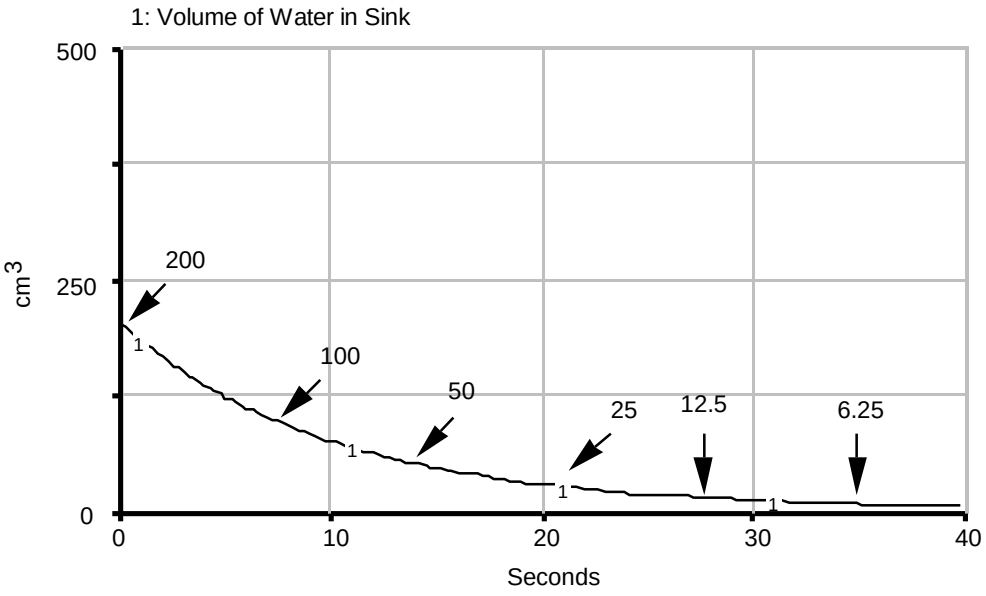


Figure 7: Exponential decay of water subject to draining

To obtain the sketch for the behavior of the system as a whole, the sketch for the 300 cm^3 of water in equilibrium, obtained in the previous example, is added to the sketch that was just obtained, representing exponential decay of 200 cm^3 of water. The result of

adding the behavior modes is shown in Figure 8. The conclusion to draw from the graph is that addition of the constant flow has shifted the equilibrium, or goal, that the system wants to reach. However, it has not changed the time constant of the feedback.

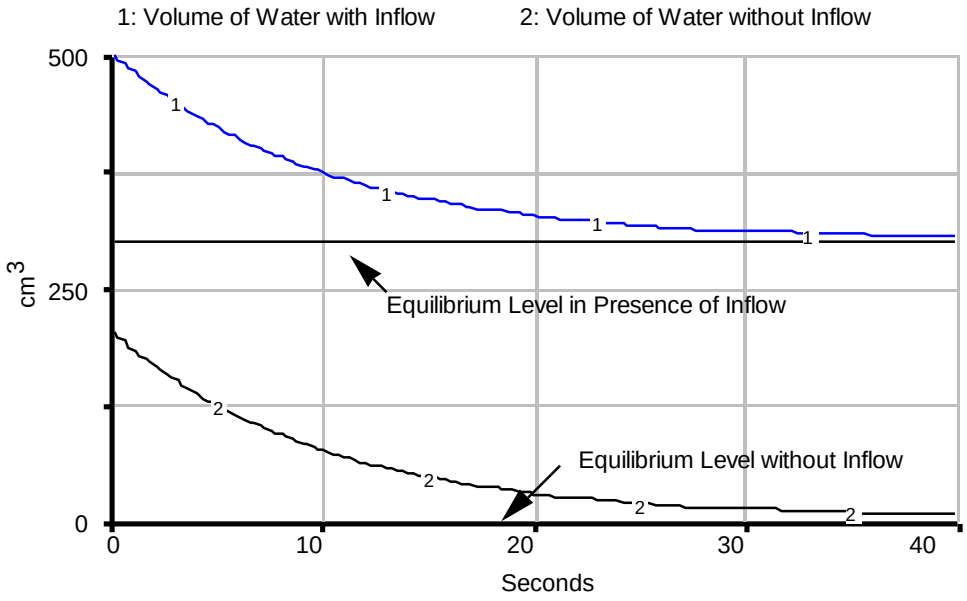


Figure 8: Mental simulation graph of water volume behavior

4. Exercise 2: Memorizing Song Lyrics

Victor loves listening to Italian opera. While he loves singing, he cannot remember the lyrics of these songs unless he listens attentively. Thus, he has decided that he will listen carefully to his favorite aria, and try to memorize each word. At first, as the song starts playing, he memorizes most words. However, as the song progresses, and Victor has already memorized many words, he starts forgetting some of the earlier words.

A. Sketch a model which shows how the stock of words that Victor remembers—while the song is being played—changes. Assume that the stream of words played is constant enough to allow modeling it as a constant inflow into Victor's consciousness. Moreover, assume that Victor forgets words at a rate proportional to the total number of words he remembers, at any given moment, and inversely proportional to some constant time-to-forget.

B. Victor comes up with a model which contains a stock of remembered words that is augmented by a constant stream of words, and decreased by a negative feedback loop which represents the words being forgotten. Victor does a variety of tests, listening to many arias, and comes to the following conclusions: for most arias a word is sung about every two seconds (0.5 words/second); for arias three minutes or longer, he remembers usually around forty-five words. Assuming his model is fairly accurate, what would the "time to forget" be? (Hint: Use the equilibrium relation for first-order systems.)

C. Using this model, how many words will Victor recall after listening carefully to a 10 minute long aria?

D. Victor eventually gets bored of listening to so much Italian opera and wants to listen to faster music. He goes to the record store and buys a Bob Dylan CD. These songs however are played at a rate of about two words per second (2 words/second), rather than one word every two seconds (0.5 words/second). Assuming that the time constant for forgetting the lyrics while the songs are playing is the same as that for the Italian arias, how would the behavior of the system, i.e. how does the stock of words he remembers while the song is being played, change? (A qualitative description is sufficient.)

E. If it takes longer to forget Bob Dylan lyrics—maybe because English is easier to remember than Italian—would the rate at which Victor forgets words be greater or less

than before? Would he remember more or fewer words than for an aria of comparable length?

_____

_____

_____

5. Review

The three steps to mentally simulating a first-order feedback system containing constant flows are as follows:

I. Calculate equilibrium:

- Sum of inflows = sum of outflows

II. Determine behavior mode:

- Equilibrium
- Diverge exponentially from equilibrium (positive feedback)
- Converge exponentially towards equilibrium (negative feedback)

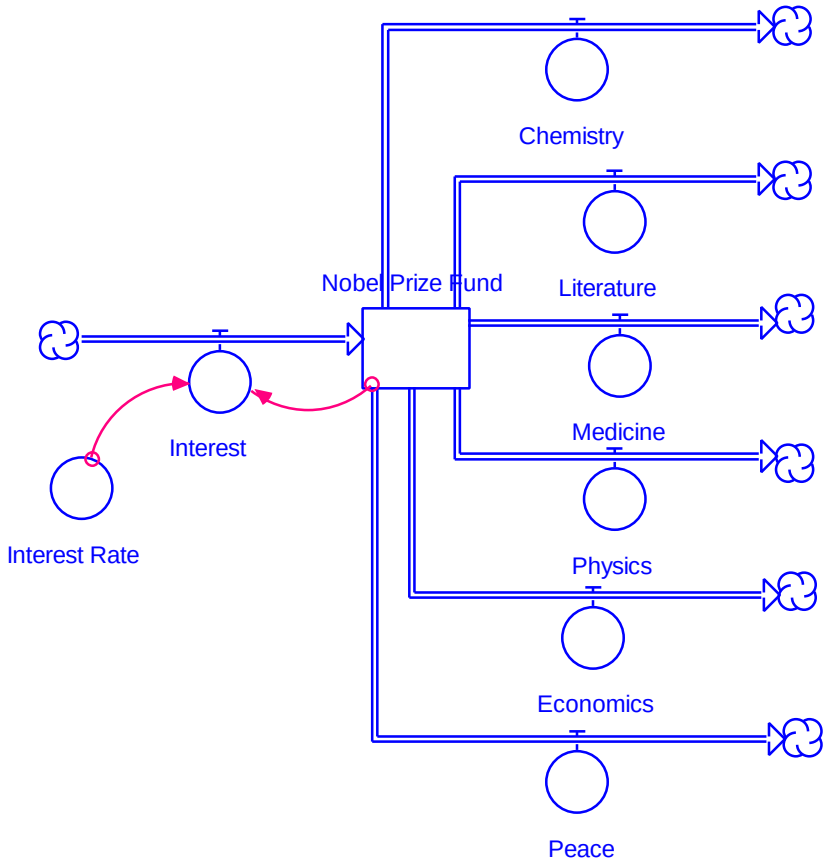
III. Sketch behavior:

1. Sketch equilibrium.
2. Sketch exponential behavior using time constant.
3. Add the behavior modes.

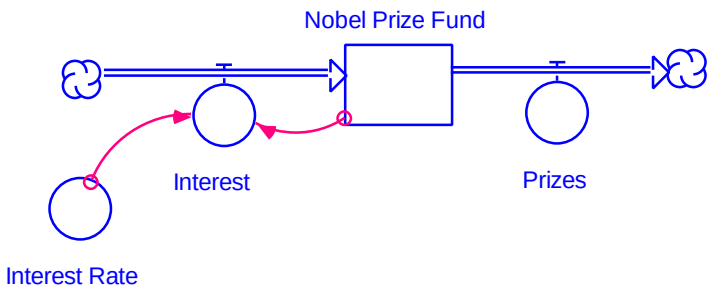
6. Solutions to Exercises

6.1 Solutions to Exercise One

A. The model contains a positive feedback loop, which represents interest payments, and six constant outflows, one for each prize.



B. This model predicts the same behavior for the bank account as the previous one. However, it is much simpler:



This model illustrates a virtue of aggregating variables in a model when possible. Doing so can simplify the model, and hence calculations, without changing the behavior of the variables being observed (such as the Nobel Prize Fund). Furthermore, this example demonstrates that the lessons we have learned for systems with one constant flow can be generalized to any first-order system containing more than one constant flow.

C. The bank account is at minimum. The removal of cash is balanced by the accrual of interest. Thus the equilibrium condition applies:

Outflow = Inflow

Removal of Cash = Accrual of Interest

Prizes = Fund * Interest Rate.

Solving this equation in terms of the Fund gives:

Fund = Prizes / Interest Rate = $6,000,000 / 0.10 = $60,000,000.

D. In order to mentally simulate the behavior of the Nobel Prize Fund when it begins at $30 million, let us follow the three steps for mentally simulating first-order systems.

1. *Calculate equilibrium.*

From the solution to Part C we know the account is at equilibrium when it contains $60,000,000.

2. *Determine the behavior mode.*

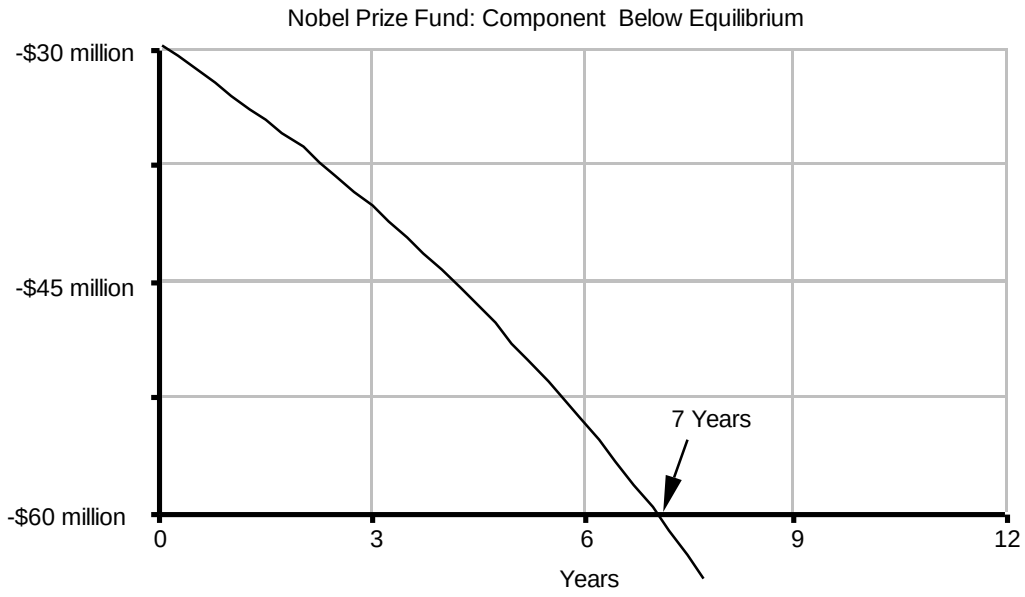
When there are only $30,000,000, the account is clearly not in equilibrium. There are *fewer* dollars than at equilibrium. Thus the behavior mode is *negative exponential growth* away from equilibrium.

3. *Sketch the behavior.*

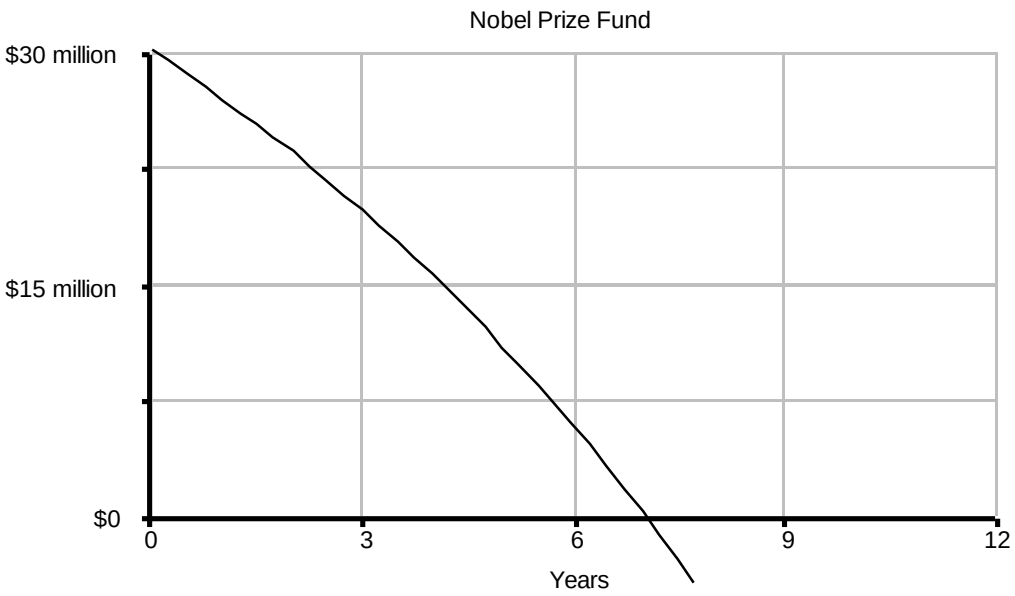
We must calculate the doubling time in order to determine by how much the fund deficit grows. By fund deficit is meant the amount by which the fund is below equilibrium. In this case the Fund's value is initially equal to the sum of the equilibrium value, $60,000,000, and the amount below equilibrium, —$30,000,000. The graph of the component of the value that is at equilibrium is a horizontal line at $60,000,000. The graph of the account component below equilibrium is negative exponential growth with the following doubling time:

Doubling Time = 0.7 / Interest Rate = 0.7 / 0.10 = 7 years.

Using the doubling time, the sketch for the component of the Fund below equilibrium is as follows:



Notice that the amount doubles in 7 years from —$30,000,000 to —$60,000,000. Now, this behavior mode, that is the behavior of the component of the Fund below equilibrium, must be added to the equilibrium graph to obtain the graph of the behavior of the Fund as a whole. Sketching the final graph amounts to shifting the graph we just obtained by $60,000,000, which is the equilibrium value resulting from addition of the constant outflow.



Because of the model's simplicity, the Fund's value is allowed to become negative! A more comprehensive model might take into account the efforts by the Nobel Prize Committee to improve the balance, either by adding funds or by reducing the value of cash prizes. Also, in reality no bank would be willing to hold an account with a negative balance.

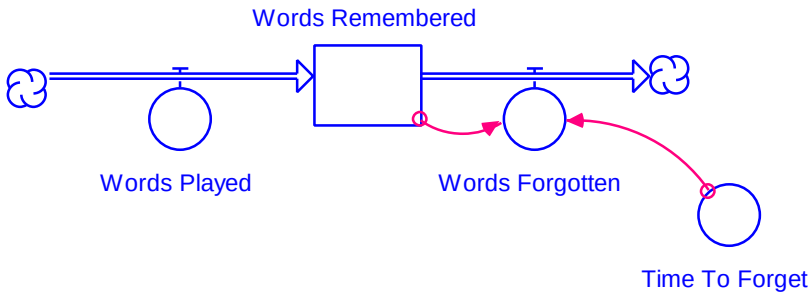
E. We use the equilibrium equation again to obtain the answer:

System Dynamics Prize = Funds available for prize * Interest rate

=$500,000 * 0.10 = $50,000.

6.2 Solutions to Exercise Two

A. Assuming that the flow of words is constant, we obtain the following model:



B. When the words are played at a rate of about one word every two seconds, he seems to reach an equilibrium of about 45 words remembered. Using the equilibrium equation below, we derive the time constant:

Words played = Words Forgotten,

= Words Remembered / Time to Forget,

Solving in terms of the time constant we obtain:

Time to Forget = Words Remembered / Words Played = 45 / 0.5 = 90 seconds.

C. Once Victor has reached equilibrium, he will remember about forty-five words, assuming that the song is still playing and the rate of words being played remains constant.

D. A faster stream of words played amounts to increasing the inflow of words entering Victor's memory. By changing the inflow, the equilibrium stock of words will shift. Solving in terms of Words Remembered, the equilibrium equation derived in part B becomes:

Words Remembered = Words Played * Time to Forget.

From the relation, increasing Words Played increases the stock of words remembered at equilibrium. Thus, Victor will remember more words when he listens to the Bob Dylan CD. Increasing the inflow of words shifts the stock of words remembered upward.

E. From the equilibrium equation in part D, increasing Time to Forget will increase the number of words Victor remembers at equilibrium. The reason is that increasing the time to forget decreases the rate at which words are forgotten. Given the same constant inflow, equilibrium shifts upward.

7. Appendix: Model Documentation

Fruit Fly Population Model

Fruit_Fly_Population(t) = Fruit_Fly_Population(t-dt) + (Reproduction_Rate - Removal_Rate) * dt

INIT Fruit_Fly_Population = 100

DOCUMENT: The Fruit Fly Population was initialized at 100 fruit flies for the equilibrium example, and 120 fruit flies for the second example, which involved exponential growth (*fruit flies*).

INFLOWS:

Reproduction_Rate = Fruit_Fly_Population * Reproduction _Ratio

DOCUMENT: Rate at which fruit flies reproduce (*fruit flies / day*).

OUTFLOWS:

Removal_Rate = 50

DOCUMENT: Fruit flies removed daily (*fruit flies / day*).

Reproduction_Ratio = 0.5

DOCUMENT: Ratio of fruit flies added to the population per day (*1 / day*).

Nobel Prize Fund Model

Nobel_Prize_Fund(t) = Nobel_Prize_Fund(t-dt) + (Interest - Prizes) * dt

INIT Nobel_Prize_Fund = 60000000

DOCUMENT: The Nobel Prize Fund contains about $60,000,000 at equilibrium (*dollars*).

INFLOWS:

Interest = Nobel_Prize_Fund * Interest_Rate

DOCUMENT: Rate of interest payments to the fund (*dollars / year*).

OUTFLOWS:

Prizes = 6000000

DOCUMENT: Amount of cash prizes awarded yearly (*dollars / year*).

Interest Rate = 0.10

DOCUMENT: Interest rate paid on bank accounts (*1 / year*).

Model for Draining Sink

Water_in_Sink(t) = Water_in_Sink(t-dt) + (Stream_In - Draining) * dt

INIT Water_in_Sink = 100

DOCUMENT: Water in the sink is at equilibrium when there are 300 cubic centimeters. For the example involving exponential decay, the water in the sink began at 500 cubic centimeters (cm^3).

INFLOWS:

Stream_In = 30

DOCUMENT: Rate at which water flows into the sink (cm^3 / s). Assumed to be constant.

OUTFLOWS:

Draining = Water_in_Sink * Draining_Fraction

DOCUMENT: Rate at which water drains from the sink (cm^3 / s).

Draining Fraction = 0.1

DOCUMENT: Fraction of volume of water which flows out the drain per second (*1 / s*).

Memorizing Song Lyrics Model

Words_Remembered(t) = Words_Remembered(t-dt) + (Words_Played -
Words_Forgotten) * dt

INIT Words_Remembered = 0

DOCUMENT: At the beginning of the song it is assumed that Victor does not know any of the lyrics (*words*).

INFLOWS:

Words_Played = 0.5

DOCUMENT: Rate at which words are played (*words / s*). Assumed to be constant.

OUTFLOWS:

Words_Forgotten = Words_Remembered / Time_To_Forget;

DOCUMENT: Rate at which Victor forgets words (*words / s*).

Time_To_Forget = 90

DOCUMENT: Time it takes for Victor to forget a word on average (*words / s*).

8. Bibliography

Forrester, Jay W. (1961). Industrial Dynamics. Portland, OR: Productivity Press.

Forrester, Jay W. (1968). Principle of Systems. Portland, OR: Productivity Press.

Goodman, Michael R. (1974). Study Notes in System Dynamics. Portland, OR: Productivity Press.

Roberts, Nancy et al. (1983). Introduction to Computer Simulation. Portland, OR: Productivity Press.

Vensim Examples:

Beginner Modeling Exercises

By Aaron Diamond

October 2001

1. Positive Feedback with Constant Outflow

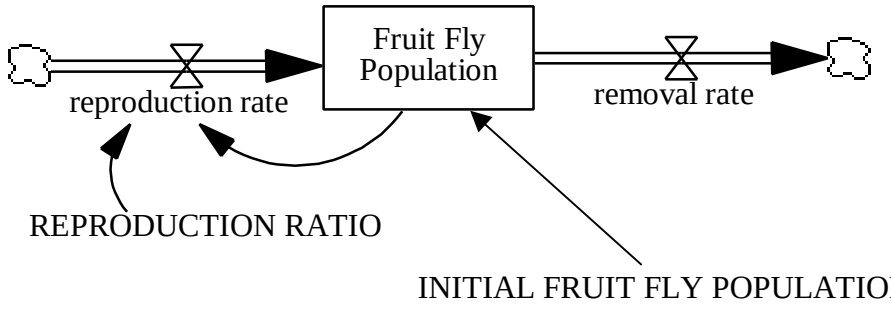


Figure 9: Vensim Equivalent of Figure 1: Fruit fly population model.

Documentation for Fruit Fly population model

(1) FINAL TIME = 4
Units: day
The final time for the simulation.

(2) Fruit Fly Population= INTEG (+reproduction rate-removal rate, INITIAL FRUIT FLY POPULATION)
Units: fruit flies
The Fruit Fly Population was initialized at 100 fruit flies for the equilibriun example, and 120 fruit flies for the second example, which involved exponential growth.

(3) INITIAL FRUIT FLY POPULATION=100
Units: fruit flies

(4) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(5) removal rate=50
Units: fruit flies/day
Fruit flies removed daily.

(6) reproduction rate=Fruit Fly Population*REPRODUCTION RATIO
Units: fruit flies/day
Rate at which fruit flies reproduce.

(7) REPRODUCTION RATIO=0.5
Units: 1/day
Ratio of fruit flies added to the population per day.

(8) SAVEPER = 1
Units: day
The frequency with which output is stored.

(9) TIME STEP = 0.0625
Units: day
The time step for the simulation.

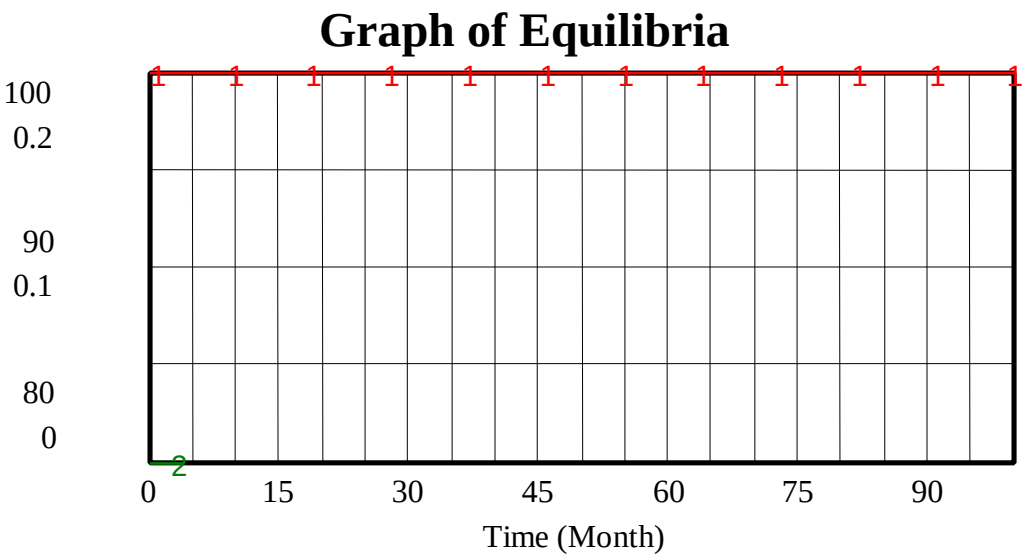


Figure 10: Vensim equivalent of Figure 2: Change in equilibrium as a result of outflow

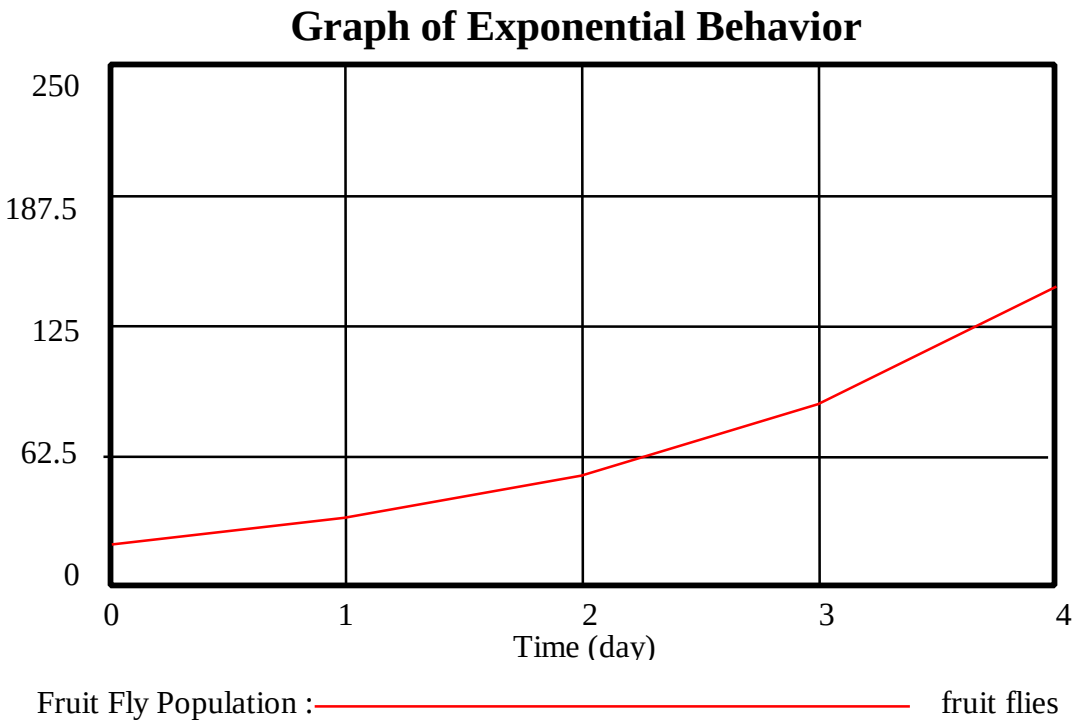


Figure 11: Vensim Equivalent of Figure 3: Exponential growth of twenty additional fruit flies

2. Exercise 1: Nobel Prize Fund

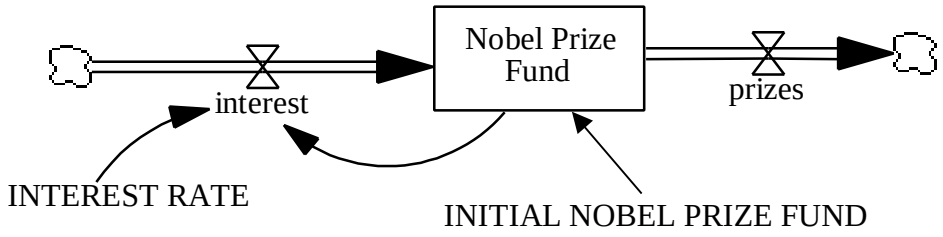


Figure 12: Vensim Equivalent of Figure 7.1, B

Documentation for Nobel Prize Fund model

(1) FINAL TIME = 12
Units: year
The final time for the simulation.

(2) INITIAL NOBEL PRIZE FUND= 6e+007
Units: dollars

(3) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(4) interest=Nobel Prize Fund*INTEREST RATE
Units: dollars/year
Rate of interest payments to the fund.

(5) INTEREST RATE=0.1
Units: 1/year
Interest rate paid on bank accounts.

(6) Nobel Prize Fund= INTEG (interest-prizes, INITIAL NOBEL PRIZE FUND)
Units: dollars
The Nobel Prize Fund contains about $60,000,000 at equilibrium.

(6) prizes=6e+006
Units: dollars/year
Amount of cash prizes awarded yearly.

(7) SAVEPER =TIME STEP
Units: year
The frequency with which output is stored.

(8) TIME STEP = 0.0625
Units: year
The time step for the simulation.

3. Negative Feedback with Constant Inflow

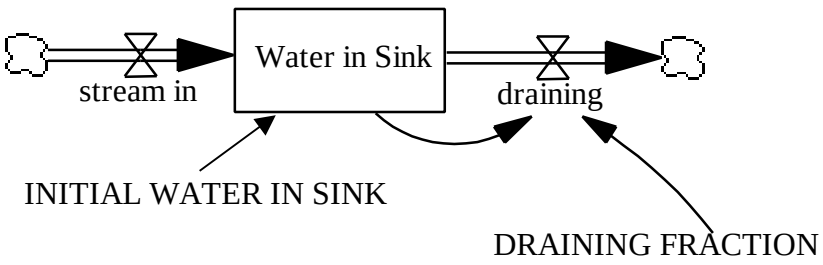


Figure 13: Vensim Equivalent of Figure 5: Model for draining sink with constant inflow

Documentation for draining sink model

(1) draining=Water in Sink*DRAINING FRACTION
Units: cm^3/s
Rate at which water drains from the sink.

(2) DRAINING FRACTION=0.1
Units: 1/s
Fraction of volume of water which flows out the drain per second.

(3) FINAL TIME = 40
Units: s
The final time for the simulation.

(4) INITIAL TIME = 0
Units: s
The initial time for the simulation.

(5) INITIAL WATER IN SINK=100

Units: cm^3

(6) SAVEPER = TIME STEP

Units: s

The frequency with which output is stored.

(7) stream in=30

Units: cm^3/s

Rate at which water flows into the sink. Assumed to be constant.

(8) TIME STEP = 0.0625

Units: s

The time step for the simulation.

(9) Water in Sink= INTEG (+stream in-draining, INITIAL WATER IN SINK)

Units: cm^3

Water in the sink is at equilibrium when there are 300 cubic centimeters. For the example involving exponential decay, the water in the sink began at 500 cubic centimeters.

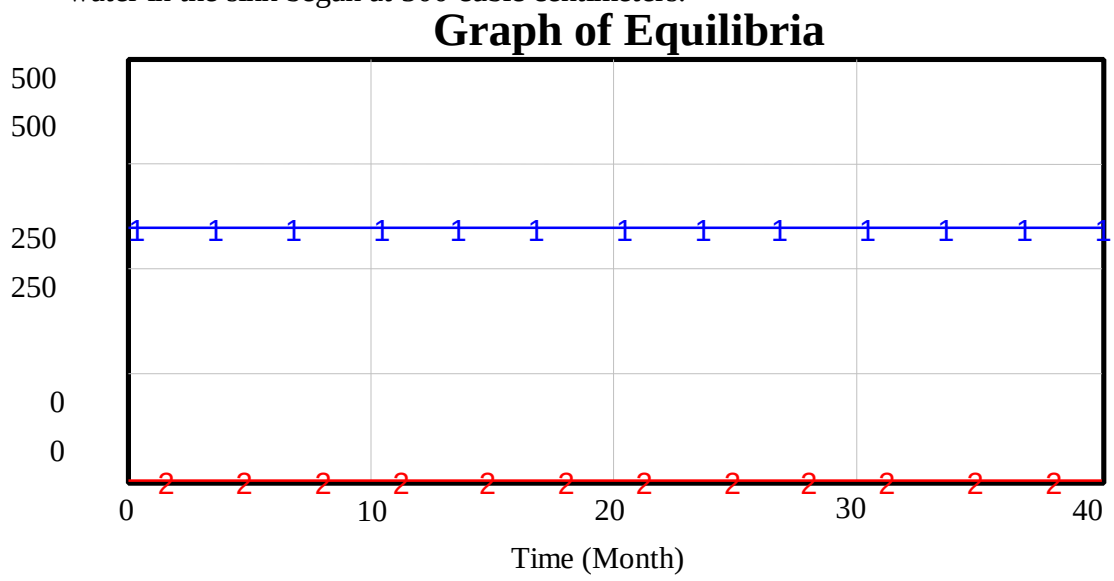


Figure 14: Vensim Equivalent of Figure 6: Equilibrium resulting from addition of inflow.

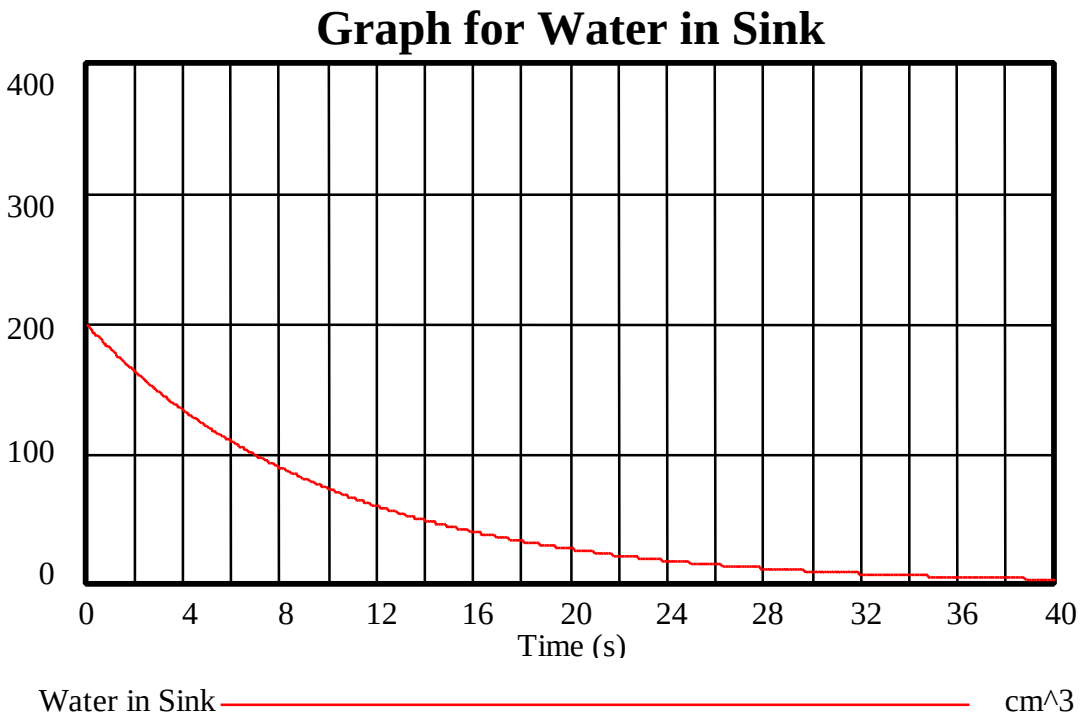


Figure 15: Vensim Equivalent of Figure 7: Exponential decay of water subject to draining.

4. Exercise 2: Memorizing Song Lyrics

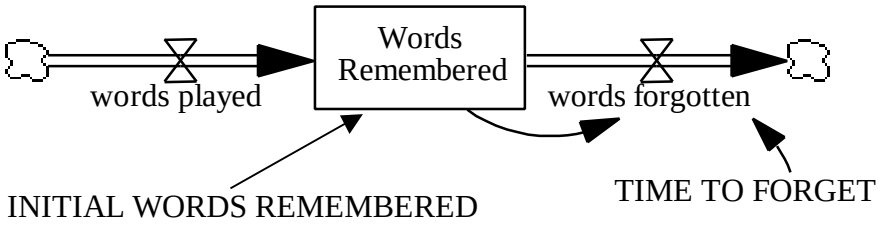


Figure 16: Vensim Equivalent of 7.2, A.

Documentation for Memorizing Song Lyrics model

(1) FINAL TIME = 100
Units: s
The final time for the simulation.

(2) INITIAL TIME = 0
Units: s
The initial time for the simulation.

(3) INITIAL WORDS REMEMBERED= 0
Units: words

(4) SAVEPER = TIME STEP
Units: s
The frequency with which output is stored.

(5) TIME STEP = 0.0625

Units: s

The time step for the simulation.

(6) TIME TO FORGET=90

Units: s

Time it takes for Victor to forget a word on average.

(7) words forgotten=Words Remembered/TIME TO FORGET

Units: words/s

Rate at which Victor forgets words.

(8) words played=0.5

Units: words/s

Rate at which words are played. Assumed to be constant.

(9) Words Remembered= INTEG (+words played-words forgotten, INITIAL WORDS REMEMBERED)

Units: words

At the begining of the song it is assumed that Victor does not know any of the lyrics.

Building the Fish Banks Model

and

Renewable Resource Depletion

Prepared for
System Dynamics Education Project
Sloan School of Management
Massachusetts Institute of Technology

Under the Supervision of
Dr. Jay W. Forrester

Compiled by
Joseph G. Whelan
July 1994

Vensim Examples added October 2001

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***Note: This paper is a combination of Building the Fish Banks Model (D-4543) and Renewable Resource Depletion (D-4263-3). The previous version was D-4448.

This paper is meant to accompany the **Fish Banks Exercises** model on the disk provided with Road Maps.

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10. VENSIM EXAMPLES **60**

This paper comes with a disk labeled “Fish Banks Exercises”. If you do not have this disk, it can be obtained free of charge from The Creative Learning Exchange.

The exercises in this paper require a Macintosh[1] Computer with 2 Mb of RAM if running System 6.0.4 and at least 4 Mb of RAM if running System 7 or higher. Also required is the STELLA[2] II software package. If you do not have STELLA, contact High Performance Systems.

***Note: The instructions given in this paper are directed toward readers using STELLA II version 3.0.1 or higher. If you have some familiarity with STELLA, you will be able to complete the modeling using an older version of the software.

The Fishing Game disk that comes with this paper contains three models. “The Fishing Game v1.02” is for STELLA II v1.02, “The Fishing Game v2.2.1” is for STELLA II v2.2.1, and “The Fishing Game v3.0.1” is for STELLA II v3.0.1 and higher. Appendices 2 and 3 explain the differences between the three versions of the game and gives some instructions specific to STELLA II v1.02 and v2.2.1. Do not open the older version of the game (v1.02 or v2.2.1) with the newer version of the software (v3.0.1) as the model will be altered.

[1]Macintosh is a registered trademark of Apple Computer, Inc.
[2]STELLA is a registered trademark of High Performance Systems, Inc..

1 INTRODUCTION

Fish Banks, Ltd.[3] is a role-playing game developed by Dennis L. Meadows at the University of New Hampshire. Dr. Meadows developed the game to inform people about using natural resources effectively and prudently. Although the game originally targeted corporate managers and public officials, anyone can benefit from the insights gained by playing the game.

In the Fish Banks, Ltd. game, teams of players manage their own fishing companies. At the beginning of the game, each fishing company has equal amounts of money and fishing ships. Each company has the same operating costs and technology. At the beginning of every simulated year, the teams make decisions about buying or selling ships, whether to fish or not, and where to fish. The object of the game for each company is to maximize profits.

In this paper, you will play The Fishing Game, a one-player game similar to Fish Banks, Ltd. by Dennis Meadows. The Fishing Game uses a system dynamics computer model. Your computer is your opponent. After playing The Fishing Game, you will build a model of a fishing system similar to that used for the game. The fishing system model can be segmented into three subsystems. You will build the three subsystems separately and observe the behavior of each subsystem. Then, you will combine the three models to form the final fishing system model.[4]

After building the final model, you will proceed to use this model to perform some policy analysis exercises to evaluate various strategies for avoiding the abuse of renewable resources.

The Fish Banks, Ltd. material, including the game board, pieces, and directions, can be purchased from:

Dennis L. Meadows, Director
Institute for Policy and Social Science Research
Hood House
Durham, NH 03824-3577

[3]Dennis L. Meadows, Institute for Policy and Social Science Research, Hood House, University of New Hampshire, Durham, NH 03824.
[4]Note: Although The Fishing Game can only be played on STELLA v2.2.1, the model can be built on any version of STELLA.

2 PLAYING THE FISHING GAME

The numerical values used in The Fishing Game as well as in the model you will build are not realistic. The values were chosen to maintain consistency with the original Fish Banks, Ltd. game developed by Dennis Meadows as well as with *Renewable Resource Depletion* (D-4263) which is the second half of this document. Nevertheless, the numbers used are altogether consistent and do not affect the qualitative behavior of the model. It may be helpful to consider the numbers as representing their values in thousands.

2.1 Role Description

You will be fishing in an area that has an optimal fish population of 1200. A natural restriction occurs once the fish population reaches 1200, so that overcrowding doesn't occur. The restriction is a reduced reproduction rate so that the population remains at 1200 while the natural habitat of the fish stays unchanged.

The presence of a fishing industry in the area disturbs the natural habitat of the fish. When the fishing industry takes only small amounts of fish from the population, the fish are able to quickly regenerate and regain their optimal population of 1200 fish. If the small depletions by the fishing industry continue steadily over time, the fish will never reach 1200, but will establish a new optimal population at less than 1200. However, if the fish population drops dramatically, the number of fish laying eggs is so small that the population takes a very long time to recover even when undisturbed by the fishing industry during the regeneration period.

The fish population is able to regenerate as shown in Figure 1 below.

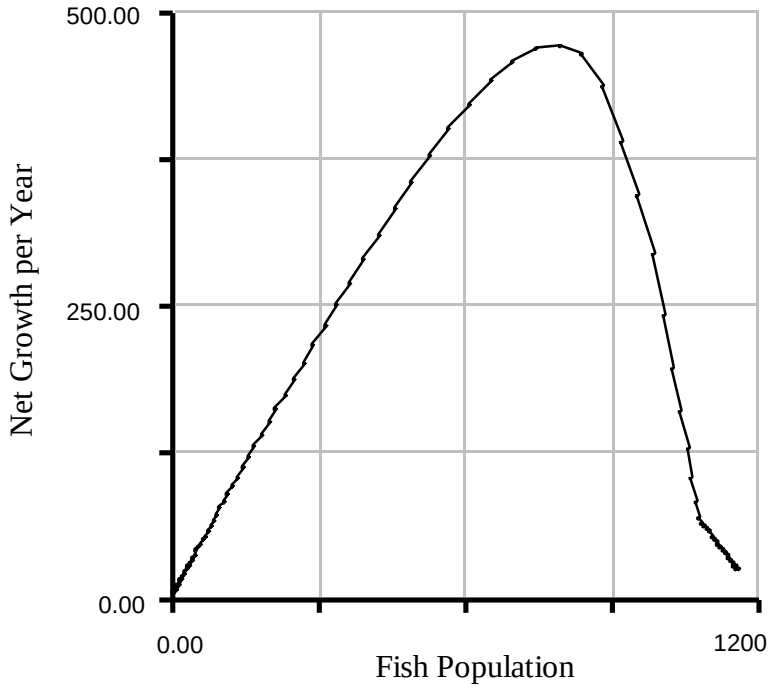


Figure 1: Net Growth per Year of Fish Population vs. Fish Population

This graph shows the net growth rate of the fish population as it varies with the size of the population. The capacity of the fish population to regenerate increases with the size of the population. As the population approaches its optimal size, the net growth of the population decreases to zero.

Your fishing ships (as well as your competitor's) are equipped with the latest in fish tracking and catching technology. As a result, you will be able to catch nearly a full load of fish even if the fish density drops by 30 or 40 percent. This characteristic of your fish catch is a result of a simplification of the model. This model assumes that your ships go out to sea once per day, catch a full load of fish and return. In real life, fishing companies would most likely increase the number of runs per day if the fish density was very high. To model this we would also have to vary the operating cost per ship. This level of complexity is unnecessary to simulate this system accurately. Figure 2 shows how your ability to catch fish varies with the fish population density.

Your goal is to maximize profits and make more money than your competitor, the computer. The game lasts for 10 years, from 1980 to 1990. The fish population at the

beginning of the game is 1000 fish. Both companies begin with zero net profit, and may go into debt to build ships. Each company begins with 2 ships each.

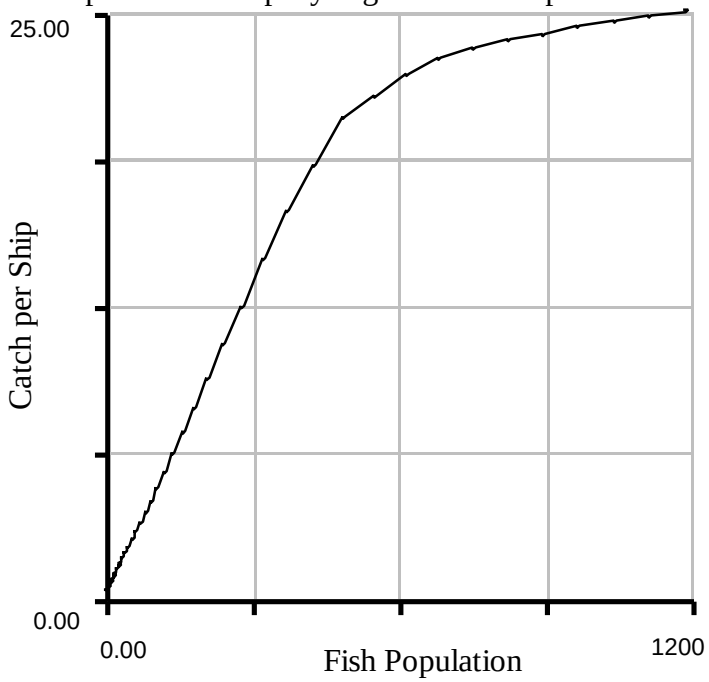


Figure 2: Catch per Ship vs. Fish Population

This graph shows the Catch per Ship as it varies with the size of the fish population. The ability of the fishing ships to catch fish does not decrease dramatically until the fish population density drops dangerously low.

2.2 Playing the Game

***Note: The Fish Banks Exercises disk that came with this paper contains three models. "The Fishing Game v1.02" is for STELLA II v1.02, "The Fishing Game v2.2.1" is for STELLA II v2.2.1 and v2.2.2, and "The Fishing Game v3.0.1" is for STELLA II v3.0.1.

Section 2.2 (this section) gives the instructions for playing the game on STELLA II v3.0.1. If you are using earlier versions of STELLA II, you should still read this section, but you should also refer to Appendices 2 and 3. Appendix 2 gives instructions specific to STELLA II v.2.2.1 and v2.2.2, and Appendix 3 gives instructions specific to STELLA II v1.02. Do not open the older version of the game (v1.02, v2.2.1 or v2.2.2) with the newer version of the software (v3.0.1) as the model will be altered. All specific instructions are given in bold.

At the beginning of each simulated year you will choose for your company how many ships either to build or to sell. You will then run the model for one year and observe the effects of your fishing on the fish population and on both players' profits. Each time you run the game, the model runs for one year and then pauses. You make your decisions for that year, and run the model for another year.

- **Make sure that STELLA II v3.0.1 is installed on your hard disk.**
- **Insert the disk labeled "Fish Banks Exercises" into your disk drive.**
- **Double-click on the disk icon labeled "The Fishing Game."**
- **In the window that appears, double-click on the icon labeled "The Fishing Game v3.0.1."**

If you have the STELLA II v3.0.1 program installed on your hard drive, the model you selected will open on your screen. If the model does not open, you need to open the STELLA program first and then open the model from within the program.

Once the model is open, you will see three sectors, each labeled “The Fish,” “You,” and “Competitor.” If the model is open and you do not see the top level of the sectors, use the arrow at the top right corner of the STELLA window to scroll up until you see it.

Figure 3: Top Level of the Three Sectors

The three sectors are the main players in the game, and the arrows with brief explanations show the relationships among them. Referring to Section 2.1 as needed, try to understand how each sector affects and is affected by the other two.

? **Looking at the three sectors, devise a plan which would maximize your profits.**
? **Using the downward arrow at the bottom right corner of the window, scroll down until you see a graph pad.**

This graph pad is where you view the progress and the final outcome of your game. It contains four graphs, one on each page, and the page number is on the bottom edge of the window. You can click on the lower left corner of the graph pad to view the different pages, one at a time. These four graphs will allow you to track your own progress as well as that of your competitor. Figure 4 contains a list of the available graphs and their contents. At the end of every simulated year, STELLA updates these graphs with current information. Please note that in graphs 3 and 4, the scales for the two elements are different.

Page:	**Graph:**	**Contents:**
1	Annual Profits	*Competitor Profit Each Year* *Your Profit Each Year*
2	Total Profits	*Competitor Total Profits* *Your Total Profits*
3	Your Catch	*Catch per Ship* *Your Total Catch*
4	Your Ships	*Your Ships* *Your Building Rate*

Figure 4: The Graphs contained in the Graph Pad

These are the graphs that are available to you as you play The Fishing Game. With these graphs you can track your performance and re-evaluate your strategy.

? **Using the downward arrow at the lower right corner, scroll down until you see a table and a slide bar.**

The table displays the current values of the following six variables: *Your Ships, Your Total Profits, Your Profit Each Year, Catch per Ship, Your Total Assets, and Competitor's Total Asset*. To complement the graphical outputs on the graph pad, this table shows numerical values. The slide bar is an input device. It lets you select a value between -10 and +10 for *Your Building Rate*.

Implement your strategy for the game by setting a value for *Your Building Rate*. Before starting the next round of the game, specify how many ships you wish to build. If you wish to decrease the size of your fleet, you may sell ships by choosing a negative number. If you do not set a value at the beginning of a year, it will automatically reset to 0, and you will not build or sell any ships. You may track your previous decisions for *Your Building Rate* by looking at page 4 of the graph pad. Figure 4 contains a list of all available graphs and their contents.

- **Develop a strategy for maximizing your profits and beating your competitor.** (You may wish to re-read section 2.1 which briefs you on the game scenario.)
- **Click and hold on the button located between -10 and +10 on the slide bar.**
- **Slide the button until the desired number of ships you wish to build the first year shows to the right of *Your Building Rate* = .**
- **Select "Run" from the Run menu or press ? -R.** The model will run for 1 year.
- **Look at the four pages of the graph and the table below the graph pad to observe the behavior of the model.**

***Note: After the first year, the Run menu will no longer contain the option "Run." Instead, you should select "Resume."

- **Continue entering values for *Your Building Rate* and running the model until the end of the simulation in 1990.**
- **Play the game a few times, revising your strategy each time to increase your profits.**

The game will automatically reset at the end of each 10 year simulation. You do not need to reset anything.

- **When you finish playing the game, select "Close" from the file menu.**
- **Click on "Don't Save."**

After playing the game a few times you may notice that if you try to beat your competitor, by the end of the game, you have nearly destroyed the fish population and thus your yearly profit is very low. In the next part of this paper, you will go through the process of building a model similar to the one used for the game. You should gain some insight into why the fish population is so depleted and how you can avoid this catastrophe.

The depletion of the fish population did not occur because you didn't play the game correctly. The problem is that you cannot maximize your profits if your only goal is to beat your competitor. Thousands of people from all over the world have played Fish Banks. Each time, the result is the same as yours: every company falls far short of its profit potential because the players harvest the entire fish population too quickly for the fish to regenerate. The companies focus on maximizing profits and making the most money. Each company tries to build as many ships as possible to catch more fish than any other company. This leads to quick short-term profit but, as companies soon find out, the future is ultimately unprofitable.

3 BUILDING THE MODEL

Part 3 of this paper challenges your modeling skills. At the beginning of each section you will read a brief written description of the subsystem you are to build. It is possible, and you should attempt, to build the model based solely on the information given in the description. Following the description are step-by-step instructions on how to build the model. If you have difficulty, skip to the end of the section and look at the picture of the model. If you still have trouble, refer to the final model with equations in Appendix 4.

In order for your model to run properly, you need to change the value of the solution interval, DT. The reason is that the default value of DT (0.25) is too large for the unusually high values for the hatch and death rates of the fish population. Since the fish population system is changing very quickly, you need to instruct STELLA to calculate the values of the model more frequently. Changing the value of DT from the default value of 0.25 to 0.1 will do just that and consequently result in a more accurate model.

The hatch and death rates are set high so that we will be able to observe the dynamics of the fish population in the time frame of the simulation. Although the values of these rates are unrealistic, they are consistent with the other variables in the model and do not affect the qualitative behavior of the model.

? **Begin a new model in STELLA.**
? **Select "Time Specs" from the Run menu.**
? **Change DT to 0.1.**
? **Select "Years" in the Unit of Time column.**
? **Click on OK.**

There is an icon in the upper-left corner of your STELLA window that has in it either a picture of a globe , or X^2. If you do not see any one of these two icons, click on the upside down triangle at the upper-left corner, and you will see, right below the two triangles, a globe , or X^2. This is the Map/Model mode toggle button. You must be in model mode to define model elements as described in this paper.

• **Click on the Map/Model mode toggle button until it looks like X^2.**

Now you are ready to model the fish population, and the next section will take you through it step by step.

3.1 Modeling the Fish Population Subsystem

3.1.1 Description

The fish population is a simple S-shaped growth system. Without a fishing industry, fish hatch, grow to maturity, lay eggs and die. The size of the population naturally remains in dynamic equilibrium at the maximum sustainable number. If the population is decreased for unnatural reasons, it will return to the equilibrium level exhibiting S-shaped growth.

The first step in building a model is to identify the stocks present in the system. In the case of the fish population subsystem, the only stock present is the size of the fish population, called *FISH*.

Once you have identified the stock, you must recognize the flows which affect it. *Fish Hatch Rate* and *Fish Death Rate* are the two flows that affect the fish population stock. The *Fish Hatch Rate* is proportional to *FISH*. On average, one female fish produces 12 offsprings per year. This means that if 50% of the fish population is female, six fish hatch each year for every living fish. In other words, the number of fish hatched per year for each member of the fish population, or the *Hatch Fraction*, is six.

The *Fish Death Rate* does not remain proportional to the fish population. The fraction of the fish population that dies each year depends on the degree of crowding of the fish population. The *Death Fraction* increases as crowding increases. When the fish population is below the *Carrying Capacity* of the area, *Death Fraction* is lower than the *Hatch Fraction*, so the population increases. As the population increases and approaches the *Carrying Capacity*, the *Death Fraction* increases and approaches the value of the *Hatch Fraction*. When the fish population is at the carrying capacity, the population enters into dynamic equilibrium. If the fish population becomes greater than the *Carrying Capacity*, the *Death Fraction* is greater than the *Hatch Fraction*, and *FISH* decreases to the equilibrium level. The *Death Fraction* in this model will be a function of the ratio between the fish population *FISH* and the *Carrying Capacity*.

3.1.2 Building the Model

- **Create the stock *FISH* in your STELLA window.**
- **Create the flow *Fish Hatch Rate* going into the *FISH* stock and the flow *Fish Death Rate* going out of the stock.**
- **Place the converters *Death Fraction* and *Hatch Fraction* next to their corresponding flows.**
- **Make connections from *FISH* and *Hatch Fraction* to *Fish Hatch Rate* and from *FISH* and *Death Fraction* to *Fish Death Rate*.**
- **Define *Fish Hatch Rate* and *Fish Death Rate:***
  Fish Hatch Rate = FISH * Hatch Fraction
  Fish Death Rate = FISH * Death Fraction
- **Enter initial values for FISH (10) and Hatch Fraction (6).**
- **Place a converter called *Carrying Capacity* next to *Death Fraction*.**
- **Set *Carrying Capacity* = 1200 (fish).**
- **Place connectors from the *FISH* stock and the *Carrying Capacity* converter to the *Death Fraction*.**

- **Double click on *Death Fraction* and enter *FISH/Carrying Capacity***
- **Click on *Become Graph***
- **Enter the proper values for the Input and Output as shown in Figure 6.**
- **Change the upper and lower limits of *Death Fraction* from 0 and 100 to 5 and 11, and *FISH/Carrying Capacity* from 0 and 100 to 0 and 2.**
- **Click on OK.**

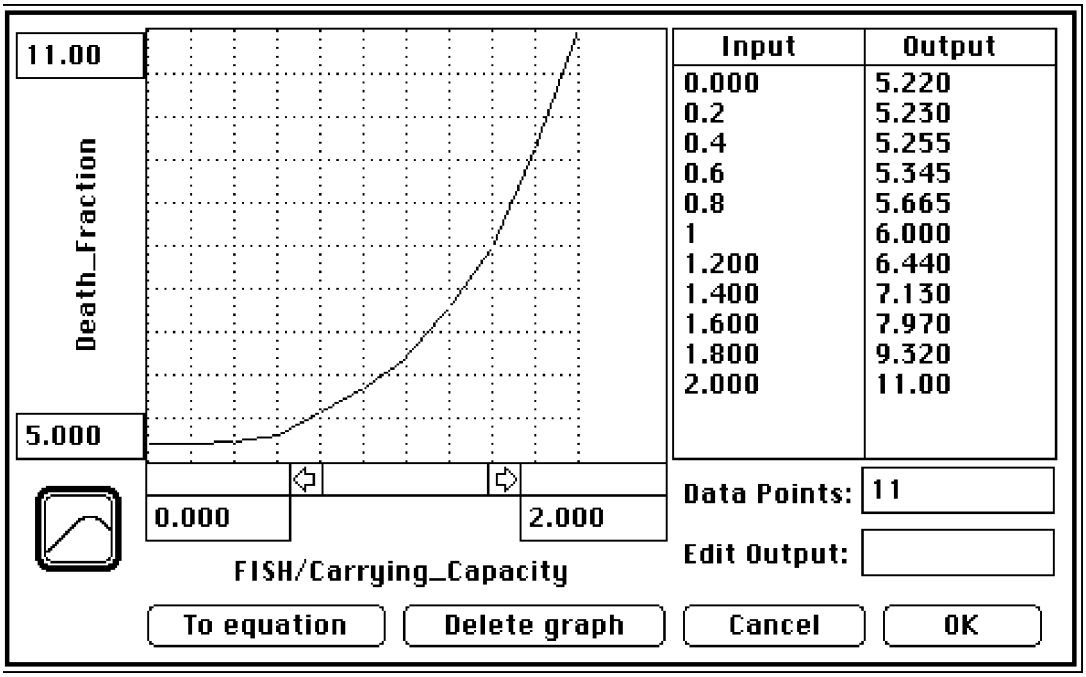


Figure 6: Graph of Death Fraction

This figure shows the graph that defines the relationship between *Death Fraction* and the fish crowding factor (*FISH/Carrying Capacity*).

The graph shown in Figure 6 covers a range extending from a fish population of zero to a fish population that is twice as large as the carrying capacity. Although it may seem extraneous to define such a large range , it is a good practice to follow. It is unlikely that the fish population will rise much above the maximum carrying capacity, but to test your assumptions about the model, you should test a few points well outside the expected operating range and see if your assumptions remain valid. In this model, we assumed that the *Death Fraction* would rise quickly as the number of fish increased above the carrying capacity. Using STELLA, we can create a simulation to test this assumption. If you dump 1000 extra fish into the area, the death rate will surge and quickly bring the

population back to equilibrium at maximum capacity. This behavior makes sense, so our assumption is probably correct.

- **Remember to set the value of DT = 0.1 before you run the model.**

Your model should now look similar to the one shown in Figure 7. The feedback loops present in the system are highlighted in Figure 7 and will not look the same in your diagram. The negative and positive signs indicate the polarities of the loops and are only for your information. They do not show on your model.

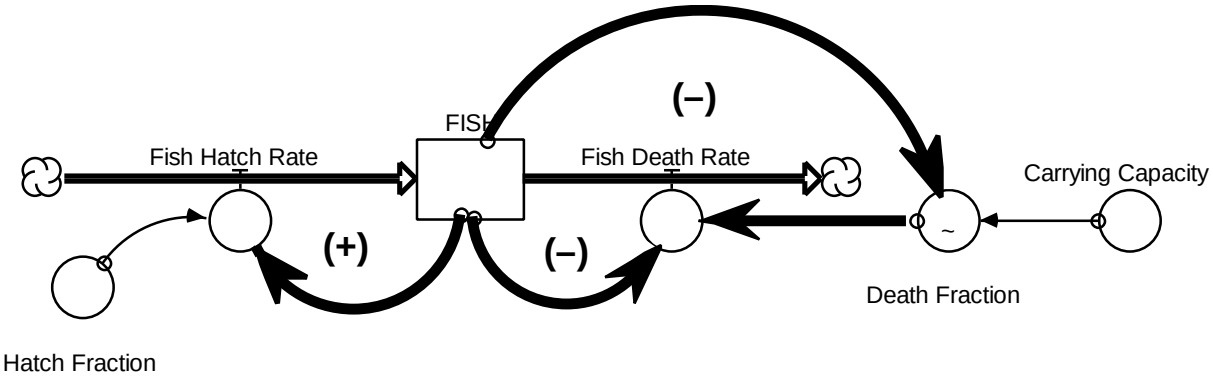


Figure 7: Modified STELLA diagram of The Fish Population Model

This modified STELLA diagram shows the model of the fish population. It has been modified to highlight the feedback present in the model.

You have now completely modeled the fish population as it behaves in the absence of a fishing industry. The questions below will help you to understand the model more insightfully. Go through the questions carefully and then compare your answers with suggested answers in Appendix 1.

3.1.3 Questions

1. Trace out the feedback loops shown in Figure 7. Which is positive? Negative? Why?

2. Do you recognize the S-shaped growth structure in the model? Can you think of other situations where similar behavior occurs?

3. Run the model. Create a graph and observe the behavior of the *FISH* stock. What is the shape of the curve? Which loop is dominant after three years? After nine years? When does the transition occur?

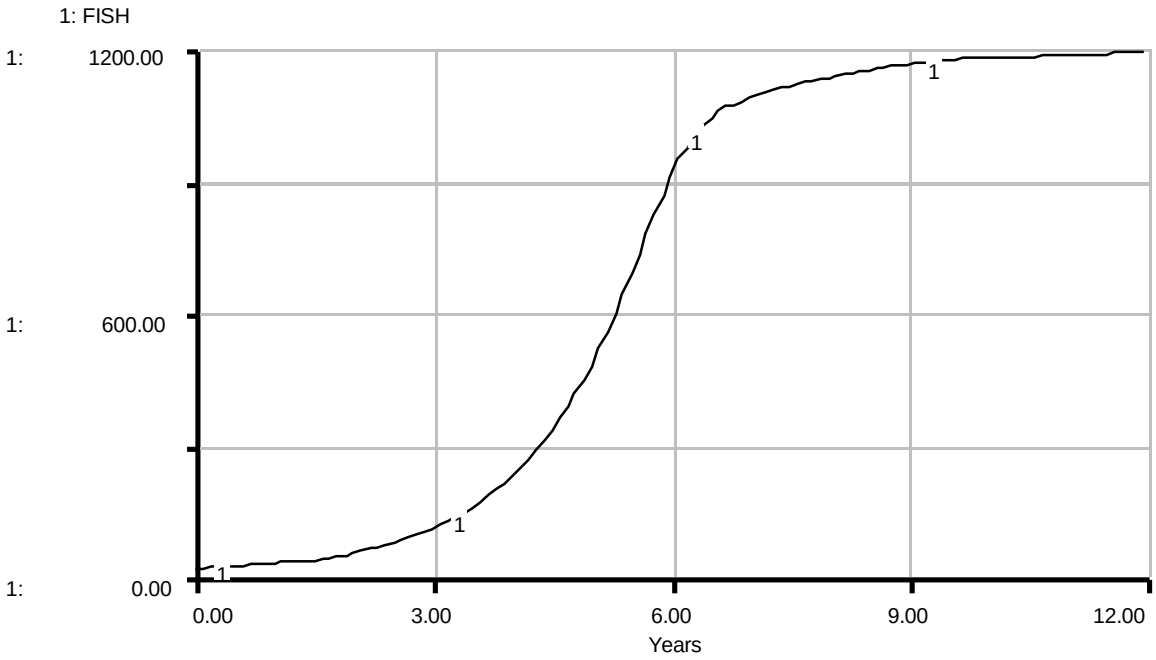


Figure 8: The Behavior of the Fish Population over time

This graph shows the S-shaped growth of the fish population as it grows from 10 fish to the maximum limit of 1200 fish in the absence of a fishing industry.

3.2 Modeling the Ship Subsystem

***Note: To facilitate the process of combining the models, you should build the Ship subsystem on the same STELLA diagram pad as the Fish Population subsystem.

3.2.1 Description

***Note: For simplicity, this model will combine all the individual fishing companies into one aggregate company. Also, we have made assumptions about the buying strategy of the fishing companies.

Each year, the company harvests the fish and reinvests 20% of its *Yearly Profits* into building more *SHIPS* at $300 apiece. The *Ship Building Rate* is the number of ships

the company can build with the fraction of the *Yearly Profits* being reinvested into ship building. The company will begin with 10 *SHIPS* and increase its fleet as long as *Yearly Profits* are positive. If the *Catch per Ship* is low, the company's *Operating Costs* may exceed the income from fishing, and the net profit for that year will be negative. *Yearly Profits* are determined by subtracting the company's *Operating Costs* from the *Revenues.* The *Operating Costs* of the company depend on the number of ships in its fleet. It costs $250 per ship to operate for one year. The company sells the *Total Catch per Year* at $20 per fish to produce *Revenues.* For this model, assume that the *Catch per Ship* is 15 fish.

Two feedback loops affect the *SHIPS* stock. The relationship between *SHIPS* and *Revenues* generates the first loop. That is, fishing with more ships leads to a larger *Total Catch per Year,* which in turn leads to larger *Revenues* and increased *Yearly Profits.* The increase in *Yearly Profits* results in an increase in the amount of money reinvested into building more ships. This is a positive feedback loop.

The relationship between the number of *SHIPS* and *Operating Costs* generates the second feedback loop. Increasing the number of *SHIPS* in operation increases *Operating Costs* which then causes *Yearly Profits* to decrease. This decrease in profits results in a decrease in the amount of money reinvested into building more ships. This is a negative feedback loop.

***Note: Since later we will be connecting this model to the Fish Population model, create *Total Catch per Year* as a flow. For now, leave it as an unconnected flow (with *Catch per Ship* and *SHIPS* as inputs.)

3.2.2 Building the Model

- **Create the stock *SHIPS* with the *Ship Building Rate* flowing into it.**
- **Set the initial value of *SHIPS* = 10 (ships).**

The *Ship Building Rate* is the number of ships the company can build with the fraction of the yearly profits being reinvested into ship building. This leads to the following equation:

*Ship Building Rate = Yearly Profits * Fraction Invested / Ship Cost*

- **Create the following converters in your model:**
  ***Ship Cost* = 300 ($)**
  ***Fraction Invested* = 0.2 (dimensionless)**
  ***Yearly Profits* (leave undefined)**
- **Connect the three new converters to *Ship Building Rate*.**
- **Define *Ship Building Rate.***

Yearly Profits are equal to *Revenues* minus *Operating Costs*. *Operating Costs* are simply the number of *SHIPS* multiplied by $250. Fish sales produce *Revenues*.

- **Create the following converters:**
  ***Operating Costs = SHIPS* * 250**
  Fish Price = 20 ($)
- **Create the flow *Total Catch per Year*, but leave it unconnected.**
- **Create the converter *Revenues.***
- **Connect *Fish Price* and *Total Catch per Year* to *Revenues.***
- **Define *Revenues:***
  Revenues = Total Catch per Year * Fish Price
- **Connect *Revenues* and *Operating Costs* to *Yearly Profits*.**
- **Define *Yearly Profits*:**
  Yearly Profits = Revenues - Operating Costs

The *Total Catch per Year* is simply the number of fishing ships present in the area times the number of fish that each ship catches.

- **Create the converter *Catch per Ship.***
- **Connect *SHIPS* and *Catch per Ship* to *Total Catch per Year.***
- **Define *Catch per Ship* and *Total Catch per Year:***
  ***Catch per Ship* = 15 (fish)**
  Total Catch per Year = SHIPS * Catch per Ship

After building the entire ship subsystem model on STELLA, compare your model to the one shown in Figure 9. If your model is the same, please continue with the rest of the paper. If it is not, correct your model and check it with the relevant equations in Appendix 4.

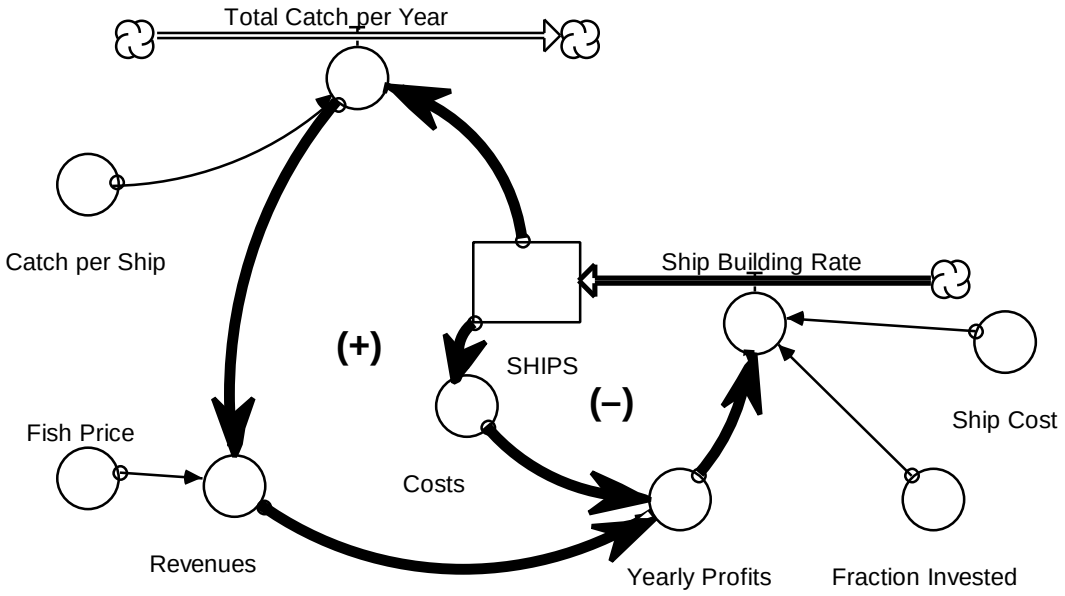


Figure 9: Modified STELLA diagram of The Ship Subsystem

This modified STELLA diagram shows the model used to represent the Ship subsystem. It has been modified to highlight the feedback loops present in the model.

3.2.3 Questions

1. Trace the feedback loops on the model in Figure 9. Which loop is positive feedback? Which is negative? Why?

2. Run the model. Look at the graphs for *SHIPS* and *Yearly Profits*. Why are they the same shape? Will they always look like this?

3. Change *Catch per Ship* to 10. What happens? Can you make *Yearly Profits* zero? What would the value of *Catch per Ship* be?

3.3 Modeling The Connection Subsystem

You have just modeled both the fish population and the fishing companies. Now it is time to put those two subsystems together into one complete model. We will analyze the connection between the two previous subsystems separately as its own subsystem.

This subsystem is perhaps the most important piece of the whole system. Its behavior is one of the main reasons why the fish population is depleted every time.

The model of this subsystem contains some of the elements that you have already included in the fish and the ships models. Since you cannot use the same element twice in STELLA, you must save the models that you have already built and begin a new model.

- **Select "Save" from the File menu.**
- **Click on "Desktop" in the Save dialogue box.**
- **Name your model and save it. (Be sure to remember the name!)**
- **Select "Close Model" from the File menu.**
- **Select "New" from the File menu to begin a new model.**

3.3.1 Description

The connection subsystem contains the link between the FISH and the SHIPS stocks. The major characteristic of this subsystem is the delay between *FISH* and *Catch per Ship*. Modern fishing ships have sonar tracking and other high tech equipment to aid in finding and catching fish. As a result, the ships are able to return with nearly a full load even if the fish population begins to drop. It is not until the fish population has dropped dangerously low that fishing companies detect any significant decline in the catch. To represent this fact in a model we will need to redefine *Catch per Ship* and have it depend on a new factor: *Density*. *Density* is the number of *FISH* in the 100 square mile *Area* where the fish population lives. The *Catch per Ship* will be 0 when *Density* is 0. As *Density* increases, *Catch per Ship* will increase sharply, leveling off at 25 fish per ship when the population is at maximum density.

This model introduces a new flow affecting the *FISH* population, *Total Catch per Year*. This flow was defined in the previous section, but it was left unconnected. In this model, and in the final model, connect *Total Catch per Year* to *FISH* as an outflow. For this model, set the initial value of *FISH* = 1000 (fish) and *SHIPS* = 10 (ships).

There is a feedback loop present in this subsystem. An initial increase in the *Total Catch per Year* increases the depletion rate of the *FISH* population. As a result, fish *Density* is lower than it would have been, and so is *Catch per Ship*. Finally, the *Total Catch per Year* decreases. This is a negative feedback loop.

3.3.2 Building the Model

- **Create the *FISH* and *SHIPS* stocks.**
- **Create *Total Catch per Year* as an outflow to the *FISH* stock.**
- **Create the following converters:**
  - ***Area* = 100 (square miles)**
  - ***Density = FISH / Area***
  - ***Catch per Ship* (Leave undefined)**
- **Define the flow Total Catch per Year:**
  - ***Total Catch per Year = SHIPS * Catch per Ship***
- **Set the initial value of *FISH* (1000 fish) and *SHIPS* (10 ships).**
- **Connect *Density* to *Catch per Ship.***
- **Double-click on *Catch per Ship*.**
- **Select *Density* from the Allowable list.**
- **Click once on Become Graph, then enter the proper ranges, Inputs and Outputs as shown in Figure 10.**
- **Click on OK.**

Make sure that your model is correct by comparing with Figure 11. If your model is structurally the same, then go on to the next section. If not, please refer to Appendix 4 and correct your model.

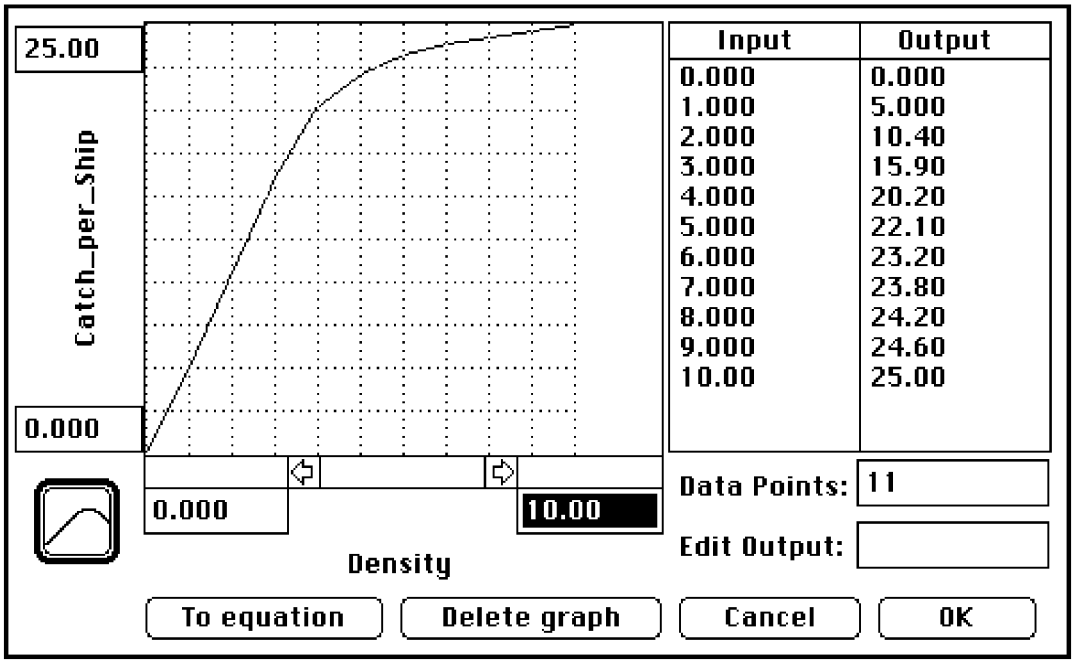


Figure 10: Catch per Ship graph

This figure shows the graph which you must enter into your STELLA model to define the relationship between *Catch per Ship* and fish *Density*.

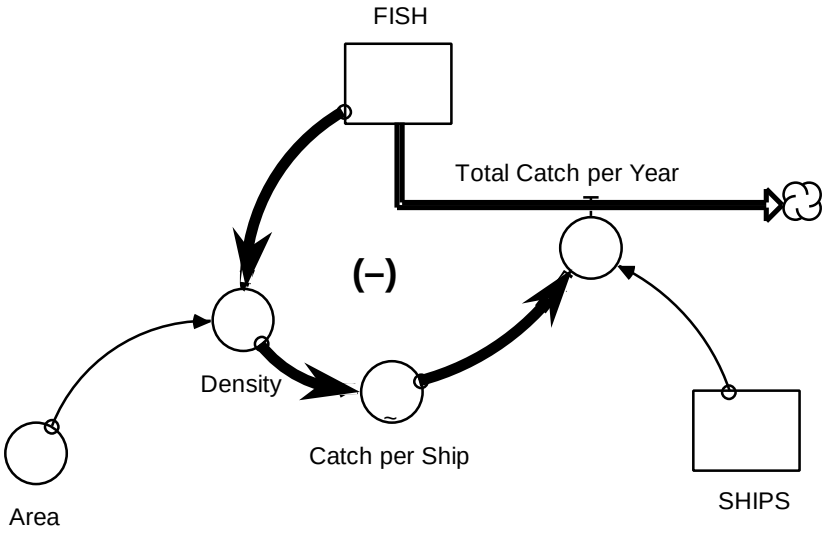


Figure 11: Modified STELLA diagram of The Connection Subsystem

This modified STELLA diagram shows the model of the feedback loop which connects the Fish and the SHIPS subsystems.

3.3.3 Questions

1. Why is this a negative loop?

2. Run the model and observe the behavior of *Total Catch per Year* and *FISH*. Notice that the fish population drops significantly before any noticeable change occurs in the catch. After one year, the *Catch per Ship* dropped by only 4% while nearly 1/4 of the fish population was eliminated. After two years, there is only a 10% drop in the *Catch per Ship*, and almost 1/2 of the fish population is gone. What is the major implication of this in the fishing industry?

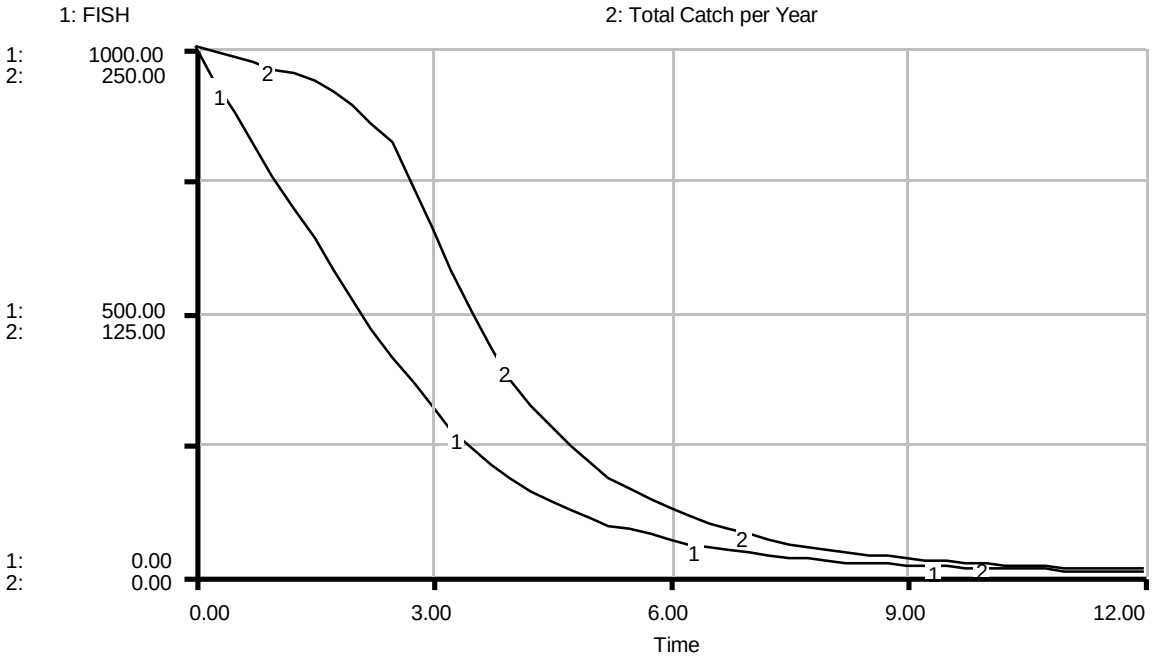


Figure 12: Graph of the Connection System behavior

This graph shows the behavior of the connection subsystem. The graph shows the delay between the *FISH* population and the *Total Catch per Year*.

3.4 Combining the Systems

You will need to close your model of the connection subsystem and open your saved model of the fish and ships subsystems. It may be helpful to tear off the Appendix and keep the picture of the final model handy for this part.

- **Select "Close Model" from the File menu.**
- **Click on "Don't Save" in the dialogue box which appears.**
- **Select "Open" from the STELLA File menu.**
- **Click on "Desktop" in the Save window.**
- **Select your saved model and open it.**
- **Highlight the entire Ship system by clicking and dragging the mouse to form a dotted outline around the system.**
- **Align the two models so that the *Total Catch per Year* is below the *Fish Death Rate*.**
- **Click on the cloud opposite the arrow of the *Total Catch per Year* flow and drag it until it is inside the *FISH* stock.**
- **Release the mouse button and click on the *FISH* stock.**
- **Move the mouse slightly until the cloud hidden by the stock becomes highlighted as well. Release the button.**
- **Click on the arrow in the upper-left corner of the STELLA window to display the equations. The *FISH* stock should now have *Total Catch per Year* listed as an outflow**
- **Click on the arrow in the upper-left corner again.**
- **Place the *Area* and *Density* converters in their proper places in the model and make their connections.**

Now that you have completed the model of a fishing industry system, feel free to play around with it and see what happens. You should check your model equations with those included in Appendix 4 to make sure they are identical. Notice that the model you have built, unlike the one-player game, does not allow for any user input other than to run the model. In your model, the profits from the previous year determine the *Ship Building Rate*. Although the reinvestment of a constant percentage of the profits into building more ships may seem oversimplified, this is similar to the growth strategy of many fishing companies. Some of the more interesting elements to observe may be *FISH*, *Yearly Profits* and *Total Profits*. After building the model you should have a good idea why it behaves the way it does. The next section of the paper will take you through a policy analysis exercise using the model you have just created. The goal of the policy analysis is to find a solution to the fish resource problem.

4 THE TRAGEDY OF THE COMMONS

As the fish population decreased, your fishing ships began returning a smaller and smaller profit. The natural reaction to this is to increase your fishing fleet to keep making the same net profit. Even if you realize the danger of over fishing the area, there are no guarantees about your competitors realizing the same thing. Short-term individual profits motivate many fishing companies. They give little concern for the resource involved. As long as this is the case, the companies will overuse the resource and greatly diminish or destroy it. This phenomenon is very common in real life, so common that it has a name: Tragedy of the Commons.[5]

Tragedy of the commons occurs when several individuals share a limited resource. It is described as follows:

> *"Individuals use a commonly available but limited resource solely on the basis of individual need. At first they are rewarded for using it; eventually, they get diminishing returns, which causes them to intensify their efforts. Eventually, the resource is either significantly depleted, eroded, or entirely used up."*[6]

This phenomenon shows up in many fields other than resource extraction. Tragedy of the commons affects companies with shared secretarial pools, departments competing for funds, businesses competing for a small number of specialized clients and many other systems.

5 DEBRIEFING

The seas have been a source of wealth for centuries. During those centuries, fishermen have been in constant battle with the seas in order to provide for their very survival. However, over the past century, man has taken the upper hand in this age-old battle. Through the use of superior harvesting technology, man has been able to rob the

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[5] Hardin, Garrett. "The Tragedy of the Commons", Science. Vol.162, December 13, 1968. pp. 1243-1248.

[6] Senge, Peter M., 1990. *The Fifth Discipline: The Art and Practice of the Learning Organization*, New York, Doubleday.

seas of greater and greater numbers of fish. As a result, there are many fisheries which have been totally destroyed or are near destruction because of over-harvesting. Man's superior intellect has destroyed stocks of renewable resources which could have remained harvestable for hundreds of generations.

The model you built in this paper is based on the game Fish Banks, Ltd. developed by Dennis L. Meadows.[7] Dr. Meadows developed the original game to address the problems inherent in the use of natural resources. The original game requires a large group of people. The model you have built gives individuals the opportunity to learn about the natural resource depletion dilemma.

The interaction between the fish population and the fishing companies is only one of the many systems that include and affect these entities. You can build models to describe the effect of weather patterns, natural predators for the fish, the local economy and many other factors. It is important to realize that the model created in this paper cannot be used to analyze the behavior of those other systems. This model addresses a specific problem: the effects of a fishing industry on an otherwise stable fish population. When used for this purpose the model describes very accurately the actual behavior of similar real-life systems.

The following quote illustrates the result of overexploitation of a fish population in real life.

> *"The Pacific Coast sardine industry had its beginnings back in 1915 and reached its peak in 1936-1937 when the fishing netted 800,000 tons. It was first in the nation in numbers of pounds of fish caught, and ranked third in the commercial fishing industry, growing $10 million annually. The fish went into canned sardines, fish bait, dog food, oil, and fertilizer. The prosperity of the industry was supported by over exploitation. The declines in the catch per ship and success per unit of fishing were compensated for by adding more ships to the fleet. The fishing industry rejected all forms of regulation. In 1947-1948 the Washington-Oregon fishery failed. Then, in 1951, the San Francisco fleet returned with only eighty tons. The fishery closed down and has never recovered..." Ecologist Robert Leo Smith.*[8]

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[7] See Footnote 3.

[8] Owen, Oliver, 1985. *Natural Resource Conservation: An Ecological Approach*, New York, MacMillan.

Now we introduce you to a paper which deals with policy analysis. Please read and do the exercises in the following pages.

Renewable Resource Depletion:

A Systems Approach to the Tragedy of the Commons
"Supplemental Materials for Fish Banks, Ltd."

This section of the paper is a shortened version of:

Halbower, Matthew C., 1992. *Renewable Resource Depletion* (D-4263-2), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, January 28, 37 pp.

In this section, you will continue to use the fish banks model you just built. The model will be used to test several strategies for renewable resource management.

6. PURPOSE OF LESSON

The purposes of the Fish Banks, Ltd. role-playing simulation and the Fish Banks computer simulation are complementary. The purpose of the role-playing simulation is to give students an understanding of the economic incentives and environmental realities from the perspective of a manager of a fishing company. From this view, the goal is to have students understand how the system can drive rational fishermen to the point of elimination of their fish supply. General knowledge of the feedback which controls the system behavior should be learned as well as the deeper understanding taken away by the firsthand experience of living through the huge initial profits followed by the sudden collapse and destruction of the fishing industry.

Beginning with the debriefing stage of the role-playing simulation and continuing through the computer simulation stage, students assume the role of scientists. Their job is to brainstorm and model (with the teacher's guidance) the feedback present in the system. The goal here is to go beyond a superficial understanding of the reasons causing the tragedy of the commons. Students can explicitly develop the structure with the help of a computer and test their model to determine whether their structure produces the behavior observed during the role-playing simulation.

Once students have developed a model which represents the system, they can assume the role of ministers in the National Fishery and Wildlife Service. In this part of their work, they conduct policy analysis on the system by changing its structure to conform to a number of regulatory policy formulations. Their responsibility and goal is to formulate a regulatory policy which will sustain a thriving national fishing industry, while keeping in mind the political, practical, and economic obstacles of various policy paths. This policy can be reinforced by computer simulation as well as written arguments supporting the policy choice and the reasons why other competing policies were not chosen.

With these general goals in mind, there are a number of specific subject areas emphasized throughout the lesson plan. The diversity of the plan hints of the interdisciplinary nature of simulations such as Fish Banks.

The main theme of the simulation stresses the necessity to understand and practice renewable resource management. From the systems perspective, this requires an understanding of the tragedy of the commons phenomenon from both an economic and environmental viewpoint. Stated simply, tragedy of the commons is a phenomenon

whereby individuals, sharing a common limited resource, each try to maximize his/her share of the resource until the resource is depleted and eventually destroyed.

Understanding the mechanics of tragedy of the commons requires knowledge of the interactions between the environmental and economic characteristics of the system. An environmental analysis can include population dynamics, carrying capacity, and maximum sustainable harvesting rate which helps explain that the earth's seemingly boundless natural resources are indeed limited. An economic analysis can show how the collective use of a renewable resource gives individuals an incentive to maximize their own profits in the short run rather than looking at the bigger picture and attempting to maximize everyone's profits, including their own. Combining the economic and environmental analysis points out why fishermen cannot see the effects of their actions on the system in the beginning. They can only see these effects after the destruction is complete.

Thus, students conduct computer simulation experiments to investigate the effectiveness of a range of regulatory policies. Policies can vary from taxation to the limitation of ships. The results of the implementation displays the effectiveness of each policy. Therefore, each student can discover, on his own, the solutions to the tragedy of the commons effect.

One of the most interesting regulatory policies deals with imposing restrictions on fishing technology.[9] Students can explore the effects of such limits on the sustainability of a fishing industry as well as ask in-depth questions concerning the effects of man's technological innovations.

Other regulatory policies include imposing taxes on ship construction, taxes on fish, forced dry-docking of ships over a certain age, ship building regulations and limits, market fish price regulation, and government fish price regulation. Of course students must always consider the political consequences of the regulatory decisions as well as the feasibility of carrying out those regulations.

In addition to exploring those computer simulation policies, the Fish Banks curriculum provides many opportunities for exploration away from the model. One of the most interesting areas to explore is the legal relationships between the economics and environmental consequences of renewable resource depletion. How does one set up a regulatory agency controlled by the state? What effects might proposed regulatory plans

[9] Such policies are already established in places like Alaska where fishing regulations require the use of gill nets which the fish can see underwater rather than the undetectable gill nets in order to reduce overharvesting.

have on individual liberty or property rights? Would assigning property rights to portions of the sea cut down on the tragedy of the commons?[10]

Finally, no matter what particular issues you wish to emphasize throughout this lesson, the nature of the work requires your students to develop group interaction and communication skills. This is true in two particular aspects. First, the computer exercises allow students to work together on the Macintosh and learn from each other. In this situation, the teacher fades into the background as the students explore the system with each other. Second, during the role-playing portion of the simulation, students must express their strategies and ideas and convince others through sound arguments. These two aspects of group interaction encourage the students to form mental models of the fishing system, to brainstorm solutions, and to verbalize complete ideas. These techniques show students how to work with complex problems and can later be applied to situations other than Fish Banks.

Dennis Meadows summed up its general purpose the best when he said, "This curriculum is specifically designed to convey the fundamental insights required for corporate managers, public officials, and private citizens to use their region's natural resources intensively without deteriorating the long-term productivity of their resource endowment."[11]

[10] On the Virginia side of the Chesapeake Bay, the state has assigned property rights to portions of the bay for the havesting of oysters in an attempt to control the problem of Tragedy of the Commons.

[11] Meadows, Dennis, Thomas Fiddaman, and Diana Shannon. Fish Banks, Ltd. A Microcomputer Assisted Simulation That Teaches Principles for Sustainable Management of Renewable Resources. p. 1.

7. FISH BANKS MODEL REVIEW

This section contains a brief review of the model you built earlier in the first part of this packet.

Model Assumptions:

The computer model contains two assumptions which are different from the assumptions in the role playing simulation. First, unlike the role-playing simulation, there is only a single fishery present within this model. All ships fish in the deep sea, and operating costs are computed accordingly.

Second, unlike the role-playing simulation, the model aggregates all shipping companies into a single stock of ships. The purpose of this is to allow the student to conduct a holistic study of the entire industry through the aggregation of the various fishing fleets. Both assumptions are meant to reduce the complexity and redundancy present within the role playing simulation.

Model Description:

Figure 13 on the next page shows the final result of combining the ship subsystem with the fish subsystem. The ships drive the depletion of the fish resource. Simple economics drives the increase in the number of ships. As the total catch per year increases, the total revenues increase since more fish are sold. Profits increase as revenues increase because more money is being made. When deciding whether or not to purchase a ship, the fisherman looks at his profits and invests some fraction of those profits in new ships thus increasing the total number of ships. This forms a positive feedback loop since an increase in ships will lead to an increase in total catch per year which leads to more revenues, more profits and ships, thus increasing the stock of ships.

The only force counteracting this positive feedback loop is the negative feedback loop formed between ships, costs, profits, and new ships. As the number of ships increases, the cost of operating those ships increases which decreases profits, new ships, and then the number of ships. This feedback loop is much less dominant than the positive feedback loop at the beginning of the simulation when a great deal of money is being made.

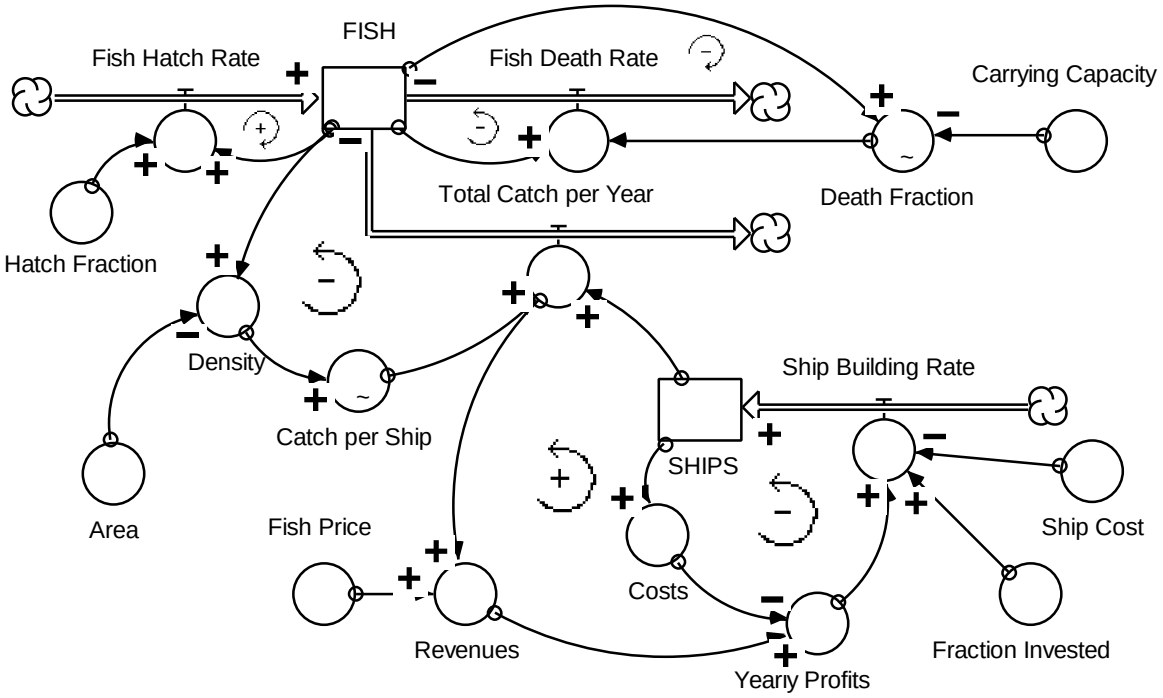


Figure 13: STELLA Diagram Depicting Structure of Fish Banks Model

8. POLICY ANALYSIS EXERCISE

The following exercises are for your students. No answer key is included because the answers vary depending on the emphasis of the lesson. Therefore, the exercises should be examined before presentation to the students so that areas of focus can be determined and original answers created. The length of time required to complete the exercises is approximately three hours, so you may wish to delete some of them. The students should work in pairs or groups of three to encourage communication and interaction. Let the students learn from each other; you may be surprised by some of their innovative suggestions.

System Dynamics in Education Project

Fish Banks Policy Analysis Exercise

Name _____

The year is 1970.

You are a manager in the national fish and wildlife service. You are charged with the responsibility of making policy suggestions regarding the management of a fishery surrounding your national borders. Fishing is an important source of economic income for your country so your decisions must insure the healthy maintenance of the fish population and fishing industry.

Luckily, you have retained some expensive consultants from MIT (paid with government money, of course). The consultants have built a model of the fish population and fishing industry in your nation. They claim that you can use the model to test the impact of different regulatory policies on the fish population and fishing economy. A diagram of the model is shown below in Figure 1.

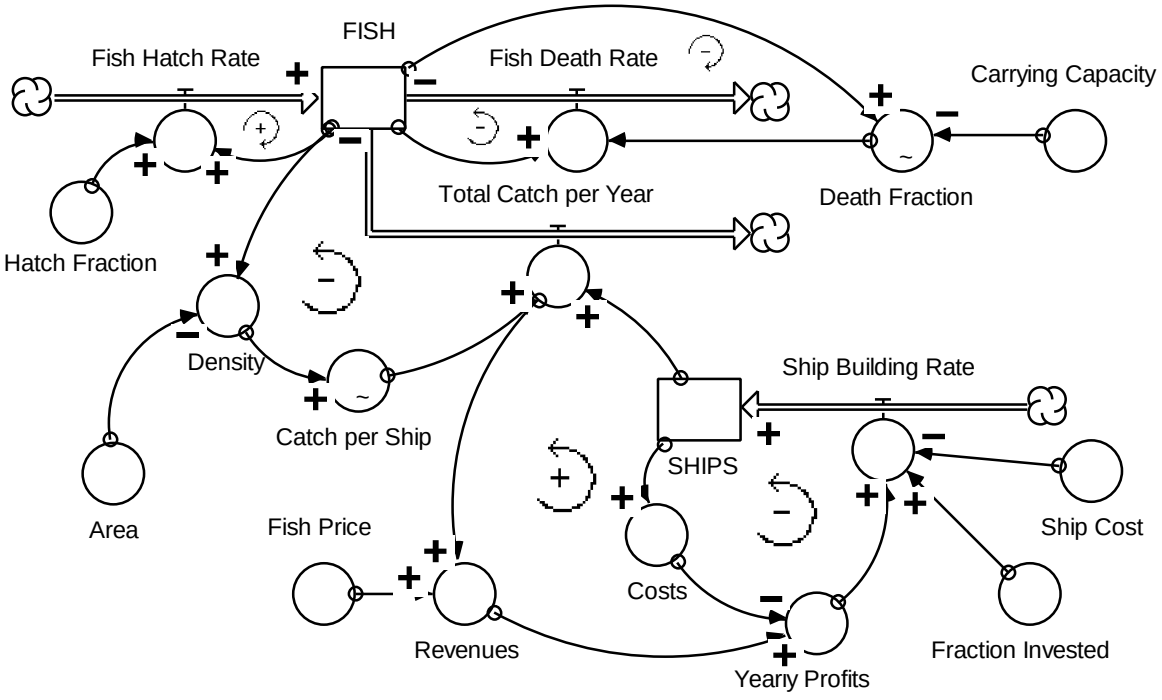


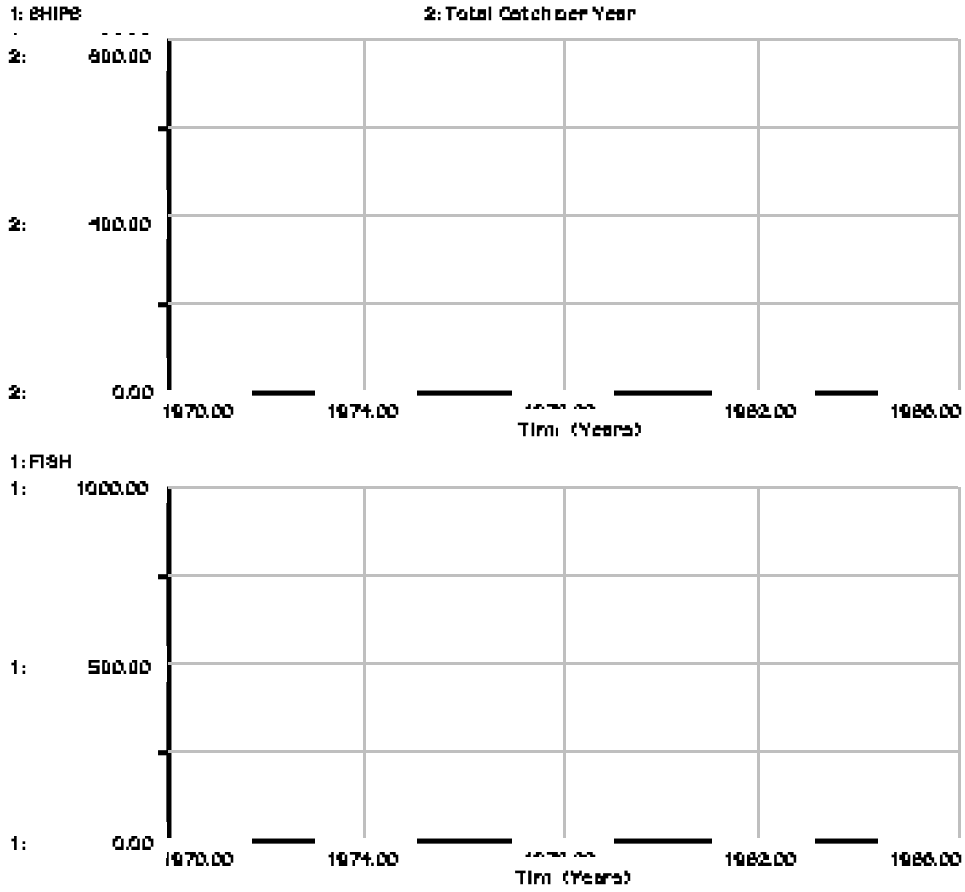
Figure 1: STELLA Diagram of Fish Banks Model

YOUR TASKS:

1. The model is currently set up to simulate the regulatory situation present within the national fishing industry as of 1970 (no regulation). To simulate the system:

- **Select Graph Pad from the Windows Menu**
- **Select Run from the Run Menu**

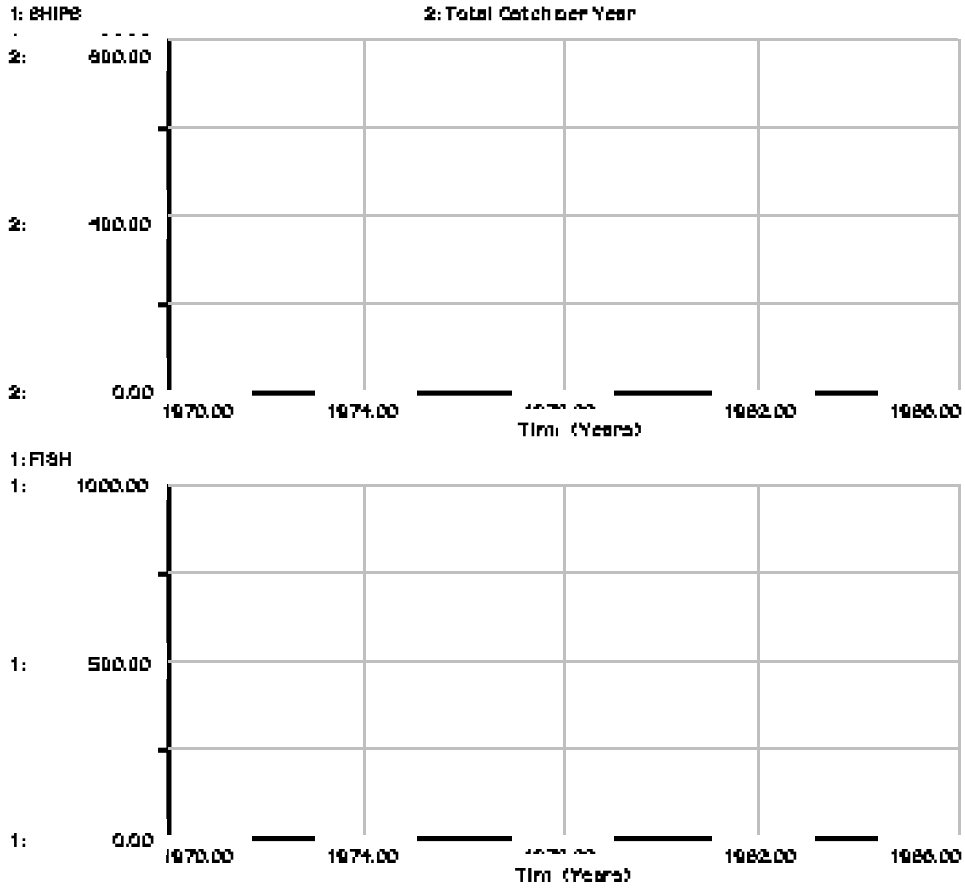
Observe the behavior of the number of ships, catch per year, fish population, and total accumulated profits on the different pages of the graph pad. The pages may be changed by clicking on the page numbers on the top right of the graph pad window. Sketch the graphs of ships, catch per year, and fish population on the axes below.



Explain in a few sentences why the system generated the exhibited behavior. Try to tell the story of what happened in the Fish Banks simulation by making reference to the feedback loops describing the system.

2. Clearly, leaving the present system undisturbed is an unacceptable situation. The fish population is decimated, the fishing industry dies, and you lose your job.

One regulatory policy which might help to avoid this situation is to levy a large tax on new ships (say $200 per ship), thereby raising the price and curtailing purchases. Sketch on the axes below the effect you believe that this policy will have on the number of ships, total catch per year, and fish population.



Try running this policy by changing the ship cost from $300 to $500.

- **Select Diagram from the Windows Menu**
- **Double-click on the converter labeled "Ship Cost"**
- **Type "500"**
- **Click the OK Button**

Once you have changed the ship cost, simulate the system:

- **Select Graph Pad from the Windows Menu**
- **Select Run from the Run Menu**

Using either a dashed line or different colored pen, sketch on these same axes: the computer simulated graphs of ships, total catch per year, and fish population.

Was this policy successful? How does the behavior generated in this simulation compare with the previous behavior? Explain the differences in behavior.

_____

_____

_____

Vary the amount of the tax and re-simulate to observe the effects on behavior. What behavior is exhibited if the tax is too high?

3. It is clear that a tax on ships seems to do nothing more than delay the destruction of the fish population. Another possible policy alternative is to keep building ships until there is a noticeable problem with the fish population. Since the only feedback signal from the fish population that fishermen receive is from the fish catch, the policy could make building ships illegal once the catch per ship decreases to an unacceptable rate.

To implement this policy, first change the ship cost back to 300 dollars per ship.

- **Select Diagram from the Windows Menu**
- **Double-click on the converter labeled "Ship Cost"**
- **Type "300"**
- **Click the OK Button**

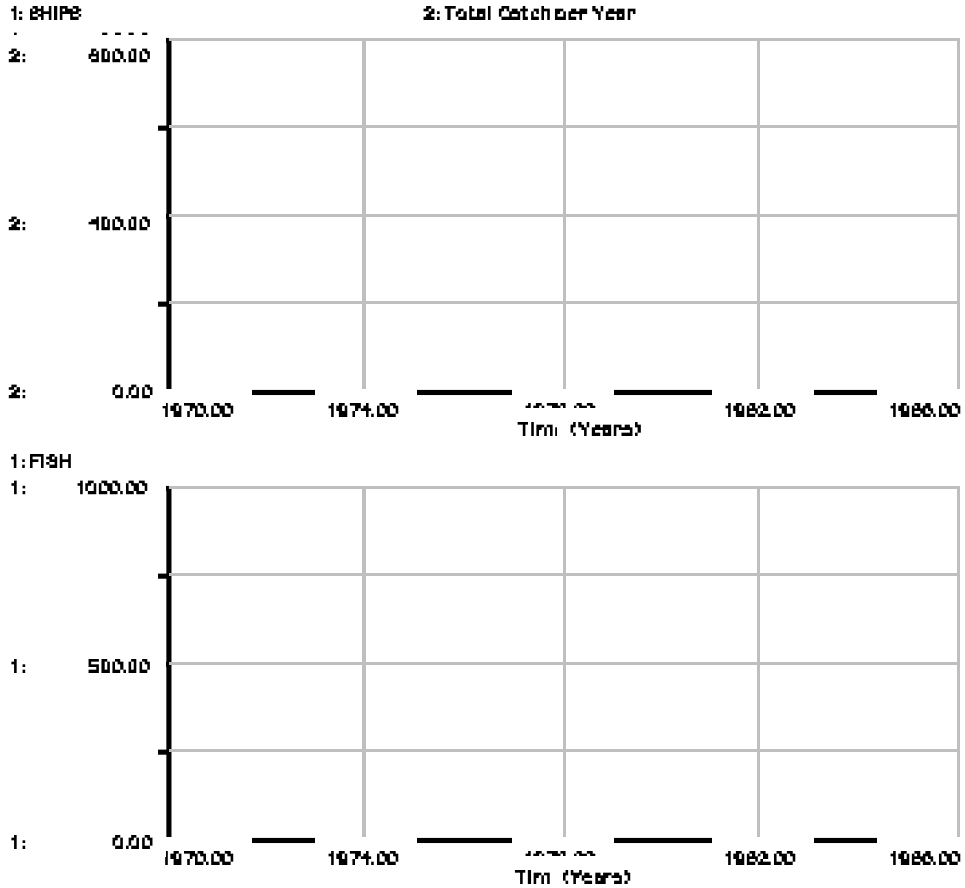
Next insert the feedback from Catch per Ship to Ship Building Rate. During good fishing, the average ship will catch 25 fish per year. See what happens when the ship building is limited after the catch drops below 22 fish per year.

- **Draw a causal connection from Catch per Ship to Ship Building Rate**
- **Insert the following equation into the Ship Building Rate:**

if Catch_per_Ship>22 then Yearly_Profits*Fraction_Invested/Ship_Cost else 0

This equation says that if the catch per ship is greater than 22 fish per year then purchase the normal number of ships (Profits*Fraction Invested/Ship Cost). However, if the catch per ship falls below 22, then build zero ships.

Graph on the axes below the effect you believe this regulation will have on the system.



Run the model and sketch the behavior on the axes above using either dashed lines or a different-colored pen.

Were your predictions correct? How does the behavior generated based on the new regulation differ from a system with no regulation? Explain any differences or lack of differences.

Was this policy successful? If not, try increasing the threshold on Catch per Ship from 22 to a higher number. At what number does the system produce a stable fish population?

What factors not present within the model might affect the catch per ship?

How easy do you think it will be for the government to collect data on the catch per ship? What problems might be encountered?

Write down a plan outlining how you would collect the data in order to decide when to limit the ship building rate.

Do you get the impression that fishing systems tend to give a great deal of warning before a catastrophe or do you feel these catastrophes come on more suddenly? Is the catch per ship an accurate indication of the fish population? Explain your reasoning by referring to the graph of Fish Catch per Ship and its relationship to fishing technology.

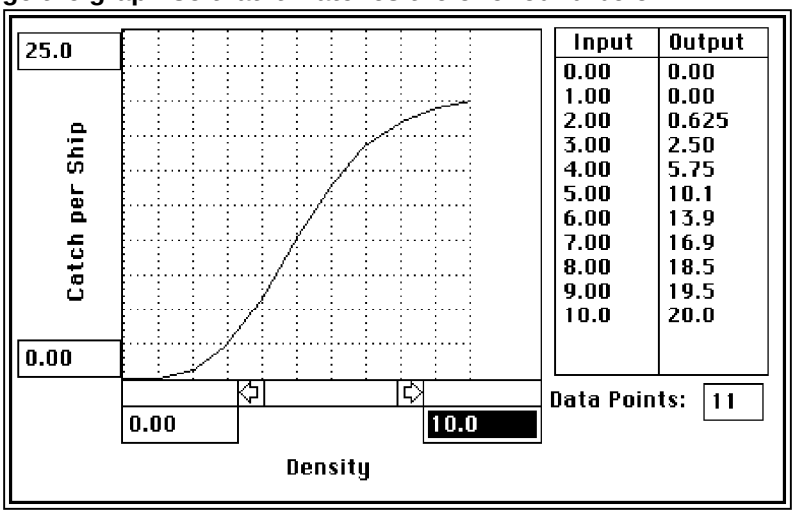
4. Another policy alternative is to make certain fishing technologies illegal by making it more difficult for ships to catch fish. Fewer fish caught means that the fish population has a better chance for survival. To implement this policy, first return the model to its original state:

• Select "Revert to Saved" under the file menu

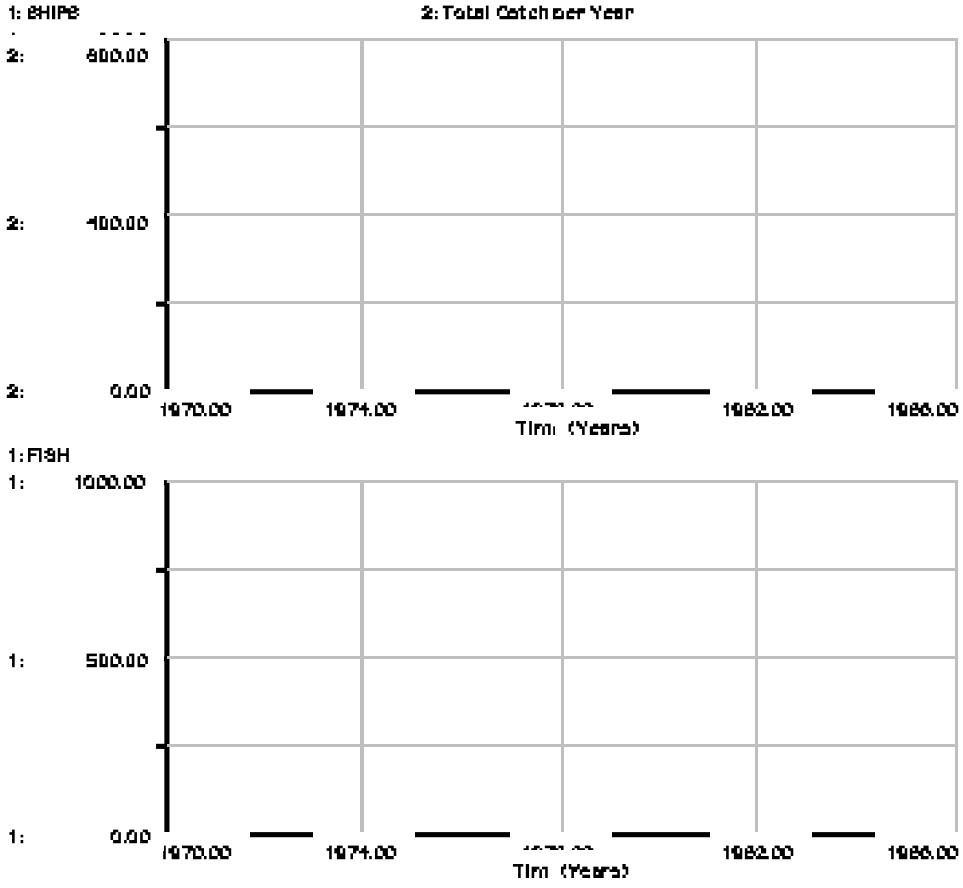
Next, change the graph defining Catch per Ship to model low fishing technology.

• Double-click on the converter labeled Catch per Ship

• Change the graph so that it matches the one found below



Once again try to predict how this policy will affect the number of ships, catch per year, and fish population by sketching on the axes below.



Run the model and sketch the computer simulated behavior on the previous axes using either dashed lines or a different colored pen.

How does this policy alternative seem to work? Were your predictions correct? If not, why were they incorrect? If you would like to run the time axis out further:

- **Select Simulation Time under the run menu and change the final time value.**

What are your accumulated profits here as compared to previous simulations? Explain why the fishermen's accumulated profits are more or less under this regulation.

Comment on the likelihood of being able to implement regulation of fishing technology. What groups would be opposed to or in support of such regulation? Do you think that your job would be in jeopardy if you proposed such a regulation?

5. Another policy might be to force all boats over a certain age have to be dry docked for safety reasons. This would create an outflow from the stock of ships. To implement this policy, first return the model to its original state:

- **Select "Revert to Saved" under the file menu**

To model the policy:

- **Select the Flow icon on the upper left portion of the STELLA Diagram Window**
- **Place the Flow icon on top of the stock labeled SHIPS**
- **Hold the mouse button down and drag the mouse an inch to the left**
- **Release the Mouse Button**
- **Type "Ship Scrapping Rate"**

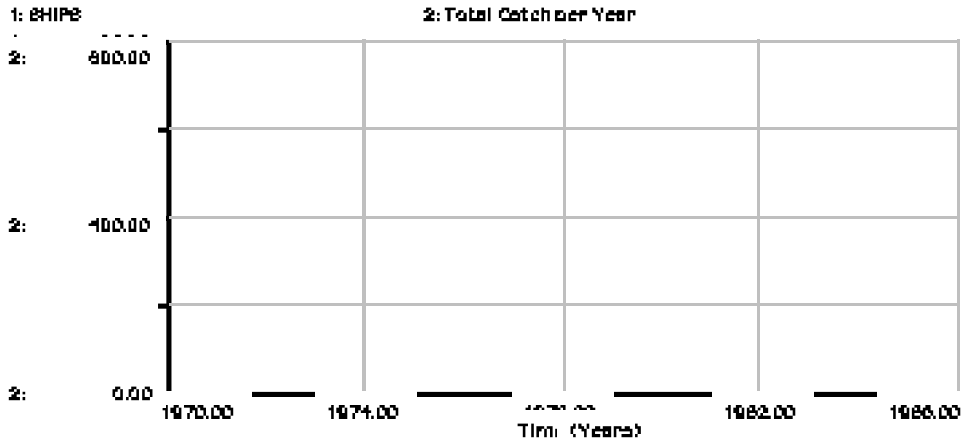
To define the flow, it is necessary to have a converter specifying the maximum age of ships:

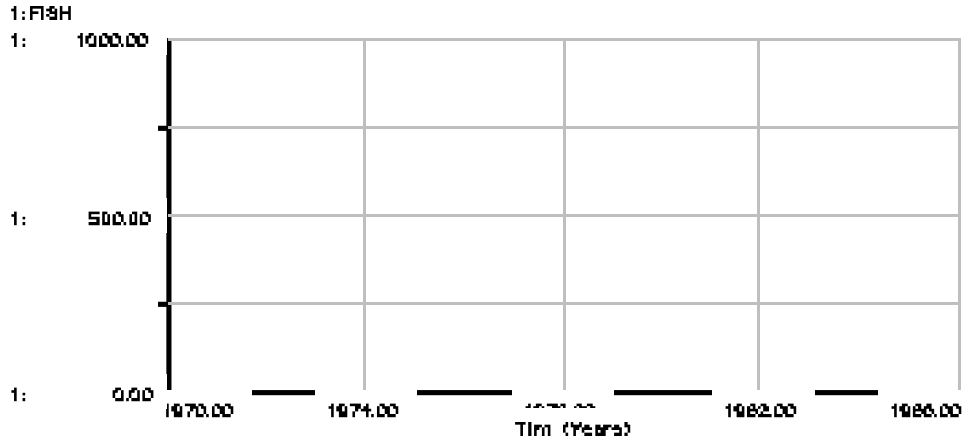
- **Select the Converter icon on the upper left portion of the STELLA Diagram Window**
- **Place the Converter icon just underneath the Ship Scrapping Rate flow**
- **Click the Mouse Button**
- **Type "Maximum Ship Lifetime"**
- **Double-click on Maximum Ship Lifetime and give it a value of 12 years**

All that remains is to determine the equation for the Ship Scrapping Rate. Assuming that the age of the ships is evenly distributed from young to old, about one-twelfth of the ships are scrapped each year due to old age. The equation defining ship scrapping rate is therefore the number of ships divided by the maximum age. To model this:

Draw a converter from SHIPS to Ship Scrapping Rate
Draw a converter from Maximum Ship Lifetime to Ship Scrapping Rate
Double-click on Ship Scrapping Rate
Define Ship Scrapping Rate to be SHIPS/Maximum Ship Lifetime

Once again try to predict how this policy will affect the number of ships, catch per year, and fish population by sketching on the axes below.





Run the model and sketch the behavior on the previous axes using either dashed lines or a different-colored pen.

Were your predictions correct? How does the behavior generated based on the new regulation differ from a system with no regulation? Try to tell the story of the differences between the two simulations by making reference to the feedback loops describing the system.

Continue to simulate the model with different maximum ship lifetimes. What happens if the maximum ship lifetime is made too short? Why?

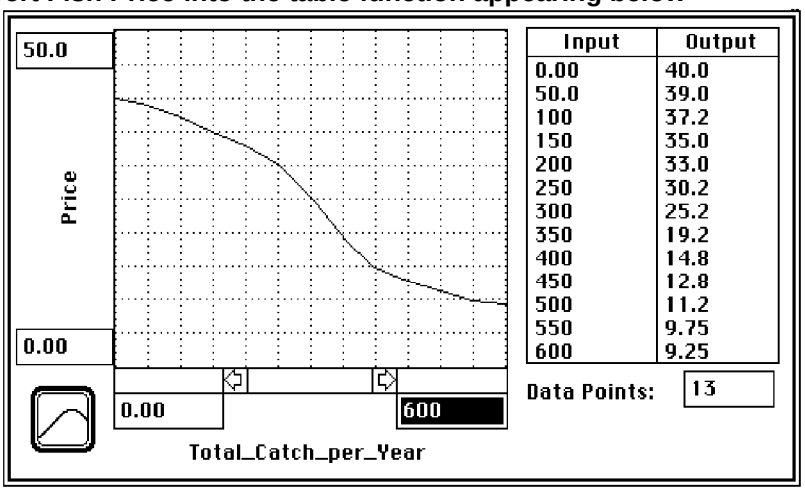
Do you think a regulatory policy such as this would be fair to the fishermen? Why?

6. Thus far, the fish price has always remained inelastic or constant relative to the actual supply of fish. In real life economics, the price of fish changes relative to the supply of fish. Thus, the price of fish will decrease if the total catch per year (supply) is higher for a particular year or increase if the total catch per year is smaller. To see how this affects the behavior of the system, first return the model to its original state:

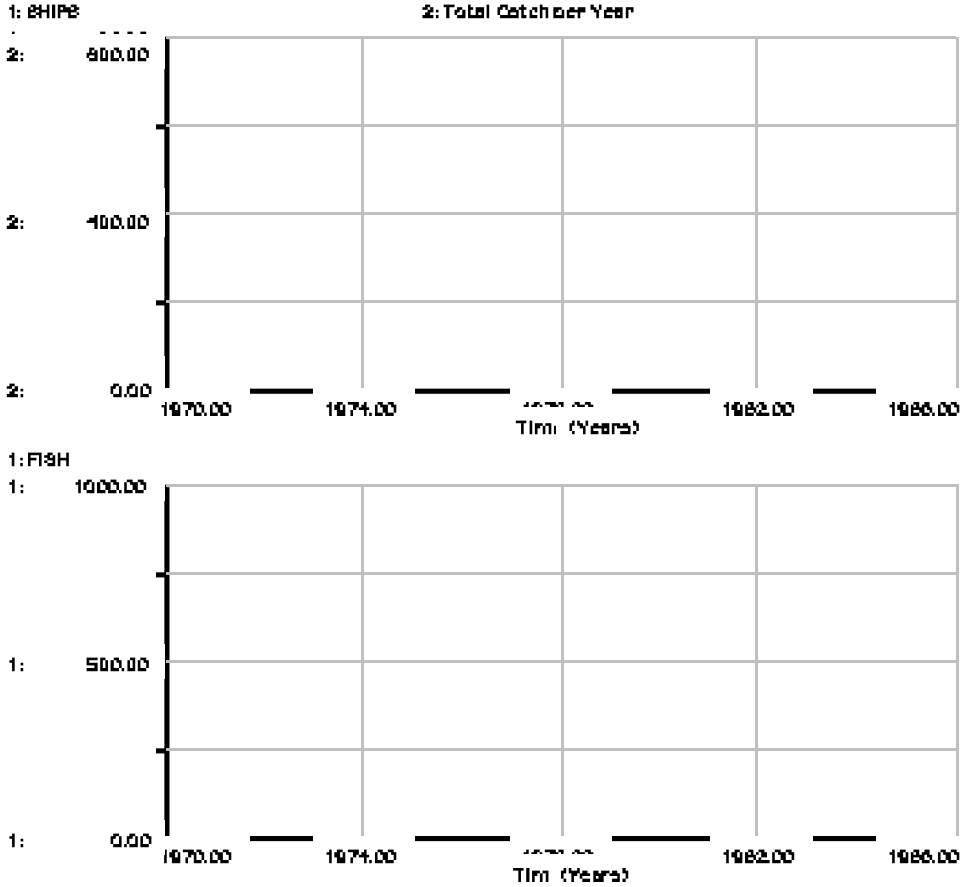
- **Select "Revert to Saved" under the file menu**

To implement the change:

- **Draw a causal connection between the Total Catch per Year and the Fish Price**
- **Convert Fish Price into the table function appearing below**



Again, place your predictions concerning the system behavior on the axes below.



Run the model and sketch the computer simulated behavior on the previous axes using either dashed lines or a different colored pen.

Were your predictions correct? Explain why the simulation generated this behavior.

Examine the graph showing the yearly profits of the fishermen. Does an extremely elastic price curve seem to be an acceptable solution?

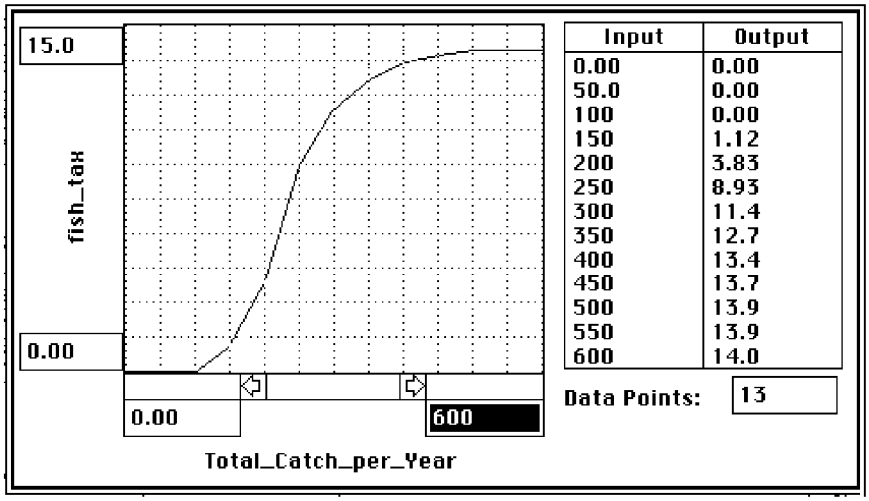
Change the price curve and re-simulate.

7. Suppose that we design an environmental tax which will tax the fishermen on a per fish basis based on the total number of fish which are caught each year. To implement this policy, first return the model to its original state:

- **Select "Revert to Saved" under the file menu**

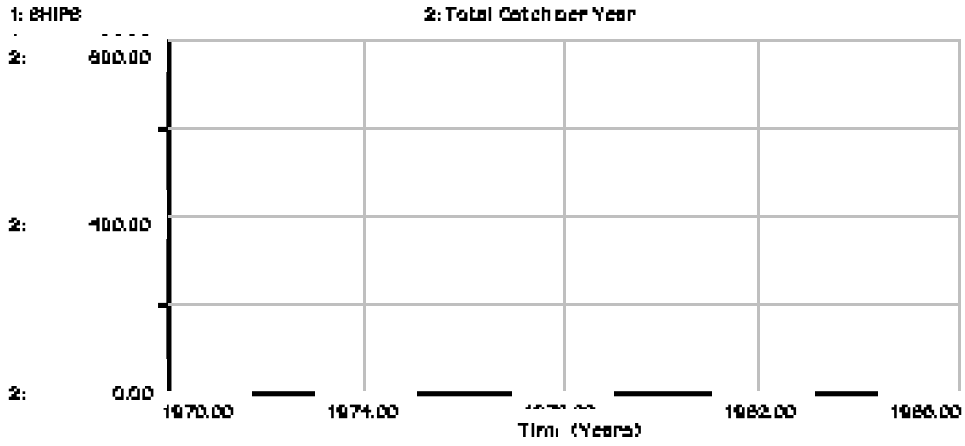
To model the policy:

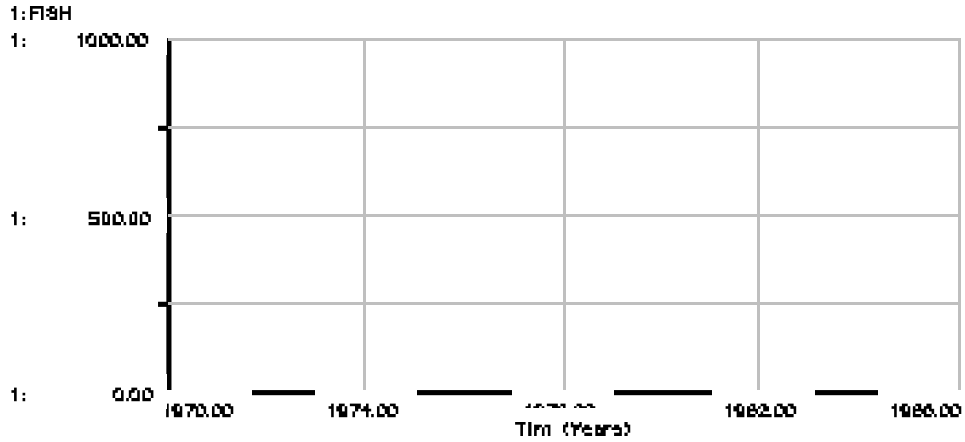
- **Create a new converter called Fish Tax**
- **Draw a causal connection from Total Catch per Year to Fish Tax**
- **Define Fish Tax as the table function shown below**



- **Draw a causal connection from Fish Tax to Costs**
- **Define Costs as "SHIPS*200 + Fish Tax"**

Again, sketch your predictions concerning the system behavior on the axes below.



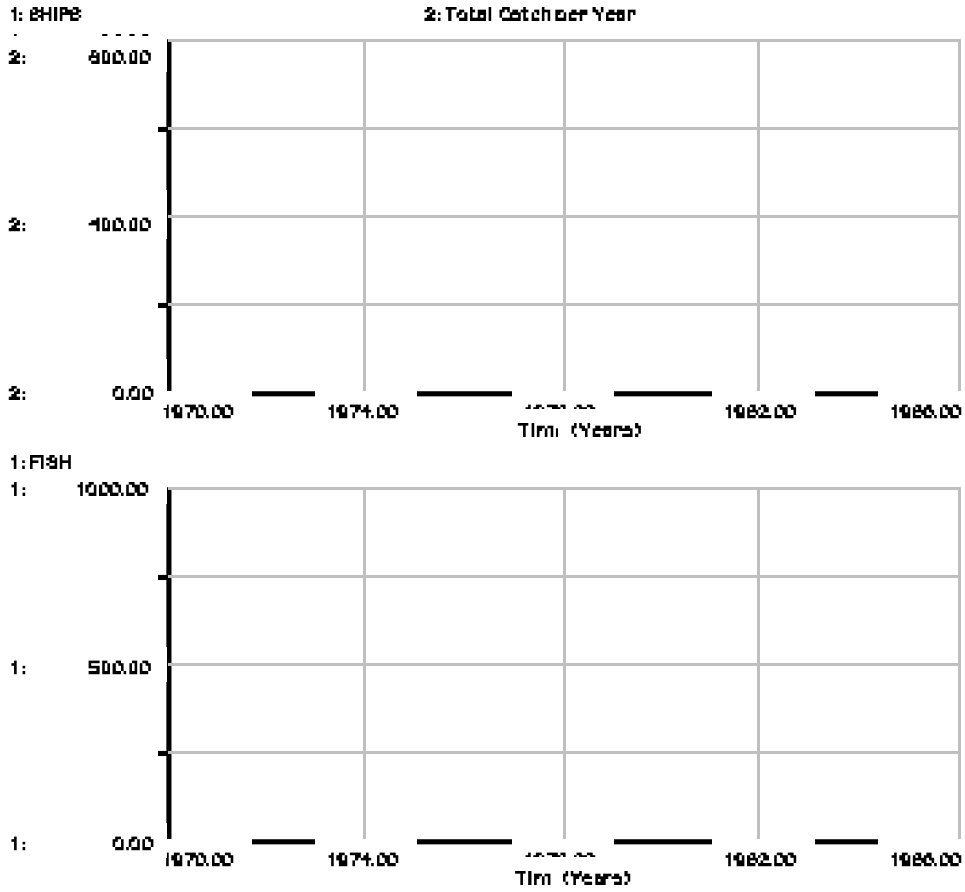


Run the model and sketch the behavior on the previous axes using either dashed lines or a different-colored pen.

Were your predictions correct? How and why does such a policy affect the fishermen's profits? Is this an acceptable policy solution?

8. Can you come up with any policies which have not been tried above? If so, list them below and attempt to model them if you have time.

9. Devise your own regulatory plan incorporating any or all of the regulatory policies tested above. Implement your plan on the model, and record the results on the graphs below.



What final policy plan would you recommend to the Wildlife and Fisheries Service? Clearly state the goals you are trying to meet and describe what an effective policy should accomplish? Justify your choice, and explain how it will work. Give your predictions regarding how your policy plan will be accepted by fishermen, politicians, and the public. Describe any possible difficulties associated with the implementation of your plan.

9 APPENDIX

9.1 Appendix 1: Selected Answers to Questions

3.1.3 Modeling the Fish Population Subsystem: Page 16

1. A positive or reinforcing loop is a loop where an initial change in an element of the loop will result in a change in the same direction of that element. Negative or balancing loops have the opposite effect. An initial change in the value of an element will result in a change in the opposite direction of that same element.

2. S-shaped growth occurs when you have a reinforcing growth process that is eventually brought under control by some growth constraint. An S-shaped growth structure consists of a positive growth loop and a negative balancing loop. Some real world examples include the spread of a rumor, infection of a population and growth of an economy.

3. The positive feedback loop is dominant for approximately four years. During this period the graph of the fish population seems to show pure exponential growth. After about five years, the dominance is shifted to the negative feedback loop. The dominance of the negative feedback loop causes the goal seeking behavior demonstrated in the last few years of the simulation.

3.2.3 Modeling the Ship Subsystem: Page 19.

1. The feedback loops are described in the explanation of the system.

2. As long as the *Catch per Ship* remains constant, as it does here, *Yearly Profits* and *SHIPS* will have the same shape curve. In the final model, they do not.

3. When *Catch per Ship* is 10, the company loses money each year, because the cost of maintaining a ship is greater than the revenue it produces. A *Catch per Ship* of about 12.5 is needed to break even. (i.e., *Yearly Profits* = 0)

3.3.3 Modeling The Connection Subsystem: Page 23.

1. The feedback loop is described in the explanation of the system.

2. The fishing companies are unaware of the plight of the fish population until it has been severely depleted. Because of this, it is impossible to protect the fish population with a reactive strategy. A proactive strategy is necessary. You will help design such a strategy in the Renewable Resource Depletion exercise. (D-4263)

9.2 Appendix 2: The Fishing Game on STELLA II version 2.2

Fishing Game v2.2.1 looks very much different from v3.0.5., but the basic ideas are exactly the same.

- **Make sure that STELLA II v2.2.1 is installed on your hard disk.**
- **Insert the disk labeled "Fish Banks Exercises" into your computer.**
- **Double-click on the disk icon labeled "The Fishing Game."**
- **In the window that appears, double-click on the icon labeled "The Fishing Game v2.2.1"**

If you have the STELLA II v2.2.1 program installed on your hard drive, the model you selected will open on your screen. If the model doesn't open, you need to open the STELLA program first and then open the model from within the program.

Your interface for the game is in a separate sector labeled "Control Panel." Figure 3 shows a picture of the control panel. The sector shown in Figure 3 is located in the upper-left corner of your STELLA diagram pad. If you do not see the sector shown in Figure 3, use the arrow bars on the bottom and right edges of the STELLA window to scroll the window until you see it.

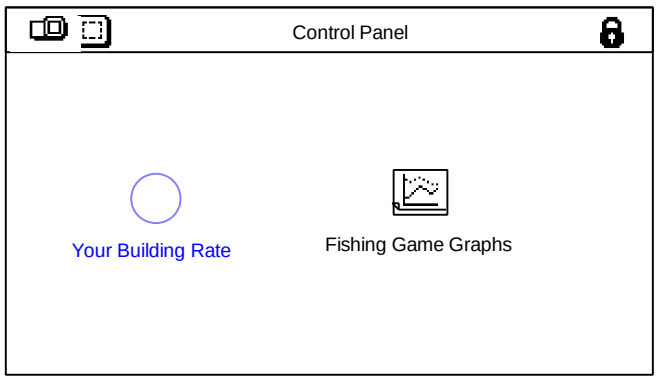


Figure 3: The Control Panel

This is the control panel through which you will control your fishing company in The Fishing Game. You can alter *Your Building Rate* each turn and observe the effects in the *FishingGame Graphs.*

? **Double-click on the icon labeled "Fishing Game Graphs" in the Control Panel.**

A graph window will appear on your screen. This window is called the graph pad, and it contains four graphs. At the top of the window is the title of the graph. The page number is on the bottom edge of the window. You can click on the lower-left corner of the graph pad to view the different pages, one at a time. These four graphs will allow you to track your own progress as well as that of your competitor. See Figure 4 on Page 8 for a list of the available graphs and the information each graph contains. At the end of each year, STELLA updates these graphs with current information. Please note that in graphs 3 and 4, the scales for the two elements are different.

? **Double-click on *Your Building Rate*.**

The dialogue box shown in Figure 5 should appear on your screen. The value for *Your Building Rate* from the previous year is highlighted. At the beginning of the game, this value will be zero.

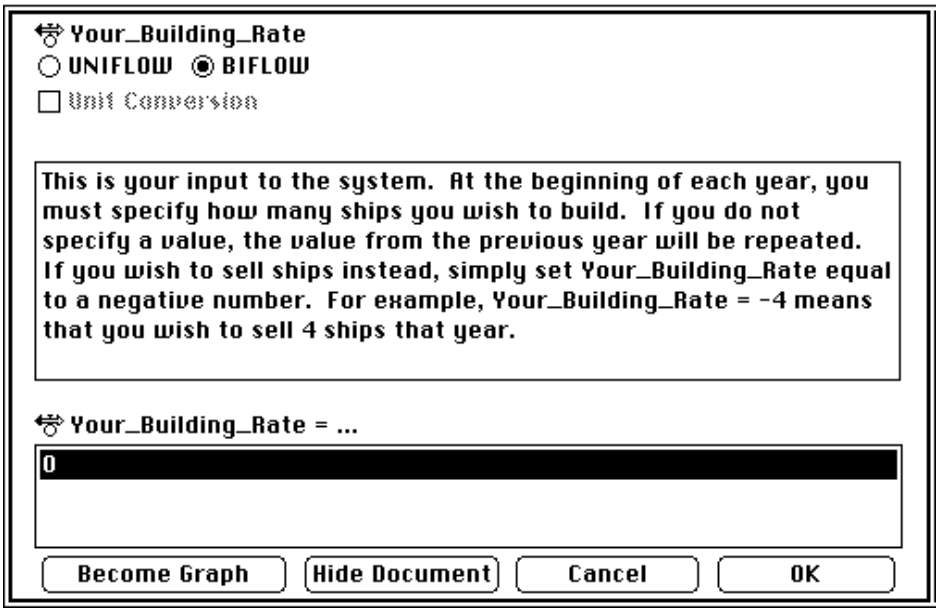


Figure 5: The input screen for *Your Building Rate*

This figure shows the dialogue box that will appear when you double-click on the *Your Building Rate* icon in the control panel. At the beginning of each round, set a value for *Your Building Rate* and then click on OK.

Implement your strategy for the game by setting a value for *Your Building Rate*. Before starting the next round of the game, specify how many ships you wish to build. If you wish to decrease the size of your fleet, you may sell ships by entering a negative number. If you do not set a value at the beginning of a year, you will build (or sell) the same number of ships as the previous year. You may track your previous decisions for *Your Building Rate* by looking at page 4 of the graph pad. Figure 4 contains a list of all available graphs and their contents.

- **Develop a strategy for maximizing your profits and beating your competitor.** (You may wish to re-read section 2.1 which briefs you on the game scenario.)
- **Enter the number of ships you wish to build the first year.**
- **Click on OK.**
- **Select "Run" from the Run menu or press Command-R.** The model will run for 1 year.
- **Double-click on the Fishing Game Graphs icon.**
- **Look at the four pages of the graph and observe the behavior of the model.**

***Note: After the first year, the Run menu will no longer contain the option “Run.” Instead, you should select “Resume.”

- **Continue entering values for *Your Building Rate* and running the model until the end of the simulation in 1990.**
- **Play the game a few times, revising your strategy each time to increase your profits.**

The game will automatically reset at the end of each 10 year simulation. You do not need to reset anything.

- **When you finish playing the game, select “Close” from the file menu.**
- **Click on “Don’t Save.”**

Please go to Section 3 now.

9.3 Appendix 3: The Fishing Game on STELLA II version 1

The Fishing Game on STELLA II version 1 is very similar to the game on version 2 because the model is the same, but playing the game is slightly different. The main reason for the difference is that STELLA II version 1 does not allow the user to input a pause interval as does version 2. Because of this, you will need to input your entire strategy for the game at the beginning, then run through all ten years of the simulation at once.

- **Insert The Fish Banks Exercises disk into your disk drive.**
- **Double-click on The Fishing Game disk icon.**
- **Open the Fishing Game v1.02 model by double-clicking on it.**

If you have STELLA II version 1 installed on your computer, the model will open. If the model doesn't open, you need to open the STELLA II program first, then open the model from within the program.

A small circular icon labeled "Your Building Rate" should appear in your STELLA window. This icon is located in the upper-left corner of the STELLA diagram pad. If you do not see it, use the scroll bars on the bottom and right edges of the STELLA window to scroll until you see it.

- **Double-click on *Your Building Rate.***

The dialogue box shown in Figure 14 should appear on your screen. The dialogue box allows you to enter your strategy for the entire simulation before running the model. Before running the simulation, set a value for *Your Building Rate* for each year of the game. You may do this directly on the graph with the mouse, or by clicking on the numbers in the right-hand column and entering new values with the keyboard. The default value is zero for each year.

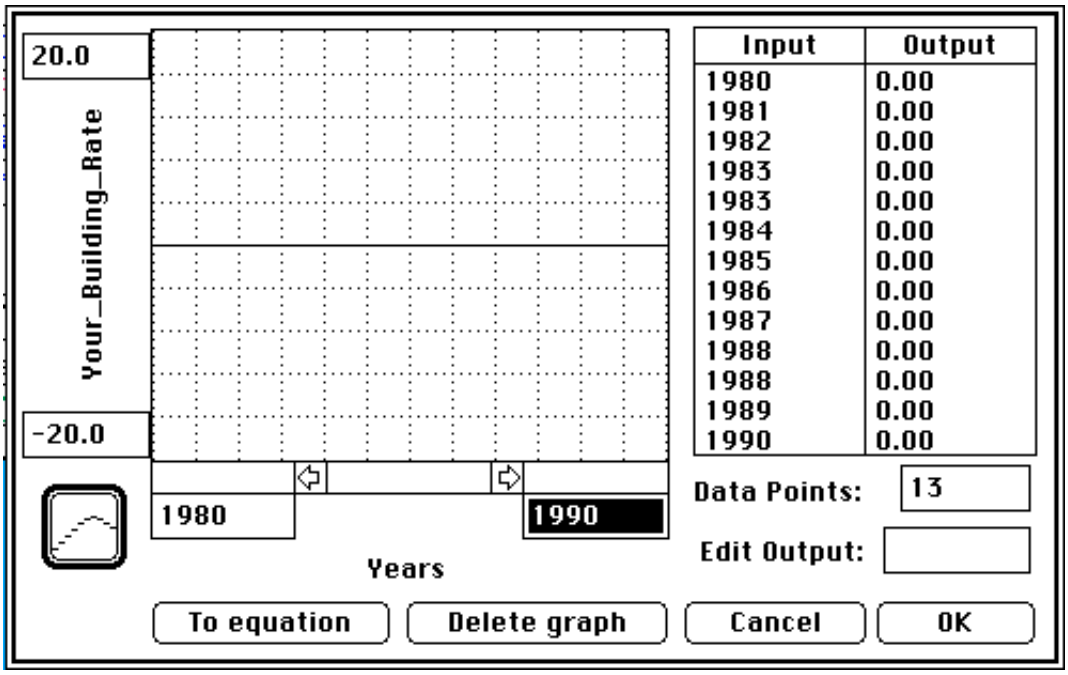


Figure 14: *Your Building Rate* dialogue box

This dialogue box allows you to enter your strategy for each year before running the simulation. Right now, it is set to build zero ships each year from 1980 to 1990.

- **Set the value of *Your Building Rate* for each year.**
- **Click on OK**
- **Run the model by selecting "Run" from the Run menu or by pressing Command-R.**

STELLA II version 1 does not use the graph icon. Instead, there is something called a "Graph Pad." The same graphs are included in both versions of the fishing game.

- **Select "Graph Pad" from the Windows menu to open the Graph Pad.**
- **Click on the numbered boxes in the upper-right corner of the Graph Pad window to view the other graphs.**
- **See Figure 4 on page 8 for a description of the available graphs.**
- **Select "Diagram" from the Windows menu to return to the model.**

9.4 Appendix 4: The Fish Banks Model

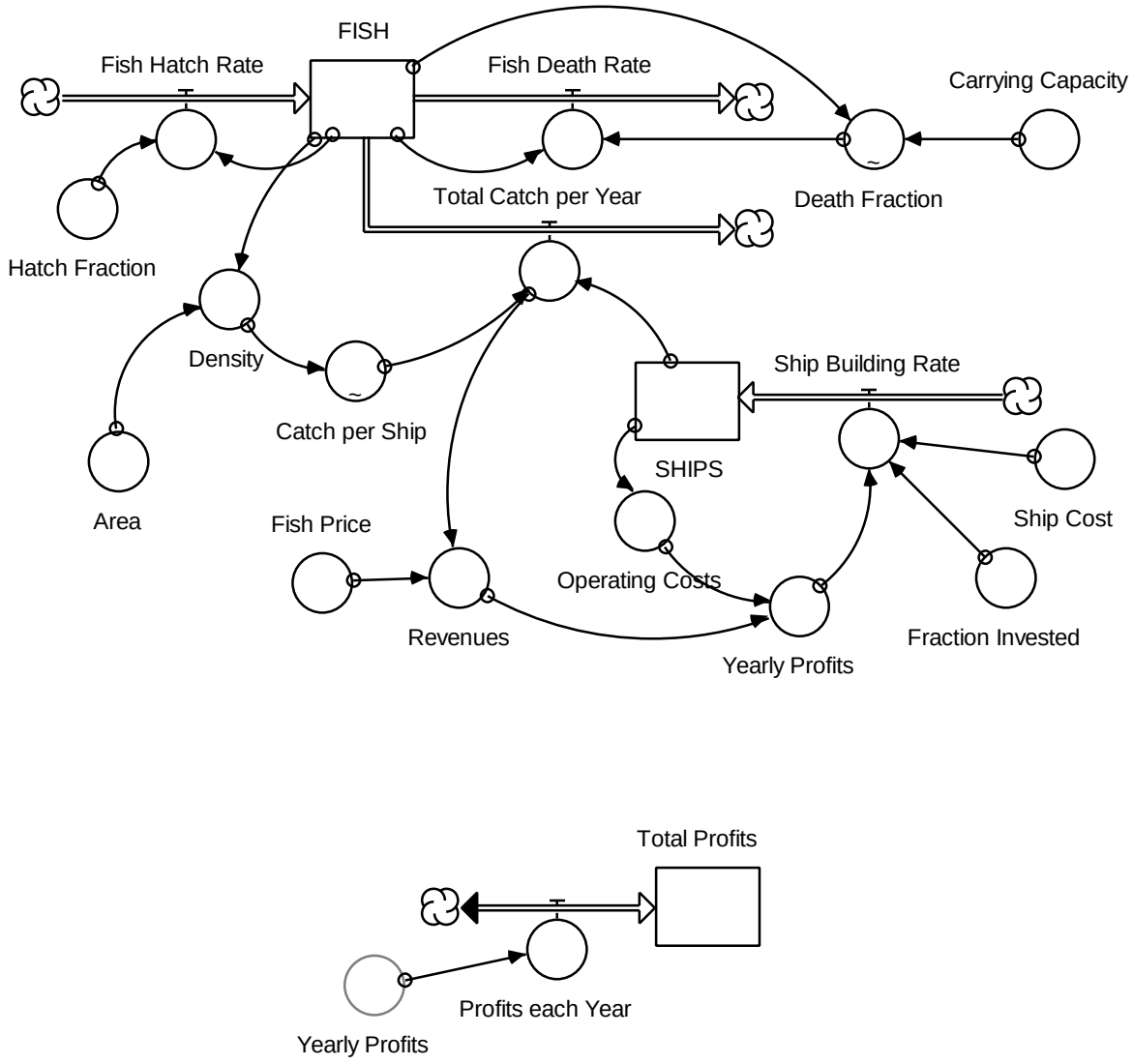


Figure 15: STELLA model of the Fish Banks system

This is the final model created by the reader. This model simulates the behavior of a fish population being depleted by a fishing industry.

The Fish Banks Model Equations:

FISH(t) = FISH(t-dt) + (Fish_Hatch_Rate- Fish_Death_Rate - Total_Catch_per_Year) * dt

INITIAL FISH = 1000

DOCUMENT: Fish are the number of fish alive within the fishing bed which is being modeled. At the beginning of the simulation there are initially 1000 fish alive in the bed.
UNITS: fish

INFLOWS:



Fish_Hatch_Rate = FISH*Hatch_Fraction
DOCUMENT: The fish hatch rate is the number of fish hatched per year. It is computed by multiplying the total fish population by a hatch fraction. The hatch fraction is the average number of offspring per year per fish.
UNITS: Fish/year

OUTFLOWS:



Fish_Death_Rate = FISH*Death_Fraction
DOCUMENT: Fish Death Rate is the number of fish per year which die from causes other than fish harvesting.
UNITS: fish/year



Total_Catch_per_Year = SHIPS*Catch_per_Ship
DOCUMENT: Total Catch per Year is the total number of fish which are harvested from the fishing waters each year. It is computed by multiplying the number of ships by the catch per ship.
UNITS: fish/year



SHIPS(t) = SHIPS(t - dt) + (Ship_Building_Rate) * dt
INITIAL SHIPS = 10
DOCUMENT: There are initially 10 ships at the beginning of simulation.
UNITS: ships

INFLOWS:



Ship_Building_Rate = Yearly_Profits*Fraction_Invested/Ship_Cost
DOCUMENT: The Ship Building Rate is the number of new ships which are built each year. It is determined by the amount of money invested each year in new ships (profits_each_year*fraction_invested) divided by the ship cost. The units are ($/year)/($/ship)=(ship/year).
UNITS: ships/year



Total_Profits(t) = Total_Profits(t - dt) + (Profits_each_Year) * dt
INITIAL Total_Profits = 0

INFLOW:

Profits_each_Year = Yearly_Profits

Area = 100
DOCUMENT: The Area is the size of the fish bed in square miles.
UNITS: square miles

Hatch_Fraction = 6
DOCUMENT: Hatch fraction is the average number of offspring per year per fish. It is the fractional increase in the population per year. A hatch fraction of 6 means that every female fish (half the fish population) will have an average of 12 offspring per year.
UNITS: 1/years

Carrying_Capacity = 1200
DOCUMENT: The carrying capacity is the total number of fish which the ocean environment can support.
UNITS: Fish

Operating Costs = SHIPS*250
DOCUMENT: The Total Cost is the number of ships times the operating cost per year of each ship. The cost per ship per year is 250 dollars.
UNITS: $/year

Density = FISH/Area
DOCUMENT: Density is the number of fish found per square mile within the fish bed.
UNITS: fish/square mile

Fish_Price = 20
DOCUMENT: Fish Price is the amount of money a fisherman can sell a fish for as soon as it is caught.
UNITS: $/fish

Fraction_Invested = .2
DOCUMENT: The Fraction Invested is the fraction of the profits each year which are invested in the building of new ships.
UNITS: (unitless)

Revenues = Total_Catch_per_Year*Fish_Price

DOCUMENT: Revenues are the moneys made each year through the sale of fish. Revenues are the number of fish sold each year times the price per fish.
UNITS: $/year

○ **Ship_Cost** = 300
DOCUMENT: The Ship Cost is the amount of money it costs to build a new fishing ship.
UNITS: $/ship

○ **Yearly_Profits** = Revenues-Costs
DOCUMENT: Profits each year are the difference between revenues and costs.
UNITS: $/year

○ **Catch_per_Ship** = GRAPH(Density)
(0.00, 0.00), (1.00, 5.00), (2.00, 10.4), (3.00, 15.9), (4.00, 20.2), (5.00, 22.1), (6.00, 23.2), (7.00, 23.8), (8.00, 24.2), (9.00, 24.6), (10.0, 25.0), (11.0, 25.3), (12.0, 25.5)
UNITS: fish/ship

○ **Death_Fraction** = GRAPH(FISH/Carrying_Capacity)
(0.00, 5.22), (0.1, 5.22), (0.2, 5.23), (0.3, 5.24), (0.4, 5.26), (0.5, 5.29), (0.6, 5.34), (0.7, 5.45), (0.8, 5.66), (0.9, 5.94), (1.00, 6.00)
UNITS: 1/years

Vensim Examples: Building the Fish Banks Model And Renewable Resource Depletion

By Aaron Diamond
October 2001

3.1.2 Building the Fish Population Subsystem model

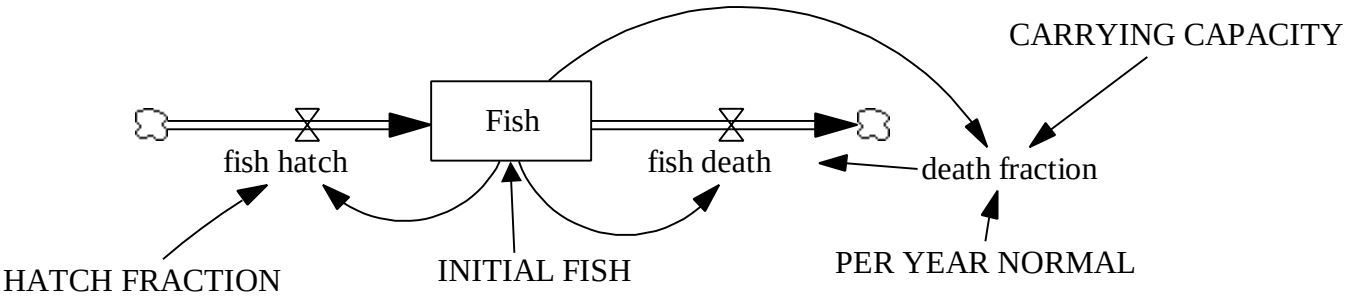


Figure 16: Vensim equivalent of Figure 7: Modified diagram of The Fish Population Model

Documentation for the Fish Population Model

(01) CARRYING CAPACITY=1200
Units: fish

(02) death fraction=Fish/CARRYING CAPACITY*PER YEAR NORMAL
Units: 1/year

(03) FINAL TIME = 100
Units: year
The final time for the simulation.

(04) Fish= INTEG (+fish hatch rate-fish death rate,
INITIAL FISH)
Units: fish

(05) fish death rate=Fish*death fraction
Units: fish/year

(06) fish hatch rate=Fish*HATCH FRACTION
Units: fish/year

(07) HATCH FRACTION=6
Units: 1/year

(08) INITIAL FISH=10
Units: fish

(09) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(10) PER YEAR NORMAL=1
Units: 1/year

(11) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(12) TIME STEP = .0625
Units: year
The time step for the simulation.

3.2.2. Building the Ship Subsystem model

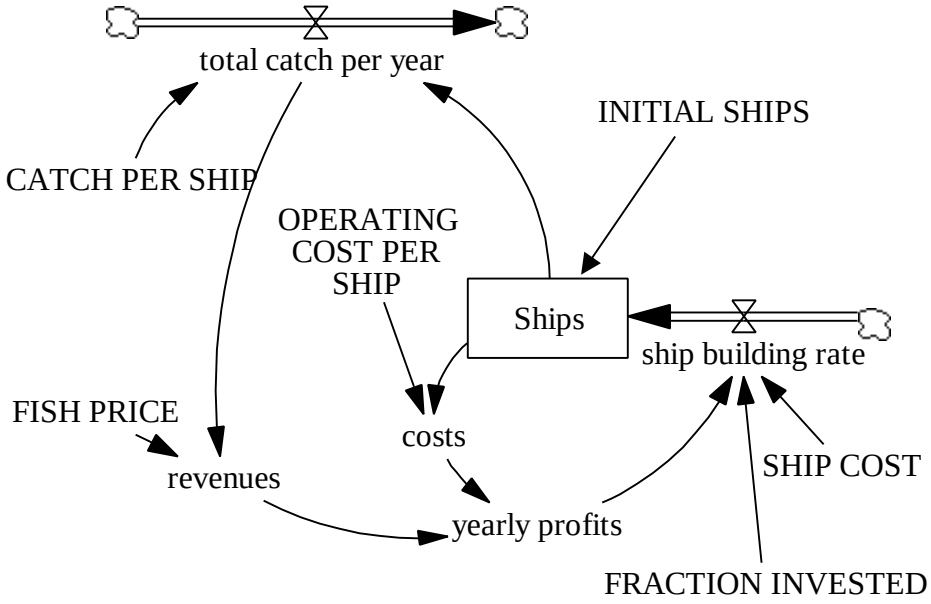


Figure 17: Vensim equivalent of Figure 9: Modified diagram of The Ship Subsystem

Documentation for the Ship Subsystem Model

(01) CATCH PER SHIP=15
Units: fish/ship

(02) costs=Ships*OPERATING COST PER SHIP
Units: $

(03) FINAL TIME = 100
Units: year
The final time for the simulation.

(04) FISH PRICE=20
Units: $/fish

(05) FRACTION INVESTED=0.2
Units: 1/year

(06) INITIAL SHIPS=10

Units: ship

(07) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(08) OPERATING COST PER SHIP=250
Units: $/ship

(09) revenues=FISH PRICE*total catch per year
Units: $

(10) SAVEPER = 1
Units: year
The frequency with which output is stored.

(11) ship building rate=yearly profits*FRACTION INVESTED/SHIP COST
Units: ship/year

(12) SHIP COST=300
Units: $/ship

(13) Ships= INTEG (ship building rate, INITIAL SHIPS)
Units: ship

(14) TIME STEP = 0.0625
Units: year
The time step for the simulation.

(15) total catch per year=Ships*CATCH PER SHIP
Units: fish

(16) yearly profits=revenues-costs
Units: $

3.3.2 Building the Connection Subsystem model

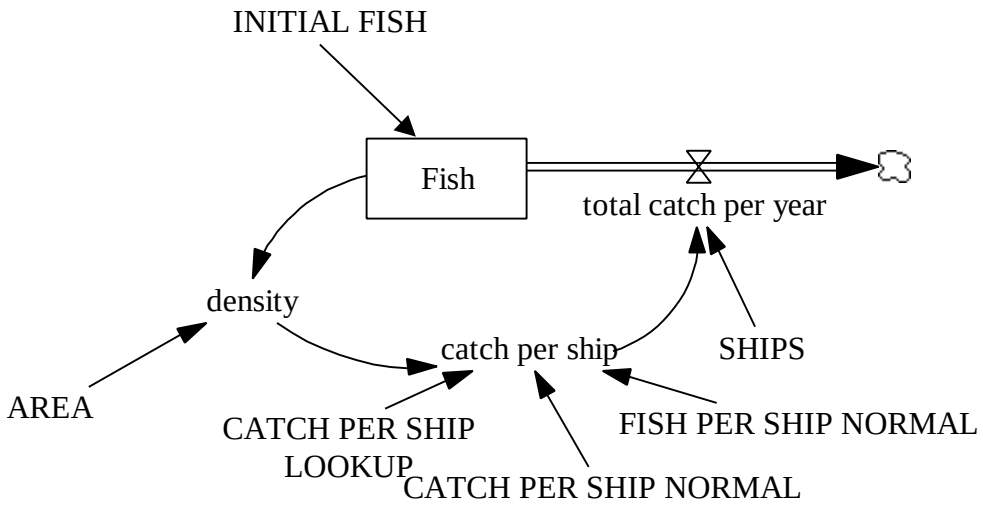


Figure 18: Vensim equivalent of Figure 11: Modified diagram of The Connection Subsystem

Documentation for the Connection Subsystem Model

(01) AREA=100
Units: square miles

(02) catch per ship=CATCH PER SHIP LOOKUP (density*CATCH PER SHIP NORMAL)*FISH PER SHIP NORMAL
Units: fish/(ship*year)

(03) CATCH PER SHIP LOOKUP=([(0,0)(10,40)],(0,0), (1,5),(2,10.4),(3,15.9), (4,20.2),(5,22.1), (6,23.2),(7,23.8),(8,24.2),(9,24.6),(10,25))
Units: dmnl

(04) CATCH PER SHIP NORMAL=1
Units: square miles/fish

(05) density=Fish/AREA
Units: fish/square miles

(06) FINAL TIME = 12
Units: year
The final time for the simulation.

(07) Fish= INTEG (-total catch per year,
INITIAL FISH)

Units: fish

(08) FISH PER SHIP NORMAL=1
Units: fish/(ship*year)

(09) INITIAL FISH=1000
Units: fish

(10) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(11) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(12) SHIPS=10
Units: ship

(13) TIME STEP = 0.0625
Units: year
The time step for the simulation.

(14) total catch per year=SHIPS*catch per ship
Units: fish/year

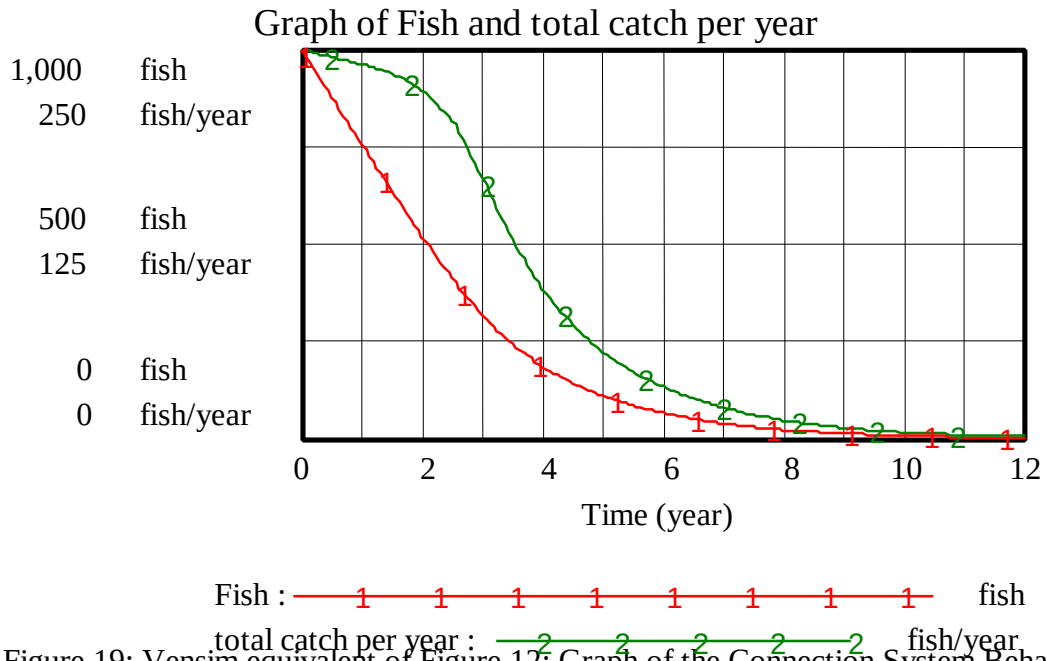


Figure 19: Vensim equivalent of Figure 12: Graph of the Connection System Behavior

7. Fish Banks Model Review

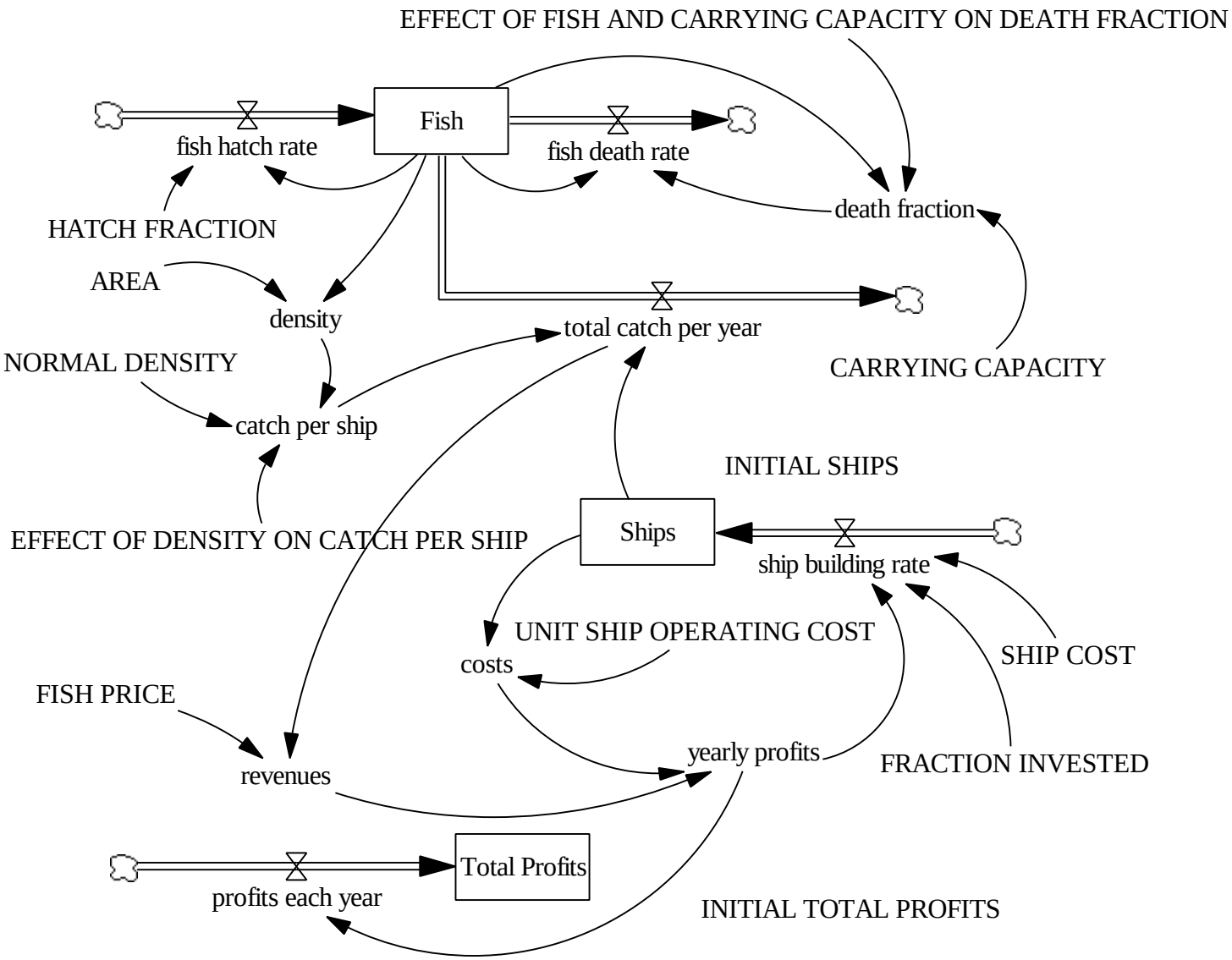


Figure 20: Vensim equivalent of Figure 13, Figure 15, and Figure 1 of the Fish Banks Policy Analysis Exercise: Vensim final model of the Fish Banks system

Documentation for Fish Banks Final Model

(01) AREA=100
Units: square mile
The Area is the size of the fish bed in square miles.

(02) CARRYING CAPACITY=1200
Units: fish
The carrying capacity is the total number of fish which the ocean environment can support.

(03) catch per ship=EFFECT OF DENSITY ON CATCH PER SHIP(density/NORMAL DENSITY)
Units: fish/ship/Year
The amount of fish caught per year.

(04) costs=Ships*UNIT SHIP OPERATING COST
Units: $/Year
The Total Cost is the number of ships times the operating cost per year of each ship.

(05) death fraction=EFFECT OF FISH AND CARRYING CAPACITY ON DEATH FRACTION(Fish/CARRYING CAPACITY)
Units: 1/Year

(06) density=Fish/AREA
Units: fish/square mile
Density is the number of fish found per square mile within the fish bed.

(07) EFFECT OF DENSITY ON CATCH PER SHIP([(0,0)-(12,26)], (0,0), (1,5), (2,10.38), (3,15.88), (4,20.25),(5,22.13),(6,23.25),(7,23.75),(8,24.25),(9,24.63),(10,25),(11,25.35),(12,25.48))
Units: fish/ship/Year

(08) EFFECT OF FISH AND CARRYING CAPACITY ON DEATH FRACTION([(0,5)(1,6)],(0,5.22),(0.1,5.225),(0.2,5.23),(0.3,5.24),(0.4,5.255),(0.5,5.29),(0.6,5.345),(0.7,5.45),(0.8,5.665),(0.9,5.94),(1,6))
Units: 1/Year

(09) FINAL TIME = 1986
Units: Year
The final time for the simulation.

(10) Fish= INTEG (fish hatch rate-fish death rate-total catch per year,

INITIAL FISH)
Units: fish
Fish are the number of fish alive within the fishing bed which is being modeled. At the beginning of the simulation there are initially 1000 fish alive in the bed.

(11) fish death rate=death fraction*Fish
Units: fish/Year
Fish Death Rate is the number of fish per year which die from causes other than fish harvesting.

(12) fish hatch rate=Fish*HATCH FRACTION
Units: fish/Year
The fish birth rate is the number of fish born per year. It is computed by multiplying the total fish population by a birth fraction. The birth fraction is the average number of offspring per year per fish.

(13) FISH PRICE=20
Units: $/fish
Fish Price is the amount of money a fisherman can sell a fish for as soon as it is caught.

(14) FRACTION INVESTED=0.2
Units: dimensionless
The Fraction Invested is the fraction of the profits each year which are invested in the purchase of new boats.

(15) HATCH FRACTION=6
Units: 1/Year
Birth fraction is the average number of offspring per year per fish. It is the fractional increase in the population per year. A birth fraction of 6 means that every female fish (half the fish population) will have an average of 12 offspring per year.

(16) INITIAL FISH=1000
Units: fish

(17) INITIAL SHIPS=10
Units: ship

(18) INITIAL TIME = 1970
Units: Year
The initial time for the simulation.

(19) INITIAL TOTAL PROFITS=0
Units: $

(20) NORMAL DENSITY=1
Units: fish/square mile

(21) Profits each Year=yearly profits
Units: $/Year
The amount of money made per year.

(22) revenues=FISH PRICE*total catch per year
Units: $/Year
Revenues are the moneys made each year through the sale of fish.
Revenues are the number of fish sold each year times the price
per fish. units: $/year

(23) SAVEPER = TIME STEP
Units: Year
The frequency with which output is stored.

(24) ship building rate=MAX(0,yearly profits*FRACTION INVESTED/SHIP COST)
Units: ship/Year
The Ship Building Rate is the number of new ships which are
built each year. It is determined by the amount of money
invested each year in new ships
(profits_each_year*fraction_invested) divided by the ship cost.
The units are ($/year)/($/ship)=(ship/year).

(25) SHIP COST=300
Units: $/ship
The Ship Cost is the amount of money it costs to build a new
fishing ship.

(26) Ships= INTEG (ship building rate, INITIAL SHIPS)
Units: ship
There are initially 10 ships at the beginning of simulation.

(27) TIME STEP = 0.25
Units: Year
The time step for the simulation.

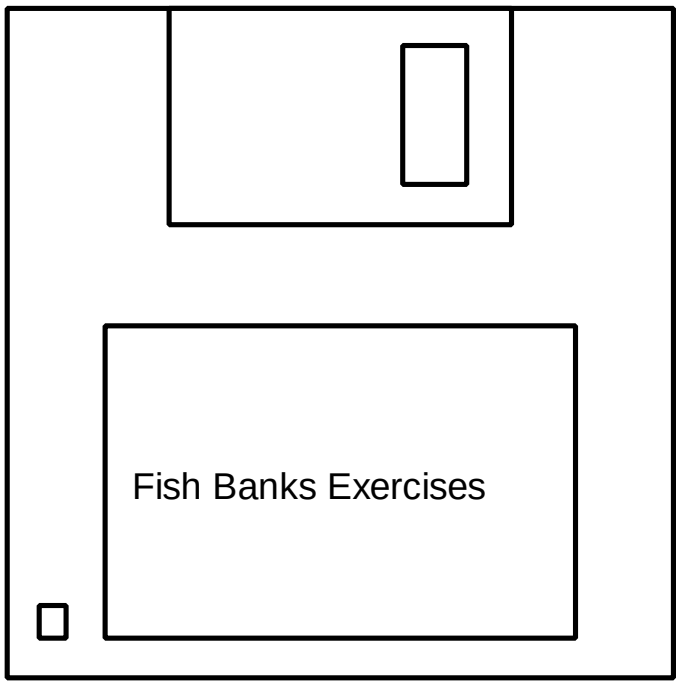
(28) total catch per year=Ships*catch per ship
Units: fish/Year
Total Catch per Year is the total number of fish which are
harvested from the fishing waters each year. It is computed by
multiplying the number of ships by the catch per ship.

(29) Total Profits= INTEG (Profits each Year,
INITIAL TOTAL PROFITS)

Units: $
Total Profit is the total amount of money made.

(30) UNIT SHIP OPERATING COST=250
Units: $/ship/Year
The unit ship operating cost is the cost of maintaining one ship for one year.

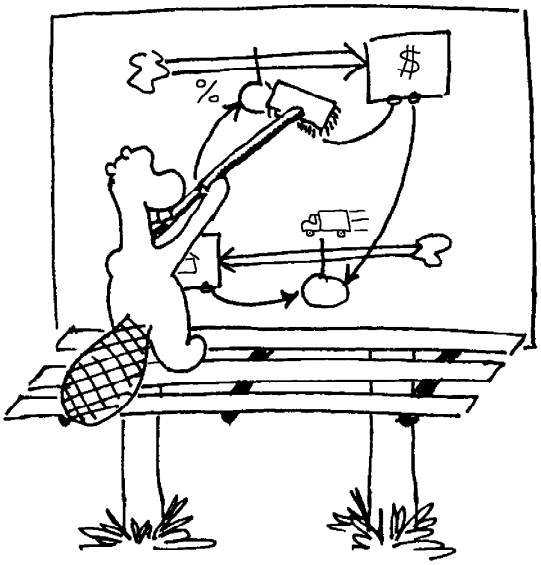
(31) yearly profits=revenues-costs
Units: $/Year
Profit each year are the difference between revenues and costs.

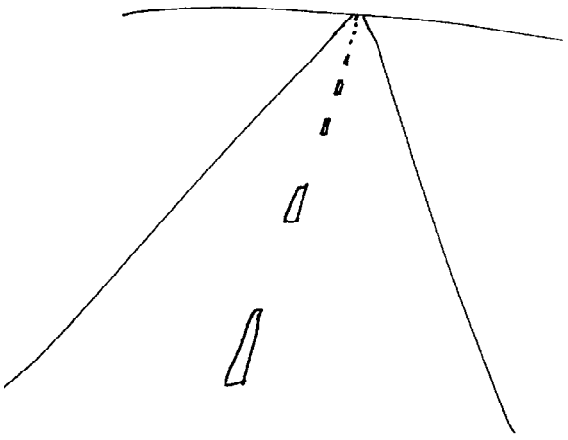


Attach Disk Here

Road Maps 5

A Guide to Learning System Dynamics





System Dynamics in Education Project

Road Maps 5

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

August 21, 1992
Latest Revision March 18, 2001

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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Five!

Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps Five is the fifth in the series of chapters in Road Maps. Road Maps One through Three gave you a broad introduction to system dynamics and focused on the building blocks of models: positive and negative feedback loops. Road Maps Four continued to broaden your understanding by introducing generic structures, policy analysis, computer simulation games, and tragedy of the commons.

In Road Maps Five you will explore and challenge the understanding of system dynamics that you have built in Road Maps One through Four. Specifically, Road Maps Five introduces the modeling of delays, mental simulation of S-shaped growth, generic structures producing S-shaped growth, the dynamics of the spread of an epidemic, and the concept of model validity. You will also practice and improve your graphical integration skills. At the end of Road Maps Five, we recommend that you read *Beyond the Limits*, an insightful book about the need for a transition to a sustainable world.

Topics Covered in Road Maps Five

An Introduction to Delays in Computer Modeling

- *Answers to Exercises for Chapter 17: Introduction To Delays* from *Introduction to Computer Simulation: A System Dynamics Modeling Approach* (D-4415)
  by Kamil Msefer and Mark Choudhari

Beginner Modeling Exercises

- *Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedbacks in First Order Systems* (D-4593-1)
  by Laughton Stanley and Helen Zhu

Transferability of Structures

- *Generic Structures: S-Shaped Growth I* (D-4432)
  by Terri Duhon and Mark Glick

Model Validity

- *Dynamic Simulation Models: How Valid Are They?* (D-4463)
by Ray C. Shreckengost

The Spread of an Epidemic: S-Shaped Growth

- *Teaching System Dynamics: Looking at Epidemics* (D-4243)
by Will Glass

Graphical Integration Exercises

- *Graphical Integration Exercises Part 3: Combining Flows* (D-4596)
by Kevin Agatstein and Lucia Breierova

Limits to Growth

- *Beyond the Limits*
by Donella Meadows, Dennis Meadows, Jorgen Randers

Things You'll Need for Road Maps Five

Modeling Software

In order to complete Road Maps Five and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is

located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

Books

You will need the following books for Road Maps Five. *Introduction to Computer Simulation,* by Nancy Roberts et al, was previously required. Please note that selected chapters from this book are available for download on the SDEP server in electronic format.

Roberts, Nancy, 1983. *Introduction to Computer Simulation.*
Portland, Oregon: Productivity Press, 562 pp.

Meadows, Donella H., Dennis Meadows, J?rgen Randers, 1992.
Beyond the Limits: Confronting Global Collapse, Envisioning a Sustainable Future.
Post Mills, VT: Chelsea Green Publishing Co., 300 pp.

To order a copy of this book, contact Chelsea Green Publishing Co. (see Appendix).

How to Use Road Maps Five

Road Maps Five explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

Introduction to Delays

- Answers to Exercises for Chapter 17: Introduction To Delays[1] from ***Introduction to Computer Simulation: A System Dynamics Modeling Approach*** by Nancy Roberts, et al.

by Kamil Msefer and Mark Choudhari

In Chapter 17 of *Introduction to Computer Modeling*, two different types of delays, material delays and information delays, are discussed. However, the chapter in the book has its models built in using the DYNAMO simulation language, and as you are probably using STELLA or iTHINK, we have translated the models for your convenience. In addition, the paper describes the use of STELLA delay functions such as SMTH1. The paper works through six modeling exercises and provides their solutions. If you understand equations written in DYNAMO, you do not have to refer to the paper when you do the exercises, unless you want to check your answers to the exercises.

The importance of delays is well presented by the following poem credited to W.L. Everitt:

There was a dachsund once so long
He hadn't any notion
How long it took to notify
His tail of an emotion.

And so it was that though his eyes
Were filled with woe and sadness
His little tail went wagging on
Because of previous gladness.

Please read and do *Chapter 17*... now.

After reading *Chapter 17*...

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[1] Kamil Msefer and Mark Choudhari, 1994. *Answers to Exercises for Chapter 17: Introduction to Delays* (D-4415) from Roberts, Nancy, David Andersen, Ralph Deal, Michael Garet, and William Shaffer, 1983. *Introduction to Computer Simulation: A System Dynamics Approach*, Portland, Oregon: Productivity Press. (800) 394-6868. 562 pp. The models in Chapter 17 are translated from DYNAMO into STELLA and answers to the exercises are provided. This translation was done for the System Dynamics in Education Project, System Dynamics Group with approval from the authors, August 5, 26 pp.

Chapter 17 explained why delay functions are used in computer modeling and gave you more experience with computer modeling. The answers to exercises also described the usage of some STELLA commands, such as SMTH1. What is the difference between a material and an information delay? When would you use them? Where would you include them in your model? Why is a delay important?

We now introduce two new system principles with examples from the chapter you've just read in *Introduction to Computer Simulation.*



System Principle #10:

First-order loops exhibit exponential behavior.

The first-order feedback loop always exhibits an exponential time shape. Take for example the solution given to Exercise 1 in *Answers to Exercises for Chapter 17* from *Introduction to Computer Simulation* (or look at your own solution). The system here is a first-order feedback loop—there is one level with feedback. Its solutions show an exponential approach to an upper value.

The solution to Exercise 5 is also a first-order feedback loop, and it also shows exponential behavior. All the first-order feedback solutions in this chapter—or anywhere—exhibit exponential growth or decay.



System Principle #11:

Levels completely describe the system condition.

Look at the solution to Exercise 7 on pp. 24-25 *of Answers to Exercises for Chapter 17: Introduction To Delays* from *Introduction to Computer Simulation*. There are two levels: **Apartments in Construction**, and **Total # of Apartments**. There are initially zero **Apartments in Construction**, and 10000 **Total # of Apartments**. This completely describes the state of system. The values of all other variables (the rate variables) can be computed from these values and the system equations alone.

Beginner Modeling Exercises

The next reading in Road Maps Five is the fifth in the series of Beginner Modeling Exercises. You have already learned how to mentally simulate positive and negative feedback systems, as well as systems with constant flows. We will now look at mental simulations of first-order systems with combined feedbacks.

- Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedbacks in First-Order Systems[2]

by Laughton Stanley and Helen Zhu

The fifth paper in the Beginner Modeling Exercises series explains mental simulation of systems with both a positive and a negative feedback loop. It describes four different examples of such systems, and discusses how to mentally simulate the behavior they produce. Exercises and their solutions are provided at the end of the paper.

Please read *Beginner Modeling Exercises Section 5* now.

After reading *Beginner Modeling Exercises Section 5...*

This paper explained that systems with combined positive and negative feedback can produce four different kinds of behavior: equilibrium, exponential growth, asymptotic growth, and S-shaped growth, and described how to use mental simulation to estimate the behavior of these systems. It is necessary to understand the concepts introduced in this paper before moving on to the next reading.

Transferability of Structures

In Road Maps Four, you learned about the generic structures in first-order positive and negative feedback systems. Generic structures are structures that are common to several systems. You will now see which generic structures produce S-shaped growth.

- Generic Structures: S-Shaped Growth I[3]

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[2] Laughton Stanley and Helen Zhu, 1996. *Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedbacks in First Order Systems* (D-4593-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 28pp.

by Terri Duhon and Marc Glick

This paper develops the concept of generic structures and presents two different systems that produce S-shaped growth. STELLA II models are used to describe the fact that generic structures are common to many systems. Causal loop diagrams are also used to explain why these specific generic structures produce S-shaped growth behavior. This paper emphasizes the transferability of these two structures to explain S-shaped growth in models of different systems.

Please read *Generic Structures: S-Shaped Growth I* now.

After reading *Generic Structures: S-Shaped Growth I*...

This paper again emphasized that in many cases different systems share the same underlying structure. Can you think of any other systems that have similar structures producing S-shaped growth? Transferability of structure is important in system dynamics because it allows you to estimate the behavior of new systems with the same underlying generic structures, based on your knowledge of other systems with which you may be familiar. It is important to understand the two structures introduced in this paper before moving to the next paper in the generic structures series.

The following system principle about levels has been introduced earlier in Road Maps. Now we discuss it again to emphasize its importance through an example in the *Generic Structures* paper that you just read.

System Principle #5:

Levels are changed only by the Rates.

Look at the model depicted in Figure 5a (and similarly in 5b) of *Generic Structures: S-Shaped Growth I*. Note that **Rabbit Population** is changed only by the rates **Births** and **Deaths**. **Rabbit Population** is increased by **Births** decreased by **Deaths**. But no other variables alter the levels. Only rates can change levels. Even the levels do not alter each other or themselves.

A level variable's current value is computed using only its previous value and the change due to the rates acting on the level. The earlier value of the level is

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3 Marc Glick and Terri Duhon, 1994. *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 22, 25 pp.

carried forward from the previous period. It is altered by rates that flow in and out of the level over the intervening time period. The present value of a level is not directly dependent on the present or previous values of any other levels.

We now introduce a new system principle.



System Principle #12:

Variables have the same units within conservative subsystems.

Recall from System Principle #6 that levels exist in conservative subsystems—the contents of stocks are neither created nor destroyed, just moved between levels via flows. Levels connected within a conservative subsystem have the same units of measure.

For example, look at the models on p.10 of *Generic Structures: S-Shaped Growth I*. The units of measure for both levels in the Epidemic model is People (both sick and healthy). The units of measure for both levels in the New Product Life Cycle model is Customers. Levels have the same units within conservative subsystems.

Model Validity

Have you ever looked at a model and wondered whether or not it was valid? How does one know whether or not the structure, components, or assumptions made while creating a model are valid? Is there any way to test a model for validity?

- Dynamic Simulation Models: How valid are they?[4]

by Ray C. Shreckengost

In this paper, the author outlines several structural, behavioral and policy implication tests that can be applied to a model to test its validity. He continues on to explain why these tests are different from the more traditional statistical tests.

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[4] Raymond C. Shreckengost, 1984. Self-Report Methods of Estimating Drug Use: Meeting Current Challenges to Validity, *Dynamic Simulation Models: How valid are they?* (D-4463), NIDA Research Monograph 57, DHHS Publication No. (ADM)85-1402, May, 13 pp. Also available upon request from the System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

This paper also emphasizes certain good modeling practices, such as the emphasis on judgment and intuition rather than data in determining a model's structure.

Please read *Dynamic Simulation Models* now.

After reading *Dynamic Simulation Models...*

The paper offered some excellent tools to check the validity of models which you may create in the future. These tests are also good for critiquing other people's models. However, remember that there is no one right way to model a situation; as mentioned in the paper, there are as many valid models as there are different fingerprints. The only thing that will determine in the end whether or not a model is valid is the usefulness of the model to the user. While modeling, it is important to think of what you are trying to demonstrate with your model, and how you can show this through your model.

The Spread of an Epidemic: S-Shaped Growth

In Road Maps Three, you saw a model that showed S-shaped growth. In Road Maps Five, you have mentally simulated a system producing this type of behavior, and you just explored the generic structures that generate it. In this section, we will model the spread of an epidemic. Such a system also shows S-shaped growth, but with a different structure than the one shown earlier.

***- Teaching System Dynamics: Looking at Epidemics*[5]**

by William Glass-Husain

This paper is a complete set of information and suggestions for teachers who want to begin system dynamics curricula in their classrooms. The paper includes notes, transparencies, worksheets, and handouts. We only ask that you read certain parts of the paper but if you are interested in the rest of the information, feel free to review the entire paper.

The paper describes The Epidemic Game, a game where a "disease" is spread by shaking hands. By playing the game, the students gain an understanding of the

[5] William Glass-Husain, 1991. *Teaching System Dynamics: Looking at Epidemics* (D-4243), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 54 pp.

dynamics of an epidemic. Afterwards, the students can model the game in STELLA and learn about the structure of the system.

Read pages 18 and 19 to learn about the game and the model. Then skip ahead to page 33 and complete all the exercises. The exercises may seem simple at first, but they are an important review of feedback loops.

This paper uses the terms "balancing loop" and "reinforcing loop" in place of "negative feedback loop" and "positive feedback loop." The different terms do not imply a different meaning; they were invented to move away from the connotations of "positive" and "negative" movement that most beginning modelers confuse with the true meanings of "positive" and "negative" feedback.

Now please read *Teaching System Dynamics: Looking at Epidemics.*

After you have read *Teaching System Dynamics...*

This paper described that the spread of an epidemic (or a rumor, joke, etc.) has a characteristic S-shape. Notice that S-shaped behavior can be caused by two different structures. This behavior is common to all systems where something spreads until it approaches some kind of limit. This limit could simply be the number of people in a room or the area of a plot of land. The paper explained why the system starts off with exponential growth and then switches to goal-seeking behavior. What is happening in the system to create this behavior?

The following System Principle is one that you have seen before. We are revisiting the principle in order to emphasize its importance.

System Principle #5:

Levels are changed only by the Rates.

Look at the model depicted on page 20 in *Appendix III* of *Looking at Epidemics*. Note that **Healthy People** and **Sick People** are changed only by the rate **New Infections**. **Healthy People** are decreased by this rate and **Sick People** are increased. No other variables alter the levels. Even the levels do not alter each other or themselves.

A level variable is computed by the change, due to rate variables, that alters the previous value of the level. The earlier value of the level is carried forward from the previous period and is altered by rates that flow in and out of the level over the intervening time period.

Graphical Integration Exercises

Road Maps Two and Three introduced you to graphical integration of constant, step function, and linearly increasing and decreasing flows. However, in the previous two papers in the Graphical Integration Exercises series, we have only looked at systems with a net flow. We will now turn to consider systems with a separate inflow and outflow.

- Graphical Integration Exercises Part 3: Combining Flows[6]

by Kevin Agatstein and Lucia Breierova

The third paper in the Graphical Integration Exercises series explains the methods of graphical integration to estimate the behavior of systems with an inflow and an outflow. It shows how to calculate the net flow, and then expands your knowledge from the first two papers to find out the value of the stock. You can then test your understanding of these concepts by doing the exercises; their solutions are again provided at the end of the paper.

Please read *Graphical Integration Exercises Part 3* now.

After reading *Graphical Integration Exercises Part 3...*

This paper developed your skills of using graphical integration as a method to estimate the behavior of the stock. It also reviewed how to find the stock ís value at a certain point in time. Doing all the exercises and understanding the concepts presented in the paper completely is necessary before moving on to the next paper in this series.

Limits to Growth

- Beyond the Limits[7]

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[6] Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part 3: Combining Flows* (D-4596), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, March 14, 32 pp.

[7] Meadows, Donella H., Dennis L. Meadows, J?rgen Randers, 1992. *Beyond the Limits: Confronting Global Collapse Envisioning a Sustainable Future*, Post Mills, VT: Chelsea Green Publ. Co., 300 pp.

by Donella Meadows et al.

Beyond the Limits is the sequel to the bestseller, *The Limits to Growth*[8], which was written twenty years ago by the same authors. We feel that this book is important enough to encourage you to read it. The following is a review of *Beyond the Limits* by which we hope to inspire you to read the book.

REVIEW: *Beyond the Limits*

In *Beyond the Limits* , a system dynamics model, WORLD3, was used to analyze the Earth's limits to population growth, nonrenewable resources, pollution, food production and industrial output. The original book that used a system dynamics model of the world was *World Dynamics*[9] by Forrester, which, along with *Limits to Growth*, sparked much controversy and public debate. This sequel provides an update of the model and its implications based on the 20 years that have passed since the publication of *Limits to Growth*.

By using the WORLD3 model, one is able to see into the future with the ability to vary global policy assumptions. The model allows one to try different things and test different scenarios. The policies that the authors test result in outcomes that range from collapse to the ability to sustain the human race. The authors discuss the importance of balancing long and short term goals in an effort to make a transition to a sustainable society. Specifically the book discusses possible changes in the situation of our society which could help bring about sustainability.

The book stresses that the driving force pushing us into a potential overshoot situation is exponential growth. As you have discovered thus far, exponential growth behavior results from a positive feedback situation.

In this sequel, the information that was included in the *Limits to Growth* is updated and a number of new insights are discussed. The depletion of the ozone layer is given as an example of an overshoot situation. The authors look at this story in some detail, and express a certain degree of hope based on human reactions to this global problem.

The entire model is also displayed in the book. It is separated into its different subsystems, and displayed in STELLA. (The original model was in

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8 Meadows, Donella H., Dennis L. Meadows, J?rgen Randers, William W. Behrens III, 1992. *The Limits to Growth*, New York, New York: Universe Books, 205 pp.

9 Forrester, Jay W., 1971. World Dynamics, (1973 second ed.). Portland, Oregon: Productivity Press. 144 pp. Second ed. has an added chapter on physical vs. social limits.

DYNAMO.) This book is intriguing, and is a good example of the power of using a system dynamics modeling approach for real world issues and problems.

In order to provide some background of the books that address the world model (and its revisions), we will include here a few of the numerous criticisms and review comments that surrounded the initial publication of *World Dynamics* and *Limits to Growth*. The issues were in the press and publications ranging from *Technology Review* to *Playboy*.

The books sparked controversy, inspired public debate, and encouraged others to pursue global models to study this issue. *The Henoit Book Review* said that "This book will open eyes, stimulate studies, start fights, and affect public policies."[10] William Bowen of *Fortune* stated in 1971 that "This year will not see the publication of a more important book than Forrester's *World Dynamics*, or a book more certain to arouse dislike."[11]

Some have centered their criticisms around the structure and assumptions of the model, while others have looked at conclusions and policy recommendations in the books. As Robert Gillette stated in *Science*, "There are too many assumptions that are not founded, and there is too high a level of aggregation in the model."[12] Robert Solow, an MIT economist, and other economists assert that the model is not valid without a pricing system. The books have been labeled "doomsday" predictions and some have claimed that the authors started with a conclusion and worked their way backwards to the structure without valid data.[13] Other criticisms are in a different light. Henry Wallich, a Yale economist, said: "This is a piece of irresponsible nonsense, a publicity stunt."[14] Wallich is also a proponent of the theory that physical (economic) growth equals progress. Martin Shubik of Yale was also quite skeptical: "This is extremely simplistic, given the current state of knowledge in the social sciences."[15]

On the other side, there are an equal number of rave reviews. Anthony Lewis of the New York Times said that "This is likely to be one of the most important documents of our age..."[16] Eliot Richardson, Secretary, HEW, said "Too thoughtful,

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[10]*Henoit Book Review*, 1972, Vol. 126 America, p. 636.

[11]William Bowen, *Fortune*, Sept. 1971, Vol. LXXXIV, No. 3, p. 131.

[12]Robert Gillette, *Science*, 10 Mar 1972, Vol. 175, No. 4026, p. 1088.

[13]Robert Solow, Proceedings of the National Academy of Sciences, "Notes on Doomsday Models", Dec. 1972, Vol. 69, No. 12, 3833.

[14]Henry Wallich, *Connecticut Review*, 1972, Vol. 6, No. 1, 7.

[15]Martin Shubnik, *Science*, 3 Dec. 1971, Vol. 174, No. 4013, p. 1014.

[16]Anthony Lewis, *The New York Times*, 4 Oct. 1973.

too thorough, too significant to ignore even if it is not entirely correct."[17] E.J. Mishan of Great Britain studied the global situation in an entirely different way and came to a similar conclusion.[18] Robert S. Saunders in the *Stanford Journal of International Studies* stated that "The book makes a number of philosophical contentions in the most dramatic and convincing way," and that "this was not a very technical or academic paper, but it is a crucial philosophical treatise about man's role on the planet."[19]

There were also a number of responses to these criticisms by the authors. These responses were often backed by a number of people who reviewed the books favorably. In response to the exclusion of prices, the authors pointed out that prices are inherent in the system because of the structure. The authors admitted that this was not a perfect model, but they asked: "Is there a better one? Produce better data." They insisted that people should awaken from the dream that growth has no limits, and anticipated people would simply not like this idea. The authors insisted that physical growth must inevitably come to an end.

Obviously, the models and books have led to much controversy and debate. We encourage you to read *Beyond the Limits*, draw your own conclusions, and join the public debate that will surround issues related to the present day version of the model.

Finishing Off Road Maps Five

Road Maps Five covered a broad spectrum of intermediate concepts in system dynamics through readings and exercises. Delays in computer models were introduced through Chapter 17 of *Introduction to Computer Simulation.* Delays are an important concept to understand in system dynamics. You continued to develop your mental simulation skills on first-order systems with combined feedbacks. Generic, or transferable, structures were developed through the explanation of S-shaped growth and two different structures that cause this common behavior. Model validity was explored in *Dynamic Simulation Models: How Valid Are They?* through various tests which can be applied to existing and future models. We also

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[17] Elliot Richardson, *Industrial Engineering*, Sep. 1972, Vol. 4, No. 9.

[18] E.J. Mishan, *Connecticut Review*, 1972, Vol. 6, No. 1.

[19] Robert S. Saunders, *Stanford Journal of International Studies*.

explained the dynamics of the spread of an epidemic. What other types of systems can be represented this way? You have also practiced your graphical integration skills in estimating the behavior of systems with independent inflows and outflows. Road Maps Five also introduced three new system principles: a) that first-order loops exhibit exponential behavior; b) that levels completely describe the system condition; and c) that variables have the same units within conservative subsystems.

Finally, you've now had some insight into a real-world controversy related to system dynamics used for policy analysis. The authors of *Beyond the Limits* have repeatedly claimed that the increasing rate of growth of world systems cannot continue as it has in the past. How do they justify that statement? Can you find evidence that supports or casts doubts on their predictions? Try looking in the newspaper for examples.

Road Maps Six describes how system dynamics can be used in economics, and introduces more generic structures. It also gives you a first experience with completely independent computer modeling.

Key Terms and Concepts:

Balancing Loop
Delays
Generic structure
Limit to Growth
Model validity
Reinforcing Loop
S-shaped growth
Transferability of structure



Answers to Exercises for Chapter 17 INTRODUCTION TO DELAYS

from

INTRODUCTION TO COMPUTER SIMULATION

written by
Nancy Roberts
David F. Andersen
Jonathan Choate
Ralph M. Deal
Michael S. Garet
William A. Shaffer

Models translated to STELLA II 3.0 and Answers provided
by
Kamil Msefer
Mark Choudhari

Prepared for
MIT System Dynamics Education Project
Under the Supervision of
Dr. Jay W. Forrester
System Dynamics Group
Sloan School of Management

August 5, 1994
Vensim Examples added October 2001

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INTRODUCTION TO COMPUTER SIMULATION can be ordered from
Productivity Press, Inc., Dept. 315
P.O. Box 13390, Portland, OR 97213

The Chapter has also been provided under Road Maps Five in pdf format.

Please read Chapter 17: Introduction to Delays from *Introduction to Computer Simulation* by Nancy Roberts et. al. However, the chapter in the book has its models built using the DYNAMO simulation language, and as you are probably using Vensim, STELLA or iTHINK, we have translated the models for your convenience. This paper contains these translated versions of the models for the first seven exercises from the book. Later, we also suggest answers to these exercises for you to measure your answers against.

NOTE FROM THE TRANSLATOR: All the models in this paper have rates that flow in both directions. This is to encourage the readers to make the direction of the flow explicit in the equation, and not in the plumbing. In DYNAMO, there is no biflow-uniflow toggle. All rates are biflows.

Example 1: The Martan Chemical Company

We present the model of dumping of the Nobug pesticide by the Martan Chemical Company, as translated from DYNAMO to STELLA. A causal loop and flow diagram of the Martan case are shown in Figure 1.

The equations for the model are as follows:

NOBUG(t) = NOBUG(t - dt) + (Dump_rate - Absorb_rate) * dt
INIT NOBUG = 0

OUTFLOWS:
ABSORB_RATE = NOBUG/NAT

NAT = 2

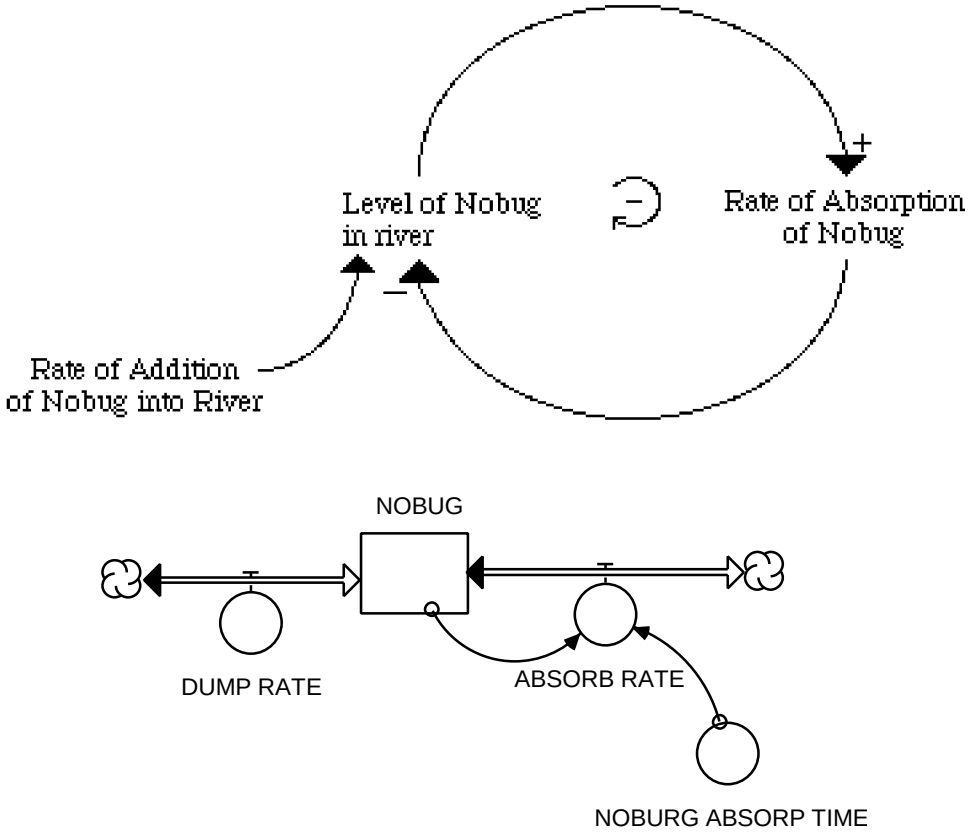


Figure 1. Diagram of the Martan Case

These equations show that the level of NOBUG in the Sparkill River is influenced by the DUMP RATE and the ABSORB RATE; and the ABSORB RATE is equal to the level of NOBUG divided by the Nobug Absorption Time NAT (2 days).

Exercise 1: Preliminary Nobug Model

a. Create a STELLA II model for the Nobug case, adding the DUMP RATE equation, and other needed specifications. Run the model, setting the initial level of NOBUG = 0 and choosing DT = 0.25 days. Set length of simulation (in the Time specs window under the Run menu) to 25 days, the range of Nobug (in the Range specs window under the Run menu) from 0 to 250 gallons and the range of the absorption rate from 0 to 100 gallons/day. What behavior does the model generate?

b. Rerun the model, setting the Nobug Absorption Time, NAT = 4. How do the results differ? Rerun the model, setting NAT = 1. How do these results differ?

USING THE PULSE FUNCTION TO REPRESENT THE DUMPING RATE

We use the corresponding STELLA functions instead of the DYNAMO functions in these exercises. The STELLA PULSE function permits modifying the model to represent the dumping of Nobug in once a week batches. The following rate equation indicates that 420 gallons of Nobug are dumped into the river on day 1 of the simulation, and 420 are dumped again at regular intervals of 7 days.

DUMP RATE = PULSE(420,1,7)

Figure 2 shows the dumping rate over the first 25 days of the simulation. Please see the chapter in the book for further reading.

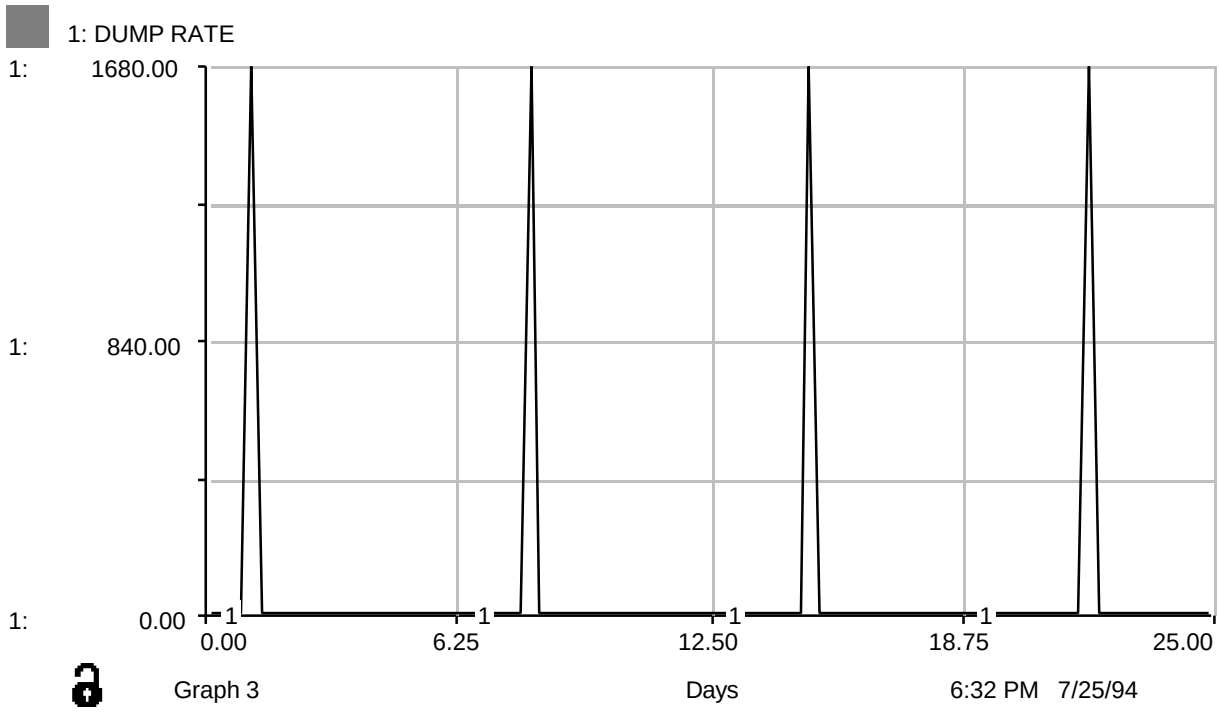


Figure 2. Dumping rate

The PULSE function can also be used to examine the response of the system

to the dumping of just one batch of NOBUG. It is possible to do this by making the following change in the equation for DUMP RATE.

DUMP RATE = PULSE(420,1,0)

This equation indicates that the dumping rate rises from zero to 1680 on day one (for only a quarter of the day)and stays at zero for the rest of the simulation.

Figure 3 shows a simulation of the Nobug system, with one batch of Nobug released into the river on day one. As can be seen, the level of NOBUG in the river rises sharply to 420 gallons on day one, and then drifts slowly toward zero.

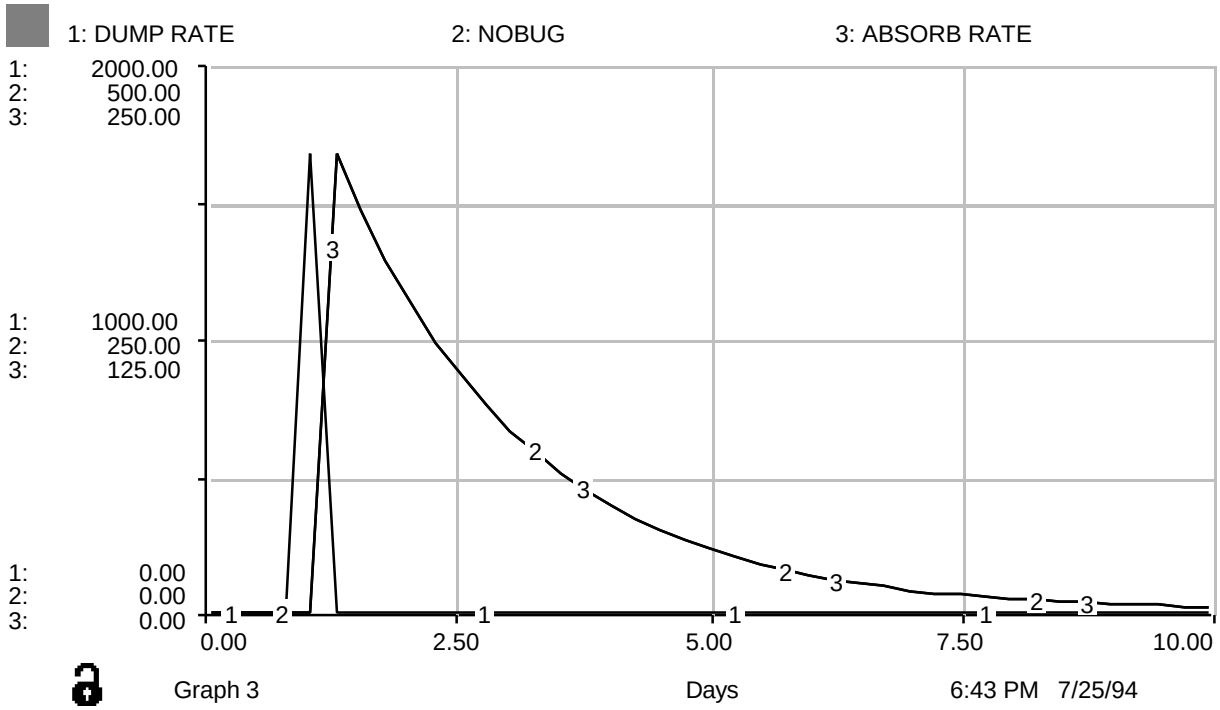


Figure 3. Release of one batch of NOBUG

The general form of the PULSE function is

PULSE(AMOUNT, FIRST, INTERVAL)

where AMOUNT indicates the amount to be inputted in the pulse; FIRST indicates the time at which the first pulse occurs; and INTERVAL indicates the time interval between pulses. Setting INTERVAL to 0, indicates to STELLA that you only want

one pulse.

Exercise 2: The Halving Time for NOBUG

Revise your STELLA equations to include a PULSE function for the DUMP RATE. Choose an interval of 0, in order to examine the effects of just one pulse.

a. What is the halving time for the amount of NOBUG in the Sparkill River?

b. Experiment with various values of NAT. How does the choice of NAT influence the halving time?

c. Using your result for part (b), select a value of NAT to produce a halving time that corresponds to the data shown below in Figure 4.

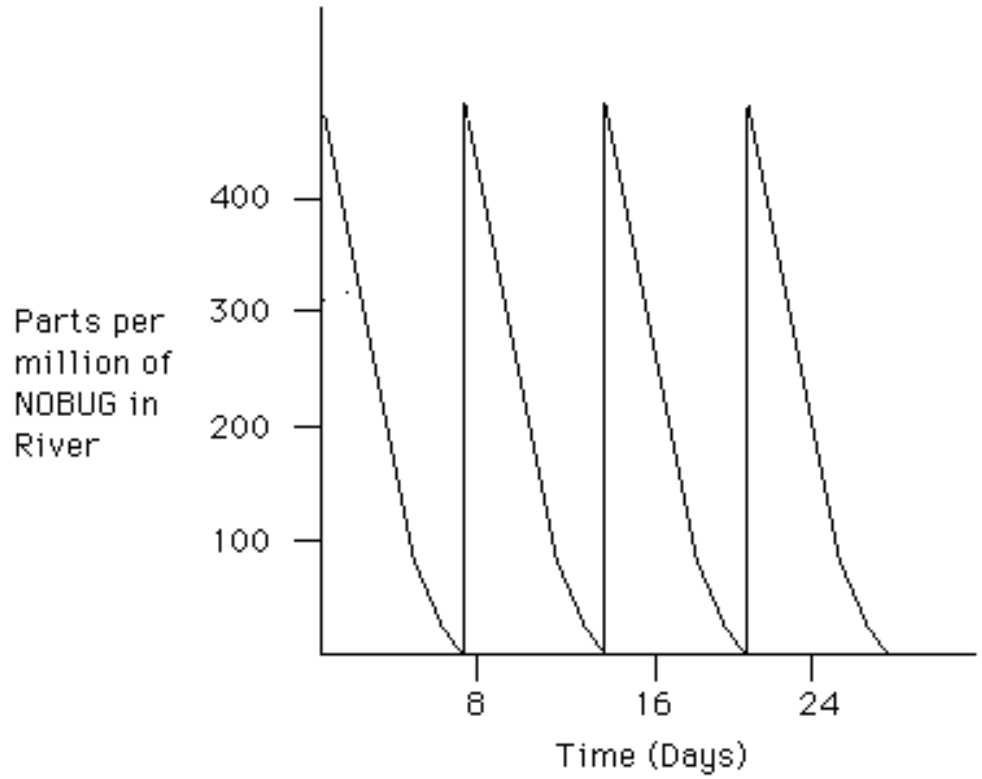


Figure 4. Amount of Nobug in the Sparkill River.

Exercises 3: Simulating Repeated Batches

a. Select NAT equal to the value you determined in Exercise 2, part (c). Use the

PULSE function to simulate the effect of dumping 420 gallons of Nobug into the river at 7-day intervals. Compare your results with the data shown in , Figure 4 above.

b. Assume Martan Chemical Inc. changed the chemical composition of Nobug, such that the halving time of Nobug was lengthened to seven days, resulting in an increase in the Nobug absorption time NAT. Change the value of NAT in your model to reflect the change in the chemical composition of Nobug. How does the system respond?

NOBUG AND MATERIAL DELAYS

Please read the chapter for an introduction to the concept of a first-order material delay.

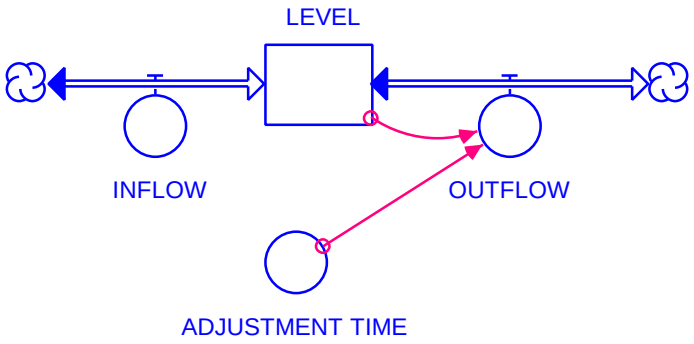


Figure 5. First-order material delay

When a first-order material delay is used in a model, there are two ways to write the equations. One approach is to simply to write out individual equations, exactly as was done in the Nobug case. But because first-order material delays are frequently used, a special STELLA function is available that can be used to replace the set of individual equations. Note that we are using the equivalent STELLA functions instead of the DYNAMO functions used in the book. The following equation can be used to indicate that the OUTFLOW rate is a first-order delayed response to the INFLOW rate, with an adjustment AT.

OUTFLOW = SMTH1(INFLOW, AT)

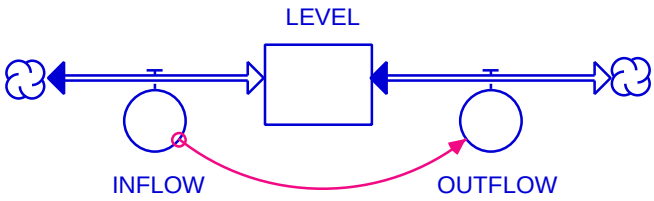


Figure 6. First-order material delay using SMTH1 function.

The SMTH1 function is simply a shorthand notation. When the model is run, STELLA will substitute the full level and rate formulation for a first-order delay, whenever the SMTH1 function appears in the model.

NOTE FROM THE TRANSLATOR:
Exercise 4 from the original text has been left out intentionally.

Exercise 5: The Mail Delay

The Nifty Department Store sends out bills to its charge-card customers once a month, and the credit department has learned that on the average, it takes about three days for the bills to arrive in the mail.

a. Draw a causal-loop diagram, flow diagram, and equations for the Nifty Department Store case. (Hint: See figure 7). Assume that NIFTY has 1000 charge customers.

b. Run the Model and examine the results.

c. How many bills take more than six days to arrive?

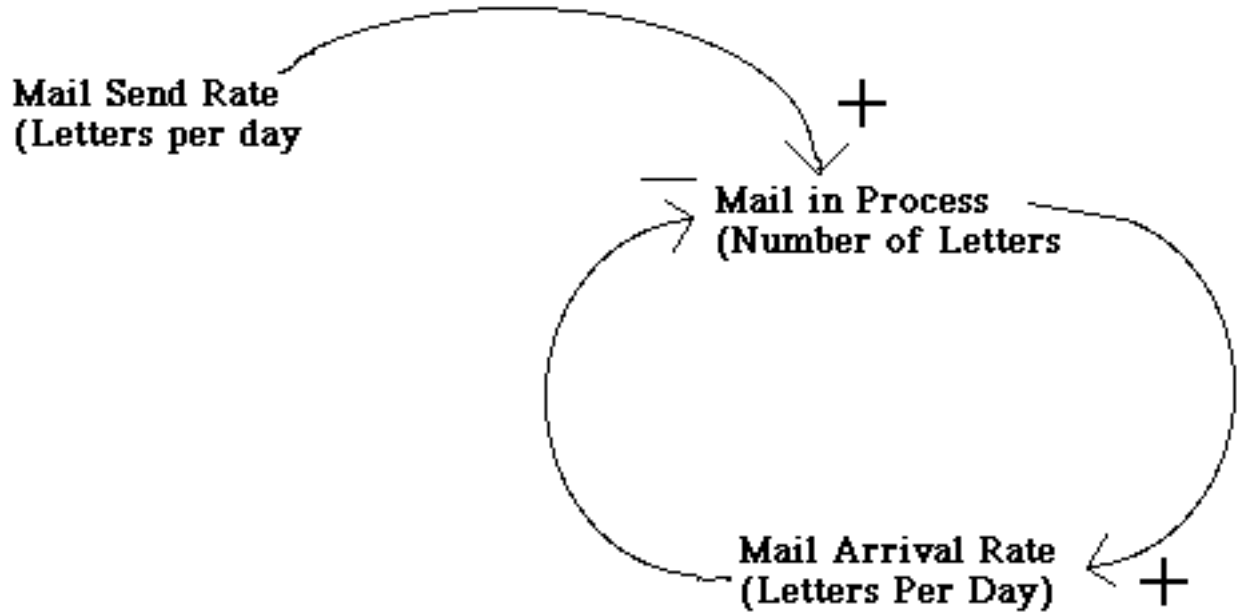


Figure 7. Causal loop representing a delay in mail delivery

EXAMPLE II: The GOAL-GAP FORMULATION, DELAYS AND CYCLES IN APARTMENTS

Please read the chapter for an introduction to this example. A causal-loop diagram and flow diagram for this system are shown in Figure 8.

Exercise 6: Apartments-Part I

a. Build the apartment model shown in Figure 8 using STELLA. Assume that the desired number of apartments is 10,000 and the time required to respond to the gap is one year. (Do not include an explicit construction delay in your formulation-it will be added in the next exercise.)

b. Run the model and examine the behavior. Determine the equilibrium value for the number of apartments.

c. Start the model in equilibrium, and use a STEP function to test the response of the system to an increase in the number of apartments desired from 10,000 to 15,000.

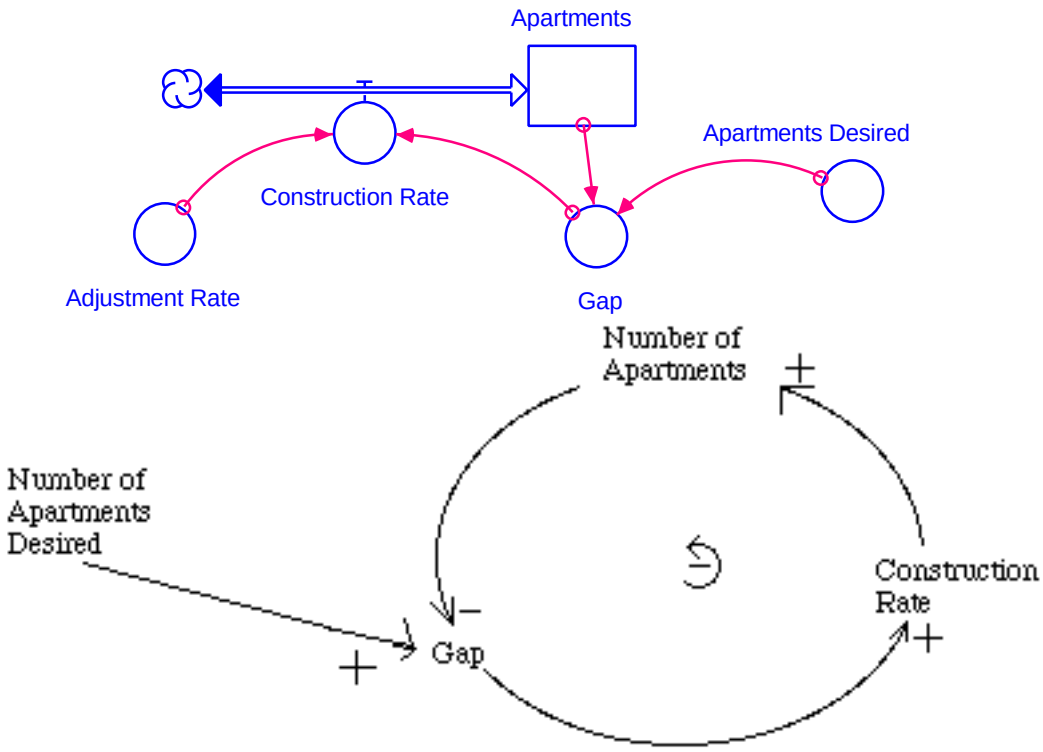


Figure 8. Diagrams expressing a housing gap

Please read the chapter for a discussion of why a delay should be added to the model. One way to do this is shown in figure 9.

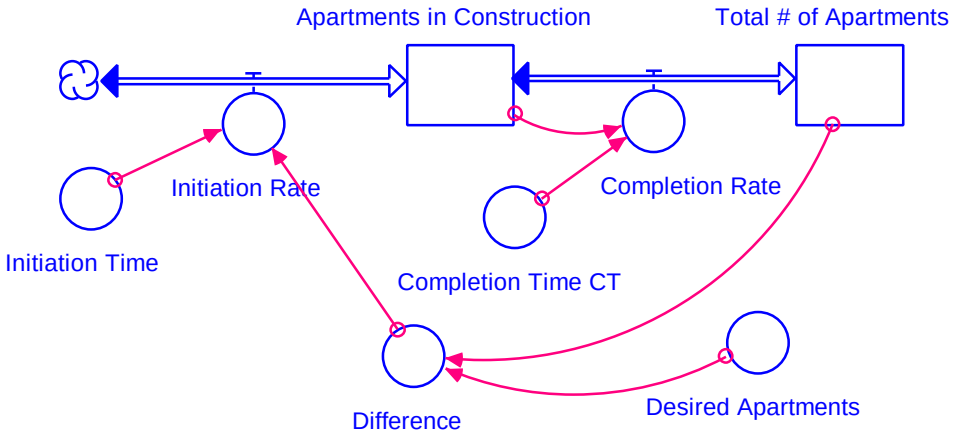


Figure 9. Apartment model with completion delay

Exercise 7: Apartments-Part II

For the questions in this exercise, please refer to the chapter.

INFORMATION DELAYS

Please read the chapter for an introduction to information delays.

EXAMPLE III: MARINA'S BAKE SHOP

Please read the chapter for an introduction to Marina and her Bake Shop.

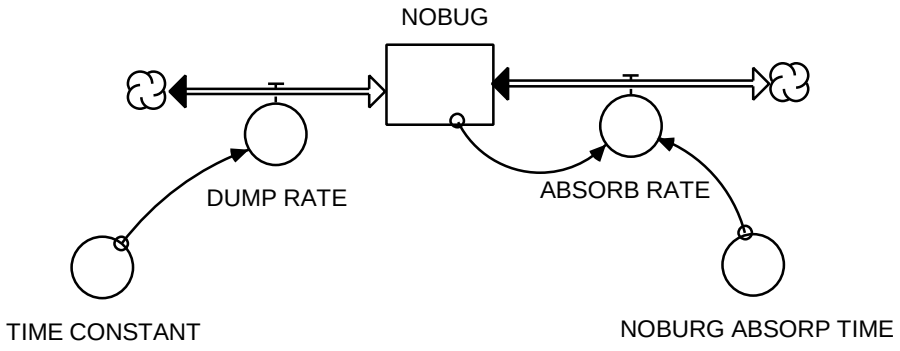
NOTE FROM THE TRANSLATOR: This paper presents only the first seven exercises from Chapter 7 in the book. We will discuss higher order delays later in Road Maps.

Now, we present our suggestions for the answers to the exercises.

Answers to Exercises

Exercise 1

a.

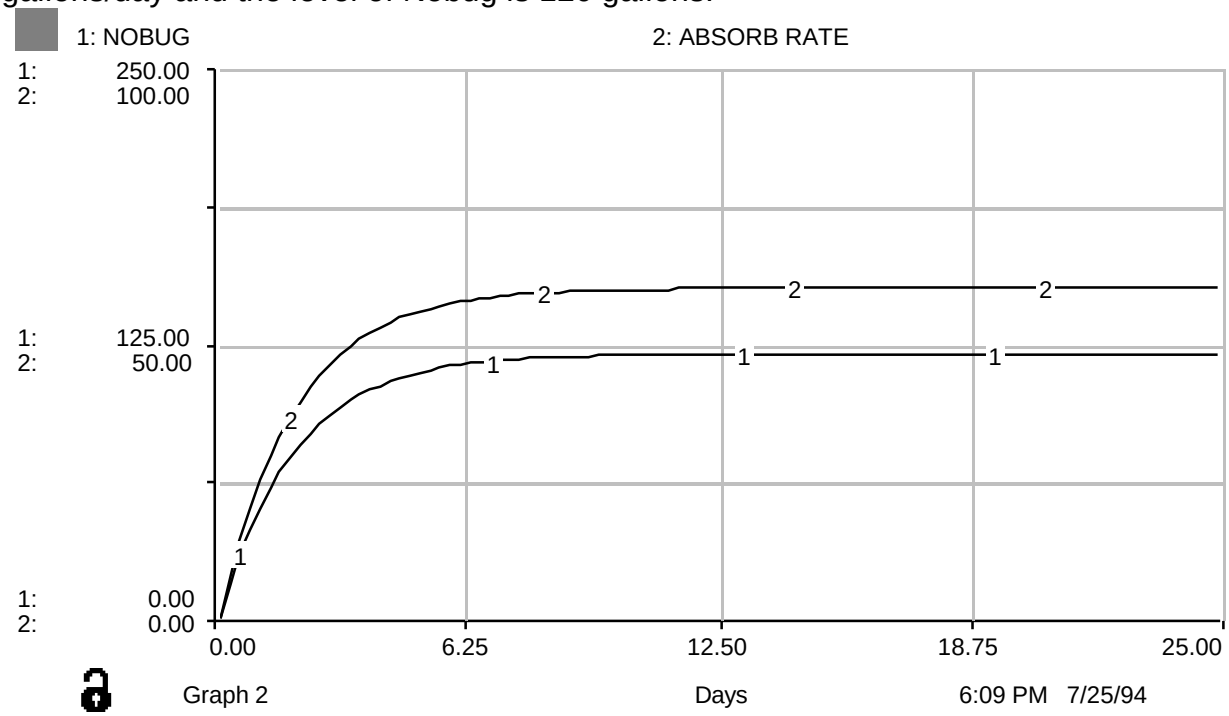


INFLOWS:
DUMP_RATE = 60/TIME_CONSTANT
DOCUMENT: The DUMP RATE is how fast Nobug is dumped into the river. In this case it is 60 gallons per day, which was given in the text.
Units: gallons/day

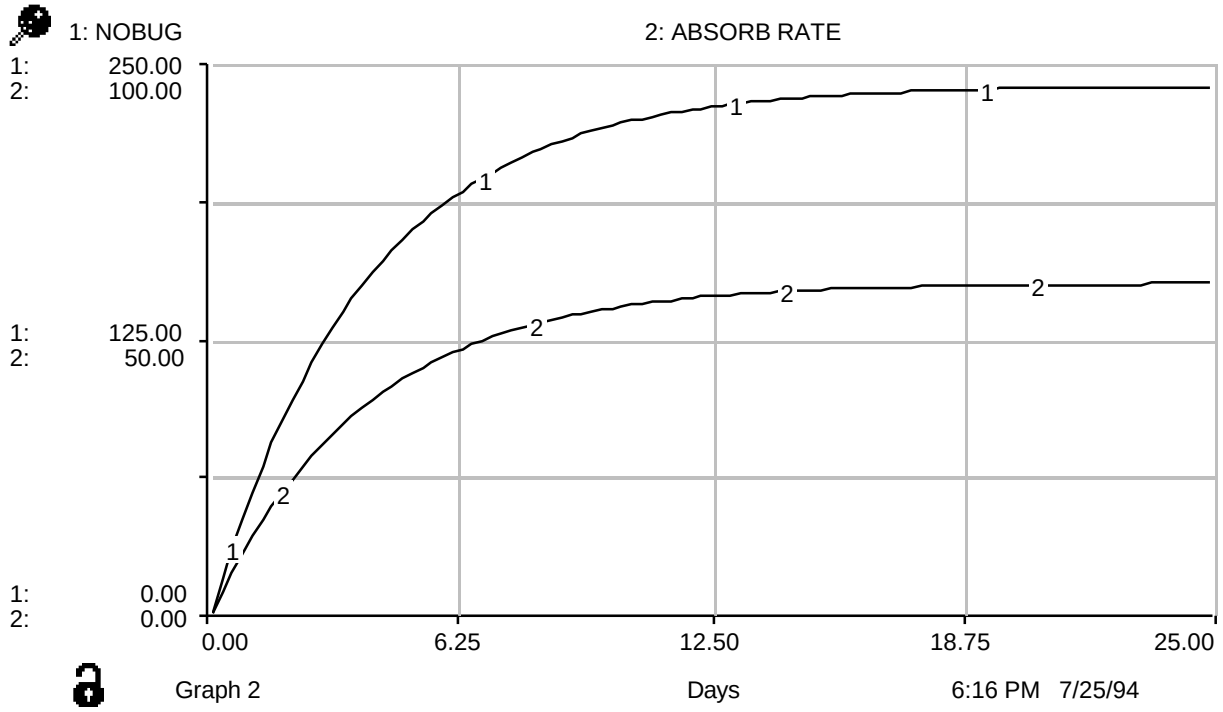
TIME_CONSTANT = 1

DOCUMENT: The time constant is the time over which the dump rate is calculated.
Units: days

The DUMP RATE is not shown in the graph below because it is constant. As the content of NOBUG increases (because of the DUMP RATE), the ABSORB RATE also increases since it is dependent on the content of NOBUG in the river. The system reaches equilibrium when the ABSORB RATE equals the DUMP RATE at 60 gallons/day and the level of Nobug is 120 gallons.

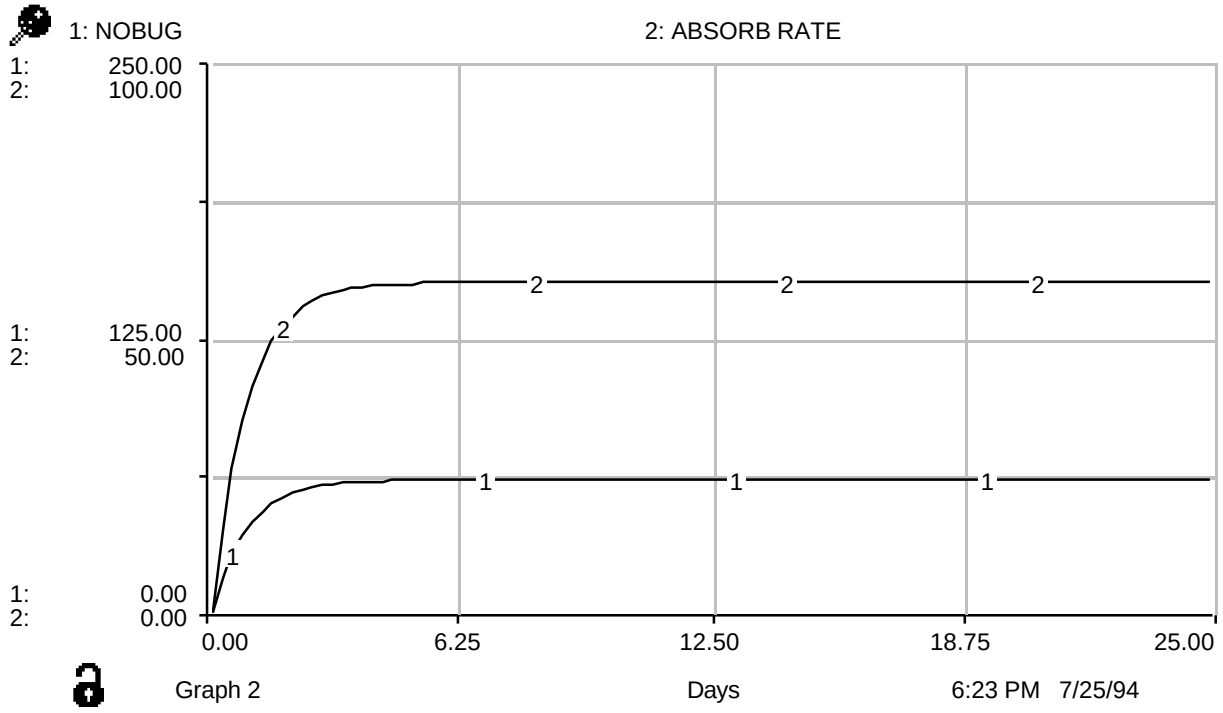


b. For NAT=4, the behavior is shown below.



When NAT=4, it takes twice as long to reach equilibrium, because the ABSORB RATE is twice as small as in part a. Also, the new equilibrium level of Nobug is at 240 gallons.

For NAT=1, the behavior is shown below.



When NAT=1, equilibrium is reached in half the time it takes in part a. That is because the ABSORB RATE is twice as large as in part a. The new equilibrium level of Nobug is at 60 gallons.

Exercise 2

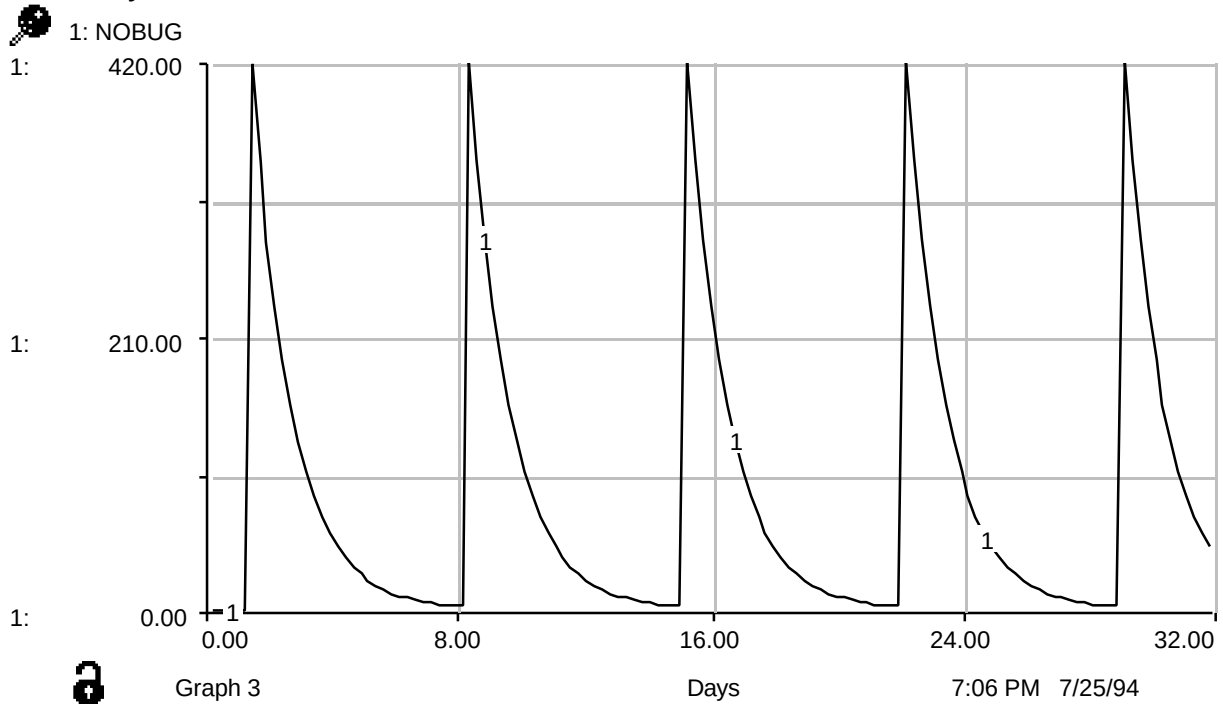
a. The level of NOBUG drops from 420 gallons at day 1 to about 210 gallons in 2.5 days. The halving time is then about 1.5 days.

b. After experimentation, you should find that an increase or decrease in the Nobug Absorption Time correspondingly increases or decreases the halving time.

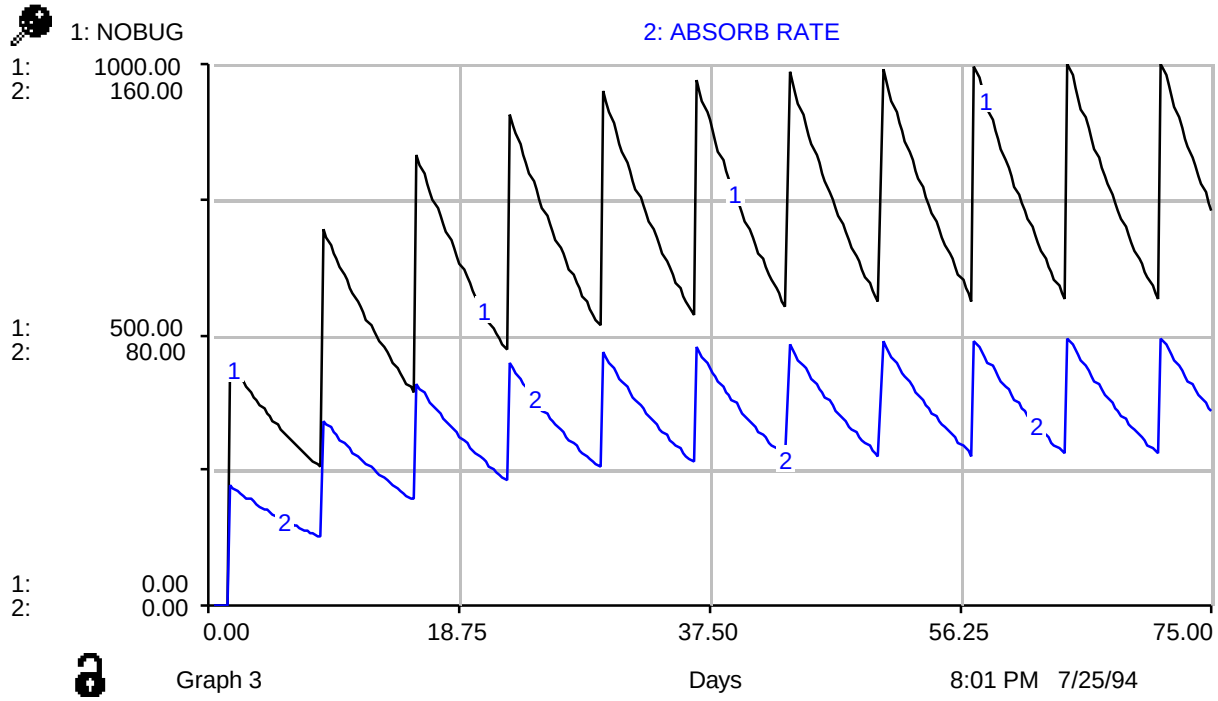
c. Figure 4 shows a halving time for NOBUG of about 1 day (ie. half-life = 1 day). The value of NAT that produces this behavior can be calculated as (half-life)/.69 or 1.4 days. With this value for NAT the outcome matches Figure 4.

Exercise 3

a. When the model is run with 420 gallons dumped by MARTAN every 7 days, with a NAT of 2 days, the output is a good match. Note that a run of NAT = 1.4 would give essentially the same run.

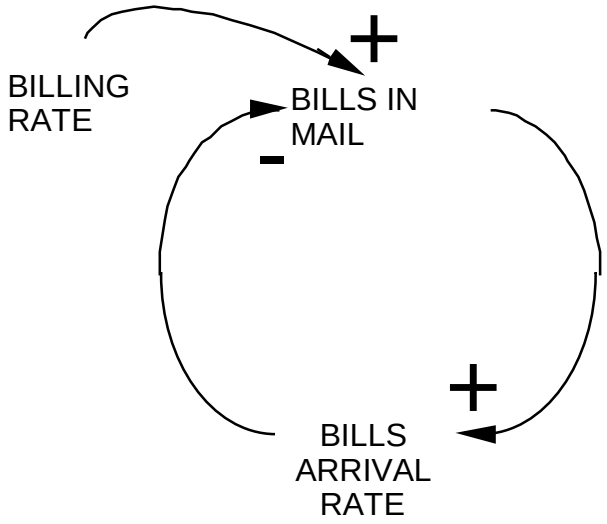


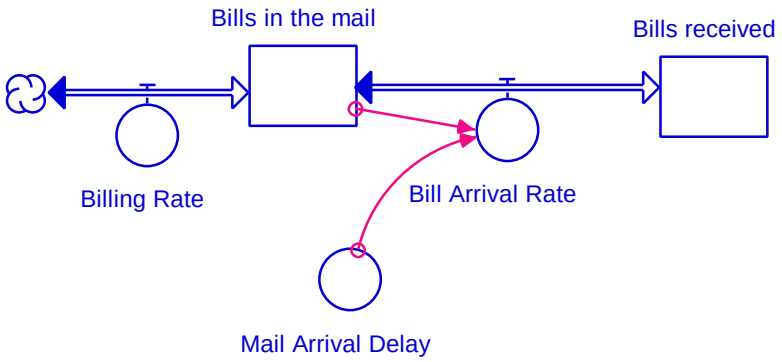
b. With the halving time of Nobug lengthened to seven days, the new value of NAT is 12 days (ie. half-life/.69). Since halving time is so long, Nobug is not eliminated from the river within the 7 days. As a result, content of Nobug continues to increase with each successive dumping of Nobug. However, the level of Nobug stabilizes again! The level stabilizes when the quantity of Nobug dumped into the river (ie. 420 gallons once a week) is equal to the quantity absorbed (ie. the area under the curve of the ABSORB RATE) during the week.


1: NOBUG
2: ABSORB RATE
1: 1000.00
2: 160.00
1: 500.00
2: 80.00
1: 0.00
2: 0.00
0.00
18.75
37.50
56.25
75.00
Graph 3
Days
8:01 PM 7/25/94

Exercise 4

a.


BILLING RATE
+
BILLS IN MAIL
-
+
BILLS ARRIVAL RATE



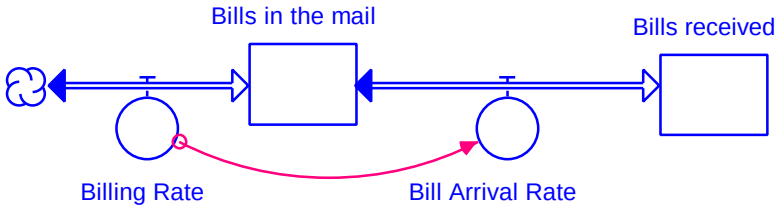
Bills_in_the_mail(t) = Bills_in_the_mail(t - dt) + (Billing_Rate - Bill_Arrival_Rate) * dt
INIT Bills_in_the_mail = 0
DOCUMENT: Initially there are no bills in the mail.
Units: bills

INFLOWS:
Billing_Rate = PULSE(1000,1,0)
DOCUMENT: 1000 bills are sent out on the first day of the month. The interval was set to 0 because in this case we are only interested in studying the first pulse.
Units: bills/day

OUTFLOWS:
Bill_Arrival_Rate = Bill_Arrival_Rate/Mail_Arrival_Delay
DOCUMENT: This is a first order material delay, where bills arrive on average 3 days after they are sent.
Units: bills/days.

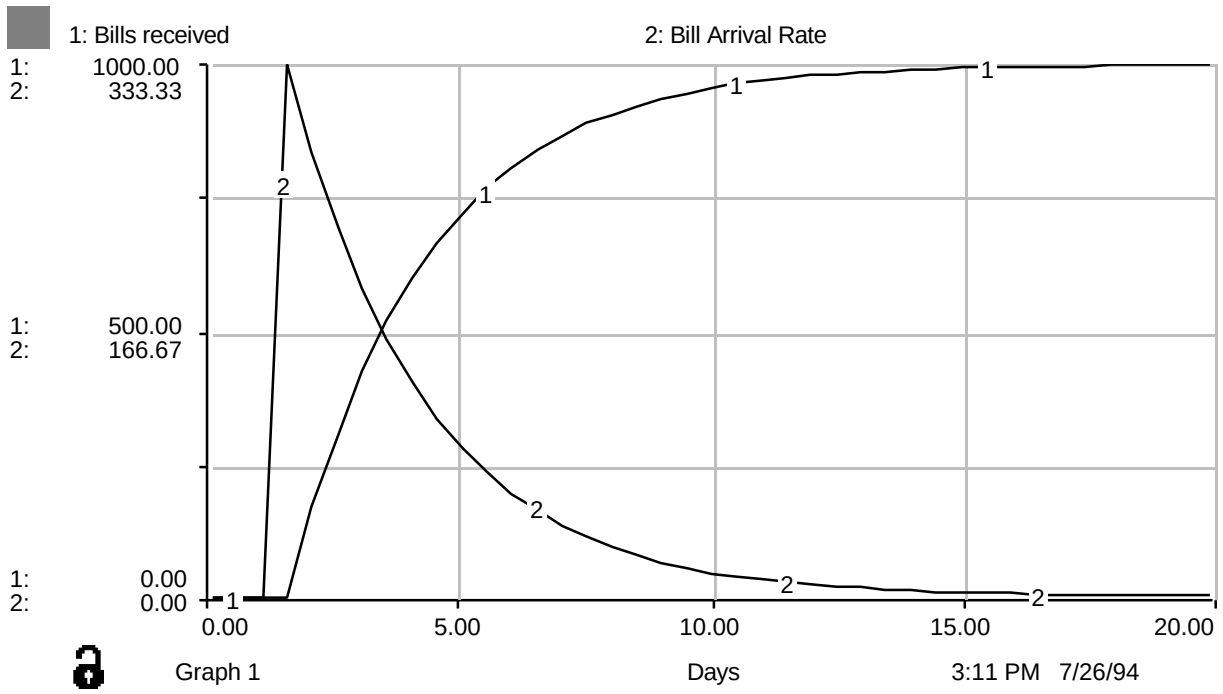
Mail_Arrival_Rate = 3
Document : Time it takes for mail to arrive to destination.
Units: days

Another way of modeling the same behavior using the SMTH1 function is shown below. The behavior of this model should be the same as the graph below.



Billing_Rate = PULSE(1000,1,0)
Bill_Arrival_Rate = SMTH1(Billing_Rate, 3)
DOCUMENT: The adjustment time or delay is 3 days.

b.

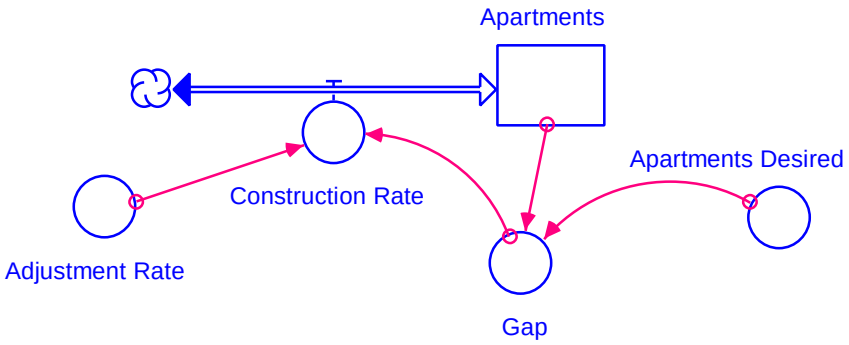


The pattern in the output is a mail arrival rate that decays exponentially from an initial peak just after the mail is sent. This is an unrealistic modeling of the mail process since no mail will be received immediately after the mail is sent.

c. From the tabular output, at the end of 6 days, 806 bills have been received or 194 bills (ie. 1000-806) will take more than 6 days to arrive.

Exercise 6

a.



Apartments(t) = Apartments(t - dt) + (Construction_Rate) * dt
INIT Apartments = 0

DOCUMENT: Apartments built so far.
Units: apartments

INFLOWS:
Construction_Rate = Gap/Adjustment_Rate

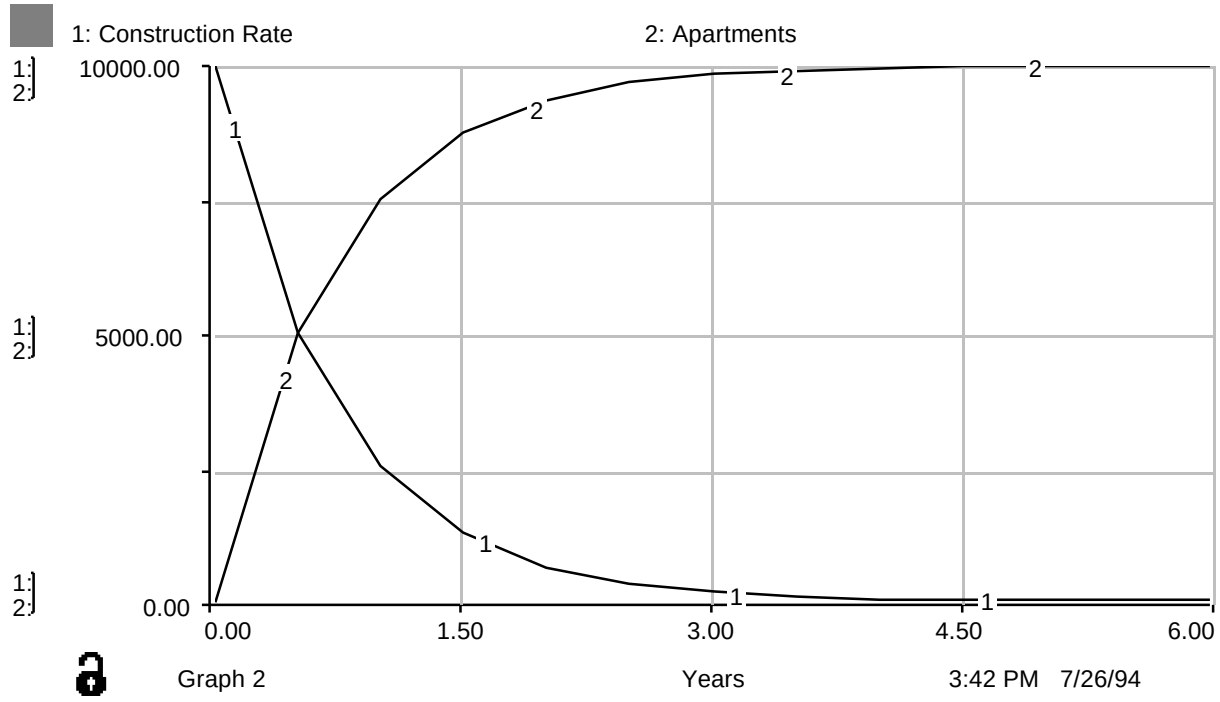
DOCUMENT: Apartments per year constructed.
Units: apartments/year

Adjustment_Rate = 1
DOCUMENT: Time to respond to the gap is one year.
Units : years

Apartments_Desired = 10000
DOCUMENT: Units : apartments

Gap = Apartments_Desired-Apartments
DOCUMENT: Difference between the number of desired apartments and actual apartments.
Units: apartments.

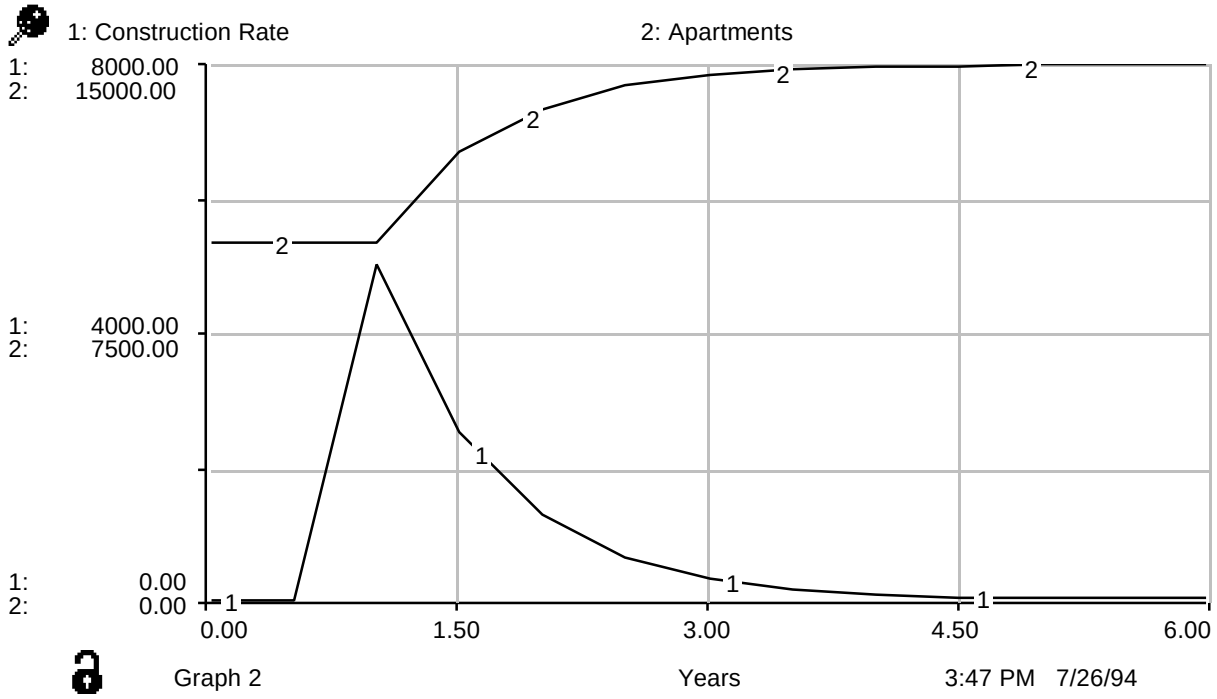
b.



From the plot, the number of apartments at equilibrium will be 10000, as expected for this simple model with no demolition process.

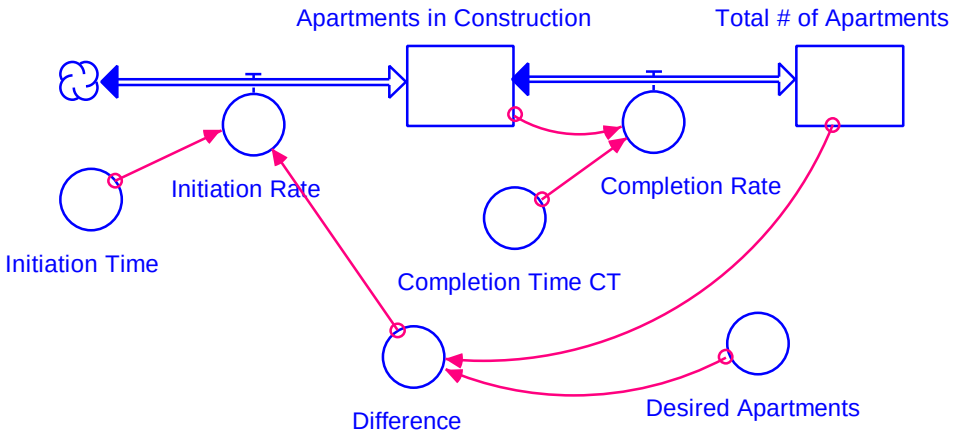
c. The equation for desired apartments should be changed to:

Apartments_Desired = 10000 + STEP(5000,1)



Exercise 7

a.



Apartments_in_Construction(t) = Apartments_in_Construction(t - dt) + (Initiation_Rate - Completion_Rate) * dt
INIT Apartments_in_Construction = 0

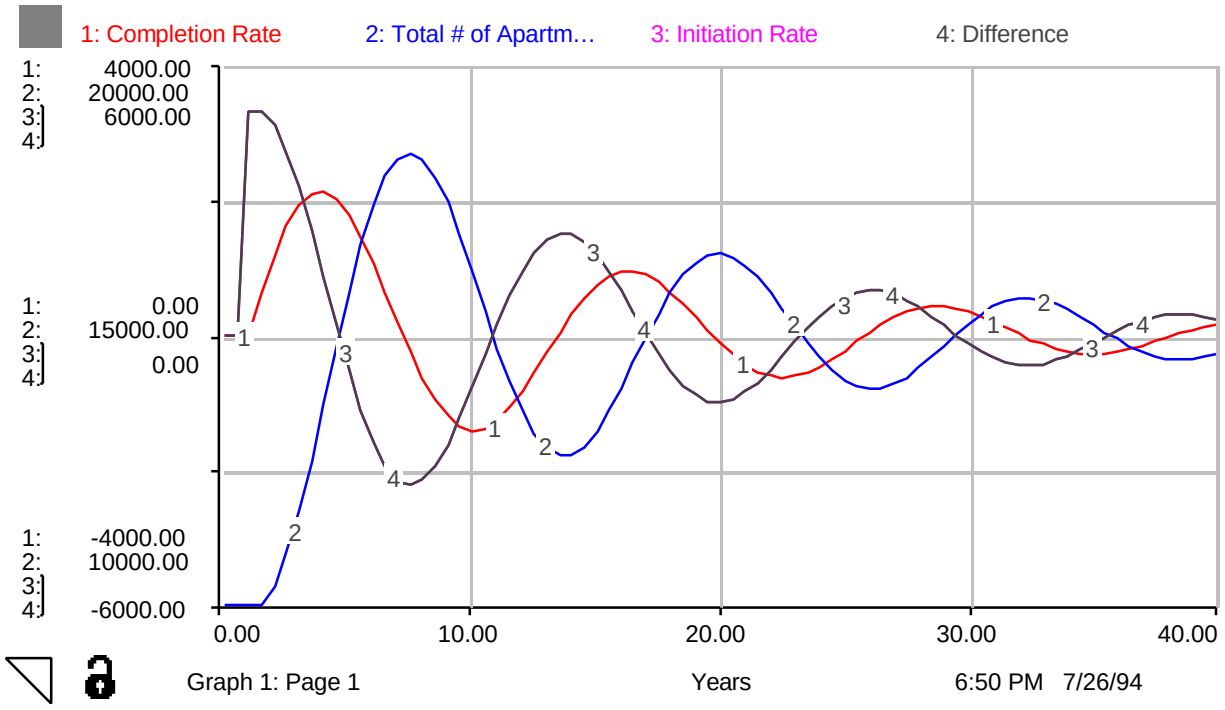
INFLOWS:
Initiation_Rate = Difference/Initiation_Time
OUTFLOWS:
Completion_Rate = Apartments_in_Construction/Completion_Time_CT
Total_#_of_Apartments(t) = Total_#_of_Apartments(t - dt) + (Completion_Rate) * dt
INIT Total_#_of_Apartments = 10000

INFLOWS:
Completion_Rate = Apartments_in_Construction/Completion_Time_CT
Completion_Time_CT = 4
Desired_Apartments = 10000+STEP(5000,1)
DOCUMENT: Units : apartments

Difference = Desired_Apartments-Total_#_of_Apartments
DOCUMENT: Difference between the number of desired apartments and actual apartments. Units: apartments.

Initiation_Time = 1

b& c. Set the Total # of Apartments equal to 10,000 to start the model in equilibrium. The STEP function is used to increase the Desired Apartments from 10,000 to 15,000. The stock for the Total # of Apartments must not be left checked as a non-negative stock in the dialog box for this stock. This is because it is possible to have a negative construction rate. See part (f) for further explanation.



c. The system generates oscillating behavior. The oscillations in this case are damped oscillations. The Total # of Apartments oscillates around 15,000 apartments and eventually converges to this value in equilibrium.

d. A delay in a negative feedback loop leads to oscillation. There are different types of oscillatory behavior- sustained, expanding and damped oscillations. The linkage between the stock for the Total # of Apartments and it's outflow causes the system to exhibit damped oscillations.

e. Try three runs with Completion Times CT of 3, 4, and 6 years. In all cases, the number of apartments will oscillate about the equilibrium value, the main effect of changing the construction delays time is to change the period of oscillation. An increase in the Completion Time increases the period of oscillation and makes it take longer for the system to achieve equilibrium. In all cases, the oscillations are damped, becoming smaller and smaller in amplitude from each cycle to the next.

f. A negative apartment construction rate appears to be physical nonsense. However, it could correspond to the destruction of buildings or to the cancellation of contracts to build, even after construction has started. The prediction was to be made for a period twice as long as your run. Since you have made several runs, you have several possibilities. Consider the case of CT (construction delay time) being 6 years. The period of oscillation for this value of CT is about 15 years so you could expect new maxima in the rate of completion of apartments at 35 and 50 years but much smaller in size than those at 5 and 20 years. A run for 50 years shows just that pattern.

Vensim Examples:
Introduction to Computer Simulation

By Lei Lei and Samitha Samaranayake
October 2001

Example 1A: Martan Chemical Company

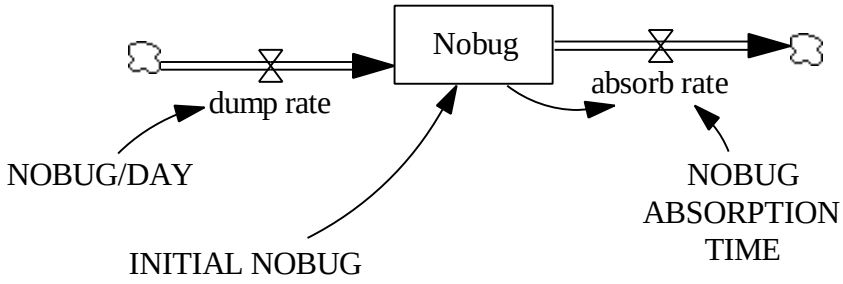


Figure 10: Vensim Equivalent of Diagram of the Martan Case without Pulse Function

Documentation for Martan Chemical Company Model

(01) absorb rate = Nobug/NOBUG ABSORPTION TIME
Units: gallons/day

(02) dump rate = "NOBUG/DAY"
Units: gallons/day
The dump rate is how fast Nobug is dumped into the river.

(03) FINAL TIME = 25
Units: days
The final time for the simulation.

(04) INITIAL NOBUG = 0
Units: gallons
The Initial amount of Nobug is 0 gallons.

(05) INITIAL TIME = 0
Units: days
The initial time for the simulation.

(06) Nobug = INTEG (+dump rate-absorb rate, INITIAL NOBUG)
Units: gallons

(07) NOBUG ABSORPTION TIME = 1

Units: day

(08) "NOBUG/DAY" = 60
Units: gallons/day

(09) SAVEPER = TIME STEP
Units: days
The frequency with which output is stored.

(10) TIME STEP = 0.25
Units: days
The time step for the simulation.

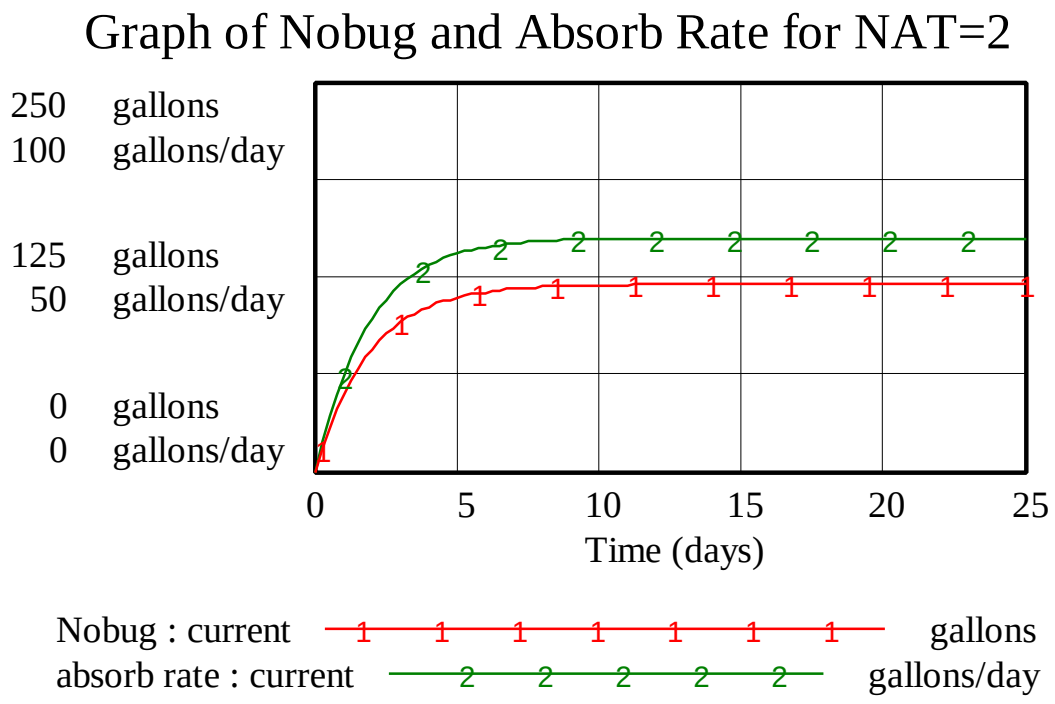


Figure 11: Vensim Equivalent of Simulation for NAT = 2

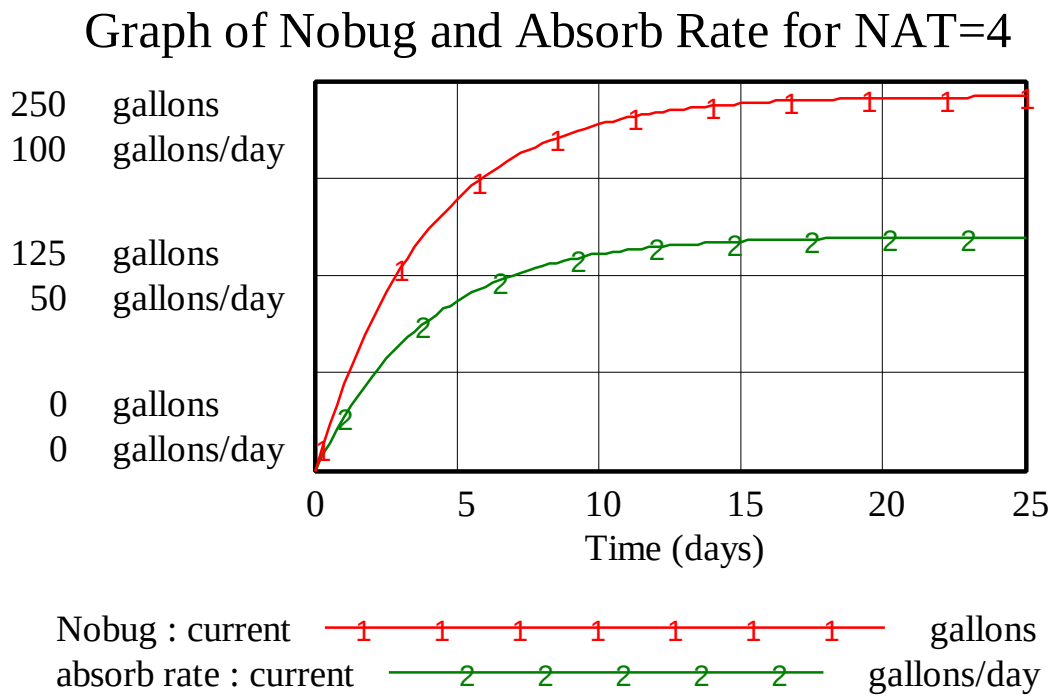


Figure 12: Vensim Equivalent of Simulation for NAT = 4

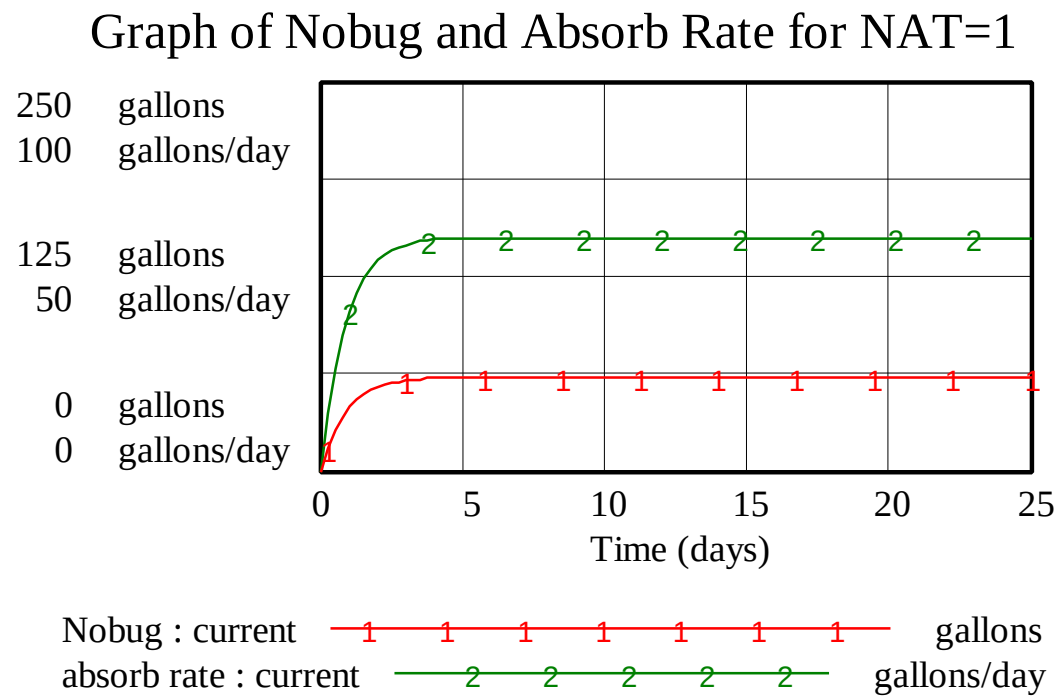


Figure 13: Vensim Equivalent of Simulation for NAT = 1

Example 1B: Dump Rate With Pulse(s)

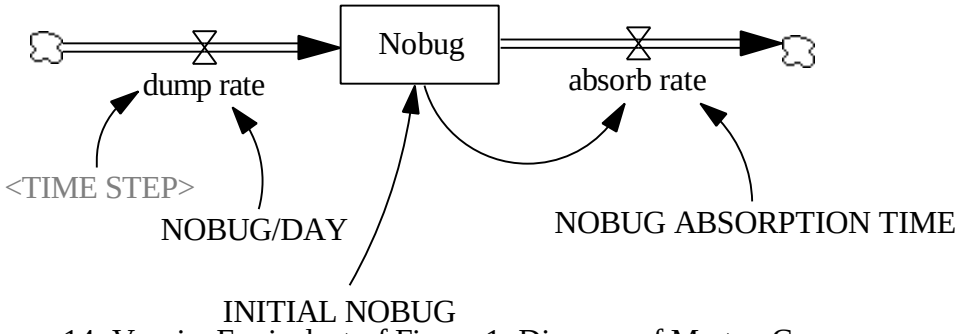


Figure 14: Vensim Equivalent of Figure 1: Diagram of Martan Case

Documentation for Dump Rate With Pulse(s) Model

(01) absorb rate = Nobug/NOBUG ABSORPTION TIME
Units: gallons/day

(02) dump rate = "NOBUG/DAY"*(PULSE(1 ,TIME STEP)+ PULSE(8 ,TIME STEP)+ PULSE(15 ,TIME STEP)+ PULSE(22 ,TIME STEP))
Units: gallons/day
420 gallons of Nobug are dumped each quarter of a day, so 1680 gallons of Nobug are dumped on the first day of each week

Note: This formulation is used because Vensim does not have a repeated pulses function. We have inserted individual pulses at intervals of 7 days instead.

(03) FINAL TIME = 25
Units: days
The final time for the simulation.

(04) INITIAL NOBUG = 0
Units: gallons

(05) INITIAL TIME = 0
Units: days
The initial time for the simulation.

(06) Nobug = INTEG (+dump rate-absorb rate, INITIAL NOBUG)
Units: gallons

(07) NOBUG ABSORPTION TIME = 7
Units: day

(08) "NOBUG/DAY" = 1680
Units: gallons/day

There are 1680 gallons of Nobug dumped per day.

(09) SAVEPER = TIME STEP
Units: days
The frequency with which output is stored.

(10) TIME STEP = 0.25
Units: day
The time step for the simulation.

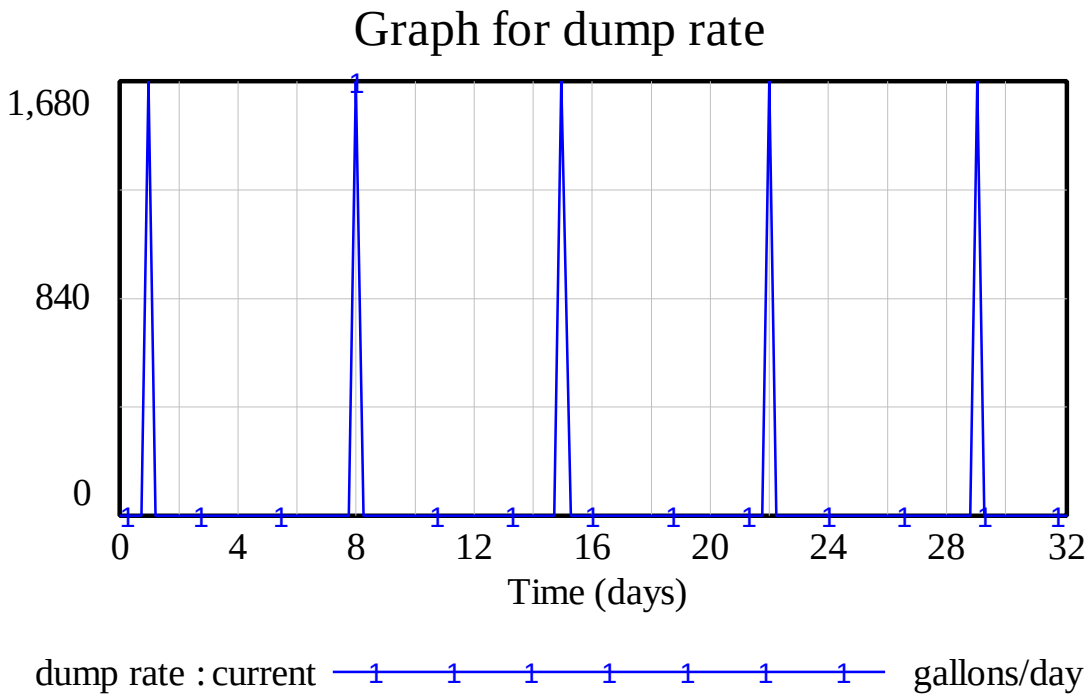


Figure 15: Vensim Equivalent of Figure 2: Simulation for dump rate

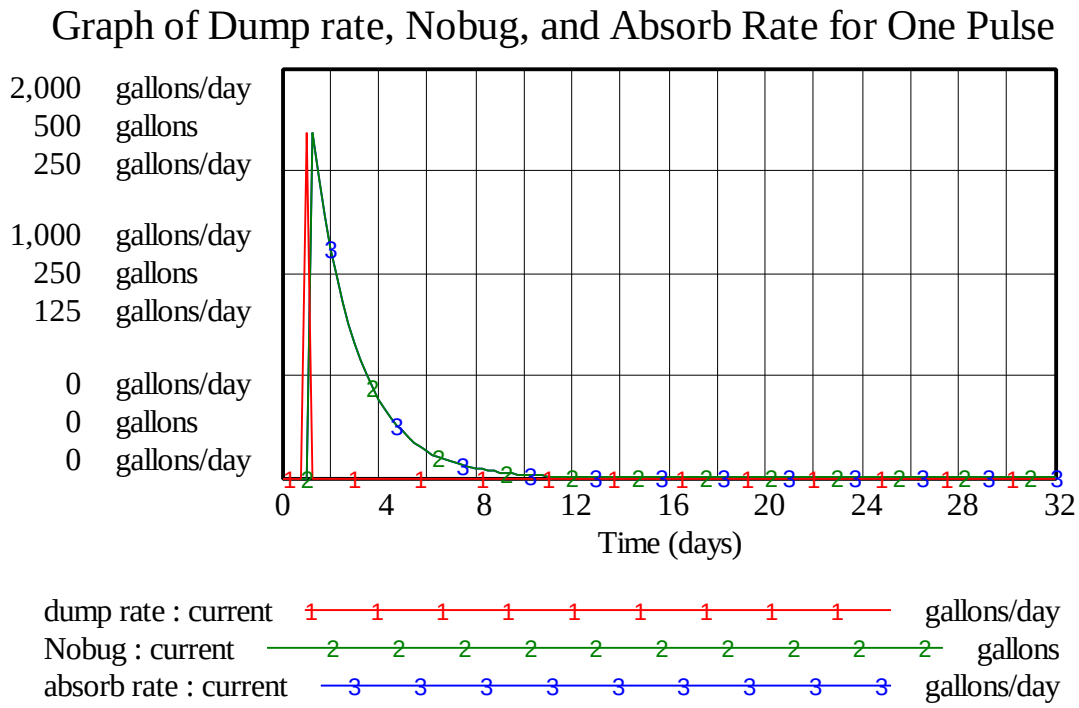


Figure 16: Vensim Equivalent of Figure 3: Simulation of release of one batch of Nobug

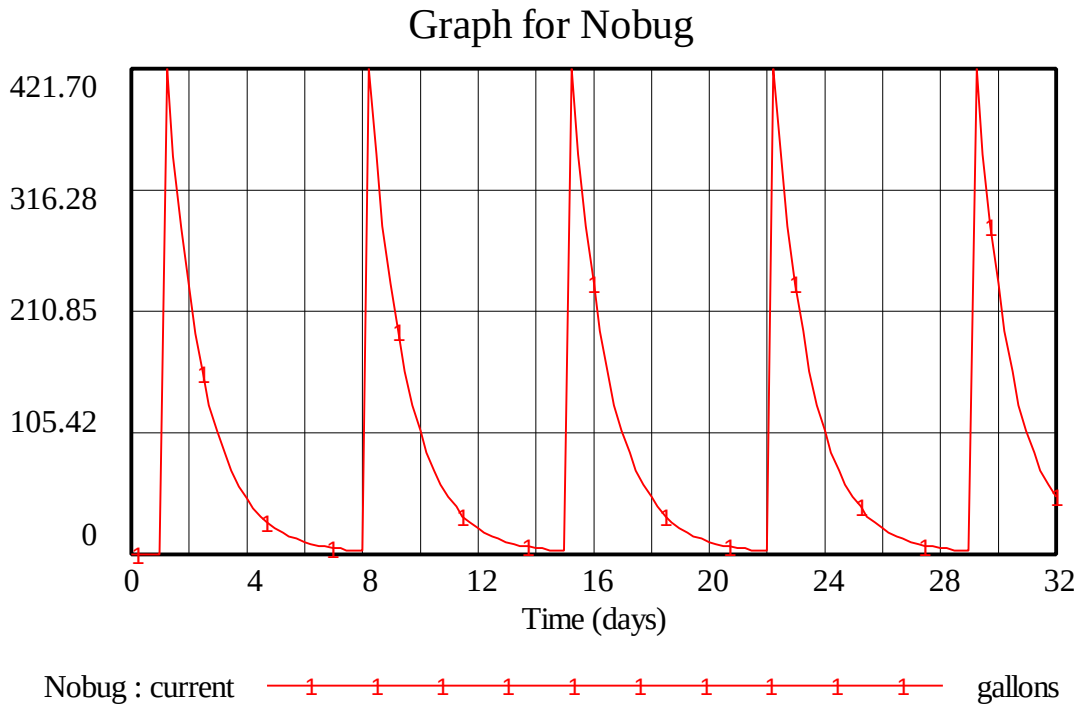


Figure 17: Vensim Equivalent of Simulation of Nobug for NAT = 1.4

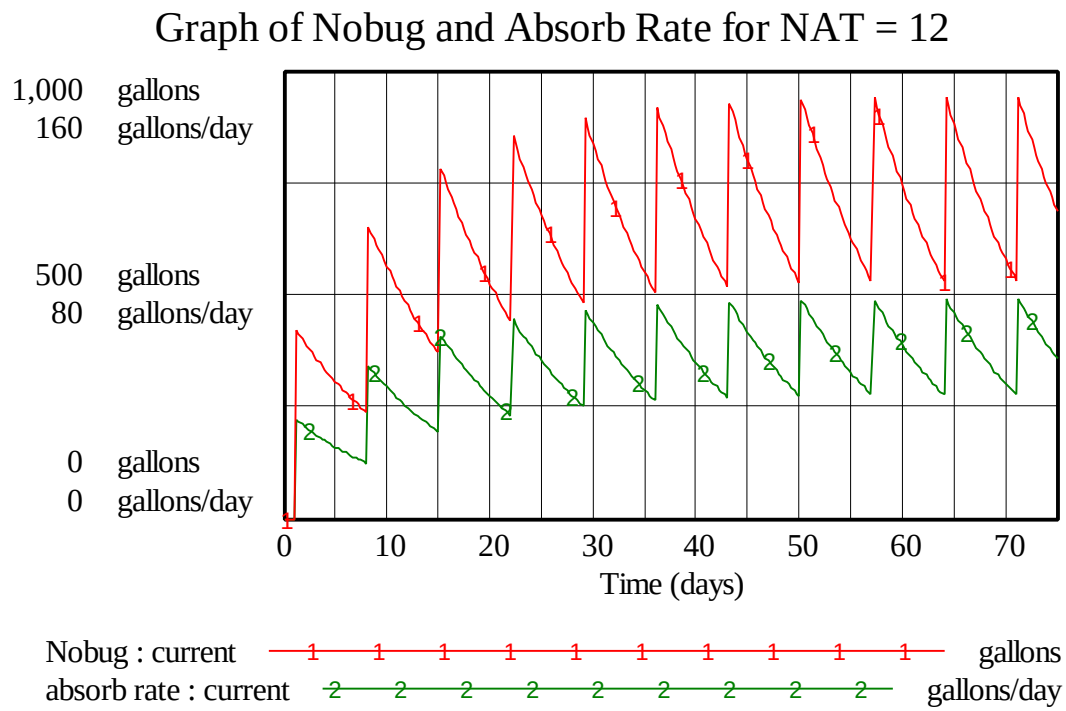


Figure 18: Vensim Equivalent of Simulation of Nobug and Absorb Rate for NAT = 12

Exercise 5: The Mail Delay

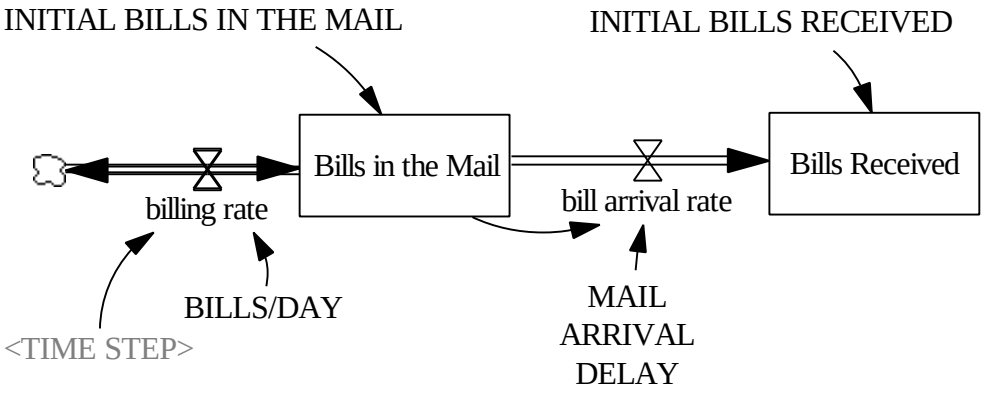


Figure 19: Vensim Equivalent of Mail Delay Model

Documentation for Mail Delay Model

(01) bill arrival rate = Bills in the Mail/MAIL ARRIVAL DELAY
Units: bills/day
This is a first order material delay, where bills arrive on average 3 days after they are sent.

(02) billing rate = "BILLS/DAY"*PULSE(1, TIME STEP)
Units: bills/day
1000 bills are sent out on the first day of the month. The interval was set to 0 because in this case we are only interested in studying the first pulse.

(03) Bills in the Mail =INTEG (+billing rate-bill arrival rate, INITIAL BILLS IN THE MAIL)
Units: bills
Initially there are no bills in the mail.

(04) Bills Received = INTEG (bill arrival rate, INITIAL BILLS RECEIVED)
Units: bills

(05) "BILLS/DAY"= 1000
Units: bills/day
1000 bills are sent out on the first day of the month.

(06) FINAL TIME = 20
Units: day
The final time for the simulation.

(07) INITIAL BILLS IN THE MAIL = 0
Units: bills
There are initially 0 Bills in the Mail.

(08) INITIAL BILLS RECEIVED = 0
Units: bills
There are initially 0 bills received.

(09) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(10) MAIL ARRIVAL DELAY = 3
Units: day
Time it takes for mail to arrive to destination.

(11) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.

(12) TIME STEP = 0.0625
Units: day
The time step for the simulation.

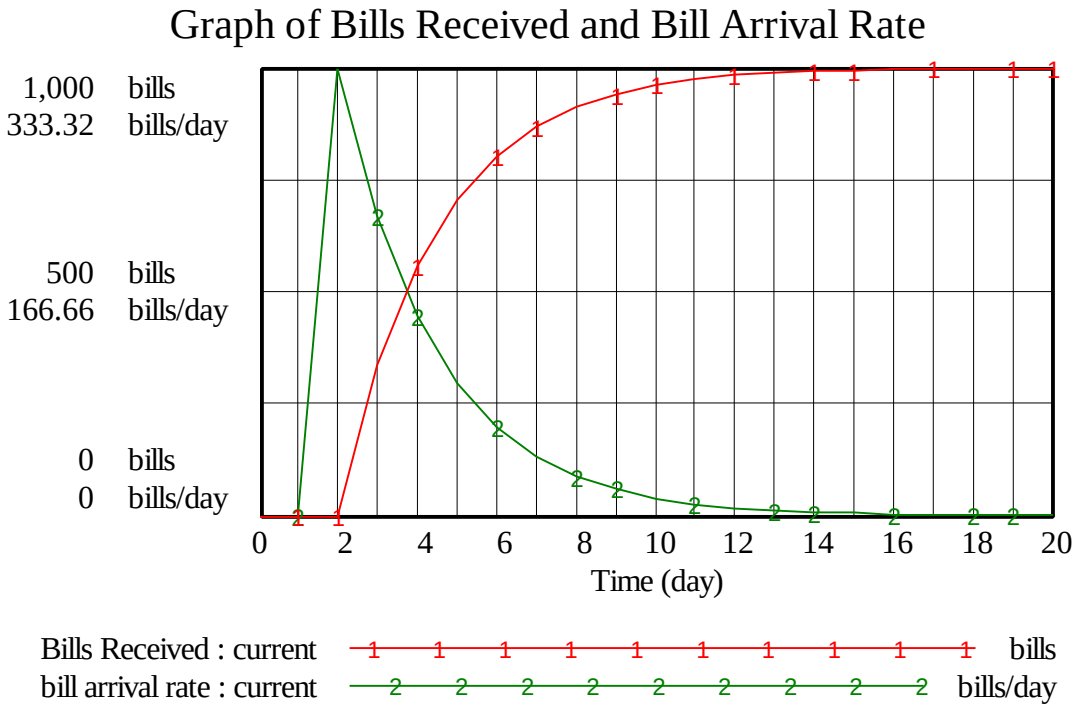


Figure 20: Vensim Equivalent of Simulation of Bills Received and Bill Arrival Rate

Exercise 6A: Apartments-Part 1

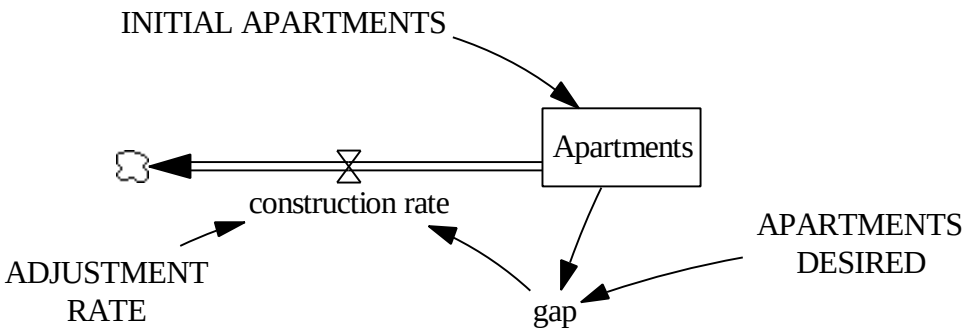


Figure 21: Vensim Equivalent of Apartments Model-Part 1

Documentation for Apartments Model-Part 1

(01) ADJUSTMENT RATE = 1
Units: year
Time to respond to the gap is one year.

(02) Apartments = INTEG (construction rate, INITIAL APARTMENTS)

Units: apartments
Apartments built so far.

(03) APARTMENTS DESIRED = 10000
Units: apartments

(04) construction rate = gap/ADJUSTMENT RATE
Units: apartments/year

(05) FINAL TIME = 6
Units: year
The final time for the simulation.

(06) gap = APARTMENTS DESIRED-Apartments
Units: apartments
Difference between the number of desired apartments and actual Apartments.

(07) INITIAL APARTMENTS = 0
Units: apartments
There are 0 apartments initially.

(08) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(09) SAVEPER =TIME STEP
Units: year
The frequency with which output is stored.

(10) TIME STEP = 0.0625
Units: year
The time step for the simulation.

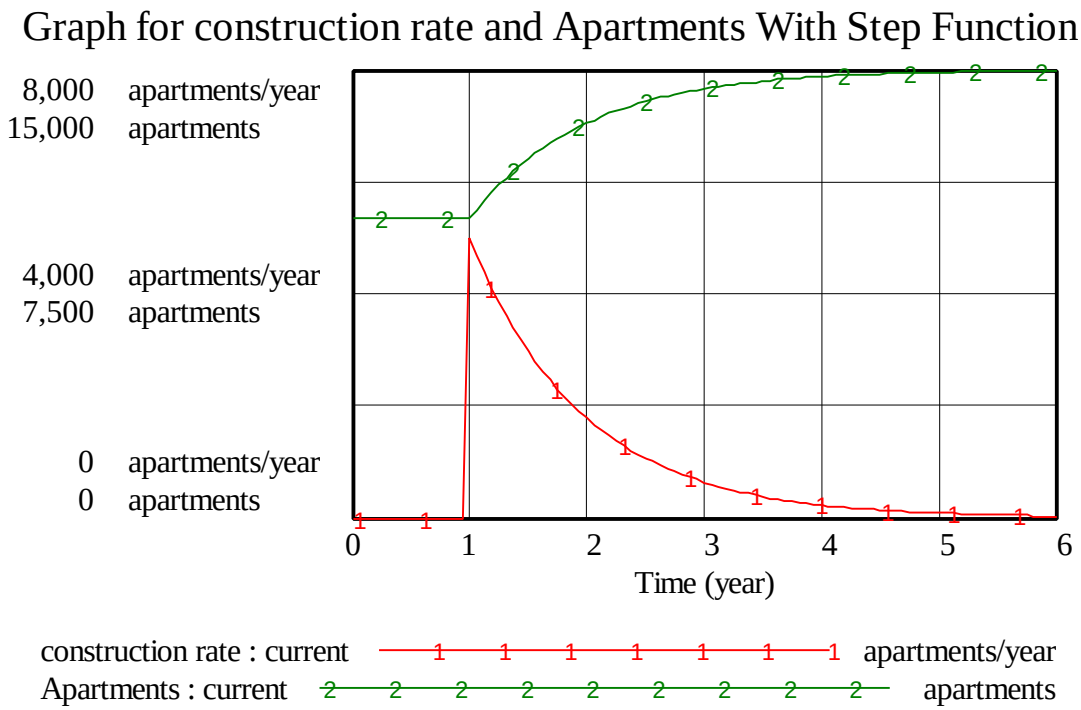


Figure 22: Vensim Equivalent of Simulation with Step Function

Exercise 6B: Apartments –Part 2

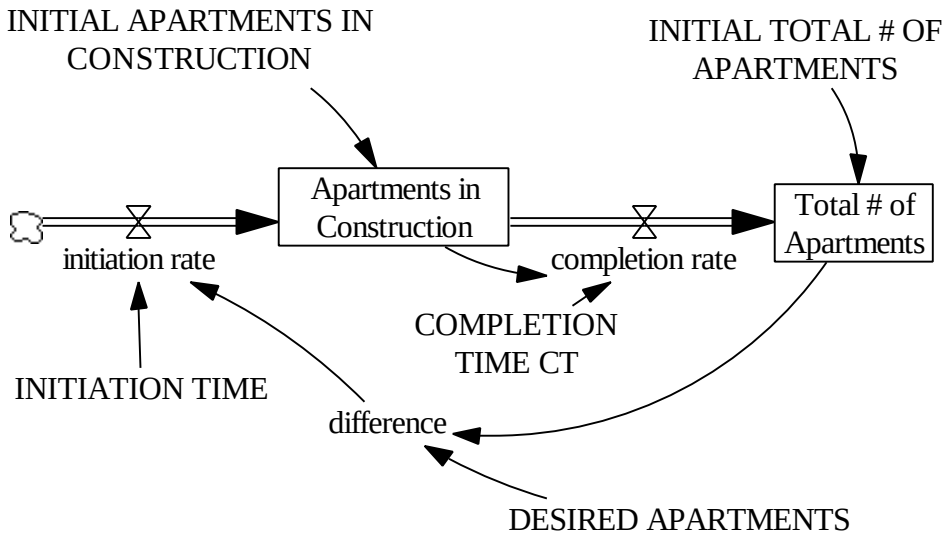


Figure 23: Vensim Equivalent of Apartments Model-Part 2

Documentation for Apartments Model-Part 2

(01) Apartments in Construction = INTEG (+initiation rate-completion rate, INITIAL APARTMENTS IN CONSTRUCTION)
Units: apartments

(02) completion rate = Apartments in Construction/COMPLETION TIME CT
Units: apartments/year

(03) COMPLETION TIME CT = 4
Units: year

(04) DESIRED APARTMENTS = 10000+STEP(5000,1)
Units: apartments

(05) difference = DESIRED APARTMENTS-"Total # of Apartments"
Units: apartments
Difference between the number of desired apartments and actual apartments.

(06) FINAL TIME = 40
Units: year
The final time for the simulation.

(07) INITIAL APARTMENTS IN CONSTRUCTION = 0
Units: apartments
There are 0 apartments initially.

(08) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(09) INITIAL TOTAL # OF APARTMENTS= 10000
Units: apartments
There are 10000 apartments initially.

(10) initiation rate = difference/INITIATION TIME
Units: apartments/year

(11) INITIATION TIME = 1
Units: year

(12) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(13) TIME STEP = 0.0625
Units: year

The time step for the simulation.

(14) Total # of Apartments = INTEG (completion rate, INITIAL TOTAL # OF APARTMENTS)

Units: apartments

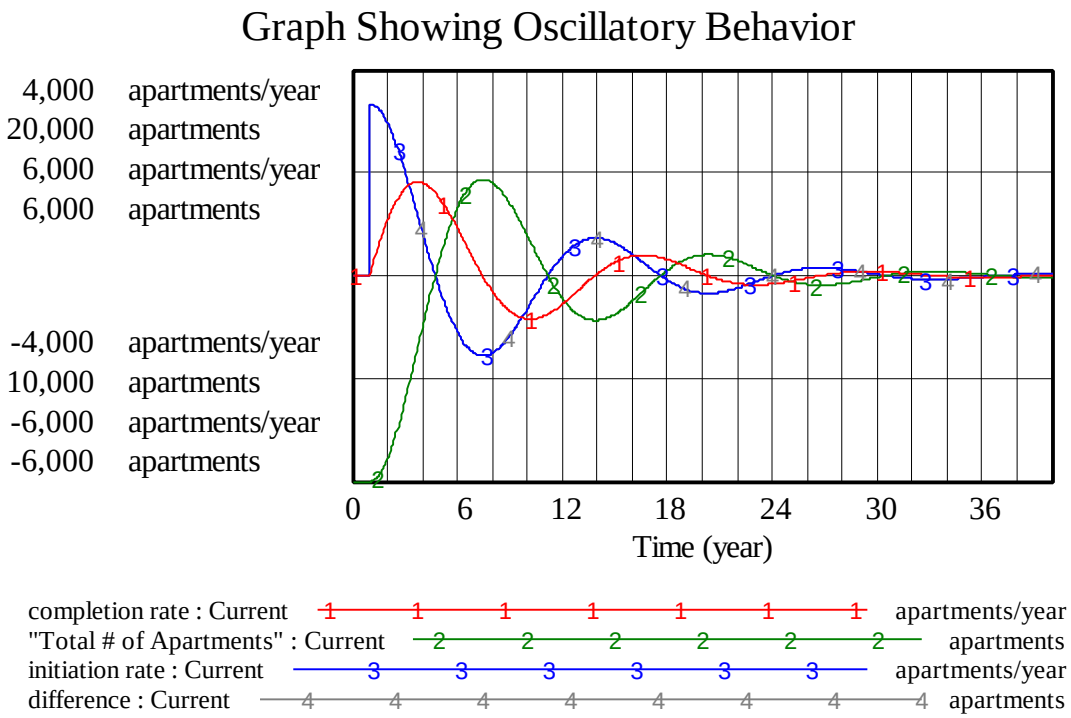
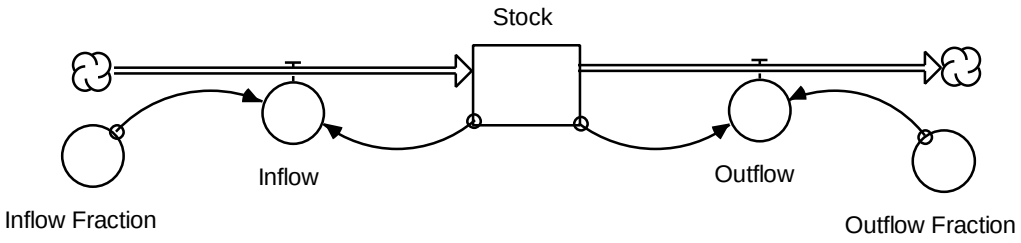


Figure 24: Vensim Equivalent of Simulation showing oscillatory behavior

Beginner Modeling Exercises

Section 5

Mental Simulation of Combining Feedback
in First-Order Systems



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Laughton Stanley
Helen Zhu
May 29, 1996
Vensim
Examples added October 2001

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1. Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool, one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on a computer. Computer simulation is a very useful tool for explaining systems; however one should also be able to use the other simulation tool of system dynamics: *mental simulation*. A strong set of mental simulation skills will enhance the ability to validate, debug, and understand dynamic systems and models.

It is assumed that the reader is familiar with the simulation of positive feedback, negative feedback, and adding constant flows.[1] To further develop these capabilities, it is necessary to study the behaviors associated with combined feedback loops in first-order systems. Here we examine four possible behaviors of this structure: equilibrium, exponential growth, asymptotic growth, and S-shaped growth. A set of exercises will reinforce understanding of these models. Solutions are included.

2. Example 1: Identical Constant Fractions

Eddie is creating a model of his nursery's tree production so that he can determine the policies that might best increase his inventory and sustain sales. Eddie looks at the behavior of the past five years and realizes that he has been using the combined feedback model shown in Figure 1. Eddie's yearly sales have been around 8% of his inventory, so his past policy was to replenish his inventory by planting the same 8% yearly.[2]

---

[1] The "Beginner Modeling Exercises, Mental Simulation" series is available in "Positive Feedback" (D-4487) by Joseph Whelan, "Negative Feedback" (D-4536) by Helen Zhu, and "Adding Constant Flows" (D-4546) by Alan Coronado. The series is part of Road Maps.

[2] All models are simplifications. The assumptions in this model about the Sales_Fraction are only an approximate method of modeling Sales. In the reality it is unlikely that Sales would remain a constant percentage of inventory, but for the purposes of this paper it is instructional to model Sales in this way.

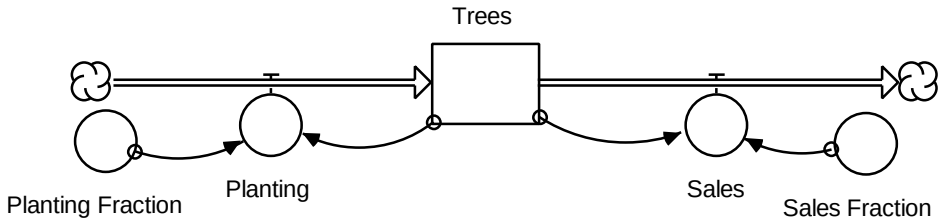


Figure 1: Eddie's Past Business Strategy

Using the mental simulation skills that he had learned from Road Maps, Eddie sets out to determine the results of his current strategy. He examines the model and recognizes the presence of both positive and negative feedback loops in the system. He ponders over how the two interact to produce the final system behavior. In attempting to figure this out, Eddie takes the following steps of mental simulation:

1. *Determine the nature of loop behavior in the system.*

   Eddie looks at his model and realizes that as the number of Trees increases, so does the Planting rate. Over time, this generates exponential behavior due to positive feedback. The outflow loop, however, shows that as the number of Trees increases, Sales also increase, causing the Trees and Sales to drop again. Under just this second pattern of behavior, the number of Trees gradually approaches zero, its goal. This goal-oriented behavior is caused by negative feedback.

   To determine the net behavior of the system, both flows must be considered. The two following equations give the values of Planting and Sales. The line below the equation gives the units for the equation.

   Planting = Trees * Planting_Fraction = Trees * 0.08
   *trees/year = trees * (1/year)*
   Sales = Trees * Sales_Fraction = Trees * 0.08
   *trees/year = trees * (1/year)*

   In this case, the two flows are equally strong because the compounding fractions are equal. That means the inflow is always equal to the outflow; thus the net flow for the system is always zero, and the stock value is always constant.

2. *Determine the initial stock value.*

   Eddie notes that he starts out with 500 trees.

3. *Determine the final stock value.*

   What does the stock do after the initial point? The inflow at any time is the stock times the Planting_Fraction, and the outflow at any time is the stock times the Sales_Fraction. Both fractions happen to be 0.08. Eddie realizes that in this case the system is always in equilibrium because the inflow and outflow start out equal and never change. Thus the value of the stock is always 500 trees.

Now that Eddie has a pretty good idea of what his Tree inventory looks like, he plots it in Figure 2.

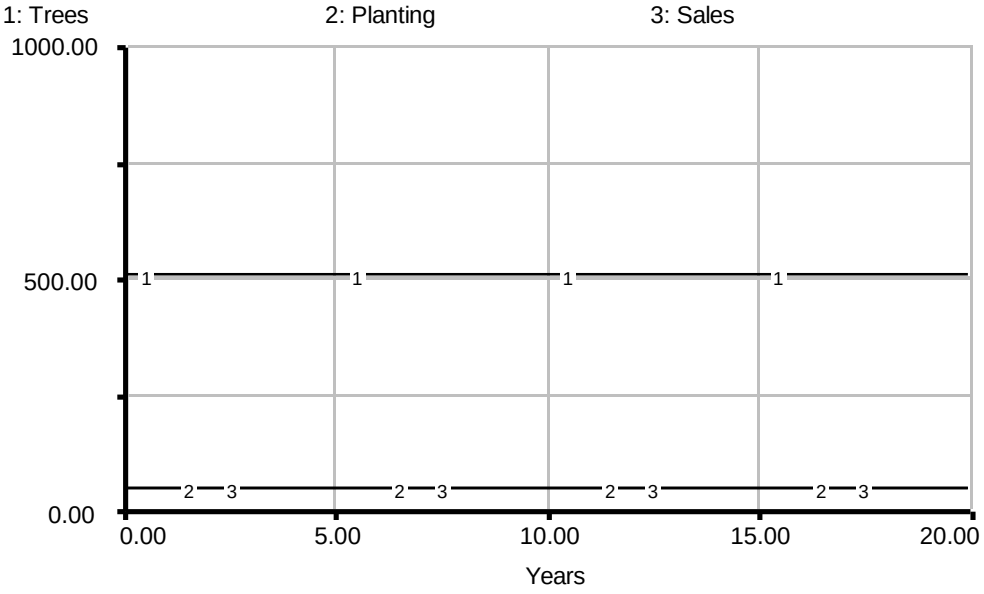


Figure 2: Eddie's Projected Inventory

3. Example 2: Different Constant Fractions

Although business is stable, Eddie would like to design a strategy to gradually increase his inventory and expand business. He realizes that if the Planting_Fraction and the Sales_Fraction are equal, there will be no growth in his Sales. Therefore he decides to increase the Planting_Fraction in the hopes that it will give him more trees to sell. Eddie sticks with the same model structure, but changes the Planting_Fraction to 0.30 so that he

will plant 30% of his inventory each year. He then mentally simulates the system behavior using the same steps:

1. *Determine the nature of loop behavior in the system.*

   The behaviors of the individual loops are the same as in Example 1 because the fundamental relations between stocks and flows have not changed. The difference between the two models is the value of the Planting_Fraction. Because the Planting_Fraction has changed, the relative strengths of the two loops are no longer equal. The larger value for the Planting_Fraction makes the Planting loop considerably stronger. Because the dominant loop is a positive feedback loop, Eddie's tree inventory will increase exponentially.

2. *Determine the initial stock value.*

   Eddie's initial inventory is 500 trees.

3. *Determine the final stock value.*

   A system reaches equilibrium when the inflow and outflow are equal. Because the system produces exponential growth, it can never reach an equilibrium. Eddie confirms this hypothesis by equating the flows:

   Inflow = Outflow
   Trees * Planting_Fraction = Trees * Sales_Fraction
   *trees * (1/year) = trees * (1/year)*
   Trees * 0.30 = Trees * 0.08
   0.30 = 0.08

   Can this be true? It cannot. Eddie is now certain the two flows are never equal (except for the trivial case where there are no trees). Figure 3 shows the computer-simulated system behavior over the next twenty years to illustrate the inequality of the flows. Notice that as the flows grow farther apart, the slope of the tree inventory line grows steeper.

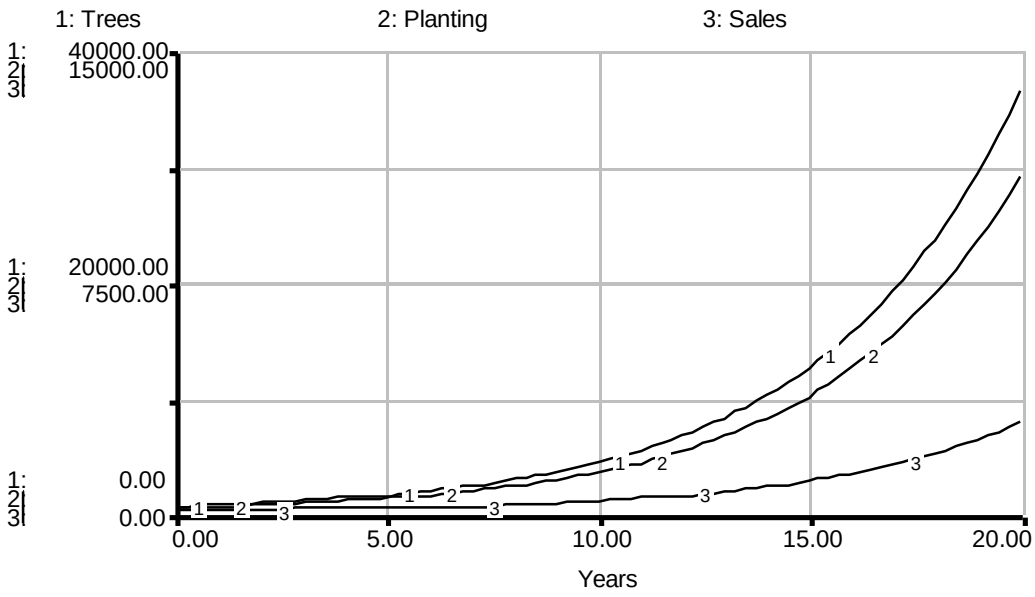


Figure 3: Model Behavior with Increased Planting_Fraction

4. Example 3: Linear Feedback

Looking over the results of his strategy, Eddie decides that it is not very practical for his nursery. He does not want his tree inventory to get out of hand because his trees could die from overcrowding and the lack of sunlight. After consulting with a botanist, Eddie decides that to make the most use out of his 6000 square yards of land, he should grow approximately 5800 trees. He proposes a hypothetical strategy designed to reach this goal over the next few years. Each year he will plant a number of trees equal to 30% of the difference between his goal of 5800 trees and his current inventory. He describes his strategy with the following model:

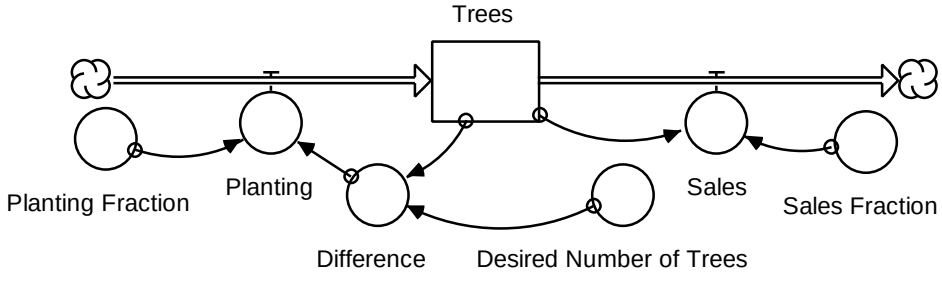


Figure 4: Eddie's Planting Regulation Strategy

Eddie uses his mental simulation steps as before to find out the results of his new strategy. He assumes that his inventory will not surpass his goal.

1. *Determine the nature of loop behavior in the system.*

   The structure of the Planting loop has changed. Now an increase in the number of trees closes the difference between the current inventory of Trees and Eddie's Desired_Number_of_Trees (5800 trees). Tree planting will slow as the inventory approaches the goal. This kind of goal-oriented behavior exemplifies a negative feedback loop. The Sales loop is also a negative feedback loop for the same reasons as before because the loop and its values have not changed.

   The general behavior of the system can be determined by examining the relative strengths of the feedback loops. Initially the Planting loop is stronger than the Sales loop. For instance, in the first year of his program Eddie will plant 1590 trees (Planting = 5300 * 0.3 = 1590) but he will sell only 40 trees (Sales = 500 * 0.08 = 40). This will not always be the case. As Eddie's tree inventory increases, the Difference between the inventory and his goal will decrease. Eventually he will be planting as many trees as he sells. At this point when inflow equals outflow, his stock will be at a stable equilibrium.

   Because the inflow loop starts off increasing the stock strongly and then gradually decreases until it reaches the strength of the negative loop, the system will show asymptotic behavior with a large initial slope that approaches zero as it goes toward the asymptote.

2. *Determine the initial stock value.*

   The initial value is still 500 trees.

3. *Determine the final stock value.*

   The final value occurs when the system is in equilibrium because the inflow and outflow are equal. Eddie quickly goes through the algebra:

   Planting = Sales
   trees/year = trees/year
   Planting_Fraction * Difference = Trees * Sales_Fraction

*(1/year) * trees = trees * (1/year)*

Substituting in numbers, he gets:

0.30 * (5800 - Trees) = 0.08 * Trees
5800 - Trees = 0.08 * Trees / 0.30
5800 = Trees + Trees * (0.08 / 0.30)
5800 = Trees * (1 + 0.08 / 0.30)
5800 ˜ Trees * 1.267
Trees ˜ 4578 at equilibrium

Although he set a goal of 5800 trees in his model, Eddie's model has an equilibrium that is much lower. The reason for this lies in the structure of the model. If there was no outflow at all, the inflow would eventually reach an equilibrium value very close to 5800. Because there is an 8% outflow, there must be an equivalent inflow to create equilibrium. This can only be true when the stock value is some distance, in this case about 1222[3] trees, away from the goal. The nature of this behavior can be seen graphically in Figure 5. Notice that the system reaches equilibrium as the two flows approach the same value.

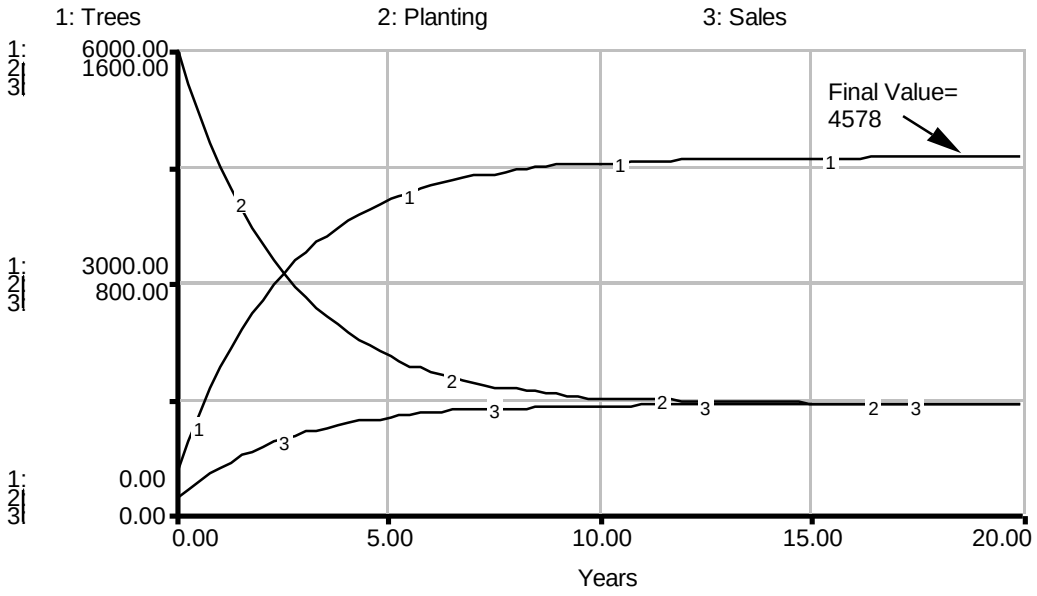


Figure 5: Stock and Flow Behavior of Eddie's Planting Regulation Strategy

[3] The goal is 5800, and equilibrium is reached at 4578 trees. Therefore, the stock is about 5800 - 4578 ? 1222 trees away from the goal.

5. Example 4: Nonlinear Feedback with Graph Function

Even though his previous model produces the goal-seeking behavior that he needs, Eddie is surprised to find the rate at which he must begin planting. Because he cannot afford to plant at a rate of 1600 trees per year without first increasing his sales, he sets out to find a different strategy. Eddie becomes curious as to what would happen if he used a nonlinear relationship to control his tree Planting. He hypothesizes that if he lowers his planting as the tree density increases, he will be able to successfully regulate the tree density. He constructs the following model:

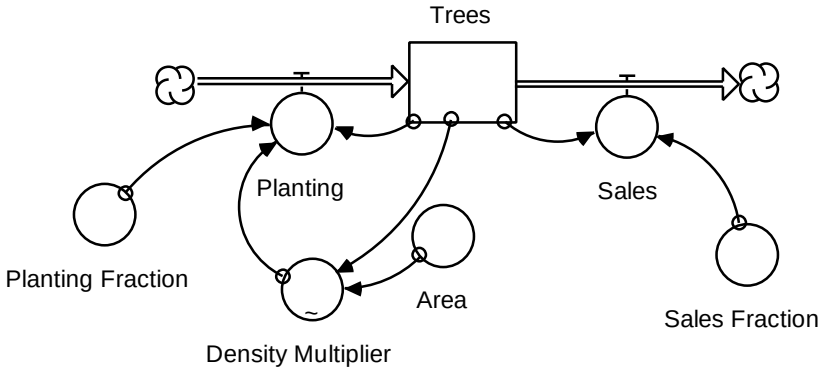


Figure 6: Model Regulating Tree Planting Rate

Eddie leaves his Sales loop the same as in Example 3, but he changes his Planting loop. Instead of having his planting be a fraction of the total number of trees, he scales this value by a multiplier that adjusts for the tree density. After studying the effects of overcrowding on tree growth, he designs the graph function in Figure 7 as his Density_Multiplier. The Density_Multiplier is designed to adjust Eddie's Planting based on the density of his trees so that he will plant fewer and fewer trees as he nears the carrying capacity of his property. The Area of his property is 6000 square yards.

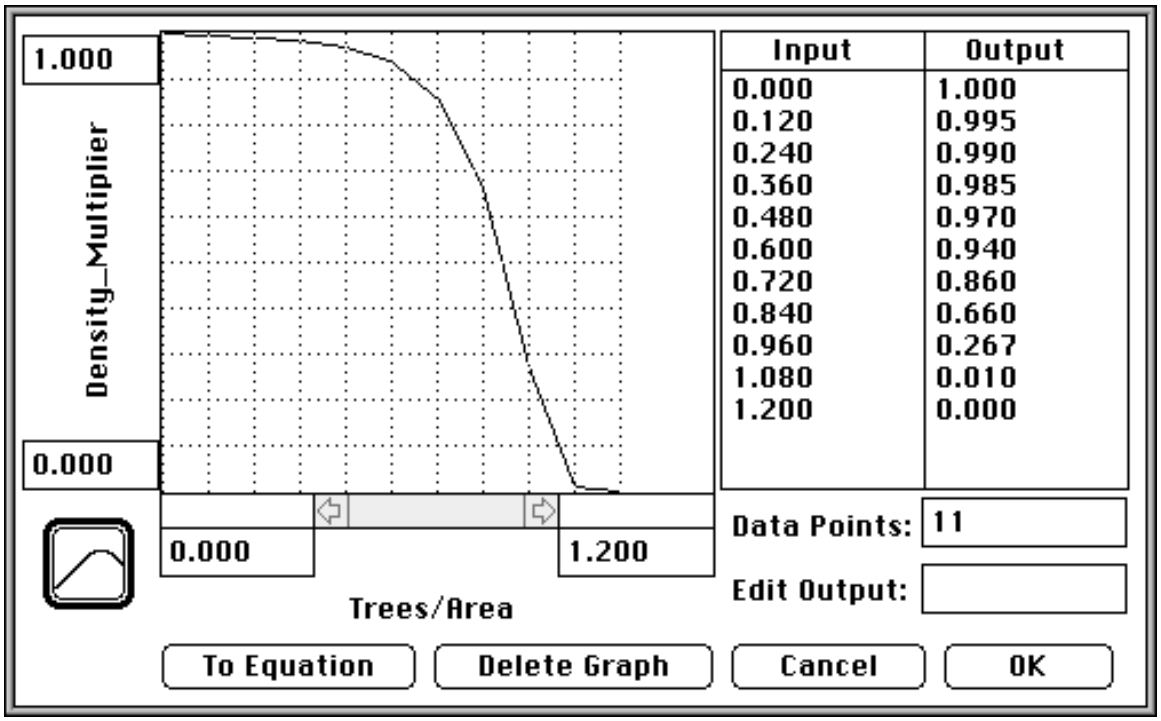


Figure 7: Graph of Eddie's Density_Multiplier

Using his mental simulation skills, Eddie looks at the model and begins to work out its behavior:

1. *Determine the nature of loop behavior in the system.*

   Eddie quickly recognizes that his Sales loop is unchanged and remains a negative feedback loop. After looking at the Planting loop, however, he is somewhat puzzled by the presence of both a Density_Multiplier and a Planting_Fraction in the loop. Looking back to his system dynamics text, he realizes that the extra elements in this loop can be seen as abstractions for a single element. In other words the Density_Multiplier and the Planting_Fraction may be lumped together into a single equation for the purposes of his calculations.

   Eddie sees that this loop has an unusual behavior because of his graph function. The 0.30 planting fraction has the tendency to increase exponentially the number of Trees planted. This is positive feedback. The Density_Multiplier, however, decreases as the number of trees increases. This is negative feedback. Being uncertain of which route the whole system will follow, Eddie writes out the possible behaviors.

Eddie assumes that at any given time, the planting loop will exhibit either positive or negative feedback behavior. Given the characteristics of the loop, Eddie envisions the following three possibilities:

? The positive feedback will dominate the loop and characterize its behavior at all times.

When considering possibilities, it is often useful to consider extreme cases. Eddie sees that if his tree density grows to a value of 1.2 trees / square yard, the planting rate will become zero. This means that Eddie can discount the possibility of simple positive feedback generating exponential growth. There can be no growth at all with a planting rate of zero.

? The negative feedback will dominate the loop and characterize its behavior at all times.

This possibility sounds plausible to Eddie. This could cause asymptotic behavior and bring his tree inventory to the desired level. Unfortunately this is very similar to example 3, which had the very large initial planting rate, and Eddie would like to find a more ideal strategy. The fact that the Density_Multiplier changes very little for densities up to .6 (trees/square yard) helps Eddie to decide that initially there will be room for exponential growth. If the Density_Multiplier is not changing up to .6 (trees/square yard) then it cannot be providing negative feedback at that point. In other words, the positive feedback portion of the loop will be stronger initially until the value of the Density_Multiplier makes a significant change.

? The loop will initially have the characteristics of positive feedback, but the dominance will shift and the loop will show negative feedback characteristics.

Eddie likes this possibility almost immediately. This loop characteristic would allow for a low initial planting rate and an asymptotic final behavior.

After considering his choices, Eddie realizes that the switch from positive to negative feedback makes the most sense. The graph of the Density_Multiplier changes very little initially, so it is reasonable to expect the positive feedback to be dominant at first. As the density increases, the Density_Multiplier dives sharply. Here it is reasonable to assume that the negative feedback will dominate the loop because of the extremely

quick change in the value of the Density_Multiplier. Eddie concludes that the result will be S-shaped growth as shown in Figure 8.

2. *Determine the initial stock value.*

   This is still 500 trees.

3. *Determine the final stock value.*

   The final value occurs when the system is in equilibrium because the inflow and outflow are equal. As before, Eddie writes down the algebraic equations involved:

   Planting = Sales
   Planting_Fraction * Trees * Density_Multiplier = Trees * Sales_Fraction
   *(1/year) * trees * (trees/trees) = trees * (1/year)*
   0.30 * Trees * Density_Multiplier = Trees * 0.08
   Density_Multiplier = (Trees * 0.08) / (Trees * 0.30)
   Density_Multiplier = 0.08/0.30
   Density_Multiplier ~ 0.267[4]

By looking back to his graph, Eddie is able to see that a Density_Multiplier value of 0.267 corresponds to a density of 0.96 trees/square yard. At this density, his nursery, with a 6000 square yard area, will hold about 5800 trees. Given the initial and final values as well as a good idea of the graph shape, Eddie is able to sketch a model of the system similar to that in Figure 8 which illustrates the behavior of the stock and flows over a span of twenty years. Notice that the system reaches equilibrium as the flows reach identical values.

[4] To solve this equation, Eddie has to solve for the value of the graph output and then to determine the value of the graph's input based on its output value. This method is necessary when working with a graph function that is not defined mathematically.

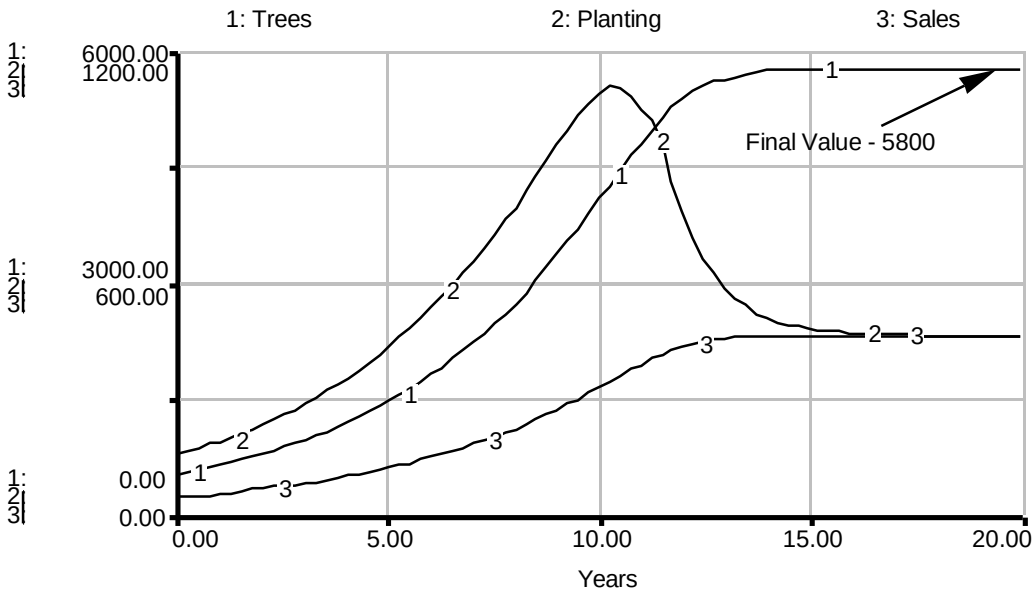


Figure 8: Behavior of Non-Linear Regulation of Planting

Completely satisfied with the simulation results tested using system dynamics, Eddie proceeds to follow his newest strategy. Sure enough, it turns out to be a success!

6. Review

Before moving on to the exercises, here is a summary of the steps of mental simulation for combined feedback in first order systems:

1. *Determine the nature of loop behavior in the system.*

   Recognize the positive and negative feedback loops and how they affect the stock behavior over time.

2. *Determine the initial stock value.*

   This is usually given.

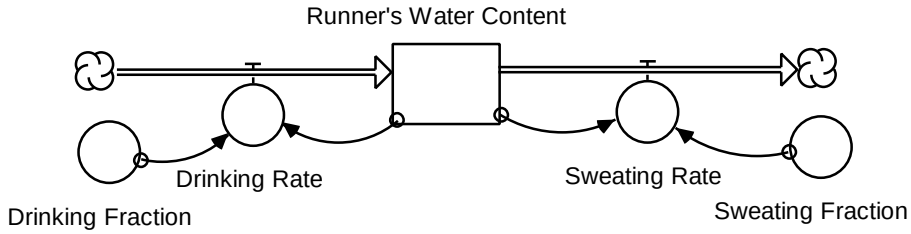
3. *Determine the final stock value.*

   This can be done by equating the rates and solving the resulting equation for the stock value. The stock will have a non-zero solution only when there is an equilibrium.

7. Exercises

7.1 Exercise 1: Running a Marathon

Henry is preparing to run the Boston Marathon. In preparation, he has devised a model for the water level in his body during the race. Henry assumes that he will sweat 3% of his available water every hour. To make up for this loss, he plans to consume water at the rate of 3% of his total water content per hour.



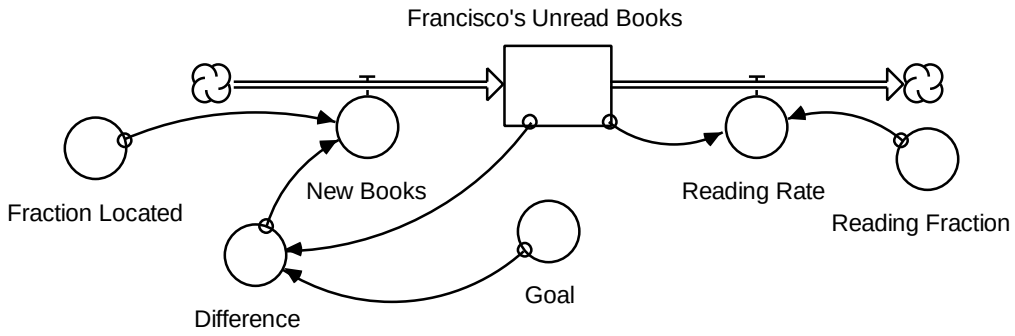
A. What are the feedback loops in this system?

B. What is the qualitative behavior of the stock over time?

C. Is the qualitative stock behavior dependent on the initial stock value?

7.2 Exercise 2: Unread Books

Francisco is a philosophy major who loves to read. His affluent Uncle Midas tries to give him enough books to keep him busy every month. Uncle Midas would like Francisco to have 15 unread books on hand at all times. When buying books, Uncle Midas makes a list of new books for Francisco and buys the books from his list that are being sold at the local bookstore. Some of the books that Uncle Midas selects are rare, however, and he often is able to find only 75% of them. Even so he manages to keep up a good flow of books for his nephew. Francisco, determined to make a dent in the shelf full of his uncle's contributions, has decided that he will read half of the books each month, no matter how many that may be. Here is a model of the number of Francisco's unread books.



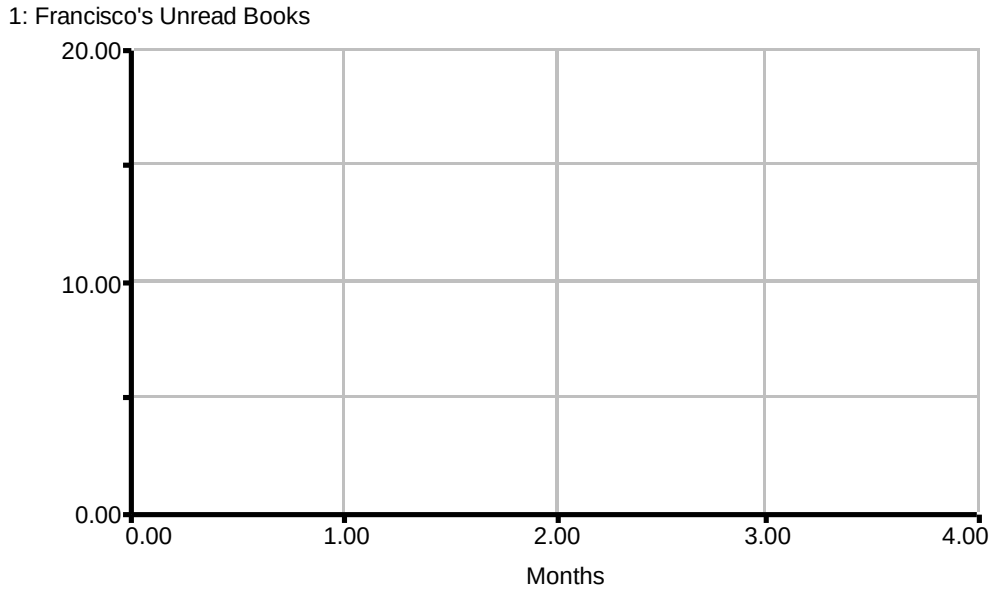
The Fraction_Located represents the average percentage of the books on the list that Uncle Midas can find in the store each month.

New_Books = Difference * Fraction_Located
*books/month = books * (1/month)*
Difference = Goal - Francisco's_Unread_Books
books = books - books
Goal = 15 *(books)*
Fraction_Located = 0.75 *(1/month)*
Reading_Rate = Francisco's_Unread_Books * Reading_Fraction
*books/month = books * (1/month)*
Reading_Fraction = 0.50 *(1/month)*
Initial number of Francisco's_Unread_Books = 20 *(books)*

A. Identify the positive and negative feedback loops in this system.

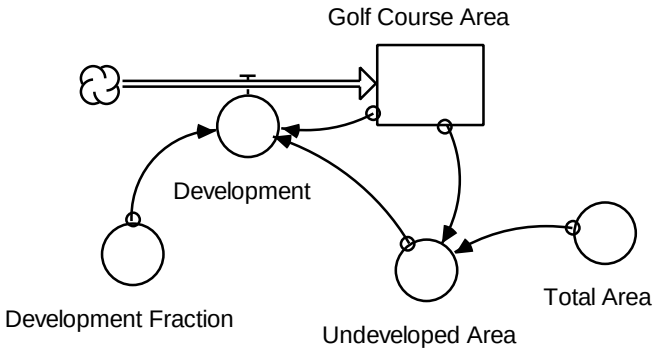
B. What is the final value of the stock?

C. Plot the general behavior of the stock on the following graph:



7.3 Exercise 3: Galt Development Company

Milton Head Island was just purchased by the Galt Development Company, which hopes to turn it into a major golf resort. The island has a total area of 1000 acres that the company would like to develop into golf courses. The company first develops two test holes on 10 acres of land to insure terrain quality. At this point, Galt Development conducts extensive market research into the growth of successful golf developments. Their statistician, fresh out of MIT and raving with the latest statistical modeling strategies, comes to the conclusion that development should be directly proportional to the product of the developed and undeveloped areas. The strategy is modeled below:



Development = Development Fraction * Undeveloped Area * Golf Course Area

*acres/month = (1/(month * acre))* acres * acres*

Development Fraction = 0.0004 *1/(month * acre)*

Undeveloped Area = Total Area - Golf Course Area

acres = acres ó acres

Total Area = 1000 *acres*

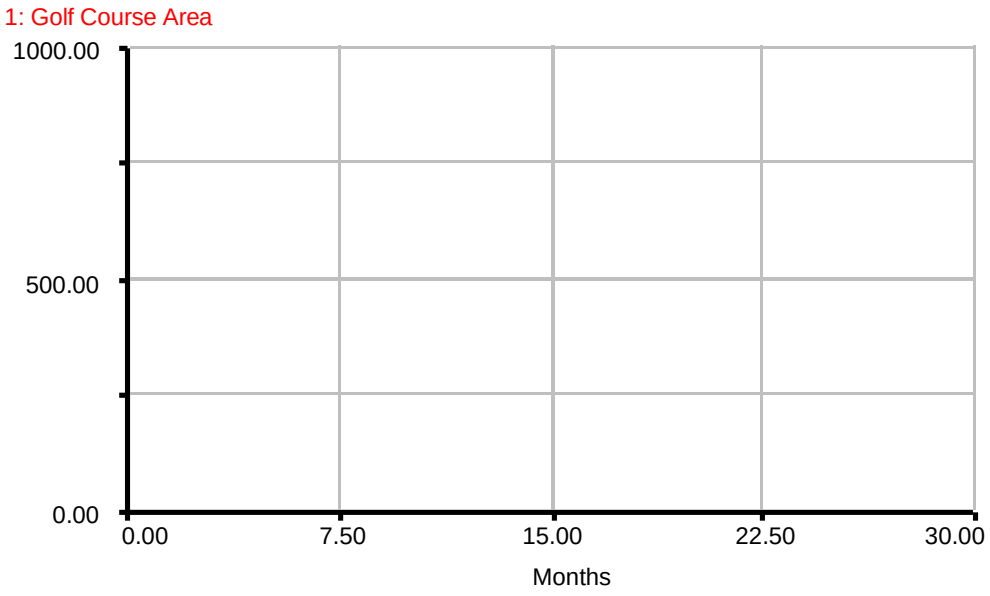
initial Golf Course Area = 10 *acres*

A. What kind of feedback behavior is contained in this system?

B. What is the final stock value?

C. What kind of behavior does the stock exhibit?

D. Plot the stock behavior on the following graph pad:



8. Solutions to Exercises

8.1 Solution to Exercise 1

A. Recognize the presence of a positive and negative feedback loop in this system. The Drinking_Rate is a positive feedback loop because the inflow of water into the runner's system increases the water in his system and thus increases the rate at which he drinks water. The Sweating_Rate is a negative feedback loop because sweating takes away water from the runner and thereby decreases his Sweating_Rate. Although these assumptions may sound unreasonable in the real world, it is the model that we are interested in understanding.

B. The important fact to notice here is that the Drinking_Fraction and the Sweating_Fraction are the same. Thus the system will start at equilibrium and never divert.

C. No. The qualitative behavior of the system (equilibrium throughout) is not dependent upon the initial value of the stock. It is caused merely by the equivalence of the inflow and the outflow.

8.2 Solution to Exercise 2

A. There are two feedback loops in this model. The feedback system of Uncle Midas's giving exhibits negative feedback because an increase in the number of Francisco's_Unread_Books will cause a decrease in Uncle Midas's New_Books rate. The Reading_Rate loop is also a negative feedback system. An increase in the Reading_Rate will decrease the number of Francisco's_Unread_Books and decrease the Reading_Rate.

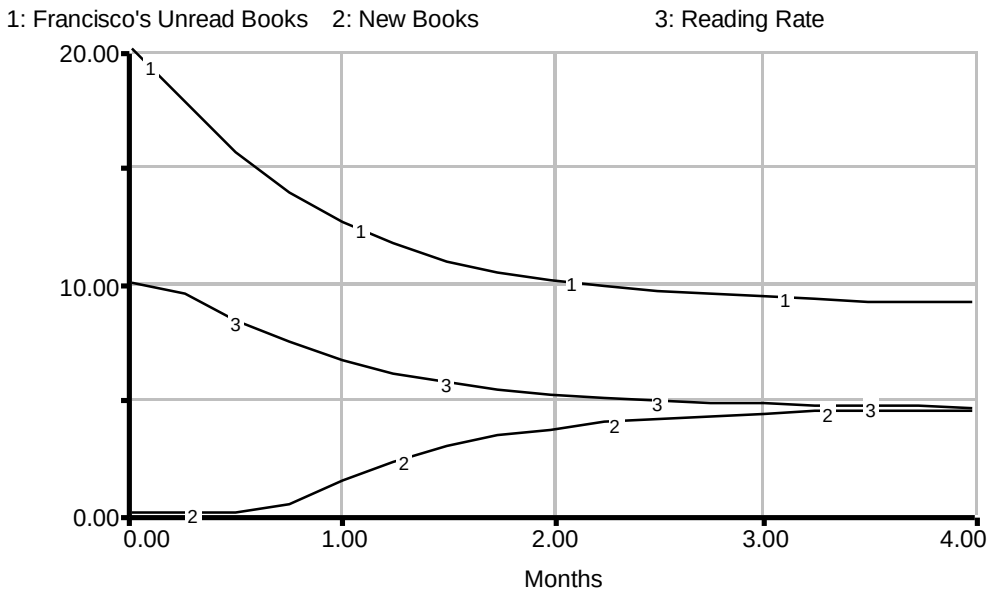
B & C. The equations given show that:

New_Books = Difference * Fraction_Located
*books/month = books *(1/month)*

Difference = Goal - Francisco's_Unread_Books
books = books - books
Goal = 15 *books*
Fraction_Located = 0.75 *(1/month)*
Reading_Rate = Francisco's_Unread_Books * Reading_Fraction
*books/month = books * (1/month)*
Reading_Fraction = 0.50 *(1/month)*
Initial number of Francisco's_Unread_Books = 20 *(books)*

In order to figure out the final value of the stock, we equate the system's flows:

Inflow = Outflow
books/month = books/month
New_Books = Reading_Rate
books/month = books/month
Difference * Fraction_Located = Reading Fraction *
Francisco's_Unread_Books
*books * (1/month) = (1/month) * books*
(Goal - Francisco's_Unread_Books) * Fraction_Located =
Reading Fraction * Francisco's_Unread_Books
(*books - books*) * *(1/month) = (1/month) * books*
(15 - Francisco's_Unread_Books) * 0.75 = 0.50 *
Francisco's_Unread_Books
(*books - books*) * *(1/month) = (1/month) * books*
(15 - Francisco's_Unread_Books) = [0.50 *
Francisco's_Unread_Books] / 0.75
(*books - books*) = [*(1/month) * (books*)] / *(1/month)*
Francisco's_Unread_Books = 15 -
[0.50 * Francisco's_Unread_Books] / 0.75
books = books — [*(1/month) * (books*)] / *(1/month)*
Francisco's_Unread_Books = -Francisco's_Unread_Books * (2/3) + 15
books = books + books
Francisco's_Unread_Books + (2/3) * Francisco's_Unread_Books = 15
books + books = books
(5/3) * Francisco's_Unread_Books = 15
books = books
Francisco's_Unread_Books = 15 * (3/5)
books = books
Francisco's_Unread_Books = 9
books = books



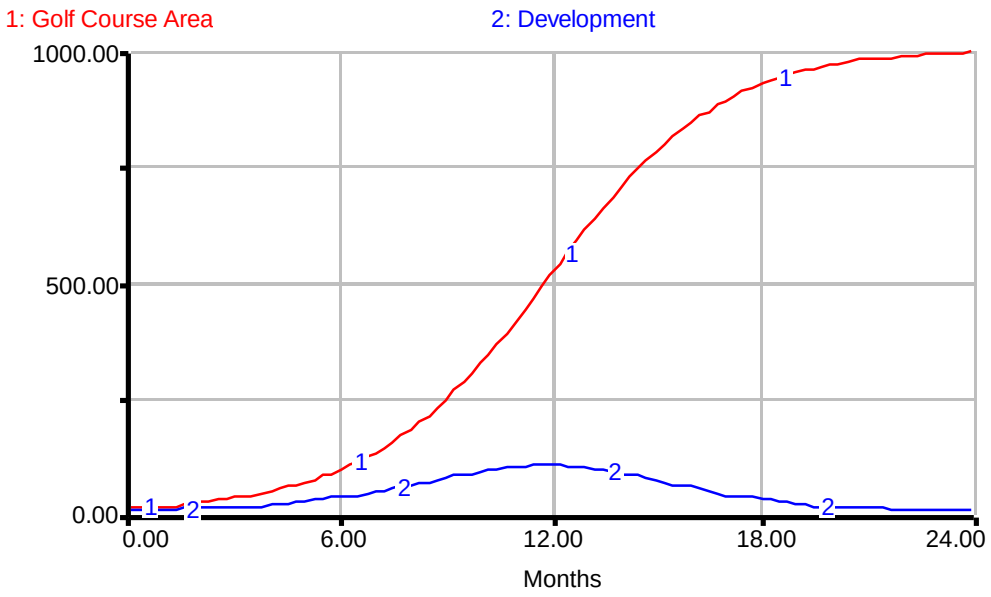
8.3 Solution to Exercise 3

A. By inspection, there are two feedback loops acting upon the system, one positive and one negative. The greater golf course area, the greater the rate of development, in turn further increasing golf course area. At the same time, stock is inversely related to the undeveloped area, which hinders the development rate.

B. When the system comes to equilibrium, undeveloped area should be 0, so all of the area should be developed. The final stock value is, therefore, 1000 acres.

C. In order to figure out what kind of behavior the stock exhibits, we need to figure out which feedback loop dominates first. In this case, at time = 0, the positive feedback loop dominates. Goal oriented behavior does not dominate until developed area becomes larger than the undeveloped area. Since the system is initially dominated by positive feedback but the dominance shifts to negative feedback, the system exhibits S-shaped behavior.

D.


1: Golf Course Area
2: Development
1000.00
500.00
0.00
0.00
6.00
12.00
18.00
24.00
Months
1
2

9. Appendix: Exponentiation

In Exercise 3, it is necessary to perform operations on the exponents so that Boat Speed may be solved for explicitly. The step is shown below:

$$(\text{Boat Speed} / 20)^{1.8} = \text{Force} / 500$$
$$\text{Boat Speed} / 20 = (\text{Force} / 500)^{5/9}$$

The change of exponents is an operation which may be performed on both sides of an equation just as one would do with addition or multiplication. In this case, the left side of the equation is raised to the 1.8 power. Since 1.8 is the equal to 9/5, it may also be said that the left side is raised to the 9/5 power. In order to solve for a variable, such as Boat Speed, explicitly, it is necessary to have the variable raised only to the first power. (Any variable raised to the first power is the variable itself.) To change $(\text{Boat Speed} / 20)^{9/5}$, one may raise that quantity to another power. In this case raising it to the 5/9 power is needed. Since raising one power to another power is equivalent to multiplying the two powers, and (9/5)*(5/9) is 1, the result is $(\text{Boat Speed} / 20)^{1}$ or just Boat Speed / 20. Since everything done to one side of the equation must also be done to the other side, the right hand side is raised to the 1*5/9 or 5/9 power.

Vensim Examples:
Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedback in First-Order Systems

By Lei Lei & Nathaniel Choge
October 2001

Example 1: Identical Constant Fractions

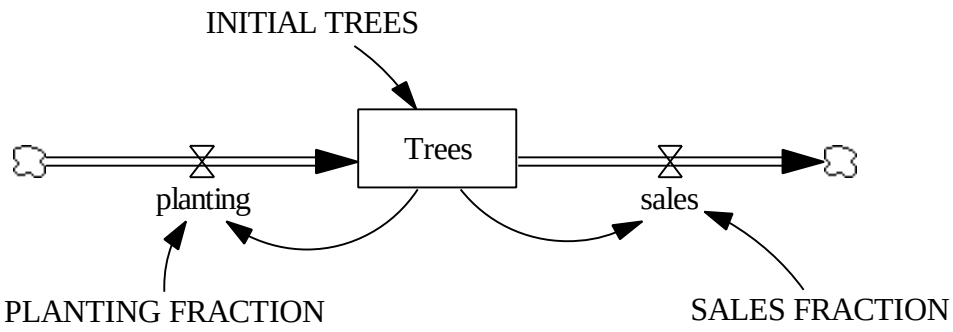


Figure 9: Vensim Equivalent of Figure 1: Eddie's Past Business Strategy

Documentation for Identical Constant Fractions Model

(01) FINAL TIME = 20
Units: year
The final time for the simulation.

(02) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(03) INITIAL TREES = 500
Units: trees

(04) planting = Trees* PLANTING FRACTION
Units: trees/year

(05) PLANTING FRACTION = 0.08
Units: 1/year

(06) sales = Trees* SALES FRACTION
Units: trees/year

(07) SALES FRACTION = 0.08
Units: 1/year

(08) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(09) TIME STEP = 0.125
Units: year
The time step for the simulation.

(10) Trees = INTEG (planting-sales, INITIAL TREES)
Units: trees

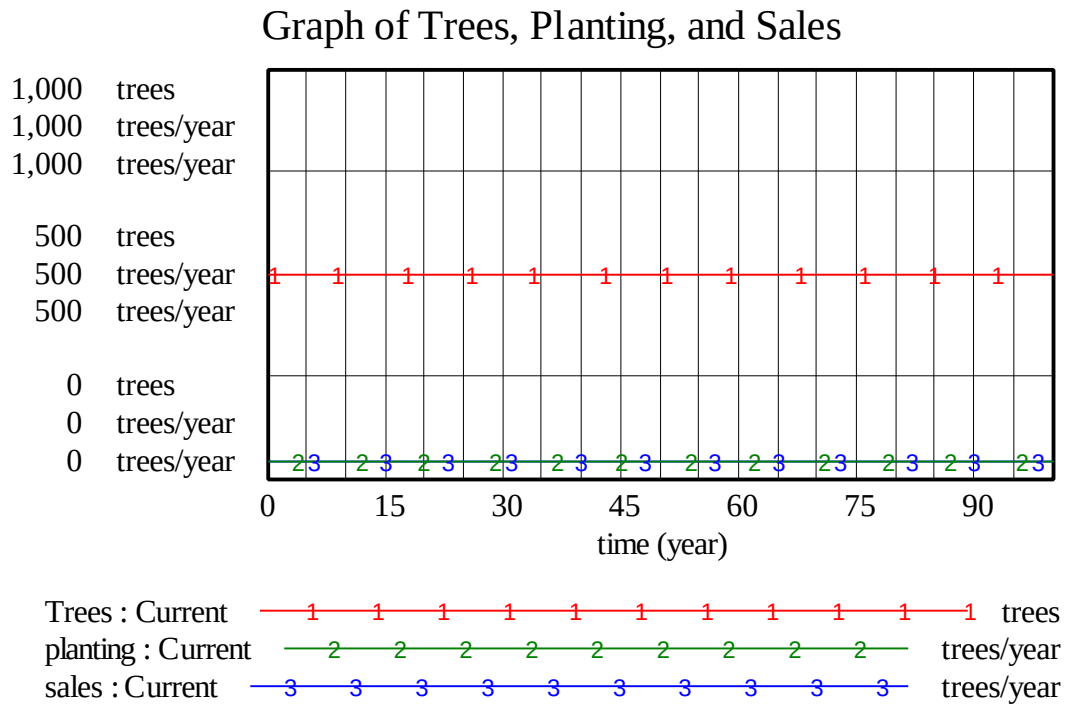


Figure 10: Vensim Equivalent of Figure 2: Eddie's Projected Inventory

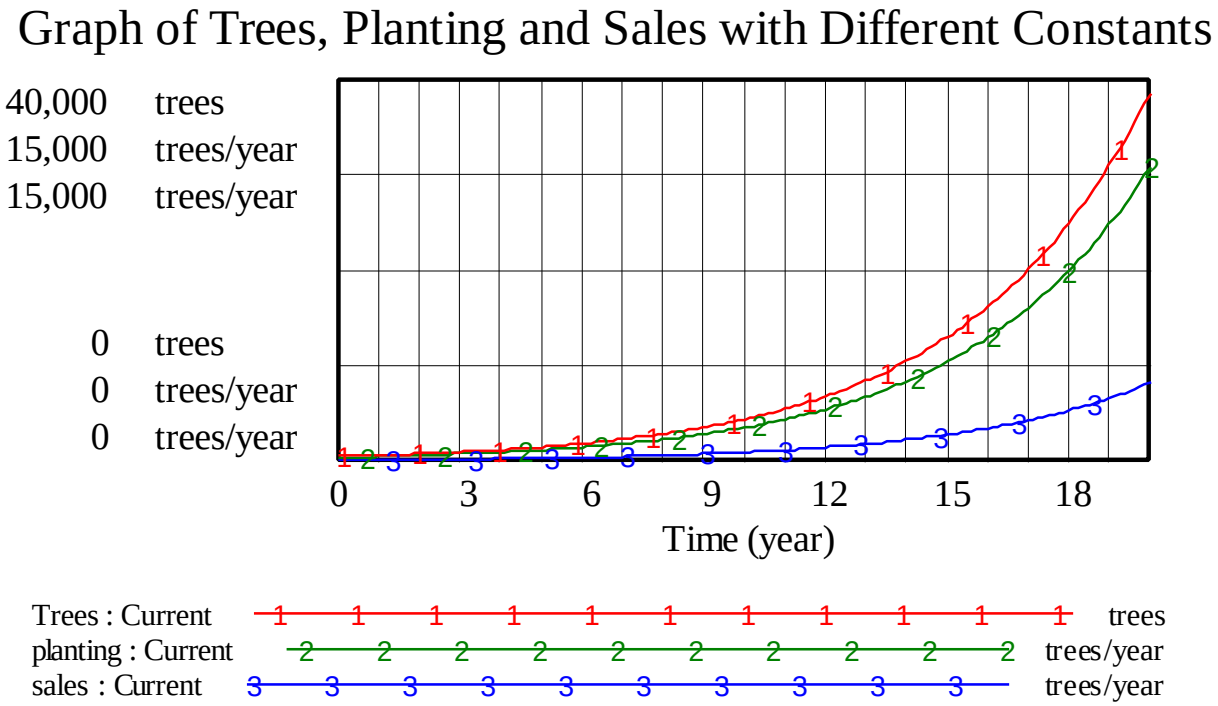


Figure 12: Vensim Equivalent of Figure 3: Model Behavior with increased PLANTING FRACTION

Example 3: Linear Feedback

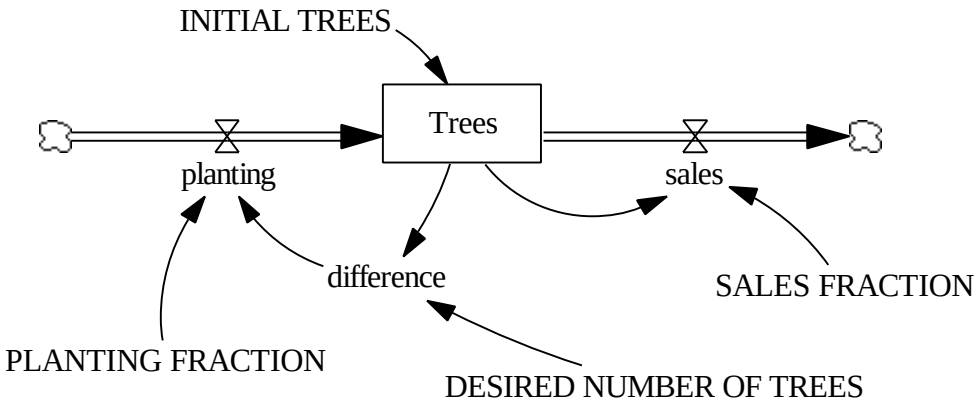


Figure 13: Vensim Equivalent of Figure 4: Eddie's Planting Regulation Strategy

Documentation of Nonlinear Feedback Model

(01) DESIRED NUMBER OF TREES = 5800
Units: trees

(02) difference = DESIRED NUMBER OF TREES-Trees
Units: trees

(03) FINAL TIME = 20
Units: year
The final time for the simulation.

(04) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(05) INITIAL TREES = 0
Units: trees

(06) planting = difference* PLANTING FRACTION
Units: trees/year

(07) PLANTING FRACTION = 0.3
Units: 1/year

(08) sales = Trees* SALES FRACTION
Units: trees/year

(09) SALES FRACTION = 0.08
Units: 1/year

(10) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(11) TIME STEP = 0.125
Units: year
The time step for the simulation.

(12) Trees = INTEG (planting-sales, INITIAL TREES)
Units: trees

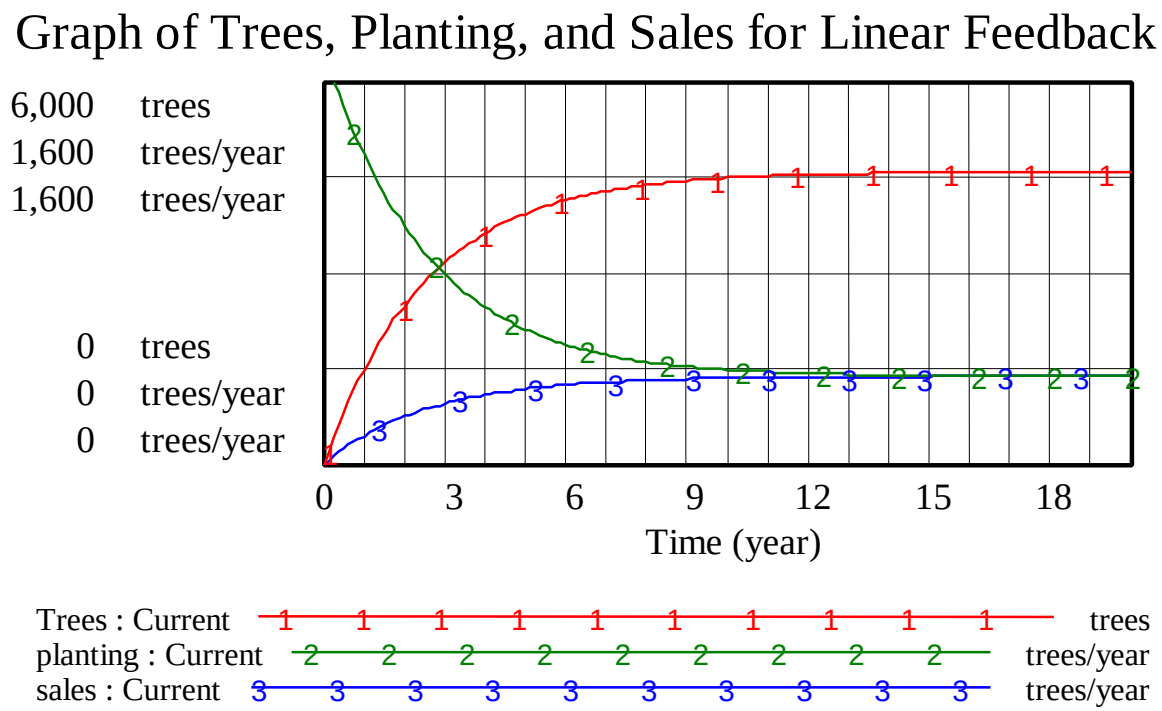


Figure 14: Vensim Equivalent of Figure 5: Stock and Flow Behavior of Eddie's Planting Regulation Strategy

Example 4: Nonlinear Feedback with Graph Function

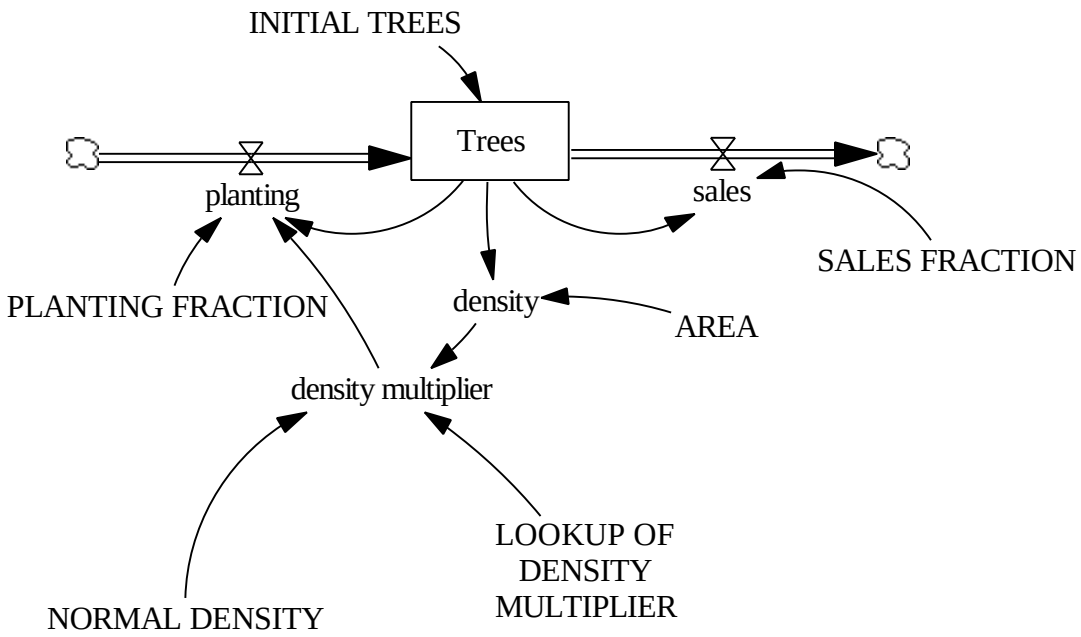


Figure 15: Vensim Equivalent of Figure 6: Model Regulating Tree Planting Rate

Documentation for Nonlinear Feedback with Graph Function Model

(01) AREA = 6000
Units: yard* yard

(02) density = Trees/AREA
Units: trees/(yard*yard)

(03) density multiplier = LOOKUP OF DENSITY MULTIPLIER(density/NORMAL DENSITY)
Units: dmnl
This is the equation for the table function.

(04) FINAL TIME = 20
Units: year
The final time for the simulation.

(05) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(06) INITIAL TREES = 500
Units: trees

(07) LOOKUP OF DENSITY MULTIPLIER
[(0,0)-(20,1)],(0,1),(1,1),(1.44,0.995),(2.88,0.985),(4.32,0.99),
(5.76,0.97),(7.2,0.94),(8.64,0.86),(10.08,0.66),(11.52,0.267),
(12.97,0.01),(14.41,0)
Units: dmnl

(08) NORMAL DENSITY = 0.0833
Units: trees/(yard* yard)
This is the initial, or normal density of trees. It is taken by dividing the intial number of Trees, 500 trees, by the AREA, 6000 square yards.

(09) planting = Trees* density multiplier* PLANTING FRACTION
Units: trees/year

(10) PLANTING FRACTION = 0.3
Units: 1/year

(11) sales = Trees* SALES FRACTION
Units: trees/year

(12) SALES FRACTION = 0.08
Units: 1/year

(13) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(14) TIME STEP = 0.125
Units: year
The time step for the simulation.

(15) Trees = INTEG (planting-sales, INITIAL TREES)
Units: trees

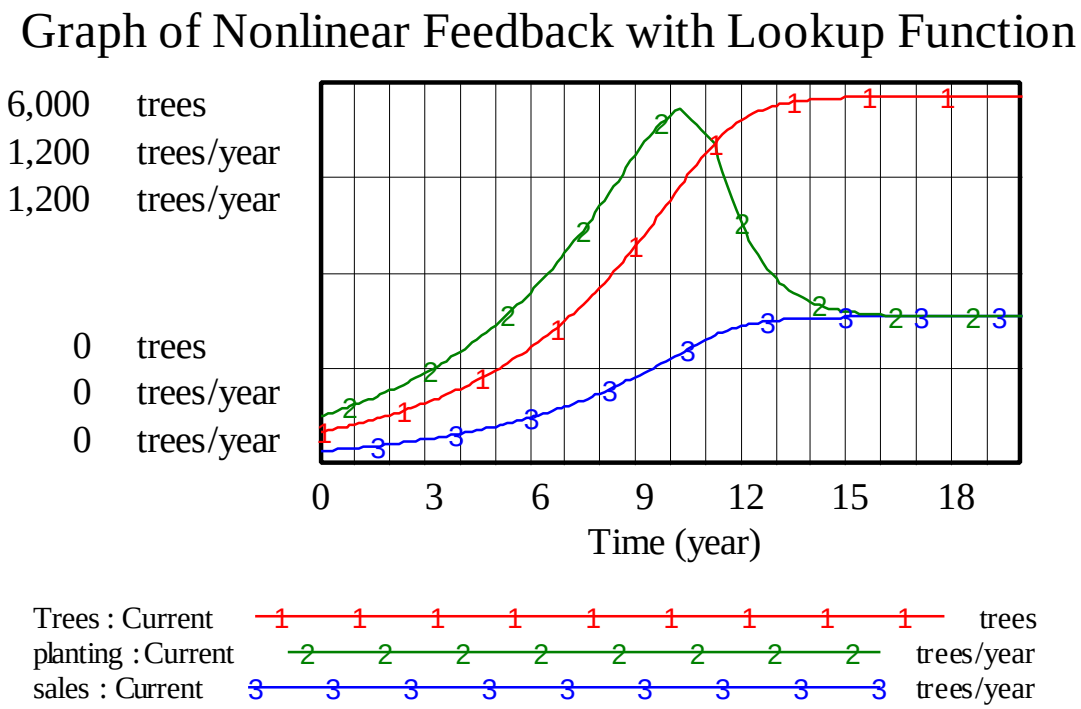


Figure 16: Vensim Equivalent of Figure 8: Behavior of Non-Linear Regulation of Planting

Exercise 2: Unread Books

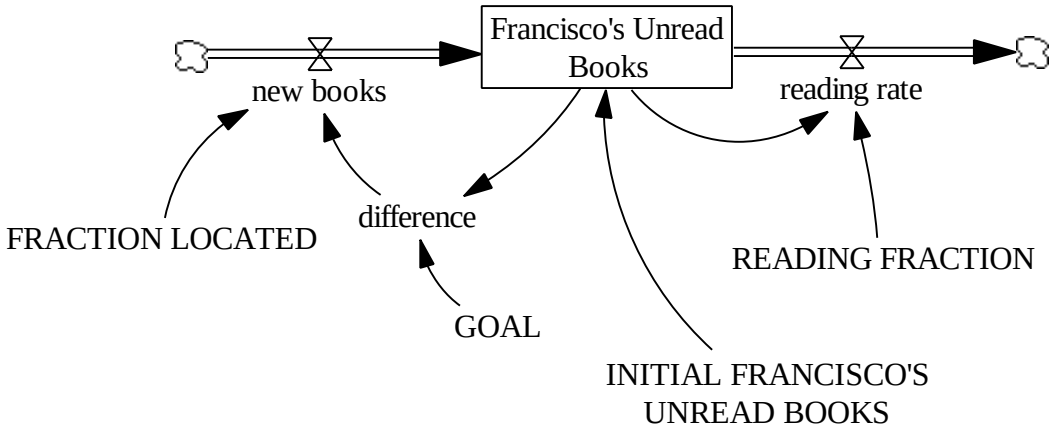


Figure 17: Vensim Equivalent of Diagram of Francisco's Unread Books

Documentation for Unread Books Model

(01) difference = GOAL-Francisco's Unread Books
Units: books

(02) FINAL TIME = 4
Units: Month
The final time for the simulation.

(03) FRACTION LOCATED = 0.75
Units: 1/Month

(04) Francisco's Unread Books = INTEG (new books-reading rate,
INITIAL FRANCISCO'S UNREAD BOOKS)
Units: books

(05) GOAL = 15
Units: books

(06) INITIAL FRANCISCO'S UNREAD BOOKS = 20
Units: books

(07) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(08) new books = difference* FRACTION LOCATED

Units: books/Month

(09) READING FRACTION = 0.5
Units: 1/Month

(10) reading rate = Francisco's Unread Books* READING FRACTION
Units: books/Month

(11) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(12) TIME STEP = 0.125
Units: Month
The time step for the simulation.

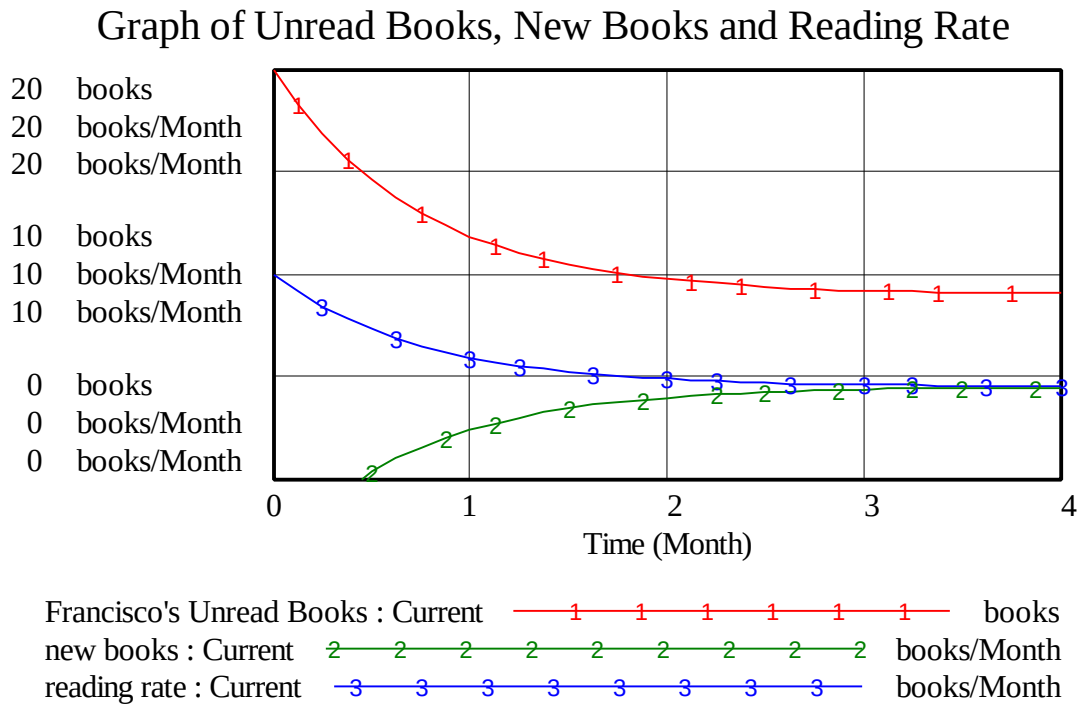


Figure 18: Vensim Equivalent of Simulation of Book Model

Exercise 3: Galt Development Company

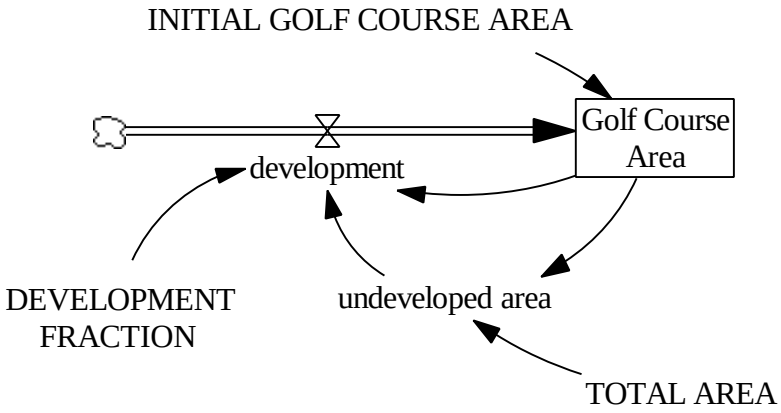


Figure 19: Vensim Equivalent of Golf Course Development Model

Documentation for Galt Development Company Model

(01) development = DEVELOPMENT FRACTION* undeveloped area* Golf Course Area
Units: acres/Month

(02) DEVELOPMENT FRACTION = 0.0004
Units: 1/(Month* acres)

(03) FINAL TIME = 24
Units: Month
The final time for the simulation.

(04) Golf Course Area = INTEG (development, INITIAL GOLF COURSE AREA)
Units: acres

(05) INITIAL GOLF COURSE AREA = 10
Units: acres

(06) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(07) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(08) TIME STEP = 0.0625

Units: Month
The time step for the simulation.

(09) TOTAL AREA = 1000
Units: acres

(10) undeveloped area = TOTAL AREA-Golf Course Area
Units: acres

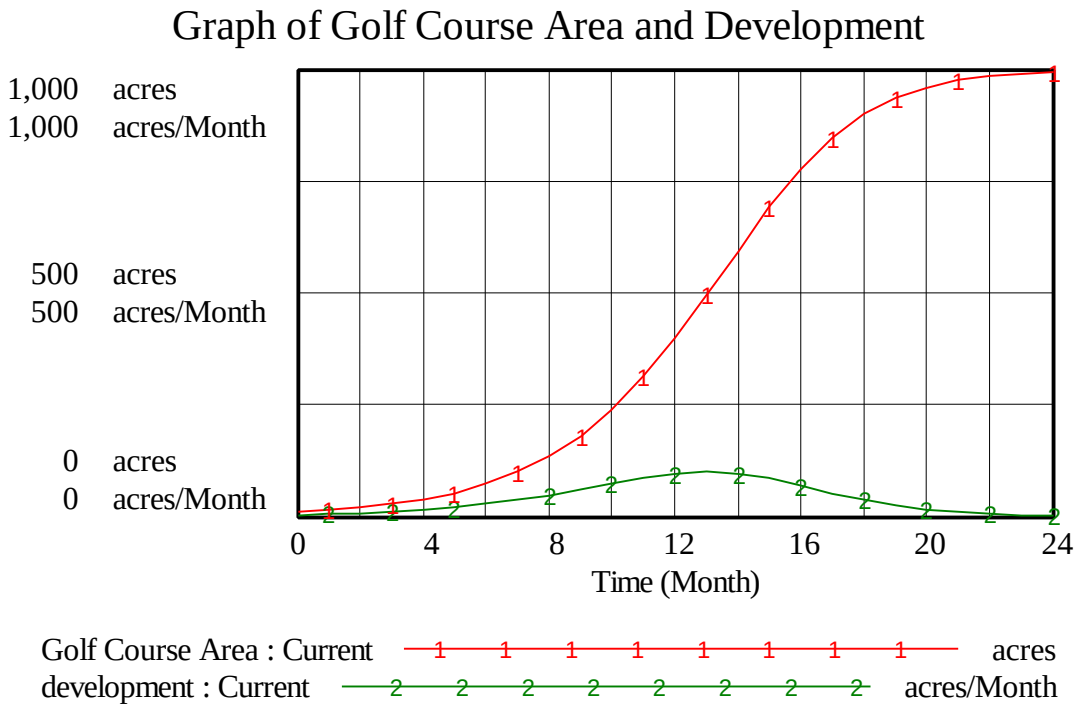


Figure 20: Vensim Equivalent of Simulation of Golf Course Area and development

GENERIC STRUCTURES: S-SHAPED GROWTH I

Produced for the
MIT System Dynamics in Education Project

Under the Supervision of
Dr. Jay W. Forrester
Sloan School of Management
Massachusetts Institute of Technology

BY

Marc Glick
Terri Duhon
August 24, 1994
Vensim Examples added October 2001

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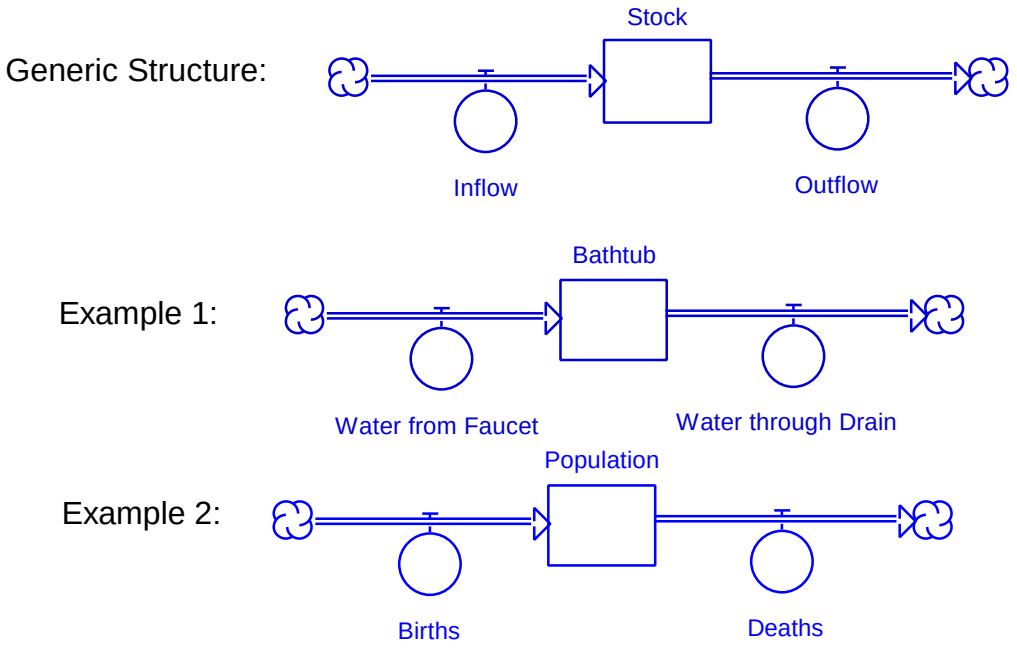
ABSTRACT

Generic Structures: S-Shaped Growth I is an introduction to the concept of generic structures, along with specific examples of such a structure. This paper assumes knowledge of STELLA[1] software, and simple system dynamics structures such as positive and negative feedback loops, exponential growth, and S-shaped growth.

Generic Structures: S-Shaped Growth I will examine two different structures that generate S-shaped growth. We will look at the multiple behaviors that each structure generates, and the simulations of each structure that produce the same behaviors. Then causal loop diagrams will be utilized to explain similarities between the structures.

INTRODUCTION

Generic structures, also known as transferable structures are structures that can be found in different situations. Generic structures facilitate learning by allowing one to transfer knowledge of one system to another. Figure 1 shows how a simple generic structure can be used to model different systems, simply by altering the names and parameter values depending on the specific system being modeled.



[1] STELLA is a registered trademark of High Performance Systems, Inc..

Figure 1 - Model of a generic structure followed by two specific models based on that structure.

Once a person understands a generic structure, that person's knowledge of other similar systems is expanded.

"A generic structure that many people are familiar with is S-shaped growth." The previous sentence is a popular misconception. S-shaped growth is commonly classified as a generic structure. However, it is not a generic structure, but rather a behavior. A structure generates a behavior, but a behavior is not a structure. They are not synonymous. Several different structures can exhibit the same behavior. Also, several different behaviors can be generated by the same structure. This can be achieved by changing either the initial values of the stock, or the parameter values. Despite the lack of direct relation between behavior and structure, the different behaviors a structure can generate are related. Additionally, different structures that exhibit similar behaviors must also be related. This is the focus of *Generic Structures: S-Shaped Growth I.*

S-SHAPED GROWTH

S-shaped growth is also called sigmoidal growth. Figure 2 shows a general example of S-shaped growth. The curve begins at less than the equilibrium amount. Initially, exponential growth is the dominant behavior of the curve. Then, at the inflection point[2], the curve begins an asymptotic approach to an equilibrium. Examples of systems that exhibit this behavior are the growth of rumors, sale of new products, epidemics, and populations with limited resources.

[2] The Inflection Point is merely the point where the transfer from exponential to asymptotic behavior occurs.

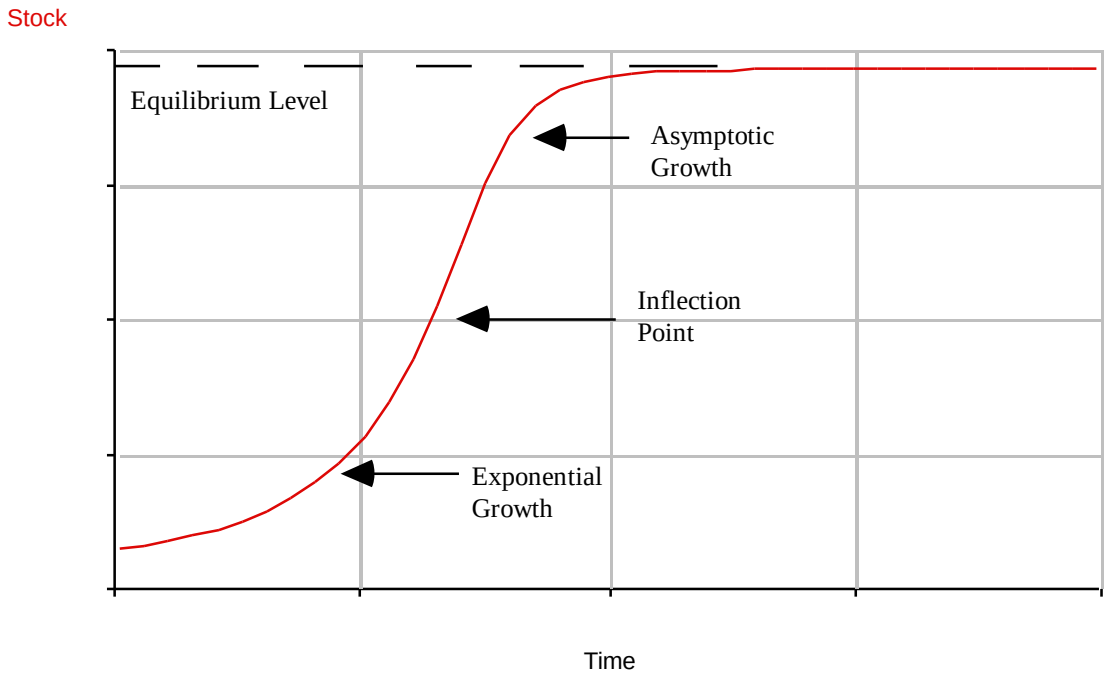


Figure 2 - STELLA generated graph of generic S-shaped growth.

The examples listed above each exhibit S-shaped growth under certain conditions, yet they cannot all be modeled by the same structure. This paper examines two distinctly different structures that produce S-shaped growth. These structures are pictured in Figures 3 and 4.

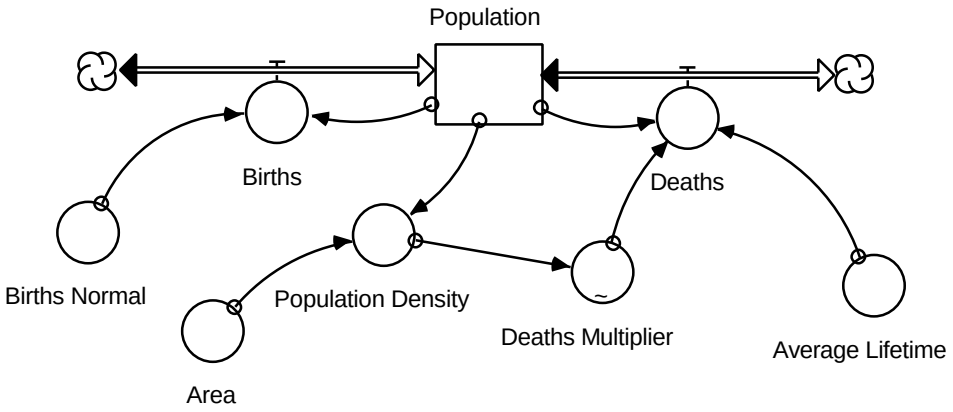


Figure 3 - S-Shaped Growth Structure 1

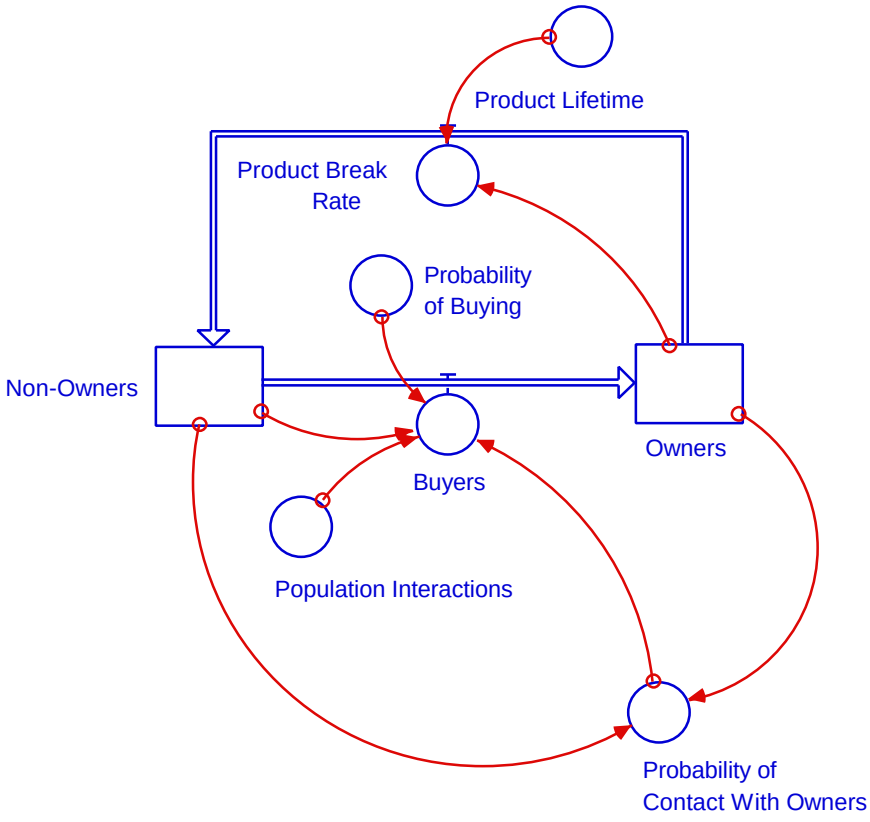


Figure 4 - S-Shaped Growth Structure 2

S-SHAPED GROWTH STRUCTURE 1

The first structure can be used to model growth with a limiting factor. Two examples of systems governed by this structure are the growth of rabbits in a limited area (Figure 5a) and the construction of buildings in a limited area (Figure 5b).

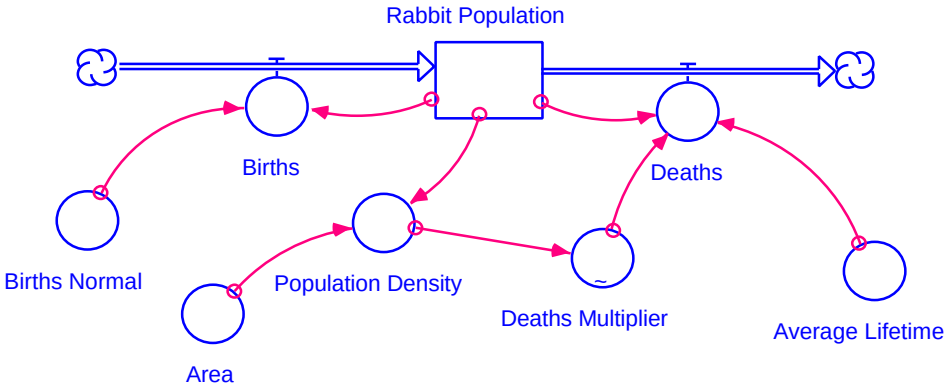


Figure 5a - Model of rabbit population growth with limited area.

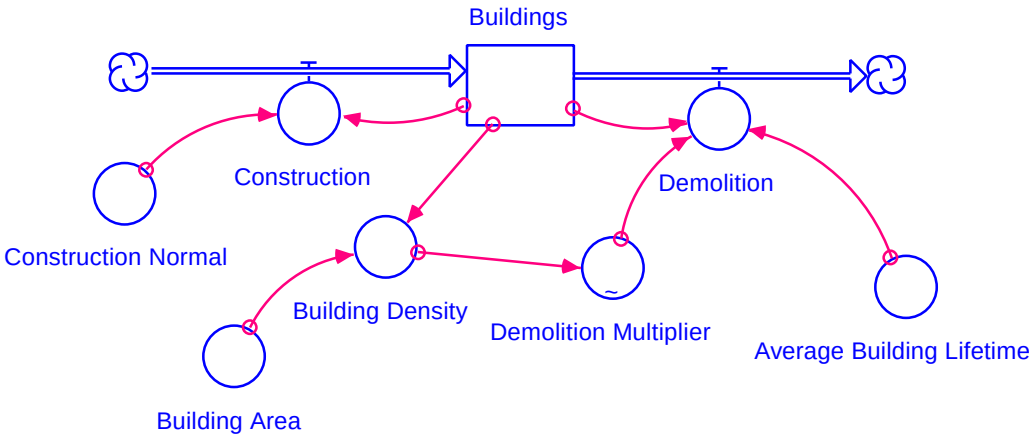


Figure 5b - Model of building construction with limited area.

In these examples, initially the positive feedback loop of either births or construction is dominant, causing the stock to grow exponentially. As the stock increases, the density also increases. At the inflection point, the negative feedback loop of death or demolition becomes more dominant than the positive feedback loop, and the stock begins increasing by a smaller amount each month. When the density reaches its maximum value, the area can no longer sustain a larger stock. Therefore the inflow must be equal to the outflow.

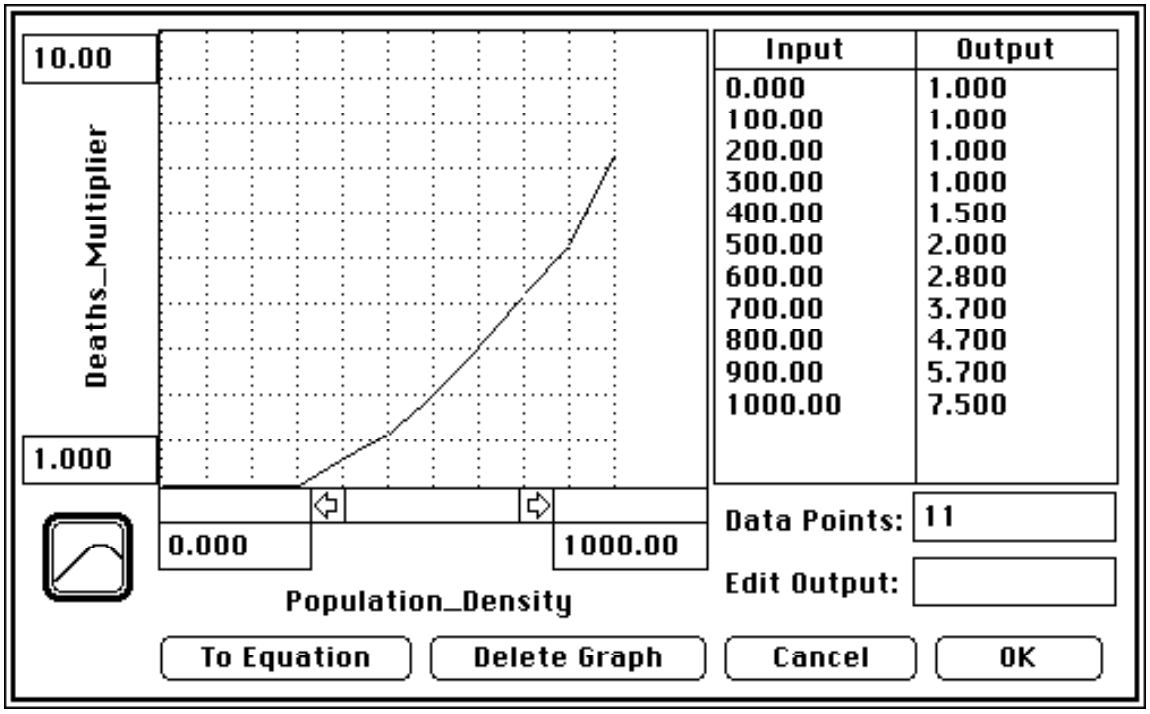
Other behaviors this model can generate are exponential growth to infinity or exponential decay to zero. Exponential growth to infinity will occur if the parameters are set at values that cause the birth rate to always exceeds the death rate. Exponential decay to zero will occur if the parameters are set at values that cause the death rate to always exceeds the birth rate. For a further explanation of this see Appendix C.

EXERCISE 1: S-SHAPED GROWTH STRUCTURE 1

S-shaped growth structure 1 (Figure 5a) produces three distinct behaviors depending on the initial value of the stock.

The model is defined as follows.

Births = Rabbit Population * Births Normal
Deaths = (Rabbit Population/Average Lifetime) * Deaths Multiplier
Area = 1 Acre
Births Normal = 1.5/Year
Average Lifetime = 4 Years
Density = Rabbit Population/Area

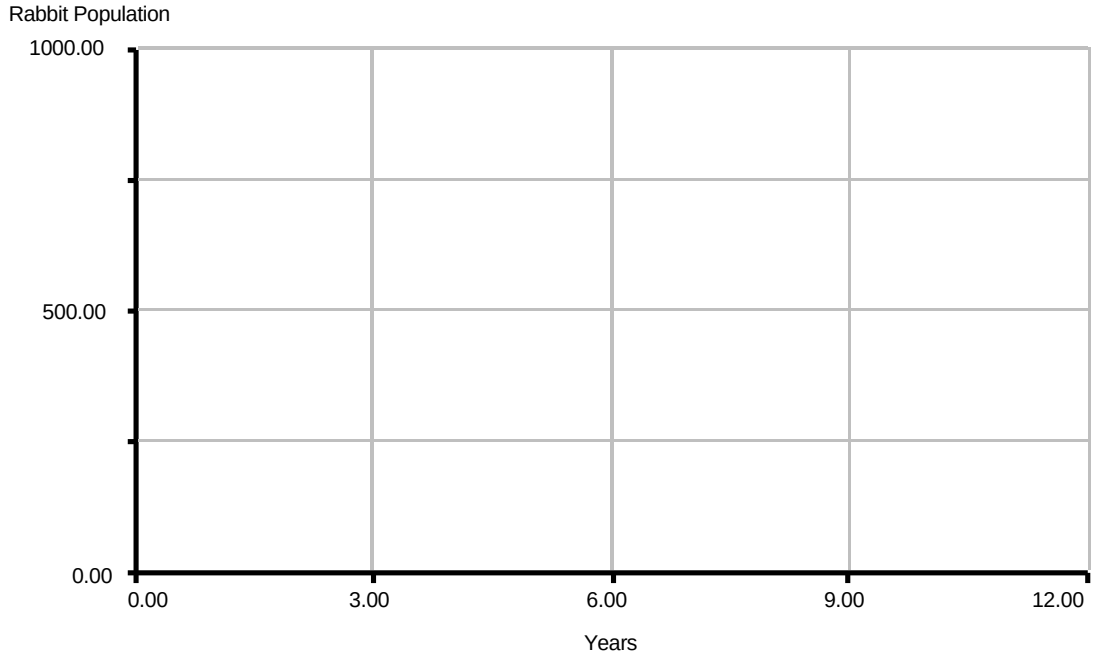


Exercise 1

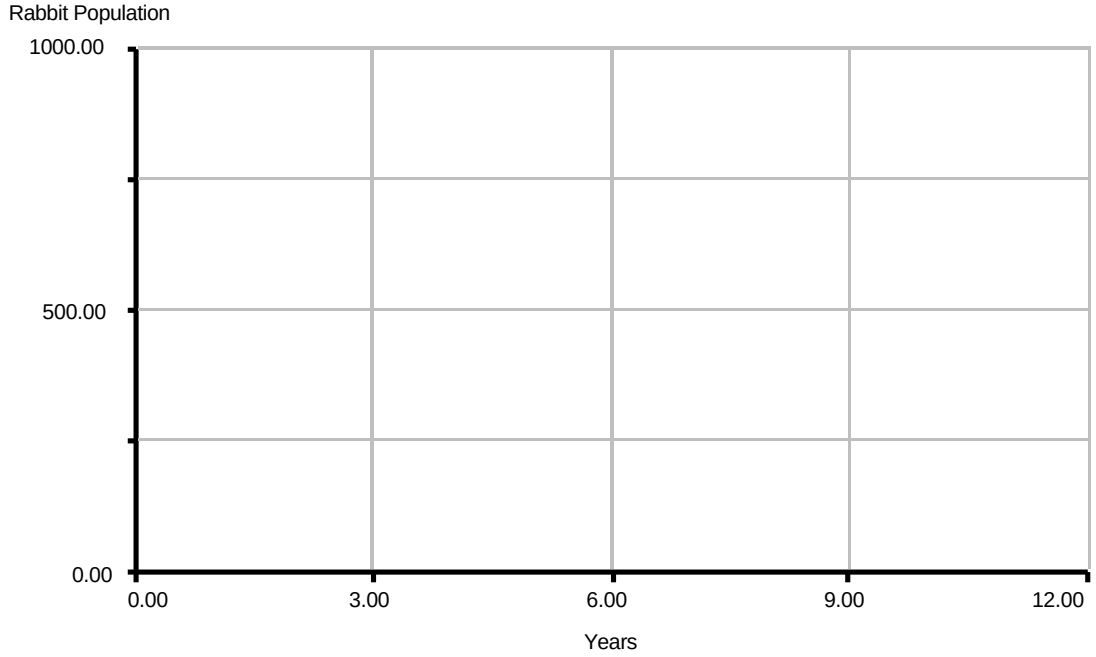
Based on the above information

a. Graph Population on graph A for initial Population equal to 2.
b. Graph Population on graph B for initial Population equal to 0.
c. Graph Population on graph C for initial Population equal to 990.

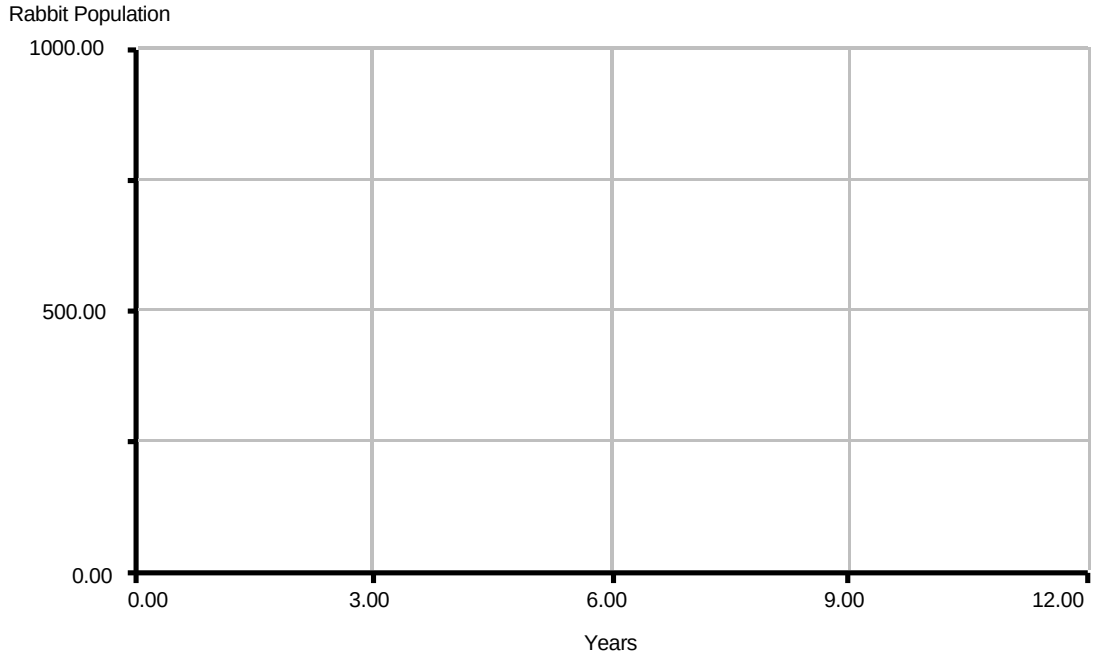
Graph A



Graph B



Graph C



S-SHAPED GROWTH STRUCTURE 2

The second S-shaped growth structure and its variations are derived from systems involving rumors, new products, and epidemics. Two examples of systems governed by this structure are epidemics (Figure 7a) and a new product life cycle (Figure 7b).

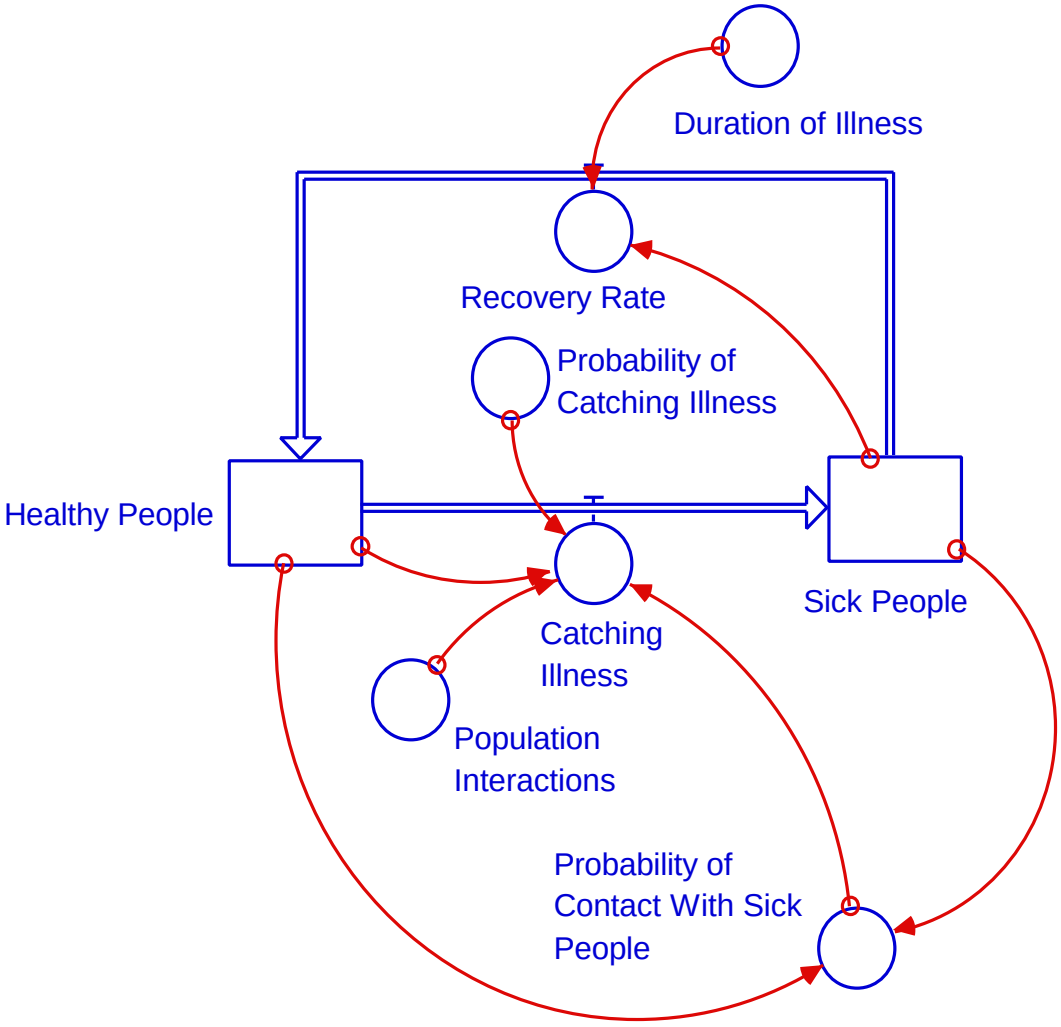


Figure 7a - Epidemic

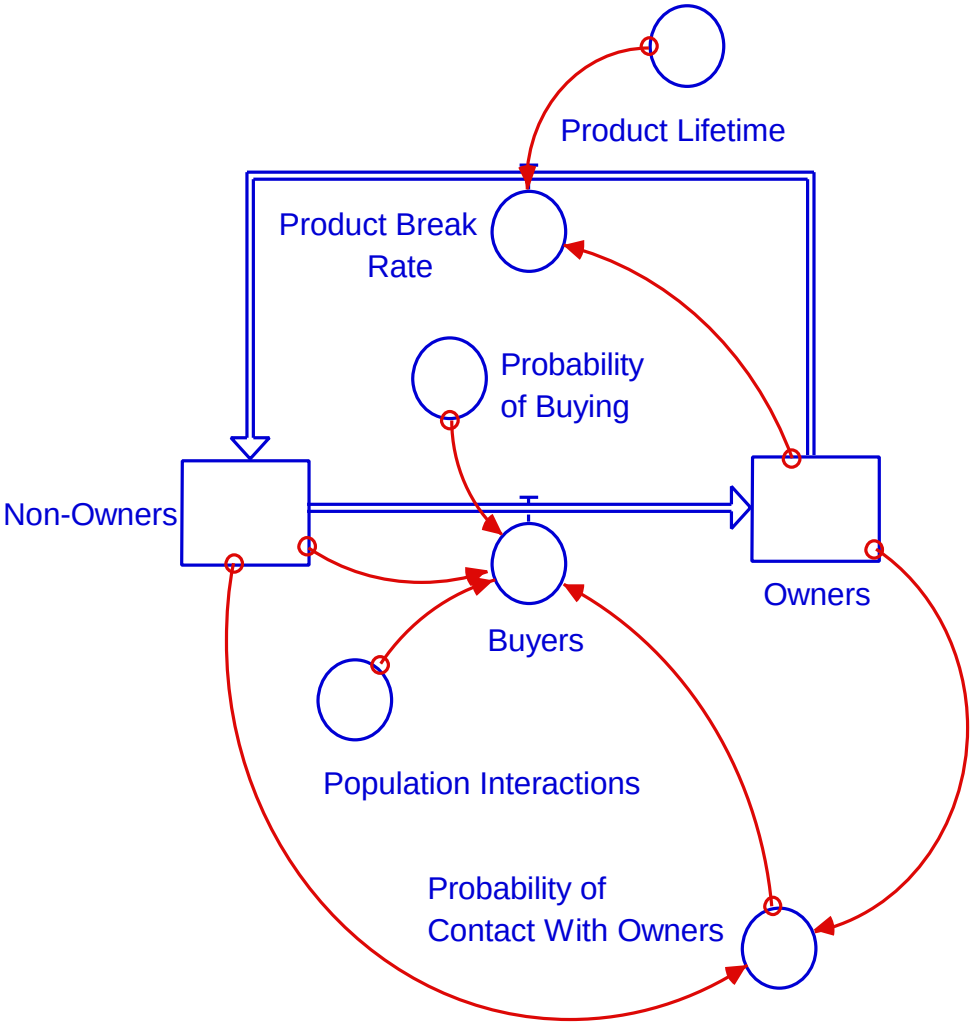


Figure 7b - New Product Life Cycle

This model is more complicated than s-shaped growth structure 1. To make the model easier to understand, a step by step explanation of the model in Figure 7a will be provided. The same reasoning will apply to an explanation of the model in Figure 7b. Figure 7a has two stocks, healthy people and sick people. The probability of a healthy person coming into contact with a sick person is proportional to the total number of sick people in the population. Therefore, the more sick people in the population the greater the chance of coming into contact with one. There is also a probability of catching the illness when you are in contact with a sick person. This is a constant value. Another value that affects whether or not a healthy person becomes sick is the total number of interactions or contacts any one person has with other people in the population per time unit. The rate at which healthy people become sick is therefore based on the probability of catching the illness when contact is made, the number of interactions each person has

with others in the population, and the probability that an interaction is with a sick person. Another factor that affects the number of sick people and healthy people is the rate at which sick people recover from their illness and re-enter the stock of healthy people, assuming no permanent immunity.

Altering the parameter values in this structure will only affect the behavior of the model by changing the time scale over which the various behaviors occur. For a further explanation of this see Appendix C.

EXERCISE 2: S-SHAPED GROWTH STRUCTURE 2

The structures in Figure 7 can produces three distinct behaviors which depend on the initial values of the stocks. We will examine these three behaviors using the epidemic model shown in Figure 7a.

The model is defined as follows:

Recovering Patients = Sick People/Duration of Illness

Catching Illness = Healthy People * Probability of Contact With Sick People *
Population Interactions * Probability of Catching Illness
Population Interactions = 10/Months
Duration of Illness = .5 Months
Probability of Catching Illness = .5
Probability of Contact With Sick People = Sick People/(Sick People + Healthy People)

Exercise 2

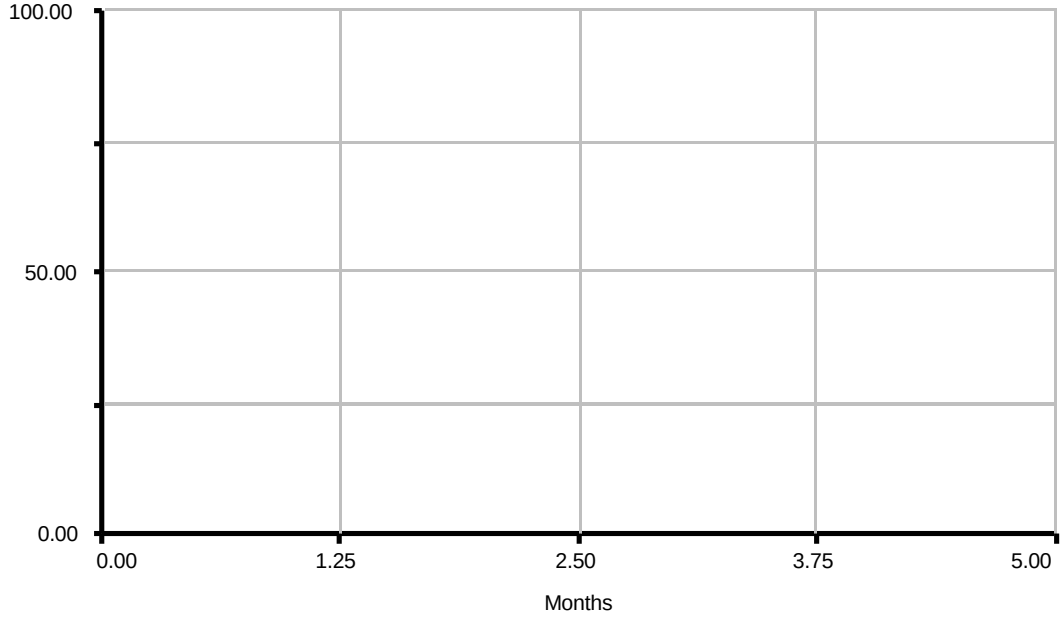
Based on the above information

a. Graph Sick People and Healthy People on graph A for initial Sick People = 10 and Healthy People =90.
b. Graph Sick People and Healthy People on graph B for initial Sick People = 0 and Healthy People =100.
c. Graph Sick People and Healthy People on graph C for initial Sick People = 55 and Healthy People =45.
d. Graph Sick People and Healthy People on graph D for initial Sick People = 50 and

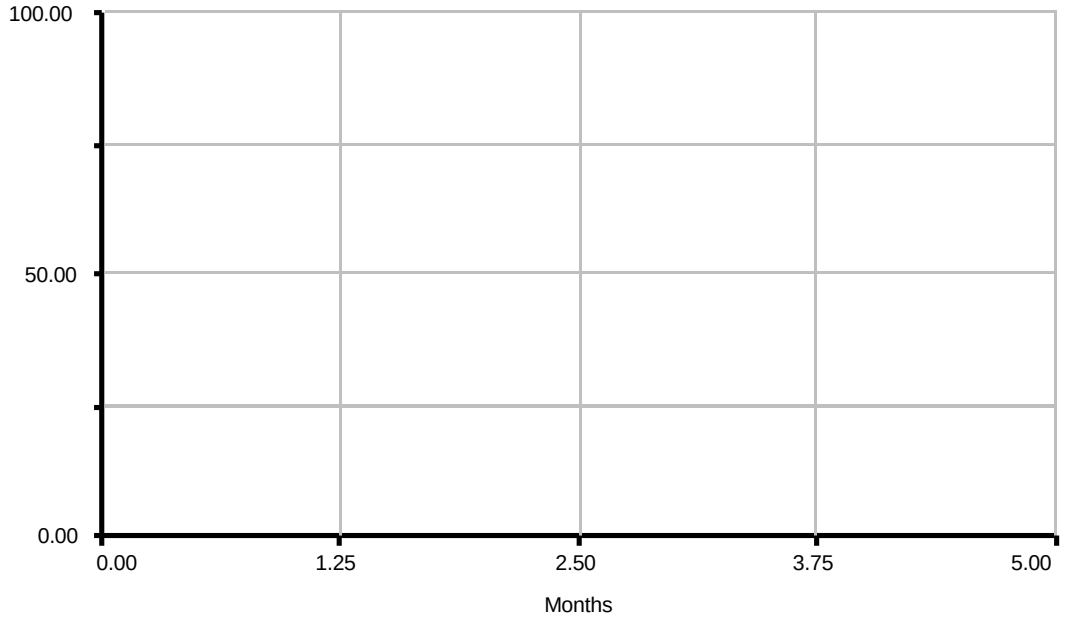
Healthy People = 50.

e. Graph Sick People and Healthy People on graph E for initial Sick People = 10 and Healthy People = 90, and Recovering Patients now set equal to a constant of zero. What type of illness is this a graph of?

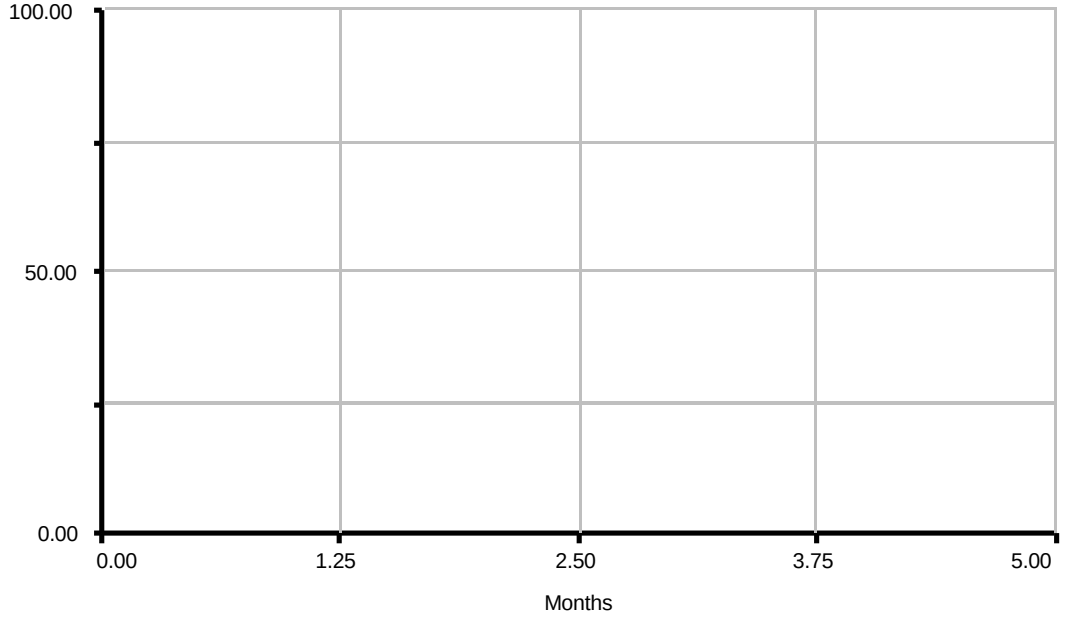
Graph A



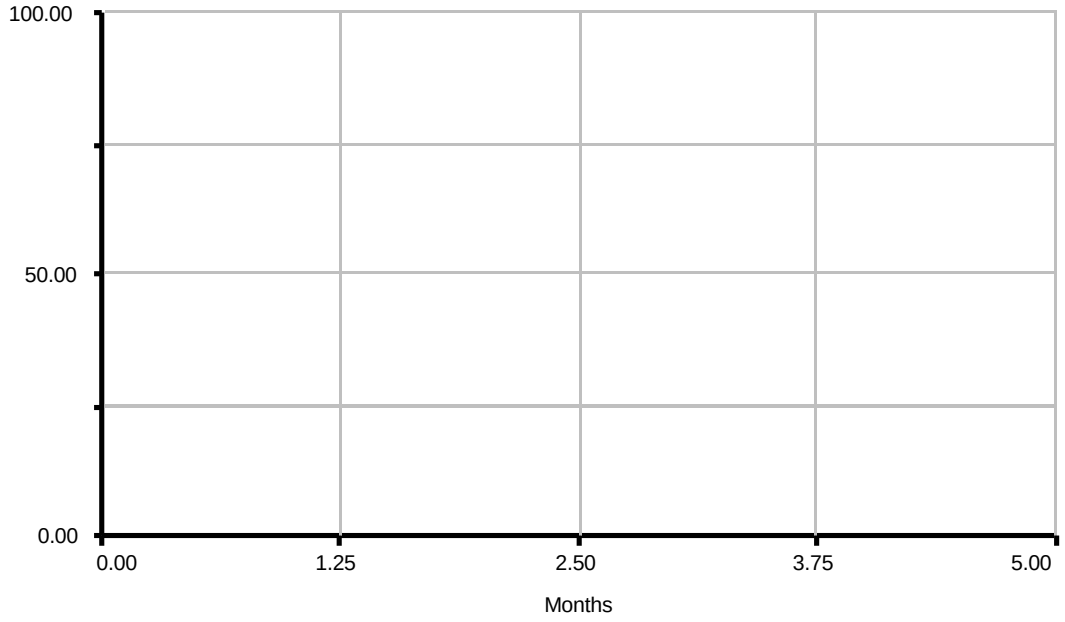
Graph B



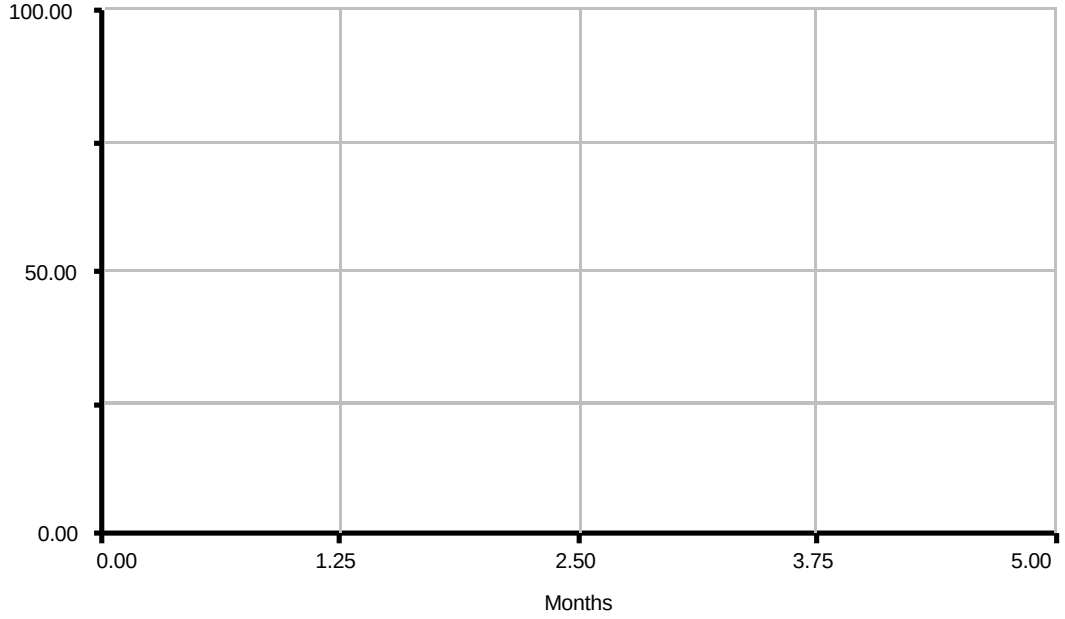
Graph C



Graph D



Graph E



STRUCTURE SIMILARITIES

Although these two structures seem very different, they are capable of producing the same behavior. We will now examine the underlying similarities of the structures.

Each has an initial dominant positive feedback loop, then a dominant negative feedback loop that creates a dynamic equilibrium. The causal loop diagrams in Figure 9 demonstrate these similarities.

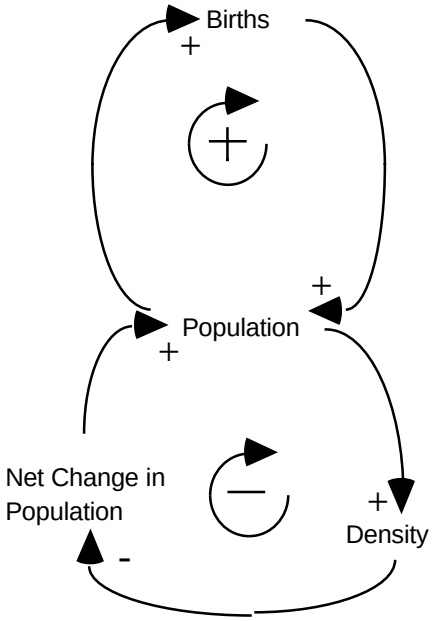


Figure 9a - Generic Structure 1 Causal Loop Diagram

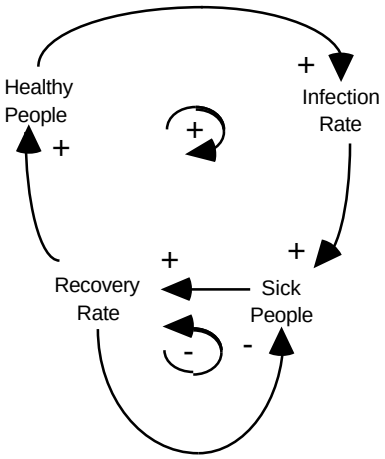


Figure 9b - Generic Structure 2 Causal Loop Diagram

The causal loop diagrams (Figures 9a and 9b) of the two generic structures are similar. This explains why both structures generate S-shaped growth behavior. This also exemplifies a problem with causal loop diagrams to which many people fall victim: Causal loop diagrams do not give an accurate representation of the structure of a system.

Causal loop diagrams are useful when describing the necessary behavior of a system, but fail when it comes to defining structure.

CONCLUSION

Generic Structures: S-Shaped Growth I provides a specific case study of generic structures as it applies to S-shaped growth behavior. It both gives examples of, and explains two generic structures for S-shaped growth. Two different structures can have similar causal loop diagrams. Since causal diagrams can be used to predict behavior, this explains why it is possible for two different structures to exhibit similar behaviors. In addition, the multiple behavior modes of each structure were examined, showing that while structure and behavior are related, they are not exclusive of each other. One structure can produce many behaviors, depending on initial conditions, and one behavior can be produced by different structures.

APPENDIX A - Documentation

S-SHAPED GROWTH STRUCTURE 1 (From Figure 5a)

Rabbit_Population(t) = Rabbit_Population(t - dt) + (Births - Deaths) * dt
INIT Rabbit_Population = 1
DOCUMENT: Units = Rabbits
The number of rabbits in the rabbit population.

INFLOWS:
Births = Rabbit_Population * Births_Normal
DOCUMENT: Units = Rabbits/Year
The number of rabbits being born each year.

OUTFLOWS:
Deaths = (Rabbit_Population/Average_Lifetime) * Deaths_Multiplier
DOCUMENT: Units = Rabbits/Year
The number of rabbits that die each year.

Area = 1
DOCUMENT: Units = Acres
The amount of land the rabbit population is living on.

Births_Normal = 1.5
DOCUMENT: Units = Fraction/Year
The number of rabbits born into the population per rabbit in the population each year.

Average Lifetime = 4
DOCUMENT: Units = Years
The number of rabbits that die per rabbit in the population per year.

Population_Density = Rabbit_Population/Area
DOCUMENT: Units = Rabbits/Acres
The number of rabbits per unit of area.

Deaths_Multiplier = GRAPH(Population_Density)
(0.00, 1.00), (100, 1.00), (200, 1.00), (300, 1.00), (400, 1.50), (500, 2.00), (600, 2.70), (700, 3.70), (800, 4.70), (900, 5.70), (1000, 7.50)
DOCUMENT: Units = Unitless
Births Multiplier is a multiplication factor which is dependent upon the population density. It converts density to a factor which affect the number of Births(i.e. it makes Births affected by density).

S-SHAPED GROWTH STRUCTURE 2 (From Figure 7b)

Healthy_People(t) = Healthy_People(t - dt) + (Recovering_Patients - Catching_Illness) * dt
INIT Healthy_People = 100
DOCUMENT: Units = People
The number of healthy people in the population, where the population is the number of healthy people and the number of sick people.

INFLOWS:
Recovering_Rate = Sick_People/Duration_of_Illness
DOCUMENT: Units = People/Month
The number of sick people recovering and becoming healthy people each month.

OUTFLOWS:
Catching_Illness = Healthy_People * Probability_of_Contact_With_Sick_People * Population_Interactions * Probability_of_Catching_Illness
DOCUMENT: Units = People/Month
The number of healthy people catching the illness and becoming sick people each month.

Sick_People(t) = Sick_People(t - dt) + (Catching_Illness - Recovering_Patients) * dt
INIT Sick_People = 1
DOCUMENT: Units = People
The number of sick people in the population.

INFLOWS:
Catching_Illness = Healthy_People * Probability_of_Contact_With_Sick_People *
Population_Interactions * Probability_of_Catching_Illness
DOCUMENT: Units = People/Month
The number of healthy people catching the illness and becoming sick people each month.

OUTFLOWS:
Recovering_Rate = Sick_People/Duration_of_Illness
DOCUMENT: Units = People/Month
The number of sick people recovering and becoming healthy people each month.

Probability_of_Contact_With_Sick_People = Sick_People/(Sick_People +
Healthy_People)
DOCUMENT: Units = Unitless
The probability that when a person in the population is in contact with another person in the population, that the other person is a sick person.

Duration_of_Illness = .5
DOCUMENT: Units = Months
The time it takes for a sick person to recover and become healthy.

Probability_of_Catching_Illness = .5
DOCUMENT: Units = Unitless
The odds of catching the illness.

Population_Interactions = 10

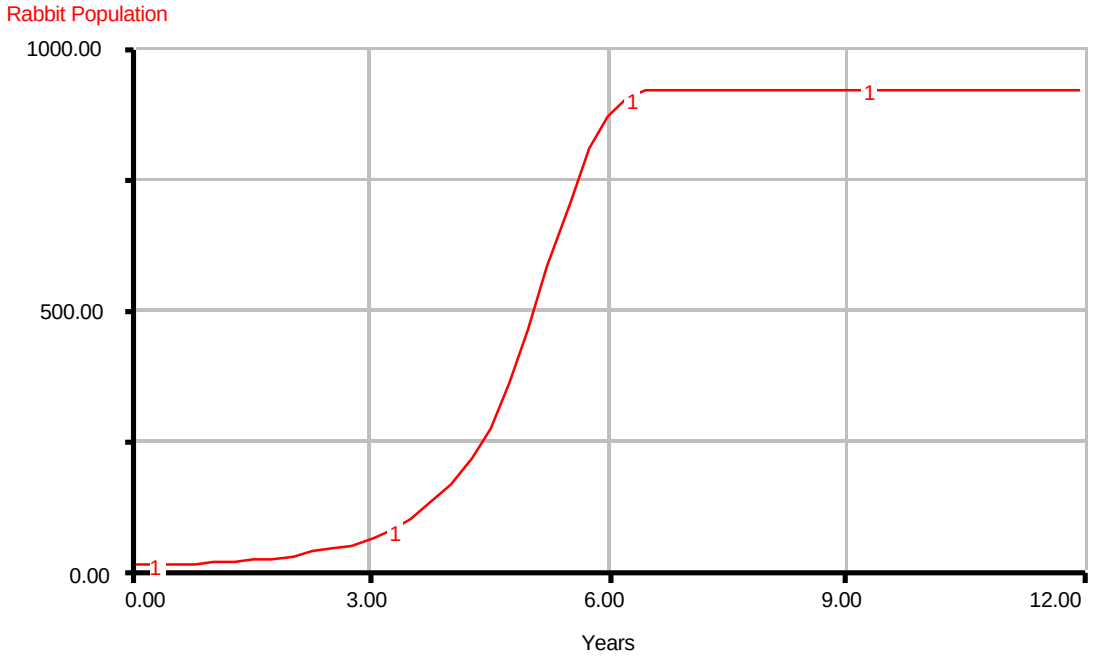
DOCUMENT: Units = Fraction/Month

The number of people a person will come into contact with each month.

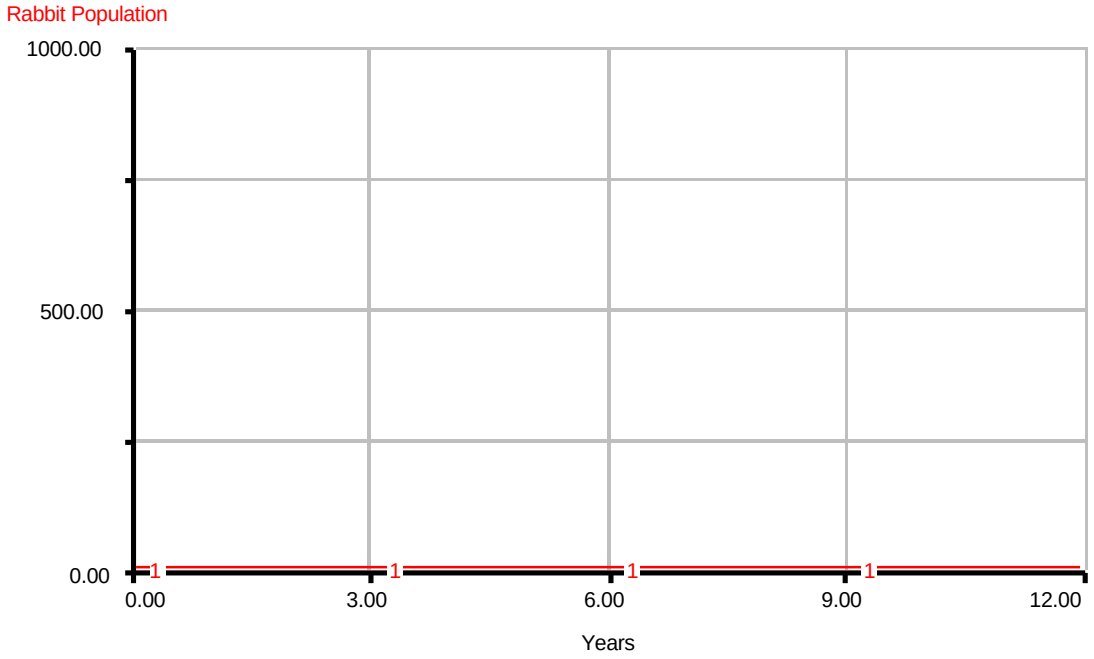
APPENDIX B - Solutions to Exercises

EXERCISE 1

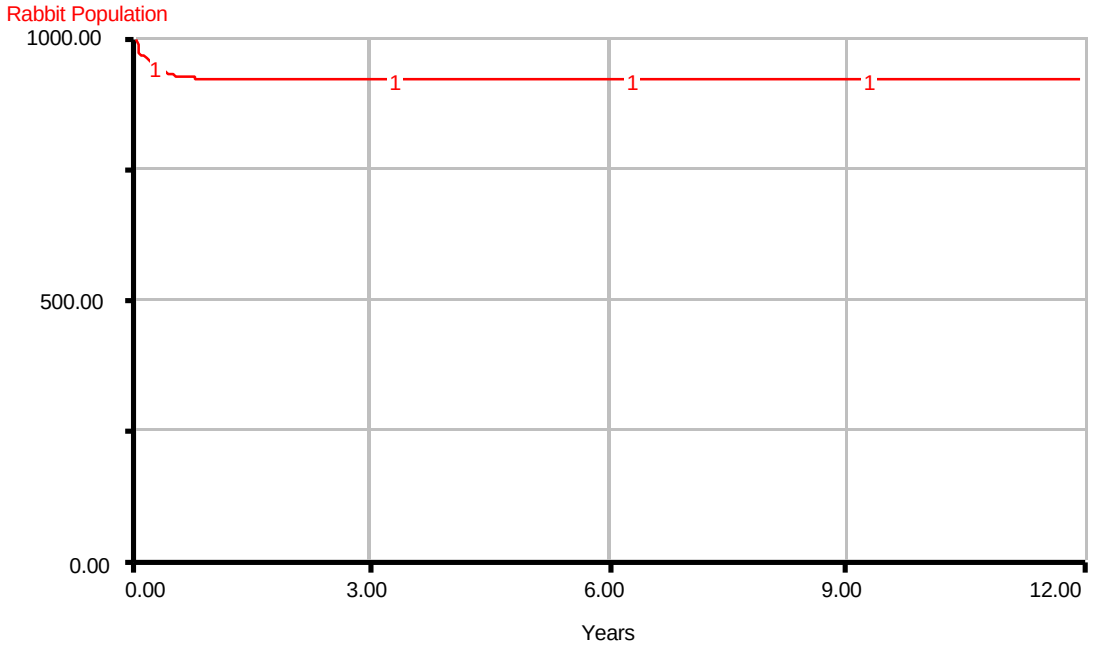
Graph A



Graph B

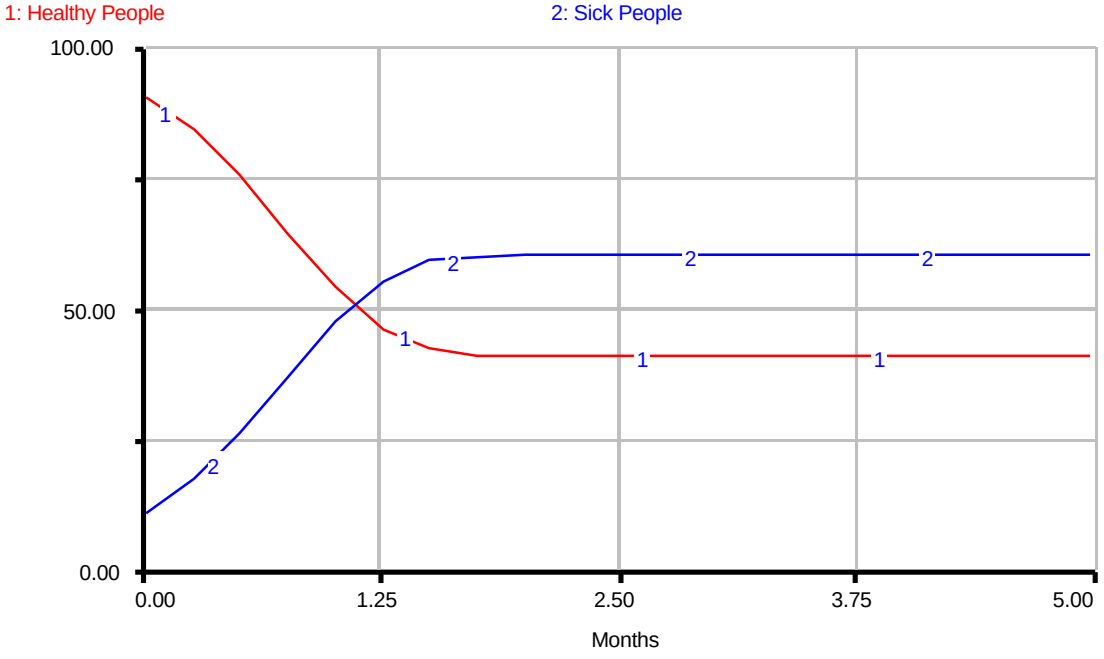


Graph C

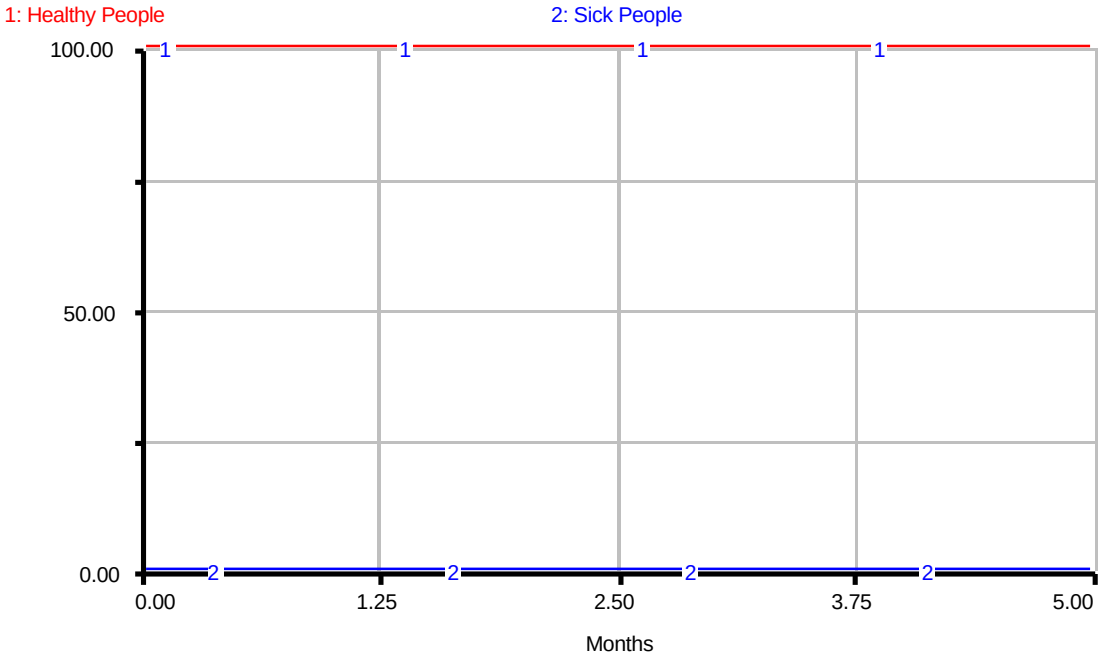


EXERCISE 2

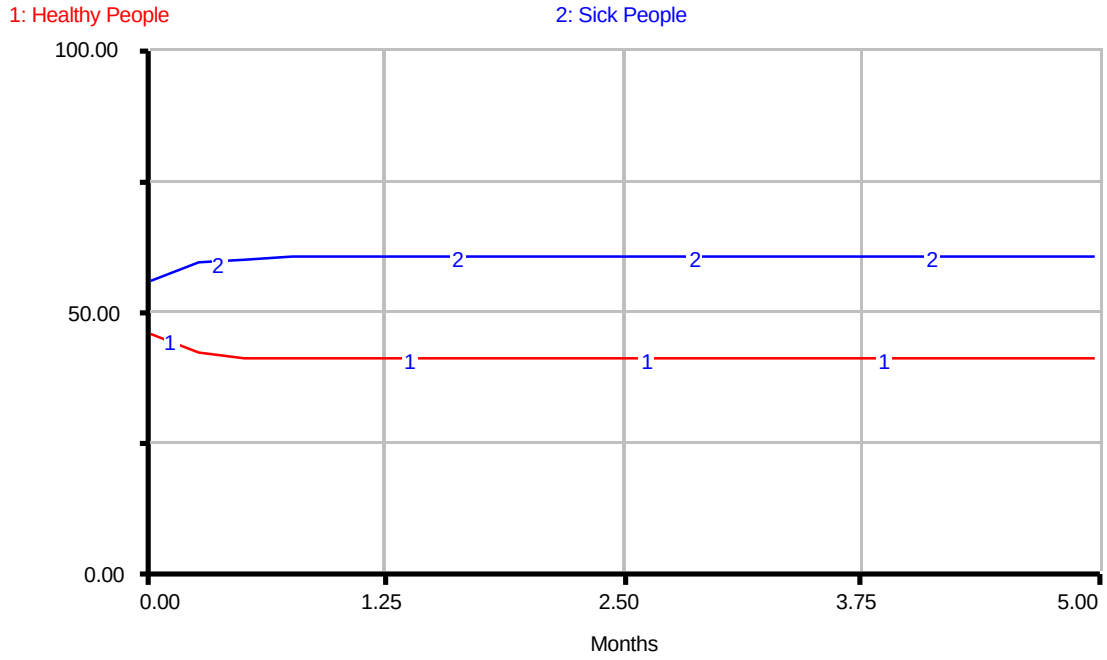
Graph A



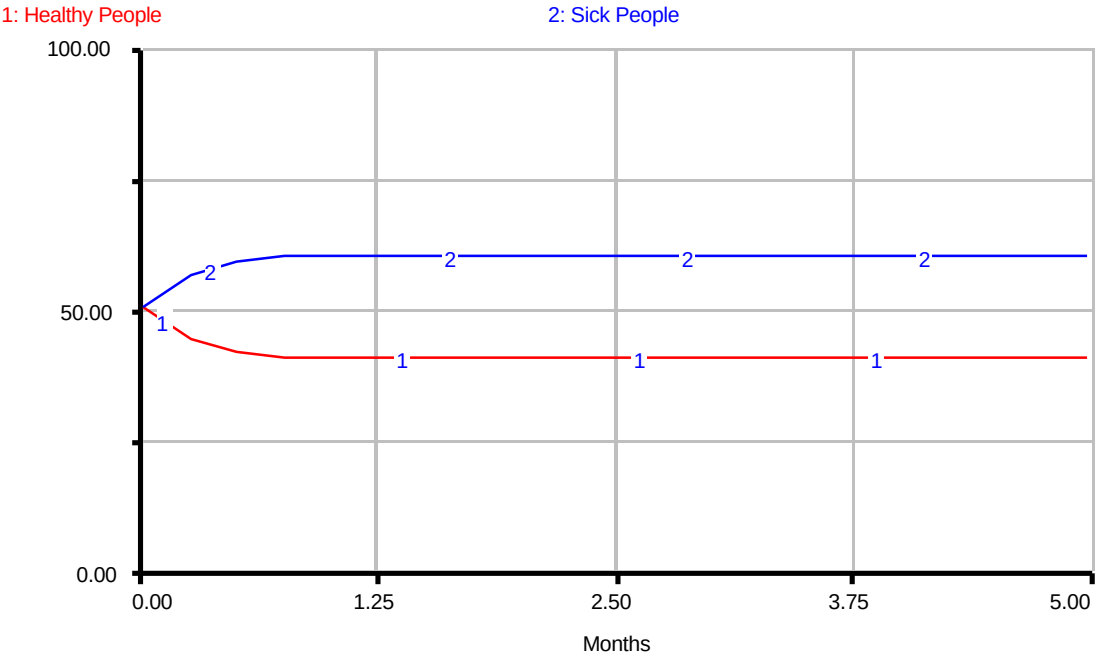
Graph B


1: Healthy People
2: Sick People
100.00
50.00
0.00
0.00
1.25
2.50
3.75
5.00
Months

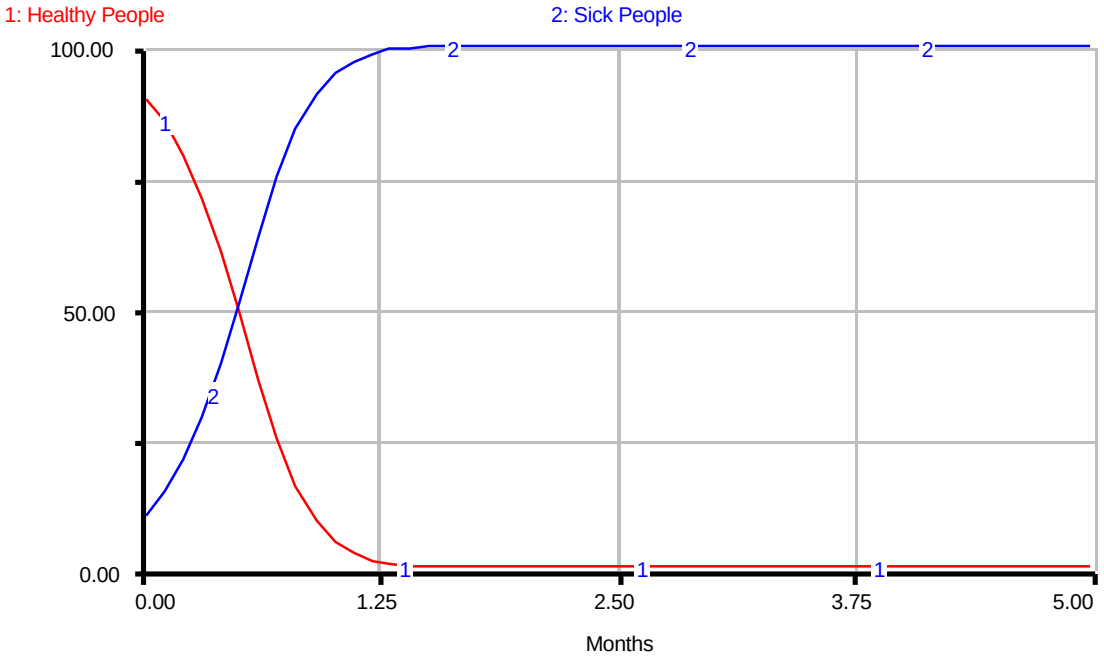
Graph C



Graph D



Graph E



The type of illness is terminal illness. This is because there is no recovery. Anyone who becomes sick stays sick.

APPENDIX C - Technical Documentation

S-Shaped Growth Structure 1

Exponential growth to infinity occurs when the parameters are set at values that cause the Birth Rate to always exceed the Death Rate.

$$\text{Birth Rate} = \text{Rabbit Population} * \text{Birth Fraction}$$

$$\text{Death Rate} = \text{Rabbit Population} * \frac{\text{Deaths Multiplier}}{\text{Average Lifetime}}$$

For the Birth Rate to always be greater than the Death Rate the following must always hold true:

$$\text{Birth Fraction} > \frac{\text{Deaths Multiplier}}{\text{Average Lifetime}}$$

Exponential decay to zero occurs when the parameters are set at values that cause the Death Rate to always exceed the Birth Rate. For this to be the case the following must always hold true:

$$\text{Birth Fraction} < \frac{\text{Deaths Multiplier}}{\text{Average Lifetime}}$$

S-Shaped Growth Structure 2

Changing parameter values for this structure will only affect the time scale over which the various behaviors occur. This can be easily understood if one examines the equation for Catching Illness.

$$\text{Catching Illness} = \text{Healthy People} * \text{Probability of Contact with Sick People} * \text{Population Interactions} * \text{Probability of Catching Illness}$$

One must now must substitute into the above equation, the equation for Probability of Contact with Sick People.

$$\text{Probability of Contact with Sick People} = \frac{\text{Sick People}}{\text{Healthy People} + \text{Sick People}}$$

$$\text{Catching Illness} = \text{Healthy People} * \frac{\text{Sick People}}{\text{Healthy People} + \text{Sick People}} * \text{Population Interactions} * \text{Probability of Catching Illness}$$

The Catching Illness equation has several constants in it. They are the Population Interactions, the Probability of Catching Illness, and Healthy People plus Sick People, which actually is total population. All these constants can be combined to form one constant.

$$\text{Constant} = \frac{\text{Population Interactions} * \text{Probability of Catching Illness}}{\text{Healthy People} + \text{Sick People}}$$

The Catching Illness equation can now be written as follows:

$$\text{Catching Illness} = \text{Healthy People} * \text{Sick People} * \text{Constant}$$

The significance of this simplified form is that it combines all the parameter values into one large constant. If parameters are changed the only thing that is affected is the constant in the Catching Illness rate. A change in this constant only affects the time scale of the behavior.

Vensim Examples:
Generic Structures:
S-Shaped Growth I

By Lei Lei
October 2001

S-Shaped Growth Structure 1

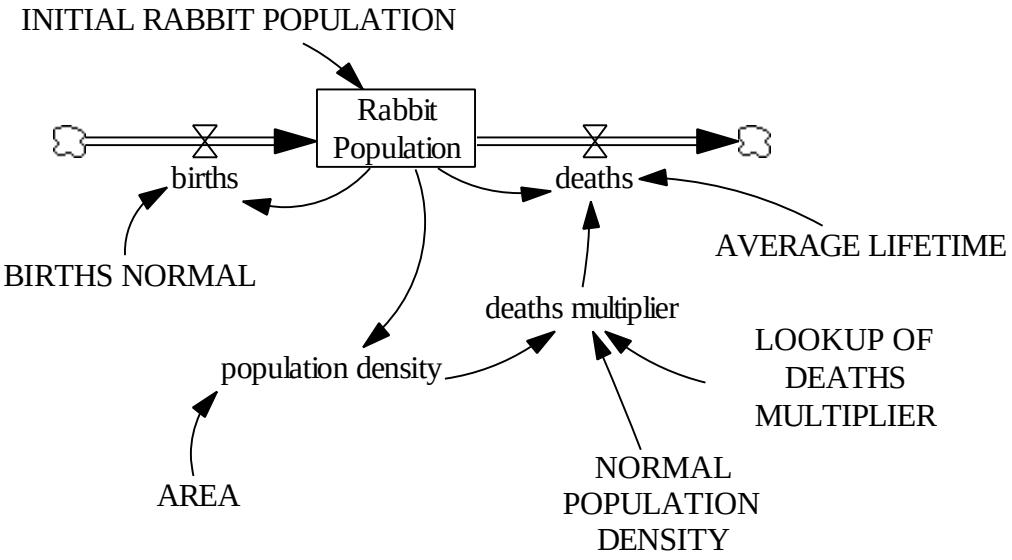


Figure 10: Vensim Equivalent of Figure 5a: Model of rabbit population growth with limited area.

Documentation for Rabbit Population Model

(01) AREA = 1
Units: acres
The amount of land the rabbit population is living on.

(02) AVERAGE LIFETIME = 4
Units: year
The number of rabbits that die per rabbit in the population per year.

(03) births = Rabbit Population* BIRTHS NORMAL
Units: rabbits/year
The number of rabbits being born each year.

(04) BIRTHS NORMAL = 1.5
Units: 1/year
The number of rabbits born into the population per rabbit in the population each year.

(05) deaths = (Rabbit Population/AVERAGE LIFETIME)* deaths multiplier
Units: rabbits/year
The number of rabbits that die each year.

(06) deaths multiplier = LOOKUP OF DEATHS MULTIPLIER(population density/NORMAL POPULATION DENSITY)
Units: dmnl
Deaths Multiplier is a multiplication factor that is dependent upon the population density. It converts density to a factor that affects the number of births (i.e. it makes births affected by density).

(07) FINAL TIME = 100
Units: year
The final time for the simulation.

(08) INITIAL RABBIT POPULATION = 1
Units: rabbits

(09) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(10) LOOKUP OF DEATHS MULTIPLIER
[(0,1)(20,25)],(0.01,1),(1,1),(2,1),(3,1),(4,1.5),(5,2),(6,2.7),(7,3.7),(8,4.7),(9,5.7),(10,7.5),(11,9),(12,10.5),(13,12),(14,14),(15,17),(16,19),(17,21),(18,23),(19,24),(20,25)Units: dmnl

(11) NORMAL POPULATION DENSITY = 100
Units: rabbits/acres
The normal population density is the initial rabbit population, 1 rabbit, divided by the area, 1 acre. So it is 1.

(12) population density = Rabbit Population/AREA
Units: rabbits/acres

(13) Rabbit Population = INTEG (births-deaths, INITIAL RABBIT POPULATION)
Units: rabbits
The number of rabbits in the rabbit population.

(14) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(15) TIME STEP = 0.0625
Units: year
The time step for the simulation.

S-Shaped Growth Structure 2

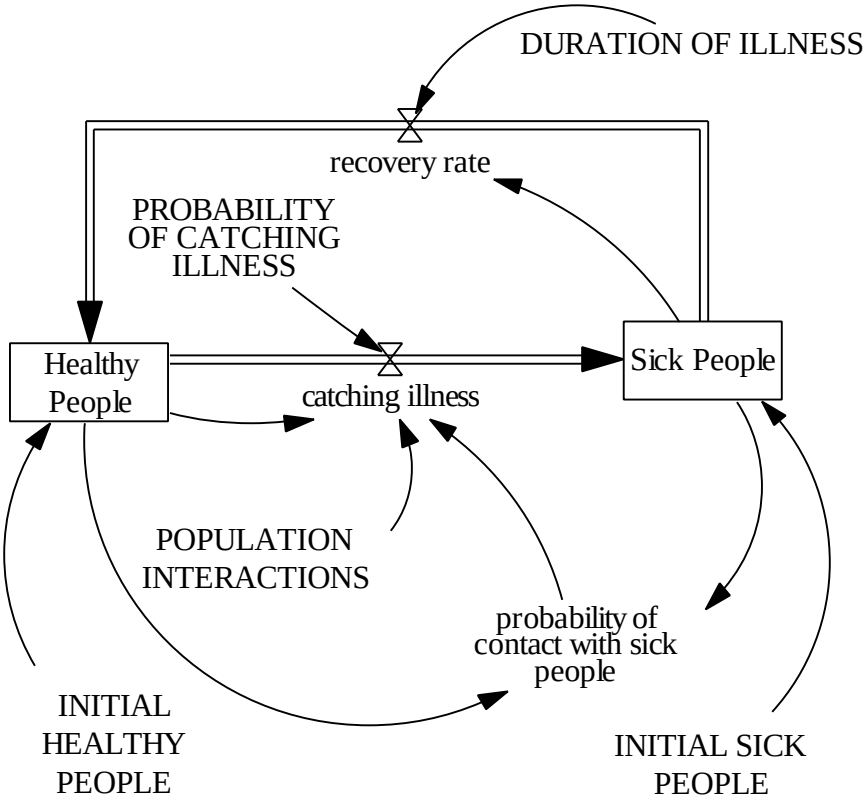


Figure 11: Vensim Equivalent of Figure 7a: Epidemic

Documentation for Epidemic Model

(01) catching illness = Healthy People* probability of contact with sick people* POPULATION INTERACTIONS * PROBABILITY OF CATCHING ILLNESS
Units: people/Month
The number of Healthy People catching the illness and becoming Sick People each month.

(02) DURATION OF ILLNESS = .5
Units: Month
The time it takes for a Sick Person to recover and become healthy.

(03) FINAL TIME = 5
Units: Month
The final time for the simulation.

(04) Healthy People = INTEG (+recovery rate-catching illness, INITIAL HEALTHY PEOPLE)
Units: people

The number of Healthy People in the population, where the population is the number of healthy people and the number of Sick People.

(05) INITIAL HEALTHY PEOPLE = 100
Units: people

(06) INITIAL SICK PEOPLE = 1
Units: people

(07) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(08) POPULATION INTERACTIONS = 10
Units: 1/Month
The number of people a person will come into contact with each month.

(09) PROBABILITY OF CATCHING ILLNESS = 0.5
Units: dmnl
The odds of catching the illness.

(10) probability of contact with sick people = Sick People/(Sick People+ Healthy People)
Units: dmnl
The probability that when a person in the population is in contact with another person in the population, that the other person is a sick person.

(11) recovery rate = Sick People/DURATION OF ILLNESS
Units: people/Month
The number of Sick People recovering and becoming Healthy People each month.

(12) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(13) Sick People = INTEG (catching illness-recovery rate, INITIAL SICK PEOPLE)
Units: people

(14) TIME STEP = 0.125
Units: Month
The time step for the simulation.

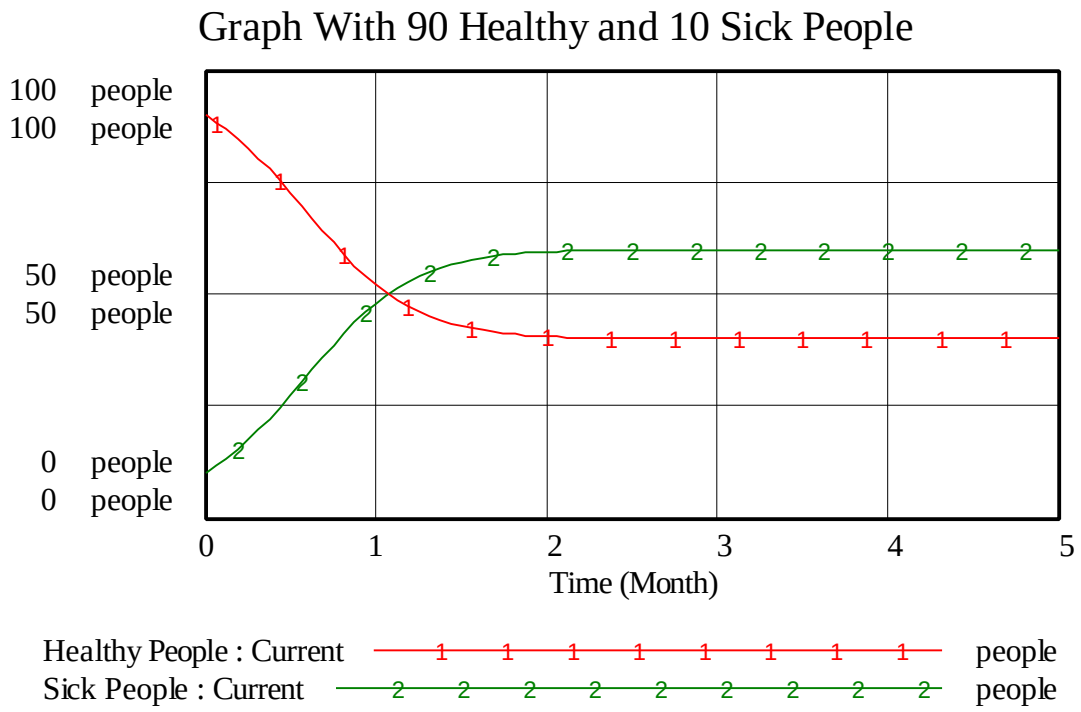


Figure 12: Vensim Equivalent of Simulation with 90 Healthy and 10 Sick People initially

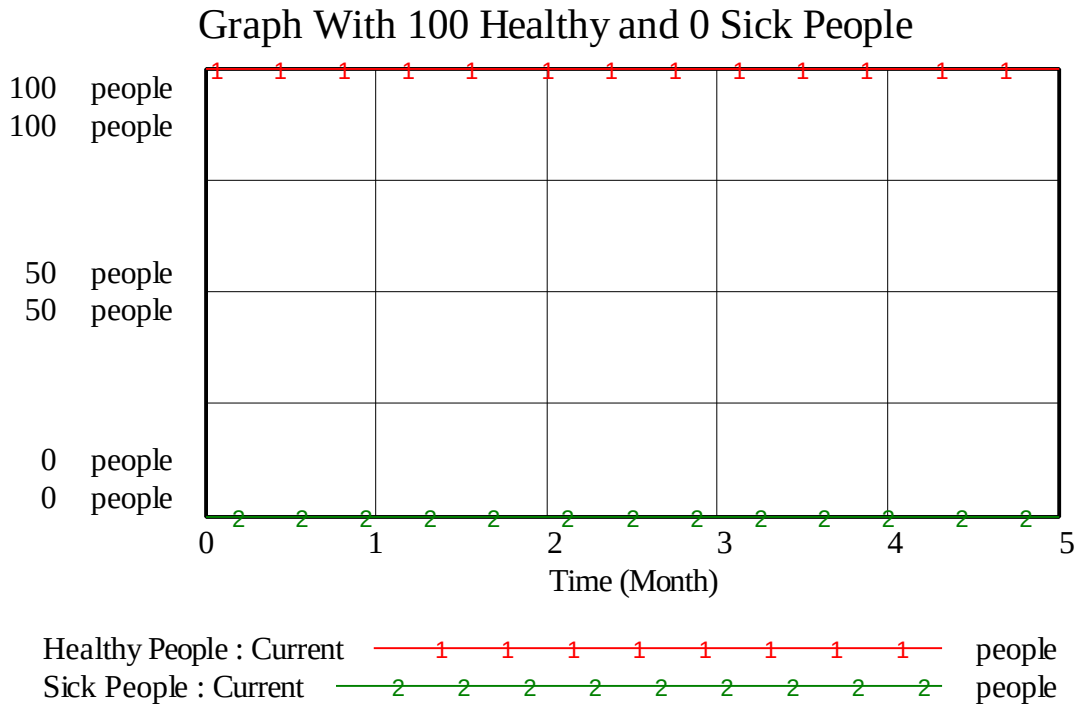


Figure 13: Vensim Equivalent of Simulation with 100 Healthy People and 0 Sick People initially

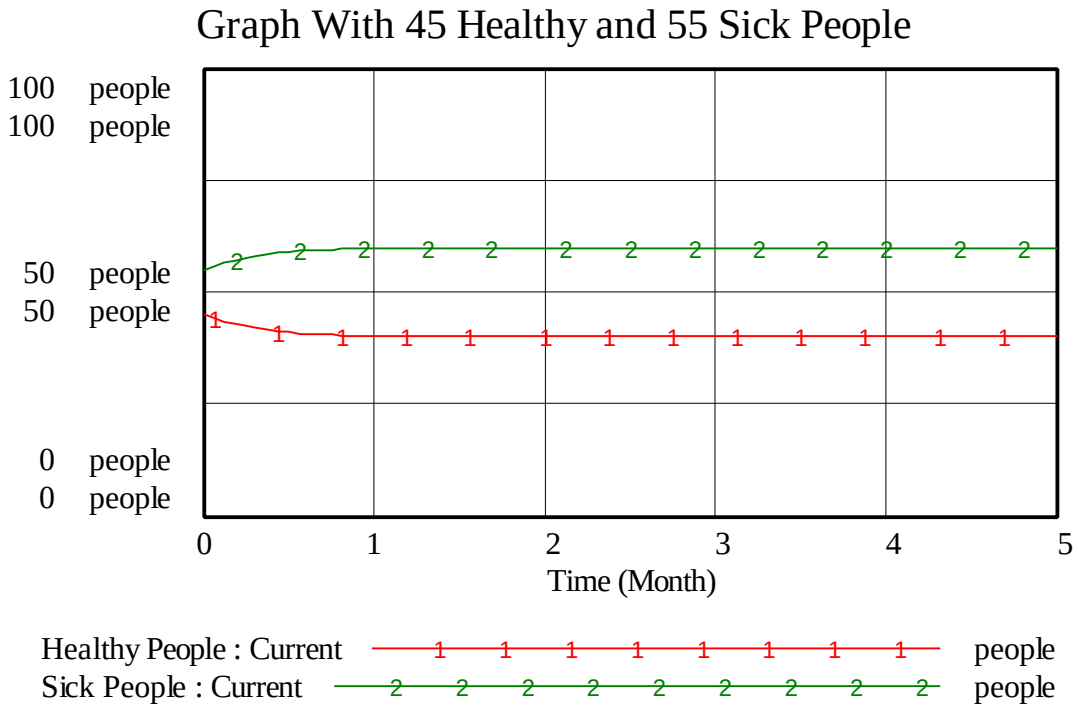


Figure 14: Vensim Equivalent of Simulation for 45 Healthy and 55 Sick People initially

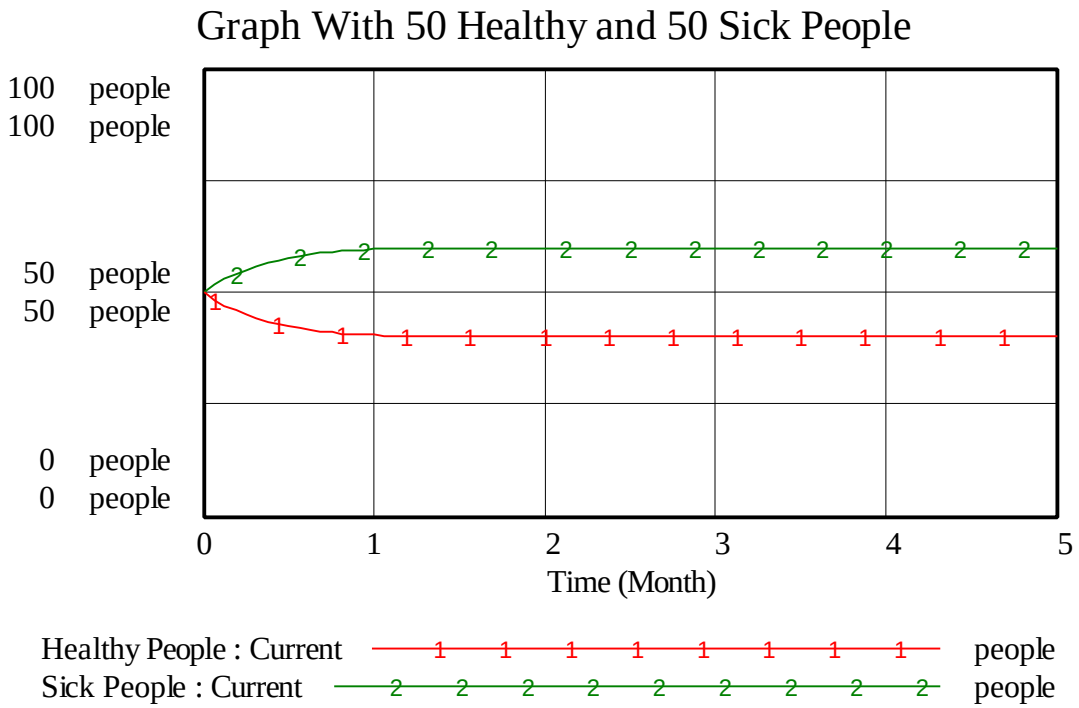


Figure 15: Vensim Equivalent of Simulation for 50 Healthy and 50 Sick People initially

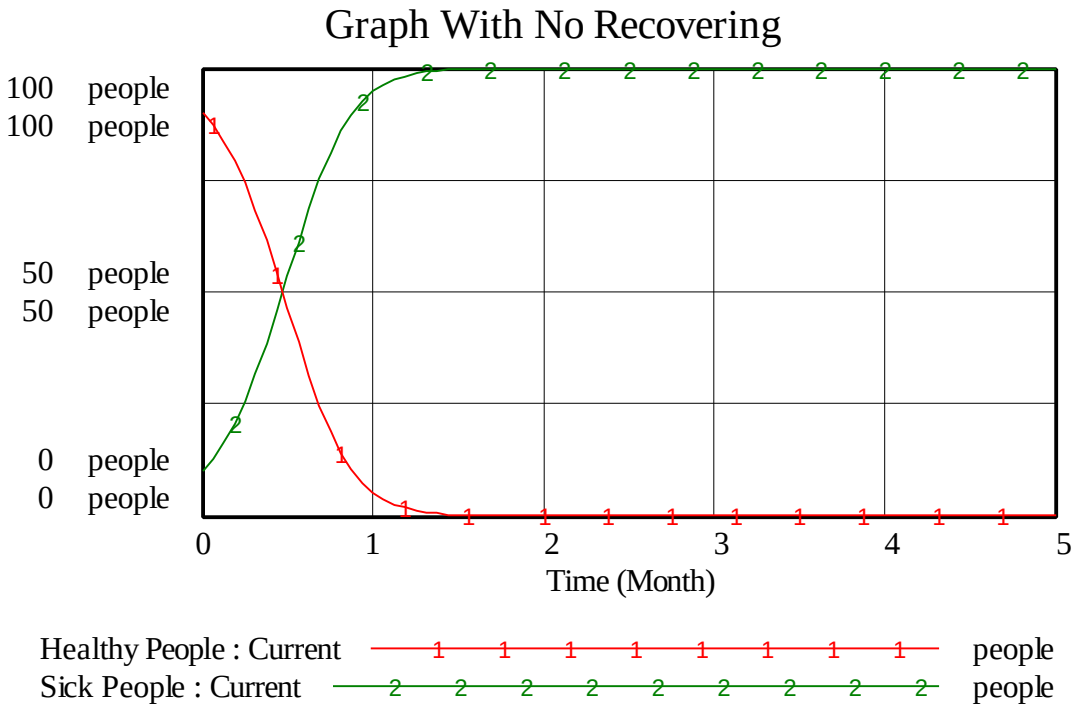


Figure 16: Vensim Equivalent of Simulation with no recovering patients

DYNAMIC SIMULATION MODELS: HOW VALID ARE THEY?

Raymond C. Shreckengost

VALIDITY, PURPOSE, AND CONFIDENCE

In a strict sense, the subject of dynamic simulation model validity can be treated thoroughly and quickly: there are no fully valid models because all models are something less than the object, or system, being modeled. For example, millions of people have a conceptual model of the President, but, like fingerprints, no two of these models is exactly the same. Further, none matches precisely every detail of the real system. The same reasoning applies to all types and kinds of models of drug abuse.

In a practical sense, we are concerned with usefulness rather than validity. Does the model serve the purpose for which it was intended? Is it helpful? Thus, the developer's or user's purposes must be kept in mind in evaluating a model's usefulness, or validity. Criticisms of models also should reflect this perspective.

Much depends on the purpose for which the model is developed -- for example, the choice of the level of detail used in the model. Just as micrometers are not used to measure inter-city distance a tenth of a mile, explicit modeling of each household would be equally absurd in a model treating the gross behavior of drug abuse systems. The selection of an appropriate level of detail, problem boundaries, and similar considerations constitute the "art" aspect of dynamic simulation model development.

Validity, or usefulness, lies in the subjective view of the user. We think of models as valid when they can be used with confidence. So, this paper focuses on how we can gain confidence in dynamic simulation models. In particular, it considers confidence or validity tests as they relate to a particular dynamic simulation method, System Dynamics (Forrester 1961, 1975; Robbers 1978).

These tests, however, are equally valid for other simulation techniques. They have evolved from nearly 30 years' experience of the inventor of System Dynamics, Jay W. Forrester (Bell and Senge 1980 Forrester and Senge 1980).

MODEL STRUCTURE TESTS

Because the foundation for model behavior is the model's structure, the first test in validating a model is whether the structure of the model matches the structure of the system being modeled. Every element of the model should have a real-world counterpart, and every important factor in the real system should be reflected in the model. Although this may seem like a simple, obvious test, it may not be so. For example, descriptions of how all of the structural parts of real systems are tied together rarely exist. More often than not, such descriptions must be based on the concepts, or mental models, of people familiar with the system. Further, important parts of some systems may lie unrecognized prior to modeling. During the development of a model dealing with the effects of heroin imports into the United States, for example, the key factor in the system, the relative abundance of heroin, was not immediately identified (Gardener and Shreckengost this volume). Thus, the art of model building may, at times, entail discovery and invention.

This approach differs strongly from "Let's collect lots of data and then see what they tell us." Structure, like many other System Dynamics model elements, exploits judgment, experience, and intuition. Data plays a secondary role.

Model Parameter Tests

The model's parameter values are a specific area for testing. Parameter values in a model often may be tested in a straightforward manner, e.g., against historical data. However, in dynamic simulation models of social systems the desired data may be unavailable, in an inappropriate form, or incorrect. There may be elements that are not usually quantified, but that are critical to the system being modeled. These elements must be included in the model. If prejudice, for example, is an important element, it must be included in the model, and its relationship to other pertinent parts of the system must be specified quantitatively. Many required parameter values may not exist and must be developed. In the heroin model, data and descriptions relating to heroin's relative abundance were initially absent. On the other hand, some available, apparently reasonable and acceptable, data on heroin imports turned out to be unreasonable and unacceptable when employed in the model. The point is that dynamic simulation model parameter values, from whatever source they may be derived, are subject to a rigorous and demanding environment. These

values contribute significantly to confidence in the model when the specified parameter values are reasonable and consistent with whatever supporting data might exist.

Boundary Adequacy Test

If a model is focused on the heroin system in New York City, it will not generate national behavior. Conversely, a national heroin model is not likely to replicate the behavior of local systems. Model boundaries must match the purpose for which the model is designed, if the model is to be used with confidence: that is, the model must include all of the important factors affecting the behavior of interest. In practice, boundaries tend to shift as the developers' and users' understanding of a problem evolves with the model's development. As model purpose shifts, changes in the model's boundaries may be required.

In many problems, a simple model with limited boundaries may be expanded, or disaggregated, from time to time, as the model is used to address problems in greater detail. When this occurs, careful attention must be given to indirect effects, which may not be obvious. Suppose, for example, a model treating United States heroin users as a homogeneous group is disaggregated to identify users of small, medium, and large amounts. This will change the user boundaries, and associated changes will be needed in the consumption boundaries.

If the model boundaries are improper, or inadequate, the model's validity is degraded. However, criticism of dynamic simulation models aimed at boundary issues frequently reflects different notions about the model's intended use or purpose. For example, criticism of the user boundary in a model treating users as a homogenous group may ignore the fact that the grouping is consistent with the purpose of the model. But, as explained above, if the purpose is to account for different classes of users, a boundary change is required to account for the change in purpose.

Extreme Conditions Test

A less obvious test relating to model structure involves the effects of extreme conditions. The ability of a model to function properly under extreme conditions contributes to its utility as a policy evaluation tool as well as user confidence. Testing to extreme conditions may easily be overlooked or brushed aside in the hectic environment of early model

development. Subsequently, this oversight may degrade model performance: subtly under normal conditions and significantly when the model is used to answer "What if?" questions that fall outside the operating regions emphasized in early development.

Again, the heroin import model provides a good example. In the past, heroin imports have been, roughly, 5 metric tons per year. They have not fallen to zero, nor have they soared to 10 or 20 tons. Consequently, during the model's development parameter values covered the range of import variations that were of immediate interest, say, 3 to 7 tons. If these initial values only were retained in later versions of the model, the model would show a residual, sizable user population even if imports were reduced to zero. At the other extreme, the number of users would reach an understated upper limit in the presence of a very large heroin supply surplus. The point is that model validity is enhanced if the region within which the model was originally designed to operate is extended so the model generates plausible behavior conditions outside the initial region. For example, the user population should be zero when imports are zero.

Tests under extreme conditions may also expose structural faults or inadequacies and incomplete or erroneous parameter values.

MODEL BEHAVIOR TESTS

Behavior Replication Test

The tests relating to model behavior are less technical and, for many users, more appealing and convincing than the structural tests. Foremost among these tests is the comparison of model behavior with the behavior of the system being modeled. A model whose behavior has little, or nothing, in common with that of the system of interest generates little, or no, confidence.

Where historical time series data are available, the model must be capable of producing similar data. That is, if the model's initial conditions are matched to the state of the system being modeled at some time in the past, the model's behavior should parallel the historical data from that time to the present. In this test, it is again important to keep in mind the purpose of the model -- including the time span of the areas of behavior that are of interest. Further, judgment must be exercised about how closely the model's behavior should match the historical data, since historical data are less than perfect and, sometimes, far from

perfect. It is not at all uncommon for; models to illuminate erroneous data. Where historical data are very poor or nonexistent, the test may be one of reasonableness.

Given the imports over 10 years or so, the heroin model (Gardiner and Shreckengost, this volume) generates heroin purity and price values that match well with the historical data for these parameters. Further, it also produces heroin-related death figures that match the historical data closely. The closeness of the model-historical correspondence is quite surprising, given the difficulties inherent in collecting and processing the data that the historical time series represents.

Purity, price, and deaths can be defined and measured with relative ease compared to the heroin user population. The model generates user population values against a strict, limited definition of a heroin user. Here, no parallel historical data exist, and the test becomes one of reasonableness considering the purpose for which the model was developed. Subsequent to its initial development, this sector of the model has been detailed to accommodate users with different consumption habits and varying responses to the abundance of heroin. Although this is intuitively more satisfying, there is still no opportunity for a confirming historical test.

Anomalous Behavior Test

When model behavior does not replicate the behavior of the real system, model structure, parameter values, boundaries, or similar factors are suspect. Something may have been omitted, improperly specified, or assigned incorrect values. In addition to being a powerful tool during model development, tests of anomalous behavior may contribute convincingly to model validity. For example, if a model behaves well except for, say, a limited period of time, and no faults can be found in the model, the error may lie in the data with which the model behavior is being compared. Or, matching the real system's purported behavior may require the inclusion of implausible structure, or parameter values, in the model. In the heroin model, the import data for 1 year were revised downward, because the consumption required to match that import level could be achieved only by an unrealistic increase in heroin user population. Whether due to faults in the model or in the real system, the resolution of the discrepancies found through the anomalous behavior test bolsters confidence and validity.

Behavior Sensitivity Test

Most, but certainly not all, social systems are stable -- bureaucracies, in particular, are frequently lampooned for their very, very stable behavior. Small, reasonable changes in a model's parameter values, then, should normally not produce radical behavior changes. If the model's behavior is not seriously affected by plausible parameter variations, confidence in the model is increased. On the other hand, dynamic simulation models are often used to search for parameters that can effect behavior changes. The criterion in the sensitivity test is that any sensitivity exhibited by the model should not only be plausible, but also consistent with observed, or likely, behavior in the real system.

Behavior Prediction Test

Dynamic simulation models are especially useful in predicting how a system would behave if various policies of interest were implemented. Dynamic simulation models offer significant advantages when used in this role; they provide a consistent basis for the predictions. This basis is a consolidation of judgment, experience, and intuition that has been tested against historical evidence, and the predicted effects of implementing alternative policies are promptly available. Confidence in the model is reinforced if the model not only replicates long-term historical behavior, but also responds similarly to existing systems in which various policies have been implemented. For example, over the years many treatment policies have been followed in drug abuse treatment centers. A generic model of such a system, tailored to match any particular center of interest, should replicate the effects produced by the policies implemented in that center.

Family Member Test

Dynamic simulation models acquire added value and confidence when they are generic, i.e., applicable to a family of similar situations, as in the case of treatment centers mentioned above. Drug abuse treatment centers have common basic features, so any one facility may be thought of as a particular case of the basic model embodying these common features. The same is true of payroll, retirement, university, village, city, region, and many other social systems or organizations.

Under these conditions, confidence is enhanced not only because the complementary systems can contribute to the robustness of the model developed for a particular member of

the family, but also because the differences among the members can be explicitly identified and defined.

Some family member applications of the heroin model, for example, are readily apparent. The structure is equally applicable to subdivisions of the United States, such as regions or cities. Further, it appears that it is also directly applicable for cocaine, and, possibly, other illegal drug systems.

Behavioral Boundary Test

Exploiting generic models, behavior prediction, and tests of extreme policies may impinge on the model boundary. Is the boundary still adequate for excursions that may extend beyond the region of operation initially envisioned for the model? In prediction, for example, the basic model may have to be revised, so that policy alternatives, or events, such as the impact of discoveries in research programs, can be introduced. In drug abuse models, the inclusion of social trends, or new domestic or foreign policies, may require boundary modifications. The behavioral boundary test is an important step in determining whether the model includes the necessary modifications.

OTHER TESTS

A third class of test -- policy implication tests -- which includes system improvement, changed behavior prediction, boundary adequacy, and policy sensitivity tests, deals with whether a real system's response to a policy change would replicate the response to the policy change predicted by a model. These tests reflect a different perspective in the application of some of the tests discussed earlier. For example, if real system behavior improves as predicted when tested in a model, was the policy change responsible for the improvement, or were other factors responsible?

This test builds confidence only after numerous real life tests have been completed. The boundary question is inverted: how would boundary changes alter the evaluation of policies and the selection of policies for implementation? These tests tend to be long term and to contribute to confidence and validity most importantly by enlarging the scope of congruence between dynamic simulation models and the systems they represent.

Checking the dimensional consistency of model equations is an additional structural test that may be ignored as trivial, or obvious, but at some peril. For example, if a model contains an equation with heroin expressed in grams on the left side of the equal sign, heroin in grams, and heroin in grams only, must fall out from the right side of the equation. Errors in dimensional consistency can easily creep into model equations during model development and, subsequently, during revisions.

Additional behavioral tests, surprise behavior, and extreme policy behavior can also contribute to confidence and validity. Surprise behavior relates to the recognition of behavior in the real system that was there all along, but not noticed until the system was modeled. Because of its emphasis on identifying the causes underlying observed behavior, System Dynamics readily leads to such discoveries. For example, in the heroin system model, such a surprise was the identification of the relative abundance of heroin as a key parameter influencing the purity, price, and heroin-related deaths that occurred in the real system. In retrospect, like many inventions and discoveries, the relationships may seem very obvious. Such new-found perspectives, of course, contribute significantly to confidence in the model. Extreme policy tests introduce radical policies into the model to see if the behavior of the model is consistent with what would be expected under these conditions. This helps affirm the model's robustness.

COMMON TESTS NOT USED

Paralleling the development of the tests described above has been a growing body of evidence and opinion that many tests commonly associated with model testing are inappropriate, inadequate, or even dysfunctional. In part, these changes derive from the philosophy underlying the System Dynamics method of dynamic simulation modeling, particularly, the notion that all important factors in the real system exerting an influence on the behavior of the system must appear in the model -- whether these factors are normally modeled or not. Further, all factors in the model must have a counterpart in the real system. Together with the dynamic, rather than static, nature of the simulation, these characteristics have shifted emphasis from more traditional, statistical tests to the kinds of tests described in this paper -- whole model tests that engage all the model variables and their relationships in the testing process.

The t-test, for example, has been shown to be of little use, and possibly misleading, in several studies (Johnson 1980, Mass and Senge 1978).

Briefly, the tests can lead to the exclusion of factors that are important to a model's behavior. Although the tests may be helpful in detecting structural flaws, they are insufficient in the absence of whole model tests. Recently, statistical tests employing Kalman filtering principles have been developed. These tests may be more useful in the development of dynamic simulation models (Peterson 1979). The greater power of these tests stems from their ability to eliminate the effects of measurement error in hypothesis testing.

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Self-Report Methods of Estimating Drug Use:
Meeting Current Challenges to Validity

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Graphical Integration Exercises

Part Three: Combined Flows

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
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March 14, 1996

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1. Abstract

This paper is the third in a series of papers on developing skills in graphical integration. It will study the use of graphical integration to estimate the behavior of simple systems with constant and linearly increasing or decreasing inflows and outflows. This paper assumes you can estimate behaviors of models with constant, step, and variable net flows. Also, you should be familiar with the terms "slope" and "area," and you should know how to calculate them for simple behaviors.[1]

2. Introduction

Let us consider a simple system, that of a bathtub with a constant inflow through a faucet. We know that if we turn on the faucet to fill a tub, go outside for five minutes and come back to find the tub one-fourth full, then it will take five more minutes to fill up halfway. It will then take five more minutes to be three-fourths full, and a total of twenty minutes for the tub to be completely full of water.

What we just did, essentially, is graphical integration without explicitly drawing graphs. We were able to perform this integration because the system in question was very simple. However, many systems are not as simple as the bathtub system. Thus, it becomes difficult to predict how they will behave. Practice in using graphical integration will make it easier to use it to better estimate the behavior of many systems.

Although we have access to sophisticated computer programs that simulate models of great complexity, it is important that we can intuitively estimate and understand what we see on the graph pad before and after running a simulation. This helps to improve confidence in the model. The series of graphical integration papers will enable us to do that. In this third paper of the series, we will graphically integrate systems with separate inflows and outflows.

---

[1] If you need to develop these skills, please refer to *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571), by Kevin Agatstein & Lucia Breierova, System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Jan., 1996.

3. Graphical Integration with Independent Inflows and Outflows

The first paper in the series, *Graphical Integration Exercises Part One: Exogenous Rates*,[2] studied graphical integration of constant flows and step functions. In the second paper in the series, *Graphical Integration Exercises Part Two: Ramp Functions*,[3] we saw how to graphically integrate linearly increasing and decreasing flow ramp functions. In both of these papers we dealt only with a single net flow. We will now turn our attention to graphical integration with both inflows and outflows to a single stock.

An example of such a system with different inflows and outflows is a ship whose hull has punctured a leak. Water rushes into the hull while the sailors furiously work at the pumps to expel the water. This is modeled in Figure 1:

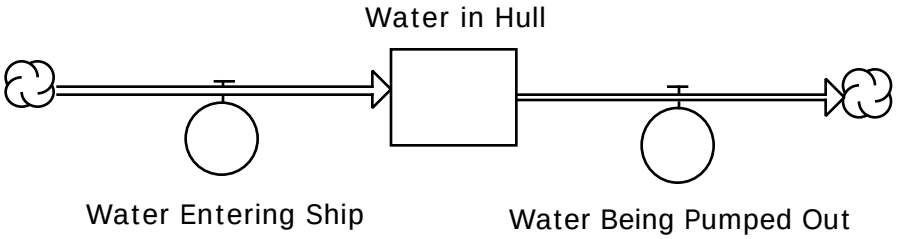


Figure 1: Model of Ships Hull.
(Note that the Inflow and Outflow are separate)

In these situations ship captains intuitively graphically integrate (without drawing the actual graphs) to determine whether or not to "abandon ship." In this paper we will quantitatively graphically integrate similar systems with separate inflows and outflows. Even though we are dealing with separate inflows and outflows, it does not mean that the material we discussed in the first two papers in the *Graphical Integration Exercises* series does not apply. In fact, as you shall soon see, graphically integrating systems with separate inflows and outflows is nearly identical to other types of graphical integration. The only difference is the first step of calculating a "net flow." Let's learn how to do that now.

---

[2] G*raphical Integration Exercises Part One: Exogenous Rates* (D-4547), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Dec 1., 1995.

[3] *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571).

3.1 Calculating a Net Flow

For graphically integrating systems with separate inflows and outflows, the first step is to calculate this single net flow. Then the problem becomes identical to the ones discussed in the previous two papers in the *Graphical Integration Exercises* series.

To calculate the net flow into a system all we need to do is subtract the outflow from the inflow. If this net flow is positive, then the stock will fill; if it is negative, the stock will empty. The net flow is written mathematically for any time interval as:

Net Flow = Change in Value of Stock **= Inflow - Outflow**

This means that the new value of the stock can be expressed as:

New Value of Stock = Initial Value of Stock + (Inflow - Outflow) * time.

Don't worry if you are confused by these formulas. Once we do the following example the concept should become much clearer.

Let us consider a system where oil is being added to an initially empty storage tank at a constant rate of 15 gallons per minute and at the same time is being pumped out of the tank at a constant rate of 5 gallons per minute. Can we estimate the behavior of the stock?

Graphically, the two rates that affect the system are shown in Figure 2.

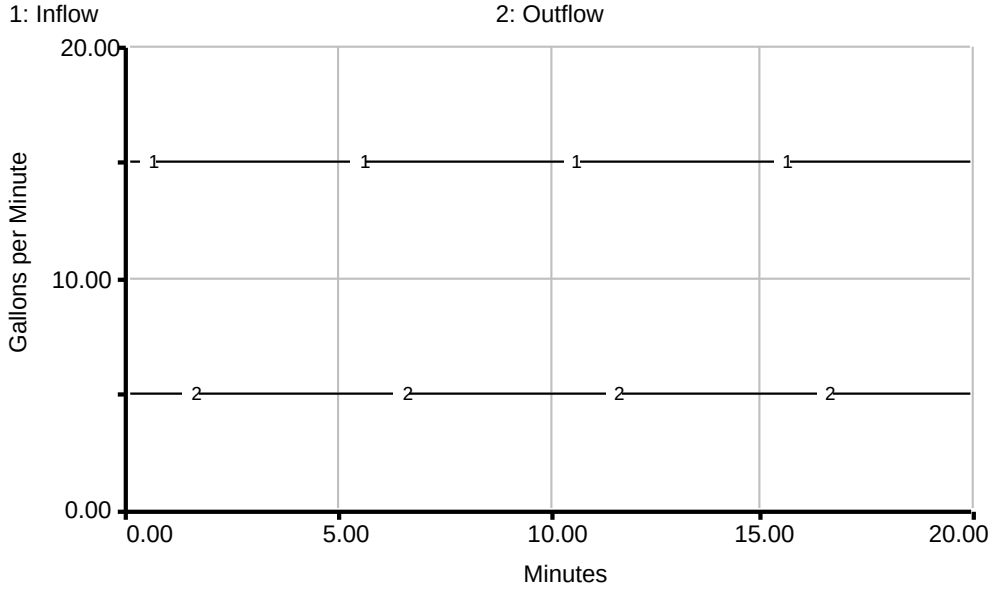


Figure 2: Flows Into and Out of the Oil Storage Tank.

The first step in estimating the behavior of this system is to determine the net flow. The net flow is simply the difference between inflow and outflow. In this system:

Initially:
Net Flow at Time 0 = Inflow at Time 0 - Outflow at Time 0
Net Flow at Time 0 = 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

At 5 min:
Net Flow at 5 min = Inflow at 5 min- Outflow at 5 min
Net Flow at 5 min = 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

At 10 min:
Net Flow at 10 min= Inflow at 10 min - Outflow at 10 min
Net Flow at 10 min= 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

The same holds true for every point in time, since both the inflow and outflow are constant. This means that we can rewrite the system with a single flow, the net flow, as opposed to the separate inflow and outflow drawn above. This is done in Figure 3.

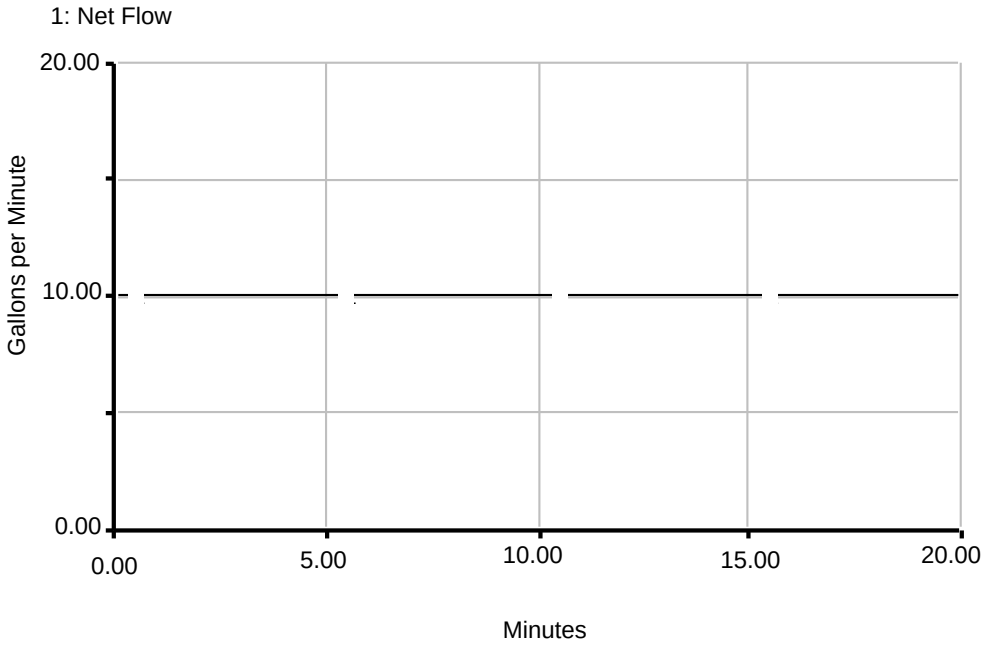


Figure 3: Net Flow for Oil Tank System

According to this graph at any given point in time there is a net addition of 10 gallons of oil per minute to the tank. This is logical since we are always adding 15 gallons per minute to the tank, and subtracting 5 gallons per minute.

Now that we have calculated the net flow, estimating the behavior of the system should be easy. We learned in *Graphical Integration Exercises Part 1: Exogenous Rates* how to graphically integrate a constant flow. For review, we will do it again here.

We know that constant net flows generate linear stock behaviors. Also, we know that for any given interval the area under the net flow graph is the change is the value of the stock for that interval. Thus from Time 0 to 5 minutes:

Change in value of Stock from 0 to 5 min = Area under net flow graph from 0 to 5 min
Change in value of Stock from 0 to 5 min = 10 gallons per minute * 5 minutes
Change in value of Stock from 0 to 5 min = 50 gallons

Now we can calculate the value of the stock at 5 minutes using:

Value of Stock at 5 min = Value of Stock at Time 0 + Change in value of Stock from 0 to 5 min
Value of Stock at 5 min = 0 gallons + 50 gallons = 50 gallons

Repeating this calculation from 5 minutes to 10 minutes, we see that:

Change in value of Stock from 5 to 10 min= Area under net flow graph from 5 to 10 min
Change in value of Stock from 5 to 10 min= 10 gallons per minute * 5 minutes
Change in value of Stock from 5 to 10 min= 50 gallons

Thus, the value of the stock at 10 minutes can be calculated as follows:

Value of Stock at 10 min = Value of Stock at 5 min + Change in value of Stock from 5 to 10 min
Value of Stock at 10 min = 50 gallons + 50 gallons = 100 gallons

The same calculation can be repeated for the remainder of the net flow graph. After doing this, we get the behavior shown in Figure 4.

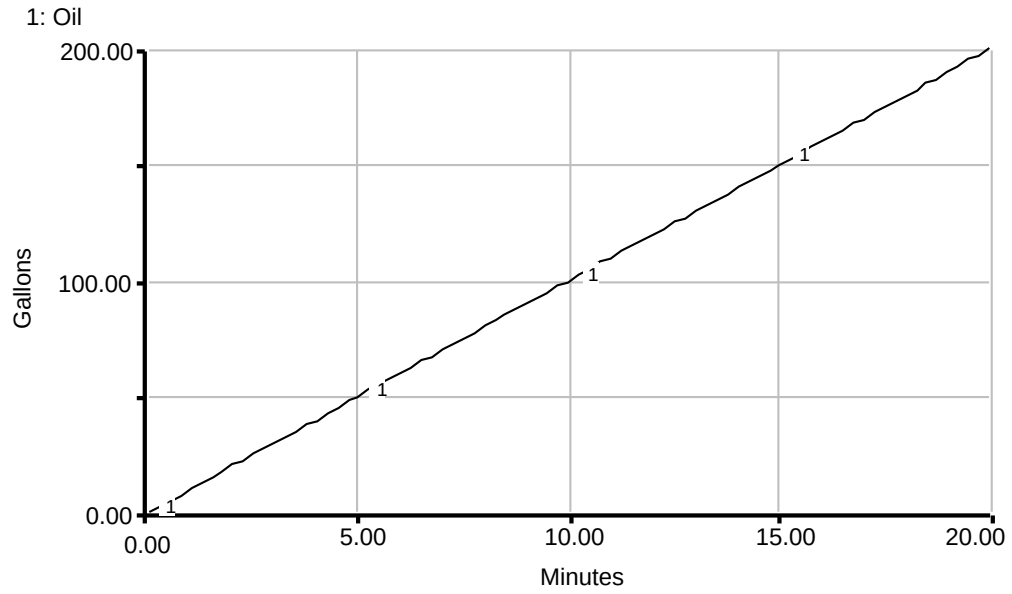


Figure 4: Behavior of the Oil Storage Tank with an Inflow of 15 Gallons per Minute and n Outflow of 5 Gallons per Minute

It might be a good idea to reflect back at this point on what the "net flow" is and how it relates to what we have studied in the two previous graphical integration papers. In these papers we always calculated the area between the flow graph and the line representing flow = 0. What we were really doing was treating the net flow as the inflow, and assuming there was no outflow. In other words, we treated the outflow as constant at zero units per time. We then found the area between the inflow and the outflow curves. In the next example we will apply this concept to graphically integrate a similar system, but without going through the trouble of calculating the net flow.

3.2 Combining Constant Flows

Let us look at another example of determining net flows when the inflow and outflow are constant. Let's imagine the same oil tank, this time initially with 25 gallons of oil in it. To this tank we are constantly adding 75 gallons per minute, and draining 50 gallons per minute. This can be represented graphically, as shown in Figure 5:

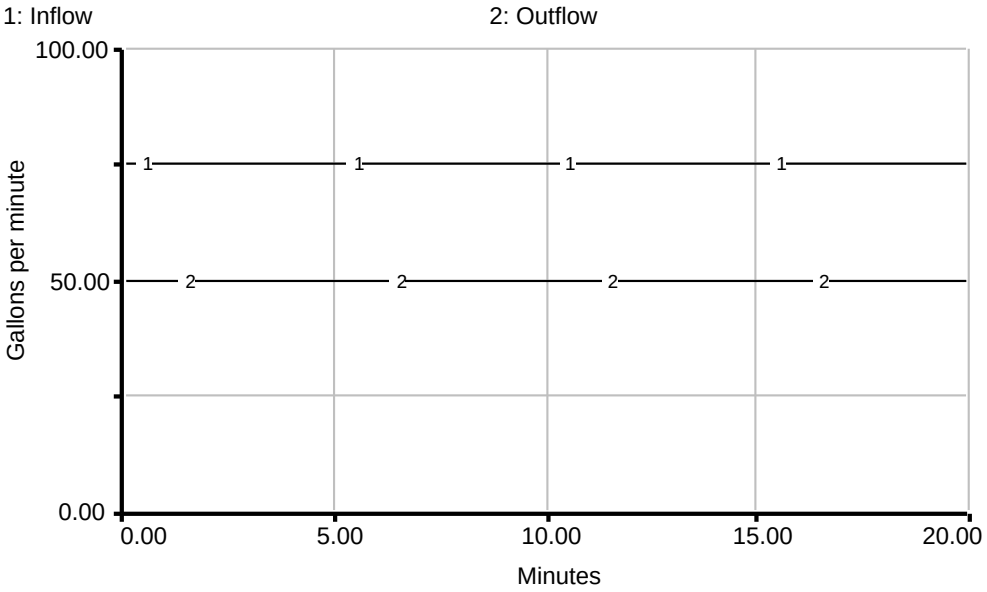


Figure 5: Inflow and Outflow to the Oil Storage Tank

To estimate the behavior of this system we could do exactly what we did in the previous example: draw the net flow graph and integrate it the same way we did in the previous two *Graphical Integration Exercises* papers. Briefly, the net flow graph would be a flat line at:

Net Flow = 75 gallons per minute - 50 gallons per minute = 25 gallons per minute.

Thus, the graph of the oil in the tank would be:

25 gallons at time 0

25 gallons + (25 gallons per minute * 5 minutes) = 150 gallons at time 5

150 gallons + (25 gallons per minute * 5 minutes) = 275 gallons at time 10,

and so on and so on.

For this example though, let us apply the fact that the change in the value of the stock is the area between the inflow and outflow curves. Thus, for Time 0 to 5 minutes:

Change in value of Stock = area between inflow and outflow graphs

Change in value of Stock = area of rectangle with a width of 5 minutes and a height of (75 gallons per minute - 50 gallons per minute)

Change in value of Stock = 5 minutes * 25 gallons per minute = 125 gallons

This is shown in Figure 6:

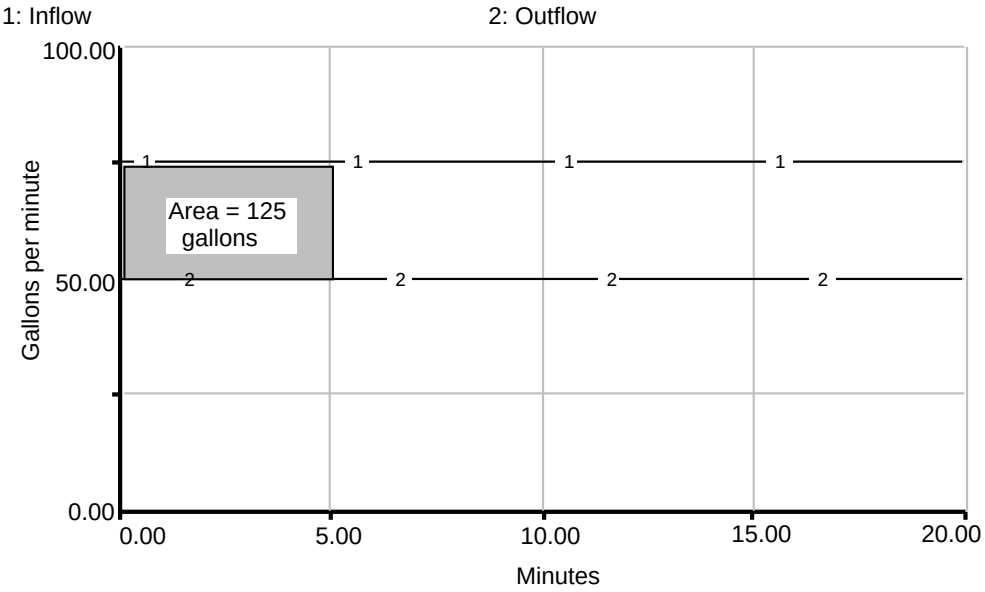


Figure 6: Graph of Flows
The area between the inflow and outflow for the first 5 minutes is shown.

Thus, we know that the value of the stock (which is the oil in the tank) at 5 minutes is:

Stock at 5 min = Initial Value of Stock + Change in value of Stock for 0 to 5 min
Stock at 5 min = 25 gallons + 125 gallons = 150 gallons.

For the time interval of 5 minutes to 10 minutes, we can repeat the calculation.

Change in value of Stock = area between inflow and outflow
Change in value of Stock = rectangle with a length of (10-5) minutes and a height of (75-50) gallons per minute
Change in value of Stock = 125 gallons

Stock at 10 min= Initial value of Stock + Change in value of Stock
= Stock at 5 min + 125 gallons
= 150 gallons + 125 gallons
= 275 gallons

Doing this calculation for time 10 minutes to 15 minutes will yield a value of the stock of 400 gallons at 15 minutes and 525 gallons at 20 minutes.

This behavior is shown graphically in Figure 7:

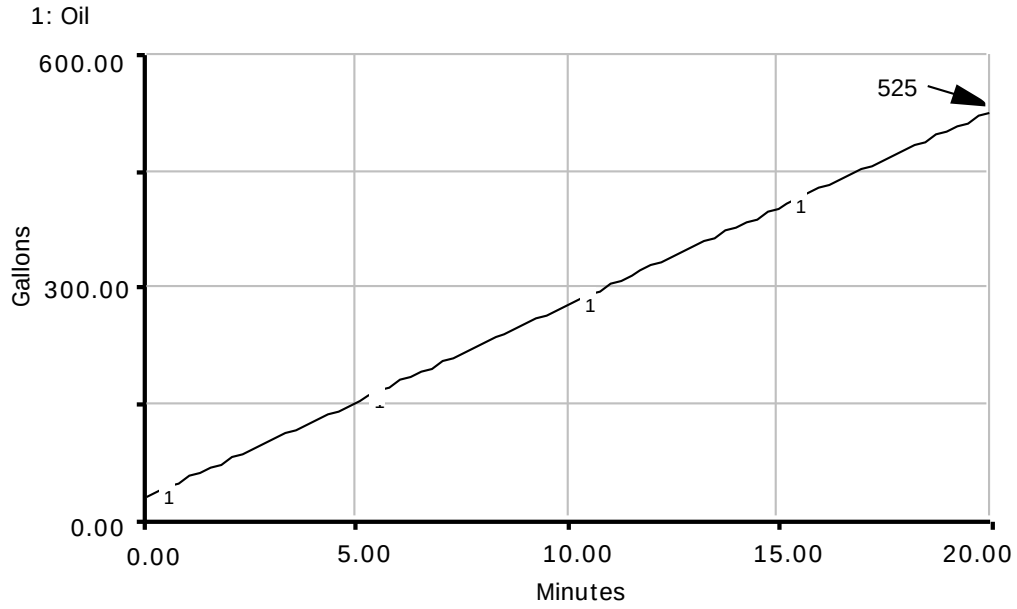


Figure 7: Behavior of the Oil Volume in the Storage Tank with Constant Inflow and Outflow

3.3 Variable Inflow with a Constant Outflow

Now that we have graphically integrated systems with constant inflows and outflows, let us look at a system that is a little more interesting. Let us consider an initially empty oil storage tank with 10 gallons per minute constantly being pumped out. Oil is added to the system initially at a rate of 20 gallons per minute which increases linearly with a slope of +1 gallon per minute per minute. Therefore at 20 minutes oil is being added at a rate of 40 gallons per minute.

These two flows are shown in Figure 8:

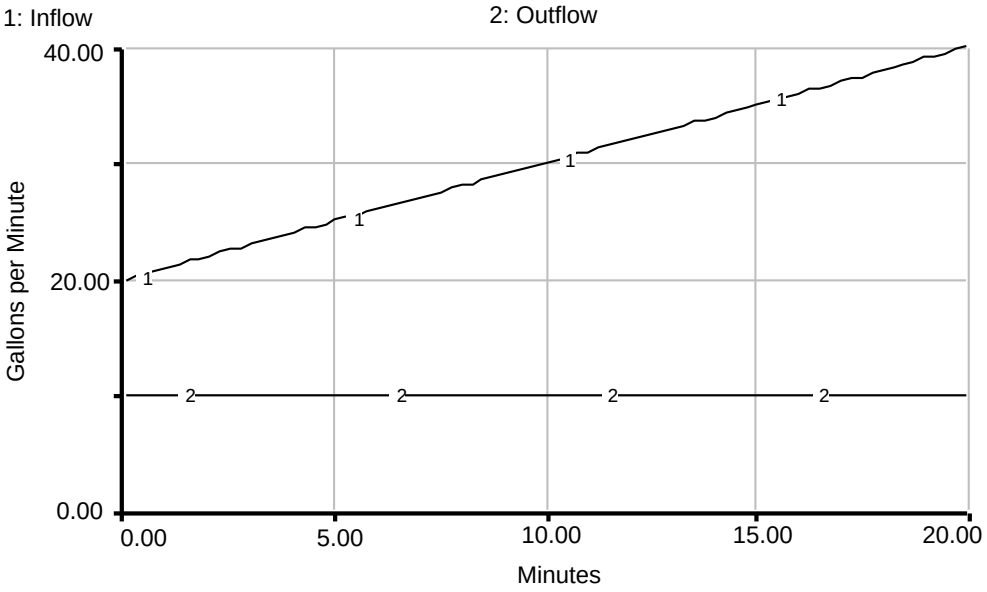


Figure 8: Graph of Inflow and Outflow for Storage Tank.

Let us solve this system by determining the net flow. (However, we could calculate the area between the Inflow and Outflow graph, like we did in the previous example. Either method is fine. You should pick the one you find easier for any given example.)

Initially:

Net flow at 0 min = Inflow at 0 min - Outflow at 0 min
= 20 gallons per min - 10 gallons per min
= 10 gallons per min

At 5 minutes:

Net flow at 5 min = Inflow at 5 min - Outflow at 5 min
= 25 gallons per min - 10 gallons per min
= 15 gallons per min

At 10 minutes:

Net flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 30 gallons per min - 10 gallons per min
= 20 gallons per min

At 20 minutes:

Net flow at 20 min = Inflow at 20 min - Outflow at 20 min
= 40 gallons per min - 10 gallons per min
= 30 gallons per min

The net flow can then be drawn as in Figure 9:[4]

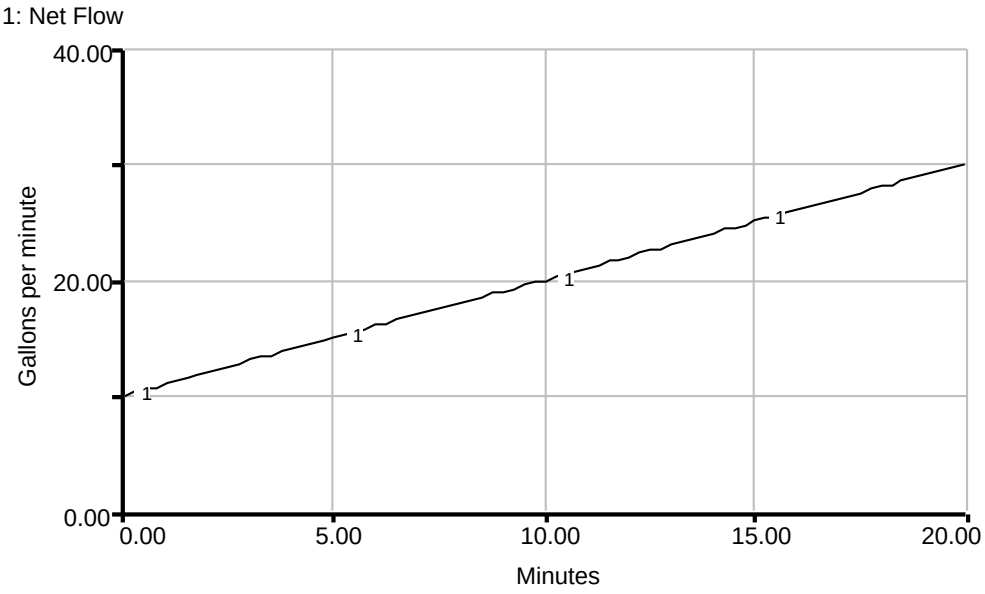


Figure 9: The Net Flow Into The Storage Tank.

In *Graphical Integration Exercises Part Two: Ramp Functions*, we went into great detail on how to integrate linearly increasing flows. We learned in that paper that the stock will display parabolic growth. We can estimate the behavior of the stock by calculating the area under the net flow graph to the line flow = 0, and adding that value to the initial value of the stock. For review purposes, let us carefully determine the behavior of the stock.

For 0 to 5 minutes:

Change in value of Stock = area under net flow graph
Change in value of Stock = area of rectangle plus area of triangle

[4] It is important to notice that the net flow is a straight line. Any time you subtract one line from another, you will get another line.

Figure 10 shows these two areas.

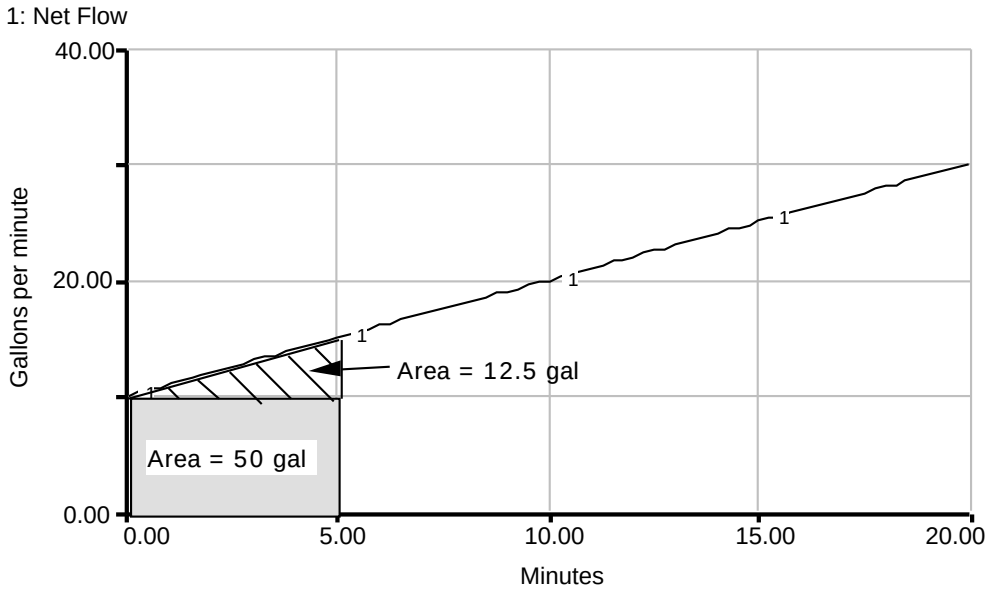


Figure 10: Calculation of Area Under Net Flow Graph

Change in value of Stock = (5 min * 10 gal per min) + (0.5 * 5 min * 5 gal per min)
= 50 gal + 12.5 gal = 62.5 gal

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 0 gal + 62.5 gal = 62.5 gallons

To calculate the value of the stock at 10 minutes, we can perform a similar process.

Change in value of stock from 5 to 10 min = area under net flow curve from 5 to 10 min

Area under curve = area of triangle + area of rectangle.

These areas are shown in Figure 11.

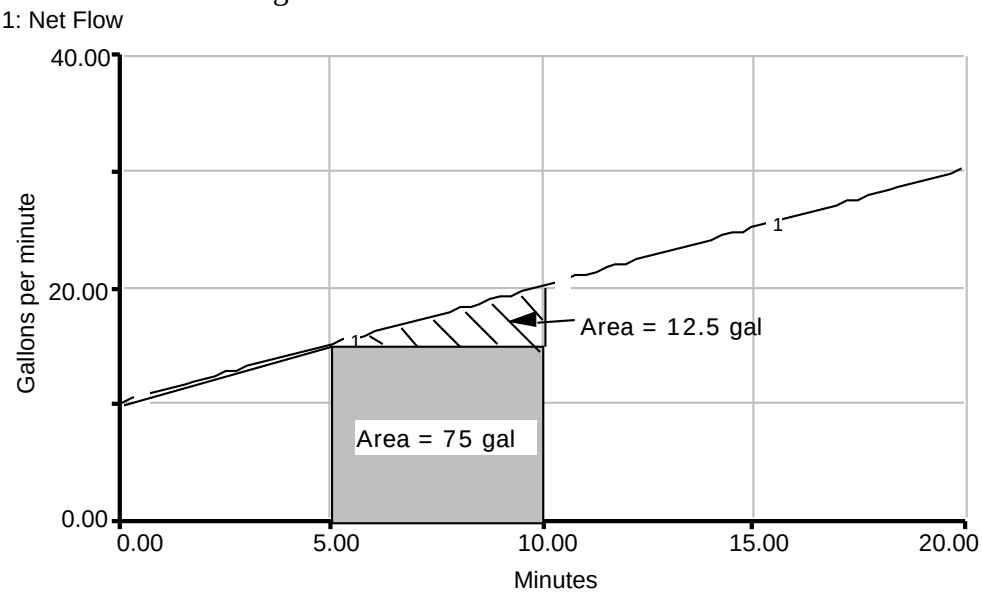


Figure 11: Calculation of Area Under Net Flow Graph

Thus, the area under the net flow graph from 5 to 10 minutes is:
Area = 75 gallons + 12.5 gallons = 87.5 gallons

The stock at 10 minutes is:
Stock at 10 min = Stock at 5 min + Change in value of Stock from 5 min to 10 min
= 62.5 gallons + 87.5 gallons = 150 gallons.

We could continue this exact same type of calculation for the rest of the net flow curve. Figure 12 shows the final result of such a calculation, the parabolic behavior of the stock (which is the amount of oil in the storage tank for this example.)

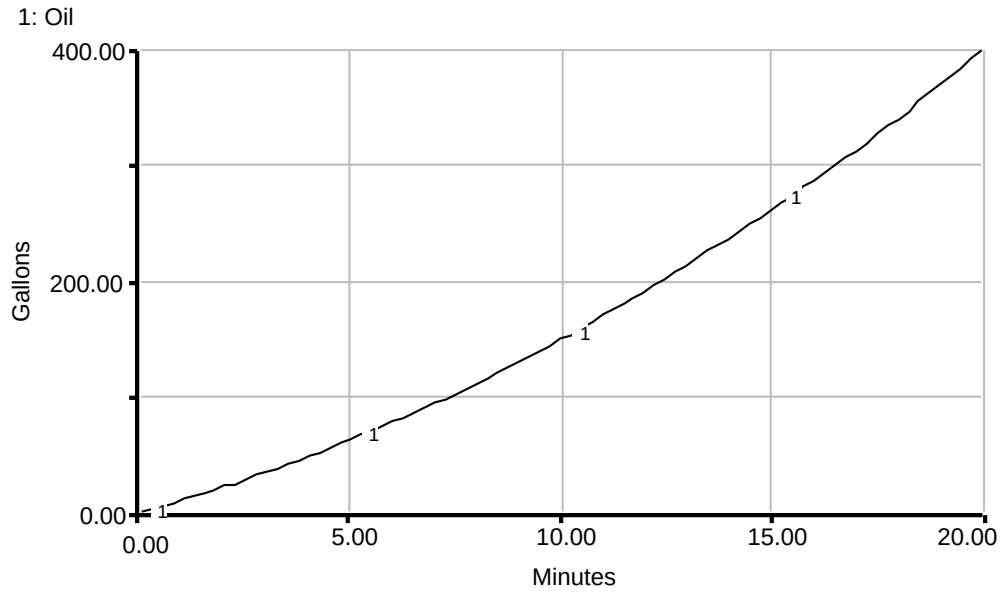


Figure 12: Behavior of the Oil Storage Tank System.

3.4 Variable Inflow and Outflow

For our fourth and final example, let us look at a system whose inflow and outflow are both changing over time. In this system the outflow, which is the oil being pumped out of our storage tank, is linearly decreasing with a slope of –2 from 40 gallons per minute to zero in 20 minutes. The inflow is constant at 20 gallons per minute for time 0 to 10 minutes and then it ramps up with a slope of +1 until 20 minutes. The oil tank is initially filled with 100 gallons of oil.

The inflow and outflow are shown in Figure 13.

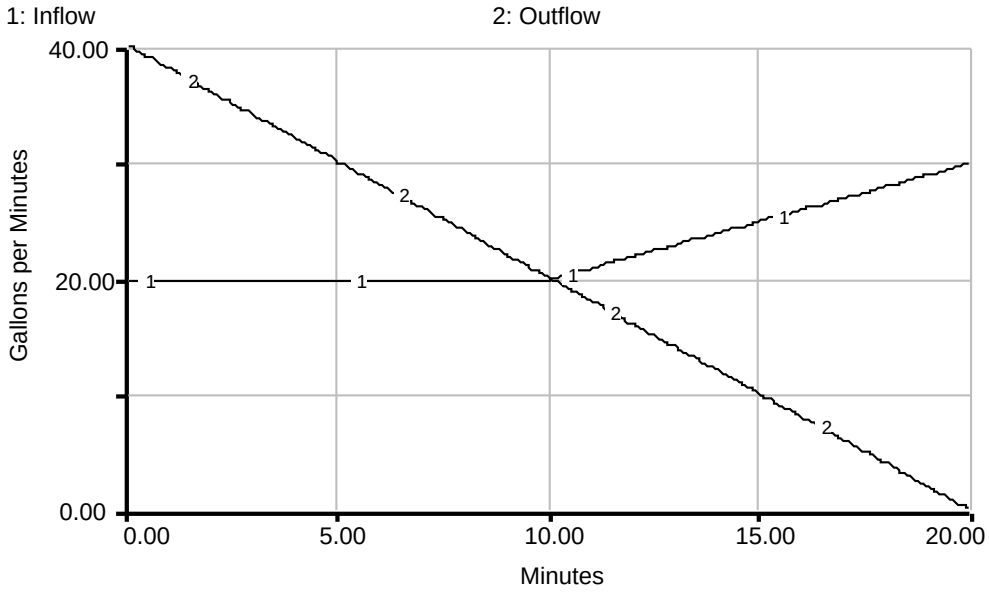


Figure 13: Graph of Variable Inflows and Outflows.

For this example we will use the net flow determination method. We could of course use the method of finding the area between the inflow and outflow graphs and get the correct answer. However, finding those areas looks a little tricky, so let's find the net flow. Remember:

Net Flow = Inflow - Outflow.

The net flow at 0 minutes is:

Net flow at 0 min = Inflow at 0 min- Outflow at 0 min
= 20 gallons per min - 40 gallons per min
= –20 gallons per minute[5]

[5] Note that the net flow is negative. All this means is that the stock (the storage tank) is being emptied, not filled.

The net flow at 10 minutes is:

Net flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 20 gallons per min - 20 gallons per min
= 0 gallons per minute

We could have done this net flow calculation for all points between 0 and 10 minutes but it is easier to simply note that any time you subtract two lines you get another line. Thus, by calculating the net flow at 0 and 10 minutes the flow can be represented as in Figure 14.

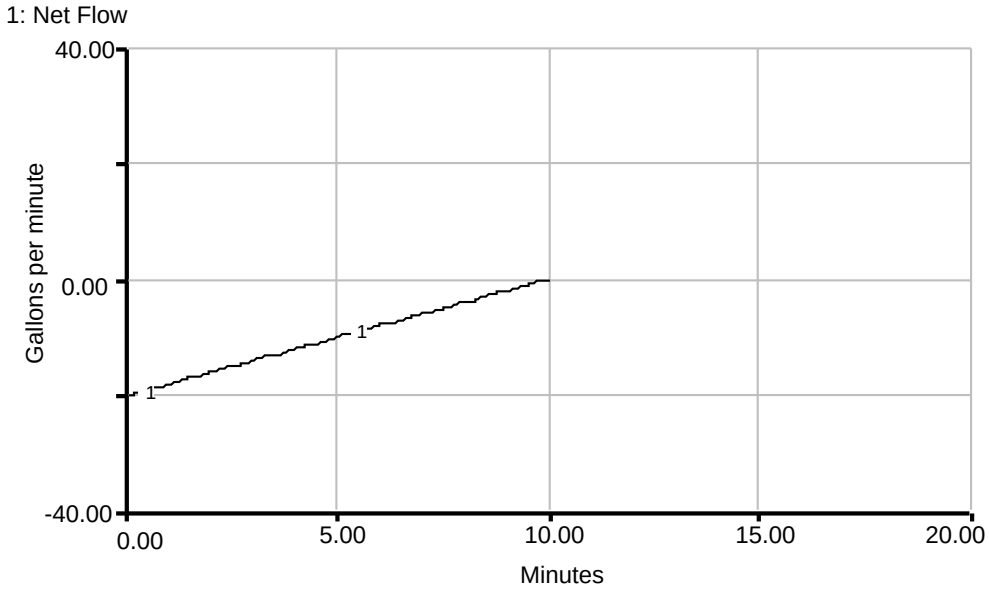


Figure 14: Graph of Net Flow from 0 to 10 Minutes.

For 10 minutes to 20 minutes we can repeat the same procedure. We already know that the net flow at 10 minutes is 0 gallons per minute.

At 15 minutes, the net flow is:

Net Flow at 15 min= Inflow at 15 min - Outflow at 15 min
Net Flow at 15 min= 25 gal per min - 10 gal per min
= 15 gallons per min

At 20 minutes, the net flow is:

Net Flow at 20 min= Inflow at 20 min - Outflow at 20 min
Net Flow at 20 min= 30 gal per min - 0 gal per min
= 30 gallons per min

With this information, and the fact that lines subtracted from one another always give us a line, we can draw the rest of the net flow graph. This is shown in Figure 15.

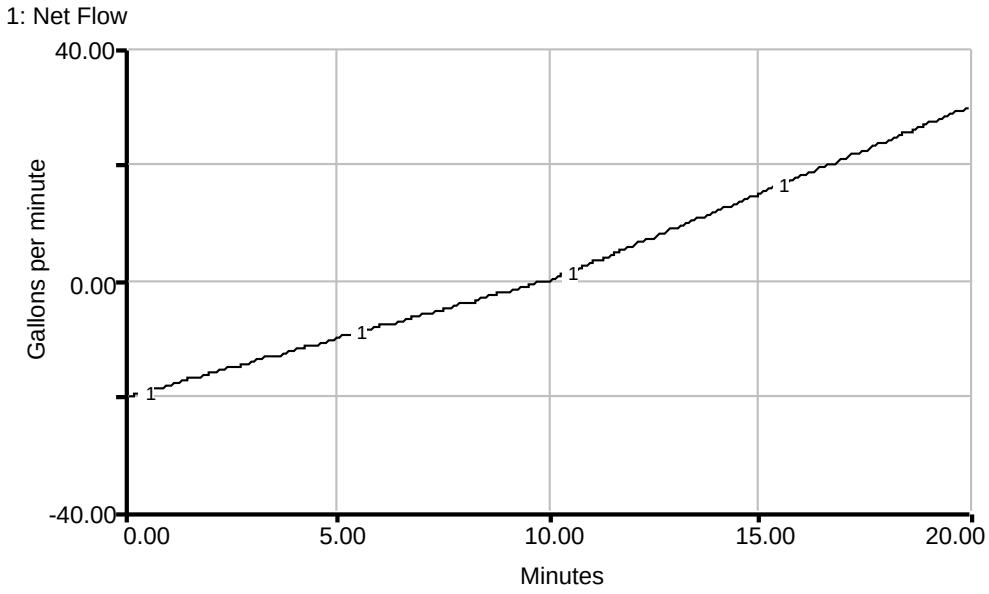


Figure 15: Graph of Net Flow

Now that we have the net flow, all we need to do to estimate the behavior of the system is calculate the areas between the net flow and the line flow = 0 gallons per minute.
While you should now be able to do this calculation yourself, Figure 16 presents the net flow graphs with all the areas calculated for you. (Remember, a negative net flow, which is an area below the line flow = 0, empties a stock, while a positive flow fills a stock.)

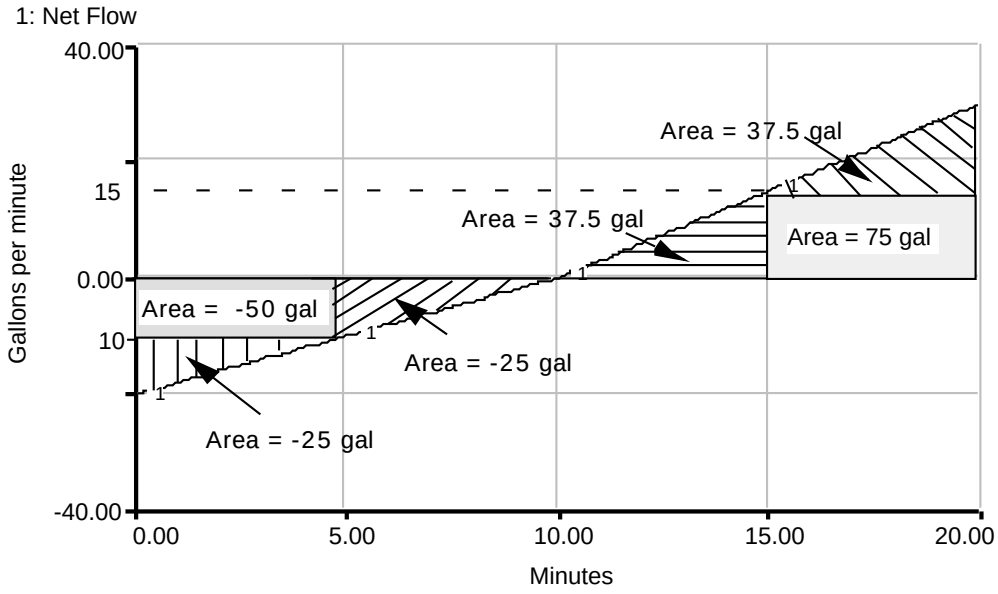


Figure 16: The Net Flow with Areas Calculated

With all the areas calculated for you, determining the behavior of the stock (the oil in the tank in this case) should be easy. For 0 to 5 minutes:

Change in value of Stock = area between Net Flow curve and Flow = 0
Change in value of Stock = Area of rectangle + Area of triangle
= (-50 gal) + (-25 gal) = –75 gall

Thus, the stock at 5 min is:

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 100 gal + (–75 gal) = 25 gal

We can repeat this type of calculation for the remainder of the net flow graph. If we do, we will see that the stock's behavior over time is shown in Figure 17.

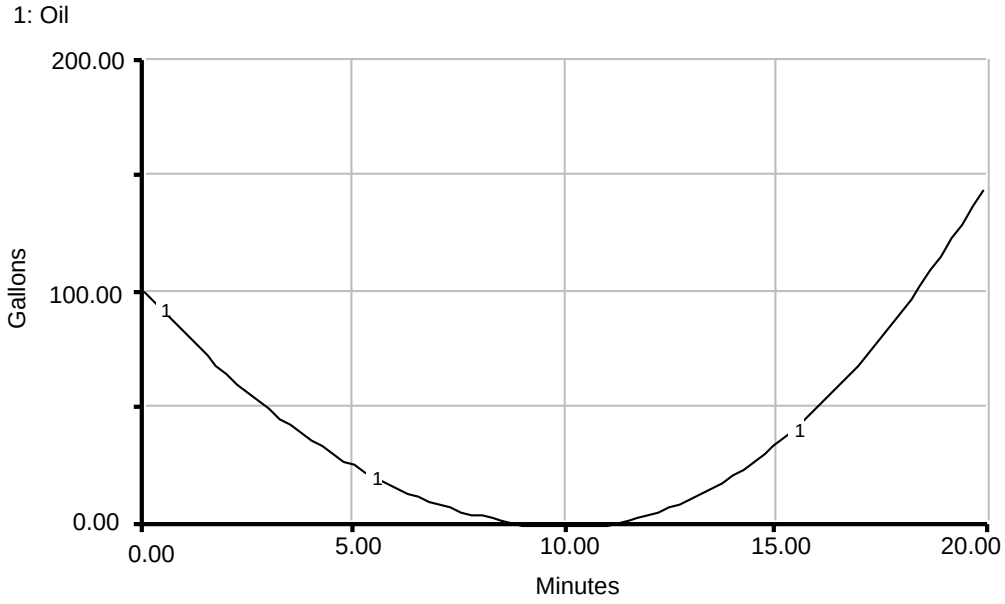


Figure 17: Behavior of the Stock with Variable Inflow and Outflow.

4. Key Ideas

Now that we have seen how stocks behave when the inflow and outflow are independent, we are ready to try some exercises. Anytime you are performing graphical integration, keep the following rules in mind:

1. **The area under the flow graph is equal to the change in the value of the stock over a period of time.**
2. **When the net flow is positive, stocks are filled; when the net flow is negative, stocks are emptied.**
3. **The value of the net flow gives the slope of the stock at that instant of time.**

4. **Constant flows cause the stock to increase linearly, with the slope of the stock equal to the value of the flow.**
5. **Linearly increasing and decreasing flows cause the stock to exhibit parabolic growth.**
6. **You can often break up complicated graphs into several small, simpler ones.**

And from this paper:

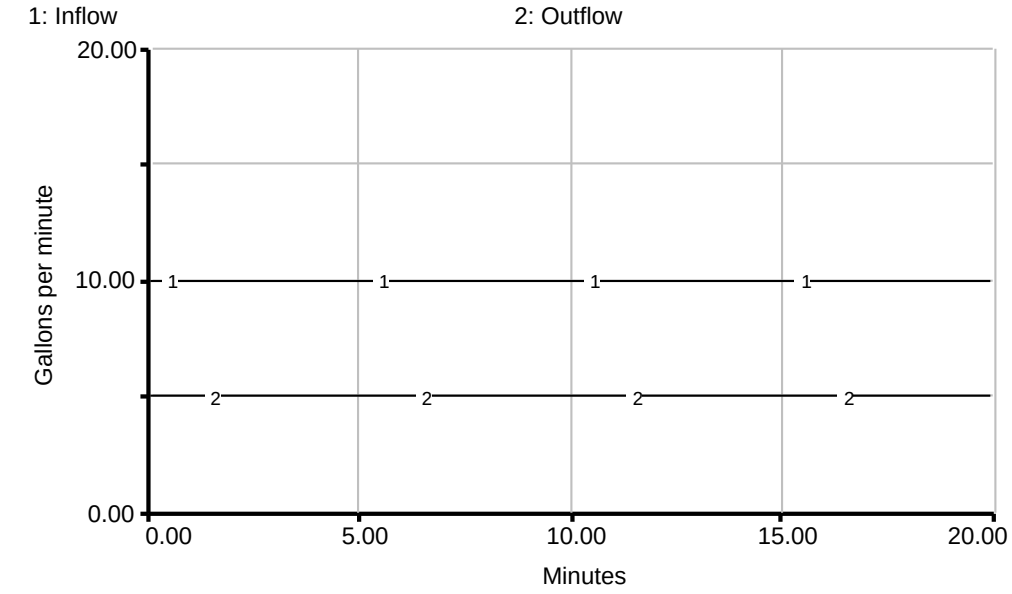
7. **Separate inflows and outflows can be used to form a net flow, which can then be graphically integrated by calculating the area between the flow curve and the line representing flow = 0. (Remember that area is the CHANGE in the value of the stock.)**
8. **Separate inflows and outflows can also be graphically integrated by simply finding the area between the two curves. (That area is the CHANGE in the value of the stock.)**

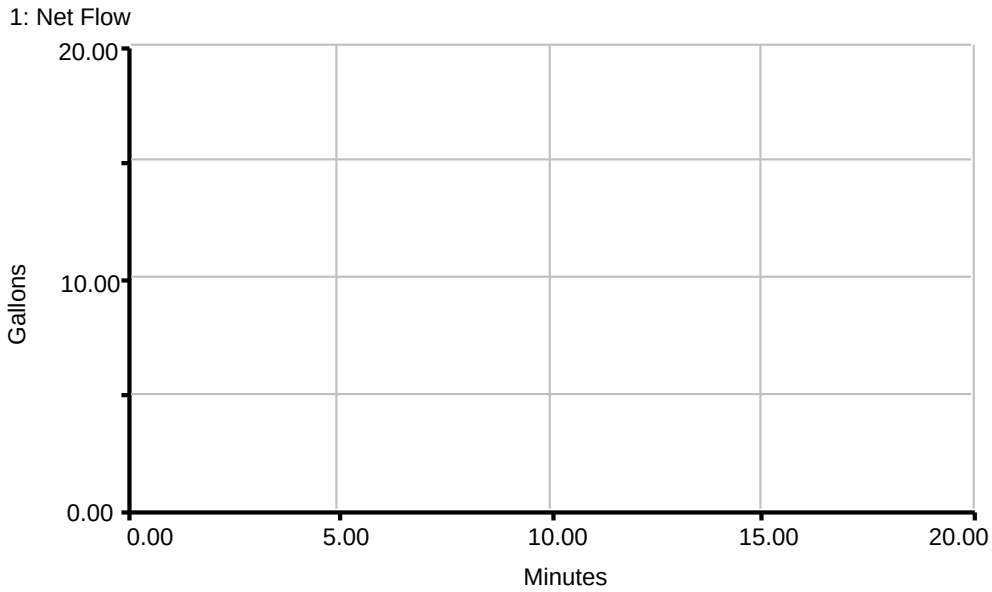
Now, let's try some exercises!!!

5. Exercises

5.1 Calculating a Net Flow

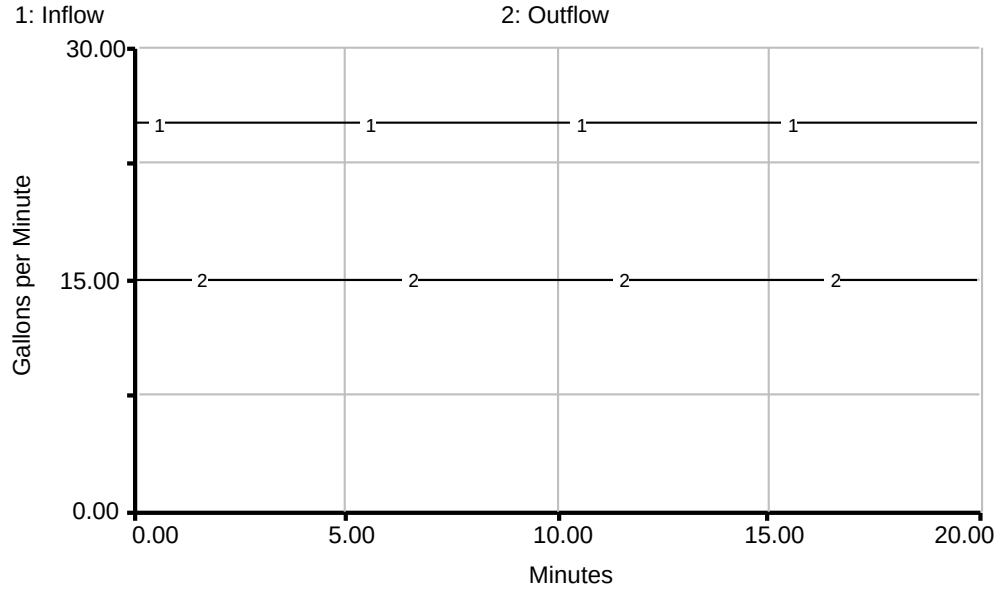
Calculate and sketch on the blank graph pad provided the net flow of oil into a storage tank where oil is being pumped in at a rate of 10 gallons per minute and being pumped out at a rate of 5 gallons per minute. These rates are shown below:

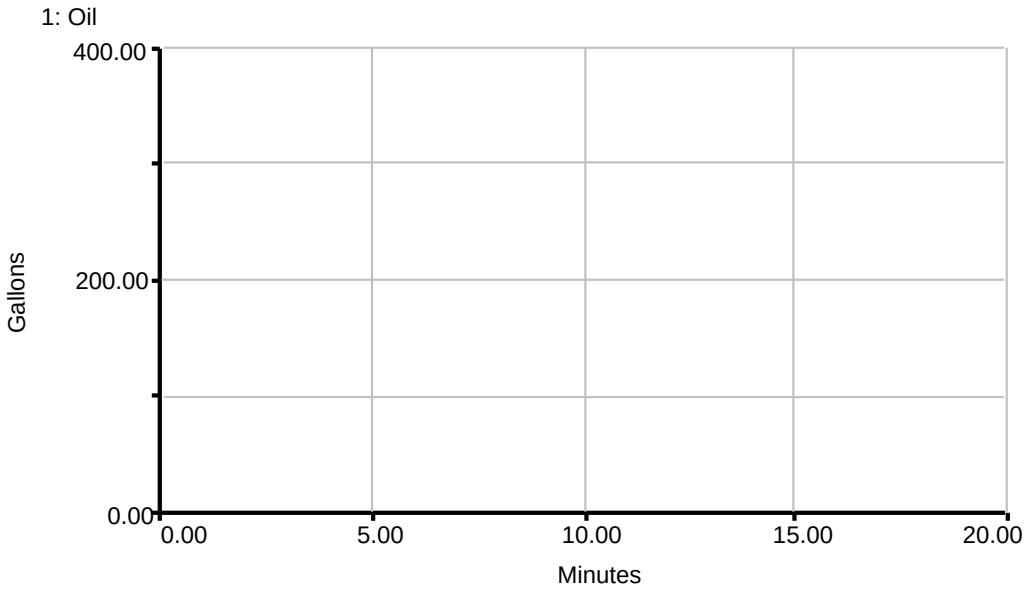




5.2 Constant Flows

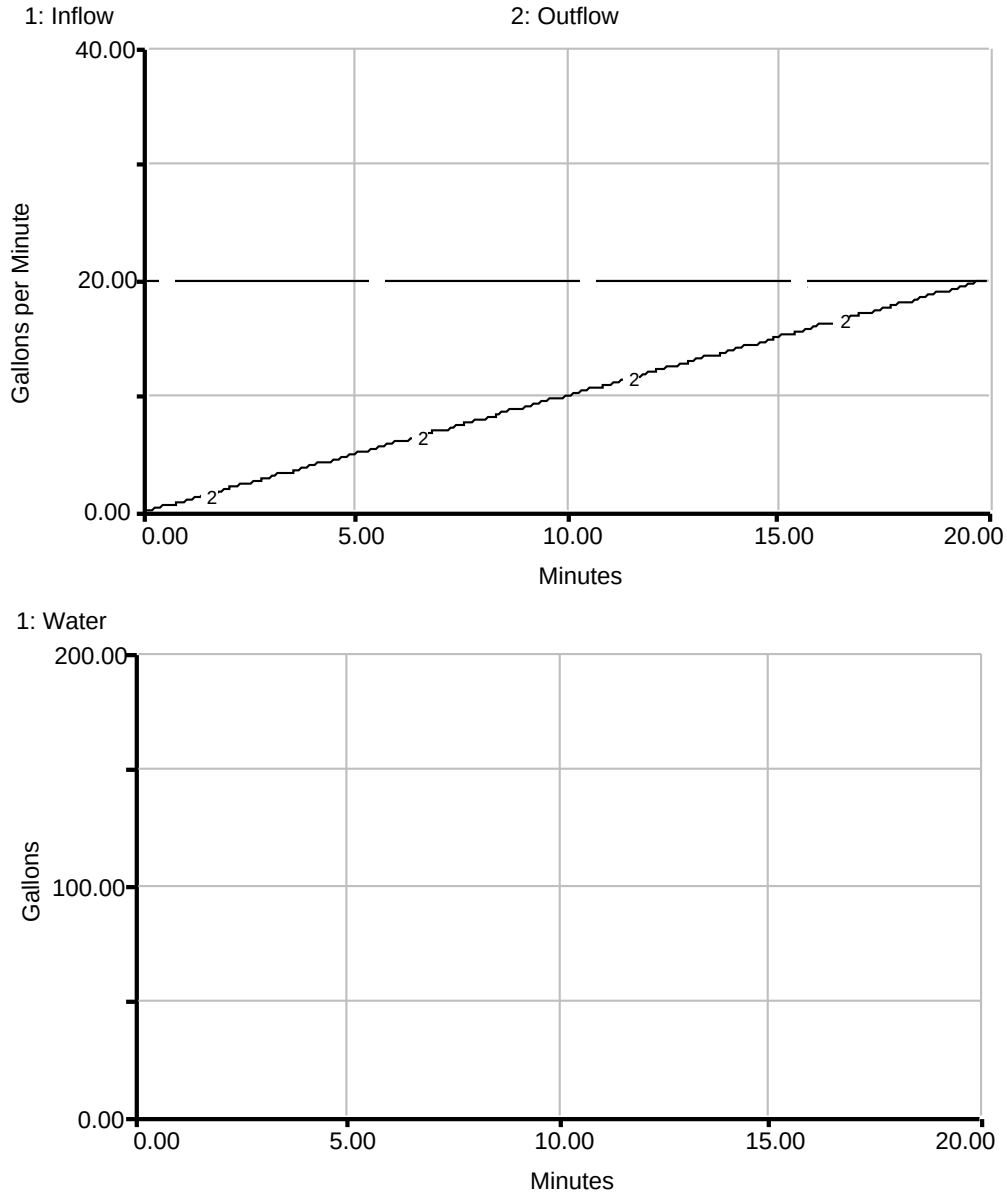
For this example, let us again look at an oil storage tank. However this time, determine the behavior of the stock (i.e. the amount of oil over time) if the pumping rate into the tank is 25 gallons per minute and oil is being drained out at 15 gallons per minute. Assume that there are initially 50 gallons of oil in the tank.





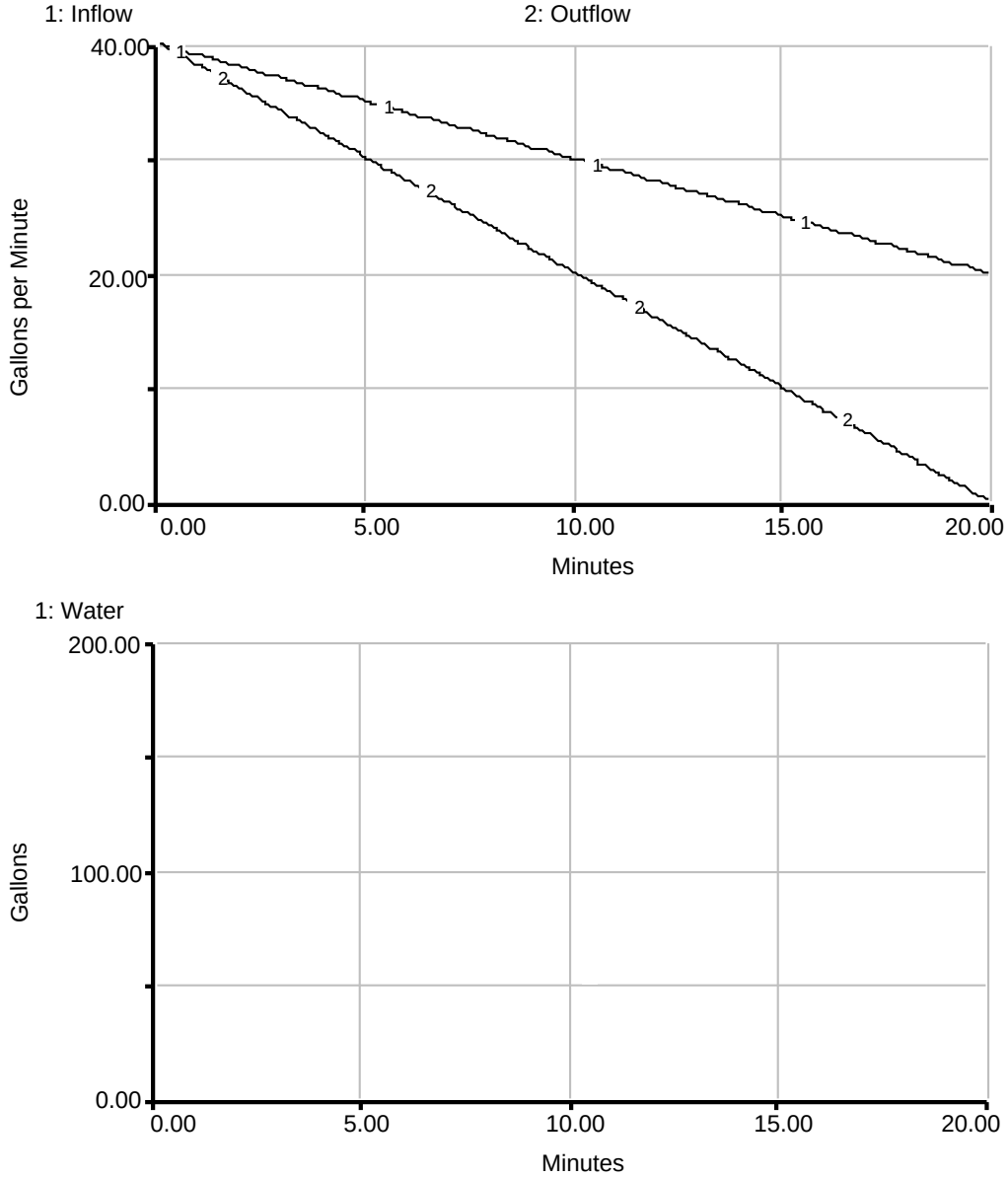
5.3 Constant Inflow, Variable Outflow

Now let us look at a slightly more complicated system. Consider a water storage tank that is filled at a constant flow of 20 gallons per minute. Water is pumped out of the tank at a linearly increasing flow, beginning at 0 gallons per minute at 0 minutes and increasing to 20 gallons per minute by 20 minutes (i.e. a slope of +1). These flows are graphed below. The tank was empty at time 0. Graph the level of water in the tank versus time.



5.4 Variable Inflow and Outflow

For the last exercise, let's consider a water storage tank whose inflow is 40 gallons per minute at 0 minutes and decreases linearly to 20 gallons per minute by 20 minutes. The outflow also starts at 40 gallons per minute, but decreases (linearly) much faster, reaching zero by 20 minutes. These flows are shown below. Determine the behavior of the stock (the amount of water in the tank) over time and graph it on the graph pad provided. Assume the tank was initially empty.



6. Conclusion

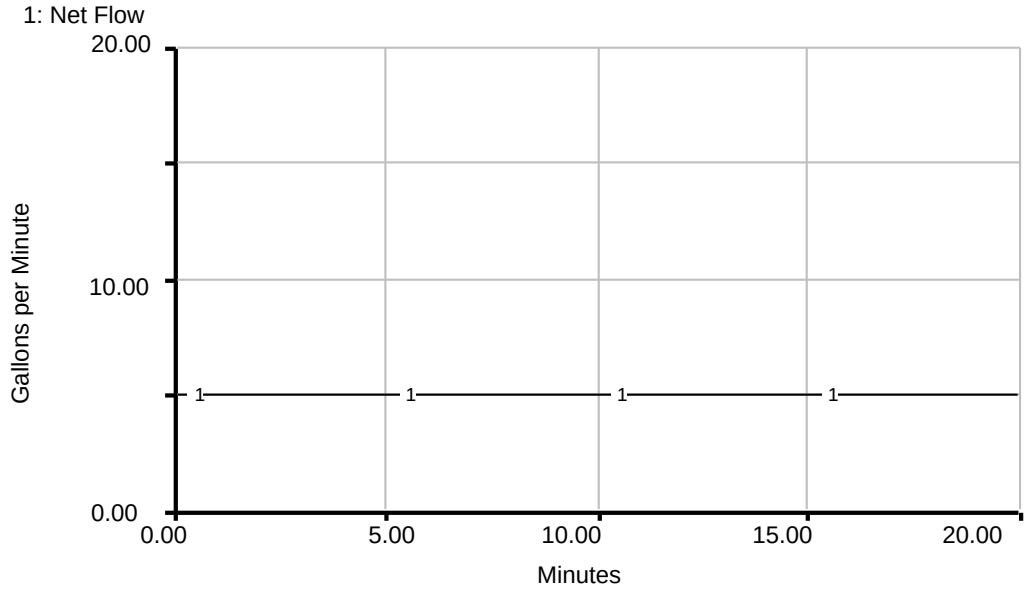
In this paper we looked at graphical integration with independent inflows and outflows. You should now feel comfortable using graphical integration as a method for estimating the behavior of systems with constant flows, step function flows, linearly increasing and decreasing flows with a single net inflow, or independent inflows and outflows.

If you do not feel you have a strong understanding of graphically integrating systems with separate inflows and outflows, please review this paper before continuing along in Road Maps.

7. Appendix: Answers to Exercises

7.1 Answer to Section 5.1

The net flow is shown below:



Remember, the net flow is simply:

Net Flow = Inflow - Outflow

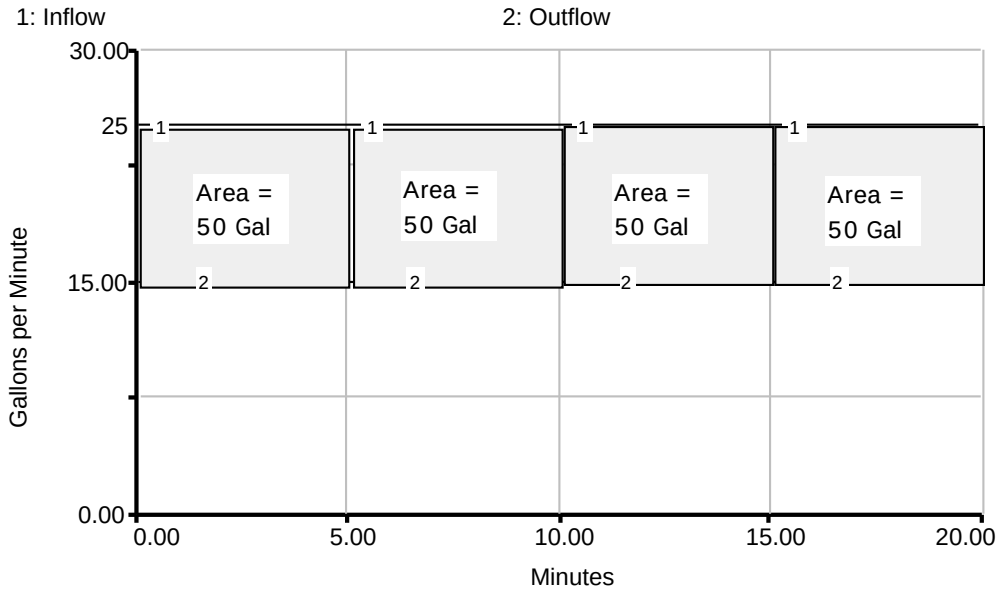
At all times in this example, the inflow was 10 gal per min and the outflow was 5 gal per min. Thus, the net flow, at all times, can be shown to be:

Net Flow = 10 gal per min - 5 gal per min = 5 gal per min

7.2 Answer to Section 5.2

For this example, let us solve it directly without calculating a net flow. Instead, we will use the fact that the area between the inflow and outflow is the change in the value of the stock. Below is the graph of the inflow and outflow divided into four sections. The area of the sections was calculated using:

Area = length * width = 5 minutes * 10 Gallons per minute = 50 gallons



Since this area is the change in the value of the stock (the amount of oil in the storage tank in this system), the value of the stock at 5 minutes can be computed from:

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
= Stock at 0 min + area between inflow and outflow curve from 0 to 5 min
= 50 gallons + 50 gallons = 100 gallons

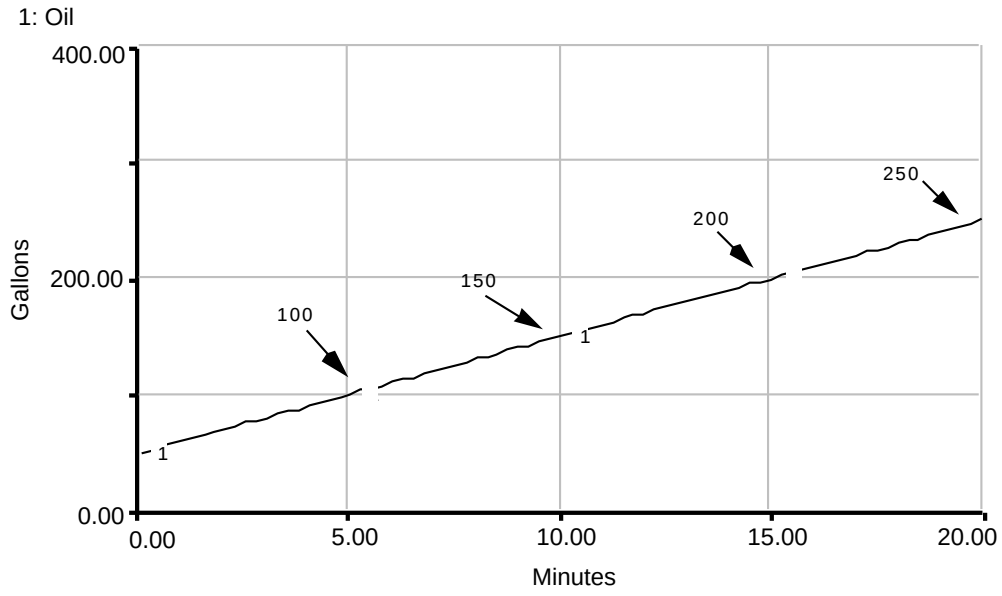
To find the value of the stock at 10 minutes, simply repeat this process:

Stock at 10 min = Stock at 5 min + change in value of Stock from 5 to 10 min
= Stock at 5 min + area between inflow and outflow curve from 5 to 10 min
= 100 gallons + 50 gallons = 150 gallons

To find the value of the stock at 15 minutes, again repeat this process:

Stock at 15 min = Stock at 10 min + change in value of stock from 10 to 15 min
= Stock at 10 min + area between inflow and outflow curve from 10 to 15 min
= 150 gallons + 50 gallons = 200 gallons

Finally, by the same method the value of the stock at 20 minutes is 250 gallons. The behavior is thus shown below.



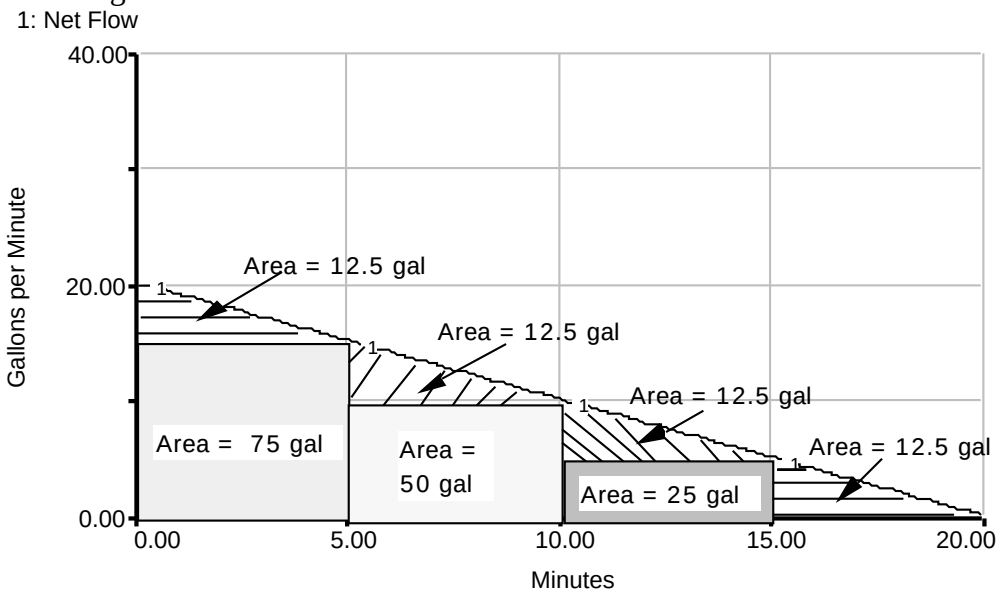
7.3 Answer to Section 5.3

Let's solve this problem by first calculating the net flow. To do this, all we need to do is use the formula:

Net Flow = Inflow - Outflow

At 0 minutes:
Net Flow at 0 min = Inflow at 0 min - Outflow at 0 min
= 20 gal per min - 0 gal per min = 20 gal per min
At 5 minutes:
Net Flow at 5 min = Inflow at 5 min - Outflow at 5 min
= 20 gal per min - 5 gal per min = 15 gal per min
At 10 minutes:
Net Flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 20 gal per min - 10 gal per min = 10 gal per min
At 15 minutes:
Net Flow at 15 min = Inflow at 15 min - Outflow at 15 min
= 20 gal per min - 15 gal per min = 5 gal per min
At 20 minutes:
Net Flow at 20 min = Inflow at 20 min - Outflow at 20 min
= 20 gal per min - 20 gal per min = 0 gal per min

Now, we can draw the net flow graph (below) and calculate the area under it to determine the change in the stock value.



Remember, the area between the net flow and the line flow = 0 is the change in the value of the stock. Thus:

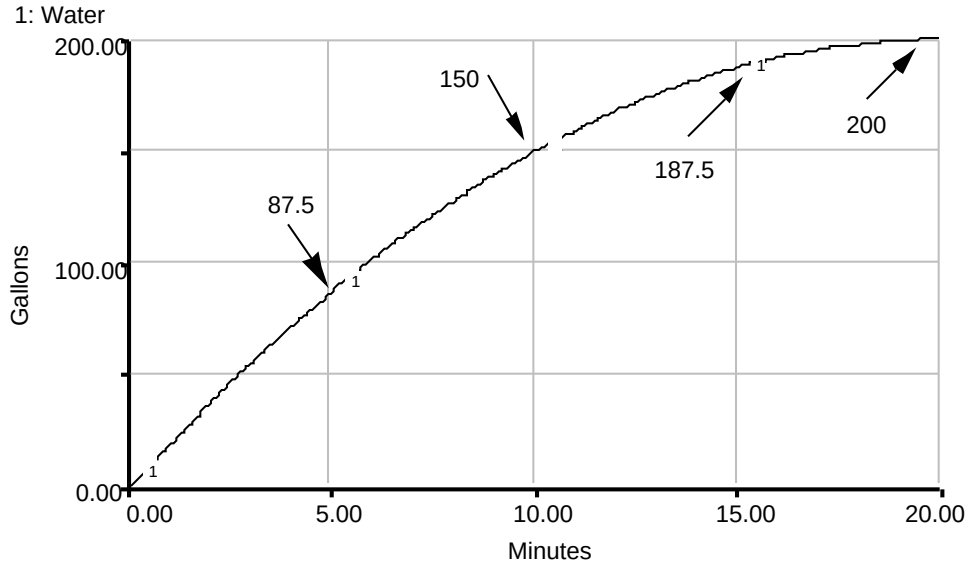
Stock at 5 min = Stock at 0 min + Change in value of Stock from 0 to 5 min
= Stock at 0 min + Area under curve from 0 to 5 min
= Stock at 0 min + (Area of Rectangle + Area of Triangle)
= 0 gal + (75 gal + 12.5 gal) = 87.5 gallons

Stock at 10 min = Stock at 5 min + Change in value of Stock from 5 to 10 min
= Stock at 5 min + Area under curve from 5 to 10 min
= Stock at 5 min + (Area of Rectangle + Area of Triangle)
= 87.5 gal + (50 gal + 12.5 gal) = 150 gallons

Stock at 15 min = Stock at 10 min + Change in value of Stock from 10 to 15 min
= Stock at 10 min + Area under curve from 10 to 15 min
= Stock at 10 min + (Area of Rectangle + Area of Triangle)
= 150 gal + (25 gal + 12.5 gal) = 187.5 gallons

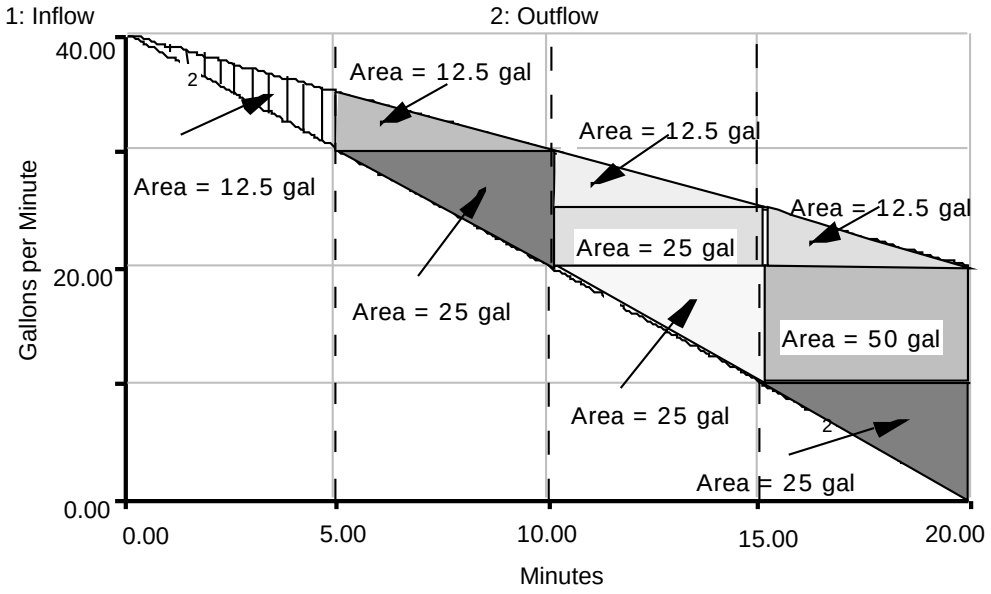
Stock at 20 min = Stock at 15 min + Change in value of Stock from 15 to 20 min
= Stock at 15 min + Area under curve from 15 to 20 min
= Stock at 15 min + Area of Triangle
= 187.5 gal + 12.5 gal = 200 gallons

We can now draw the behavior of the stock by plotting these points and connecting them.



7.4 Answer to Section 5.4

Let us solve this problem by finding the area between the inflow and outflow curves, and not bother calculating a net flow. Below is a picture of the flows with the area under the curve divided into rectangles and triangles. Also, the areas of these rectangles and triangles are calculated.



We will use these areas to calculate the change in the value of the stock for various time periods, just as we did for the previous examples.

To calculate the value of the stock at 5 minutes, we can use:

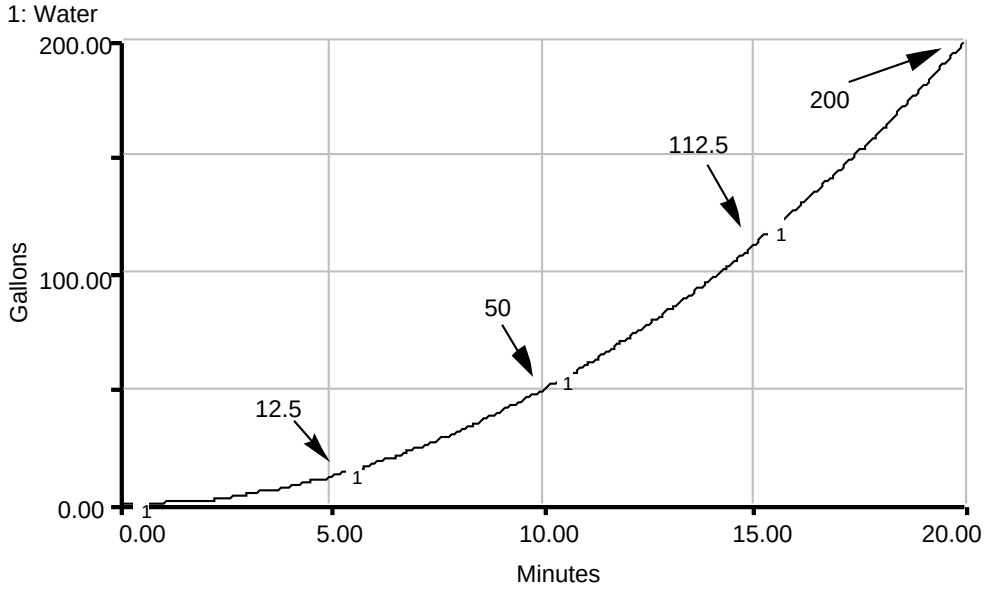
Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 0 gallons + Area between inflow and outflow
= 0 gallons + Area of Triangle
= 0 gallons + 12.5 gallons = 12.5 gallons

Stock at 10 min = Stock at 5 min + change in value of Stock from 5 to 10 min
Stock at 10 min = 5 gallons + Area between inflow and outflow
= 12.5 gallons + Area of Top Triangle + Area of Bottom Triangle
= 12.5 gallons + 12.5 gallons + 25 gallons = 50 gallons

Stock at 15 min = Stock at 10 min + change in value of Stock from 10 to 15 min
Stock at 15 min = 50 gallons + Area between inflow and outflow
= 50 gallons + Area of Top Triangle + Area of Rectangle + Area of Bottom Triangle
= 50 gallons + 12.5 gallons + 25 gallons + 25 gallons
= 112.5 gallons

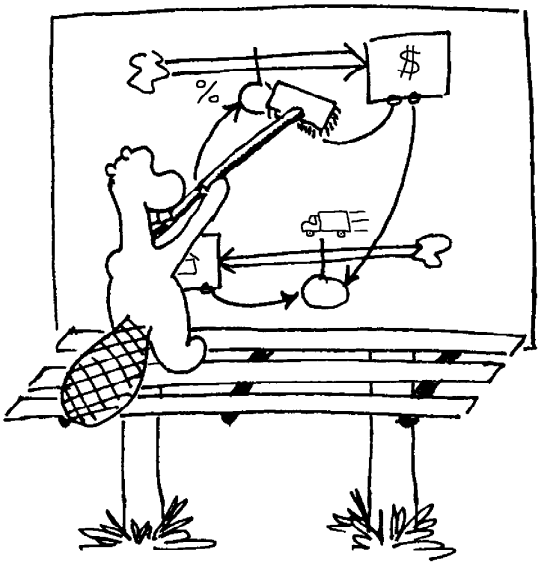
Stock at 20 min = Stock at 15 min + change in value of Stock from 15 to 20 min
Stock at 20 min = 112.5 gallons + Area between inflow and outflow
= 112.5 gallons + Area of Top Triangle + Area of Rectangle + Area of Bottom Triangle
= 112.5 gallons + 12.5 gallons +50 gallons + 25 gallons
= 200 gallons

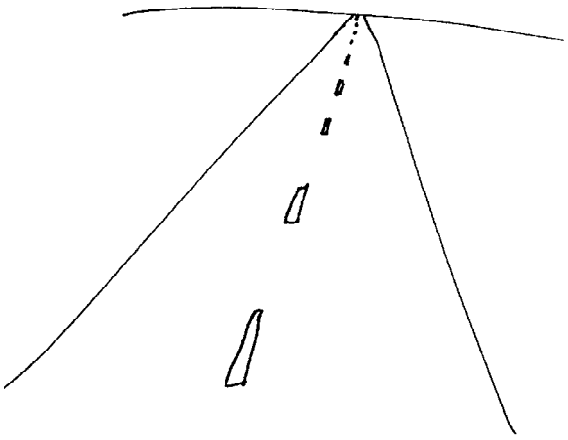
This behavior is shown below.



Road Maps 6

A Guide to Learning System Dynamics





System Dynamics in Education Project

Road Maps 6

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

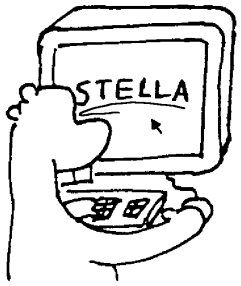
July 18, 1993
Latest Revision March 18, 2001

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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Six!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. Road Maps Six is the sixth in the series of chapters in Road Maps. Road Maps One through Three gave you a broad introduction to the field of system dynamics, focusing on understanding the structure and behavior of systems through positive and negative feedback loops. Road Maps Four introduced generic (or transferable) structures, and gave you a taste of policy analysis using the Fish Banks model. Road Maps Five introduced the use of delays in computer models, explained the transferable structures causing S-shaped growth, looked at the spread of an epidemic, and started exploring model validity.

In Road Maps Six, you will be able to apply your skills to modeling economic supply and demand. We introduce a transferable structure that produces oscillation, with two real-life oscillating systems presented as examples, and we develop the study of generic structures producing S-shaped growth. Road Maps Six also provides exercises that allow you to create your own models from given information. Finally, *Systems thinking: critical thinking skills for the 1990s and beyond* focuses on addressing current global problems using critical thinking skills acquired through systems thinking.

Topics Covered in Road Maps Six

System Dynamics in Economics

- *Economics Supply and Demand* (D-4388)
  by Kamil Msefer and Joseph Whelan

Transferability of Structures

- *Generic Structures in Oscillating Systems I* (D-4426-1)
  by Celeste Chung

-*Exploring S-Shaped Growth* (D-4476)
by Leslie Martin

Modeling Exercises

- *Modeling Exercises: Section 1* (D-4421)
by Joseph Whelan

Critical Thinking Skills

- *Systems thinking: critical thinking skills for the 1990s and beyond* (D-4565)
by Barry Richmond

Things You'll Need for Road Maps Six

Modeling Software

In order to complete Road Maps Six and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on

Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

How to Use Road Maps Six

Road Maps Five explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Most of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times.

Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about the principle. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

System Dynamics in Economics

An important application of system dynamics is to economics. Here, we propose that a system dynamics perspective on supply and demand can describe mechanisms which most classical economics texts fail to explain.

- *Economics Supply and Demand*[1] by Kamil Msefer and Joseph Whelan

In this paper, we look at one of the fundamental tenets of economics: supply and demand theory. First, the classical economic model of supply and demand is explained. Next, a system dynamics approach is proposed as a way of better understanding the mechanism of supply and demand. With this background, you will then build a simple model and complete some exercises to improve your understanding of these concepts.

Please read *Economics Supply and Demand* now.

After reading *Economics Supply and Demand...*

This paper explained the basics of supply and demand as well as the distinction between classical economic theory and the premise that system dynamics makes about supply and demand. The model you built demonstrates the dynamics of supply and demand in action. We encourage you to continue to add to this model as you make observations about how a real-life economic system might behave to

---

[1] Kamil Msefer and Joseph Whelan, 1995. *Economic Supply and Demand* (D-4388), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 36 pp.

certain changes. For instance, how would you account for the effect of available inventory on demand?

The following system principle reviews an important concept introduced in the paper you just read.



System Principle #13:
Solution interval DT is in all level equations and no others.

The DT (also called the solution interval, period of measurement, delta time, or time step) is the time period in which the level is changed by the rate. In the Supply and Demand model, measurements are made weekly; thus DT = 1 week. DT is multiplied by the current rate and added to the existing level value—determining the level value after the next DT of time. This process can be seen in the level equations included within the model equations (found in the Appendix). The DT is essential to the level equation.

Transferability of Structures

In Road Maps Chapter 5, you continued to develop the idea of generic, or transferable, structures through *Generic Structures: S-Shaped Growth I*. The idea of a transferable structure is important in system dynamics because it allows you to estimate the behavior of systems from different disciplines through understanding the behavior of its underlying generic structures.

? ***Generic Structures in Oscillating Systems I***[2]

by Celeste Chung

This paper introduces oscillation as a behavior that can be modeled using a transferable structure. A swinging pendulum and employment instability are two

[2] Celeste Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 17, 24 pp.

real-life scenarios used to illustrate the transferability of this oscillation structure. While reading this paper, try to think of other systems which oscillate.

Please read and do the exercises in *Generic Structures in Oscillating Systems I* now. The following tips may help you.

Tips for modeling in *Generic Structures in Oscillating Systems I.*

In order to observe the sustained oscillations in your model of the system, choose the Runge-Kutta integration method instead of the usual Euler's method. To do this, open the Time Specs dialog box from the Run menu, and select the Runge-Kutta 4 integration method.

It is also necessary to make sure that both of your stocks can become negative, and that the flows are biflows. This is an important modeling rule which should be followed when building any model.

Also, to make the curves look smoother, choose a smaller solution interval, such as DT = 0.1, from the Time Specs dialog box.

After reading and doing the exercises in *Generic Structures in Oscillating Systems I*...

In addition to the first-order positive and negative feedback loops introduced in Road Maps Four, we have so far presented three different transferable structures: two that produce S-shaped growth (from Road Maps 5), and one that produces sustained oscillation. Modeling the oscillation structure on STELLA by yourself will improve your understanding of the generic structure. Experiment by changing variables like time gap or initial velocity, and see if the changes in behavior fit your mental model.

The following system principles which were introduced in the generic structures paper you just read are important concepts to remember in thinking systemically.



System Principle #14:

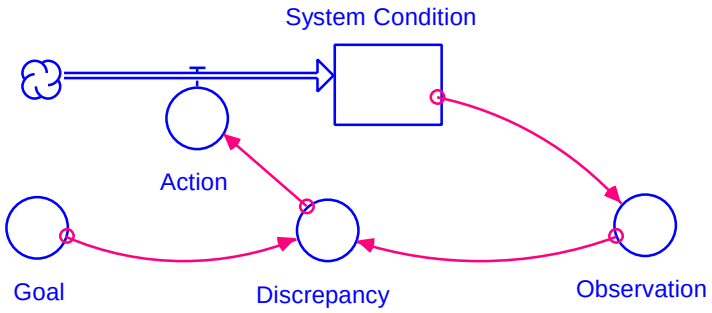
Simple, second-order negative loops exhibit sinusoidal oscillation.

All of the systems in *Generic Structures in Oscillating Systems I* are second-order negative loops, and they all oscillate. The oscillation is independent of the values of parameters; it is due to having the same qualitative structure. Any second-order, negative loop with no minor loops oscillates as a sustained sinusoid.



System Principle #15:

Goal, observation, discrepancy, and action create a system substructure.



A policy or rate equation recognizes a local goal toward which that decision point strives, compares the goal with the apparent system condition to detect a discrepancy, and uses the discrepancy to guide action.

Notice Figures 3, 5, and 7 of *Generic Structures in Oscillating Systems I*. In Figure 3, and the accompanying text, we see that the goal is **Desired Position** (namely x=0), the observation is **Position**, the discrepancy is measured in **Gap** between **Desired Position** and **Position**, and the action taken is to alter **Change in Velocity**, which changes **Velocity** and consequently **Position**. Similar processes occur within the systems seen in Figures 5 and 7. This system substructure occurs often and will be further explored later in Road Maps.

***- Exploring S-Shaped Growth*[3]**

by Leslie Martin

In Road Maps Five, we introduced two generic structures that produce S-shaped growth. The following paper describes an example of the same type of behavior and provides exercises to experiment with S-shaped growth. Positive and negative loops are shown to interact with each other to generate S-shaped behavior.

Please read *Exploring S-Shaped Growth* now.

After reading *Exploring S-Shaped Growth...*

This paper discussed a generic structure producing S-shaped growth, and provided several examples of real-world systems that exhibit S-shaped growth. Can you think of any other systems with the same structure that produce S-shaped growth? It is important to remember that generic structures are useful because they let us apply our knowledge of one system to understand the behavior of other systems with the same structure.

Modeling Exercises

So far, we've guided you through the model-building process. We gave you the levels and rates, suggested links and hinted at equations. Now, in the following model exercises, you will formulate your own models using given descriptions of systems.

***- Modeling Exercises: Section 1*[4]**

by Joseph Whelan

This is the first in a series of papers on modeling exercises to be included in Road Maps. Since this is your first independent modeling experience so far in Road Maps, the first part of *Modeling Exercises* addresses the process of model building and includes a some guidelines to follow. Then you are given three exercises, each with a description of a system which you are to model. You will then build a model

---

[3] Leslie Martin, 1996. *Exploring S-Shaped Growth* (D-4476), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 40 pp.

[4] Joseph Whelan, 1994. *Modeling Exercises: Section 1* (D-4421), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 19, 37 pp.

to simulate the system and use the model to help answer some exploration questions. At the end of the paper is one possible model for each of the system descriptions as well as answers to the exploration questions.

Please read *Modeling Exercises: Section 1* now.

After completing *Modeling Exercises: Section 1*...

This paper was your first opportunity in Road Maps to create a simple model yourself. It is necessary to understand all the questions and the models before moving on to *Modeling Exercises: Section 2* which will be in a later chapter of Road Maps. The modeling skills you have accumulated so far should be sufficient for you to begin exploring and creating models of your own.

Now is a good time to discuss a recurrence of the following system principle. Note the application of this principle in Model 1 of *Modeling Exercises* which you just completed.



System Principle #7:

Rates depend only on Levels and Constants.

Look at the author's solution to Model 1 in *Modeling Exercises* (the model diagram is on p. 20). Note that the **Hatch Rate** is influenced only by the level **Penguins** and a constant, **Hatch Fraction**. **Death Rate** also only depends on the level and a constant. **Death Rate** may appear to depend on **Death Fraction**, but it is only a converter. The true dependence must be traced back to **Penguins** and **Area**.

The value of a rate variable depends only on present values of level variables and constants. No rate variable depends directly on any other rate variable. The rate equations (or policy statements) of a system are of simple algebraic form; they do not involve time or the solution interval; they are not dependent on their own past values.

Another principle that we have encountered before in Road Maps is now discussed further in the context of Model 2 of *Modeling Exercises.*



System Principle #8:

Decisions are always made within feedback loops.

Look at the author's solution to Model 2 in *Modeling Exercises* (the model diagram is on p. 25 and the equations are on pp. 28 - 30). The house heating/cooling appliances control the **Heating** and **Cooling** in/out flows for **Heat in House**. The decision of what action to make—whether the heater or air conditioner should be on or off—is dependent upon information in the system. In this case the decision depends upon the **Measured House Temperature** and the appliance settings.

Now notice that the action made—as a result of the decision—will change **Heat in House**; the house either heats up or cools down. This change in the stock affects the **House Temperature**, which affects the **Measured House Temperature**, which in turn will affect our decision!

Every decision process is made within at least one feedback loop.

We now introduce a new system principle.



System Principle #16:

Level variables and Rate variables must alternate.

Look at the author's solution to Model 3 in *Modeling Exercises* (on p. 31). Trace out the loop starting with the level **Predator Population** and continuing with the rate **Prey Deaths**. Note that as you move through the loop, the level and rate variables alternate. Next comes **Prey Population**, then **Predator Deaths**, and finally **Predator Population** again, for one loop. Converters are algebraically part of the rates to which they are connected.

Any path through the structure of a system encounters alternating level and rate variables. Recall **System Principles #5** and **#7**: Levels depend only on rates

and Rates depend only on levels (and constants), respectively. Thus for any loop in a system, if we start at a level variable, the next variable we reach cannot be another level; the next variable must be a rate.

Likewise, if we move through any loop starting from a rate variable, the next variable cannot be another rate (or even a constant, they are not influenced by any variable); the next variable must now be a level. This could be the same level, or it could be a new one. Try tracing out a loop in your own solution to the modeling exercise, and see for yourself!

Critical Thinking Skills

The world is becoming an ever-increasingly complex and interconnected place. With this growing complexity, it is imperative that we as global citizens keep pace with problems that arise as a result. How can we do this?

- Systems thinking: critical thinking skills for the 1990s and beyond[5]

by Barry Richmond

Dr. Barry Richmond is the managing director and founder of High Performance Systems, the company which produces STELLA II and *ithink* software. In his paper, he advocates the teaching of systems thinking through learner-centered learning as well as the application of systems thinking to the acquisition of various critical thinking skills (which he outlines). He argues that by their very nature, systems thinkers understand the interdependence of systems and can therefore "experience the morning after" in terms of predicting the outcome of current situations. As you read his paper, ask yourself whether you agree or disagree with what he says based on your system dynamics experience so far.

Please read *Systems thinking...* now.

After reading *Systems thinking...*

We now review a system principle which was introduced earlier in Road Maps.

---

[5] Barry Richmond, 1993. *Systems thinking: critical thinking skills for the 1990s and beyond* (D-4565), System Dynamics Review Vol. 9, no. 2 (Summer 1993), pp. 113-133.



System Principle #12:

Variables have the same units within conservative subsystems.

All levels within conservative subsystems—levels connected by flows—must have the same units of measure. The rates of the flows connecting these levels have units of measure equal that of the levels divided by time.

Figures 12 and 13 in *Systems thinking: critical thinking skills for the 1990s and beyond* readily demonstrate this principle. **Empty Bottles, Bottles being Filled**, and **Inventory of Filled Bottles** all have units of [bottles]. **New empties**, **bottles on conveyer**, **and filled bottles** all have units of [bottles/time]. All together, these connected variables form a conservative subsystem. However, **Liquid in Vat** is not a part of this subsystem; it does not have the same units of measure (instead, it has units [liters]). This is why it is incorrect **for liquid into bottles** to flow into **Bottles being filled**: the flow's units are incorrectly matched.

Finishing off Road Maps Six

Road Maps Six emphasized that system dynamics has a wide variety of applications. For example, system dynamics can facilitate and improve understanding of economics.

The concept of a transferable structure is crucial in system dynamics. Oscillation is a common behavior in systems, and the transferable structure which causes it will appear time and time again. What other common behaviors are caused by a transferable structure? We will introduce more of these structures in future Road Maps. In Road Maps Six, you have also seen another structure that produces S-shaped growth. You also got your first taste of independent modeling through *Modeling Exercises: Section 1*.

In Road Maps Six, we presented four new system principles: a) that the solution interval DT is in all level equations but no others; b) that simple, second-order negative loops exhibit sinusoidal oscillation; c) that goal, observation, discrepancy, and action create a system substructure; and d) that level variables and rate variables must alternate.

Finally, Road Maps Six also explained that system dynamics can help in finding long-range solutions to global problems in the years to come.

Key Terms and Concepts:

closed-loop thinking
continuum thinking
demand
dimensional inconsistency
diminishing marginal utility
dynamic thinking
generic thinking
multiplier
operational thinking
oscillation
scientific thinking
structural thinking
supply
time constant
transferability



ECONOMIC SUPPLY & DEMAND

by
Joseph Whelan
Kamil Msefer

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Professor Jay W. Forrester

January 14, 1996
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1. ABSTRACT

The main purpose of this paper is to discuss supply and demand in the framework of system dynamics. We first review classical supply and demand. Then we look at how to model supply and demand using system dynamics. Finally, we present a few exercises that will improve understanding of supply and demand and help improve system dynamics modeling skills.

2. INTRODUCTION

This paper emerged as an attempt to use system dynamics to model supply[1] and demand. Classical economics presents a relatively static model of the interactions among price, supply and demand. The supply and demand curves which are used in most economics textbooks show the dependence of supply and demand on price, but do not provide adequate information on how equilibrium is reached, or the time scale involved. Classical economics has been unable to simplify the explanation of the dynamics involved. Additionally, the effects of excess or inadequate inventory are often not discussed.

In the real world, the market price is affected by the inventory of goods held by the manufacturers rather than the rate at which manufacturers are supplying goods.[2] If the manufacturers are supplying goods at a rate equal to the consumer demand, the static classical theory would propose that the market is in equilibrium. However, what if there is a tremendous surplus in the store supply rooms? The manufacturers will lower the price and/or decrease production to return inventory to a desired level.

This paper introduces a model that incorporates elements from classical economics as well as several real-world assumptions. This model will be used to examine some of the interactions among supply, demand and price.

---

[1] Supply and production are very similar terms and are often used interchangeably.

[2] Low, Gilbert W. (1974). Supply and Demand in a Single-Product Market (Exercise Prepared for the Economics Workshop of the System Dynamics Conference at Dartmouth College, Summer 1974) (Department Memorandum No. D-2058). M.I.T., System Dynamics Group.

3. CONVENTIONAL SUPPLY AND DEMAND

3.1 Introduction

This section deals with supply and demand as sometimes taught in high-school economics classes. The following descriptions of supply and demand assume a perfectly competitive market, rational consumers, and free entry and exit into the market. Economists also make the simplification that all factors other than price which affect the quantity of goods sold and purchased are held constant. Economists argue that this is a valid assumption because changes in price occur much more quickly than changes in other factors that may affect supply or demand. Examples of these other factors include changes in taste, changes in the state of the economy and long-term changes in production capacity (such as the construction of a new factory).

3.2 Demand

Demand is the rate at which consumers want to buy a product. Economic theory holds that demand consists of two factors: *taste* and *ability to buy*. Taste, which is the desire for a good, determines the willingness to buy the good at a specific price. Ability to buy means that to buy a good at specific price, an individual must possess sufficient wealth or income.

Both factors of demand depend on the market price. When the market price for a product is high, the demand will be low. When price is low, demand is high. At very low prices, many consumers will be able to purchase a product. However, people usually want only so much of a good. Acquiring additional increments of a good or service in some time period will yield less and less satisfaction.[3] As a result, the demand for a product at low prices is limited by *taste* and is not infinite even when the price equals zero. As the price increases, the same amount of money will purchase fewer products. When the price for a product is very high, the demand will decrease because, while consumers may wish to purchase a product very much, they are limited by their *ability to buy*.

The curve in Figure 1 shows a generalized relationship between the price of a good and the quantity which consumers are willing to purchase in a given time period. This is known as a *simple demand curve*.

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[3] This behavior toward aquiring additional increments of a good is called *diminishing marginal utility*.

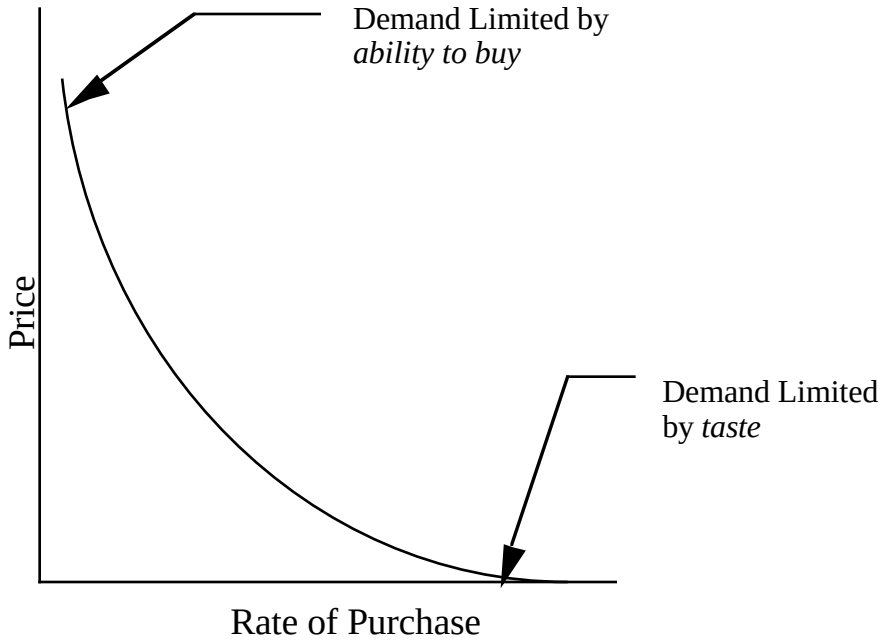


Figure 1: Demand Curve[4]

This curve shows the rate at which consumers wish to purchase a product at a given price.

The simple demand curve seems to imply that price is the only factor which affects demand. Naturally, this is not the case. Recall the assumption made by economists that the other factors which influence changes in demand act over a much larger time frame. These factors are assumed to be constant over the time period in which price causes supply and demand to stabilize.

[4] The reader should note that the convention in economic theory is to plot the price on the vertical axis and the rate of purchase on the horizontal axis.

3.3 Supply

Willingness and ability to supply goods determine the seller's actions. At higher prices, more of the commodity will be available to the buyers. This is because the suppliers will be able to maintain a profit despite the higher costs of production that may result from short-term expansion of their capacity[5].

In a real market, when the inventory is less than the desired inventory, manufacturers will raise both the supply of their product and its price. The short-term increase in supply causes manufacturing costs to rise, leading to a further increase in price. The price change in turn increases the desired rate of production. A similar effect occurs if inventory is too high. Classical economic theory has approximated this complicated process through the supply curve. The supply curve shown in Figure 2 slopes upward because each additional unit is assumed to be more difficult or expensive to make than the previous one, and therefore requires a higher price to justify its production.

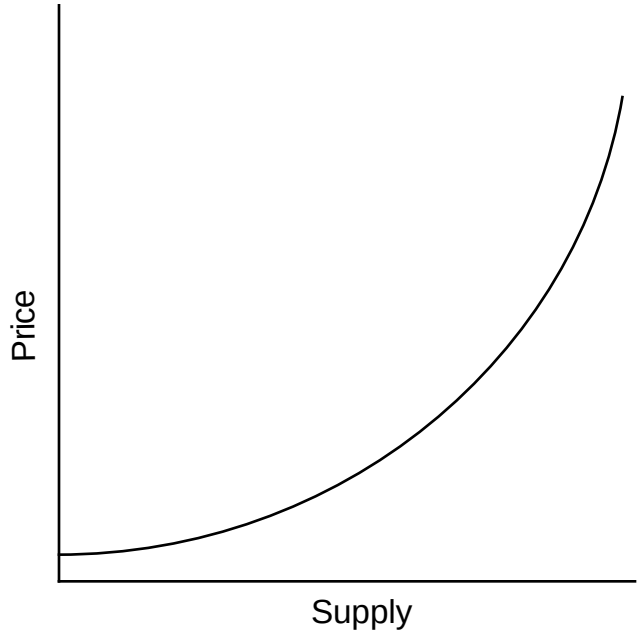


Figure 2: Supply Curve
At high prices, there is more incentive to increase production of a good. This graph represents the short-term approximation of classical economic theory.

[5]Short-term expansion can be achieved by giving workers overtime hours, contracting to an outside source, or increasing the load on current equipment. These types of changes increase per-unit supply costs.

3.4 Interaction Between Supply and Demand

Demand is defined as the quantity (or amount) of a good or service people are willing and able to buy at different prices, while supply is defined as how much of a good or service is offered at each price. How do they interact to control the market?

Buyers and sellers react in opposite ways to a change in price. When price increases, the willingness and ability of sellers to offer goods will increase, while the willingness and ability of buyers to purchase goods will decrease. To illustrate more clearly how the market works, we will look at the following example from the clothing industry.

Table 1 is called a schedule of demand and supply. For each price, it indicates how much clothing is demanded by the consumers per week, and how much clothing is supplied per week. Notice that as price decreases, demand increases and supply decreases. Eventually demand exceeds supply.

Demand and Supply Schedules

Price	Quantity Demanded (per week)	Quantity Supplied (per week)
$50	10	100
$45	14	97
$40	18	94
$35	22	89
$30	28	84
$25	35	77
$20	45	68
$15	57	57
$10	73	40
$5	100	0

Table 1: Demand and Supply Schedules

For each price, the schedule above indicates the quantity (in articles per week) of clothing demanded and supplied.

The market will reach equilibrium when the quantity demanded and the quantity supplied are equal. At $15, supply and demand are equal at 57 articles of clothing per week. To better understand the dynamics involved, suppose that one article of clothing was selling for $30. Producers would be willing to supply 84 articles of clothing per week, but consumers would only be buying 28 articles per week. As a result, the producers would have excess inventory piling up very quickly. In order to get their inventory back to the desired level, the suppliers would have to decrease production and reduce the price. Eventually, the quantity demanded and quantity supplied meet at 57 articles per week at a price of $15.

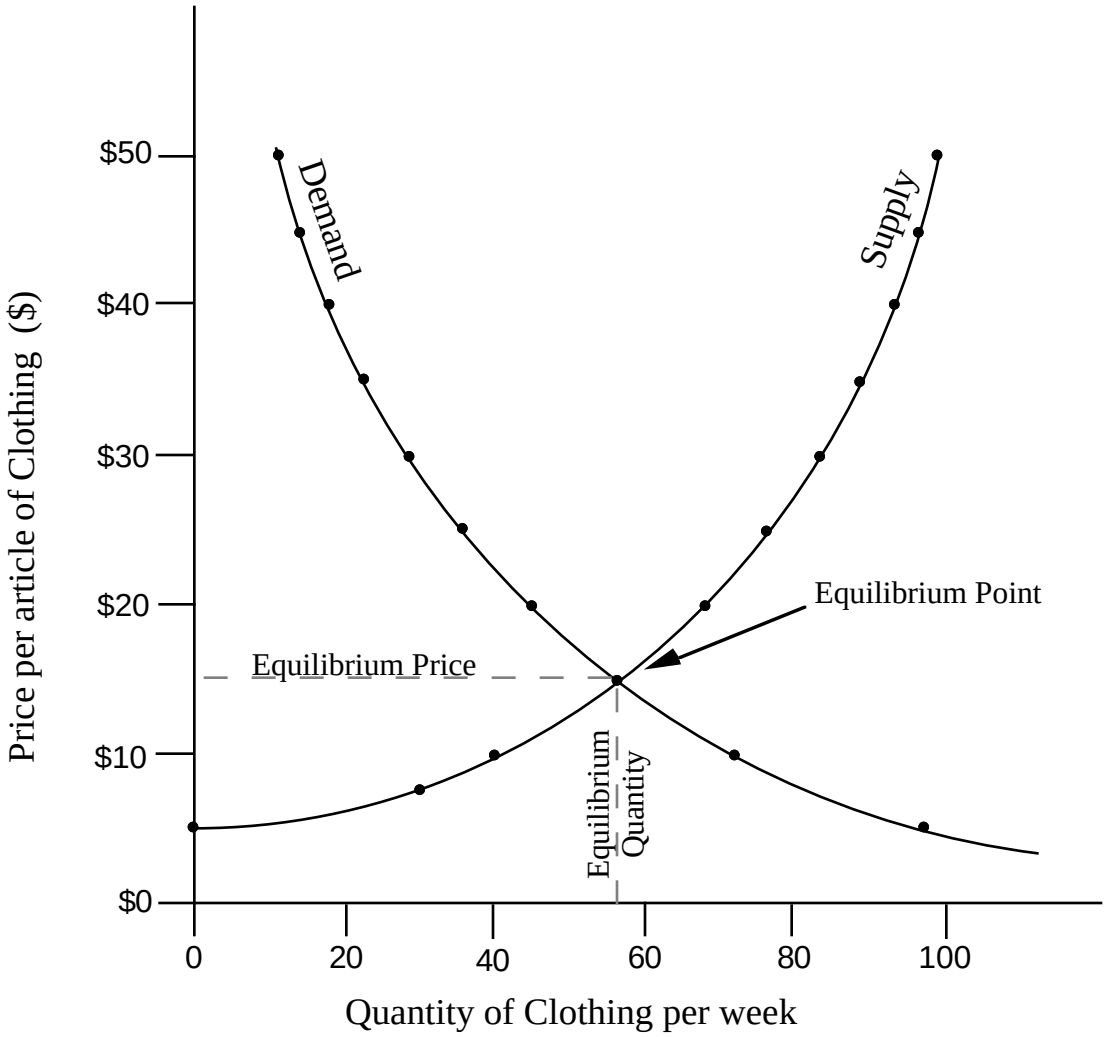


Figure 3: Demand and Supply Curves
These curves were plotted from the data for the clothing market included in Table 1.

Figure 3 plots the demand and supply curves from the data in Table 1. Notice that at $15 the supply and demand curves meet.

4. A SYSTEM DYNAMICS APPROACH TO SUPPLY AND DEMAND

4.1 Introduction

Classical economic theory presents a model of supply and demand that explains the equilibrium of a single product market. The dynamics involved in reaching this equilibrium are assumed to be too complicated for the average high-school student. Economists hold the view that price determines both the supply and the demand. Equlibrium economics defines only the intersection of the supply and demand curves, not how that intersection is reached.

On the other hand, system dynamicists believe that the availability of a product, rather than its rate of production, affects the market price and demand. This means that the inventory (or backlog) of a product is a major determinant in setting price and regulating demand. This model is a hybrid of both views in that it introduces the dynamic effects of inventory into a model that generally replicates the economists' static explanation of supply and demand.

To explore the dynamics of supply and demand we will use the clothing market as an example. Because of a very aggressive marketing campaign, demand for clothes has increased. How will the suppliers and consumers react?

To study the behavior of the market, we will look at its three major components: supply, demand, and price. There will be a series of exercises to help you understand the model. We will first look at consumer demand.

4.2 Demand

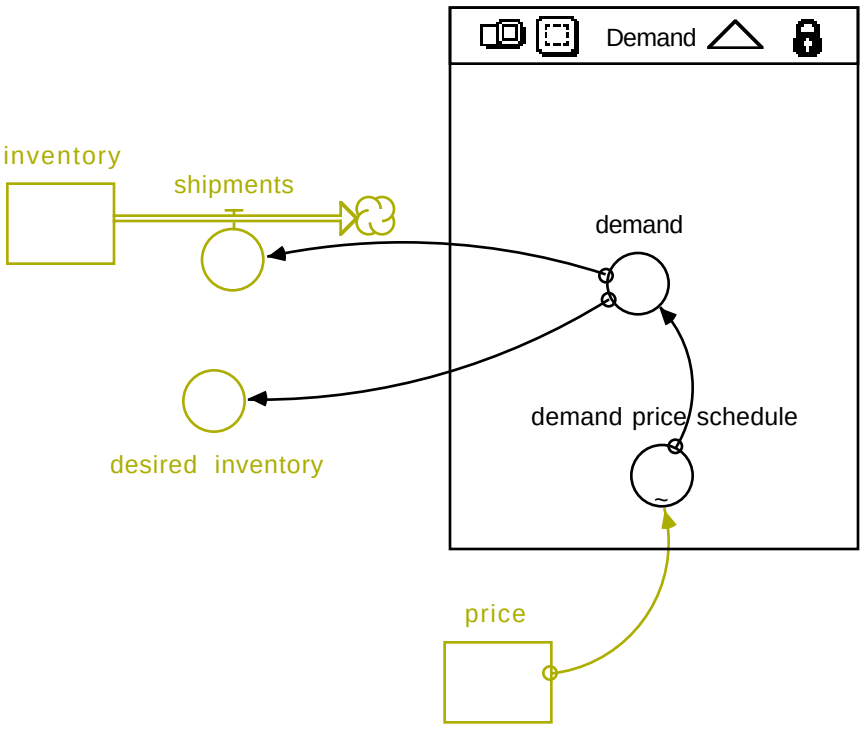


Figure 4: Demand Sector

Demand in this model obeys one simple rule. It is the *demand* as dictated by the *demand price schedule.* The *demand price schedule* is a demand curve that indicates what quantity consumers are willing to buy at a given price. The *demand* directly affects two things. First, it determines the outflow to the *inventory* stock of the suppliers. This model assumes that the rate of *shipments* from the inventory is equal to the *demand*. Additionally, the *demand* sets the size of the supplier's *desired inventory*.

Figure 5 shows the simple demand curve used in the *demand price schedule* graphical function. This curve is somewhat different from the curve shown in Figure 1. Because STELLA and system dynamics standard practice require the input to a graphical function to be on the horizontal axis, it was necessary to reverse the axes. In the curve shown in Figure 5, price is on the horizontal axis instead of the vertical. As discussed earlier, the curve shows that consumers are willing to buy more if the price is lower.

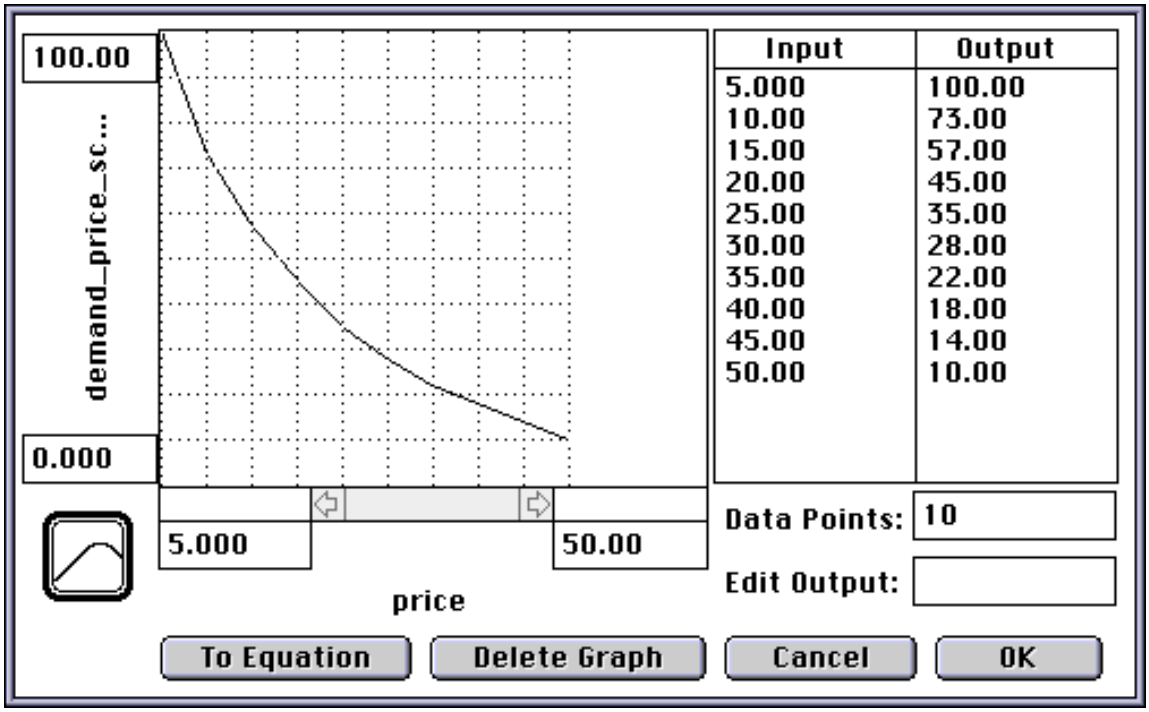


Figure 5: Demand Price Schedule
This curve is a simple demand curve from classical economics with the axes reversed.

4.3 Supply

Below is the supply sector of the model. To simplify the model, we combined the inventories of all the suppliers into one large inventory. The inflow *supply* represents the total production of goods to inventory. The outflow to this stock, *shipments*, is equal to the *demand*.

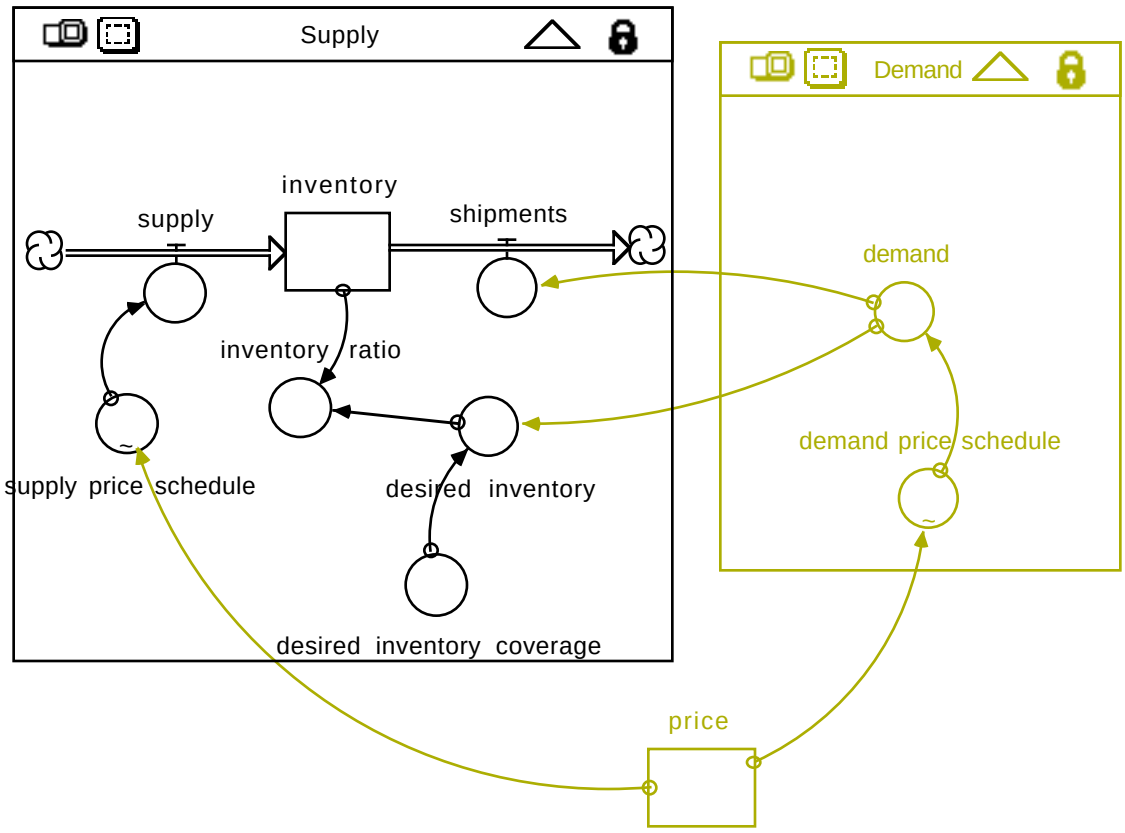


Figure 6: Supply Sector

In Section 3 (Conventional Supply and Demand, page 8) there was no discussion of inventory. Basic classical economic theory does not specifically address the effects of excess or inadequate inventory. This model includes these effects. The *inventory* stock represents the total quantity of clothing in the warehouses of all suppliers.

The *shipments* flow is equal to the weekly demand for clothing. *Desired inventory* is the quantity of clothing the suppliers would like to have in *inventory*. The suppliers like to have enough *inventory* to cover several weeks of *demand*. Therefore, *desired inventory* is the product of *desired inventory coverage* and *demand*. The *inventory ratio* is the ratio of *inventory* to *desired inventory*. The *inventory ratio* will be used to determine *price* later in the modeling process.

Before clothes can be stored in the warehouses, they need to be produced. The *supply* flow is determined by the *supply price schedule.* The *supply price schedule* is a supply curve which indicates how much the producers are willing to produce for each price they receive in the market. The source for this relation is the supply curve provided by classical economic theory[6]. As with the demand curve, the axes must be reversed so that *price* can be an input to the graphical function. The curve used in the *supply price schedule* is shown in Figure 7 below.

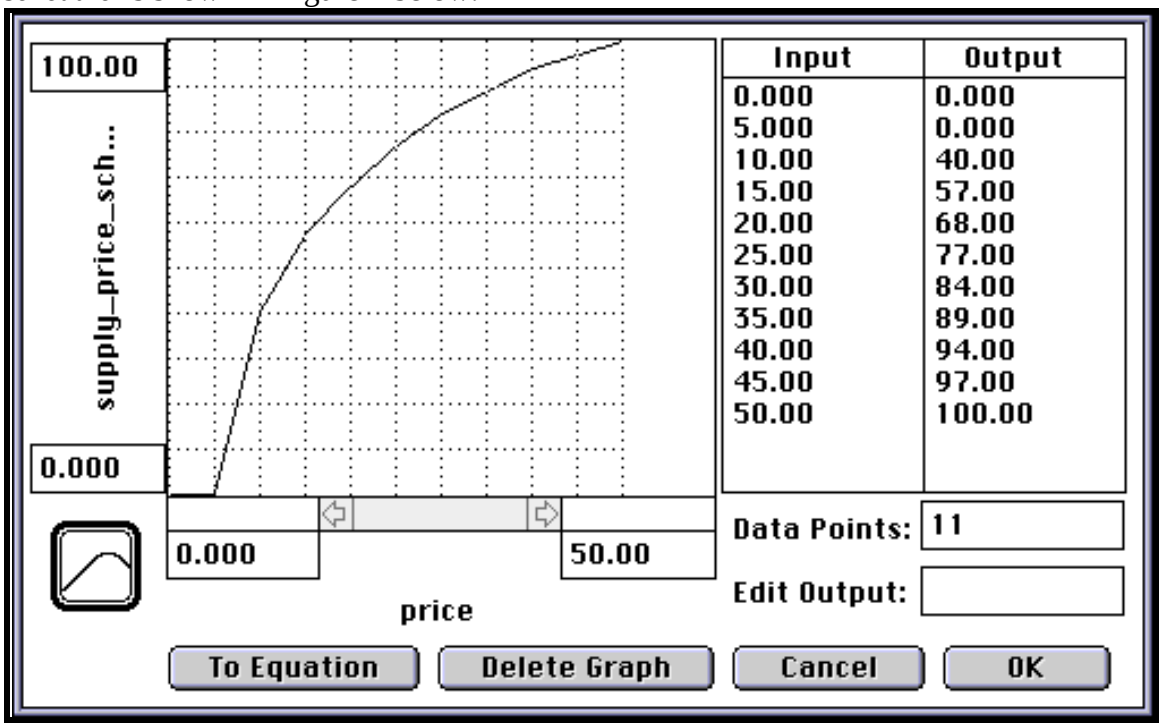


Figure 7: Supply Price Schedule

This curve was derived by taking the simple supply curve from classical economics and reversing the axes. This curve shows the rate of production for a given price.

Below a certain price, the incentive to produce is zero because manufacturers cannot cover the costs of production. As the price rises above that cutoff, supply will increase rapidly. At higher prices, the additional cost of increasing the supply begins to outweigh the benefits of selling at a higher price. As the supply rate continues to

---

[6] The reader should note that this curve is being used in place of more complictated dynamic structure. There is no real-world causal relation between the price and the supply rate of a product. The information contained in the graph is an approximation for the behavior that would be produced by including structure which includes the effects of varying production capacity and employment.

increase, it takes larger and larger increases in the market price to justify further increases in supply.

4.4 Interaction between Supply and Demand

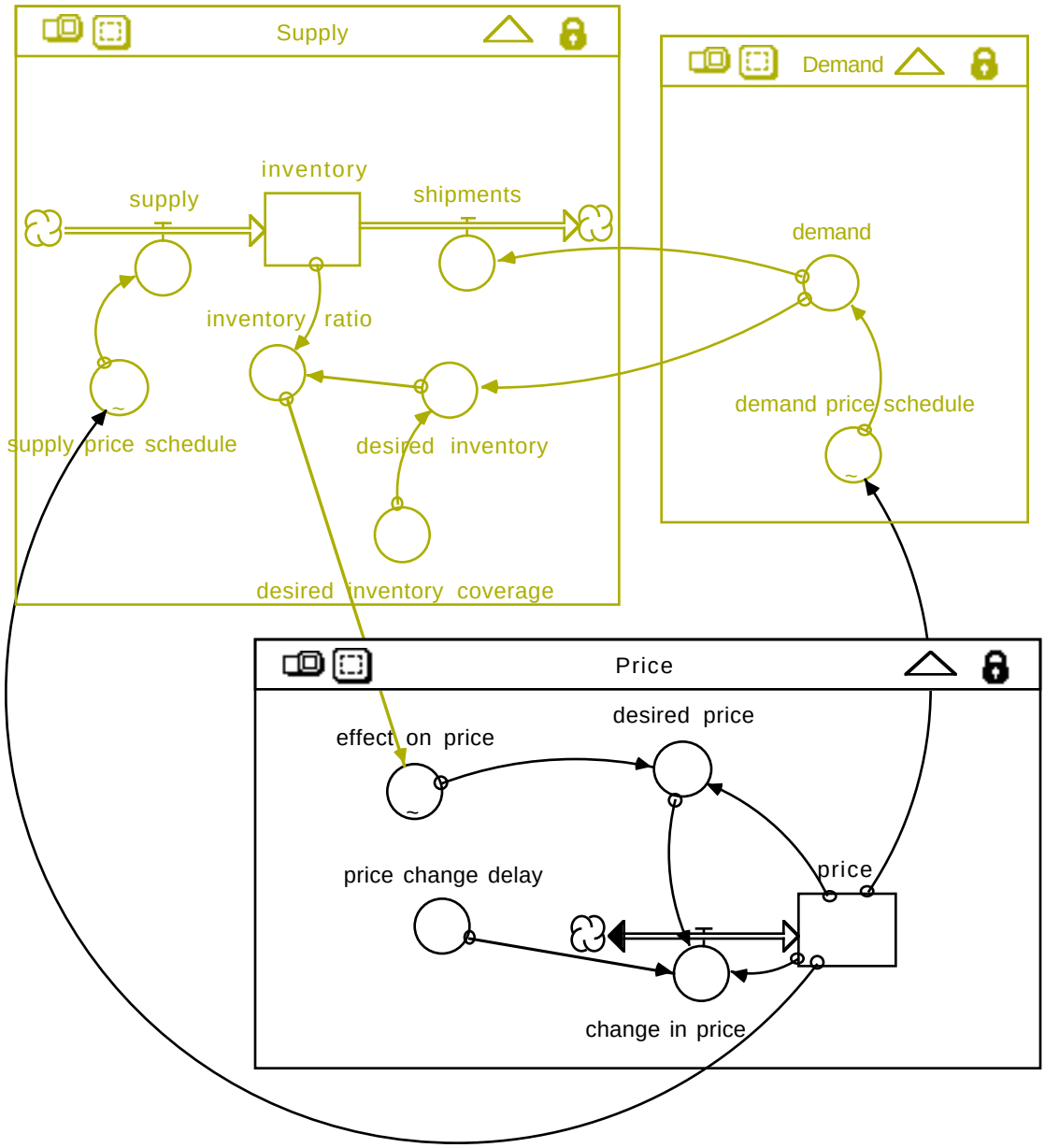


Figure 8: Price Sector

Price affects supply and demand as determined by the *supply price schedule* and the *demand price schedule*. When *price* is high, *demand* is low and *supply* is high. When *price* is low, *demand* is high and *supply* is low. We assumed that the only direct action by a manufacturer to bring inventory to the desired level is to vary price.

The action of the suppliers to regulate the *price* based on the *inventory ratio* is shown in Figure 9. Recall that the *inventory ratio* is defined as the ratio of *inventory* to *desired inventory*.

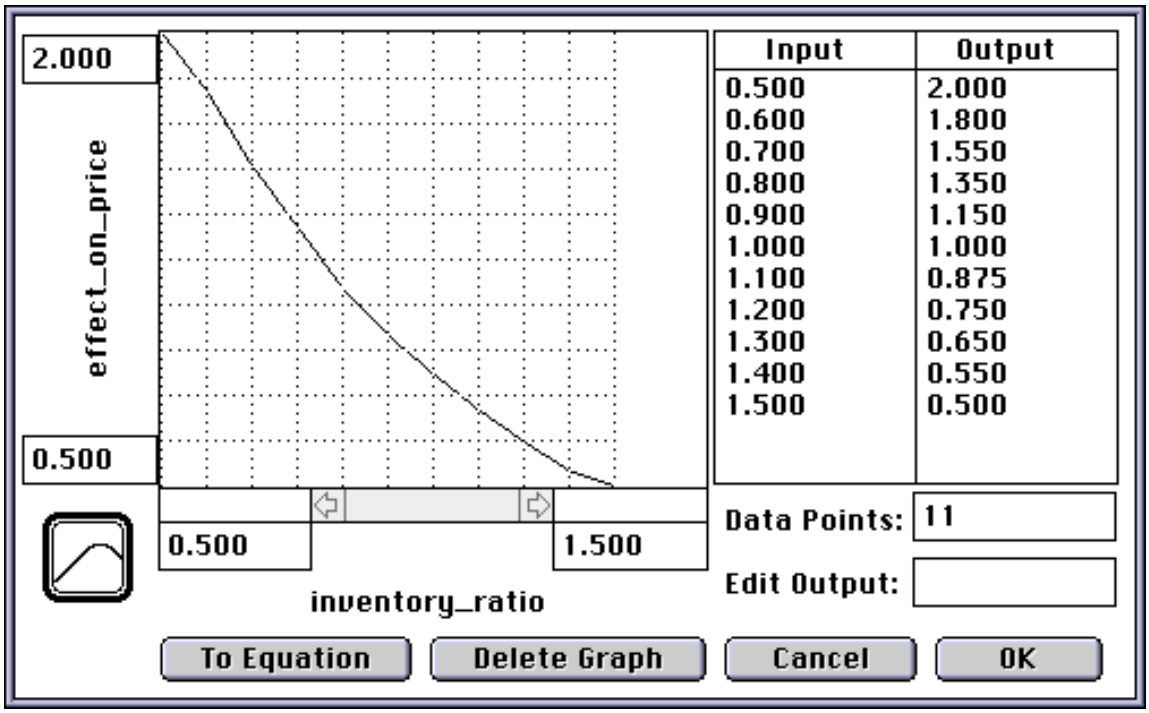


Figure 9: Effect on Price graphical function
When there is excess inventory, the price is lowered and when there is inadequate inventory, the price is raised.

The graphical function shown in Figure 9 represents the action of suppliers to regulate their inventory. When inventory is below the desired inventory, then the inventory ratio is less than one. The graph in Figure 9 shows that an inventory ratio less than 1 gives a value for *effect on price* that is greater than one. This causes the *price* to increase. The increase in *price* causes the *supply* to increase and the *demand* to decrease through their respective price schedules and brings the *inventory* closer the desired value. Multiplying the output of the *effect on price* converter and actual *price* returns *desired price*.

Price was modeled as a stock because prices cannot change instantaneously. People do not have immediate and exact information on the supply (inventory) and demand of the commodity in question. Additionally, when the information becomes available, it takes time to make a decision about changing the price.

5. TESTING THE MODEL

Putting the model together, we get the following:

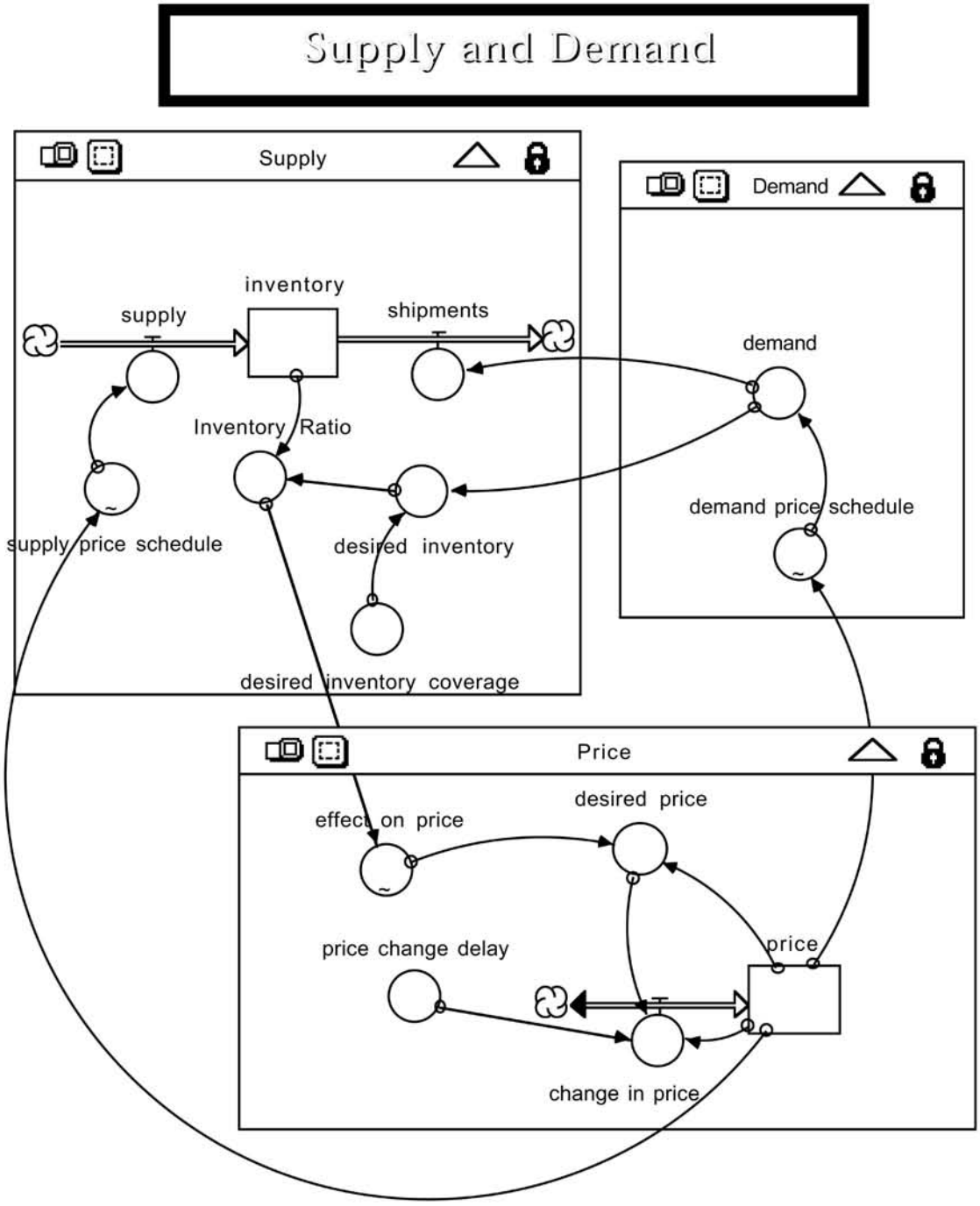


Figure 10: The complete Supply and Demand model

At this point, you may wish to build the STELLA model of supply and demand. The exercises that follow do not require you to run the model, but you may wish to perform some simulations of your own. The complete model equations are included in the appendix, beginning on page 33.

You will analyze three scenarios in this section. The first scenario will be a base case run to observe the response of the model to a step increase in demand. Then you will analyze how the behavior of the system varies from the base case when you change the desired inventory coverage and the price change delay. Solutions start on page 28.

5.1 Increase in Demand

For the base case run, assume the following conditions:

- initial *price* = $15 per article of clothing
- *desired inventory coverage* = 4 weeks
- *price change delay* = 15 weeks

#1: What should *inventory* be in order for the system to be in equilibrium? (Hint: look at the *supply price schedule* and the *demand price schedule*) _____

Discuss your reasoning below.

_____

_____

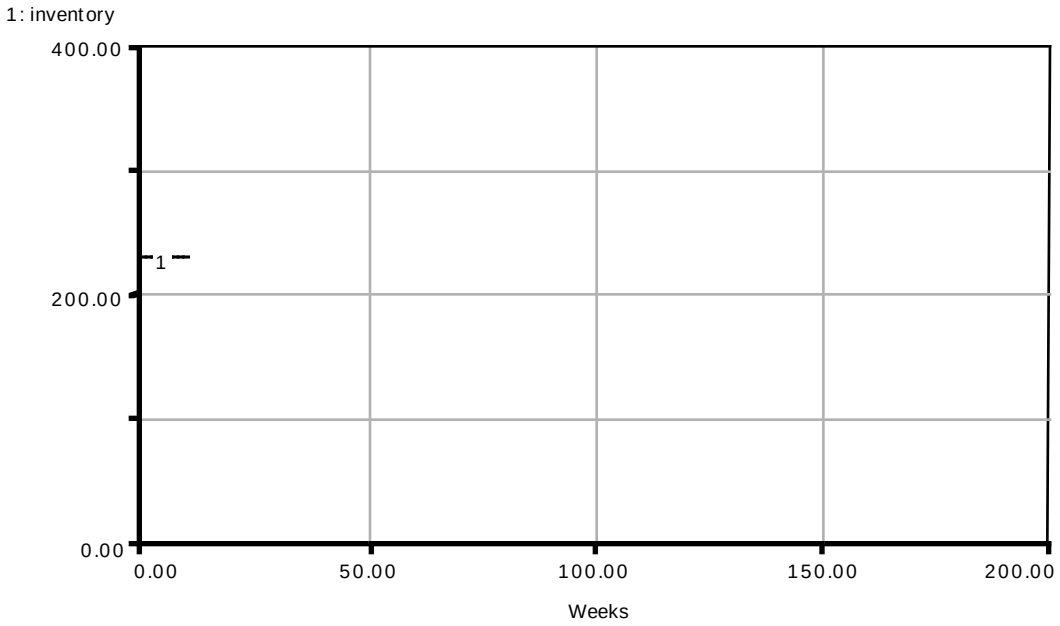
_____

_____

_____

#2: Assume that the system is in equilibrium. *Price* and *inventory* remain the same until the tenth week, at which time there is a permanent increase in *demand* of 10 units. (At each price, the consumer demand is 10 articles per week higher.) What are the new equilibrium values for *price* and *inventory*? _____

#3: Draw below what you think will happen to *inventory* in response to an increase in demand.



Explain your reasoning:

_____

_____

_____

_____

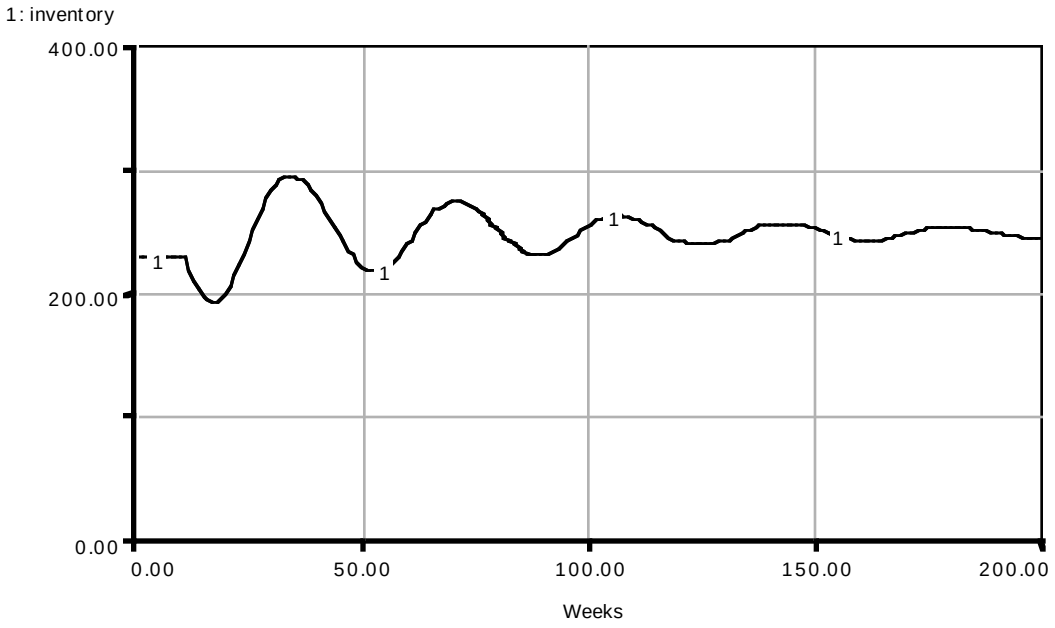
_____

5.2 Desired Inventory Coverage

We will now explore the response of the system to an increase in demand for different values of the *desired inventory coverage* (the number of weeks of desired inventory coverage). Presently *desired inventory coverage* is 4 weeks and the system is in equilibrium. The response of the system to a step increase in demand with *desired inventory coverage* = 4 weeks is shown below.

#4: If we change *desired inventory coverage* to 6 weeks, how would the system react to the same increase in demand? On the graph below, draw the expected behavior.

#5: Now, draw on the same graph what you expect the behavior of *inventory* will be if *desired inventory coverage* is equal to 2 weeks and there is an increase in demand.



Explain your reasoning:

_____

_____

_____

_____

_____

5.3 Price Change Delay

We are now ready to discuss the effect of varying the *price change delay*. The delay obviously affects how quickly the price changes, and in the following exercises we will see how well you can predict the behavior of price when that delay is modified.

#6: Currently, the *price change delay* is 15 weeks. Assuming everything else remains unchanged (system in equilibrium), would *price* or *inventory* change over time if the delay is suddenly shortened? _____

Why or why not?

_____
_____
_____
_____
_____

#7: Would you expect the system to reach equilibrium more quickly when the *price change delay* is equal to 30 weeks or 15 weeks? _____

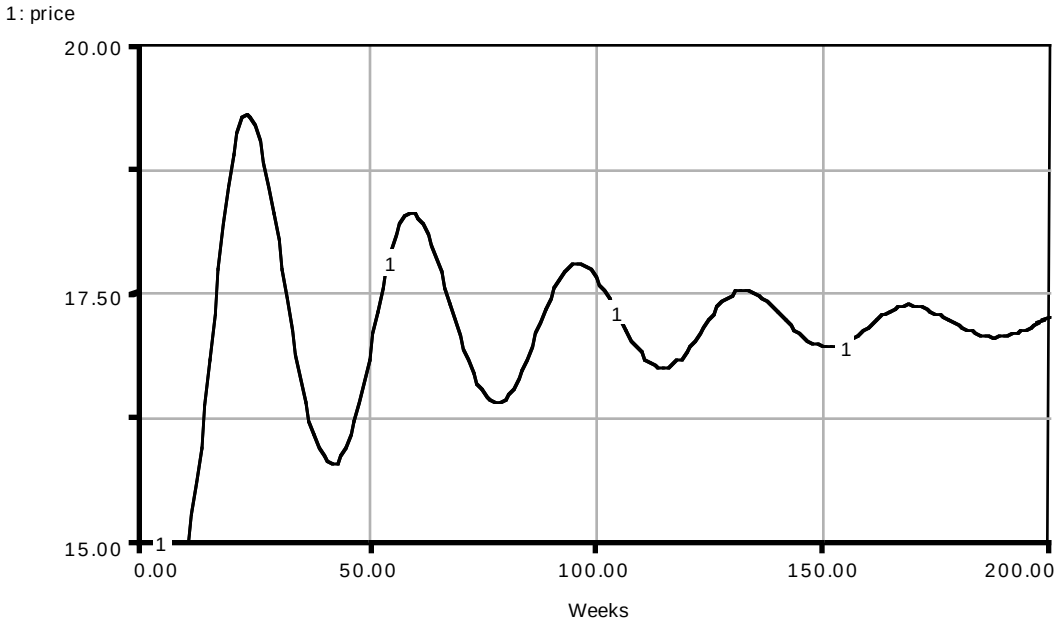
Explain your reasoning:

_____
_____
_____
_____
_____

The graph below shows the response of the system to a step increase in demand when the *price change delay* is 15 weeks.

#8: If the *price change delay* is changed to 5 weeks, how would the system react to an increase in demand? On the graph below, draw the expected behavior of *price*.

#9: Now, draw on the same graph what you expect the behavior of *price* to be if *price change delay* is changed to 30 weeks and there is an increase in demand.



Explain your reasoning:

_____

_____

_____

_____

_____

5.4 Further Exploration

Naturally, no system dynamics model is ever complete. We believe that the model presented is adequate for the purposes of this paper, but there are many possibilities for enhancing it. One possibility is to include the effect of available inventory on demand. The current model assumes that if a product is not currently available, the consumer will simply place an order and wait for the product to arrive, creating negative inventory, or backlog. You may also wish to include structure for increasing the capacity of the supplier. This would allow for increased production without raising the per-item cost. You could also experiment with the dynamics of a non-durable good market (i.e., food). The possibilities are unlimited and it will help enhance your modeling skills.

6. SOLUTIONS TO EXERCISES

6.1 Increase in Demand

#1: In equilibrium, all stocks must remain constant. The *price* will remain constant when the *inventory ratio* is one. Therefore, in equilibrium, the *inventory* is equal to the *desired inventory*. By looking at the supply and demand curves contained in the graphical functions of *Demand Price Schedule* and *Supply Price Schedule* we can see that the equilibrium price is $15 when *demand* and *production* both equal 57 articles of clothing per week. Since the *desired inventory coverage* is 4 weeks, the equilibrium *inventory* is 228 articles of clothing.

#2: An increase in *demand* of 10 articles of clothing per week means that the demand curve in the *demand price schedule* is shifted up by 10. An easy way to figure out the new equilibrium *price* is to plot the supply curve and the new demand curve on the same graph and find the intersection. Doing this shows that the new equilibrium *price* will be about $17 with *production* and *demand* slightly less than 62 articles per week. The new equilibrium *inventory* is then 62*4 or 248 articles of clothing.

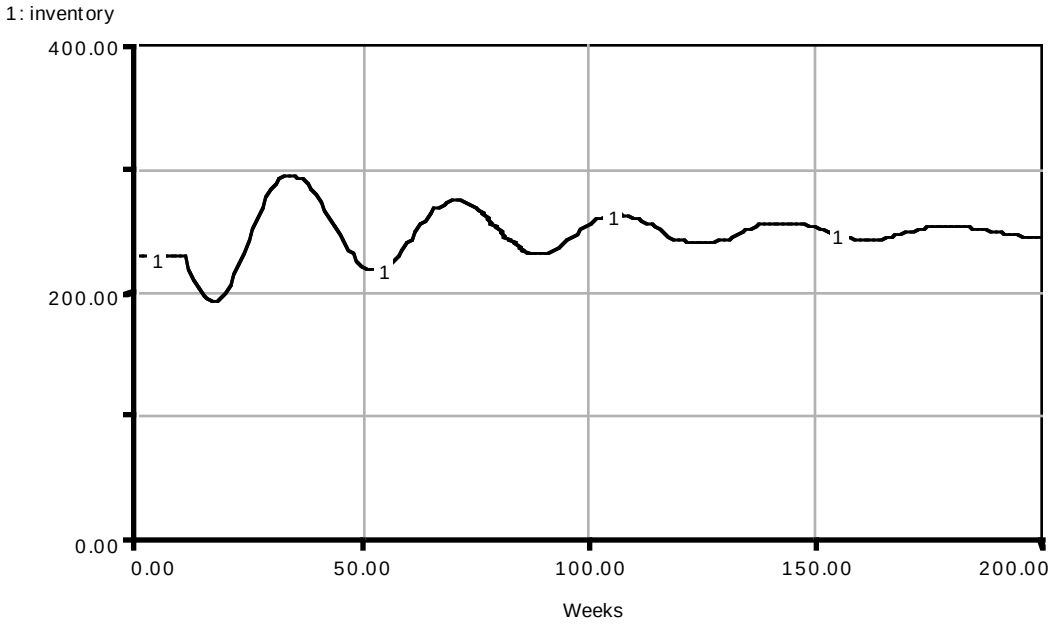


Figure 11: Step in Demand

Inventory decreases at first due to increased demand. It then overshoots and oscillates to a new equilibrium.

#3: The increase in *demand* causes the *desired inventory* to immediately shoot up by 40 articles of clothing (10 articles per week * 4 weeks of coverage). At the same time, *inventory* begins to drop because *shipments* are higher than *supply*. These cause the *inventory ratio* to drop, resulting in an increase in *price*. The *price* increase causes the *demand* to fall and *supply* to increase allowing the *inventory* to catch up to the *desired inventory*. However, the *price* continues to rise until the *inventory* has overshot its equilibrium value. At this point the inventory ratio becomes positive, causing the *price* to begin falling. Although the price is falling, it remains above its equilibrium value causing the inventory to continue increasing beyond its equilibrium. Eventually, the price falls below the equilbrium price and causes the inventory to begin decreasing, but the inventory again overshoots and the system oscillates to its new equilibrium with inventory equal to about 248. A graph of this behavior is shown in Figure 11.

6.2 Desired Inventory Coverage

#4 & #5: The *desired inventory coverage* affects the size of the *desired inventory.* The response of the system to an increase in demand was very different for the three values of *desired inventory coverage*. When the *desired inventory coverage* was 2 weeks, the inventory seemed to exhibit sustained oscillation. As the coverage was increased to 4 and 6 weeks, the reaction to an increase in demand was smaller and stabilized more quickly. This behavior shows that there is a tradeoff when considering the size of inventory coverage. When the *desired inventory coverage* is high, inventory remains fairly stable and is not greatly affected by changes in demand. Unfortunately, maintaining a large inventory can be costly. Lower values for *desired inventory coverage* are less costly to maintain, but react dramatically to changes in demand. Figure 12 shows the model runs for *desired inventory coverage* equal to 2, 4 and 6 weeks. (Curves 1, 2 and 3 respectively.)

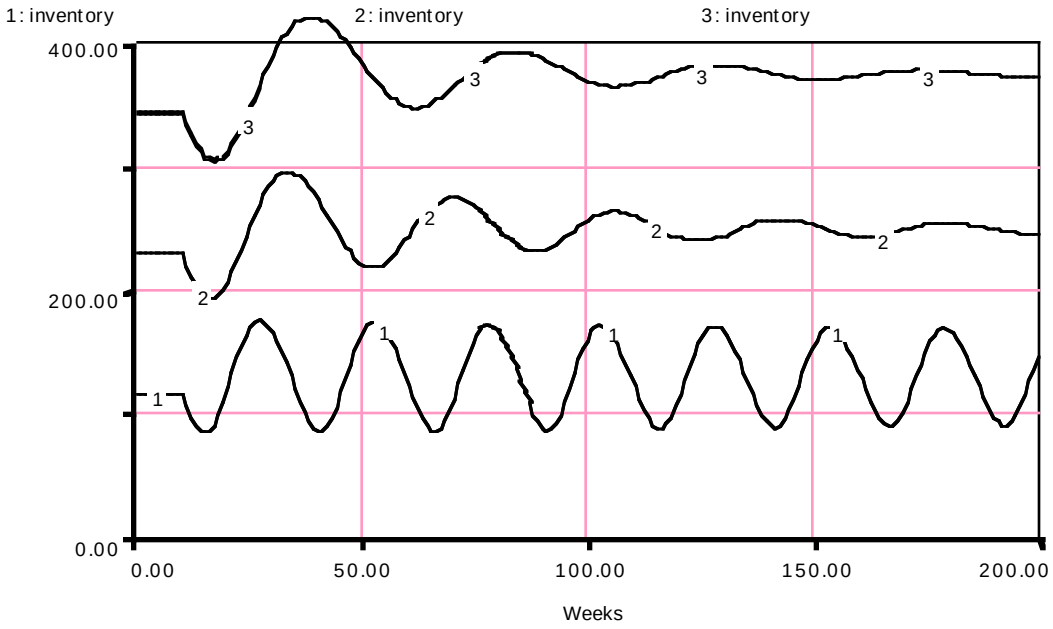


Figure 12: Change in desired inventory coverage
Variations in desired inventory coverage have a large effect on the behavior of the system. Higher coverage allows the inventory to maintain stable, but is costly to maintain.

6.3 Price Change Delay

#6: The *price change delay* does not affect the equilibrium state of the model. This delay only comes into effect when the *price* is changing. Any change in the *price change delay* will not affect the model if it is already in equilibrium.

#7: When the model is knocked out of equilibrium, the *price change delay* affects how it approaches its new equilibrium. When the *price change delay* is short (5 weeks), the *price* changes rapidly and overshoots its equilibrium value. The *price* also converges on its equilibrium quickly. As the *price change delay* increases (15 weeks and 30 weeks), the changes are more gradual, the overshoot smaller and the equilibrium takes longer to reach.

#8, #9: The graphs below show the reaction of *price* to an increase in *demand* when the *price change delay* is 5, 15 and 30 weeks.

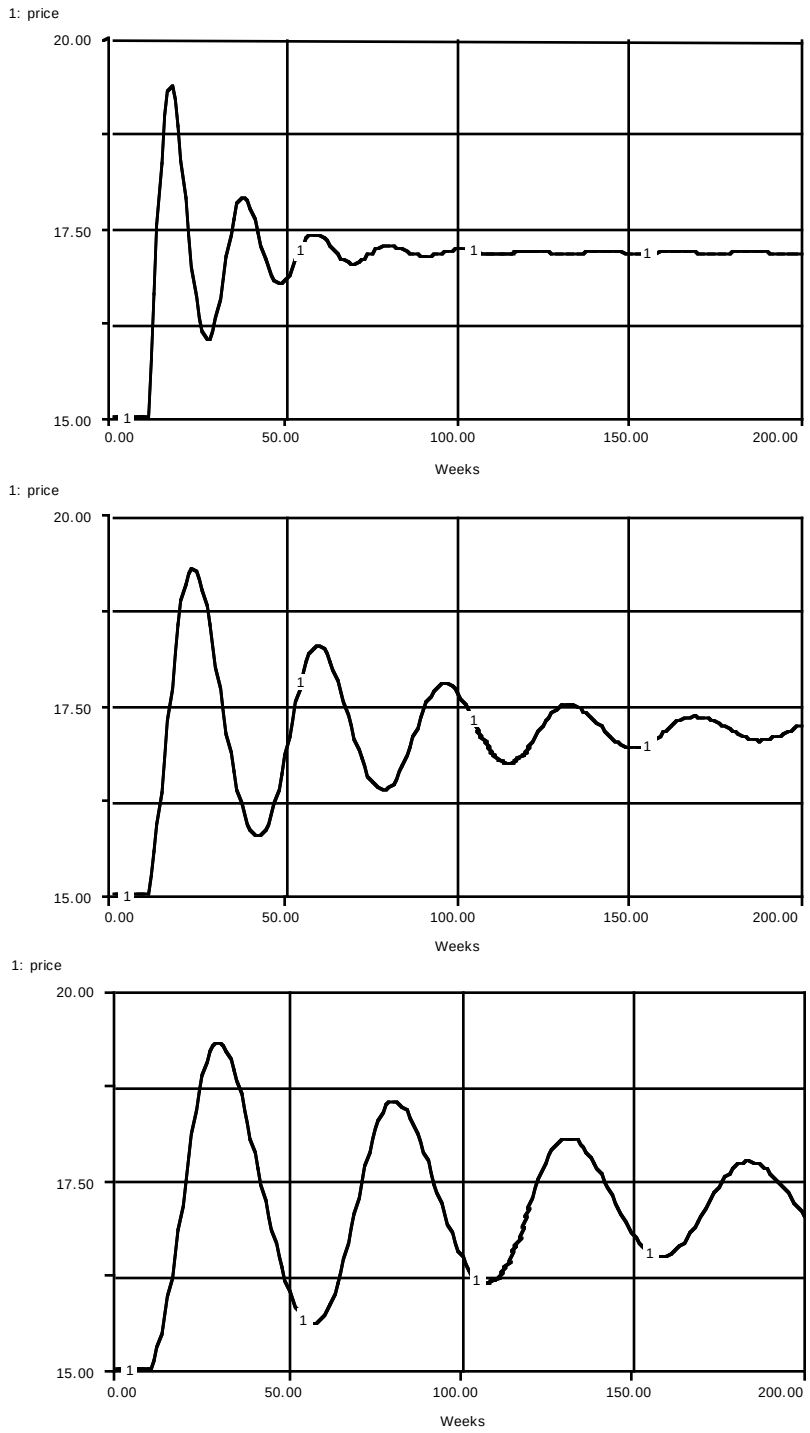


Figure 13: Variations in price change delay

7. APPENDIX

7.1 Model Equations

Demand Sector

demand = demand_price_schedule+step(10,10)
DOCUMENT: This is the rate at which consumers wish to purchase clothing from the company. The step function is used to jar the system out of equilibrium.
UNITS: shirts per week

demand_price_schedule = GRAPH(price)
(5.00, 100), (10.0, 73.0), (15.0, 57.0), (20.0, 45.0), (25.0, 35.0), (30.0, 28.0), (35.0, 22.0), (40.0, 18.0), (45.0, 14.0), (50.0, 10.0)
DOCUMENT: This is based on the simple demand curve. At some particular price, the consumers are willing and able to purchase clothing at a certain rate; the lower the price, the higher the demand.
UNITS: shirts per week

Price Sector

price(t) = price(t - dt) + (change_in_price) * dt
INIT price = 15
DOCUMENT: Price is modeled as a stock in order to model the delays inherent in changes in price.
UNITS : dollars per shirt

INFLOWS:

> **change_in_price** = ((desired_price)-price)/price_change_delay
> DOCUMENT: Change in price can be either positive or negative depending on the effect_on_price. If the effect_on_price > 1, then price will increase. If the effect_on_price < 1, then the price decreases. If effect_on_price = 1, price remains same. Price changes slowly, so we divide the change by price_change_delay.
> UNITS: price/week or ($/shirt)/week

desired_price = effect_on_price*price
DOCUMENT: This is the equilibrium price as set by the inventory_ratio. The actual price will reach this value after a delay specified by the price_change_delay.
UNITS: dollars per shirt

price_change_delay = 15
DOCUMENT: Prices do not change instantaneously. This value determines how quickly price can change.
UNITS : weeks

effect_on_price = GRAPH(inventory_ratio)
(0.5, 2.00), (0.6, 1.80), (0.7, 1.55), (0.8, 1.35), (0.9, 1.15), (1, 1.00), (1.10, 0.875), (1.20, 0.75), (1.30, 0.65), (1.40, 0.55), (1.50, 0.5)
DOCUMENT: This graphical function regulates price change. When the inventory > desired_inventory then the inventory_ratio is >1 and price must be reduced. When the inventory ratio is <1, price must be increased.
UNITS: dimensionless

Supply Sector

inventory(t) = inventory(t - dt) + (supply - shipments) * dt
INIT inventory = desired_inventory
DOCUMENT: Inventory is the stock of produced clothing in the company's warehouse.
UNITS: shirts

INFLOWS:

supply = supply_price_schedule
DOCUMENT: The price supply schedule is based on a classical short-term supply curve. The company uses this algorithm to set a desirable supply rate for a given price. This curve is being used in lieu of more complitated dynamic structure.
UNITS = shirts per week

OUTFLOWS:

shipments = demand
DOCUMENT: This is equal to the demand. The shipments deplete the inventory.
UNITS: shirts per week

desired_inventory = demand*desired_inventory_coverage
DOCUMENT: Desired inventory is how much inventory the suppliers would like to have. It is calculated as how many weeks worth of demand they would like to store in inventory.
UNITS: shirts

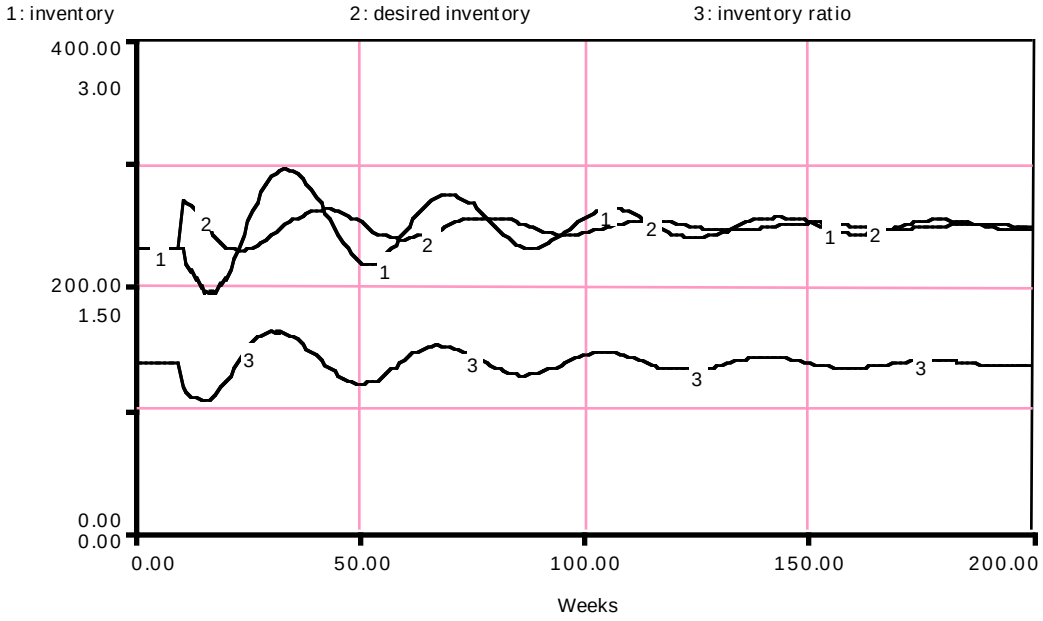
desired_inventory_coverage = 4
DOCUMENT: This value sets how many weeks of demand the suppliers would like to keep in inventory.
UNITS: weeks

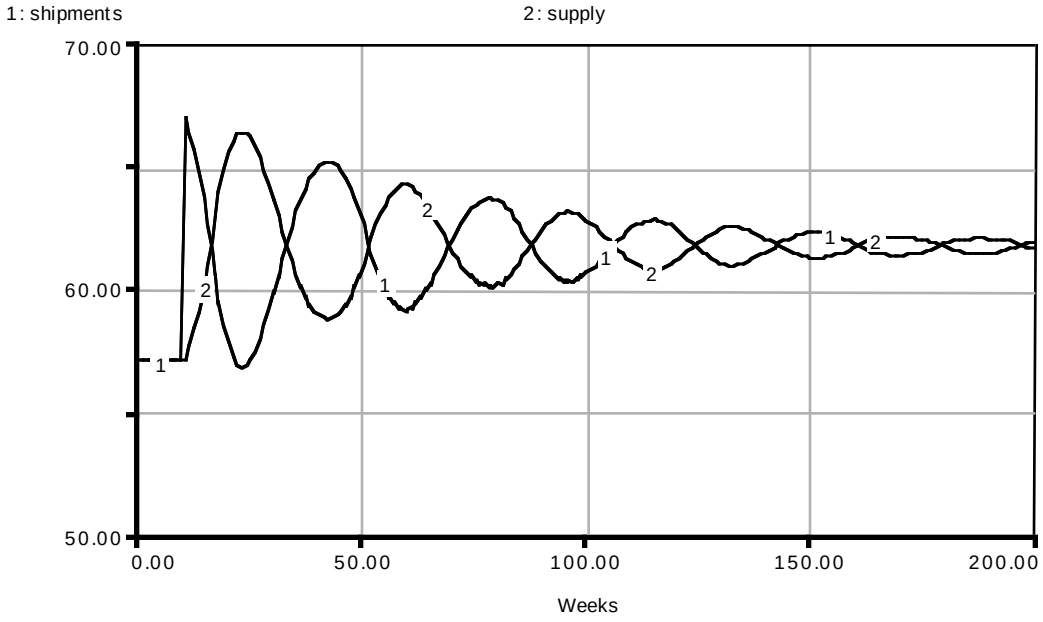
inventory_ratio = inventory/desired_inventory
DOCUMENT: This is the ratio of inventory to desired inventory.
UNITS: dimensionless

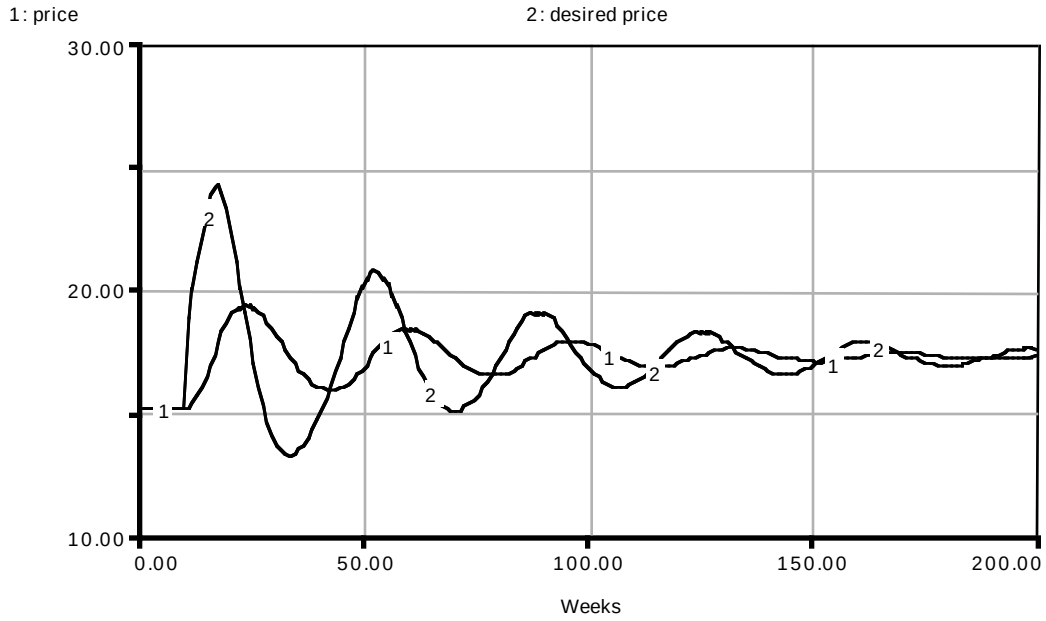
supply_price_schedule = GRAPH(price)
(0.00, 0.00), (5.00, 0.00), (10.0, 40.0), (15.0, 57.0), (20.0, 68.0), (25.0, 77.0), (30.0, 84.0), (35.0, 89.0), (40.0, 94.0), (45.0, 97.0), (50.0, 100)
DOCUMENT: This is a short-term supply curve. At higher prices there is more incentive to produce, more producers can enter the market, etc. This is why this curve points upward as price increases.
UNITS: shirts per week.

7.2 Typical Model Behavior

These graphs represent the behavior of the model set up with the values in the equations listed above. The model is initialized in equilibrium and there is a step increase in demand of 10 units/week after 10 weeks.






1: price
2: desired price
30.00
20.00
10.00
0.00
50.00
100.00
150.00
200.00
Weeks

GENERIC STRUCTURES IN OSCILLATING SYSTEMS I

Produced for the
MIT System Dynamics in Education Project

Under the Supervision of
Dr. Jay W. Forrester
Sloan School of Management
Massachusetts Institute of Technology

BY

Celeste V. Chung
June 17, 1994

Vensim Examples added in October 2001

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OSCILLATION

Celeste V. Chung[1]

INTRODUCTION

Bob, Ann, and Sam meet for dinner after a hard day at work. Bob, a high school physics teacher, tells them about his difficulty in explaining the pendulum problem to his class. Ann, a plant manager, does not understand his situation and has little interest in physics. Interrupting, she talks about her problems in regulating her inventory of widgets and her hiring/firing of factory workers.

Sam, a mutual friend, listens to both stories. On the surface, both problems seem to differ drastically. What similarities could there be between physics and manufacturing?

However, as Sam listened more closely, he began to notice similarities in the behavior of the two scenarios. Both seem to have fluctuating behavior. Sam recalled the system dynamics principles he learned in college. Forming a simple mental model, he pictured a two-level structure that could drive either example.

A two-level structure can display many modes of behavior. The following will focus on one specific mode of behavior - sustained oscillation.

A simple model demonstrating sustained oscillation contains two stocks, each influencing the flow to the other. The dependency of each flow on the opposite stock is such that the system never comes to rest.

---

[1] Celeste V. Chung is an undergraduate at the Massachusetts Institute of Technology, majoring in Management Science, June 1994. She is a participant of the System Dynamics in Education Project under Professor Jay W. Forrester.

Sam first explains the driving forces of oscillation in the pendulum model. He then transfers information from the pendulum model to the employment instability model.[2]

Through the pendulum and employment instability examples, you will learn about a simple two-level structure generating sustained oscillation. As you progress from the first example to the second, you may recognize a striking resemblance in the two models, with differences in only names and parameter values. The underlying structure causing the sustained oscillation remains the same for both models.

[2] Note: This recurring systems principle, *transferable* or *generic structures*, is prevalent throughout Road Maps.

THE PENDULUM MODEL

Bob's high school physics class covers the following simple pendulum system, consisting of a mass attached to a rigid rod pivoting at the ceiling as shown in Figure #1. In a world with no external forces, such as friction, the force of gravity and tension are the sole forces acting upon the mass. If this pendulum system is initially in equilibrium[3] and no additional forces are exerted on it, the mass will stay in its equilibrium position forever.[4]

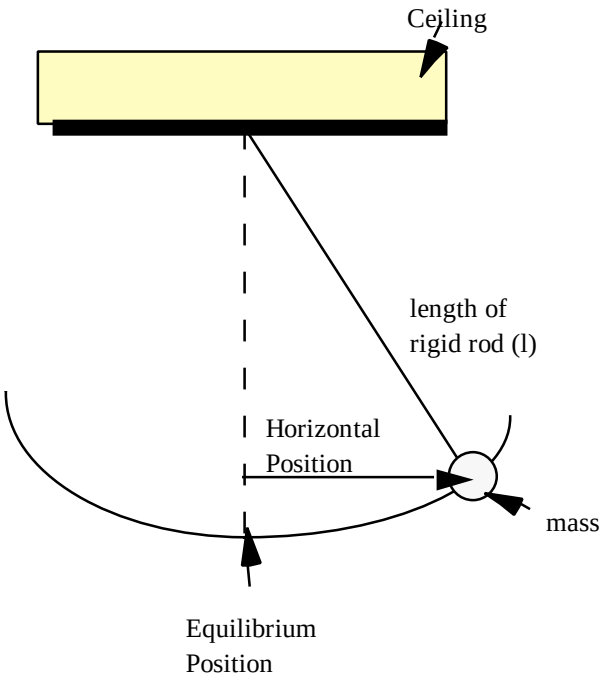


Figure #1: Mass and rigid rod pivoted at the ceiling

If the mass is pulled right or left, gravity provides a restoring force to drive the mass toward its equilibrium position. The tension force and the gravity, shown in Figure #2 on the next page, act on the mass to produce the motion. For this exercise, we will assume that the swing of the pendulum is small, around 10-20 cm for a 1 meter pendulum. For such small oscillations, we can assume the motion of the mass to be essentially horizontal. We will refer to the horizontal position of the mass with the variable X.

---

[3] Equilibrium is a state of balance between opposing forces or actions.

[4] This is Newton's First Law — *A body at rest remains at rest and a body in motion remains in motion unless acted upon by an external force.*

Assuming that the equilibrium position is defined as X = 0, Figure #2 (a) shows the mass in a positive position. Gravity, the restoring force, impels the mass toward its equilibrium position. The mass overshoots to the negative position shown in Figure #2 (b). Here, the position is negative, and gravity again attempts to drive the mass back to the equilibrium position. The mass reaches equilibrium and overshoots to the positive position. Without any external forces such as friction, this back and forth or oscillatory motion would continue forever. For a more in-depth explanation, see Appendix 1.

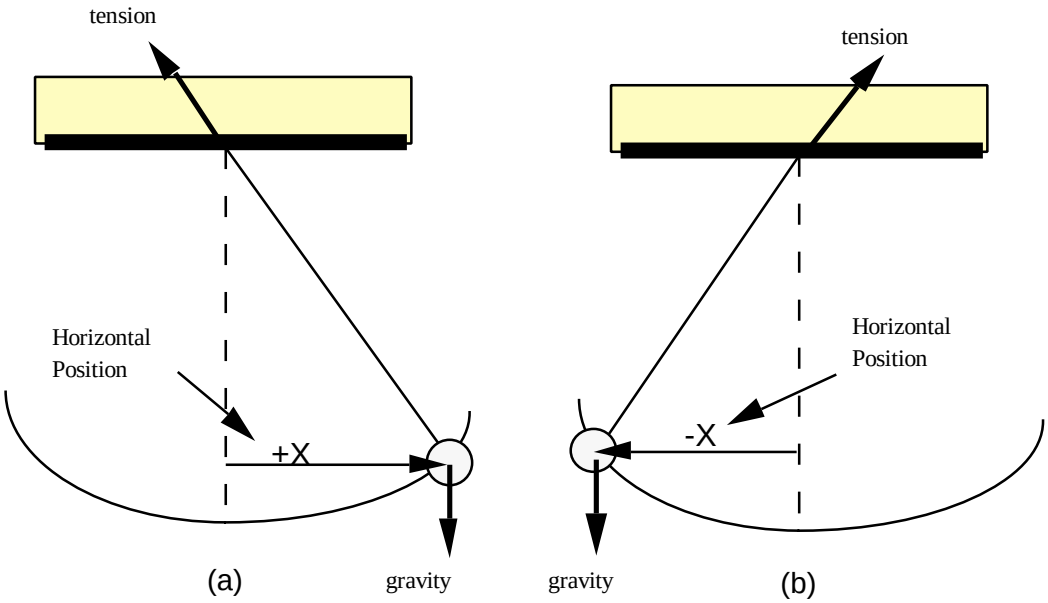


Figure #2: (a) Mass has positive position (displacement); (b) Mass has negative position

Sam's two level mental model of this simple pendulum system is modeled with the stock and flow diagram in Figure #3 on the following page. The **Position** stock represents the position mentioned earlier, with zero representing the **Desired Position**. Positive and negative positions represent the mass's horizontal displacement right and left of the vertical equilibrium line, respectively. The **Change in Position** is equal to the **Velocity**.

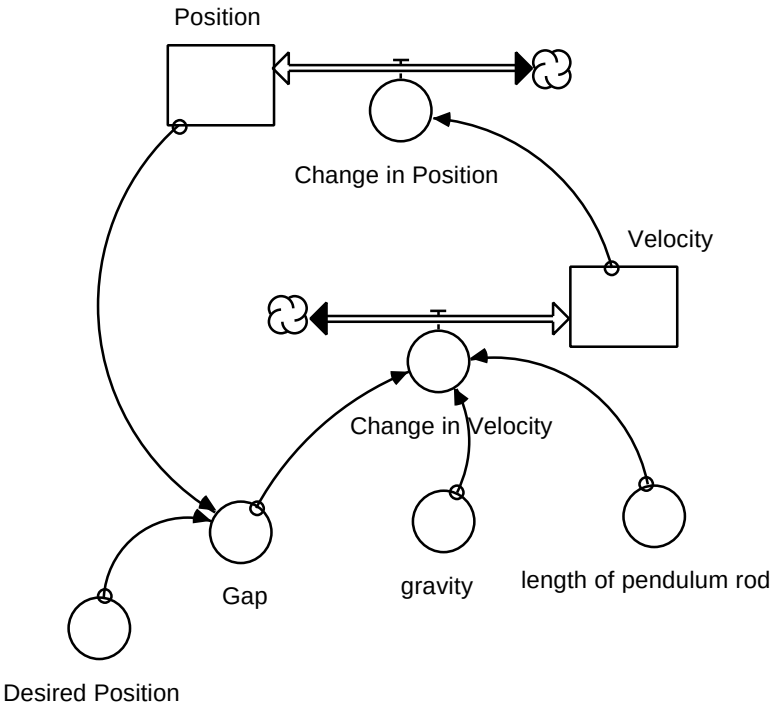


Figure #3: Pendulum Model

The **Gap** is the difference between the Desired Position and the current Position. The **Gap**, along with the influence of **gravity** and the **length of pendulum rod**, determines the **Change in Velocity**. The effect of gravity and the length of the rod can be explained by using a bit of physics.

The **Change in Velocity** is:

$$\textbf{Change in Velocity} = \frac{\textbf{gravity}}{\textbf{length of pendulum rod}} \textbf{Gap}$$ [5]

The **Change in Velocity** is the acceleration measured in meters per second per second. The sustained oscillatory behavior shown in Figure #4 is driven by the Change in Position's dependence on Velocity and the Change in Velocity's dependence on Position.

A derivation of the above equation is included in Appendix 1, but those of you who are unfamiliar with physics can still mentally justify the equation with a bit of intuition:

[5]For those more interested in the physics aspect, please note that this is a simplified relationship for the purpose of this model.

Picture a pendulum with a meter long rod with a weight attached to the end. What happens if you pull the weight 20 cm to the right? The weight will swing back and forth slowly until it stops due to friction. If there were no friction, it would never stop. Picture that same pendulum but with only a 6 inch rod. What happens now if you pull the weight to the right? The weight still swings back and forth, but more quickly.

Picture the same pendulum on the moon. In the low gravity environment, the pendulum would swing much slower. If it were on the sun, the increased gravity would cause it to swing faster. You should be able to see that all of this information is summed up in the equation for **Change in Velocity.**

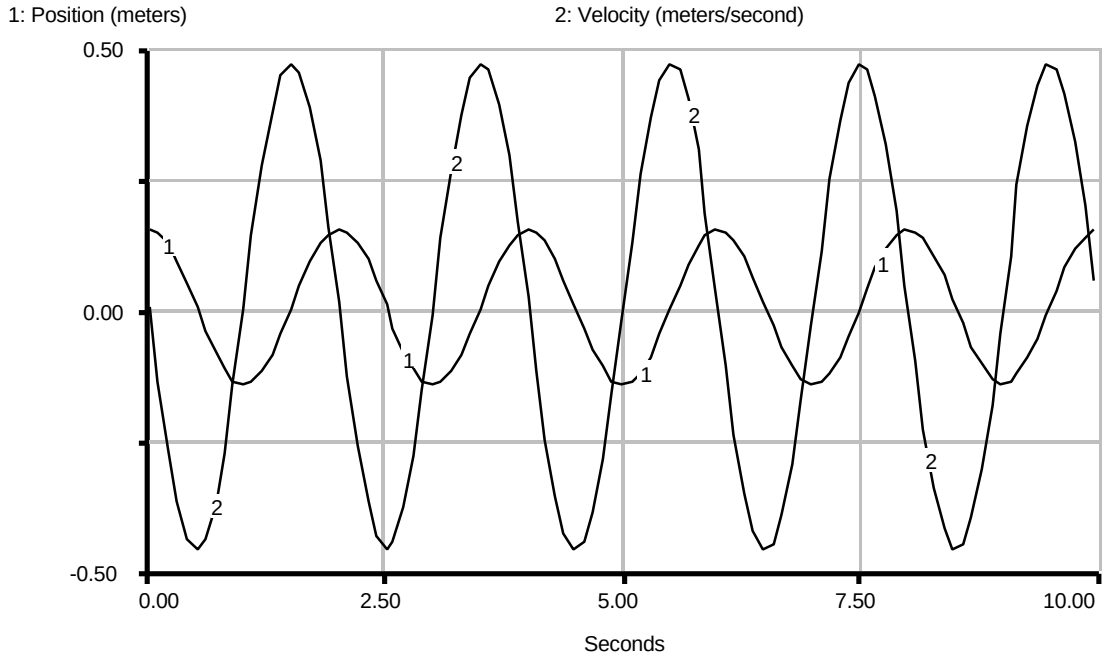


Figure #4: Sustained Oscillation of Position and Velocity

The two-level Pendulum Model is such that the largest Change in Position occurs each cycle at exactly the point where the Change in Velocity equals zero. When one stock is ready to come to rest, the other stock is moving at its fastest.[6] This is the underlying force driving the sustained oscillation.

[6] STELLA II Manual. High Performance Systems, Inc. 45 Lyme Road. Hanover, NH. 03755. 1992. p. 99.

TRANSFERABLE STRUCTURES

The dinner is ending, and Bob, Ann, and Sam order coffee. At this point, Ann nods her head. She understands the physics and the underlying structure of the oscillating pendulum. However, she does not understand how a pendulum can have the same structure as her employee instability problem.

Sam explains the idea of transferable structures. Changing the parameter names and values transforms the Pendulum Model into the Employment Instability model as shown in Figure #5. The oscillatory behavior in the Employment Instability model takes months to complete each cycle instead of seconds. Thus, the parameter values must change to realistically represent the system.

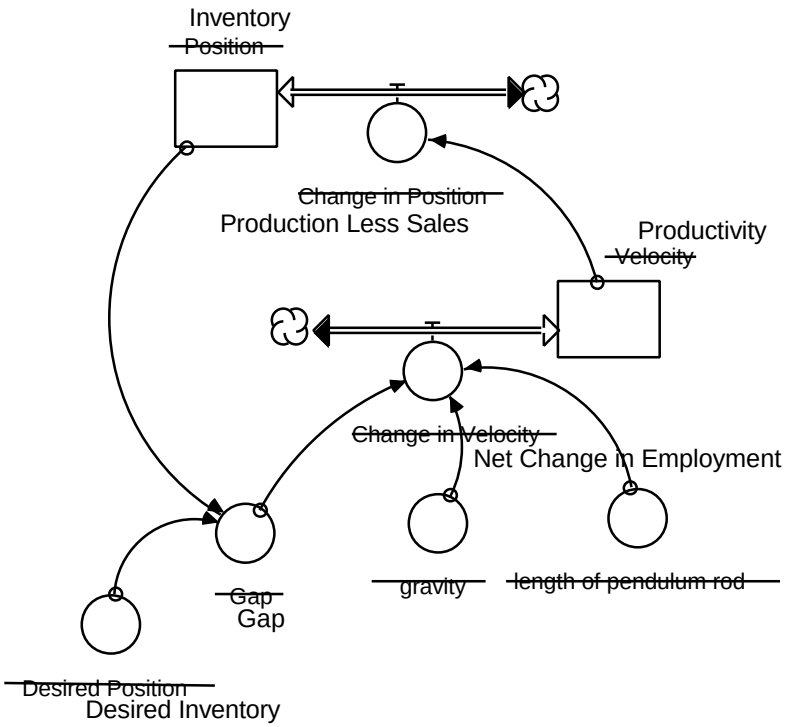


Figure #5: Changing the Parameter Names

Note that the Employment Instability model explained further in the next section has a few different converters not shown in this figure. Although the two models are not identical, the underlying structure is the same. The simplified generic structure which underlies both models is shown in Figure #6. Many

converters can be used to model each specific situation or story but the structure will still remain unchanged.

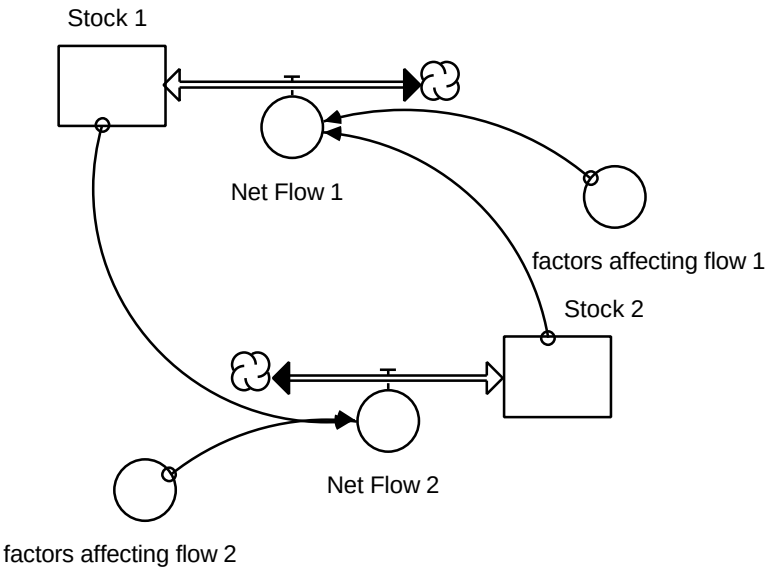


Figure #6: Generic Structure for Sustained Oscillation

Ann quickly protests. What could physics and manufacturing possibly have in common? Sam quickly reassures her that her particular problem is different, and thus the parameter names and values are specific to her situation. However, the underlying nature of the two situations is the same and thus the model structure is unchanged. By transferring what she learned from exploring the Pendulum Model, she can easily understand the driving forces of her system.

EMPLOYMENT INSTABILITY MODEL

Figure #7 shows the same underlying two-level structure as the Pendulum model. The parameter names were changed in Figure #5 from the Pendulum model to the Employment Instability model in Figure #7.

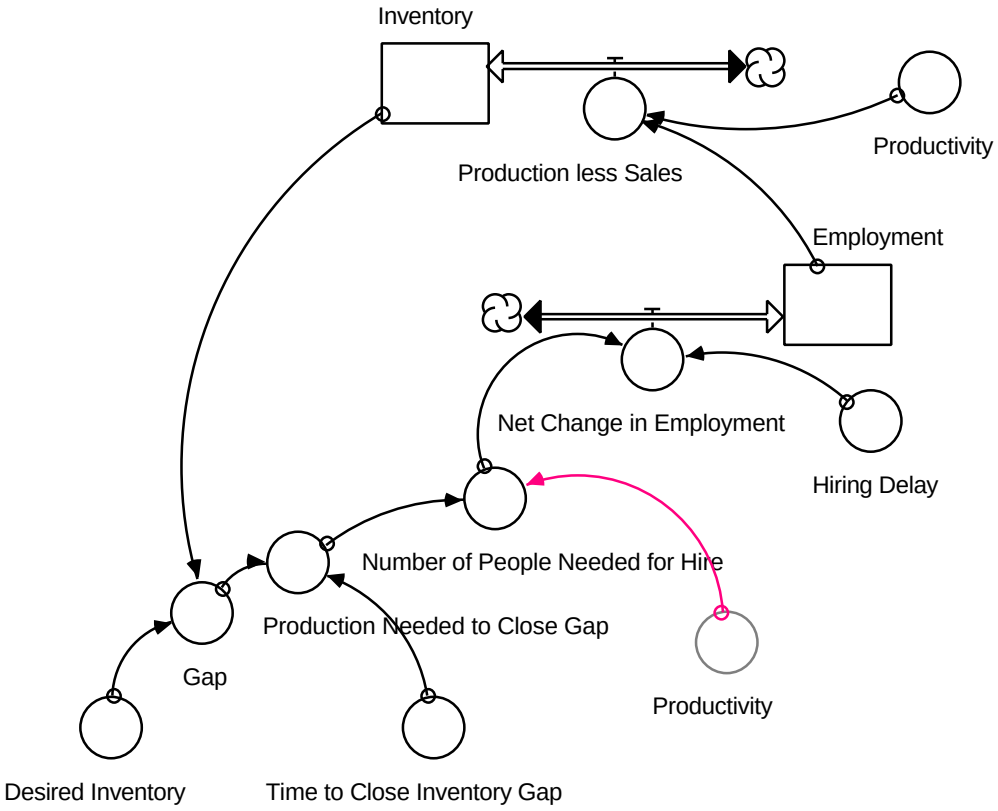


Figure #7: Employment Instability Model

What is the driving force behind Ann's Employment Instability problem? Let's first look at the **Inventory**. The Inventory is regulated by **Production less Sales**. **Sales** is constant at 20,000 Widgets per year. Each employee's **Productivity** is 100 Widgets per year, the number each person can make per year. Thus the **Production** is equal to **Productivity** * **Employment**, the number of employees. Production less Sales, which is the change in Inventory per year is

Production less Sales = Production - Sales

Production less Sales = (Productivity • Employment) - 20,000

Net Change in Employment is the net effect of all hiring and firing in the firm. The **Gap** is the discrepancy between **Desired Inventory** and I**nventory**. The **Time to Close Inventory Gap** is a parameter determined by Ann's firm. The **Time to Close Inventory Gap** and the **Gap** determine the **Production Needed to Close Gap**.

$$\textbf{Production Needed to Close Gap} = \frac{\textbf{Gap}}{\textbf{Time to Close Inventory Gap}}$$

If the urgency to ship widgets is high, the time to close the inventory gap is low, and the production needed to close the gap is high. If there is a lot of time to close inventory gap, the production does not need to be as high.

The Production Needed to Close Gap and the Productivity determine the **Number of People Needed for Hire**. Since we know how many widgets we need to produce in a certain amount of time, we can determine how many people we need to hire by knowing the productivity.

$$\textbf{Number of People Needed for Hire} = \frac{\textbf{Production Needed to Close Gap}}{\textbf{Productivity}}$$

Hence, the equation for **Net Change in Employment** is:

$$\textbf{Net Change in Employment} = \frac{\textbf{Number of People Needed for Hire}}{\textbf{Hiring Delay}}$$

The **Hiring Delay** is the time it takes to hire and train new employees. In this case, it is 3 months or 1/4 years. This model produces the oscillatory behavior shown in Figure #8.

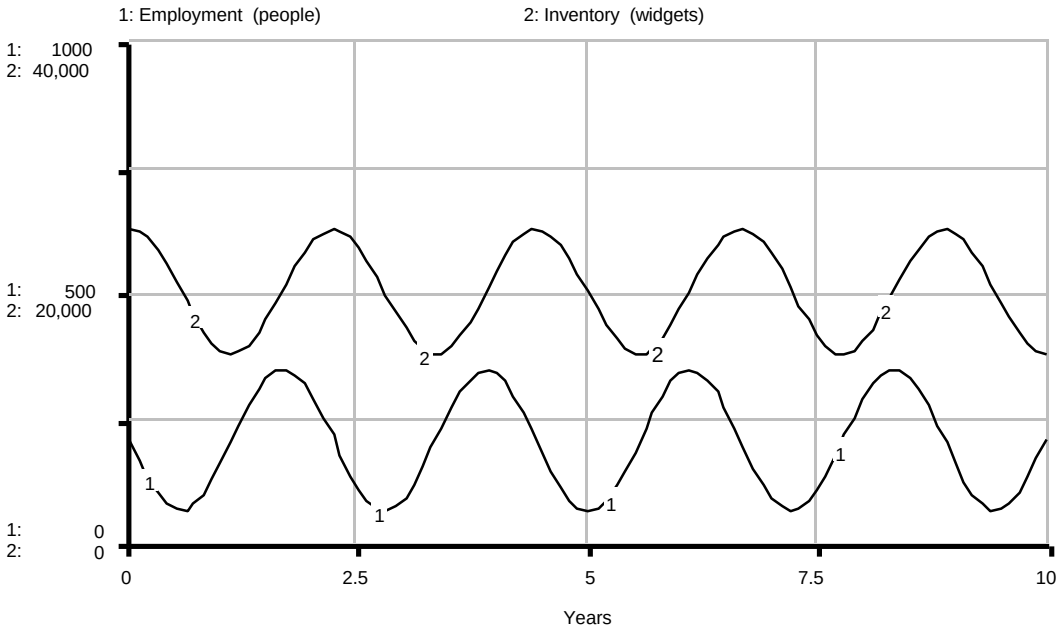


Figure #8: Sustained Oscillation of Inventory and Employment
NOTE: Inventory and Employment are on different scales.

Note the similarity between this graph and the Pendulum Model graph. The shape of the behavior looks the same. However, the time scale differs greatly. The Employment Instability Model goes through one cycle in about two years while the Pendulum Model goes through one in about 2 seconds. The magnitudes of the stocks are also different.

CONCLUSION

Both Bob and Ann were stunned by Sam's explanation. While Ann's problems as a plant manager and Bob's difficulty as a physics teacher seemed completely different, they found that they had a lot in common.

They learned that this two-level generic structure drove the oscillating behavior in both of their systems. Sam taught them that by first understanding the underlying structure of this two-level model, both of them were able to transfer the knowledge from their own situation to the other. They utilized the system dynamics principle of *transferable structures*.

Sam not only taught them about transferable structures. He also showed that sustained oscillation can be generated by a simple two-level structure. The two-level models are such that in each cycle the highest rate of change of one flow occurs at the moment when the opposite flow equals zero. This is the underlying force driving the sustained oscillation.

After this short lesson on sustained oscillation and transferable structures, Ann and Bob returned to their respective jobs the following day. Both were able to use the newly developed models to simulate classroom experiments and test management strategies quickly and accurately. System dynamics gave them insight into their problems so they could begin forming effective manufacturing policies and physics lesson plans for the future.

APPENDIX

Appendix 1: A Further Physics Explanation

Figure #9 describes the arc motion in detail. The mass is initially pulled to the position shown in Figure #9(a). T is the tension force, the force from the rigid rod's attachment to the ceiling, and g is gravity. These two forces act together to produce a net restoring force represented by the bold arrow. As the mass moves, the position X changes and thus the direction of the restoring force also changes to Figure #9(b).

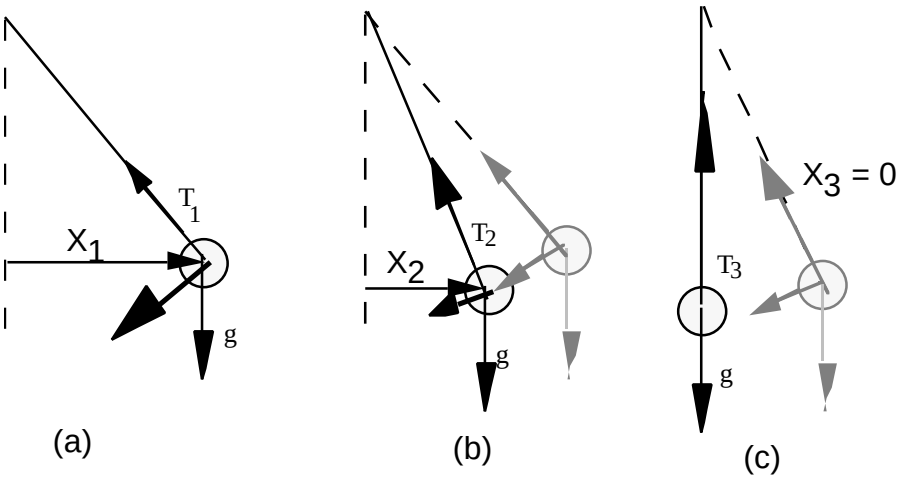


Figure #9: Mass with tension and gravity forces acting upon it.

Figure #9(b) shows the pendulum an instant later. The restoring force is more horizontal and smaller. This trend continues until the mass reaches the equilibrium position as shown in Figure #9(c). Here, the tension force and gravity exactly balance each other so the net restoring force is zero. However, the velocity has accumulated such that the momentum drives the mass horizontally and overshoots to the other side.

On the negative displacement side, the mass would similarly increase to its full negative position, as was previously shown in Figure #2(b). Again, gravity and tension work together to drive the mass back to the equilibrium position. The mass reaches equilibrium and overshoots to the positive position. Without any external forces, this back and forth or oscillatory motion would continue forever.

Derivation of motion equations:

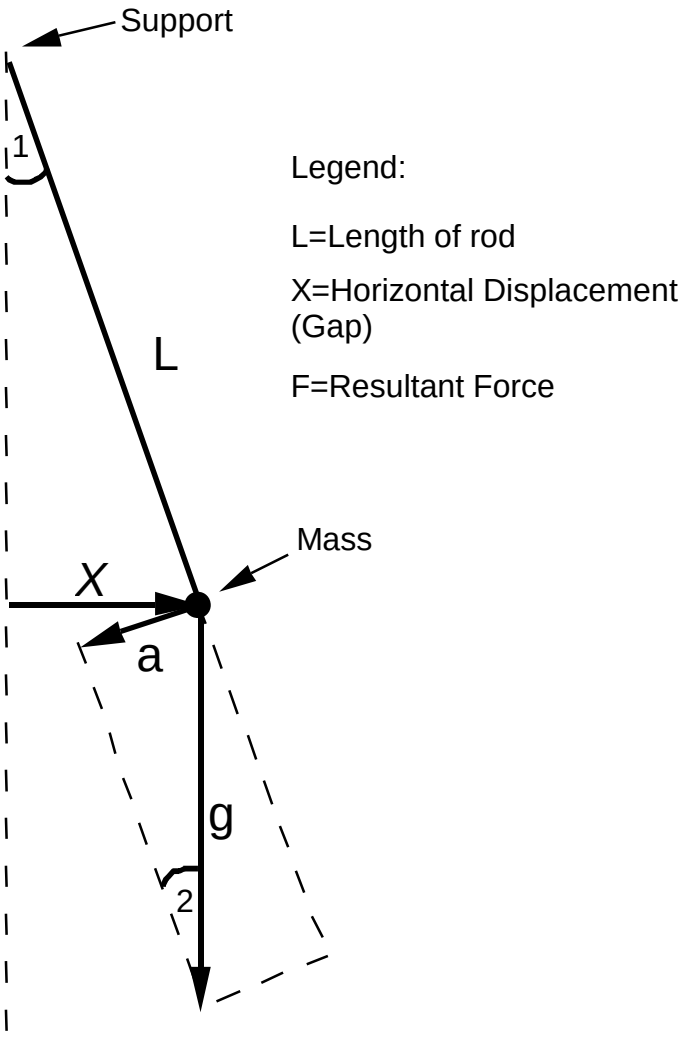


Figure #10: Diagram of pendulum

Figure #10 shows a diagram of the pendulum. The gap ***X*** represents the horizontal displacement of the pendulum from the equilibrium position. To simplify the physics of this system assume that the pendulum only has small oscillations. For our pendulum this means about 10-20 cm. For such small oscillations, the motion of the mass on the end of the pendulum is essentially horizontal.

The diagram in Figure #9 contains two triangles. One is formed by the pendulum's displacement *X* and its length L. The other triangle is formed by the acceleration of gravity *g* and the acceleration of the mass *a*. A bit of simple

trigonometry will show that angle 1 = angle 2; and the two triangles are similar. By similar triangles:

$$\frac{X}{L} ? \frac{a}{g} \quad (1)$$

This can be rearranged to:

$$a ? \frac{g}{L} ? X \quad (6)$$

In our model:

a = Change in Velocity

g = gravity

L = length of pendulum rod

X= Gap

So we get our equation for change in velocity:

Change in Velocity = (gravity/length of pendulum rod) * Gap

NOTE: Several simplifications are included in this derivation in the interest of making it understandable to someone with little or no physics experience. Those of you with more experience may wish to perform a more rigorous derivation on your own and compare the results with this simplified model.

Appendix 2: The Pendulum Model

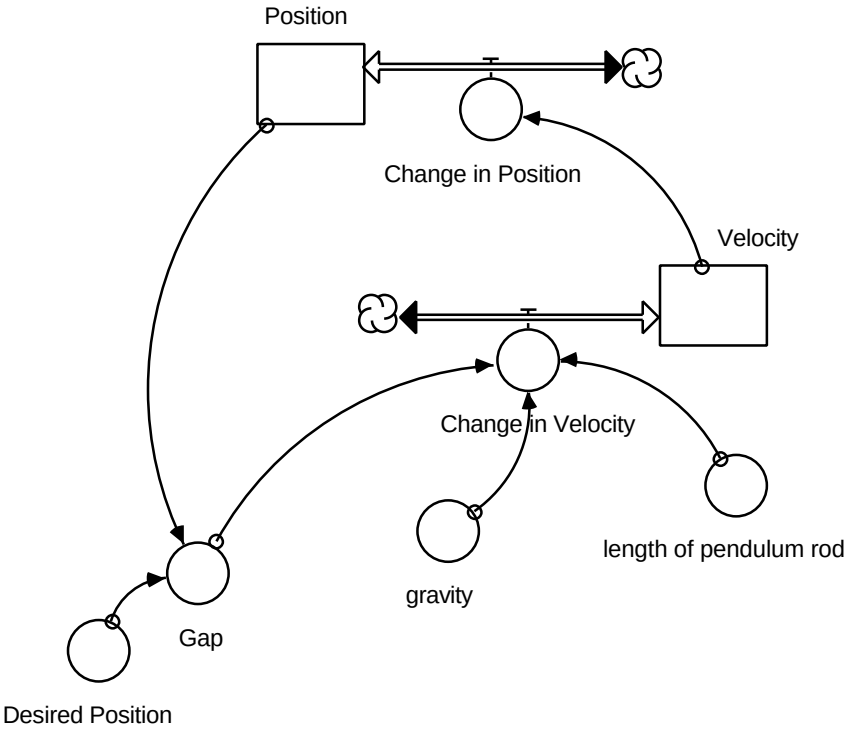


Figure #3: Pendulum Model

The Pendulum Model Equations

Position(t) = Position(t - dt) + (Change_in_Position) * dt
INIT Position = .15

DOCUMENT: The Position represents the horizontal displacement. Initially, the mass is displaced 15 centimeters.
UNITS: meters

INFLOWS:
Change_in_Position = Velocity
DOCUMENT: The Change in Position is the Velocity.
UNITS: meters/second

Velocity(t) = Velocity(t - dt) + (Change_in_Velocity) * dt
INIT Velocity = 0

DOCUMENT: UNITS: meters/second

INFLOWS:

Change_in_Velocity = (gravity/length_of_pendulum_rod)*Gap
DOCUMENT: UNITS: meters/second per second

Desired_Position = 0
DOCUMENT: This is the equilibrium position of the pendulum.
UNITS: meters

Gap = Desired_Position-Position
DOCUMENT: The gap is the difference in the Desired Equilibrium Position and the current Position.
UNITS: meters

gravity = 9.8
DOCUMENT: UNITS: meters/second per second

length_of_pendulum_rod = 1
DOCUMENT: units: meters

Appendix 3: The Employment Instability Model

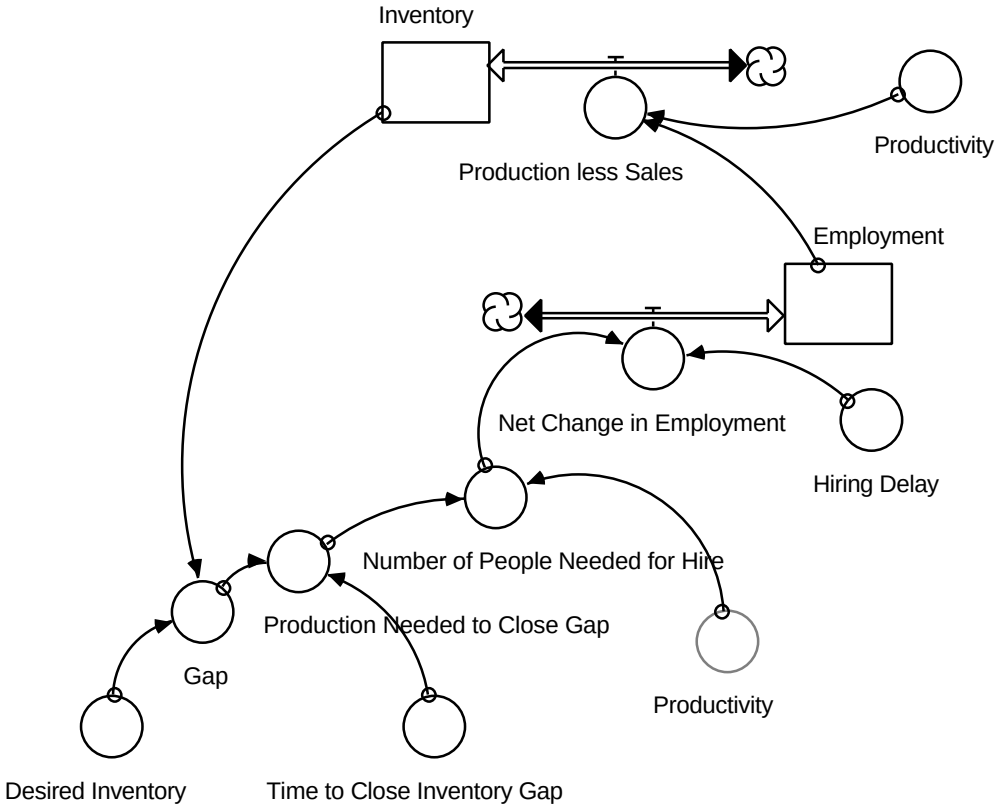


Figure #7: Employment Instability Model

The Employment Instability Model Equations

Employment = 200
UNITS: people

INFLOWS:

Net_Change_in_Employment =
Number_of_People_Needed_for_Hire/Hiring_Delay
DOCUMENT: Hiring Less Firing is the number of people needed for hire divided by the hiring delay.
UNITS: people / year

Inventory = 25000
DOCUMENT: Inventory is the amount of widgets on hand.
UNITS: widgets

INFLOWS:

Production_less_Sales = (Employment*Productivity) - 20000
DOCUMENT: Production less Sales is the change in inventory per year. Each employee produces 100 widgets per year and 20,000 widgets are sold.
UNITS: widgets/year

Productivity = 100
DOCUMENT: Productivity is the number of widgets each employee makes per year.
UNITS: widgets/(person*year)

Desired_Inventory = 20,000
DOCUMENT: The desired inventory is 20,000 widgets, which is the yearly sales.
UNITS: widgets

Gap = Desired_Inventory-Inventory
DOCUMENT: The Gap is the difference in Desired Inventory and the current Inventory.
UNITS: widgets

Production_Needed_to_Close_Gap = Gap/Time_to_Close_Inventory_Gap
DOCUMENT: The Production of Widgets needed to close the inventory gap is the number of widgets needed divided by the time desired to close the inventory gap.
UNITS: widgets/year

Number_of_People_Needed_for_Hire = Production_Needed_to_Close_Gap/Productivity
DOCUMENT: The number of people needed for hire is the desired production needed to close the inventory gap divided by the productivity, the number of widgets each person can produce per year.
UNITS: people

Time_to_Close_Inventory_Gap = .5
DOCUMENT: Time to Close Inventory Gap is the desired time with which the plant manager would like the inventory discrepancy to be fixed.
UNITS: years

○ **Hiring Delay** = 1/4
DOCUMENT: Time to adjust is the time required to hire and train new employees to work to full productivity. Here, the time is three months or 1/4 of the year.
UNITS: years

Vensim Examples:
Generating Structures in Oscillating Systems I

Lei Lei
Samitha Samaranayake
October 2001

The Pendulum Model

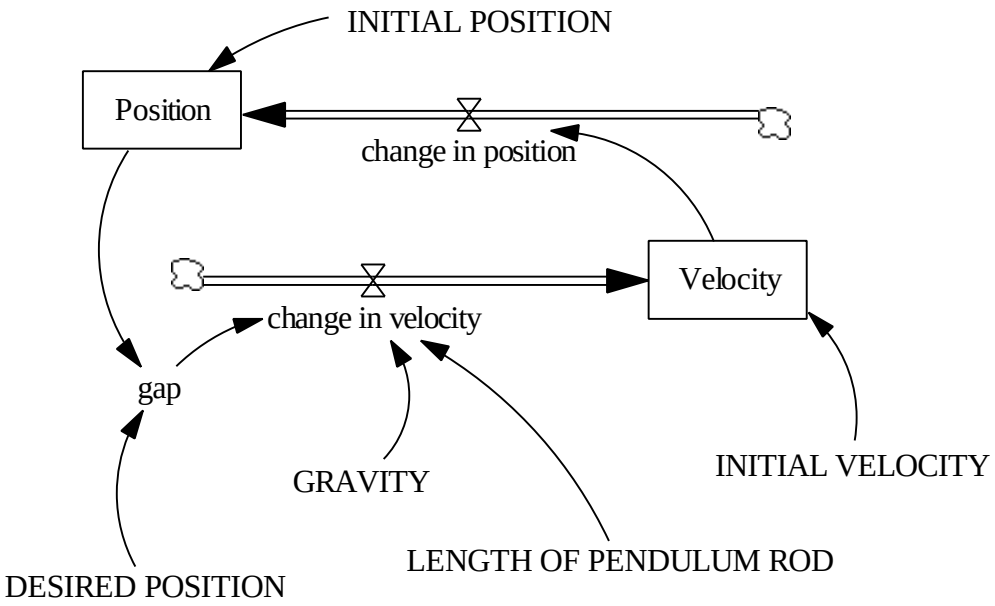


Figure 11: Vensim Equivalent of Figure 3: Pendulum Model

Documentation for Pendulum Model

(01) change in position = Velocity
Units: meters/second
The Change in Position is the Velocity.

(02) change in velocity = GRAVITY*gap/LENGTH OF PENDULUM ROD
Units: meters/(second*second)

(03) DESIRED POSITION = 0
Units: meters
This is the equilibrium position of the pendulum.

(04) FINAL TIME = 10
Units: second

The final time for the simulation.

(05) gap = DESIRED POSITION-Position
Units: meters
The gap is the difference in the Desired Equilibrium Position and the current Position.

(06) GRAVITY = 9.8
Units: meters/(second*second)

(07) INITIAL POSITION = 0.15
Units: meters

(08) INITIAL TIME = 0
Units: second
The initial time for the simulation.

(09) INITIAL VELOCITY = 0
Units: meters/second

(10) LENGTH OF PENDULUM ROD = 1
Units: meters

(11) Position = INTEG (change in position, INITIAL POSITION)
Units: meters
The Position represents the horizontal displacement. Initially, the mass is displaced by initial position.

(12) SAVEPER = 1
Units: second
The frequency with which output is stored.

(13) TIME STEP = 0.03125
Units: second
The time step for the simulation.

(14) Velocity = INTEG (change in velocity, INITIAL VELOCITY)
Units: meters/second

As you would notice when you simultate the model, the graph for position shows exapanding oscilations. This behaviour is contrary to what is expected. The Euler integration method causes integration errors that result in these expanding oscilations. The Runge-Kutta integration method can be used to solve this problem. The following section explains how to detect such an error and correct it using Runga-Kutta integration.

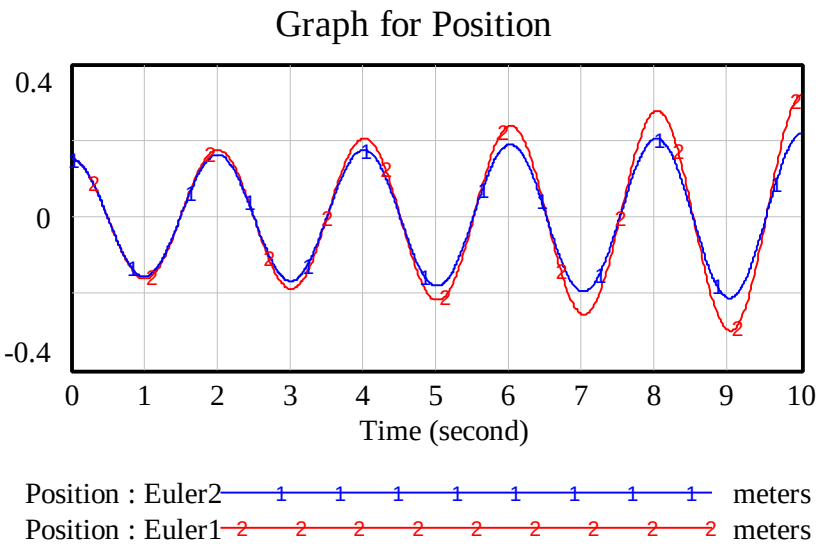


Figure 12: Behaviour of the system under different time steps with Eular integration

The time step used when simulating a model should be lowered until two successive time steps give the same behaviour. This shows that the time step used is small enough to eliminate integration errors. The two smallest time steps provided by Vensim are used in the above example. The graph still shows a difference in the behaviour of the model. Therefore, we can deduce that the integration is still faulty. Runga-Kutta integration can be used in such instances to get a accurate integration.

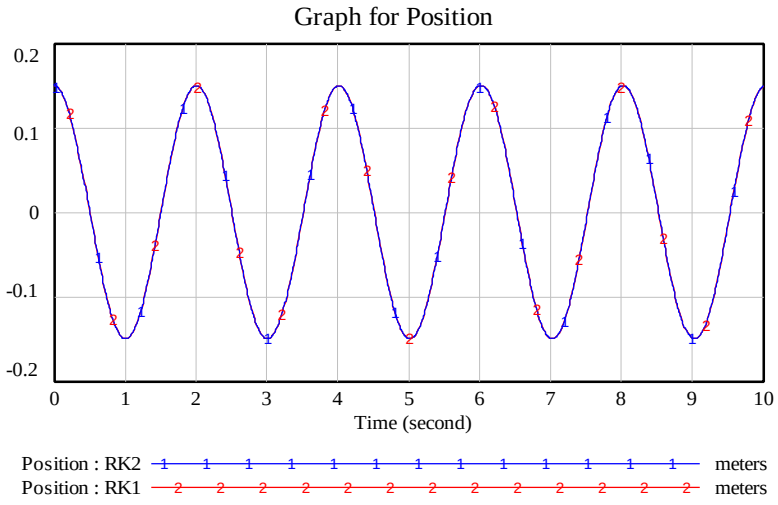


Figure 13: Behaviour of the system under different time steps with Runga-Kutta integration

When using the Runga-Kutta integration method the behavious of the system does not change with the time step. Therefore, we can assume that there are no integration errors.

To change from Euler Intergration to Runga-Kutta Integration in Vensim, press the “Simulation Control” button, which is left of the “Set” button in the menu at the top of the main page. In th new window, under “Integration Technique” check the “RK4-Auto” option, then “Simulate” to exit.

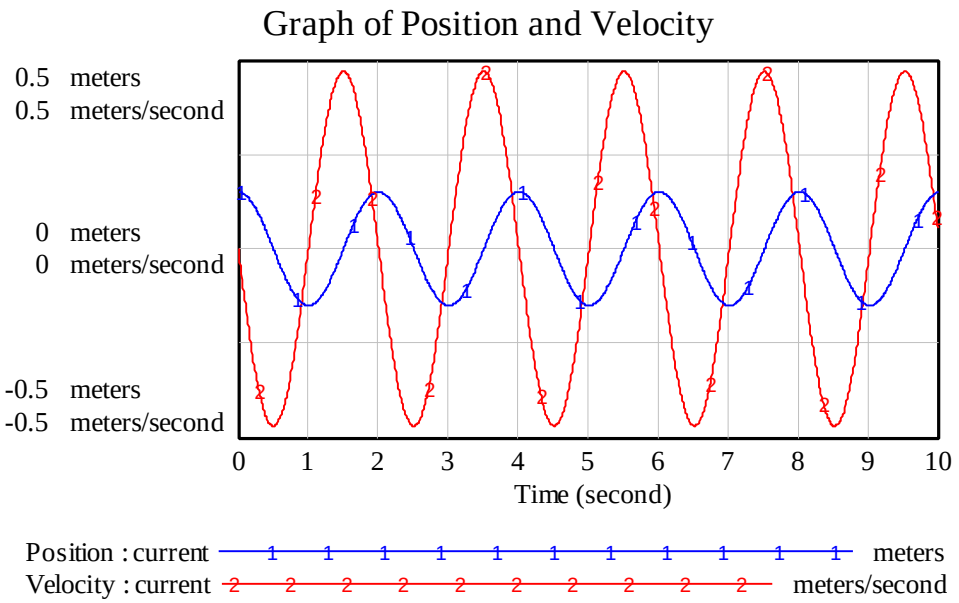


Figure 14: Vensim Equivalent of Simulation for Position and Velocity

The Employment Instability Model

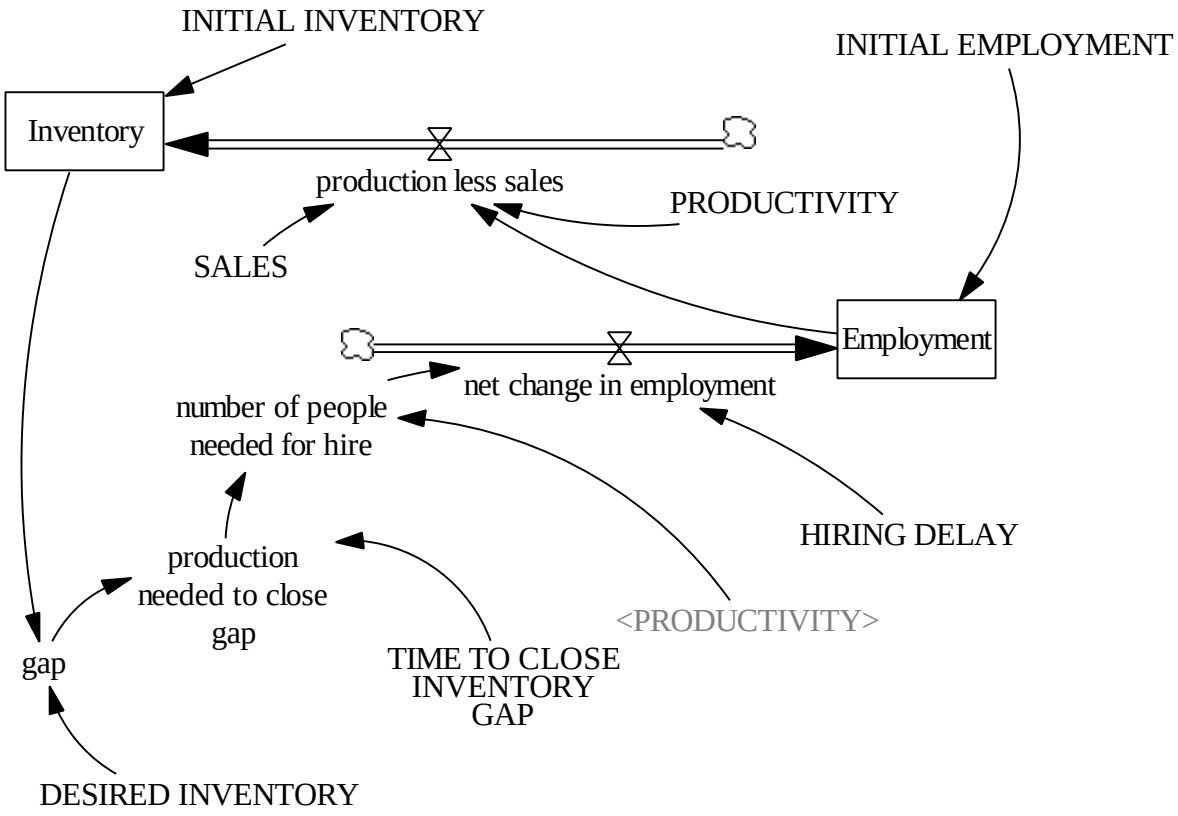


Figure 15: Vensim Equivalent of Figure 7: Employment Instability Model

Documentation for Employment Instability Model

(01) DESIRED INVENTORY = 20000
Units: widgets
The desired inventory is 20,000 widgets, which is the desired sales.

(02) Employment = INTEG (net change in employment, INITIAL EMPLOYMENT)
Units: people

(03) FINAL TIME = 10
Units: year
The final time for the simulation.

(04) gap = DESIRED INVENTORY-Inventory
Units: widgets
The Gap is the difference in Desired Inventory and the current Inventory.

(05) HIRING DELAY = 0.25
Units: year
Time to adjust is the time required to hire and train new employees to work to full productivity. Here, the time is three months or 1/4 of the year.

(06) INITIAL EMPLOYMENT = 200
Units: people

(07) INITIAL INVENTORY = 25000
Units: widgets

(08) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(09) Inventory = INTEG (production less sales, INITIAL INVENTORY)
Units: widgets
Inventory is the number of widgets on hand.

(10) net change in employment = number of people needed for hire/HIRING DELAY
Units: people/year
Hiring less firing is the number of people needed for hire divided by the hiring delay.

(11) number of people needed for hire =
production needed to close gap/ PRODUCTIVITY
Units: people
The number of people needed for hire is the desired production needed to close the inventory gap divided by the productivity, the number of widgets each person can produce per year.

(12) production less sales = (Employment* PRODUCTIVITY)-SALES
Units: widgets/year
Production less Sales is the change in inventory per year. Each employee produces a number of widgets per year equal to the productivity and the number of widgets sold is equal to the sales.

(13) production needed to close gap = gap/TIME TO CLOSE INVENTORY GAP
Units: widgets/year
The Production of Widgets needed to close the inventory gap is the number of widgets needed divided by the desired Time to Close the Inventory Gap.

(14) PRODUCTIVITY = 100
Units: widgets/(people* year)
Productivity is the number of widgets each employee makes per year.

(15) SALES = 20000
Units: widgets/year

(16) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(17) TIME STEP = 0.0625
Units: year
The time step for the simulation.

(18) TIME TO CLOSE INVENTORY GAP = 0.5
Units: year
Time to Close Inventory Gap is the desired time with which the plant manager would like the inventory discrepancy to be fixed.

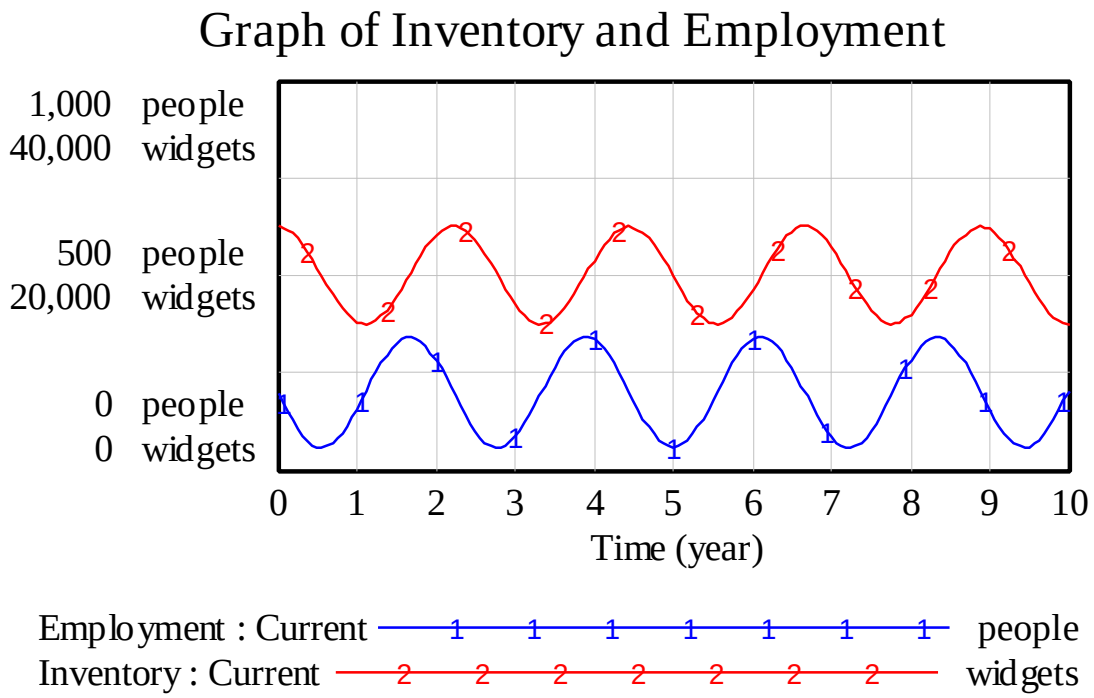


Figure 16: Vensim Equivalent of Figure 8: Sustained Oscillation of Inventory and Employment

Exploring

S-Shaped Growth

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Leslie A. Martin
October 3, 1996

Vensim Examples added October 2001

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1. Introduction

One spring a young couple and small child arrive in Provence, France, carrying bulging suitcases, ardent enthusiasm, and two abnormally large rabbits. They dream of settling down in the countryside and living off of the land. Jean, the husband, is unable to contain the excitement in his voice as he unveils his prized possession to his new neighbors: a carefully folded sheet of paper covered with ink-blotched calculations. Proud to have stumbled upon a get-rich-quick phenomenon known by the name of exponential growth, he explains how his two rabbits will become four then eight then sixteen and soon hundreds upon hundreds... and he is thought to be a madman.

So that summer the rabbits become intimate and produce more rabbits which in turn produce even more rabbits. By mid-summer Jean and his wife have more rabbits than they can control. Rabbits are everywhere. Jean and his wife are ecstatic. They dream eagerly of all the profits they will soon make. But one week Jean realizes that some of his rabbits are dying. They are dying of dehydration. The faster, stronger rabbits quickly lap up the small pools of rainwater in the meadow and leave nothing for the others. Jean prays for a rain storm, but the skies in Provence burn with a throbbing sun. Jean and his wife have to travel several miles to a well to get more water. The young couple finds itself unable to bring back enough water to care for all the rabbits. More die. Jean panics. Now the death rate is so high that for each young rabbit that is born a weak, older rabbit dies. Jean's population has stopped growing. A month later, a year later, he still has the same number of rabbits. The villagers mock the young couple for having bragged about such unrealistic dreams. A disillusioned Jean learns the hard way about limits to growth.

True, sustained exponential growth cannot exist in the real world. Eventually all exponential, amplifying processes will uncover underlying stabilizing processes that act as limits to growth. The shift from exponential to asymptotic growth, or from positive to negative feedback, is known as sigmoidal, or S-shaped, growth.

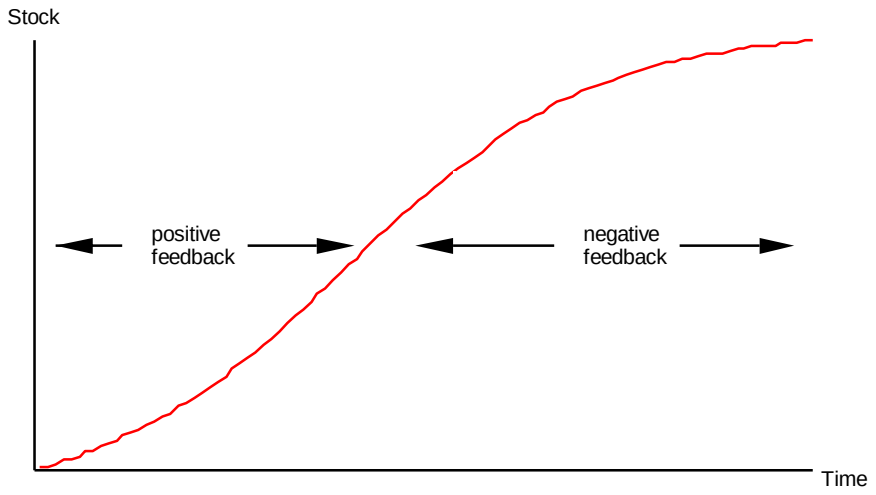


Figure 1: S-shaped growth

Figure 1 displays a typical S-shaped growth curve. Positive feedback, which generates exponential growth, is tapered by negative feedback, which produces stabilizing growth. S-shaped growth can be observed in a wide variety of phenomena. The spread of fads, rumors, or even a religion is characterized by S-shaped growth. Attention span, concern, and interest also exhibit S-shaped growth. Market saturation and epidemics are classic examples of S-shaped behavior. The cellular growth of a plant and physical and intellectual development in small children, along with the body's immune response, are all subject to S-shaped growth. This paper will begin by exploring population dynamics, taking as an example Jean's population of rabbits.

Exploring S-Shaped Growth will first study the shifting loop dominance that produces S-shaped growth. It will then present a generic structure of stocks and flows associated with S-shaped behavior. Finally this paper will examine several different examples of systems which generate S-shaped growth, emphasizing how a whole range of very distinct phenomena can produce similar behavior.

2. The Generic Structure

2.1 Shifting Loop Dominance

Shifting loop dominance produces S-shaped growth. A system that will exhibit S-shaped growth starts out in a positive feedback loop. A large increase in the positive loop awakens a dormant negative loop. The negative loop does not just spontaneously appear. It is present the entire time, but its strength depends on the strength of a variable in the positive loop. When the positive loop begins to amplify all the variables involved in its cycle, the negative loop is also amplified until the dominance shifts and the negative loop takes over. Figure 2 represents shifting loop dominance.

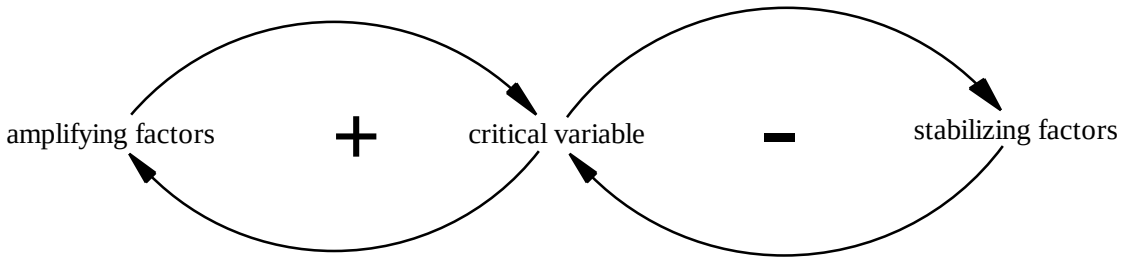


Figure 2: Shifting loop dominance

The critical variable in the behavior of Jean's rabbit population is the number of rabbits that roam around his farm. As the population of rabbits increases, so does the number of rabbit births. More rabbits, more happy couples, more baby rabbits... Births reinforce a positive feedback loop. A negative loop, however, lies dormant. As the population increases the total supply of water stays fixed, so the amount of water available for each rabbit decreases. When the amount of water per rabbit drops low enough, the rabbits will no longer have enough water to sustain themselves, and the weaker rabbits will start to die. The negative loop reduces the population growth rate until the amount of water per rabbit is just large enough to support the rabbit population.

Systems that exhibit S-shaped growth behavior are characterized by constraints, or limits to growth. In the case of Jean's rabbits, the system's constraint is a fixed supply of water. The constraint fixes the maximum number of rabbits that Jean's farm in Provence can support. Likewise, the constraint to the spread of a rumor is the number of people who could potentially be reached. That number could be the total student body on a college campus or a nation of TV viewers, depending on how juicy and pertinent the gossip. The constraint for the spread of an epidemic would be the total population exposed to the disease. Similarly, a company's product can saturate a consumer market. The constraint of the consumer market would be the size of the product's particular market. Exponential growth cannot last forever.

2.2 Generic Structure for S-Shaped Growth

Several different stock and flow structures produce S-shaped growth behavior.[1] Figure 3 represents an intuitive generic structure that explicitly shows both the feedback loops and the constraint on the system. See Appendix 5.1 for documented equations for the generic structure.

[1] See also Marc Glick and Terri Duhon, 1994. *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics is Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 24, 30p. and William Glass-Husain, 1991. *Teaching System Dynamics: Looking at Epidemics* (D-4243-3), System Dynamics is Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 58p.

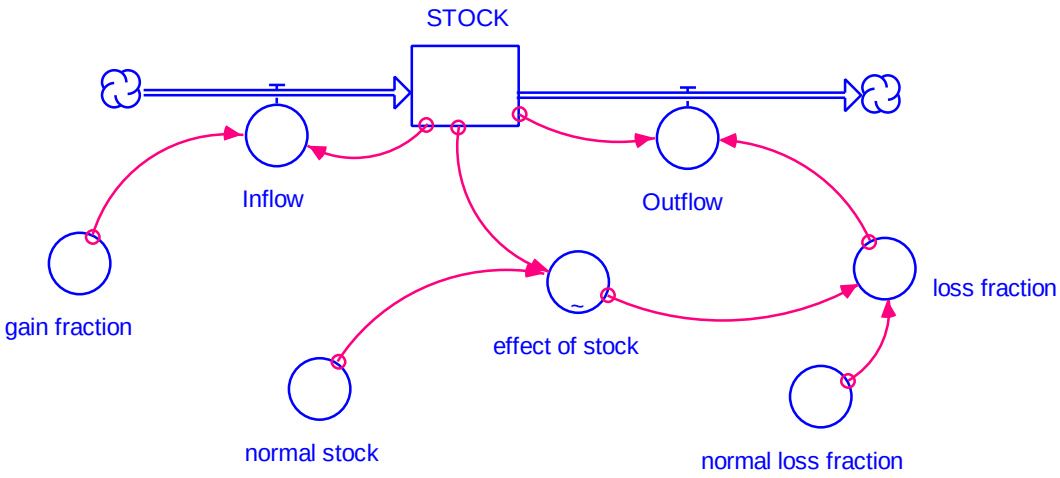


Figure 3: Generic structure that produces S-shaped growth

The model in Figure 3 is composed of three feedback loops. The positive feedback loop is associated with the Inflow to the STOCK. The Inflow, the product of the STOCK and the constant gain_fraction, drives the STOCK into exponential growth.

Two negative feedback loops regulate the Outflow from the STOCK. A connector ties the current value of the STOCK to the Outflow from the STOCK. The Outflow shapes a negative loop. The second negative loop, which passes through the loss_fraction, is responsible for shifting loop dominance.

The STOCK grows initially only if the gain_fraction starts off larger than the loss_fraction. When the gain_fraction is greater than the loss_fraction, the Inflow is greater than the Outflow, and the system experiences exponential growth. The STOCK, however, cannot grow forever. As the STOCK increases, so does the effect_of_stock multiplier. The effect_of_stock multiplier determines the effect of the STOCK on the loss_fraction. When the effect_of_stock multiplier assumes values larger than 1, the loss_fraction increases. The effect_of_stock multiplier will eventually become large enough for the loss_fraction to equal the gain_fraction. Then the Outflow will catch up to the Inflow and growth will cease. The system will reach equilibrium.

Even though the positive feedback loop that originally determines the system's growth is initially stronger than the negative loop, it is at a disadvantage when compared to the negative loop. The strength of the positive loop is constant. The negative loop, on the other hand, becomes stronger with the growth of the STOCK. The Outflow from the STOCK is the product of the STOCK and a *variable* loss_fraction. Here lies the key to the shifting loop dominance: the loss_fraction increases as the STOCK increases. When the STOCK is small the negative feedback loop goes almost unnoticed. But as the STOCK gets larger, the negative loop becomes stronger and stronger. Finally the negative feedback loop drives the system to equilibrium.

What type of curve is in the effect_of_stock table function? The curve in the effect_of_stock table function is determined by the characteristics of the constraint acting upon the specific system in question. In Figure 3 the constraint is modeled as a normal_stock converter which determines how much the STOCK can grow. As the value of the STOCK becomes large with respect to the normal_stock, the loss_fraction increases steadily due to the increased presure of the constraint. So the curve in the effect_of_stock table function increases as the ratio of the STOCK to the normal_stock increases. A sample curve is included in Figure 4.

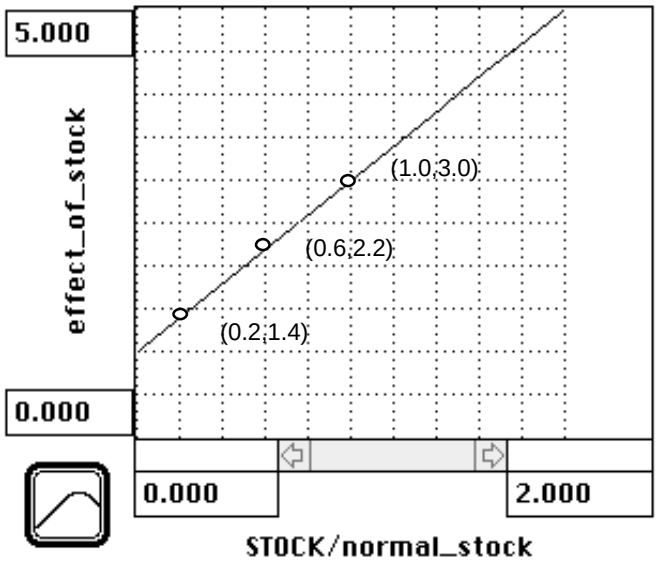


Figure 4: Sample effect_of_stock curve

The effect_of_stock curve outputs a fraction steadily increasing from 1. When the STOCK is very small, the multiplier is close to 1. The loss_fraction is almost identical to the normal_loss_fraction. The positive loop easily dominates. As the STOCK increases, however, so does the effect_ of_stock multiplier. When the STOCK is 20% of the normal_stock, the ratio of STOCK to normal_stock is 0.2. The effect_of_stock multiplier outputs a value of 1.4. So the loss_fraction equals 1.4 times the normal_loss_fraction. When the STOCK is 60% of the normal_stock, the effect_of_stock multiplier outputs a value of 2.2. So the loss_fraction is 2.2 times the normal_ loss_fraction. As the STOCK grows the loss_fraction grows, catching up to the gain_fraction.

Equilibrium occurs when the loss_fraction reaches the gain_fraction. In the generic structure model in Figure 3, the gain_fraction is set to equal 3 times the normal_loss_fraction. So the STOCK reaches equilbrium when the loss_fraction is equal to 3 times the normal_loss_fraction. When does the effect_of_stock multiplier reach the value of 3? The table function outputs a 3 when the ratio of STOCK to normal_stock is 1. So the system finally reaches equilibrium when the STOCK grows to the size of the normal_stock.

Note that the equilibrium value of the STOCK is determined by the relative values of the gain_fraction and normal_loss_fraction and the steepness of the curve in the effect_of_stock multiplier. If the curve in the table function is very steep, then a small growth in STOCK has a strong effect on the loss_fraction. The loss_fraction grows quickly and the Outflow from the STOCK catches up with the Inflow before the STOCK has the chance to become very large. The STOCK ceases to grow before it reaches the size of the normal_stock. On the other hand, if the ratio of the gain_fraction to the normal_loss_fraction is very large, then the loss_fraction has much catching up to do. If the gain_fraction was 5 times the normal_loss_fraction, then equilibrium will not be reached until the effect_of_stock multiplier outputs a value of 5. The effect_of_stock

table function in Figure 4 does not output a value of 5 until the ratio of STOCK to normal_stock has a value of 2. In other words, the STOCK would need to be twice as large as the normal_stock for the system to reach equilibrium.

2.2.1 Applying the Generic Structure to the Rabbit Population Model

Figure 5 is a model of Jean's rabbit population. Assemble the model using STELLA, paying careful attention to the relationship between the variables in the rabbit population and the corresponding variables in the generic structure. See Appendix 5.2 for documented equations of the rabbit population model.

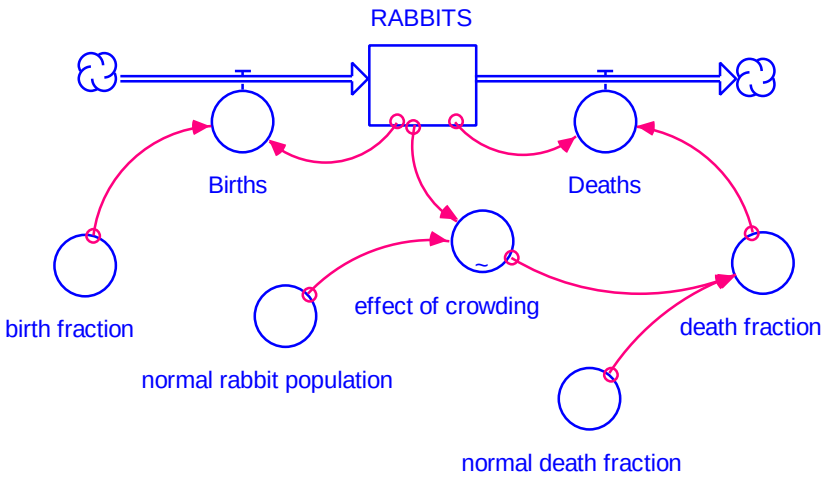


Figure 5: Stock and flow diagram of the rabbit population

Three distinct feedback loops are at work in the population model. The left hand side of the model contains the positive feedback loop. The right hand side of the model displays the two negative loops. The normal_death_fraction is equal to one third of the birth_fraction. Because the birth_fraction is larger than the death_fraction, Births will initially predominate. As more RABBITS are born there are more and more RABBITS ready to give birth to even more RABBITS.

The positive loop cannot go unchecked forever because of the constraint on the system: the environment can only support a maximum rabbit population. The total supply of water available to the rabbits is fixed. The ratio of RABBITS to the normal_rabbit_population determines the effect_of_crowding on the death_fraction. Figure 6 is a plot of the effect_of_crowding table function. The curve in the table function is a representation of the specific constraint of a constant water supply. When there are few rabbits, crowding has no effect on the death_fraction. Water is in abundance. The table function outputs a value of 1, so the death_fraction is equal to the normal_death_fraction. When the rabbit population becomes very large, however, the death_fraction starts to increase as the effect_of_crowding table function outputs a value larger than 1. As many rabbits as possible are drawing from the constant water supply, and the environment's maximum capacity for rabbits is finally reached. The effect_of_crowding table function outputs a value of 3. So the death_fraction equals 3 times the normal_death_fraction. The death_fraction has caught up to the birth_fraction. The number of Deaths reaches the number of Births and the rabbit population system stabilizes at equilibrium.

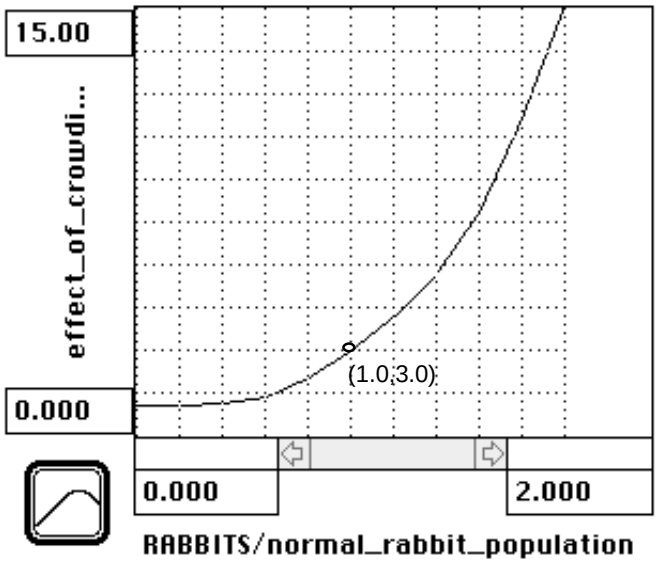


Figure 6: Effect_of_crowding table function

2.2.2 Running the Rabbit Population Model

Set up a graph for the population of RABBITS. Plot the birth_fraction, normal_death_fraction, and death_fraction on another graph. Create a converter that calculates the Net Flow into the stock of RABBITS. The Net Flow is Births (Inflow) minus Deaths (Outflow). Set up a graph for Births, Deaths, and the Net Flow over time. Then run the model over a 24-month time period.

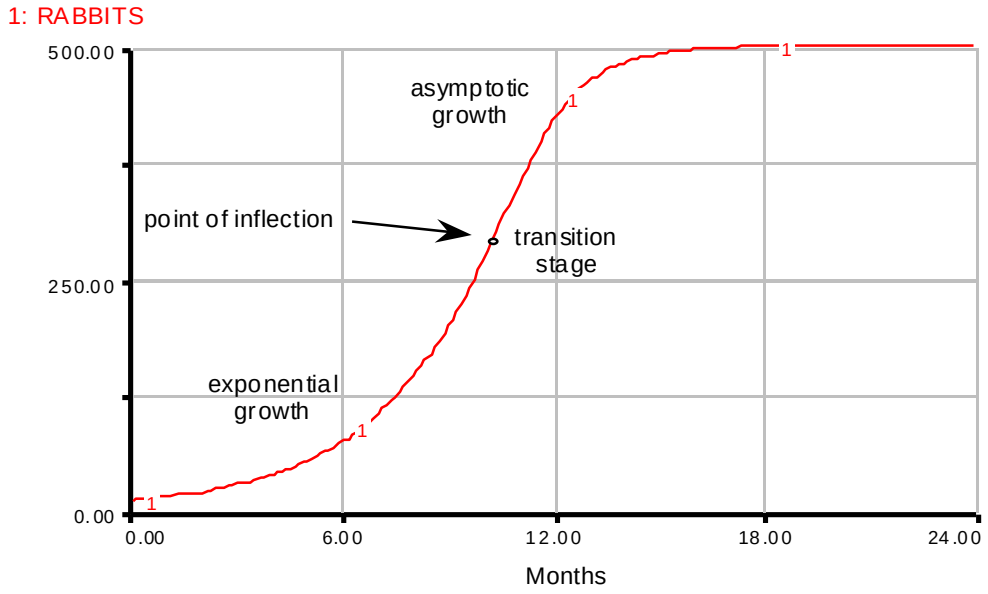


Figure 7: Rabbit population model: stock

Several interesting insights about the nature of S-shaped growth unfold with the examination of the graphs in Figures 7 through 9. First look at the stock, RABBITS, in Figure 7. Notice how RABBITS grows exponentially in the first months. The initial growth is due to the positive feedback loop. Rabbits make rabbits who make more rabbits. Around month 10, however, the curve switches from positive to negative concavity. The rabbit population is starting to feel the constraint of an environment in which resources are limited. Exponential growth has tapered into asymptotic growth. The point on the curve at which the switch occurs is called the "point of inflection." It is sometimes difficult to tell exactly where the point of inflection lies by simply looking at the

graph. Instead of determining the exact value of the point of inflection, it is often practical to simply designate the general region between exponential and asymptotic growth, the "transition stage."

The system's final state is equilibrium. The rabbit population stabilizes as the negative loop generates asymptotic growth. By month 15 the system has almost reached equilibrium.

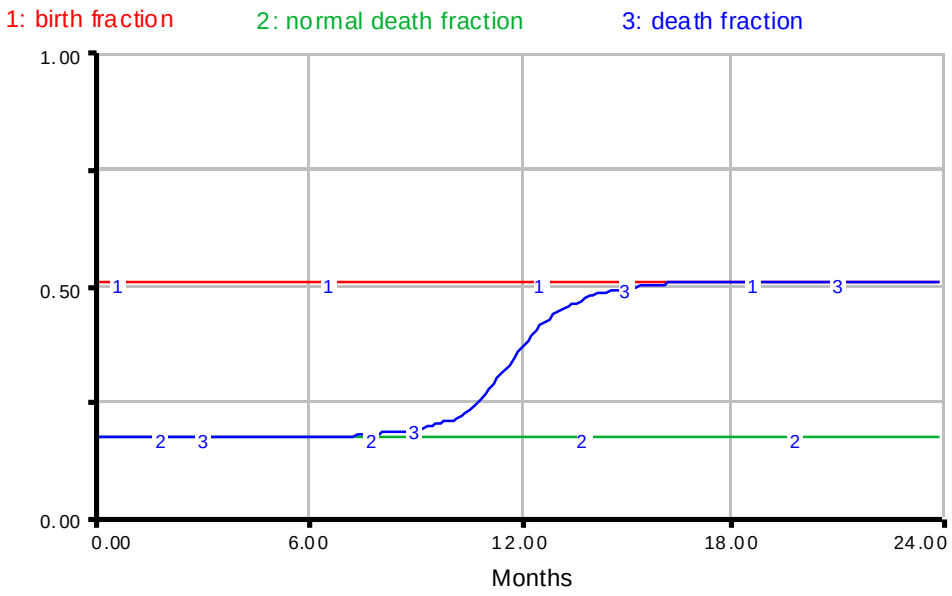


Figure 8: Rabbit population model: fractions

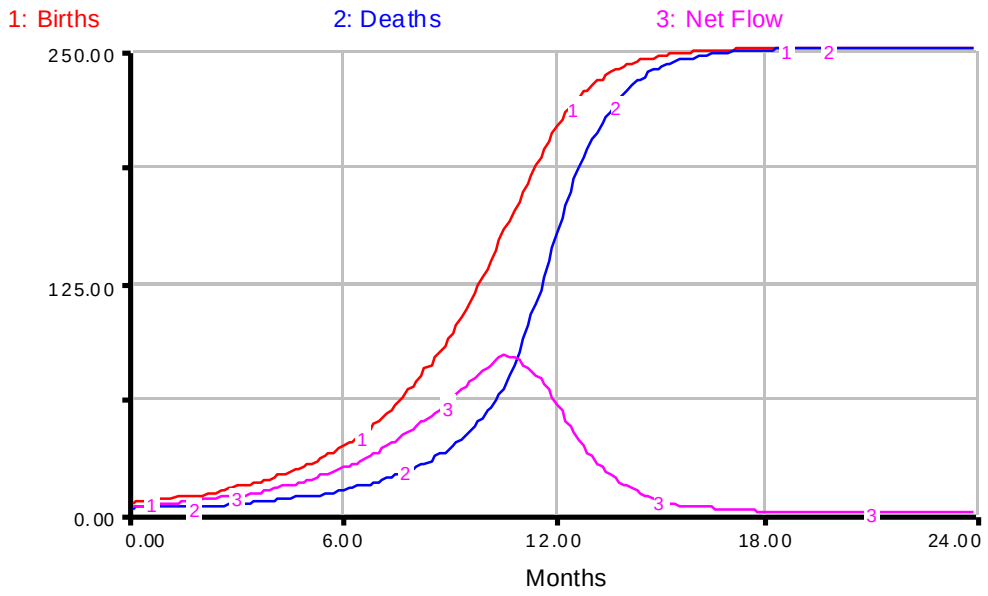


Figure 9: Rabbit population model: flows

Figures 8 and 9 depict the dynamics behind the shift in loop dominance. Examine Figure 8. At first the death_fraction is equal to the normal_death_fraction, so birth_fraction is initially three times the death_fraction. Because the birth_fraction is larger than the death_fraction, the number of Births surpasses the number of Deaths, as seen in Figure 9. Compare the behavior of the Net Flow curve in Figure 9 to the behavior of the rabbit population in Figure 7. When the Net Flow has a positive slope the number of RABBITS grows exponentially.

The death_fraction in Figure 8 starts to increase around month 8. The number of Deaths increases, but the number of Births increases even faster. Growth is still exponential.

Around month 10 a shift occurs. The Net Flow reaches a maximum. Its slope is no longer positive, but zero. When the Net Flow is at a maximum, the gap between Births and Deaths is the largest. Meanwhile, in Figure 7, the growth of the rabbit population experiences a point of inflection. Exponential growth tapers into asymptotic growth. The system switches from positive to negative feedback.

The death_fraction continues to increase in Figure 8 around month 12, approaching the gain_fraction. The number of Deaths is catching up to the number of Births. The Net Flow has a negative slope, so negative feedback causes the number of RABBITS to stabilize towards equilibrium. Although the slope of the Net Flow is negative, the Net Flow itself never goes negative. The number of Deaths never rises higher than the number of Births. So when the Net Flow has a negative slope the rabbit population does not experience decay but stabilizing *growth*.

By month 15 the system is close to equilibrium. The rabbit population stabilizes at equilibrium when the number of Deaths reaches the number of Births. The death_fraction equals the birth_fraction. Births and Deaths cancel each other. The total number of RABBITS remains constant.

There are two possible equilibria in systems that exhibit S-shaped growth. The first occurs before any growth takes place. Visualize the little farm in Provence just before Jean arrived with his rabbits. No rabbits are born. None die. Births and Deaths are both zero, and the system is at equilibrium. Only two rabbits, however, are needed to set the entire system into motion. Twenty months later the system reaches another equilibrium. The number of Births and Deaths each month are no longer zero. They are, however, equal. So the Net Flow of Births minus Deaths is zero. At the second equilibrium, many rabbits are born and just as many die. In all, 500 rabbits roam around Jean's small farm. The first equilibrium is an unstable equilibrium; the second is stable. An *unstable* equilibrium is easily perturbed, easily set into motion, like a penny balanced on its edge. A *stable* equilibrium guarantees that there will be no more motion, in population dynamics, no more growth. A penny lying flat on its side is in stable equilibrium.

The behavior obtained from the rabbit population model illustrates the characteristics which determine growth. Changes in the net flow into a stock change the

growth experienced by the stock. When the net flow has a positive slope, positive feedback loops are dominant and the stock experiences exponential growth. When negative loops dominate, the slope of the net flow becomes negative, and the stock is driven into asymptotic growth. The inflection between exponential and asymptotic growth occurs when the slope of the net flow is zero. At the shift, the net flow is at a maximum. The stock ceases to grow when the net flow is zero.

The observations on the dynamics of Net Flow are summarized in Figure 10.

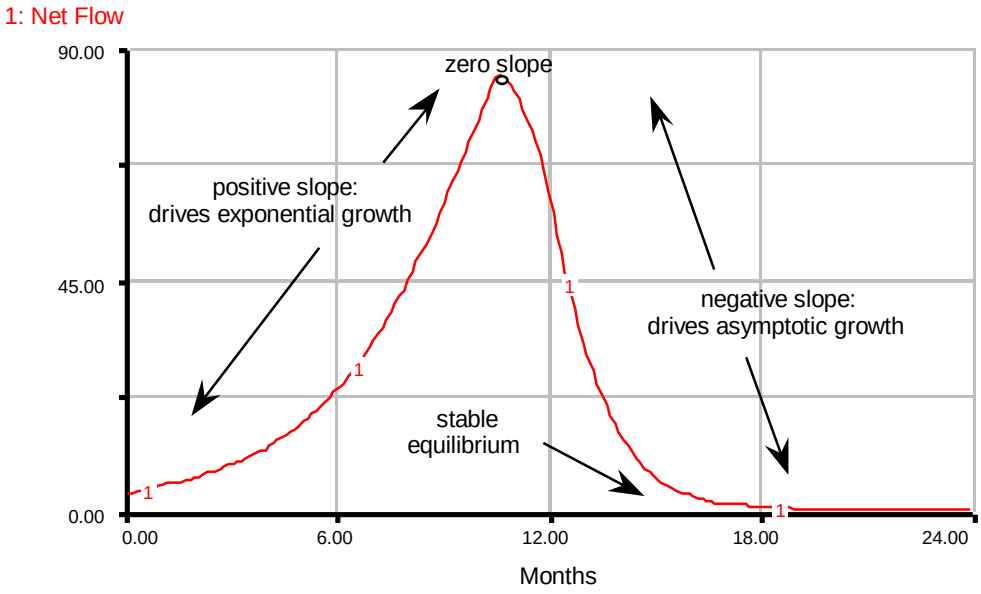


Figure 10: Summary of observations

3. Examples

Many different systems exhibit S-shaped growth. The generic structure for S-shaped growth presented in Section 2 helped explore and build intuition about the dynamics behind S-shaped growth. The generic structure explicitly shows the feedback loops and constraint on growth which cause shifting loop dominance. In most systems, however, feedback loops are less apparent and constraints on growth are often hidden or implied. The models in the next three examples will not resemble the generic structure at a first glance. The same underlying dynamics, however, are at play. The following examples will examine the principles of S-shaped growth from different perspectives; emphasizing that the same basic dynamics cause a whole range of different phenomena to produce identical behavior.

3.1 Cabbage Patch Kids

Fads exhibit the characteristics of S-shaped growth. Bell bottoms, charm bracelets, hula hoops, mood rings, Rubik's Cubes, and Cabbage Patch Kids all hit the market with a bang. People rushed to stores to buy the latest fad. All of your friends had charm bracelets, you needed one too. The number of people who would puzzle over Rubik's Cubes grew exponentially, at first. After a while though, the market for such products saturated. You have a Rubik's Cube on your coffee table, another in your office, maybe one in the bathroom. By now you have already resorted to pulling off the colored stickers and gluing them back on so that your centerpiece won't be an embarrassment to your intellect. Your Cabbage Patch Kids have taken over your bed. More mood rings? Well, you have only ten fingers...

Modeling the Cabbage Patch Kids mania will help develop a better understanding of the dynamics behind S-shaped growth. In 1980 Coleco invented Cabbage Patch Kids, fabric dolls designed to look just like their little "parent." In the Babyland General

Hospital, employees dressed in hospital gowns sewed the Cabbage Patch Kids by hand. The dolls were then given names and delivered with birth certificates and adoption papers. The dolls soon became extremely popular. Toy stores were often unable to cope with the demand for Cabbage Patch Kids. Orders that had been on backlog for weeks were greeted by mobs of parents eager to obtain the cherished dolls for their children. After a while though, most children whose parents could afford Cabbage Patch Kids (they often cost over $100) already had them. The fad died. As the small parents grew up, Cabbage Patch Kids were eventually relegated to shelves and toy chests.

Please consider the following questions before continuing with the example.

- What critical variables are involved in the Cabbage Patch Kid system?

  _____

- What constraints are acting on the system?

  _____

- What positive and negative loops determine the system's behavior?

  _____

Some important variables are the number of children who own a Cabbage Patch Kid (CPK), the number of children who have yet to buy a CPK, and how long it takes a child to purchase a doll. To simplify the model, assume that each child only buys one doll.

The constraint, the fixed value, is the market—the total number of children who can afford to buy a doll—approximated at one million children.

The system's behavior is controlled by two major loops. The positive loop represents the popularity of the dolls which grows by word of mouth. The more children own CPKs, the more popular the CPKs become. The negative loop is caused by the constraint of a fixed market. When more children buy CPKs, fewer children remain who have yet to buy a CPK of their own, and fewer purchases are made.

Figure 11 is a possible model of the Cabbage Patch Kids mania. Documented equations of the model can be found in Appendix 5.2.

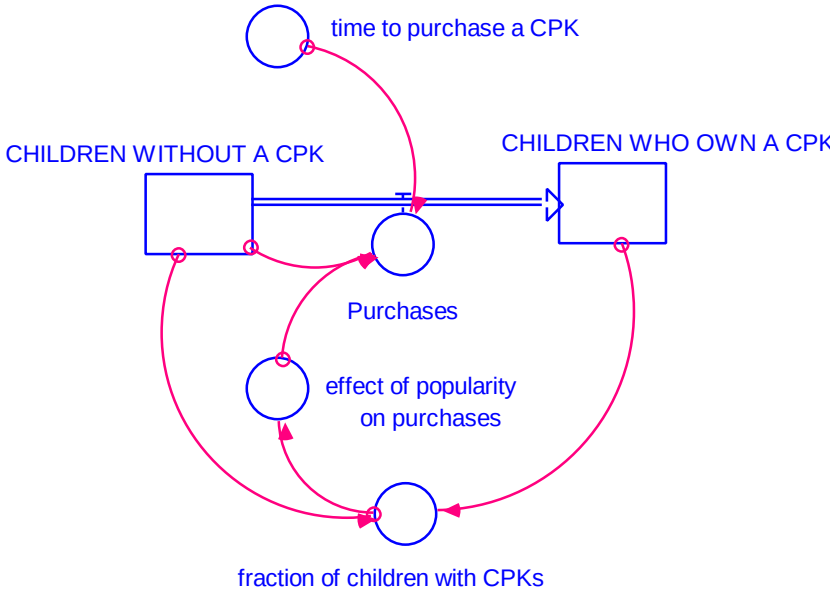


Figure 11: Cabbage Patch Kid model

The positive feedback loop in the Cabbage Patch Kid model is due to the increasing popularity of the dolls. The fraction_of_children_with_CPKs increases proportionally as the number of CHILDREN_WHO_OWN_A_CPK grows. The popularity of the dolls is directly related to the percentage of the population that owns the dolls (excluding real-world perception delays for the simplicity of the model.) An increase in the number of child-owners increases the effect_of_popularity_on_purchases, which is equal to the fraction_of_children_with_CPKs. The popularity multiplier then increases the flow of Purchases, which in turn raises the number of CHILDREN_ WHO_OWN_A_CPK.

The model in Figure 11 emphasizes the flow of children from one status to another. First they long for a doll, then they own one. As the number of CHILDREN_WHO_OWN_A_CPK increases, the number of CHILDREN_WITHOUT_

A_CPK decreases. The flow of Purchases is a fraction of the remaining market over time. The number of CHILDREN_WHO_ OWN_A_CPK determines the popularity of the dolls. The popularity in turn convinces CHILDREN_WITHOUT_A_CPK to make purchases. The number of Purchases is calculated by multiplying the effect_of_ popularity_on_purchases multiplier by the number of CHILDREN_WITHOUT_A_CPK and dividing by the time_to_purchase_ a_CPK. The time_to_purchase_a_CPK is the average time for children to convince their parents to buy the expensive dolls plus the average time for the parents to find a doll which resembles their child.

In order to observe the behavior of the Cabbage Patch Kid phenomena, run the model in Figure 11 over a 24-month time period. Graph CHILDREN_WHO_ OWN_A_CPK on one graph and Purchases on another. Then answer the questions below.

? When does the loop dominance shift?

_____

? What is the approximate value of the stock at the point of inflection?

_____

? When does the Cabbage Patch Kid system reach equilibrium?

_____

? How many children own Cabbage Patch Kids at equilibrium?

_____

? How many purchases are made when the system is at equilibrium?

_____

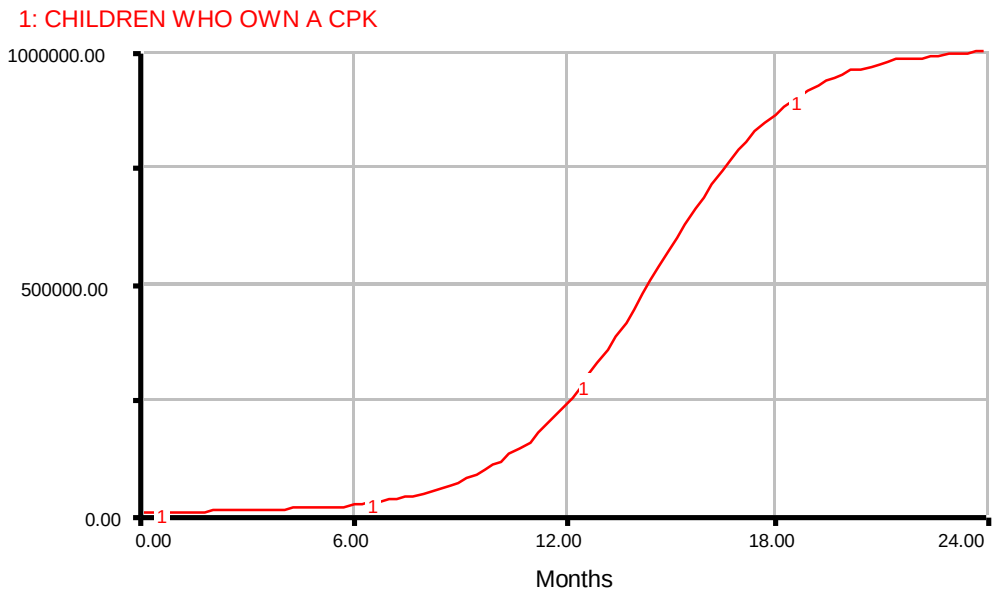


Figure 12: Cabbage Patch Kids: stock over time

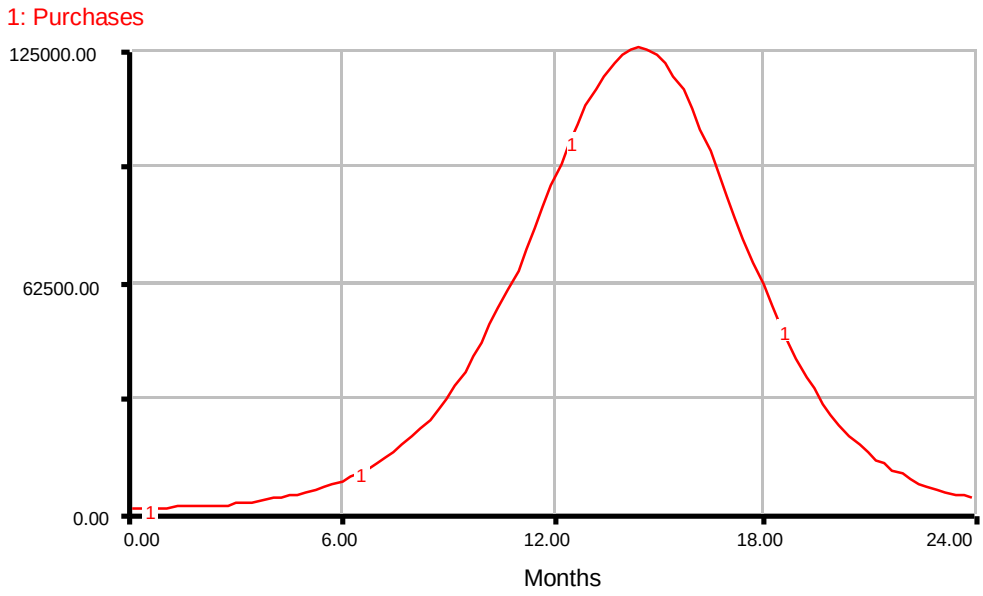


Figure 13: Cabbage Patch Kids: net flow over time

When does the loop dominance shift? When Purchases has a positive slope the system is experiencing positive feedback. On the other hand, when Purchases has a negative slope the system is undergoing negative feedback. The transition occurs when the slope of Purchases is zero, when Purchases reaches a maximum. Note in Figure 13

that Purchases reaches a maximum around month 15. The loop dominance shifts around month 15.

Examine Figure 12 to obtain the value of the stock of CHILDREN_WHO_OWN_A_CPK at the point of inflection. The curve shifts from exponential to asymptotic growth around month 15. The shift in growth corresponds to the shift in loop dominance. Around month 15, the stock of CHILDREN_WHO_OWN_A_CPK is approximately 500,000 children.

The system approaches its stable equilibrium by month 24. The asymptotic growth in Figure 12 stabilizes as the total number of CHILDREN_WHO_OWN_A_CPK approaches 1 million, exactly the total population of children. At the same time in Figure 13, at month 24, the number of Purchases goes to zero. When the net flow into a stock is zero, the system stabilizes.

3.2 Protestantism

In 1517 Martin Luther posted 95 Theses on the door of a church in a small German town. The Theses were a list of grievances against the Roman Catholic Church. This event marked the beginning of a period in history called the Reformation in which the religion of the Protestants spread across Europe.

Many factors contributed to the rise of Protestantism in the early 16th century. The Roman Catholic Church was corrupt. The Popes had been engaged for the last hundred years in a series of papal wars. At times, several contenders fought for the Papacy. Martin Luther revolted specifically against the practice of indulgences, a process by which penitents could buy forgiveness for their sins.

Protestantism reverted to apostolic poverty and simplicity. Protestants believed in the priesthood of all believers. Martin Luther translated the Bible from Latin (which only the educated people understood) to everyday German so that the Word of God would be accessible to all. Henry VIII, the king of England, severed with the Roman Catholic

Church in 1534 over the issue of divorce and created a branch of Protestantism, the Anglican Church of England.

As more and more people converted to Protestantism, however, the Roman Catholic Church began to fight back. The Counter Reformation, a period of bloody religious civil wars, began. The Inquisition, established in 1542, persecuted the Protestants under the pretense of "suppressing the heretics." Many intellectuals fled to England and Switzerland. In countries such as France, Protestants had to meet in secret, fearing for their lives.

The spread of the Protestant religion exhibited S-shaped growth. Two important stocks are the number of Protestants and the number of non-Protestants.

? What variables drive the positive feedback loop?

_____

? What variables drive the negative feedback loops?

_____

The appeal of Protestantism, spread by word of mouth, determines the positive loop. The word of mouth loop is strengthened as the number of Protestants increases, which in turn engenders more conversions. The corresponding negative loop is due to saturation, the constraint of a fixed population. The fewer non-Protestants remaining, the fewer conversions. Persecution accounts for the second negative loop. The threat of Protestantism causes the Roman Catholic Church to retaliate with the Inquisition. Persecution dissuades many people from turning to Protestantism. The more Protestants, the larger the perceived threat, the more persecution, the fewer conversions.

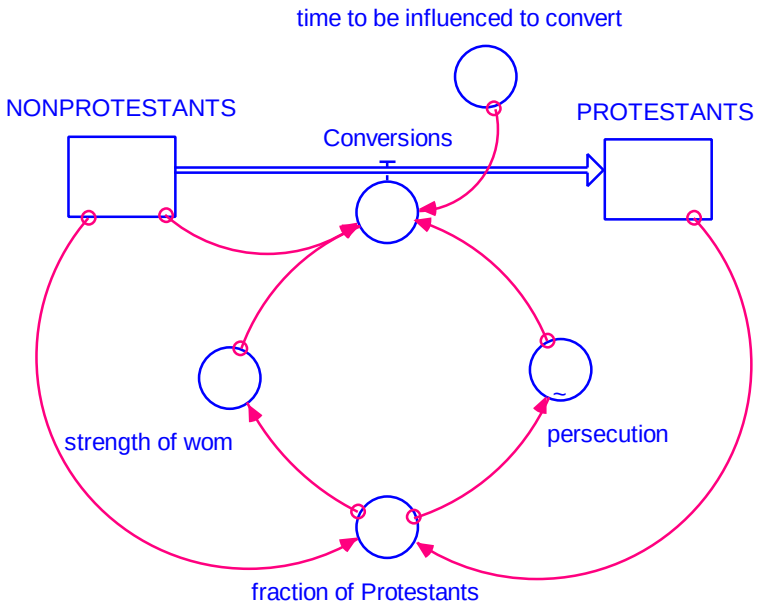


Figure 14: Protestantism model

Figure 14 depicts the Protestantism model. The persecution table function can be found, along with documented equations of the Protestantism model, in Appendix 5.4. Trace through the positive and negative loops in Figure 14. The positive loop is determined by the spread of Protestantism by word of mouth. As the number of PROTESTANTS increases, the fraction_of_Protestants in the total population correspondingly increases. The strength_of_wom (where wom stands for word of mouth) is equal to the fraction_of_Protestants in the total population. The strength_of_wom multiplier increases the number of Conversions, which in turn raises the number of PROTESTANTS.

The outflow from the stock of NONPROTESTANTS, Conversions, depends on the number of NONPROTESTANTS. Here lies a negative saturation loop. When many people are susceptible to being converted, many NONPROTESTANTS, the number of Conversions is large. When no potential converts remain, however, no amount of persuasion will generate new converts. As the number of PROTESTANTS increases, the number of NONPROTESTANTS decreases, and the number of Conversions subsequently decreases.

Persecution also decreases the number of Conversions. The threat of Protestantism increases as the number of PROTESTANTS grows. Persecution and bloody civil war begins when the PROTESTANTS become numerous enough to pose a significant threat to the established regime. In Figure 14, persecution is a multiplier that depends on the fraction_of_Protestants in the population. When the fraction is small, persecution is very low. As the fraction increases, persecution sharply rises. The number of Conversions is negatively affected by the persecution multiplier. The effect of persecution on Conversions is (1 – persecution). So when the number of PROTESTANTS increases, persecution increases, and the number of Conversions decreases.

Figure 14 is a model with three feedback loops of varying strengths. Which will be dominant and when? Computer simulation is the only way to correctly estimate a complex model's behavior. Run the model in Figure 14 over a period of 100 years, from 1500 to 1600. Graph the number of PROTESTANTS. Graph Conversions over time. Plot the persecution converter along with the strength_of_wom converter. Make an additional graph which includes both PROTESTANTS and NONPROTESTANTS on the scale.

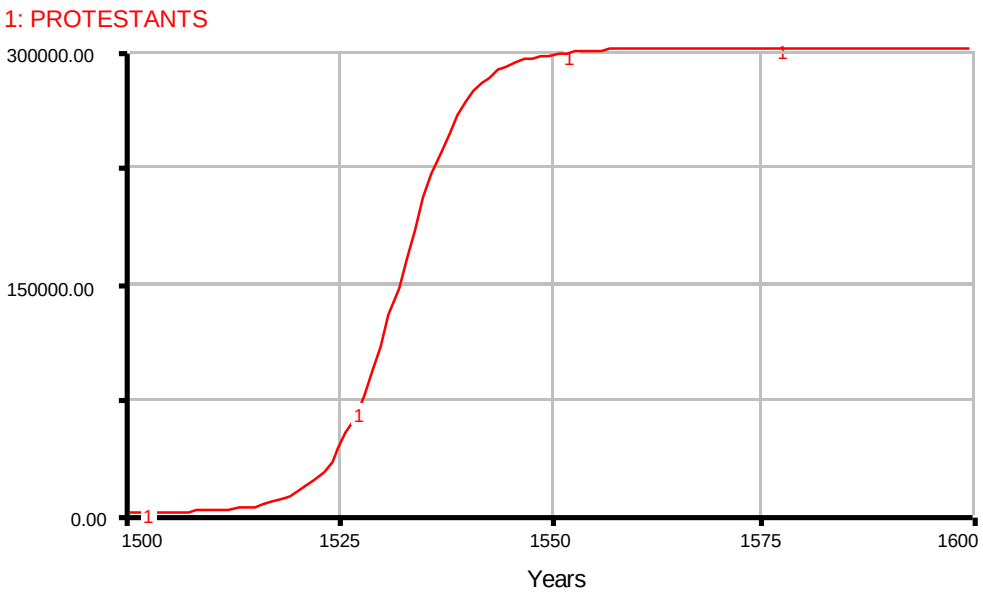


Figure 15: Protestantism: stock over time

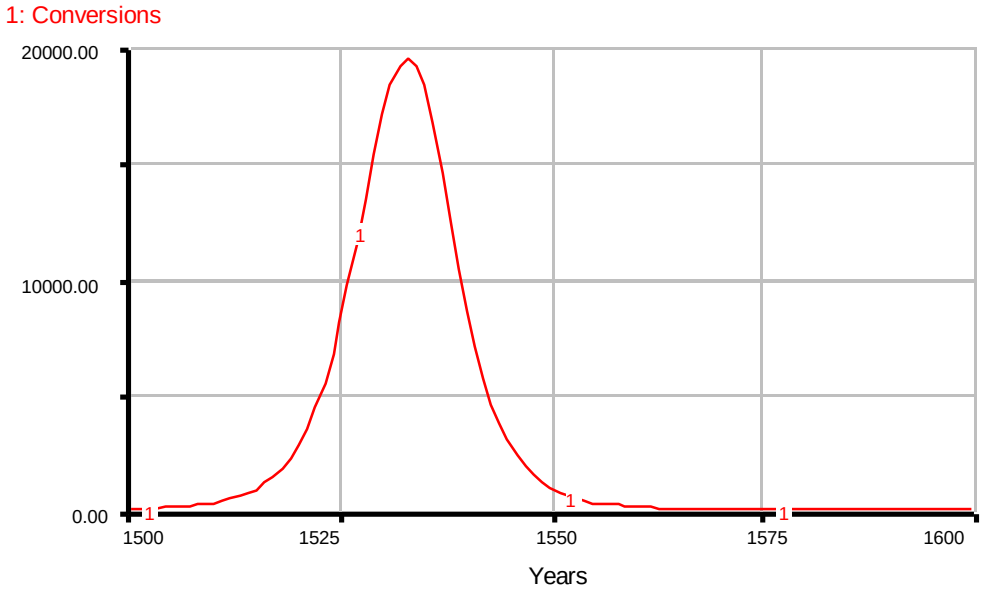


Figure 16: Protestantism: net flow over time

? When is Protestantism most popular? (When is the Conversions rate largest?)

_____

Examine Figure 16. Few people converted at the beginning of the century. Lutheran thought was just beginning to take root. As word of mouth causes the intrinsic appeal of Protestantism to entice people to convert, the number of Conversions grows. By 1535 Conversions reaches its maximum of 20,000 people per year. In Figure 15 the number of PROTESTANTS grows exponentially over the first 35 years. The shift from exponential to asymptotic growth occurs when almost a sixth of the population has been converted. At the inflection point the number of Protestants has become large enough to pose a threat to the established Roman Catholic Church.

? What causes the decrease in the rate of Conversions?

The Inquisition, officially established in 1542, is part of a growing wave of persecution that seriously drops the number of Conversions. Furthermore, with the growth of the number of PROTESTANTS, the number of NONPROTESTANTS is declining. A decrease in the number of NONPROTESTANTS also reduces the rate of conversion. Which negative loop is responsible for the shift in loop dominance seen in Figure 15?

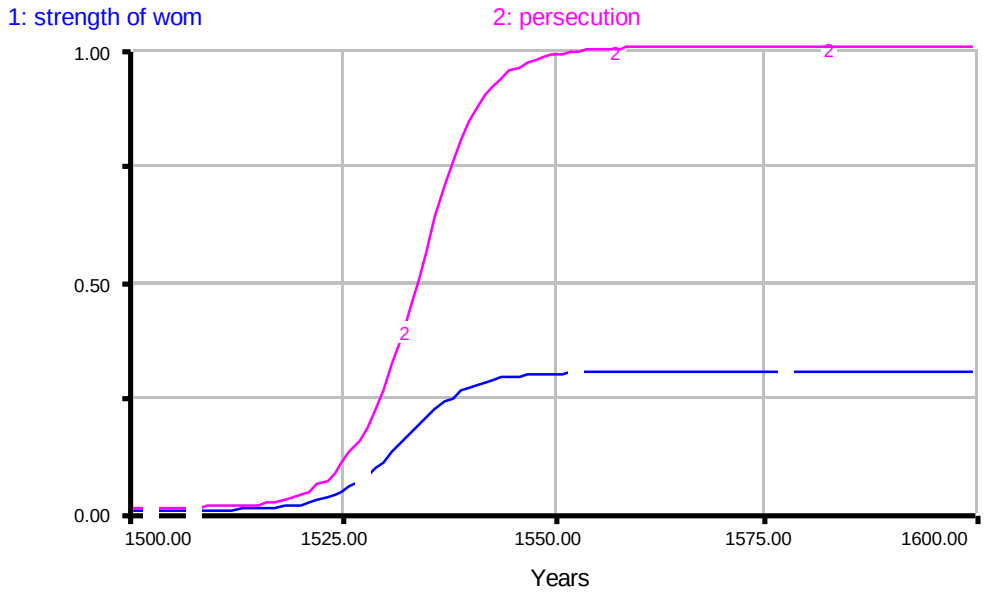


Figure 17: Protestantism: multipliers

Figure 17 sheds some light on the question of which negative loop is responsible for shifting loop dominance. The effect of persecution is much stronger than the effect_of_wom. Recall that Conversions is calculated by multiplying the number of NONPROTESTANTS by the strength_of_wom and (1 – persecution) and dividing by a time constant. In Figure 17, persecution rises sharply around 1535. As persecution approaches a value of 1, (1 – persecution) approaches zero. When the number of PROTESTANTS becomes large enough for the fraction_of_Protestants to cause such a threat that persecution reaches a value of one, the Inquisition is in full swing. Once persecution reaches a value of one, (1 – persecution) equals zero and the rate of Conversions becomes zero. Because the net flow into the stock of PROTESTANTS has become zero, growth stops.

Look at Figure 18 and notice that the entire European population does not convert to Protestantism, contrary to how the entire potential market bought Cabbage Patch Kids. Persecution drives the rate of Conversions to zero before Protestantism has reached out to the entire population. The number of PROTESTANTS stabilizes towards a constant number: 300,000 people, almost one third of the entire population. By 1600, no new conversions take place. The system has stabilized at equilibrium.

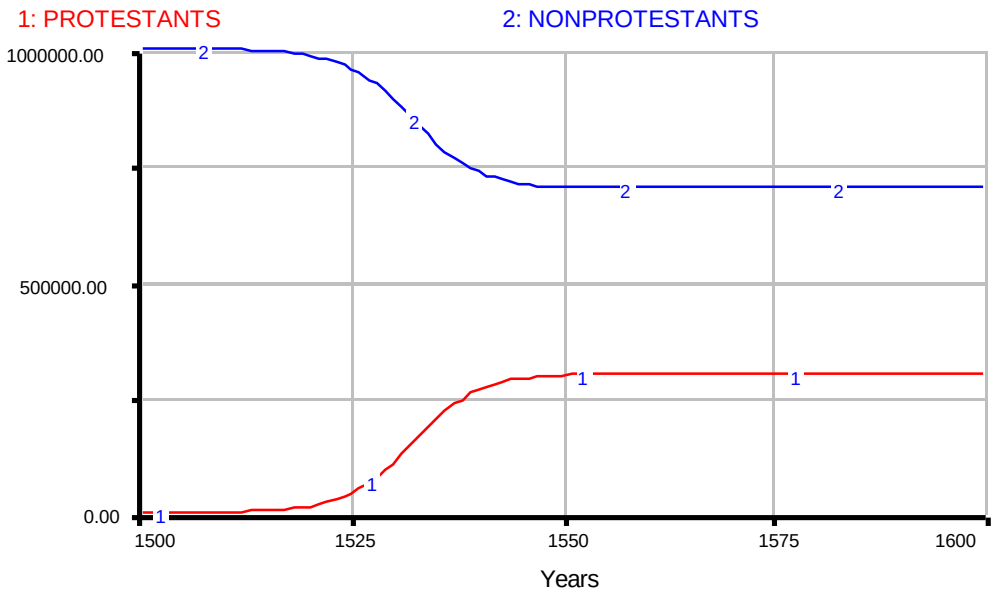


Figure 18: Protestantism: both stocks over time

In the Protestantism example, more than one negative feedback loop was present. Yet the system still generated S-shaped growth. The number of feedback loops is not a determining factor in the dynamics of S-shaped growth. What is important then? The next example shall address this question.

3.3 Immune Response

By the year 2000 as many as 40 million people worldwide will have become infected with HIV, the human immunodeficiency virus, which causes AIDS: Acquired Immune Deficiency Syndrome. AIDS is caused by a breakdown of the body's immune defense system. This example will first examine the human immune system. Then the model will be slightly modified to describe what happens when HIV impairs the human immune system and leaves it vulnerable to a great variety of otherwise harmless infections.

3.3.1 A Healthy Immune System

The immune system is powerful because of its specificity (cells attack specific invaders) and its memory (cells are ready to launch a rapid attack if the same type of invader returns again). When a foreign cell enters the human body, the intruder begins to multiply and diffuse through the blood stream. Each foreign cell has its own specific antigens. An antigen is a large molecule with a distinct configuration that triggers an immune response. Helper T cells, a specific type of white blood cells, roam around the human body, looking for foreign antigens.

Once a helper T cell recognizes a foreign antigen, it activates B cells, another type of white blood cells, that begin to proliferate rapidly. Most B cells produce plasma cells which secrete antibodies that diffuse into the blood stream. The antibodies bind to the antigens, marking them for destruction. Other B cells are reserved as memory cells. When a similar antigen invades the body a second time, the memory B cells are ready to attack. An immune response which releases antibodies is called an "antibody-mediated" immune response.

If the foreign invader is intracellular (like a virus) rather than extracellular (like bacteria) then helper T cells activate a "cell-mediated" immune response. Once a helper T cell recognizes the foreign intruder, it activates killer T cells, yet another type of white

blood cells, that begin to proliferate rapidly. Killer T cells directly attack and destroy the foreign invaders. Again, memory T cells remain, prepared for any future attacks.

T cells and B cells are also known as white blood cells. In general, white blood cells respond to a foreign invasion by proliferating and producing antigen-specific defenders that destroy the foreign invaders. The number of white blood cells in a human body is indicative of how strong a battle the immune defense system is fighting at that time.

Figure 19 shows a simplified model of the immune system. Like all models, it is a simplification of one small part of a much larger system.

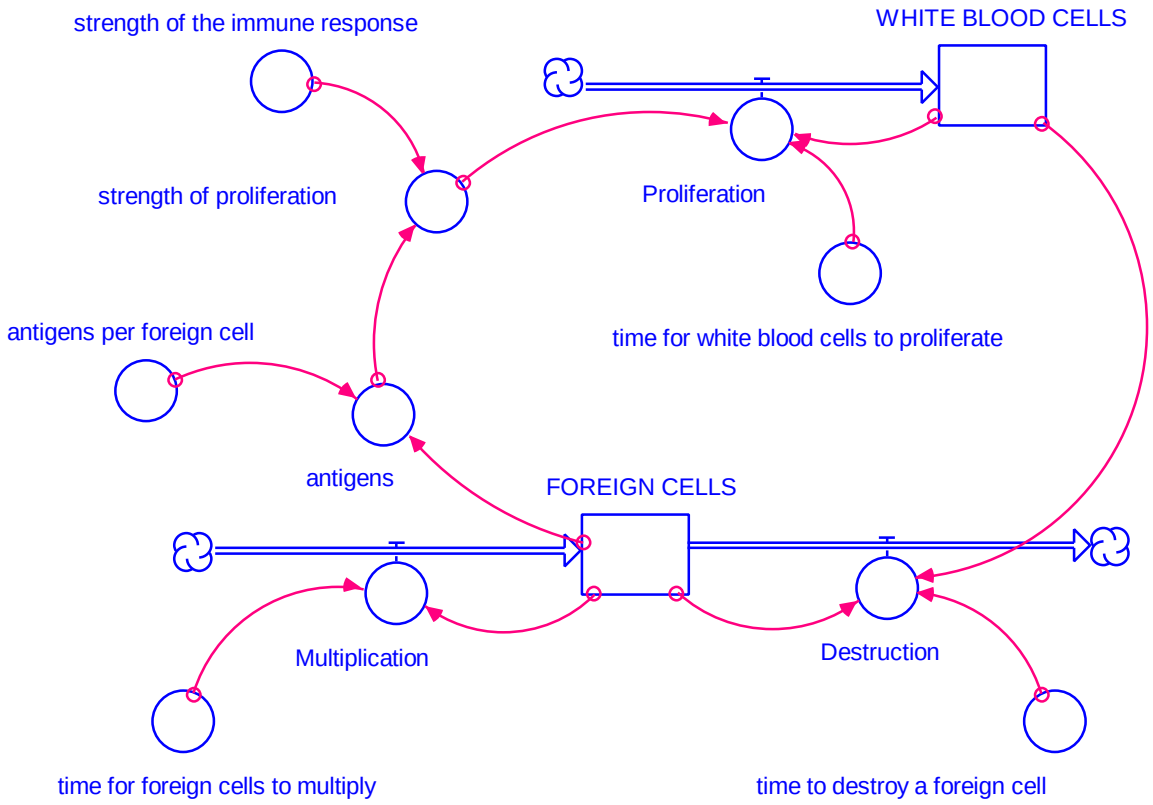


Figure 19: Immune response model

The proliferation of WHITE_BLOOD_CELLS is activated when helper T cells come into contact with antigens. The presence of a very small number of antigens is unlikely to set off a massive immune response. However, FOREIGN_CELLS multiply rapidly, much faster than rabbits. The strength_of_the_immune_response reflects the fraction of antigens that are detected by helper T cells. As the number of antigens grows, more antigens are detected, and the WHITE_BLOOD_CELLS proliferate more rapidly. Increasing Proliferation is the source of the positive loop that drives the growth of the WHITE_BLOOD_CELLS.

A negative loop comes into play as the WHITE_BLOOD_CELLS perform their duty and destroy FOREIGN_CELLS. As the number of WHITE_BLOOD_CELLS grows exponentially, antibodies or killer T cells are produced, and the number of FOREIGN_CELLS is drastically reduced. The immune response senses the absence of FOREIGN_CELLS—helper T cells come into contact with fewer and fewer antigens—and white blood cell Proliferation drops off, leaving the body with a good number of WHITE_BLOOD_CELLS ready to fend off a second possible invasion.

Run the model in Figure 19 over a period of 32 hours. Graph both WHITE_BLOOD_CELLS and FOREIGN_CELLS on the same plot. Combine the flows Multiplication, Destruction, and Proliferation on one graph. Documented equations for the immune response model can be found in Appendix 5.5.

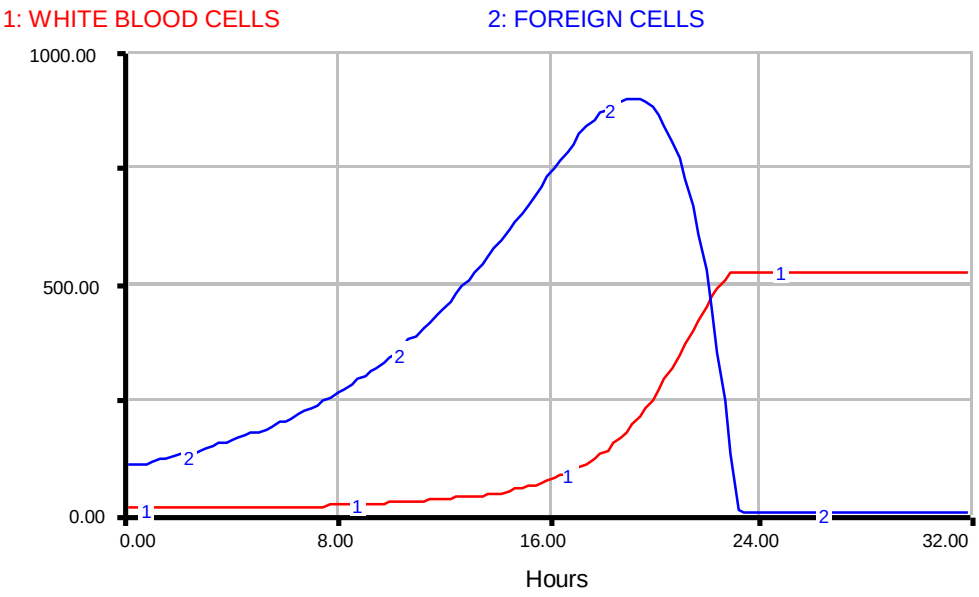


Figure 20: Immune response: stocks over time

Figure 20 depicts the growth of WHITE_BLOOD_CELLS and FOREIGN_ CELLS in an individual with a healthy immune system. Even though FOREIGN_CELLS and WHITE_BLOOD_CELLS have the same doubling time, they do not rise at the same rate because white blood cell Proliferation, unlike foreign cell Multiplication is also dependent on the number of antigens in the system.

The maximum number of FOREIGN_CELLS is larger than the maximum number of WHITE_BLOOD_CELLS because of the delay caused by the strength_of_the_immune_response. Helper T cells only come across approximately 1 out of every 1000 antigens. The WHITE_BLOOD_CELLS need to catch up on the FOREIGN_CELLS' "head start." When the hundred foreign cells per microliter of blood attack, only a few antigen-specific WHITE_BLOOD_CELLS (say, ten per microliter) are present to ward off the invasion. Notice, though, that the stock of WHITE_BLOOD_CELLS does not have an outflow. The white blood cells will eventually die, with the exception of the memory cells which will continue to reproduce, but, during the 24 hours of warfare, their forces are not depleted. The foreign cells, however, are being destroyed. Each white blood cell produces an antigen-specific

antibody or killer T cell that destroys a foreign cell each hour. The number of FOREIGN_CELLS may reach a higher maximum, but the WHITE_BLOOD_CELLS are killing machines.

Notice that the number of FOREIGN_CELLS reaches a maximum before WHITE_BLOOD_CELLS reaches equilibrium. The maximum number of FOREIGN_CELLS corresponds to the point of inflection on the WHITE_BLOOD_ CELLS curve. At the inflection point the number of WHITE_BLOOD_CELLS has become large enough to be effective in destroying FOREIGN_CELLS. As the number of FOREIGN_CELLS shoots down, the number of WHITE_BLOOD_CELLS no longer needs to grow, and their count stabilizes at an equilibrium value.

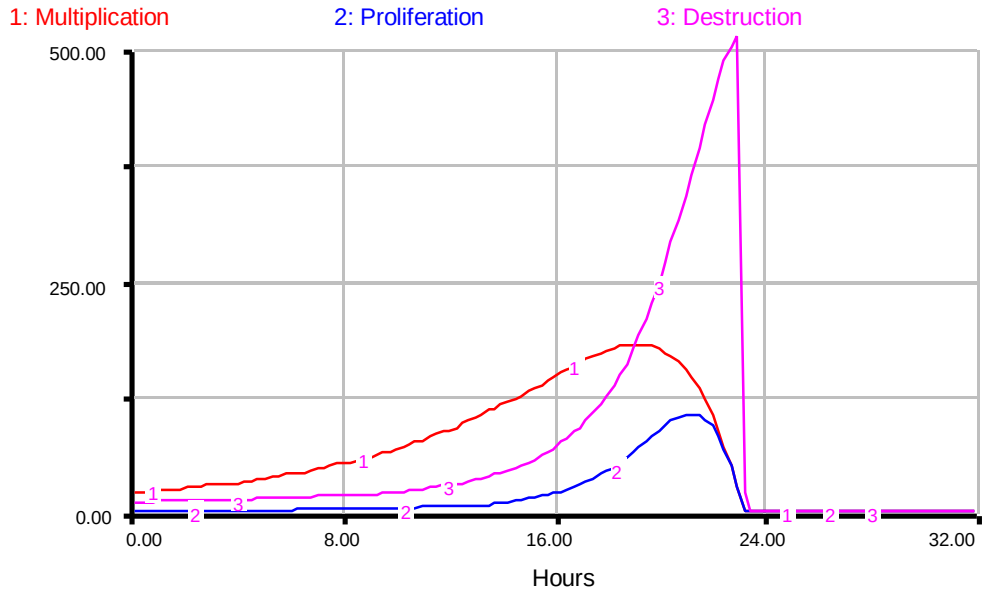


Figure 21: Immune response: net flows versus time

The curves in Figures 20 and 21 do not seem as smooth as the ones from the previous models. Destruction of FOREIGN_CELLS depends only indirectly on the number of FOREIGN_CELLS remaining. Destruction has no exponential smoothing; it continuously destroys FOREIGN_CELLS until none remain.

The immune response system after an invasion of FOREIGN_CELLS is an example of stable equilibrium. If similar invaders were to attempt to enter the body after the immune response, they would be recognized by antigen-specific memory cells and immediately destroyed. Although many of the WHITE_BLOOD_CELLS will eventually die off, memory B cells or memory T cells remain in the blood stream.

The immune response system is in unstable equilibrium before the invasion. Proliferation is zero. Once a few FOREIGN_CELLS enter, however, the system is triggered and a massive immune response ensues. Eventually the equilibrium becomes stable, and any future attacks of that specific type of foreign cell are guaranteed to have little effect.

3.3.2 An Immune System Affected by HIV

Section 3.3.1 discussed the dynamics of an immune response in a healthy immune system. HIV, however, is unlike any normal virus. It attacks the immune system itself.

The effect of HIV on the human immune system can be modeled by changing the strength_of_the_immune_response constant in the model in Figure 19. HIV cripples the immune system by sabotaging the genetic material of helper T cells. The helper T cell population is depleted. In a healthy body, helper T cells come across and recognize, say, one out of every 1000 antigens present per microliter of blood. When the immune system is impaired, however, the ratio goes down. With fewer functioning helper T cells on patrol, antigens are not detected as easily. The foreign cells have time to multiply before the immune response kicks in.

Cut the strength_of_the_immune_response by a factor of 10 and see what happens...

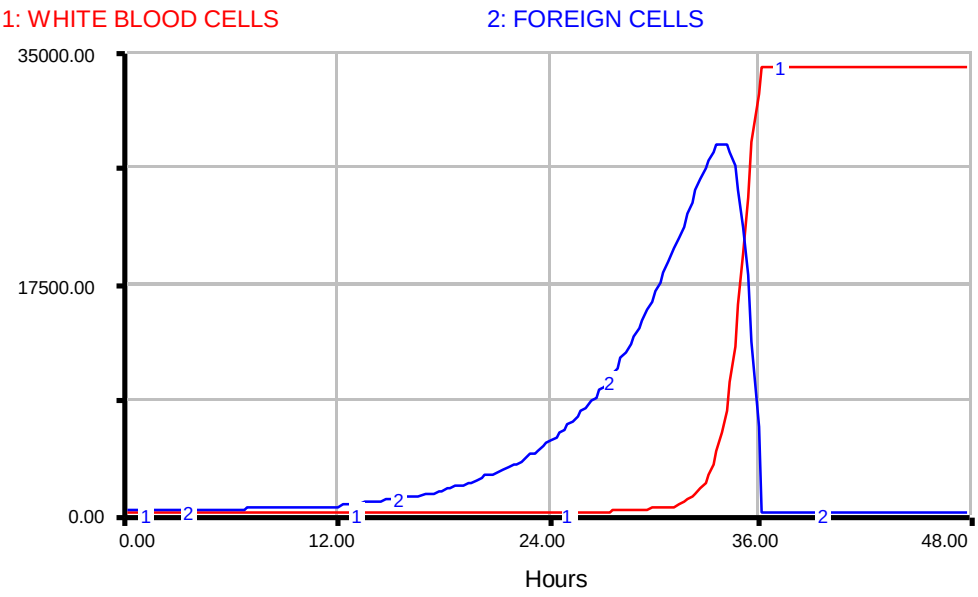


Figure 22: Strength_of_the_immune_response = 1 / 10,000

Figure 22 is a graph of the growth of WHITE_BLOOD_CELLS and FOREIGN_ CELLS in an HIV positive individual. A functioning immune system destroys the foreign invasion by hour 24. The body withstands 900 FOREIGN_CELLS per microliter of blood. When the number of helper T cells is depleted by a factor of 10, however, the same 100 FOREIGN_CELLS multiply to become an army of 30,000 FOREIGN_CELLS that it takes WHITE_BLOOD_CELLS 36 hours to destroy!

HIV itself replicates quickly. It spreads through the body until it has impaired the immune system to the point where the body succumbs to any small foreign attack. It is then that an HIV positive patient is diagnosed as having AIDS. When the strength_of_the_immune_response has a value of 1/1500 or 1/2000 the body is still able to ward off invasions. However, as the strength_of_the_immune_response gets smaller and smaller, the effects are disastrous. The body can only stand the presence of a finite number of FOREIGN_CELLS before it succumbs. What would happen if 10,000 FOREIGN_CELLS were the maximum number of FOREIGN_CELLS that a human body could support? The body would collapse by hour 28 before the immune response had the time to produce enough specific antibodies or killer T cells to ward off the invasion.

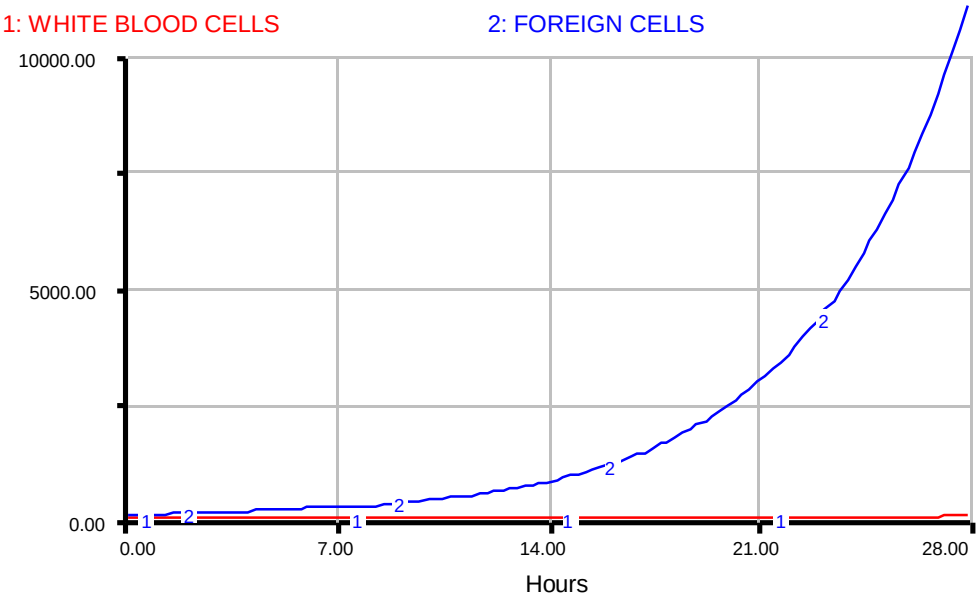


Figure 23: Impaired immune response — detail

Figure 23 shows in detail what happens. The number of FOREIGN_CELLS grows exponentially. The number of WHITE_BLOOD_CELLS also grows exponentially, but at such a slow rate that their presence is almost negligible. When the number of FOREIGN_CELLS reaches 10,000 cells per microliter, the system comes to a halt—the human being infected with AIDS has been destroyed by a foreign invasion. The halt occurs around hour 28. Fewer than 100 WHITE_BLOOD_CELLS have been produced to fight off the invasion.

A victim of AIDS has virtually no immune response system left. A small attack of FOREIGN_CELLS is allowed to grow unhindered because the body is unable to produce enough antibodies or killer T cells in time. How does the behavior of the impaired immune response model reflect the dynamics of S-shaped growth? The positive loop that causes the WHITE_BLOOD_CELLS to proliferate is so weak that the stock never grows large enough to counter the foreign invasion and shift the system into negative feedback. The system collapses under the strain of exponential growth before the natural immune response can act as a constraint. It is not the number of stocks and flows, nor the number

of positive or negative loops that determines the characteristics of S-shaped growth—it is the strength of those loops that matters.

4. Conclusion

Many systems produce S-shaped growth behavior. It is does not matter how many positive or negative feedback loops contribute to a certain system. The number of stocks is also unimportant. A shift in loop dominance is the determining factor for S-shaped growth. When a system shifts from positive to negative feedback, its behavior shifts from exponential to asymptotic growth. The strength of these loops determines the characteristics of S-shaped growth.

Exponential growth cannot be sustained forever. Constraints will always act as limits to growth.

5. Appendix: Model Equations

5.1 S-Shaped Growth Generic Structure

□ **STOCK(t)** = STOCK(t - dt) + (Inflow - Outflow) * dt
INIT STOCK = 10
DOCUMENT: Units: widgets

☞ INFLOWS:
Inflow = gain_fraction*STOCK
DOCUMENT: Units: widgets/time

☞ OUTFLOWS:
Outflow = loss_fraction*STOCK
DOCUMENT: Units: widgets/time

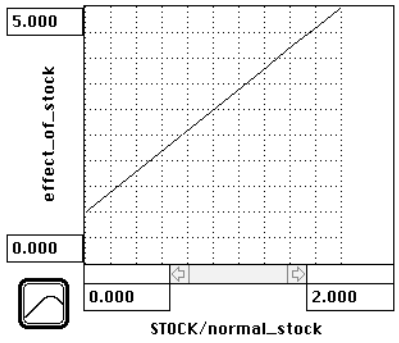
○ **gain_fraction** = *a constant*
DOCUMENT: Units: 1/time
The gain_fraction must be greater than the normal_loss_fraction in order for the STOCK to grow initially.

○ **loss_fraction** = normal_loss_fraction*effect_of_stock
DOCUMENT: Units: 1/time
The loss_fraction is a normal_loss_fraction times a multiplier which depends on the relative size of the STOCK. The loss fraction, as it grows, determines the shifting loop dominance.

○ **normal_stock** = *a constant*
DOCUMENT: Units: widgets

○ **normal_loss_fraction** = *a constant*
DOCUMENT: Units: 1/time
The normal_loss_fraction must be less than the gain_fraction in order for the STOCK to grow initially. The normal_loss_fraction represents the initial loss_fraction. If no constraints were acting upon the system, the loss_fraction would equal the normal_loss_fraction.

⊘ **effect_of_stock** = GRAPH(STOCK/normal_stock)
(0.00, 1.00), (0.2, 1.40), (0.4, 1.80), (0.6, 2.20), (0.8, 2.60), (1.00, 3.00), (1.20, 3.40), (1.40, 3.80), (1.60, 4.20), (1.80, 4.60), (2.00, 5.00)
DOCUMENT: Units: dimensionless
The simplest curve that generates S-shaped growth is a straight line that is linearly increasing. When the STOCK is 0, the multiplier is 1.



5.2 Rabbit Population Model

□ **RABBITS(t)** = RABBITS(t - dt) + (Births - Deaths) * dt
INIT RABBITS = 10
DOCUMENT: Units: rabbits

INFLOWS:
Births = birth_fraction*RABBITS
DOCUMENT: Units: rabbits/month

OUTFLOWS:
Deaths = death_fraction*RABBITS
DOCUMENT: Units: rabbits/month

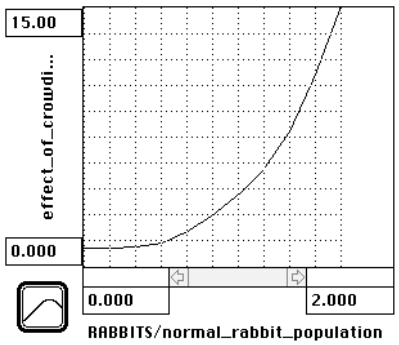
○ **birth_fraction** = 0.5
DOCUMENT: Units: 1/month
The birth fraction is the rate at which each rabbit can reproduce.

○ **death_fraction** = normal_death_fraction*effect_of_crowding
DOCUMENT: Units: 1/month
The death fraction is the actual rate at which each rabbit dies.

○ **normal_rabbit_population** = 500
DOCUMENT: Units: rabbits
The maximum rabbit population is the maximum number of rabbits for which Jean can provide water. It is fixed (a constant) because Provence has a dry, arid climate and Jean only has a fixed supply of water available to him. (In the story Jean de Florette by Marcel Pagnol, however, Jean's supply of water was not fixed but depletable. So as the number of rabbits grew, the water was all used up and all the rabbits died. The dynamics of overshoot and collapse will be explored in Road Maps 9.)

○ **normal_death_fraction** = (0.5)/3
DOCUMENT: Units: 1/month
The normal_death_fraction is the rate at which each rabbit would die if it had an unlimited supply of food and water. It is three times less than the birth_fraction.

⊘ **effect_of_crowding** = GRAPH(RABBITS/normal_rabbit_population)
(0.00, 1.00), (0.2, 1.00), (0.4, 1.10), (0.6, 1.30), (0.8, 2.00), (1.00, 3.00), (1.20, 4.20), (1.40, 5.62), (1.60, 7.80), (1.80, 11.1), (2.00, 15.0)
DOCUMENT: Units: dimensionless
Crowding has no effect on the death fraction as long as the number of rabbits on Jean's farm does not reach more than half the number that he can support. As the number of rabbits reaches the maximum rabbit population, the multiplier exponentially increases from 1 to 3.



5.3 Cabbage Patch Kids Model

□**CHILDREN_WHO_OWN_A_CPK(t)** = CHILDREN_WHO_OWN_A_CPK(t - dt) + (Purchases) * dt
INIT CHILDREN_WHO_OWN_A_CPK = 1000
DOCUMENT: Units: children

☞INFLOWS:
Purchases = effect_of_popularity_on_purchases * CHILDREN_WITHOUT_A_CPK / time_to_purchase_a_CPK
DOCUMENT: Units: children/month

□**CHILDREN_WITHOUT_A_CPK(t)** = CHILDREN_WITHOUT_A_CPK(t - dt) + (- Purchases) * dt
INIT CHILDREN_WITHOUT_A_CPK = 1,000,000
DOCUMENT: UNITS: children
The total population considered here is the total population that is susceptible to buying a doll. (ie. it excludes children who can't afford the dolls)

☞OUTFLOWS:
Purchases = effect_of_popularity_on_purchases * CHILDREN_WITHOUT_A_CPK / time_to_purchase_a_CPK
DOCUMENT: Units: children/month

○**effect_of_popularity_on_purchases** = fraction_of_children_with_CPKs
DOCUMENT: Units: dimensionless
As fraction of children with dolls increases, a corresponding fraction representing the effect_of_popularity_on_purchases increases from 0 to 1.

○**fraction_of_children_with_CPKs** = CHILDREN_WHO_OWN_A_CPK / (CHILDREN_WHO_OWN_A_CPK + CHILDREN_WITHOUT_A_CPK)
DOCUMENT: Units: dimensionless
The fraction_of_children_with_CPKs is the percentage of the total population suceptible to buying dolls that actually owns a doll.

○**time_to_purchase_a_CPK** = 2
DOCUMENT: Units: months
The time_to_purchase_a_CPK is the average time for children to convince their parents to buy a doll plus the average time for the parents to find a doll which resembles their child.

5.4 Protestantism Model

□**NONPROTESTANTS(t)** = NONPROTESTANTS(t - dt) + (- Conversions) * dt

INIT NONPROTESTANTS = 1000000
DOCUMENT: Units: people

OUTFLOWS:
Conversions = NONPROTESTANTS * strength_of_wom * (1 - persecution) / time_to_be_influenced_to_convert
DOCUMENT: Units: people/year
Conversions is rate at which NonProtestants, usually Roman Catholics, converted to become Protestant. Conversions is the product of the number of people left who have yet to convert, the fraction representing the strength of word of mouth, the opposite of the fraction representing the strength of persecution, all over the time it takes to convert.

PROTESTANTS(t) = PROTESTANTS(t - dt) + (Conversions) * dt
INIT PROTESTANTS = 100
DOCUMENT: Units: people
The initial value of 100 people represents the original converts (and friends) of Martin Luther.

INFLOWS:
Conversions = NONPROTESTANTS*strength_of_wom * (1 - persecution) / time_to_be_influenced_to_convert
DOCUMENT: Units: people/year

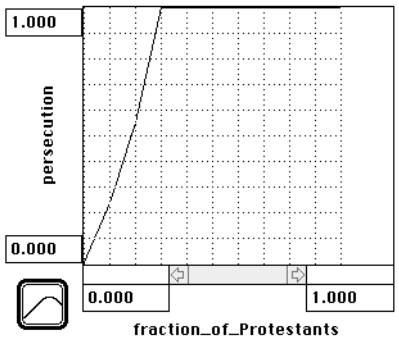
fraction_of_Protestants = PROTESTANTS / (PROTESTANTS + NONPROTESTANTS)
DOCUMENT: Units: dimensionless
The fraction_of_Protestants is the percentage of Protestants in the total population

strength_of_wom = fraction_of_Protestants
DOCUMENT: Units: dimensionless
The strength of word of mouth is equal to the number of Protestants spreading that word of mouth.

time_to_be_influenced_to_convert = 4
DOCUMENT: Units: years
The time_to_be_influenced_to_convert is the time for a nonProtestant to hear about Protestantism and decide to adopt its principles.

persecution = GRAPH(fraction_of_Protestants)
(0.00, 0.005), (0.1, 0.235), (0.2, 0.545), (0.3, 1.00), (0.4, 1.00), (0.5, 1.00), (0.6, 1.00), (0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
DOCUMENT: Units: dimensionless
Persecution is negligeable when there are relatively few Protestants. As the number of Protestants becomes threatening, it shoots up to the point of cutting off

future conversions. When the Inquisition and religious civil wars started in Europe, there were very few new conversions.



5.5 Immune Response Model

☐**FOREIGN_CELLS(t)** = FOREIGN_CELLS(t - dt) + (Multiplication-Destruction)*dt
INIT FOREIGN_CELLS = 100
DOCUMENT: Units: foreign cells
100 is an approximate number of foreign cells that invade one microliter of blood.

☞INFLOWS:
Multiplication = FOREIGN_CELLS/time_for_foreign_cells_to_multiply
DOCUMENT: Units: foreign cells/hour

☞OUTFLOWS:
Destruction = IF (FOREIGN_CELLS = 0) THEN 0 ELSE
(WHITE_BLOOD_CELLS / time_to_destroy_a_foreign_cell)
DOCUMENT: Units: foreign cells/hour
Destruction is the average rate at which white blood cells destroy foreign cells.

☐**WHITE_BLOOD_CELLS(t)** = WHITE_BLOOD_CELLS(t - dt) + (Proliferation)*dt
INIT WHITE_BLOOD_CELLS = 10
DOCUMENT: Units: white blood cells
Approximately 10 antigen-specific white blood cells are present in one microliter of blood before an antigen invasion is recognized. White blood cells is the name given to the cells that constitute the human immune response: helper T cells, killer T cells, B cells, memory cells...

☞INFLOWS:
Proliferation = strength_of_the_immune_response * WHITE_BLOOD_CELLS / time_for_white_blood_cells_to_proliferate
DOCUMENT: Units: white blood cells/hour

Proliferation is determined by multiplying a normal proliferation rate (white blood cells/time for white blood cells to proliferate) by the variable strength of proliferation.

- **antigens** = FOREIGN_CELLS * antigens_per_foreign_cell
  DOCUMENT: Units: antigens
  Antigens are macromolecules on foreign cells that, when identified by helper T cells, trigger an immune response .

- **antigens_per_foreign_cell** = 2
  DOCUMENT: Units: antigens/foreign cell
  Approximately 2 antigens are attached to each foreign cell.

- **strength_of_proliferation** = strength_of_the_immune_response * antigens
  DOCUMENT: Units: dimensionless
  Strength_of_proliferation is a multiplier that increases the proliferation rate as the number of recognized antigens increases.

- **strength_of_the_immune_response** = 1/1000
  DOCUMENT: Units: dimensionless
  The strength_of_the_immune_response is a fraction representing the ability of helper T cells to recognize antigens and activate the proliferation of white blood cells. When someone is infected with HIV, the human immune system is crippled and the strength_of_the_immune_response is severely weakened.

- **time_for_foreign_cells_to_multiply** = 5
  DOCUMENT: Units: hours

- **time_for_white_blood_cells_to_proliferate** = 5
  DOCUMENT: Units: hours

- **time_to_destroy_a_foreign_cell** = 1
  DOCUMENT: Units: (white blood cells/foreign cell) * hours

Vensim Examples:
Exploring S-Shaped Growth

Lei Lei & Nathaniel Choge
October 2001

S-Shaped Growth Generic Structure

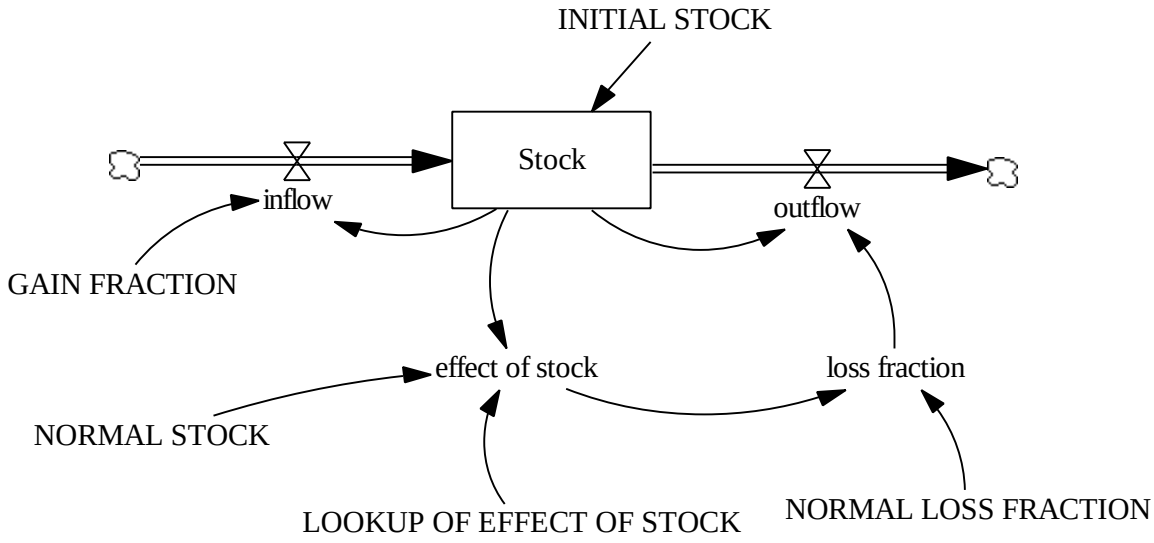


Figure 24: Vensim Equivalent of Figure 3: Generic Structure that produces S-shaped Growth

Documentation for S-Shaped Generic Structure Model

(01) effect of stock = LOOKUP OF EFFECT OF STOCK (Stock/NORMAL STOCK)
Units: dmnl

(02) FINAL TIME = 100
Units: time
The final time for the simulation.

(03) GAIN FRACTION = a constant
Units: 1/time
The GAIN FRACTION must be greater than the normal loss fraction in order for the Stock to grow initially.

(04) inflow = GAIN FRACTION* Stock
Units: widgets/time

(05) INITIAL STOCK = 10
Units: widgets

(06) INITIAL TIME = 0
Units: time
The initial time for the simulation.

(07) LOOKUP OF EFFECT OF STOCK
([(0,0)-(10,10)],(0,1),(0.2,1.4),(0.4,1.8),(0.6,2.2),(0.8,2.6),(1,3),
(1.2,3.4),(1.4,3.8), (1.6,4.2),(1.8,4.6),(2,5))
Units: dmnl
The simplest curve that generates S-shaped growth is a straight line that is linearly increasing. When the Stock is 0, the multiplier is 1.

(08) loss fraction = NORMAL LOSS FRACTION* effect of stock
Units: 1/time
The loss fraction is a NORMAL LOSS FRACTION times a multiplier that depends on the relative size for the Stock. The loss fraction, as it grows, determines the shifting loop dominance.

(09) NORMAL LOSS FRACTION = a constant
Units: 1/time
The NORMAL LOSS FRACTION must be less than the GAIN FRACTION in order for the Stock to grow initially. The NORMAL LOSS FRACTION represents the initial loss fraction. If no constraints were acting upon the system, the loss fraction would equal the NORMAL LOSS FRACTION.

(10) NORMAL STOCK = a constant
Units: widgets

(11) outflow = loss fraction* Stock
Units: widgets/time

(12) SAVEPER = TIME STEP
Units: time
The frequency with which output is stored.

(13) Stock = INTEG (inflow-outflow, INITIAL STOCK)
Units: widgets

(14) TIME STEP = 1
Units: time
The time step for the simulation.

Applying the Generic Structure to the Rabbit Population Model

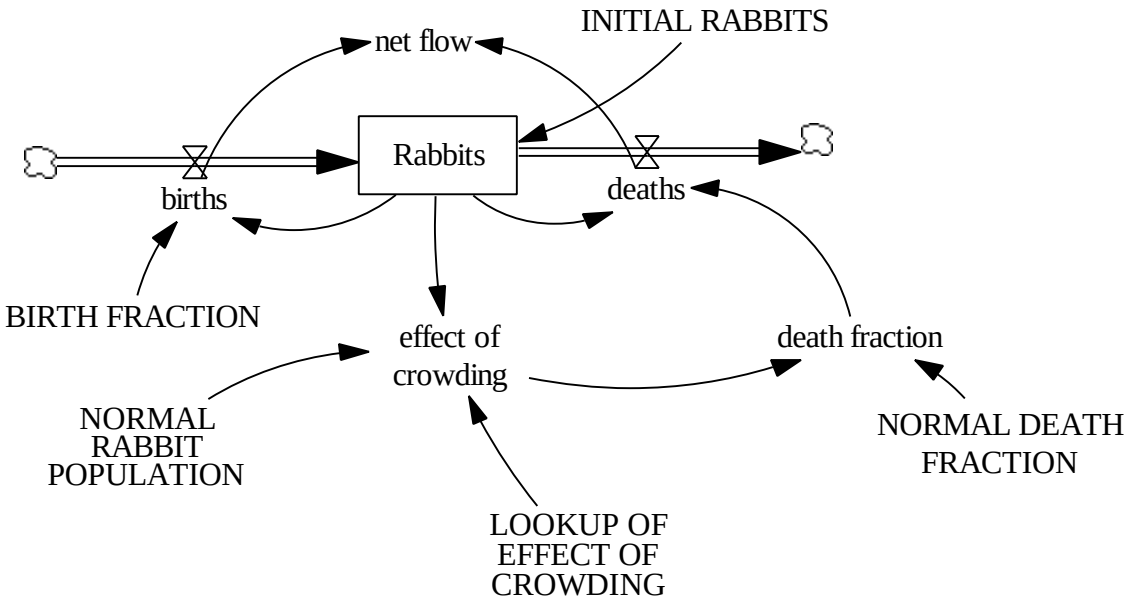


Figure 25: Vensim Equivalent of Figure 5: Stock and flow diagram of the rabbit Population

Documentation for Rabbit Population Model

(01) BIRTH FRACTION = 0.5
Units: 1/Month
The BIRTH FRACTION is the rate at which each rabbit can reproduce.

(02) births = BIRTH FRACTION* Rabbits
Units: rabbits/Month

(03) death fraction = NORMAL DEATH FRACTION* effect of crowding
Units: 1/Month
The death fraction is the actual rate at which each rabbit dies.

(04) deaths = death fraction* Rabbits
Units: rabbits/Month

(05) effect of crowding = LOOKUP OF EFFECT OF CROWDING (Rabbits/NORMAL RABBIT POPULATION)
Units: dmnl

This is the equation that shows that the effect of crowding is a function of the rabbit population.

(06) FINAL TIME = 24
Units: Month
The final time for the simulation.

(07) INITIAL RABBITS = 10
Units: rabbits

(08) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(08) LOOKUP OF EFFECT OF CROWDING
([(0,0)-(2,15)],(0,1),(0.2,1),(0.4,1.1),(0.6,1.3),(0.8,2),(1,3),(1.2,4.2),
(1.4,5.62),(1.6,7.8),(1.8,11.1),(2,15))
Units: dmnl
Crowding has no effect on the death fraction as long as the number of rabbits on Jean's farm does not reach more than half the number that he can support. As the number of Rabbits reaches the maximum rabbit population, the multiplier exponentially increases from 1 to 3.

(10) net flow = births-deaths
Units: rabbits/Month
This is the net flow of rabbits/month.

(11) NORMAL DEATH FRACTION = 0.167
Units: 1/Month
The NORMAL DEATH FRACTION is the rate at which each rabbit would die if it had an unlimited supply of food and water. It is three time less than the birth fraction.

(12) NORMAL RABBIT POPULATION = 500
Units: rabbits
The maximum rabbit population is the maximum number of rabbits for which Jean can provide water. It is fixed (a constant) because Provence has a dry, arid climate and Jean only has a fixed supply of water available to him. (In the story Jean de Florette by Marcel Pagnol, however, Jean's supply of water was not fixed but depletable. So as the number of rabbits grew, the water was all used up and all the rabbits died. The dynamics of overshoot and collapse will be explored in Road Maps 9.)

(13) Rabbits = INTEG (births-deaths, INITIAL RABBITS)

Units: rabbits

(14) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(15) TIME STEP = 0.0625
Units: Month
The time step for the simulation.

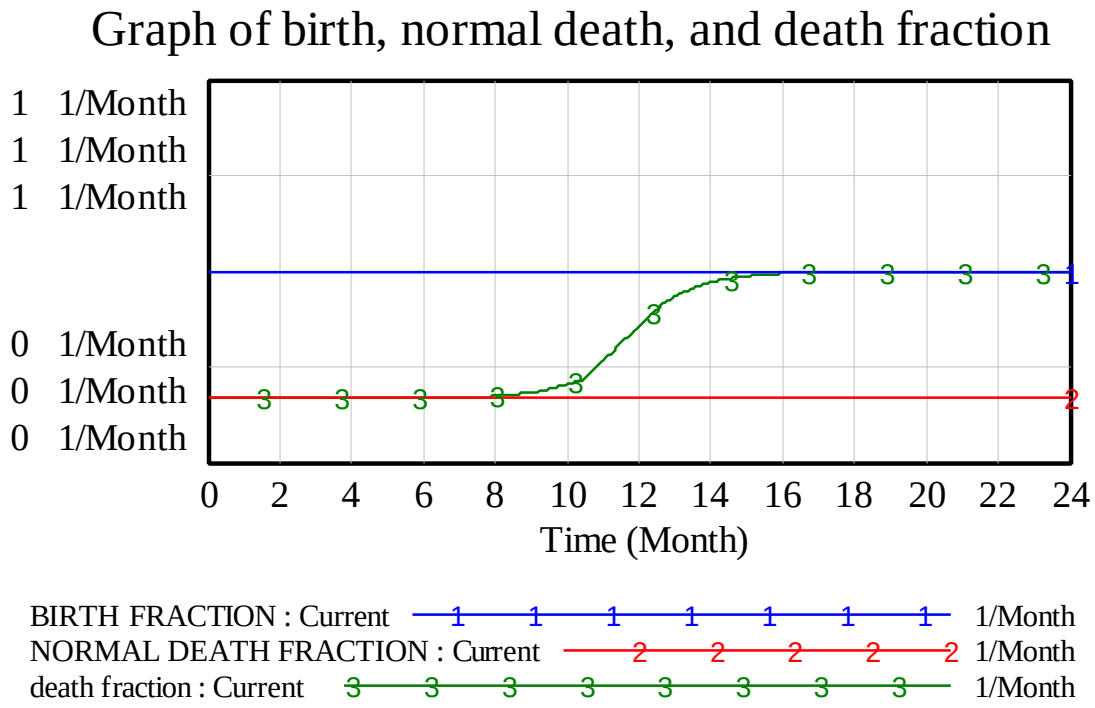


Figure 26: Vensim Equivalent of Figure 8: Rabbit population model: fractions

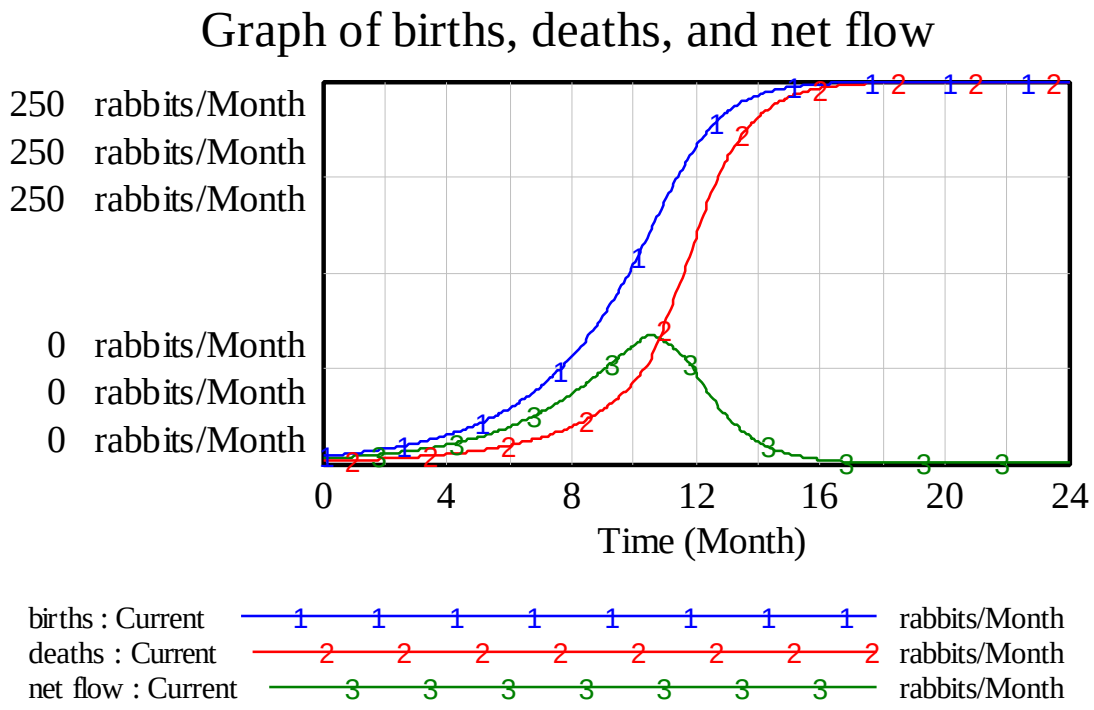


Figure 27: Vensim Equivalent of Figure 9: Rabbit population model: flows

Cabbage Patch Kids

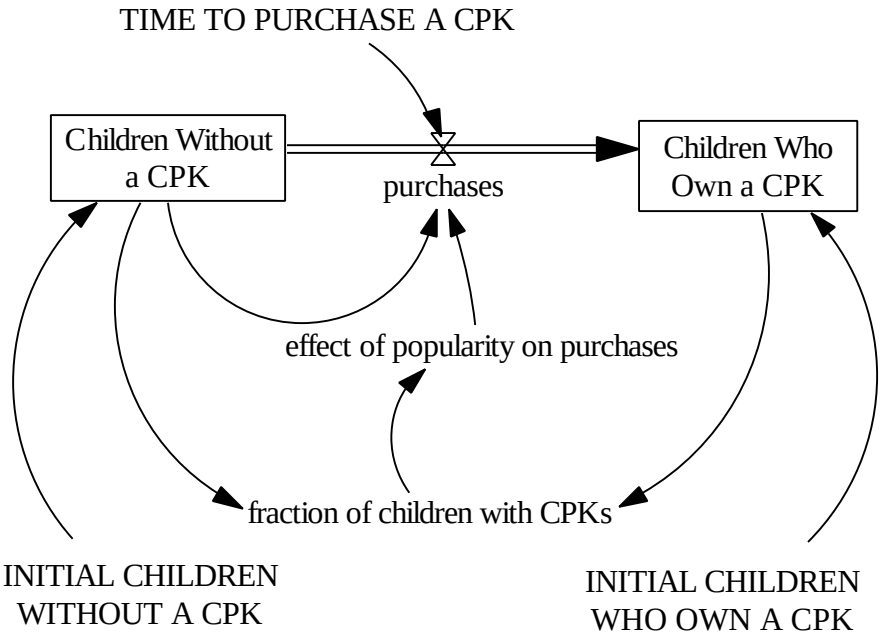


Figure 28: Vensim Equivalent of Figure 11: Cabbage Patch Kid Model

Documentation for Cabbage Patch Kids Model

(01) Children Who Own a CPK = INTEG (+purchases, INITIAL CHILDREN WHO OWN A CPK)
Units: children
Number of children who own a Cabbage Patch Kid.

(02) Children Without a CPK = INTEG (-purchases, INITIAL CHILDREN WITHOUT A CPK)
Units: children
The total population considered here is the total population that is susceptible to buying a doll. (i.e. it excludes children who can't afford the dolls)

(03) effect of popularity on purchases = fraction of children with CPKs
Units: dmnl
As fraction of children with dolls increases, a corresponding fraction representing the effect or popularity on purchases increases from 0 to 1.

(04) FINAL TIME = 24
Units: Month
The final time for the simulation.

(05) fraction of children with CPKs =
Children Who Own a CPK/(Children Who Own a CPK+Children Without a CPK)
Units: dmnl
The fraction of children with CPKs is the percentage of the total population susceptible to buying dolls that actually owns a doll.

(06) INITIAL CHILDREN WHO OWN A CPK = 1000
Units: children

(07) INITIAL CHILDREN WITHOUT A CPK = 1e+006
Units: children

(08) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(09) purchases = effect of popularity on purchases* Children Without a CPK/TIME TO PURCHASE A CPK
Units: children/Month

(10) SAVEPER = TIME STEP

Units: Month
The frequency with which output is stored.

(11) TIME STEP = 0.0625
Units: Month
The time step for the simulation.

(12) TIME TO PURCHASE A CPK = 2
Units: Month
The time to purchase a CPK is the average time for children to convince their parents to buy a doll plus the average time for the parents to find a doll that resembles their child.

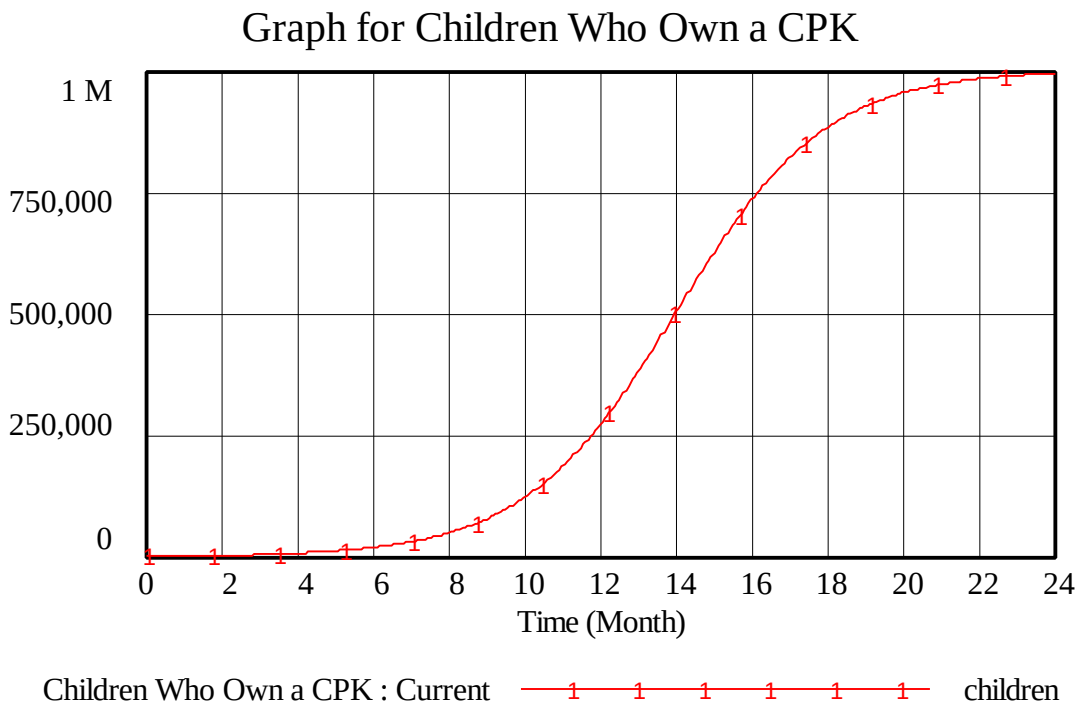


Figure 29: Vensim Equivalent of Figure 12: Cabbage Patch Kids: stock over time

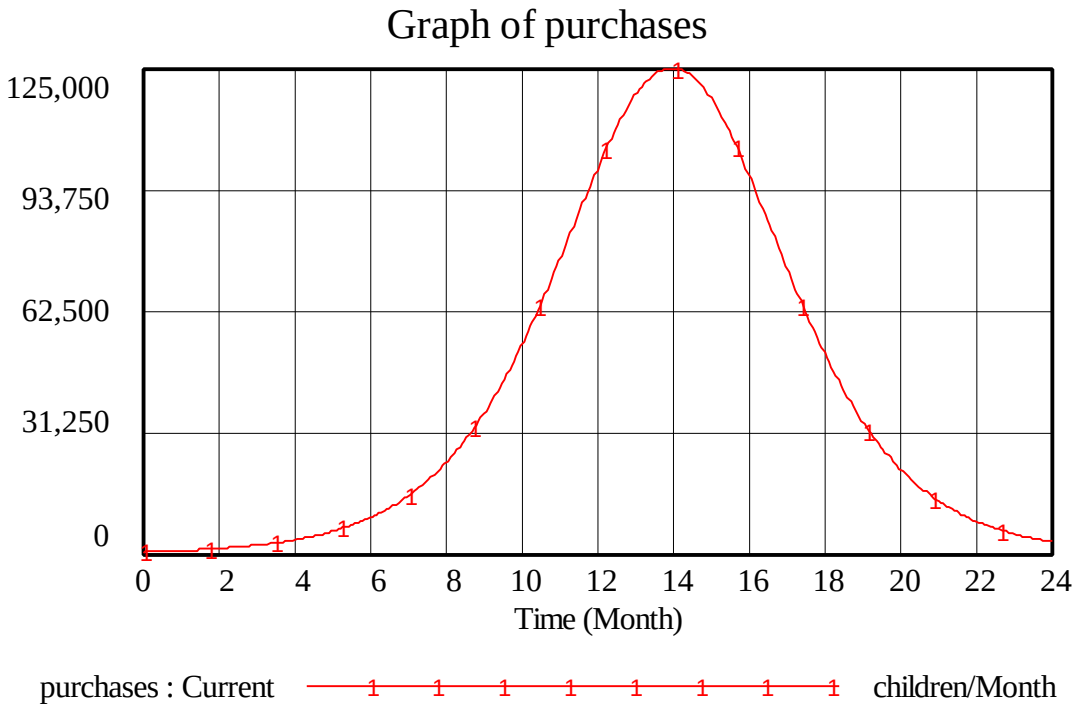


Figure 30: Vensim Equivalent of Figure 13: Cabbage Patch Kids: net flow over time

Protestantism

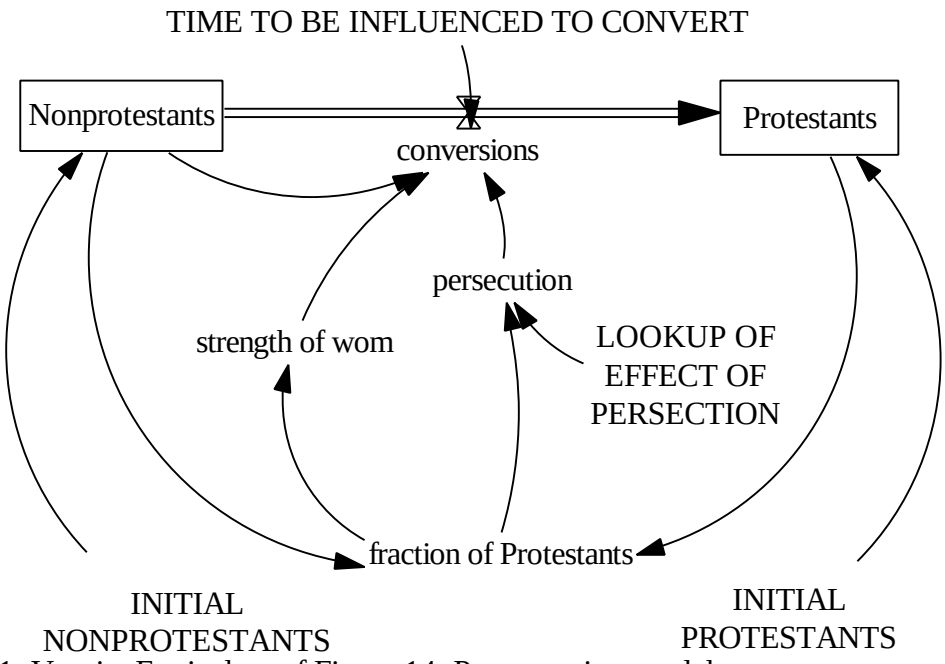


Figure 31: Vensim Equivalent of Figure 14: Protestantism model

Documentation for Protestantism Model

(01) conversions = Nonprotestants * strength of wom *
(1-persecution)/TIME TO BE INFLUENCED TO CONVERT
Units: people/year
Conversions is rate at which Nonprotestants, usually Roman Catholics, converted to become Protestant. Conversions is the product of the number of people left who have yet to convert, the fraction representing the strength of word of mouth, the opposite of the fraction representing the strength of persecution, all over the time it takes to convert.

(02) FINAL TIME = 1600
Units: year
The final time for the simulation.

(03) fraction of Protestants = Protestants/(Nonprotestants+Protestants)
Units: dmnl
The fraction of Protestants is the percentage of Protestants in the total population.

(04) INITIAL NONPROTESTANTS = 1e+006
Units: people

(05) INITIAL PROTESTANTS = 100
Units: people

(06) INITIAL TIME = 1500
Units: year
The initial time for the simulation.

(06) LOOKUP OF EFFECT OF PERSECTION
([(0,0)-(1,1)],(0,0.005),(0.1,0.235),(0.2,0.545),(0.3,1),(0.4,1),(0.5,1),
(0.6,1),(0.7,1),(0.8,1),(0.9,1),(1,1))
Units: dmnl
Persecution is negligible when there are relatively few Protestants. As the number of Protestants becomes threatening, it shoots up to the point of cutting off future conversions. When the Inquisition and religious civil wars started in Europe, there were very few new conversions.

(08) Nonprotestants = INTEG (-conversions, INITIAL NONPROTESTANTS)
Units: people

(09) persecution = LOOKUP OF EFFECT OF PERSECTION (fraction of Protestants)
Units: dmnl
This equation shows that persecution is a function of the fraction of Protestants.

(10) Protestants = INTEG (conversions, INITIAL PROTESTANTS)

Units: people
The initial value of 100 people represents the original converts (and friends) of Martin Luther.

(11) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(12) strength of wom = fraction of Protestants
Units: dmnl
The strength of word of mouth is equal to the number of Protestants spreading that word of mouth.

(13) TIME STEP = 1
Units: year
The time step for the simulation.

(14) TIME TO BE INFLUENCED TO CONVERT = 4
Units: year
The time to be influenced to convert is the time for a Nonprotestant to hear about Protestantism and decide to adopt its principles.

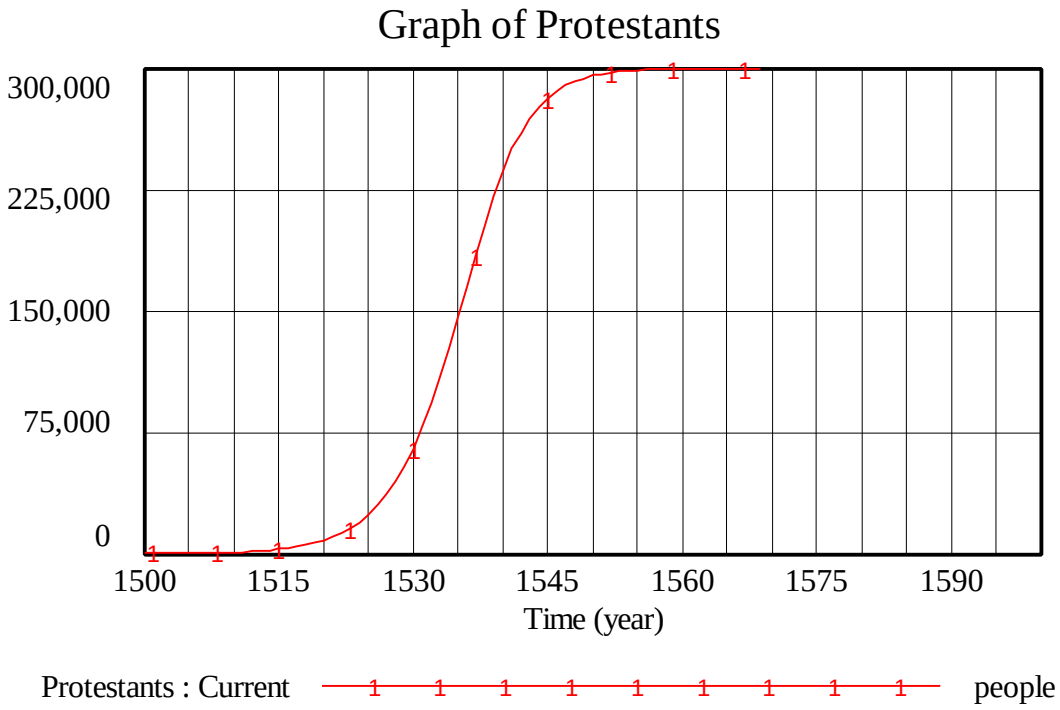


Figure 32: Vensim Equivalent of Figure 15: Protestantism: stock over time

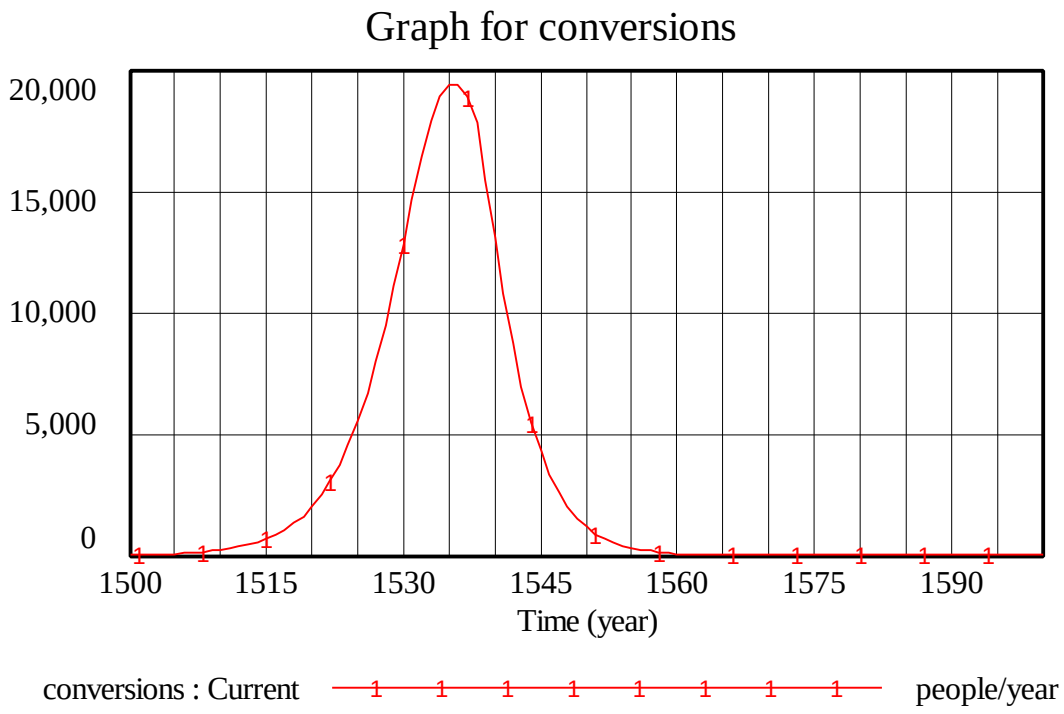


Figure 33: Vensim Equivalent of Figure 16: Protestantism: net flow over time

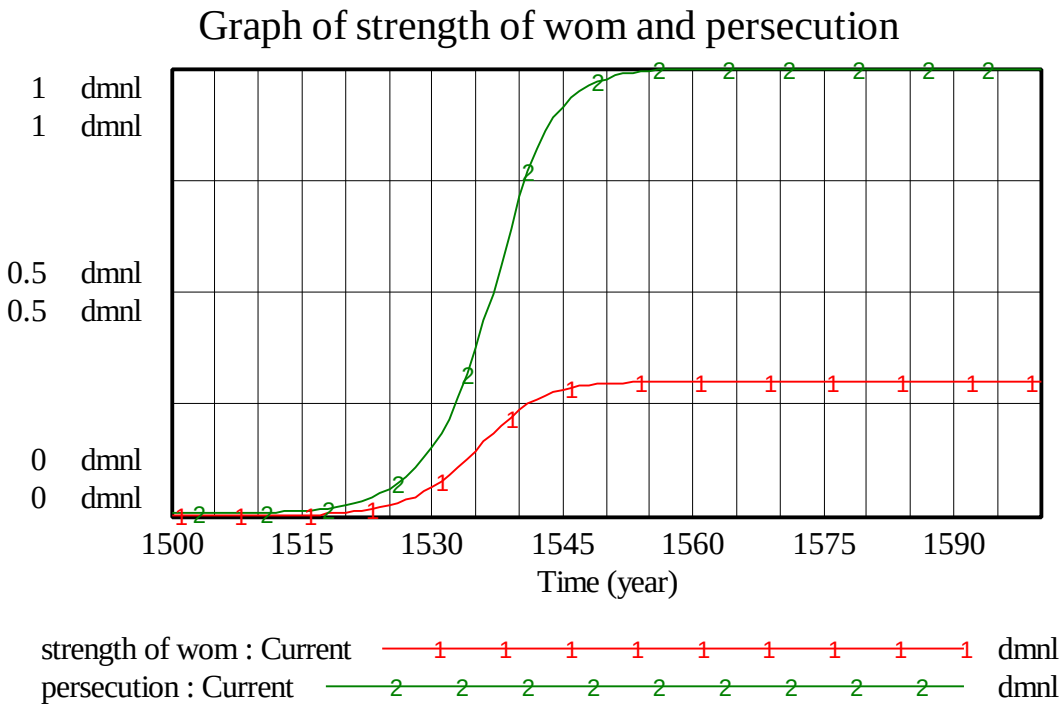


Figure 34: Vensim Equivalent of Figure 17: Protestantism: multipliers

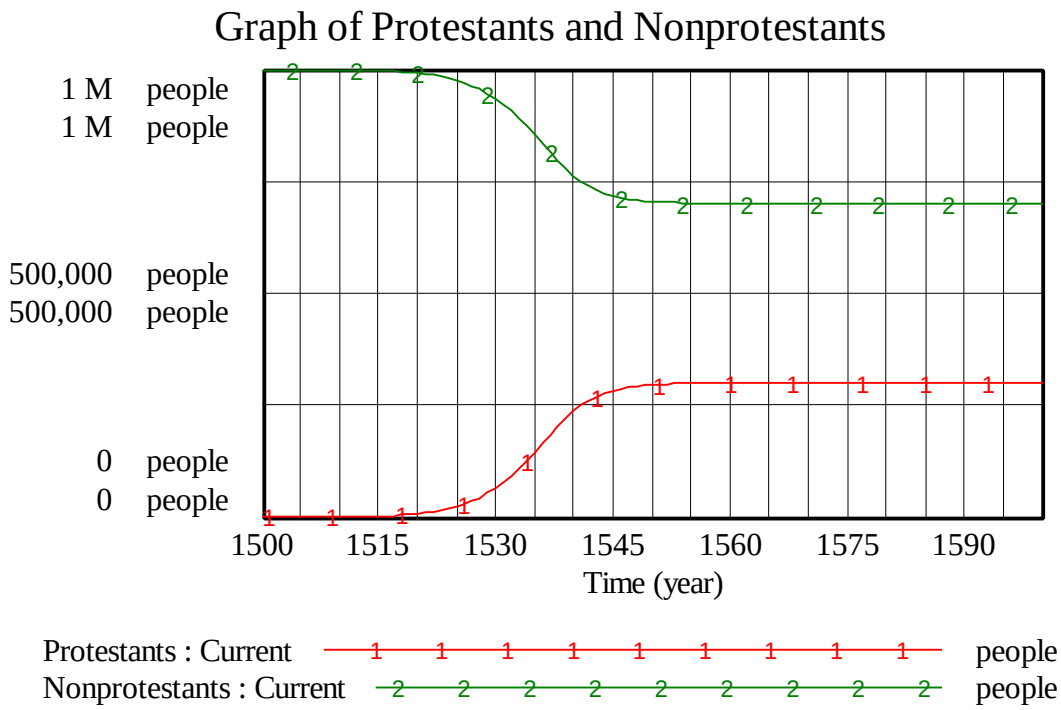


Figure 35: Vensim Equivalent of Figure 18: Protestants: both stocks over time

Immune Response

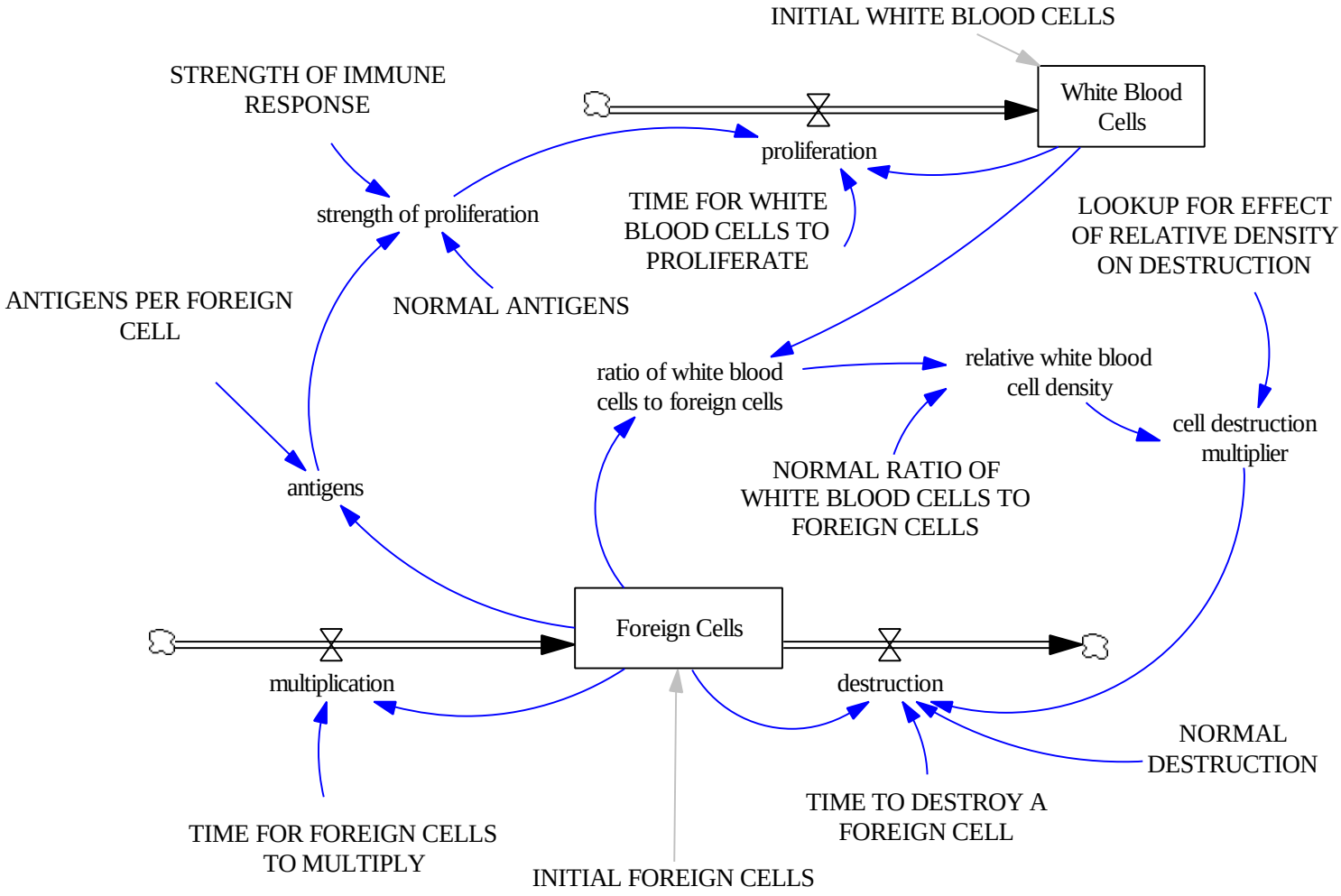


Figure 36: Vensim Equivalent of Figure 19: Immune response model

NOTE: Differences between the Stella and Vensim versions of the Immune System Model are described below the model documentation.

Documentation for Immune Response Model

(01) antigens = Foreign Cells*ANTIGENS PER FOREIGN CELL
Units: antigens
Antigens are macromolecules on foreign cells that, when identified by helper T cells, trigger an immune response.

(02) ANTIGENS PER FOREIGN CELL = 2
Units: antigens/foreign cells
Approximately 2 antigens are attached to each foreign cell.

(03) cell destruction multiplier = LOOKUP FOR EFFECT OF RELATIVE DENSITY ON DESTRUCTION(relative white blood cell density)
Units: dmnl
A dimensionless multiplier produced by the lookup table and the ratio of foreign cells to white blood cells. This multiplier shows the effect of relative white blood cell density on the number of foreign cells destroyed.

(04) destruction = Foreign Cells/TIME TO DESTROY A FOREIGN CELL*cell destruction multiplier*NORMAL DESTRUCTION
Units: foreign cells/hour
Destruction is the average rate at which white blood cells destroy foreign cells.

(05) FINAL TIME = 48
Units: hour
The final time for the simulation.

(06) Foreign Cells= INTEG (+multiplication-destruction, INITIAL FOREIGN CELLS)
Units: foreign cells
100 is an approximate number of foreign cells that invade one microliter of blood.

(07) INITIAL FOREIGN CELLS =100
Units: foreign cells

(08) INITIAL TIME = 0
Units: hour
The initial time for the simulation.

(09) INITIAL WHITE BLOOD CELLS = 10
Units: white blood cells

(10) LOOKUP FOR EFFECT OF RELATIVE DENSITY ON DESTRUCTION(
[(0,0)-(2,2)],(0,0),(0.4,0.625),(1,1.125),(1.5,1.25),(2,1.25))
Units: dmnl
This lookups gives the relationship between the ratio of white blood cells to foreign cells (relative white blood cell density) and the number of foreign cells destroyed. As the ratio gets higher, the number of foreign cells destroyed increases. However, when the ratio is above one, the multiplier is at the value that produces (cell destruction multiplier*NORMAL DESTRUCTION) = 1, since we cannot destroy more foreign cells than are present in the blood.

(11) multiplication = Foreign Cells/TIME FOR FOREIGN CELLS TO MULTIPLY
Units: foreign cells/hour

(12) NORMAL ANTIGENS = 1

Units: antigens
This is the normal number of antigens in the blood. i.e. without any foreign cells invading.

(13) NORMAL DESTRUCTION = 0.8
Units: dmnl
This is a multiplier that gives the proportion of Foreign Cells that are found and destroyed by White Blood Cells when the "relative white blood cell density" is equal to 1. i.e. the proportion of Foreign Cells killed at normal white blod cell concentration.

(14) NORMAL RATIO OF WHITE BLOOD CELLS TO FOREIGN CELLS = 1
Units: white blood cells/foreign cells
This gives the normal number of white blood cells for each red blood cells in blood uninfected by antigens.

(15) proliferation = strength of proliferation*White Blood Cells/TIME FOR WHITE BLOOD CELLS TO PROLIFERATE
Units: white blood cells/hour
Proliferation is determined by multiplying a normal proliferation rate (white blood cells/time for white blood cells to proliferate) by the variable strength of proliferation.

(16) ratio of white blood cells to foreign cells = White Blood Cells/Foreign Cells
Units: white blood cells/foreign cells
Ratio of White Blood Cells to Foreign Cellls in the blood.

(17) relative white blood cell density = ratio of white blood cells to foreign cells/NORMAL RATIO OF WHITE BLOOD CELLS TO FOREIGN CELLS
Units: dmnl
This gives a measure of the density of white blood cells with respect to the normal proportions of foreign cells and white blood cells.

(18) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.

(19) STRENGTH OF IMMUNE RESPONSE = 0.0001
Units: dmnl
The strength of the immune response is a fraction representing the ability of helper T cells to recognize antigens and activate the proliferation of white blood cells. When someone is infected with HIV, the human immune system is crippled and the strength of the immune response is severely weakened.

(20) strength of proliferation = STRENGTH OF IMMUNE RESPONSE*antigens/NORMAL ANTIGENS
Units: dmnl
Strength of proliferation is a multiplier that increases the proliferatin rate as the number of recognized antigens increases.

(21) TIME FOR FOREIGN CELLS TO MULTIPLY = 5
Units: hour
Time it takes for white blood cells to multiply after detection of the antigens.

(22) TIME FOR WHITE BLOOD CELLS TO PROLIFERATE = 5
Units: hour

(23) TIME STEP = 0.0625
Units: hour
The time step for the simulation.

(24) TIME TO DESTROY A FOREIGN CELL = 1
Units: hour
Time it takes to destroy a foreign cell.

(25) White Blood Cells= INTEG (proliferation, INITIAL WHITE BLOOD CELLS)
Units: white blood cells
Approximately 10 antigen-specific white blood cells are present in one microliter of blood before an antigen invasion is recognized. White blood cells is the name given to the cells that constitute the human immune response: helper T cells, killer T cells, B cells, memory cells....

Differences between the Vensim and Stella Immune System Models:

1. The differences between the Vensim and Stella versions all centre around the Foreign cell destruction. In the Stella version the destruction of foreign cells was limited only by the fact that the stock of "Foreign Cells" was set to **non-negative.** There was no first order control on the number of foreign cells being killed, since in any timestep it was possible to kill a number of foreign cells higher than the actual number of foreign cells present in the blood. This is incorrect modeling practise.
2. To fix the problem in the Vensim version, we have introduced new structure to ensure that the number of foreign cells destroyed is always less than or equal to the number of foreign cells present in the blood. The ratio of White Blood Cells to Foreign Cells gives a relative cell density that is the input to a lookup table. The lookup table shows how the relative cell density affects Foreign Cells destruction through the LOOKUP FOR EFFECT OF RELATIVE DENSITY ON DESTRUCTION. The output multiplier from this lookup table multiplied with the NORMAL DESTRUCTION and "Foreign Cells/TIME TO DESTROY A FOREIGN CELL" gives a rate of destruction that depends directly on the ratio of white blood cells to foreign cells for relative cell densities less than 1. However, for relative cell densities greater

than 1 i.e. more white blood cells than foreign cells in the blood, the table function begins to limit the destruction of foreign cells destroyed as they become more scarce, producing a model that more acurately represents the real system.

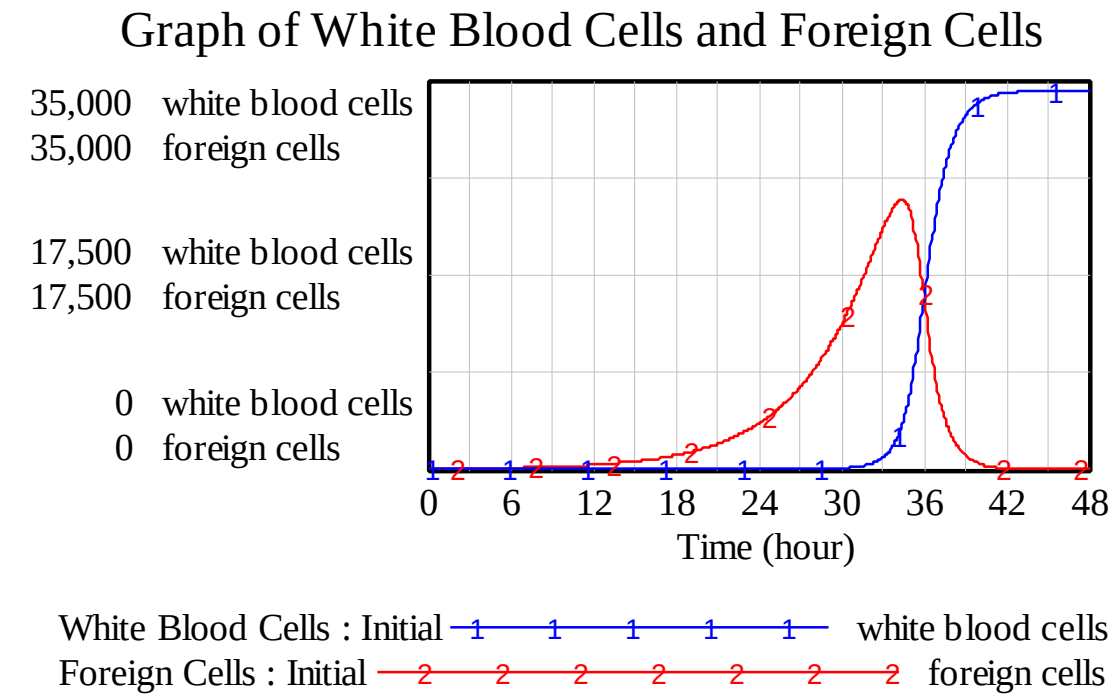


Figure 37: Vensim Equivalent of Figure 20: Immune response: stocks over time

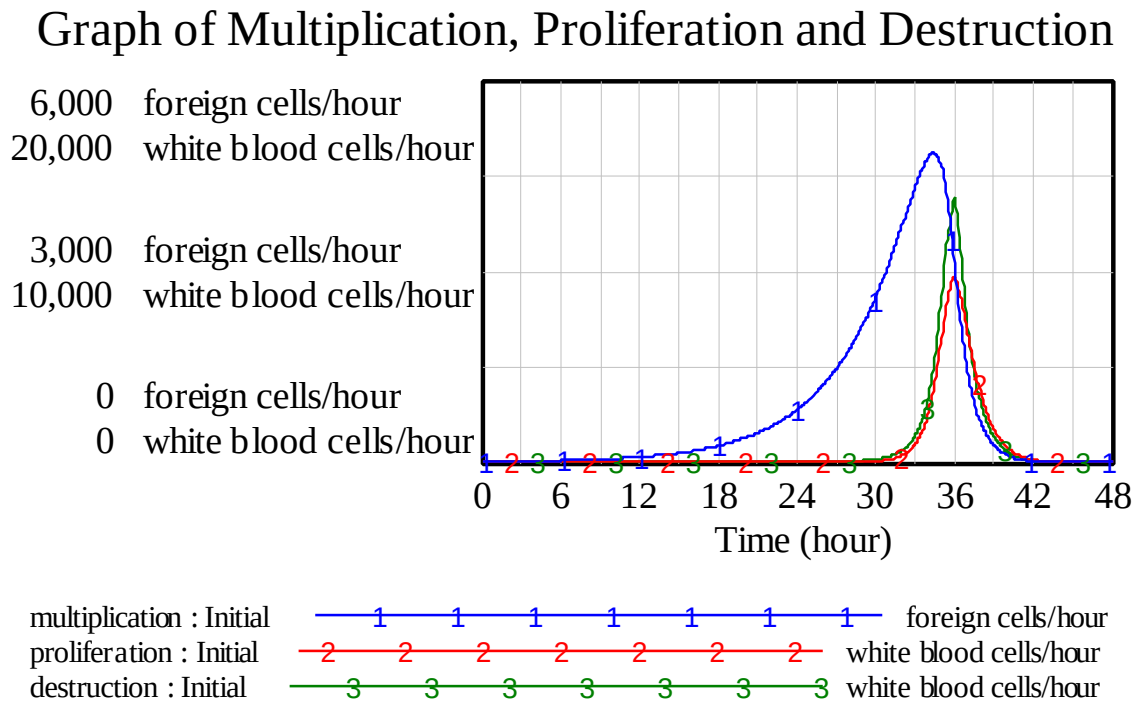


Figure 38: Vensim Equivalent of Figure 21: Immune response: net flows versus time

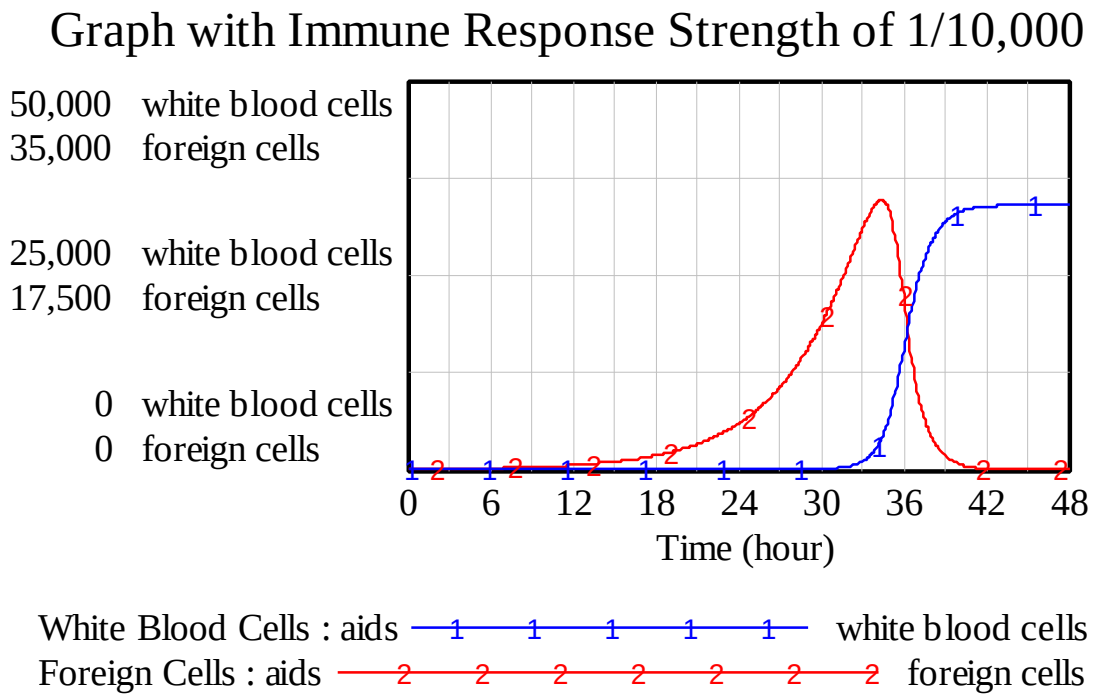


Figure 39: Vensim Equivalent of Figure 22: Strength of the immune response = 1/10,000

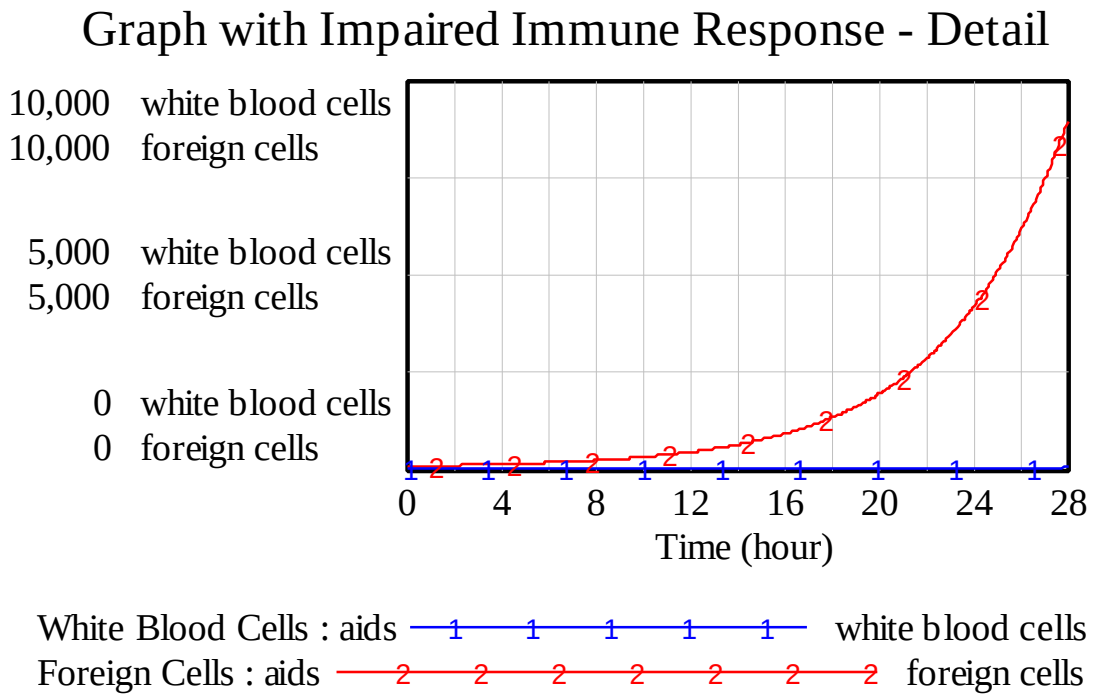
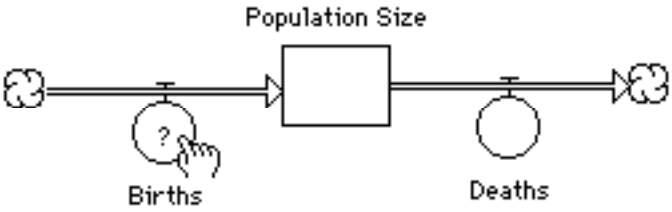


Figure 40: Vensim Equivalent of Figure 23: Impaired immune response - detail

Modeling Exercises

Section 1



Prepared for
MIT System Dynamics Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Joseph G. Whelan
May, 1994

Vensim Examples added October 2001

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The exercises in this paper require a Macintosh[1] Computer with 2 Mb of RAM if running System 6.0.4 and at least 4 Mb of RAM if running System 7 or higher. Also required is the STELLA[2] II software package. If you do not have STELLA, it can be obtained from High Performance Systems:

High Performance Systems, Inc.
45 Lyme Road
Hanover, NH 03755
(800) 332-1202

Abstract

This paper is the first in a series of papers designed to utilize and enhance the modeling capabilities of the reader. This series of papers is part of a project called Road Maps. Road Maps is a self study guide to lead the reader through the process of learning system dynamics. Road Maps is currently being worked on by the System Dynamics in Education Project at MIT under Jay Forrester, the founder of system dynamics.

The beginning of this paper outlines the importance and the process of model building. This paper also contains three separate modeling exercises. Each exercise begins with a brief description of the system after which the reader constructs a system dynamics model of the system. The reader then runs the model under varying conditions and answers some questions. This paper is not intended as an introduction to system dynamics or model building. The reader is expected to have prior experience with both. Solutions to all the exercises are included at the end. The modeling exercises are presented approximately in order of difficulty, but need not be done in this sequence.

[1]Macintosh is a registered trademark of Apple Computer, Inc.
[2]STELLA is a registered trademark of High Performance Systems, Inc..

Introduction

Modeling is a crucial element of system dynamics. We use mental models every day of our lives to make decisions without addressing the model being used. Creating and simulating models using STELLA or other modeling software forces us to solidify our mental models and allows us to observe the behavior of complex systems where mental models are inadequate. One drawback of using a computer to simulate systems is that the computer will always do exactly what you tell it to do. Your computer won't say "Hey wait a minute, you can't have a negative population," or "You can't make the number of sick people greater than the total population." You must be the one to generate and verify a valid model.

To make system dynamics modeling as useful as possible, a modeler must acquire many skills. Below is a list of some important skills:

- **Have a general idea of how the system should behave to test the accuracy of the computer simulation.**
- **Generate a valid model of the system to be analyzed.**
- **Enter the model correctly into the computer.**
- **Make the model clear so that others can understand its structure.**

The exercises you encountered in Road Maps I-IV have prepared you for the task of creating your own models. This paper will allow you to exercise some of the skills you have already learned and enable you to acquire new skills as well. Future parts of Road Maps will contain more sections of the Modeling Exercises series (this is the first section) enabling you to further increase your modeling skills.

How to Build Models

Building a model can be a very rewarding, yet challenging process. There is no set way to build a model. Each model that you build will be different, and likewise the steps you follow in building your models will vary. In some cases, you may be able to jump directly into STELLA and start mapping. In other cases the modeling process may take a considerable amount of time.

Below is a rough guideline for how to go about the process of model building. It is not intended as a "cookbook" for model building but rather as a way to get you started.

1. Define the purpose. What you want to do with your model is entirely up to you. However, you must define the purpose for your model before you can begin to build it. Do you want to explore different management policies for a park? Perhaps you merely wish to teach your third graders about population dynamics. You must have a goal in mind to decide how complex to make your model. This purpose will also guide you in your decisions regarding how much certainty you require in your numbers, where to set your model boundary and many other aspects of your model.

2. Define the system you wish to model. You will have to decide exactly what is "inside" and "outside" your system before you begin modeling. You can't include the whole world in your model, but neither can you make everything exogenous[3]. You must find a happy medium.

3. Identify key variables in the system. What factors have the most effect on your system? Be careful in naming your model variables. Use names that can be associated with a real quantity such as fish, cows or account balance. Model elements can also represent soft variables, but be sure to use something that can be scaled, like happiness. Don't use vague names like mental attitude or lifestyle.

4. Describe the behavior of the key variables. How do the key variables behave in real life? You may not know an exact answer to this, but you should at least define some constraints. (i.e., population can't be negative or infinite) This is an important step because it helps you evaluate and improve your model later on.

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[3]Exogenous inputs are factors that are not enclosed by the system boundary. They affect the system, but are not affected by the elements within the system. Exogenous inputs are either constant, or dependent on time only.

5. Identify the stocks and flows in your system. What are the levels of your system? What flows affect them?

6. Map the stocks and flows on your STELLA diagram pad. Begin building the model in STELLA with the stocks and flows.

7. Define the flows. Recall that the flows are defined only by the stocks in the system and constants. There may be several intermediate steps, but each flow must be dependent only on the stocks and the constants. Model these relationships in your STELLA model.

8. Include quantitative information. Once you have built a preliminary flow diagram for the model in STELLA you should enter the equations for the model elements. Many relationships are simple and intuitive. Sometimes, the relationship between two variables is too complicated for a simple equation. If so, you will have to define a graph to represent the relationship.

9. Run the model. Now is the time to test your model. Did it behave as expected? Is the behavior possible? You may need to return to one of the previous steps at this point and correct any problems you come across.

10. Evaluate the model. Your model may not have produced a reasonable behavior. Look at each relationship in the model to see if you have made a mistake. Here is a list of common mistakes:

- The solution interval (DT) is too large.
- A flow that should be bi-directional is defined as a one way flow[4].
- A stock is incorrectly defined as non-negative.
- Incorrect flow direction. (An outflow is modeled as an inflow or vice versa.)

[4]It is generally good practice to define all flows as bi-directional and allow all stocks to become negative. You should select the bi-directional flow option for all your flows and de-select the non-negative option for all your stocks. This allows the modeler to see the behavior of the system without any unnatural constraints. If a stock in your model is inappropriately becoming negative, examine the causes of this behavior from a structural point of view and correct the model rather than clicking the non-negative box. This principle also applies to constraints on flow direction.

- Incorrect Equations. (Don't be too quick to guess the equation for a relationship. Dividing instead of multiplying (or a similar mistake) is easy to do and will make the model behave incorrectly.)

11. Improve the Model. Try to find causes of any anomalous behavior, then modify the model so it produces an appropriate behavior. Once you get an appropriate behavior, try a few different scenarios to test your model under varying conditions. You may also wish to add additional structure to the model.

Now, return to the first step and see if your model accomplished what you wanted it to. You may find that once you have a working model it doesn't accomplish for you what you had intended. In that case, go back and change the model. There is no set point at which your model is "done" or "perfect". It can only be "good for what I wanted it for."

NOTE: The book, An Introduction to Systems Thinking, that came with your STELLA software provides a more in-depth look at the process of model building. You may want to read the section entitled "The Modeling Process" before proceeding with exercises that follow.

Building Models

This paper contains three exercises. Each begins with a written description. All of the information necessary to build a model may not be in the written description, so you will have to make some decisions for parameter values and other variable relationships. Suggested answers are included in the Appendix.

- Read the Description.
- Use the steps outlined previously to guide you in building a STELLA model.
- Follow the directions and answer the questions in the exploration that follows.

Model 1: An Ecological System[5]

Photo of penguins removed due to copyright restrictions.

This model description is taken from an ecology textbook. A causal-loop diagram was made for this system in Exercise 1 of Study Notes in System Dynamics[6] from Road Maps Part II. The exercises included after the system description will help you in formulating and testing your model.

[5]Photograph © Doug Allen/Oxford Scientific Films

[6]Goodman, Michael R.(1974) *Study Notes in System Dynamics*, Portland, OR: Productivity Press, pp. 137-147.

Description:

> ...At a critical time in the life history of a given population, a physical factor such as light or a nutrient may be significant as a regulatory agent; at another time, parasitism, predation, or competition, or even some other physical factor may become the operative factor. As complex and as variable as the niche of any species is, it is unlikely that the regulation comes about by any single agency. However, there does appear to be considerable and mounting evidence, both empirical and theoretical, to suggest that populations are self-regulating through automatic feedback mechanisms. Various mechanisms and interactions appear to operate both in providing the information and in the manner of responding to it, and with the exceptional case of a catastrophe, the stimulus to do so appears to depend on the density of the population. The end effect is one of avoiding destruction of a population's own environment and thereby avoiding its own extinction.[7]

You should be able to build a simple, one level model based on the above description. Use the steps outlined in the previous section to guide you through the process. Remember that you have been provided with a general guideline, not a cookbook. It is not necessary to follow the steps exactly.

You may wish to refer to the causal-loop diagram you created for the exercise in Road Maps Part II. In building your model, you may wish to generate a graphical function to show the relationship between the density of the population and the death rate. After you have built your model, proceed to the exploration section below:

Exploration:

1. This system can be modeled with only one level. What physical quantity does the level represent? What flows affect the level?

2. Describe the state of equilibrium for your model. What is the size of the level? What are the flow rates? Does this seem like a plausible equilibrium? (Note: If birth rate = death rate = 0, the model would be in equilibrium, but it would not be very helpful.)

3. Simulate a catastrophe by adding another outflow to the population. Use the PULSE function in STELLA to cause the catastrophe to kill half the population (i.e. the size of the pulse will be half the equilibrium population) Is the system "self-regulating through automatic feedback" as described in the passage?

4. Add another flow taking away from the population stock labeled *Parasites*. Set the catastrophe flow equal to 0. What is the effect of a constant flow out of the population? What is the new equilibrium?

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[7]Kormondy, E.J. (1969) *Concepts of Ecology*, Englewood Cliffs: Prentice-Hall, p.110.

Could you have predicted the new equilibrium? Compare this behavior to the one-time catastrophe response.

Model 2: A Heating/Cooling System

This is a description taken from Systems 1: An Introduction to Systems Thinking[8]. This book was used in Part I of Road Maps.

Description Part A:

> One of the most common mechanical feedback systems is the heating system found in most homes and buildings. Once a temperature has been set on the thermostat, the system will try to keep the temperature in the house as close to that level as possible. If the temperature drops below that level, the thermostat responds by turning the furnace on. The furnace produces heat, which warms the air back up. Eventually the temperature rises above the desired level and the thermostat shuts the furnace off. Then, if it is colder outside than inside, the house will cool down again until the thermostat turns the furnace on, and the whole cycle is repeated.[9]

You should include the effects of both the environment and the furnace on the room temperature. While the furnace is either on or off, the effect of the environment is somewhat more complicated. You may wish to refer back to the cooling cup of coffee model you built in Road Maps Part III[10]. A similar goal-gap mechanism is at work here.

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[8]Kauffman, Draper L. J. (1980) *Systems 1: An Introduction to Systems Thinking*, Minneapolis, MN: S. A. Carlton, Publishers.
[9]ibid., Page 6
[10]Nancy Roberts, David F. Andersen, Ralph M. Deal, Michael S. Garet, William A. Shaffer (1983) *Introduction to Computer Simulation: A System Dynamics Modeling Approach*. pp. 267-272 (Chapter 15).

Hints:

• Both the furnace and the outside environment affect the state of the house by adding and removing energy in the form of *heat*.

• Changes in the house's *temperature* are proportional to the change in the amount of *heat*[11] in the house.

• The rate at which *heat* is transferred from the environment to the house is directly proportional to the difference in temperature between the house and the environment.

• The furnace does not react instantly to changes in room temperature. You need to add a delay between the actual house temperature and the temperature registered by the thermostat.

Follow the model building guideline provided to you earlier to build a STELLA model of this system. After building the model, proceed with the following exercises.

Exploration Part A:

1. What is the level in this system?

2. What rate(s) affect the level?

3. How are the rates determined?

4. Run your STELLA model. What happens if the outside temperature is lower than the thermostat setting? What if it's higher?

5. What happens if it is very, very cold outside? Does this make sense?

[11]Since all we care about for this system is *changes* in energy, we can set zero reference anywhere we want. For example, you could set the zero energy point to correspond to a house temperature of 75 degrees.

Now let's add an air conditioning system to take care of the situation where the outside temperature is higher than the inside temperature.

Description Part B:

> "What about hot days? A heating system can't do anything to keep a house cool, but al least the thermostat can shut the furnace off and keep it from making the house even hotter. Of course, we could add an air conditioner to the system and connect it to the thermostat. Now if we set the heating system for 70 degrees and the cooling systems form 78 degrees, the thermostat will turn the furnace on if the temperature drops below 70 degrees and the air conditioner on if it rises above 78 degrees. This changes a one-directional feedback system which keeps us from getting cold into a two-directional system which will keep the house comfortable in any weather.[12]

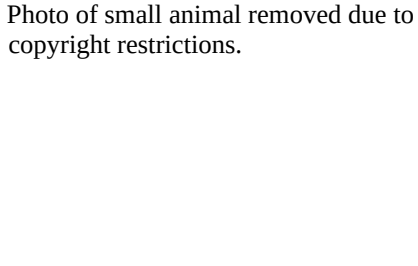
Exploration Part B:

6. Now, run the model again with the external temperature higher than the room temperature. Is the problem solved? What happens if the heater setting is higher than the air conditioner setting?

[12]Kauffman, Draper L. J. (1980) *Systems 1: An Introduction to Systems Thinking*, Minneapolis, MN: S. A. Carlton, Publishers. Page 6.

Model 3: A Predator Prey System[13]

Description:



> Interesting and only partly understood density variations are those which are not related to seasonal or obvious annual changes, but which involve regular oscillations or cycles of abundance with peaks and depressions every few years, often occurring with such regularity that population size may be predicted in advance. The best known cases concern mammals, birds, insects, fish and seed production in plants in northern environments. Among the mammals, the best-studied examples exhibit either a 9 to 10 year or a 3 to 4 year periodicity... Peaks of abundance often are followed by "crashes," or rapid declines...[14]

You should be able to model this phenomenon by connecting two simple population models and making them interdependent. Your two populations will be similar to the population model you built in the first exercise. In this case, you should make the death fraction of the predators dependent on the prey density. The number of prey killed per predator should also depend on the prey density. You may want to used the Define Graph option in STELLA to make graphical functions for these relationships.

Exploration:

1. Did your model produce the characteristic oscillations? What is the relationship between the predator and the prey oscillations?

2. What are the periods of oscillation for your two populations? Are they the same? Can they be different?

[13]Fox photograph © Leonard Lee Rue III/Animals Animals

[14]Odum Eugene P. (1971) *Fundamentals of Ecology*, Philadelphia: W.B. Saunders Co., pp. 190-192.

3. Can you make one or both populations in your model go to infinity? (This may have happened in the early stages of your model.) Why does this happen? Obviously this won't happen in real life; what is your model missing?

4. Add a PULSE function to the predator death rate and kill half the predators a few years into the simulation. What are the short-term (1-2 cycles) effects on the predator and prey populations? What are the long-term (more than 5 cycles) effects on the two populations?

5. Assume the predator population represented insects and the prey population represented a crop. What would be the result of spraying insecticides on your crop? If you looked only at the short-term results, what might be your reaction to this?

Solutions

This section contains one possible model solution for each of the descriptions in Modeling Exercises: Section 1. I wish to stress that the models and equations outlined here represent only one of many possible solutions. There is more than one right answer for all modeling exercises. Your model may have more complexity, but it should be similar. If your model displays plausible behavior and you can explain each relation in the model, then the model is probably useful. You will be learning more about testing whether or not your model is adequate in future sections of Road Maps.

Model 1: An Ecological System

For the first modeling exercise, I have included my progress through the steps outlined in the How to Build Models section included earlier in this paper. For this simple system, your results will probably be similar.

1. Define the purpose: My purpose for building this model is to simulate the behavior described in the reading passage. I also wish to learn about population dynamics and why this behavior occurs.

2. Define the system: I wish to model the population. I will use penguins in this case.

3. Identify key variables: The key variables mentioned in the passage are: Density, the population, and external regulatory agents. I will also include the hatch rate and the death rate.

4. Describe the behavior of the key variables: I want the population to center on some standard value. It should change up or down, depending on what external events affect the population, but when these events end, the population should return to the standard value.

5. Identify the stocks and flows: The one stock in my model is the population of penguins. It is increased by the hatch rate flow and decreased by the death rate flow.

6. Map the stocks and flows on your STELLA diagram pad. See the model in Figure 1.

7. Define the flows: The hatch rate is equal to the size of the penguin population multiplied by a hatch fraction. The death rate is similar, defined by the population times a death fraction. To include the regulatory effects of density, I made the death fraction dependent on density. The hatch fraction is constant.

8. Include quantitative information: Since the purpose of this model does not require much accuracy in the numerical data of the model, I did not research the reproductive characteristics of penguins. Instead, I simply chose an initial population of 800 penguins in an area of 10 acres. I let the hatch fraction be equal to .2 and the death fraction varied from .08 for an extremely small population to .294 for crowded conditions. (The model equations that follow contain more detailed information)

9. Run the model: The results of running the model are included in the exploration answers below.

10. Evaluate the model: The model displayed behavior consistent with the description upon which it was based. Running the scenarios suggested in the exploration helped to test the robustness of the model.

11. Improve and expand upon the model: The model served its purpose so far. From here, you could add the effects of a limited food supply, or show the effect of density on the hatch fraction.

Solutions to Exploration Questions:

1. This model is centered around a level that represents the total population being analyzed. I decided to examine a population of penguins for my model. The population level is increased by the hatch rate and decreased by the death rate. For this model, I made the hatch fraction constant at 0.2. The death rate is dependent on the density of the population. The values of the parameters in the system (area, hatch fraction, initial population, etc.) are somewhat arbitrary. Your values will be different, but you should have a similar structure.

Model Diagram:

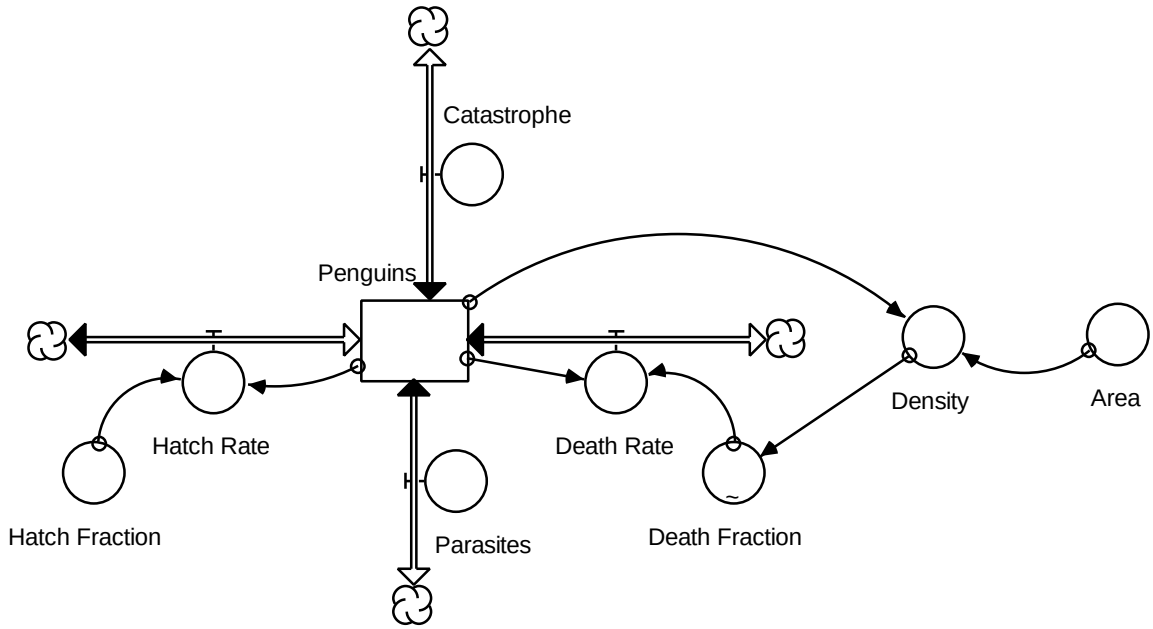


Figure 1: STELLA Diagram of the Population System

This population system is a simple, one-level model with a constant hatch fraction and a death fraction dependent on density.

To represent the dependency of the death fraction on density, I created the following graphical function:

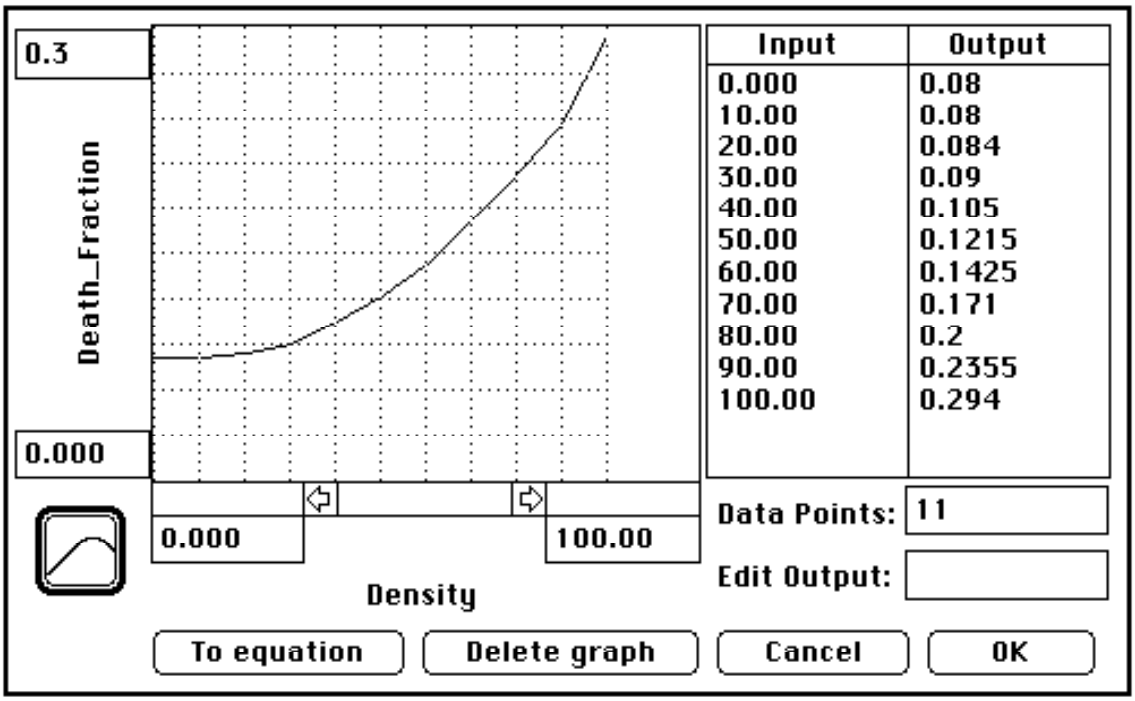


Figure 2: Death Fraction graphical function

This graph determines the dependency of the death fraction on the density. When density is very low, the death fraction is constant at 0.08, less than the hatch fraction. As the density increases, the death fraction also increases.

2. Equilibrium will occur when the hatch rate and the death rate are equal. For my model, this occurred when the population was equal to 800 penguins. The density at this point was 80 penguins per acre, which gives us a death fraction of 0.2 (see Figure 2) This makes sense because the hatch fraction is a constant at 0.2.

3. To simulate a catastrophe, I use the PULSE function in STELLA as follows:

Catastrophe = PULSE(400,5,0)

This caused a decrease in the population from 800 to 400 after 5 years. The density dropped down to 40 penguins per acre, causing the death fraction to fall below the hatch fraction and the population slowly rose back up to its equilibrium value of 800.

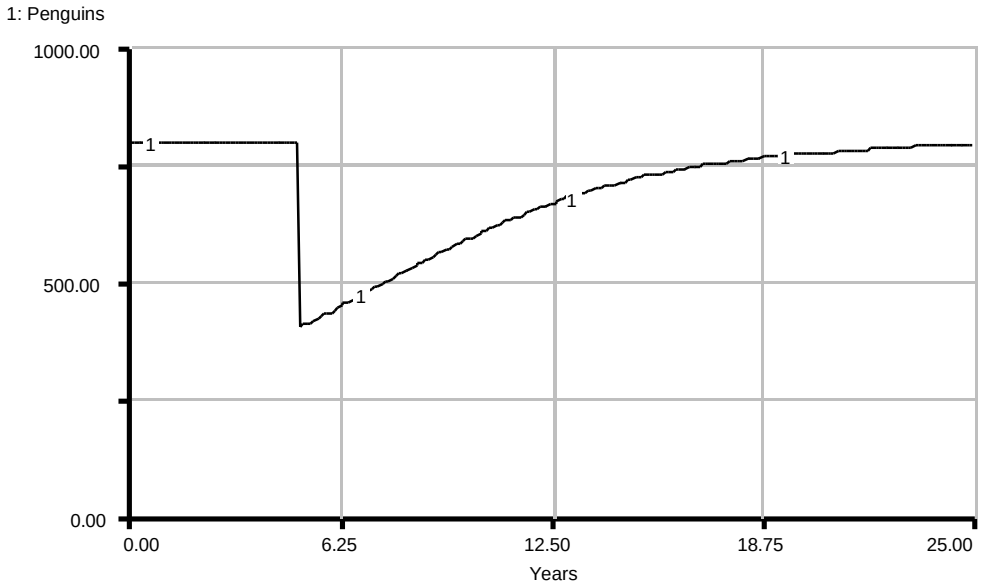


Figure 3: Graph of population response to a one time catastrophe

4. The response to the constant outflow was very different. I added a constant outflow labeled Parasites and set it to 20 penguins/year. The population dropped to a new equilibrium at approximately 700 penguins. Notice that the new equilibrium is lowered significantly by a loss of only 20 extra penguins per year. The reason for this is that the population must drop down to a level where the hatch rate is larger than the death rate by 20 penguins per year. The new equilibrium will vary depending on the parameters in your model. If both your hatch and death fractions are high, the drop will be less substantial. It will also be less if the dependency of your death fraction on density is steeper.

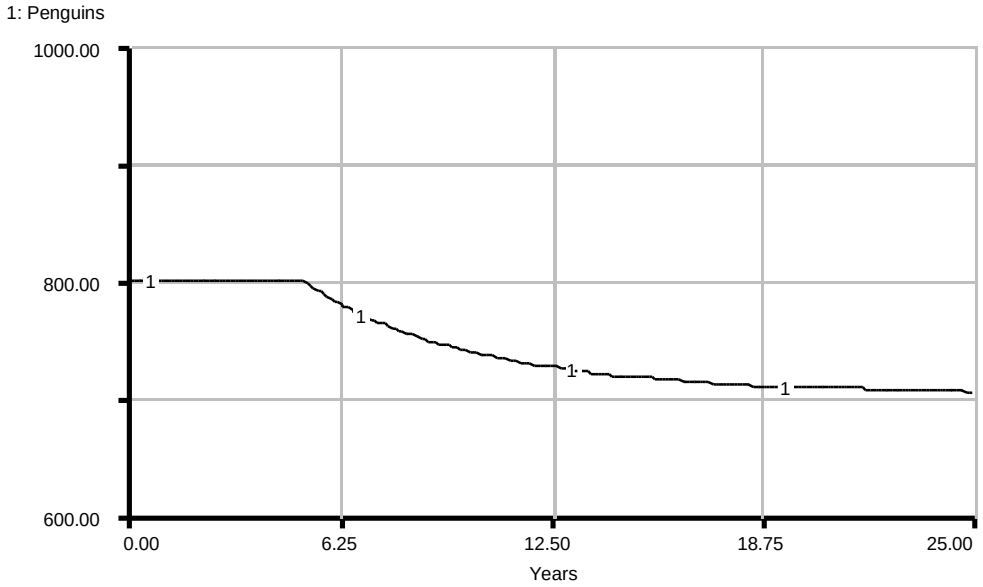


Figure 4: Graph of population response to a constant outflow.

Model Equations:

Penguins(t) = Penguins(t - dt) + (Hatch_Rate - Death_Rate - Catastrophe - Parasites) * dt
INIT Penguins = 800

DOCUMENT: This stock represents the number of penguins in the population. It is increased by hatching and decreased by deaths.
UNITS: penguins

INFLOWS:
Hatch_Rate = Penguins*Hatch_Fraction
DOCUMENT: This is the hatch rate for the population. It is determined by multiplying the population by a hatch fraction.
UNITS: penguins/year

OUTFLOWS:
Death_Rate = Penguins*Death_Fraction
DOCUMENT: This is the death rate of the population. It is determined by multiplying the population by a death fraction.
UNITS: penguins/year

Catastrophe = PULSE(400,5,0)
DOCUMENT: This flow was used to explore the effects of a sudden drop in the population.
UNITS: penguins/year

Parasites = 20
DOCUMENT: This flow is used to represent a constant outflow from the population.
UNITS: penguins/year

Area = 10
DOCUMENT: This is the total land area occupied by the population.
UNITS: acres

Density = Penguins/Area
DOCUMENT: This is the population density. It affects the death fraction of the population.
UNITS: penguins/acre

Hatch_Fraction = .2
DOCUMENT: This represents the fraction of the population that reproduces each year. If the hatch fraction is 0.2, then the hatch rate will be 20% of the population.
UNITS: 1/years

Death_Fraction = GRAPH(Density)
(0.00, 0.08), (10.0, 0.08), (20.0, 0.085), (30.0, 0.09), (40.0, 0.1), (50.0, 0.12), (60.0, 0.14), (70.0, 0.17), (80.0, 0.2), (90.0, 0.24), (100, 0.3)
DOCUMENT: The death fraction is dependent on the density of the population. The higher the density, the larger the fraction of the population that dies each year.

UNITS: 1/years

Model 2: A Heating/Cooling System

Model Diagram:

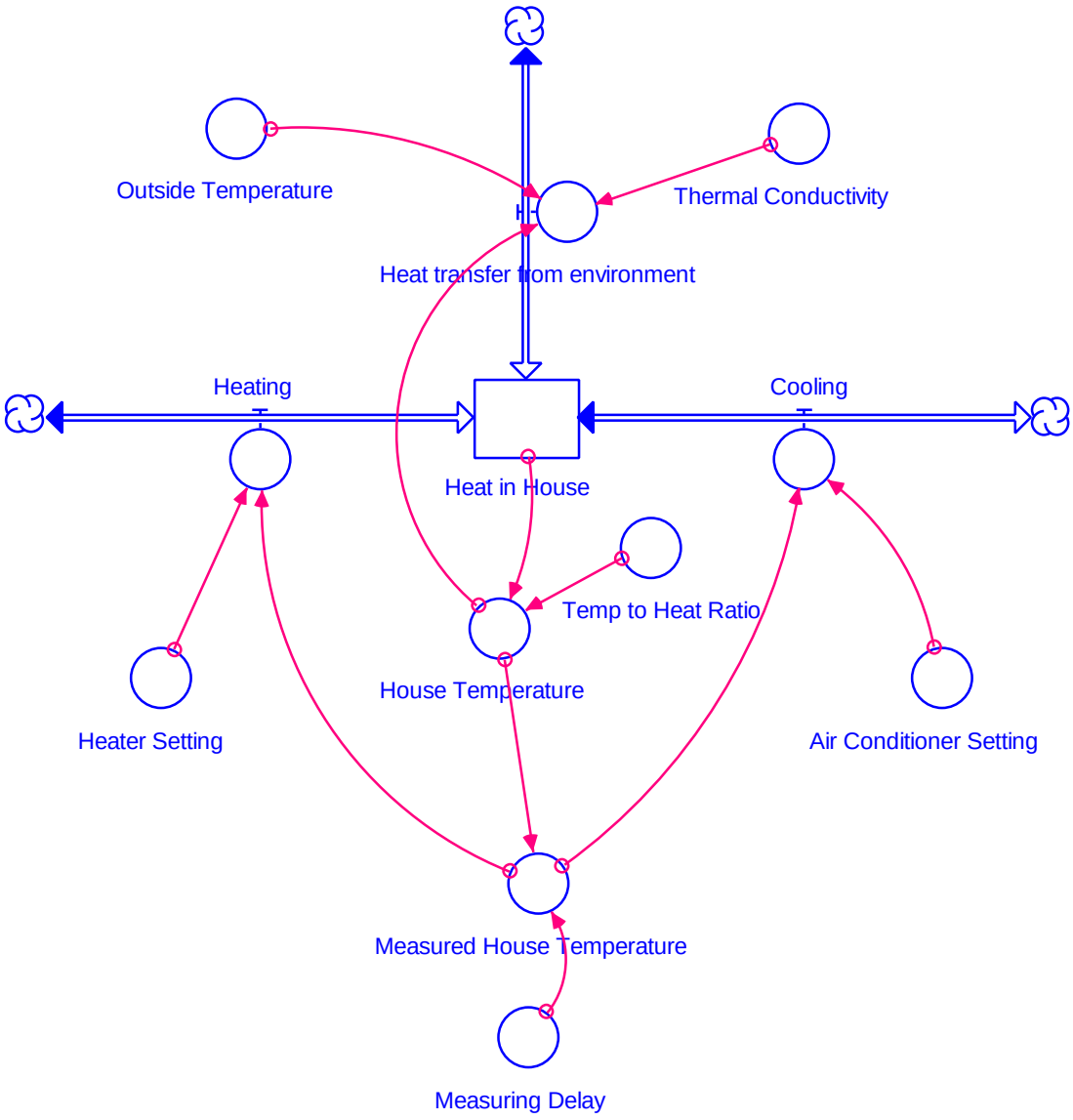


Figure 5: A Heating/Cooling System Model

The heat in the house is affected by the external environment as well as the heating and cooling systems. The heat transfer from the environment is dependent on the difference in temperature between the house and the environment. The heating and cooling systems are either on or off dependent on the measured house temperature and their respective settings.

1. The level in this model is the amount of heat in the house. The heat is then used to calculate the temperature using the relation given in the hints.

$$\text{Temperature} \propto \text{Heat in house}$$

I used this to define the house temperature. I assumed that the heat in the house was equal to zero when the temperature was 75 degrees Fahrenheit.

$$\text{House Temperature} = \frac{\text{Heat In House}}{\text{Temp to Heat Ratio}} + 75$$

2. The house temperature is affected by the heat transferred from the environment, and the heating system.

3. The heating system will add heat to the house at a rate of 100,000 joules per hour whenever the measured temperature in the house is lower than the heater setting temperature. I used the following relation to define the energy transferred from the environment to the house:

$$\text{Energy Transfer} \propto (\text{External Temperature} - \text{House Temperature})$$

So, the heat transfer from the environment was defined as follows:

$$(\text{Outside Temperature} - \text{House Temperature}) \times \text{Thermal Conductivity}$$

The Thermal Conductivity accounts for the ability of the whole house to transfer heat to and from the environment. In effect, the lower the value, the better insulated the house.

4. If the outside temperature is lower than the thermostat setting, the house temperature will cycle around the heater setting value. The cycling occurs because of the delays involved with measuring and changing the house temperature. If the outside temperature is higher than the heater setting, then the house will gradually heat up to the outside temperature. Heating will remain at 0 because the measured house temperature is above the heater setting.

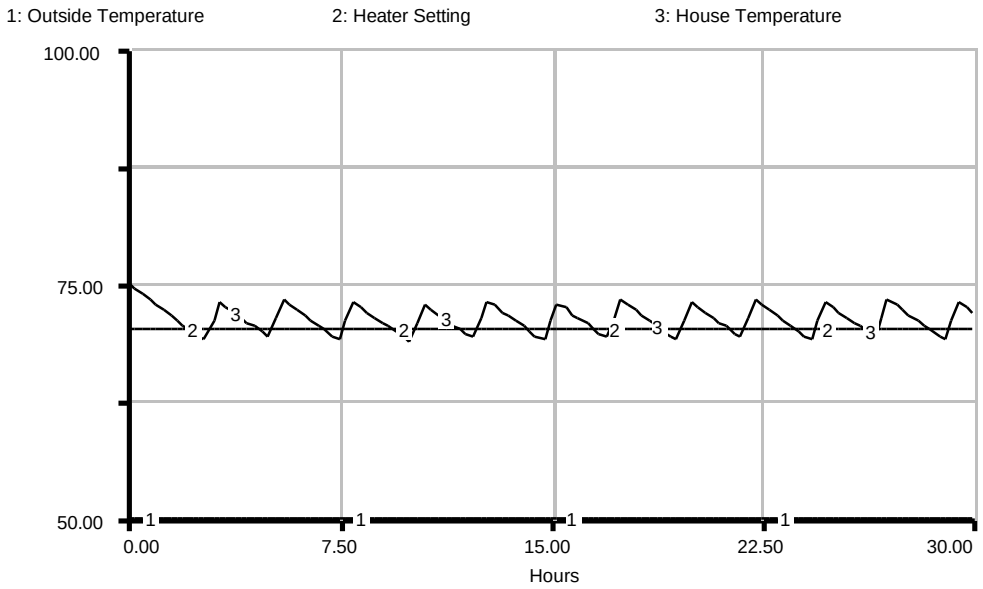


Figure 6: Graph of house temperature with heating system only.

5. The heating system will be unable to maintain the temperature of the house if heat is being removed from the house faster than the heating system can add it back in. In my model this occurs when the external temperature is more than 100 degrees below the heater setting temperature. This value is dependent on the power of the heating system as well as the thermal conductivity of the house.

The cooling system is very similar to the heating system. It will remove heat from the house at a rate of 100,000 joules per hour whenever the measured temperature in the house is higher than the air setting.

6. Now, the system will cycle about the air setting temperature if the outside temperature is higher than the measured house temperature. (See Figure 7) You should notice that the cycling period is longer. This occurs because the outside temperature is closer to the room temperature so heat transfer with the environment is slower. If the outside temperature is between the two settings, the house temperature will converge on it and remain at that temperature.

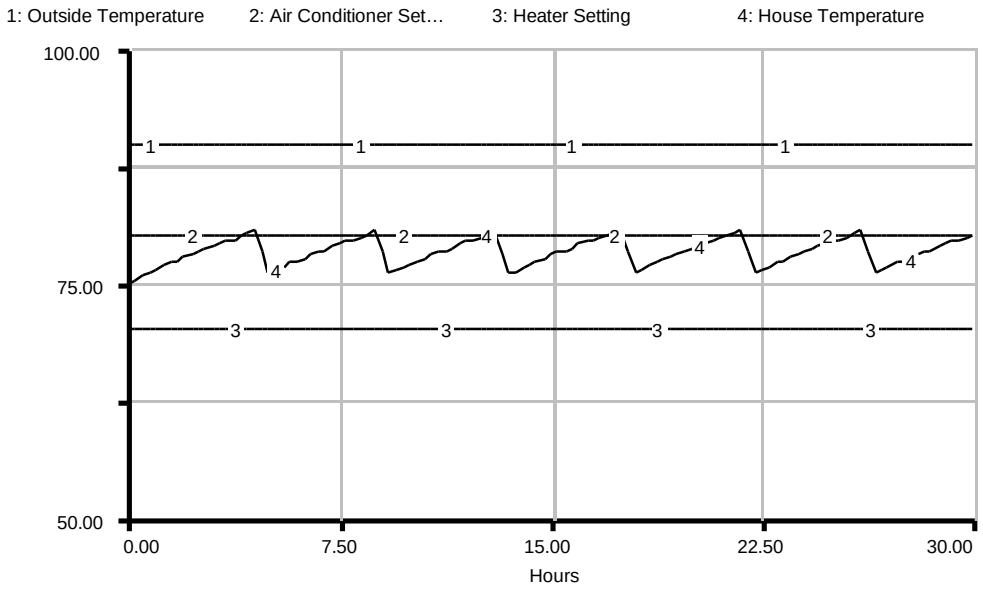


Figure 7: Graph of house temperature with both heating an air conditioning.

In this run, the outside temperature is higher than the air conditioner setting and the temperature cycles around the air conditioner setting temperature.

Setting the heater setting higher than the air conditioner setting gave some interesting results. When the outside temperature was between the two settings, the house temperature converged on it as before except that both the heater and the air conditioner were on. (Somewhat expensive) When the outside temperature was higher than both settings, the house temperature rose slowly to the heater setting because only heat transfer from the environment was affecting it. (heating and cooling balanced out) Then, when the temperature rose slightly above the heater setting the heater shut off and the temperature rapidly dropped until the heating system turned back on after a delay. The response to an outside temperature below both settings was similar. There was a cycle of slow decreases until the air setting temperature was reached, followed by rapid increases as the cooling system turned off.

Model Equations:

Heat_in_House(t) = Heat_in_House(t - dt) + (Heating + Heat_transfer_from_environment - Cooling) * dt
INIT Heat_in_House = 0

DOCUMENT: This is the amount of heat in the house. It is affected by the heating and air conditioning systems as well as the heat transferred from the environment. The initial amount of heat in the house is set to zero as a reference point. This amount of heat is assumed to correspond to a temperature of 75 degrees Fahrenheit.
Units: Joules

INFLOWS:
Heating = IF Measured_House_Temperature < Heater_Setting THEN 100000 ELSE 0
DOCUMENT: This is the heat transferred to the house by the heating system. If the measured house temperature is lower than the Heater_Setting, then Heating = +100,000 Joules per hour. If the house temperature is higher, then Heating = 0
Units: Joules/hour

Heat_transfer_from_environment = (Outside_Temperature-House_Temperature)*Thermal_Conductivity
DOCUMENT: This is the amount of heat transferred into the house from the environment. It is proportional to the temperature difference between the house and the environment.
Units: Joules/hour

OUTFLOWS:
Cooling = IF Measured_House_Temperature>Air_Conditioner_Setting THEN 100000 ELSE 0
DOCUMENT: This is the heat removed from the room by the air conditioning system. If the measured house temperature is higher than the Air_Setting, then Cooling = +100,000 joules per hour. If the house temperature is lower, then Cooling = 0
Units: Joules/hour

Air_Conditioner_Setting = 80
DOCUMENT: This is the thermostat setting for the cooling system.
Units: Degrees

Heater_Setting = 70
DOCUMENT: This is the thermostat setting for the heater system.
Units: Degrees

House_Temperature = (Heat_in_House/Temp_to_Heat_Ratio)+75
DOCUMENT: This is the temperature in the house. Changes in the temperature are proportional to changes in the house's energy. The change in temperature is calculated by multiplying the energy by the temp to heat ratio. The actual temperature is calculated by adding 75 (the reference temperature corresponding to zero energy.
Units: degrees

Measured_House_Temperature = SMTH1(House_Temperature,Measuring_Delay)
DOCUMENT: This is the room temperature as perceived by the thermostat. It is calculated by smoothing the actual room temperature with a delay.
Units: degrees

Measuring_Delay = .2
DOCUMENT: This is the delay involved in measuring the room temperature.
Units: hours

Outside_Temperature = 50

DOCUMENT: This is the temperature of the environment external to the house. The house temperature would eventually reach this temperature without a heating/cooling system.
Units: Degrees

Temp_to_Heat_Ratio = 10000
DOCUMENT: This is the ratio between temperature and energy in the house. This number tells how many joules of energy are required to heat the house by 1 degree.
UNITS: Joules/Degree

Thermal_Conductivity = 1000
DOCUMENT: This factor represents how easily energy is transferred from the outside environment to the house.
UNITS: Joules/hour/degree

Model 3: A Predator/Prey System

This is the model I came up with to simulate the predator/prey system.

Model Diagram:

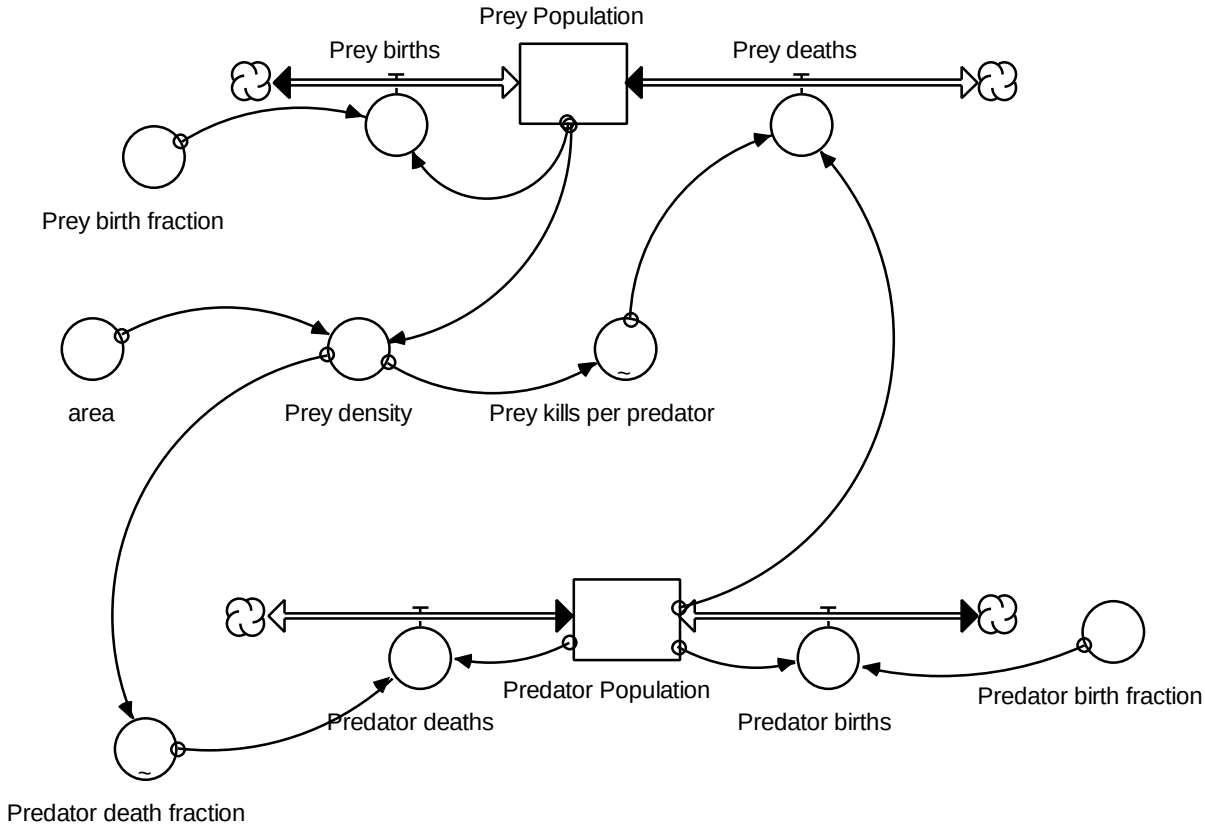


Figure 4: A Predator/Prey System Model

Two population subsystems interact to produce oscillation. The predator deaths and the prey kills per predator vary with the prey density. Both predator and prey birth fractions are constant.

I used the following graphical functions to define the dependencies of prey kills per predator and predator death fraction on prey density.

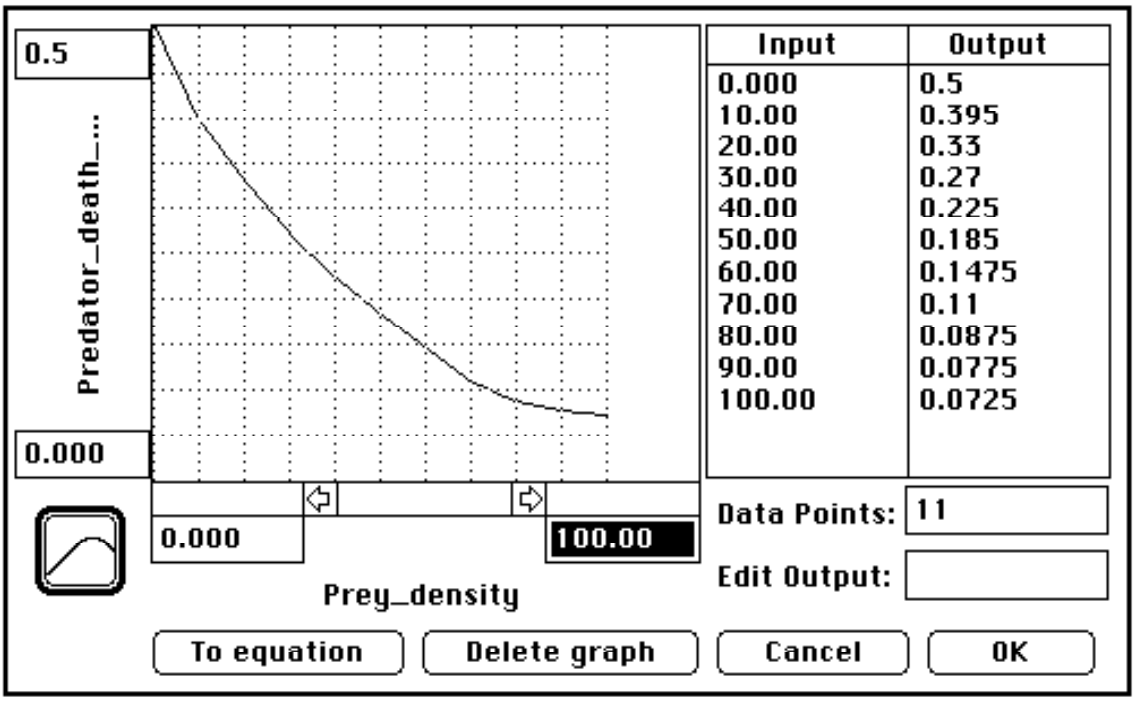


Figure 5: Predator Death Fraction

The predator death fraction varies with prey density. When there are no prey, the predators die very rapidly. As the prey density increases, the number of predators that starve decreases until the death fraction levels off at .0725.

This graph shows that the predator death fraction varies widely with the prey density. When the number of prey in the area is small, the predators are unable to catch very many and their survival rate drops. When there are no prey at all, the predators are dying twice as fast as they are being born. As the density of the prey increases, the death fraction of the predators decreases. Eventually, the predators reach a point where they are no longer at risk of dying of starvation, but rather from other factors. This is represented by the leveling-off of the predator death fraction graph when prey density is high.

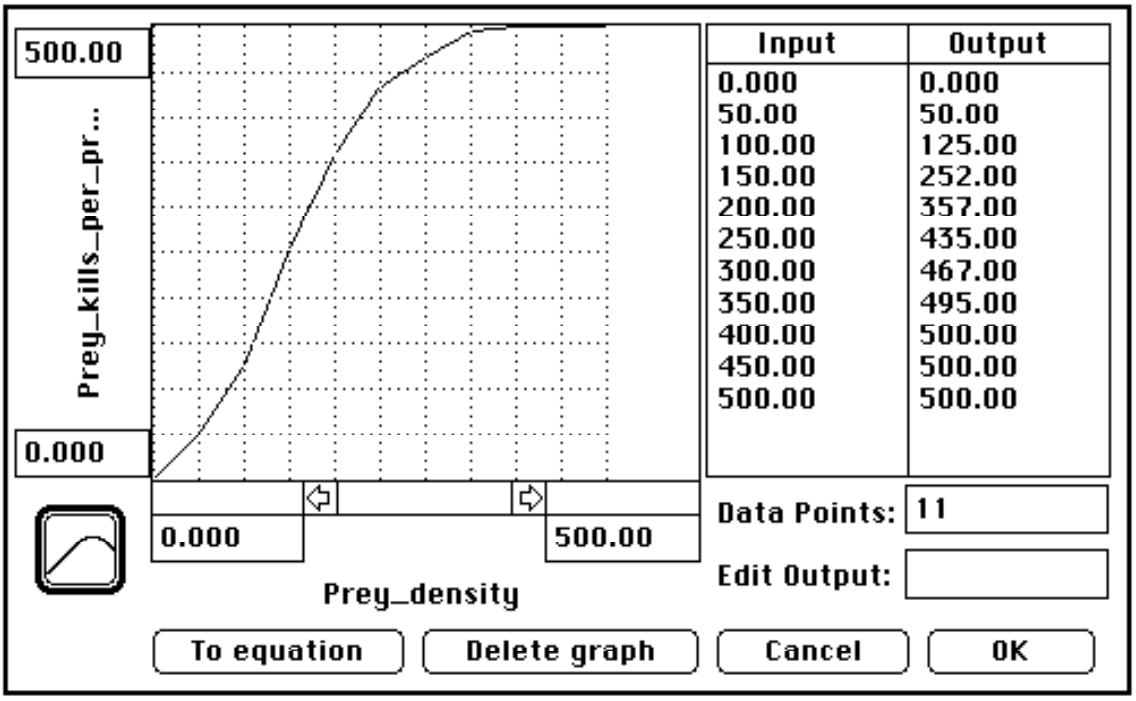


Figure 6: Prey kills per predator

When there are no prey, the predators don't catch any. As the prey density rises, so does the catch per predator until the predators are satiated at 500 prey per year.

This graph shows that prey kills per predator varies with the prey density. Naturally, when there are no prey, the density is 0 and prey kills per predator is also 0. When the prey density begins to rise, prey kills per predator rises quickly, and then levels off. This leveling off represents the point at which the predators are satiated and do not wish to eat any more prey regardless of how abundant they are.

1. You should notice that the prey population is decreasing the fastest when the predator population is highest and increasing the most when the predator population is lowest.

2. Both populations in my model oscillated with a period of about 14 years. Both species must oscillate with the same period because they are interdependent. The behavior of the prey is directly dependent on the predators and vice-versa.

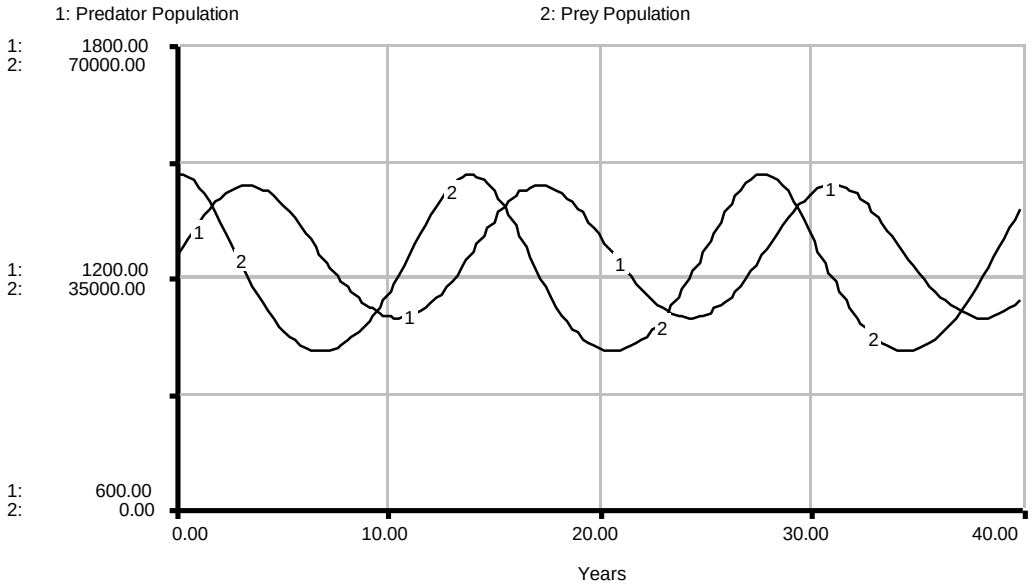


Figure 7: Graph of predator-prey system behavior

This graph shows a response of the predator prey system. The period is about 14 years. Notice how the predator population lags behind the prey population. (Please note that the scales for the two populations are different.)

3. In my model, the system became unstable if I started the prey population above 580,000. Both populations would increase exponentially without bound. Beginning with a large prey population meant that the predators were catching as many prey as possible, but this was inadequate given the tremendous birth rate of the prey. Because the prey birth fraction is larger than the predator birth fraction, the predators will never catch up. In real life this would never happen. Other limiting factors like food and water would restrict the prey population until the predators caught up.

4. I used the following function to kill half of my predators in year 15:

PULSE(Predator_Population/2,15,0)

This resulted in an instant decrease in the predator population followed by a large increase in the prey population. In the next cycle, the predator population greatly overshot its previous maximum and the prey population fell below its previous minimum value. After a few more cycles, both populations returned to their previous oscillation with the same period and amplitude.

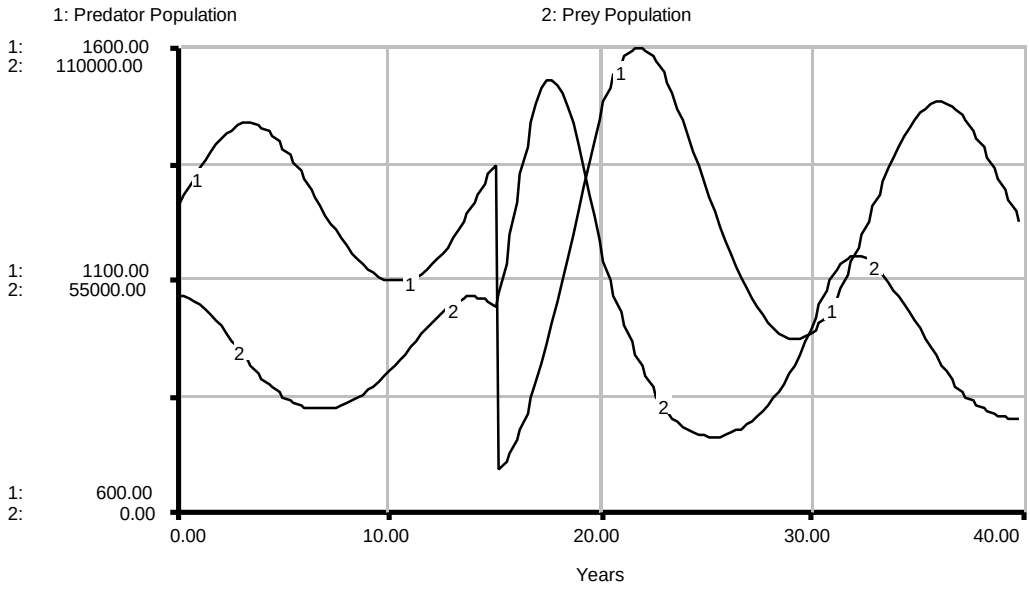


Figure 8: Graph of predator-prey system behavior

In this run, half of the predator population was eliminated in year 15. Just following the shock, the prey population rose quickly. However, the next few cycles had a much greater amplitude than before the shock. Eventually, the system returns to the original oscillation behavior.

5. This model shows that spraying crops with insecticide can have some unwanted consequences. Initially, spraying has the effect decreasing the insect population and increasing crop yield. But, a few years later, the insect population is higher than ever and crop yield is worse than it has ever been. The immediate reaction to this effect might be to spray with an even larger amount of insecticide. Using the model, you can see that this will result in even greater extremes than the first time. The farmer can end up in a vicious cycle of larger and larger sprayings of insecticide and greater and greater fluctuations in his crop yield and the size of the insect population.

Model Equations:

Predator_Population(t) = Predator_Population(t - dt) + (Predator_births - Predator_deaths) * dt
INIT Predator_Population = 1250

DOCUMENT: Initially, there are 1250 predators.

Units: Predators

INFLOWS:
Predator_births = Predator_Population*Predator_birth_fraction
DOCUMENT: A compounding process is used to depict predator births. The births flow is defined as the product of predator population and their birth fraction.

Units: Predators/year

OUTFLOWS:
Predator_deaths = Predator_Population*Predator_death_fraction+PULSE(Predator_Population/2,15,1000)
DOCUMENT: This is a draining process. Some fraction of the Predator population will die each year. The PULSE function is used to jolt the system out of its initial steady-state.

Units: Predators/year

Prey_Population(t) = Prey_Population(t - dt) + (Prey_births - Prey_deaths) * dt
INIT Prey_Population = 50000

DOCUMENT: Initially, there are 50,000 prey in the ecosystem.

Units: Prey

INFLOWS:
Prey_births = Prey_Population*Prey_birth_fraction
DOCUMENT: A compounding process is used to depict prey births. The births flow is defined as the product of Prey population and their birth fraction.

Units: Prey/year

OUTFLOWS:
Prey_deaths = Predator_Population*Prey_kills_per_predator
DOCUMENT: Each predator has a productivity, given by prey kills per predator.

Units: Prey/year

area = 1000
DOCUMENT: The area of the ecosystem is defined as 1000 acres.

Units: acres

Predator_birth_fraction = .25
DOCUMENT: On average, each Predator will generate .25 offspring per year.

Units: 1/years

Prey_birth_fraction = 1.25
DOCUMENT: Each prey in the ecosystem is assumed to produce 1.25 offspring (on average) per year.

Units: 1/years

Prey_density = Prey_Population/area
DOCUMENT: This equation calculates the average density of prey in the ecosystem.

Units: Prey/acre

Predator_death_fraction = GRAPH(Prey_density)
(0.00, 0.5), (10.0, 0.395), (20.0, 0.33), (30.0, 0.27), (40.0, 0.225), (50.0, 0.185), (60.0, 0.147), (70.0, 0.11), (80.0, 0.0875), (90.0, 0.0775), (100, 0.0725)
DOCUMENT: Predator mortality -- the fraction of the predator population that dies each year, is assumed to depend on prey density. With fewer prey, a greater fraction of the predator population will starve.

Units: 1/years

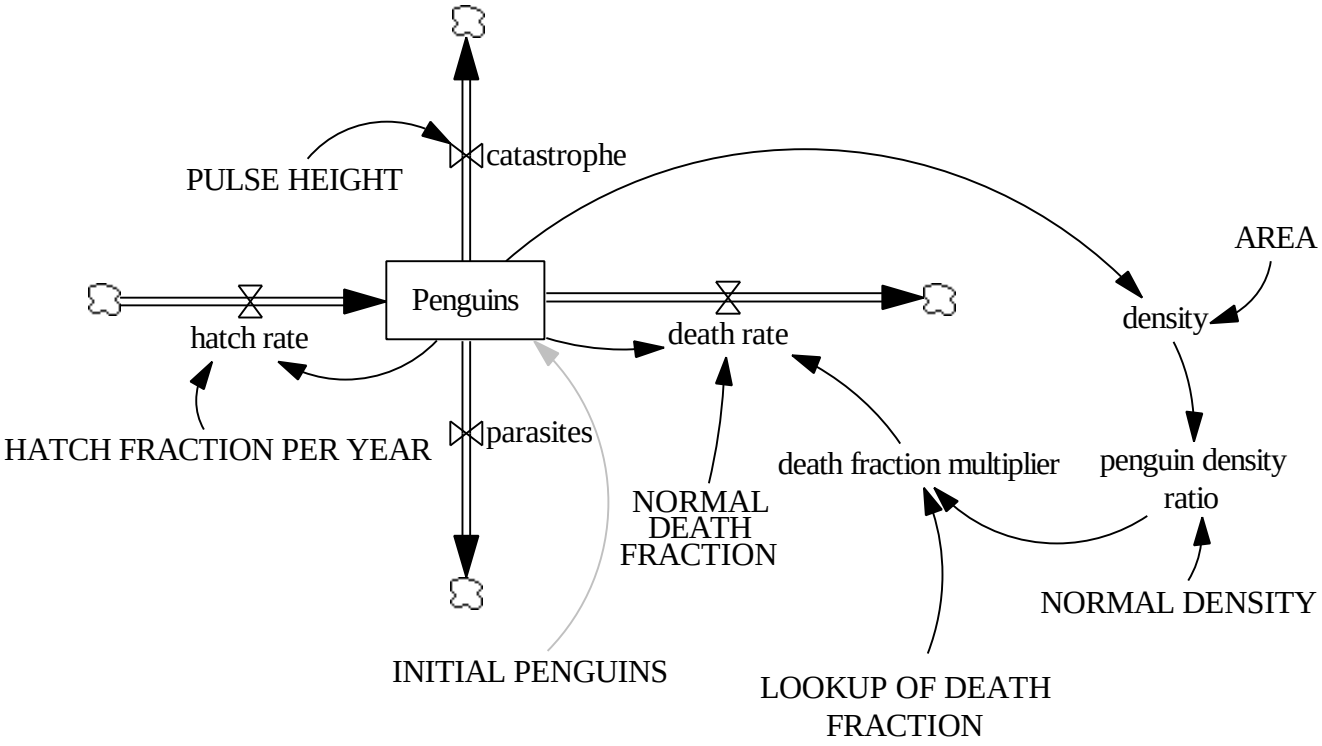
Prey_kills_per_predator = GRAPH(Prey_density)
(0.00, 0.00), (50.0, 50.0), (100, 125), (150, 252), (200, 357), (250, 435), (300, 467), (350, 495), (400, 500), (450, 500), (500, 500)
DOCUMENT: The number of prey killed per predator (per year) increases with the density of prey in the ecosystem. Note that when density is 0 (there are no prey), kills per predator per year must be 0.

Units: Prey/year/Predator

Vensim Examples:
Modeling Exercises
Section 1

Lei Lei and Nathaniel Choge
October 2001

Model 1: An Ecological System



Documentation for Ecological System:

(01) AREA = 10
Units: acres
This is the total land area occupied by the population.

(02) catastrophe = PULSE HEIGHT*PULSE(5,1)
Units: penguins/year
This flow was used to explore the effects of a sudden drop in the population.

(03) death fraction multiplier = LOOKUP OF DEATH FRACTION(penguin density ratio)
Units: dmnl
This is the equation that gives the death fraction is a function of the penguin density ratio.

(04) death rate = Penguins*death fraction multiplier*NORMAL DEATH FRACTION
Units: penguins/year
This is the death rate of the population. It is determined by multiplying the population by a death fraction, times a death rate multiplier which serves to produce the correct dimensions for the flow.

(05) density = Penguins/AREA
Units: penguins/acres
This is the population density. It affects the death fraction of the population.

(06) FINAL TIME = 25
Units: year
The final time for the simulation.

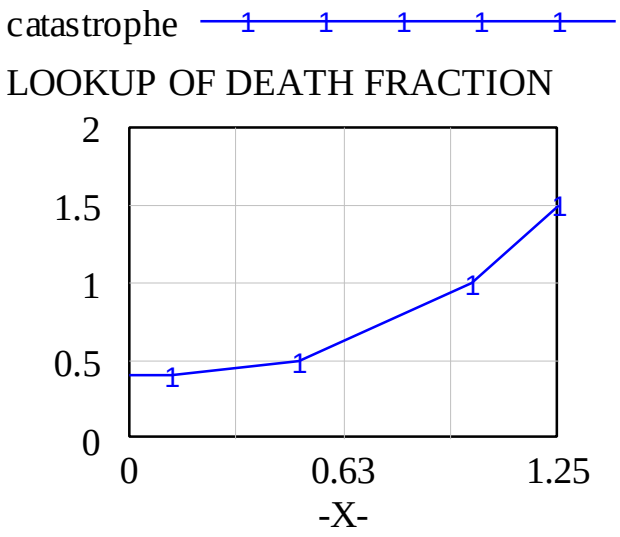
(07) HATCH FRACTION PER YEAR = 0.2
Units: 1/year
This represents the fraction of the population that reproduces each year. If the hatch fraction is 0.2, then the hatch rate will be 20% of the population. To obtain 0.2 we assume that half of the penguins are female and that each female produces an offspring every 2.5 years.

(08) hatch rate = Penguins*HATCH FRACTION PER YEAR
Units: penguins/year
This is the hatch rate fo the population. It is determined by multiplying the population by a hatch fraction.

(09) INITIAL PENGUINS = 800
Units: penguins
The system starts with 800 penguins.

(10) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(11) LOOKUP OF DEATH FRACTION([(0,0)-(1.25,1.5)],(0,0.4),(0.125,0.4),(0.375,0.45),(0.5,0.5),(1,1),(1.25,1.5))
Units: dmnl
The death fraction is dependent on the density of the population. When the penguin density ratio is 1, the density is equal to the normal density and the death fraction is 0.2. The higher the penguin density ratio, the larger the fraction of the population that dies each year.



(12) NORMAL DEATH FRACTION = 0.2
Units: penguins/year
This multiplier produces the correct dimensions for death rate. Since "Penguins" has units of Penguins and "death fraction" is dimensionless we need to divide by year to produce the correct dimensions for "death rate."

(13) NORMAL DENSITY = 80
Units: penguins/acres
This variable is introduced in order to make the modeldimensionless. We assume that the initial population divided by the area represents the normal density of penguins.

(14) parasites = STEP(20 , 5)
Units: penguins/year
This flow is used to represent a constant outflow from the population.

(15) penguin density ratio = density/NORMAL DENSITY
Units: dmnl
This ratio will be used as a dimensionless input to the lookup table.

(16) Penguins= INTEG (+hatch rate-catastrophe-death rate-parasites,
INITIAL PENGUINS)
Units: penguins
This stock represents the number of penguins in the population. It is increased by hatching and decreased by deaths.

(17) PULSE HEIGHT = 400
Units: penguins/year
This is the number of penguins removed after 5 years due to catastrophe.

(18) SAVEPER = TIME STEP
Units: year

The frequency with which output is stored.

(19) TIME STEP = 0.0625
Units: year
The time step for the simulation.

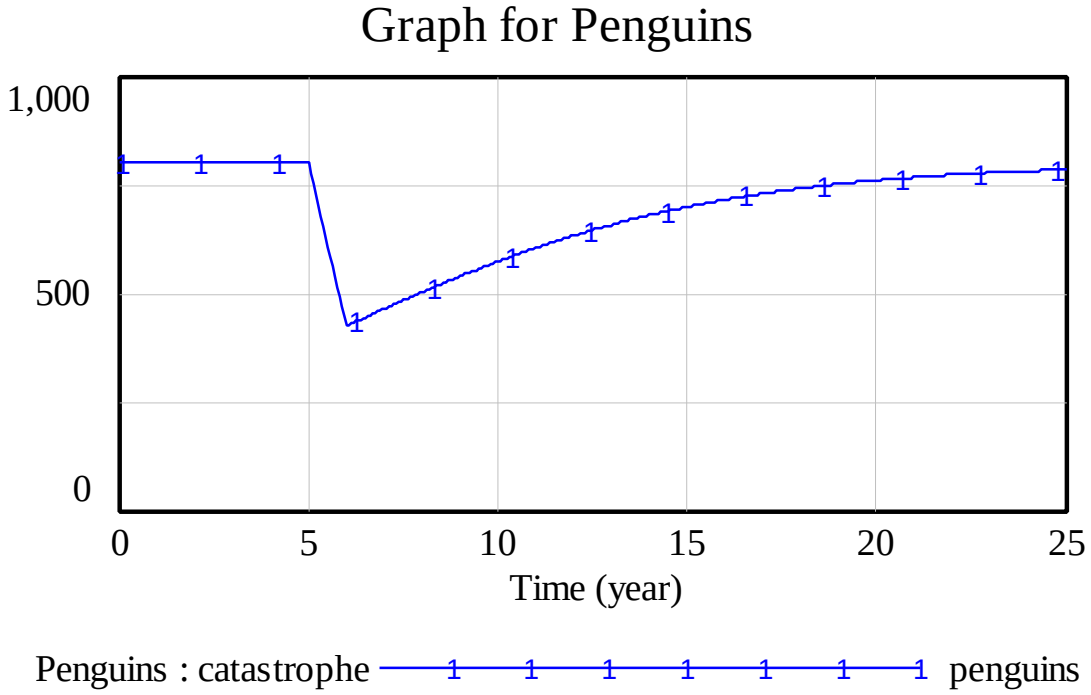
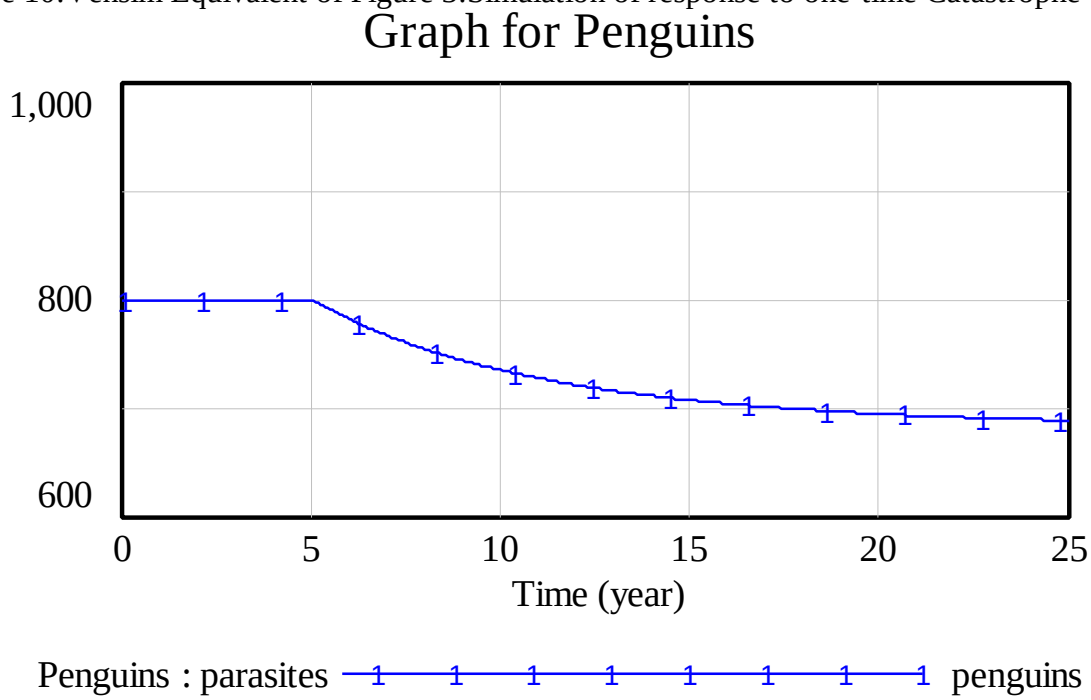


Figure 10:Vensim Equivalent of Figure 3:Simulation of response to one-time Catastrophe



11: Vensim Equivalent of Figure 4: Simulation of response to constant outflow

Model 2: Heating/Cooling System

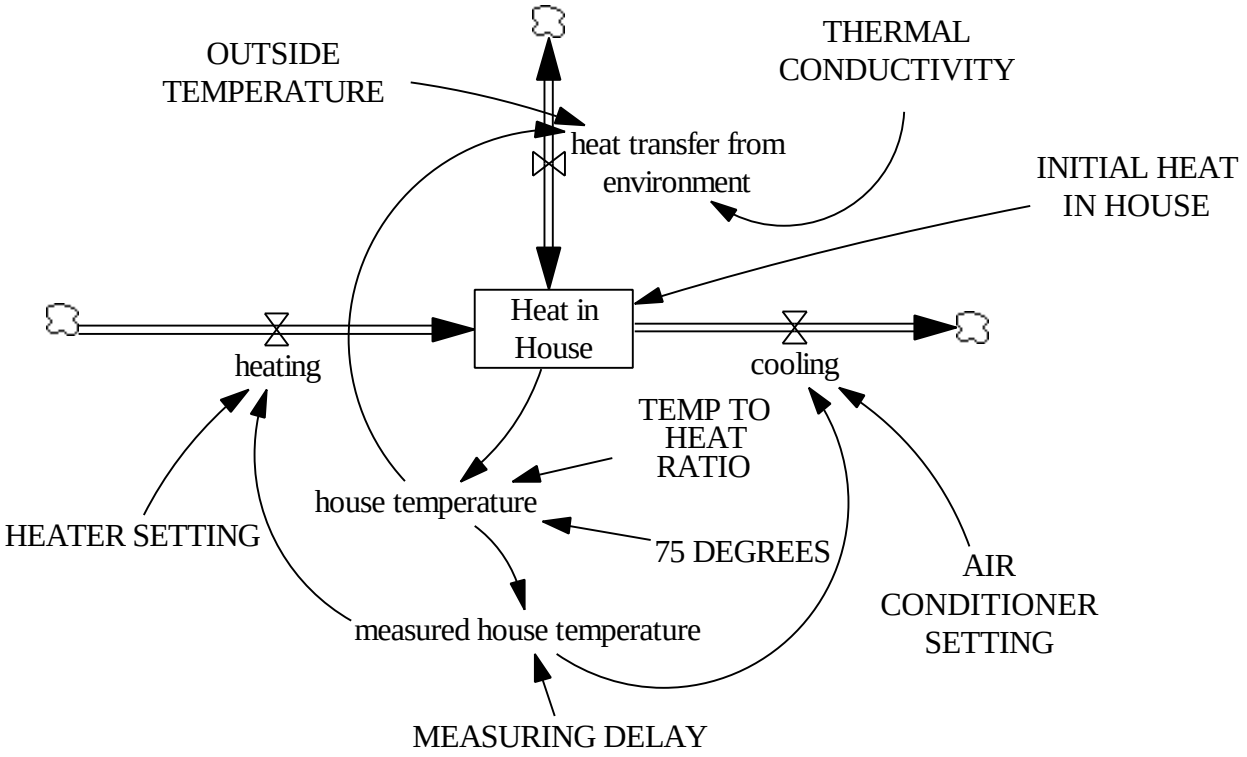


Figure 12: Vensim Equivalent of Figure 5: Heating/Cooling System Model

NOTE: Differences between the Stella and Vensim versions of the Heating/Cooling System Model are described below the model documentation.

Documentation for Heating/Cooling System Model

(01) "75 DEGREES" = 75
Units: degrees
This is the initial temperature of the house. It is the temperature that determines the reference heat in house for our simulation. The reference is the heat that we take as already present in the house as the base for our simulation.

(02) AIR CONDITIONER SETTING = 80
Units: degrees
This is the thermostat setting for the cooling system.

(03) cooling = IF THEN ELSE(measured house temperature>AIR CONDITIONER SETTING, 100000 , 0)
Units: btu/hour

This is the heat removed from the room by the air conditioning system. If the measured house temperature is higher than the air setting, then cooling = 100,000 joules per hour. If the house temperature is lower, then cooling =0

(04) FINAL TIME = 30
Units: hour
The final time for the simulation.

(05) Heat in House= INTEG (+heating-cooling+heat transfer from environment, INITIAL HEAT IN HOUSE)
Units: btu
This is the amount of heat in the house relative to the heat in the house at 75 degrees. It is affected by the heating and air conditioning systems as well as the heat transferred from the environment. The initial amount of heat in the house is set to the heat at 75 degrees by setting "INITIAL HEAT IN HOUSE" to zero.

(06) heat transfer from environment = (OUTSIDE TEMPERATURE-house temperature)*THERMAL CONDUCTIVITY
Units: btu/hour

(07) HEATER SETTING = 70
Units: degrees
This is the thermostat setting for the heater system.

(08) heating = IF THEN ELSE(measured house temperature<HEATER SETTING,100000 , 0)
Units: btu/hour
This is the heat transferred to the house by the heating system. If the measured house temperature is lower than the heater setting, then heating = 100,000 btu per hour. If the house temperature is higher, then heating = 0.

(09) house temperature = (Heat in House/TEMP TO HEAT RATIO)+"75 DEGREES"
Units: degrees
This is the temperature in the house. Changes in the temperature are proportional to changes in the house's energy. The change in temperature is calculated by multiplying the energy by the temp to heat ratio. The actual temperature is calculated by adding 75 degrees (the initial reference temperature).

(10) INITIAL HEAT IN HOUSE = 0
Units: btu

(11) INITIAL TIME = 0
Units: hour
The initial time for the simulation.

(12) measured house temperature = SMOOTHI(house temperature, MEASURING DELAY, 75)
Units: degrees
This is the room temperature as perceived by the thermostat. It is calculated by smoothing the actual room temperature with a delay.

(13) MEASURING DELAY = 0.2
Units: hour
This is the delay invloved in measuring the room temperature.

(14) OUTSIDE TEMPERATURE = 90
Units: degrees
This is the temperature of the environment external to the house. The house temperature would eventually reach this temperature without a heating/cooling system.

(15) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.

(16) TEMP TO HEAT RATIO = 10000
Units: btu/degrees
This is the ratio between temperature and energy in the house. This number depends on the size and insulative properties of the house. It tells how many btus of energy are required to heat the house by 1 degree.

(17) THERMAL CONDUCTIVITY =1000
Units: btu/hour/degrees
This factor represents how easily energy is transferred from the outside environment to the house.

(18) TIME STEP = 0.0625
Units: hour
The time step for the simulation.

Differences between the Vensim and Stella Heting/ Cooling System Models:
5/8/2001

1. The units for heat have been changed from joules to BTU to use the common terms of the air conditioning field.
2. All units for temperature have been set to degrees fahrenheit to get rid of the dimensional inconsistencies in the original paper and asumption was made that heat in the house was equal to zero when the temperature was 75 degrees Fahrenheit which is phsycally incorrect. In the Vensim version we take the heat in the house at 75 degrees Fahrenheit as the reference heat for our simulation. i.e. Heat in House is the amount of heat in the house relative to the heat in the house at 75 degrees and not the absolute amount of heat.

Please note outside temperature is set to 50 degrees for this run only.

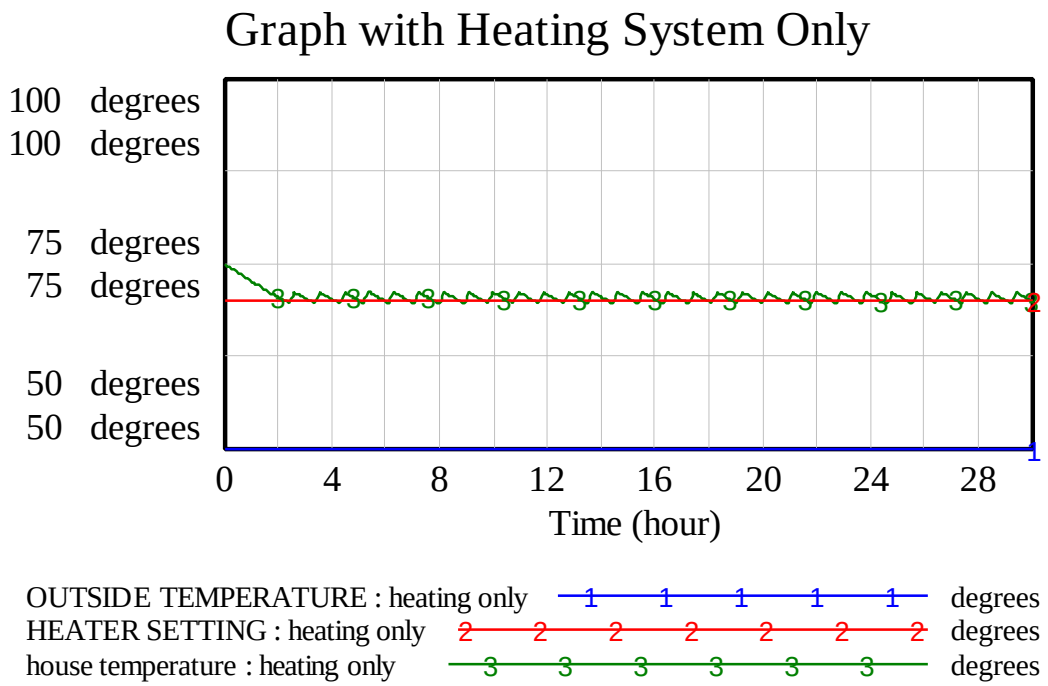


Figure 13: Vensim Equivalent of Figure 6: Simulation with heating system only

Please note outside temperature is set to 90 degrees for this run only.

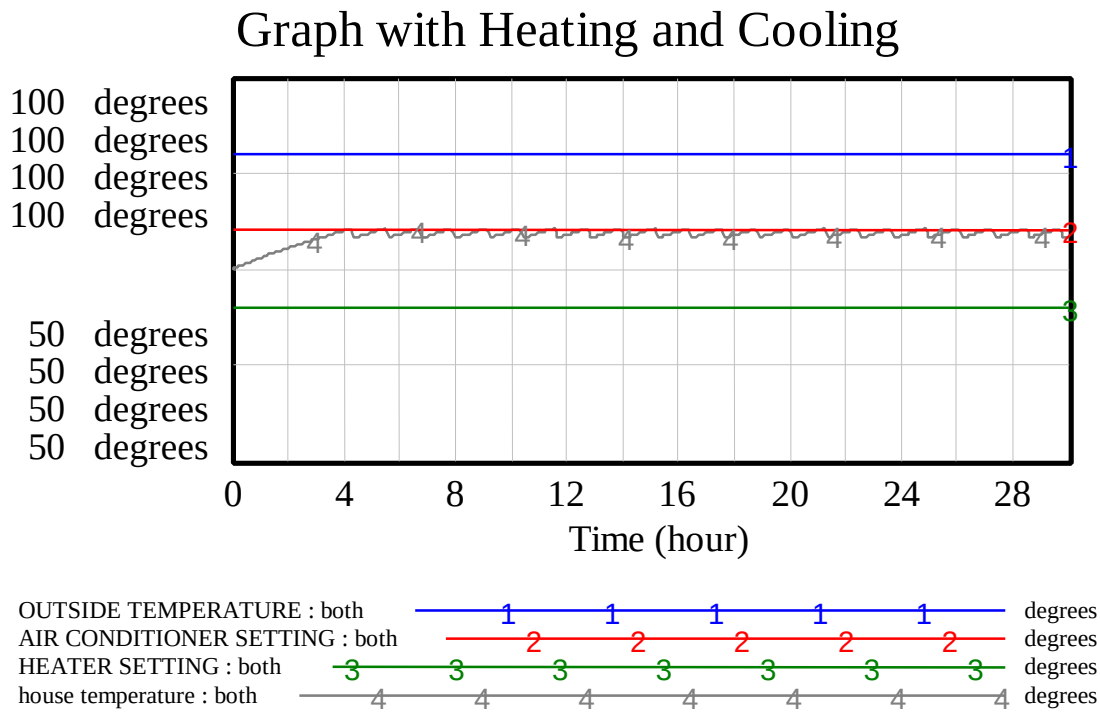
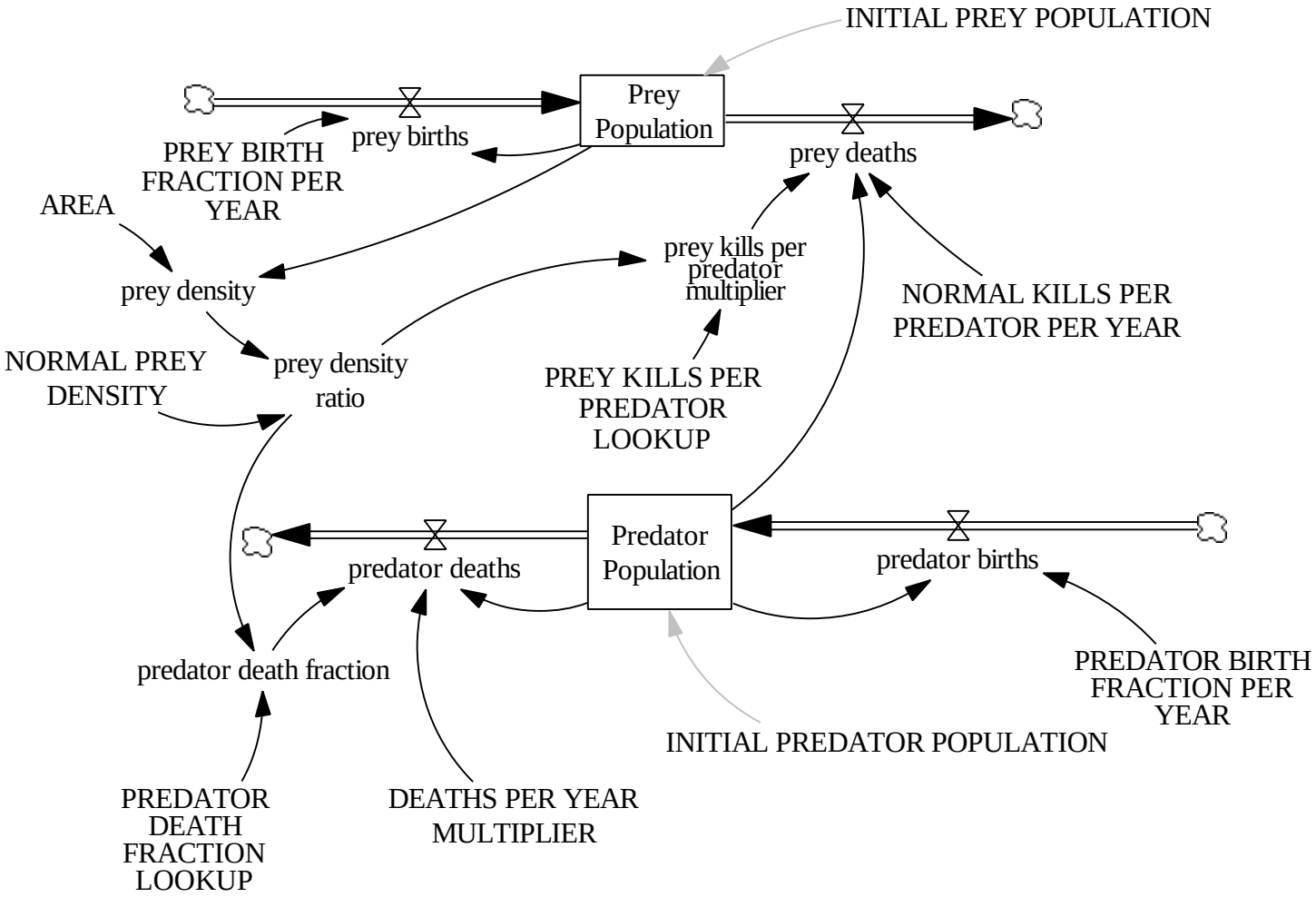


Figure 14: Vensim Equivalent of Figure 7: Simulation with Heating and Cooling Systems.

Model 3: A Predator/Prey System



Documentation for Predator/Prey Model:

(01) AREA = 1000
Units: acres
The area of the ecosystem is defined as 1000 acres.

(02) DEATHS PER YEAR MULTIPLIER = 1
Units: 1/year
This multiplier produces the correct dimensions for predator deaths. Since Predator Polulation" has units of Predators and "predator death fraction" is dimensionless we need to divide by year to ptoduce the correct dimensions for "predator deaths."

(03) FINAL TIME = 40
Units: year
The final time for the simulation.

(04) INITIAL PREDATOR POPULATION = 1250
Units: predators
Initially there are 1250 prey in the ecosystem.

(05) INITIAL PREY POPULATION = 50000
Units: prey
Initially, there are 50,000 prey in the ecosystem.

(06) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(07) NORMAL KILLS PER PREDATOR PER YEAR = 50
Units: prey/(predator*year)
At the normal prey density, each predator is able to kill 50 prey in the year. This represents the level of kills at which the predator death fraction is not affected by scarcity of prey.

(08) NORMAL PREY DENSITY = 50
Units: prey/acres
The normal density of prey per acre.

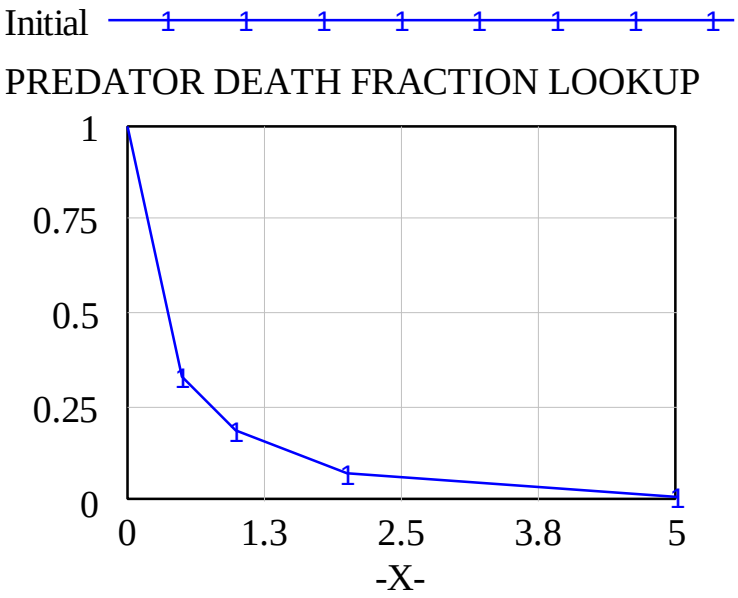
(09) PREDATOR BIRTH FRACTION PER YEAR = 0.25
Units: 1/year
On average, each Predator will generate .25 offspring per year. This is obtained by assuming that half the predators are female and that each female has a cub every two years.

(10) predator births = Predator Population*PREDATOR BIRTH FRACTION PER YEAR
Units: predators/year
A compounding process is used to depict predator births. The births flow is defined as the product of predator population and their birth fraction.

(11) predator death fraction = PREDATOR DEATH FRACTION LOOKUP (prey density ratio)
Units: dmnl
This equation relates predator death fraction as a function of relative prey density.

(12) PREDATOR DEATH FRACTION LOOKUP([(0,0)-(5,1)],(0,1),(0.5,0.33),(1,0.185),(2,0.07),(5,0.01))

Units: dmnl
Predator mortality--the fraction of the predator population that dies each year-- is assumed to depend on prey density. With fewer prey, a greater fraction of the predator population will starve. At normal "prey density ratio" the death fraction is equal to 0.185, which we take as the normal death fraction of predators per year. The .0185 is obtained by giving each predator a life expectancy of about five and a half years. At zero prey density, the death fraction is 1, since the predator cannot survive with no food. When prey density is five times the normal, the death fraction of predators is only 0.01



(13) predator deaths = Predator Population*predator death fraction*DEATHS PER YEAR MULTIPLIER+Predator Population/2*PULSE(15 , 1)
Units: predators/year
This is a draining process. Some fraction of the Predator population will die each year. A death faction of .5 would mean that half of the predator death population dies each year. The PULSE function is added in the second model run to jolt the system out of its initial steady-state and show the effect of a sudden decrease of predators to the ecosystem.

(14) Predator Population= INTEG (predator births-predator deaths, INITIAL PREDATOR POPULATION)
Units: predators
Initially, there are 1,250 predators.

(15) PREY BIRTH FRACTION PER YEAR =1.25
Units: 1/year
Each prey in the ecosystem is assumed to produce 1.25 offspring (on average) per year. This is obtained by assuming half the prey are female and that the females on average have 2.5 offspring per year.

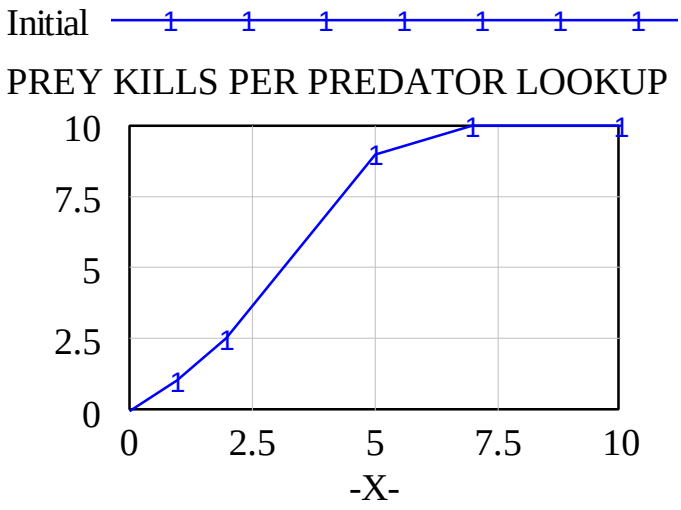
(16) prey births = Prey Population*PREY BIRTH FRACTION PER YEAR
Units: prey/year
A compounding process is used to depict prey births. The births flow is defined as the product of Prey Population and their birth fraction.

(17) prey deaths = Predator Population*NORMAL KILLS PER PREDATOR PER YEAR*prey kills per predator multiplier
Units: prey/year
Each predator kills a number of prey per year, which depends on the "normal kills per predator" times a multiplier affected by the relative scarcity or abundance of prey.

(18) prey density = Prey Population/AREA
Units: prey/acres
This equation calculates the average density of prey in the ecosystem.

(19) prey density ratio = prey density/NORMAL PREY DENSITY
Units: dmnl
Ratio of prey density to NORMAL PREY DENSITY of the area. This ratio is used to provide a dimensionless input to the lookup table function for "prey kills per predator lookup."

(20) PREY KILLS PER PREDATOR LOOKUP([(0,0)-10,10)],(0,0),(1,1),(2,2.5),(5,9),(7,10),(10,10))
Units: dmnl
The number of prey killed per predator per year increases with the density of prey in the ecosystem. Note that when density is 0 (there are no prey), kills per predator must be 0. The number of prey killed per predator increases as the "prey density ratio" increases, however when the ratio reaches values above 7, the prey kills per predator multiplier stays at 10 because we assume the predators cannot kill more than 500 prey each per year.



(21) prey kills per predator multiplier = PREY KILLS PER PREDATOR LOOKUP (prey density ratio)
Units: dmnl
This equation says that prey kills per predator is a funciton of the prey density.

(22) Prey Population= INTEG (prey births-prey deaths,INITIAL PREY POPULATION)
Units: prey
Initially, there are 50,000 prey in the ecosystem. Predator population incereases with births and deacreases with deaths. The formulation assumes no other outflows or inflows.

(23) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(24) TIME STEP = 0.0625
Units: year
The time step for the simulation.

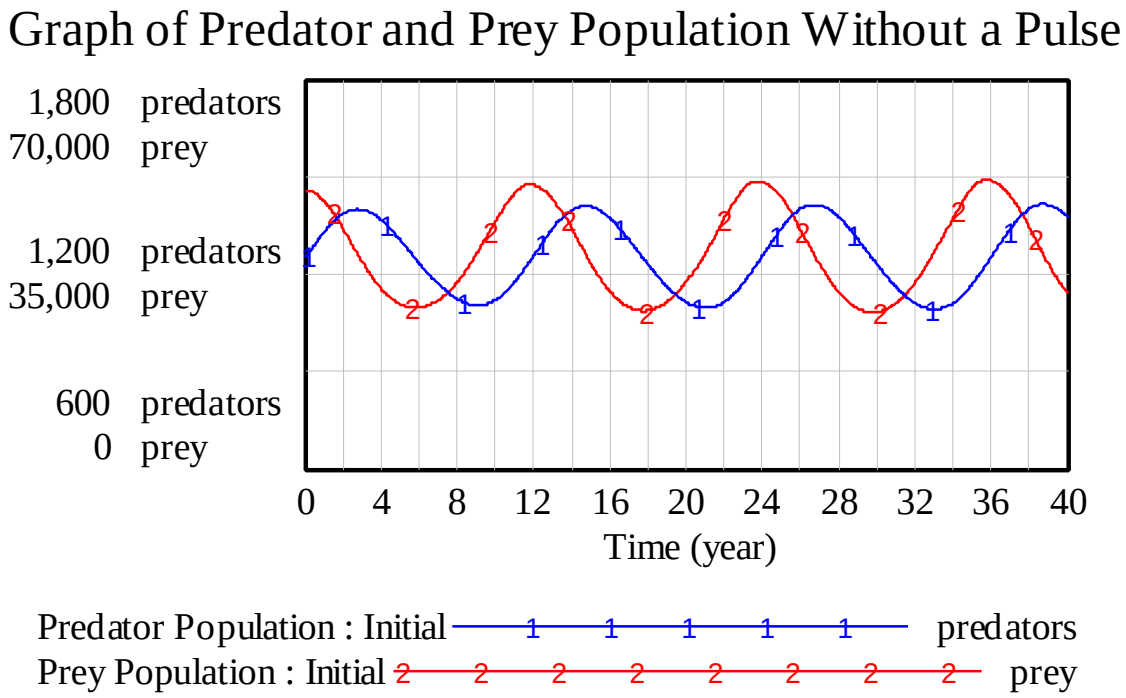
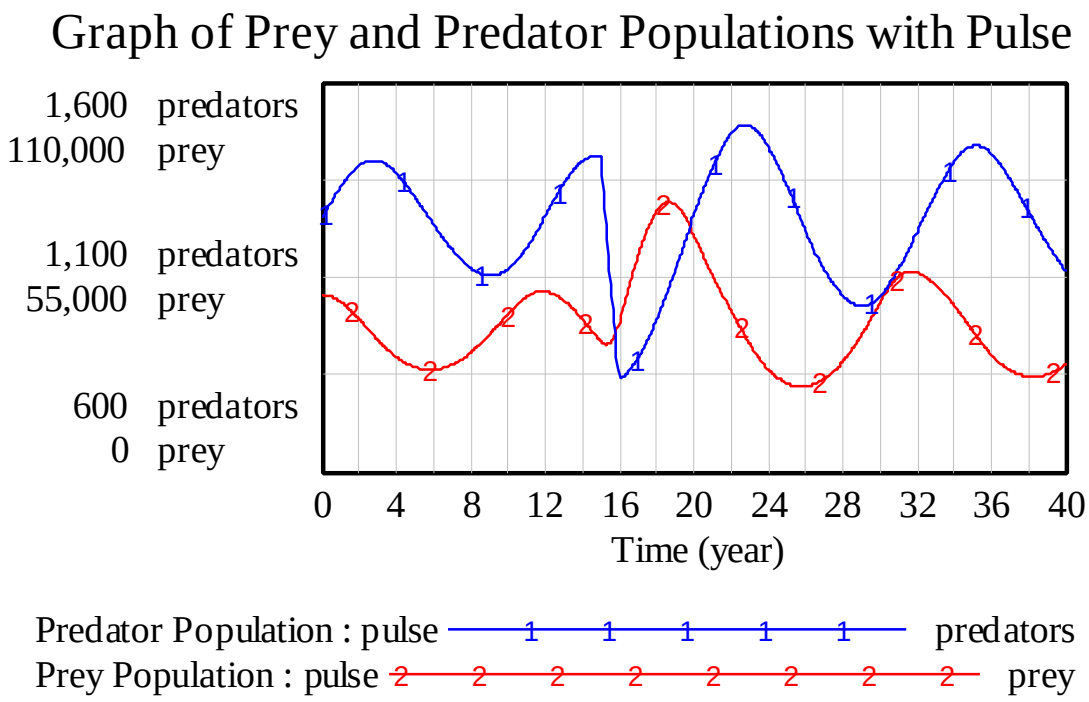


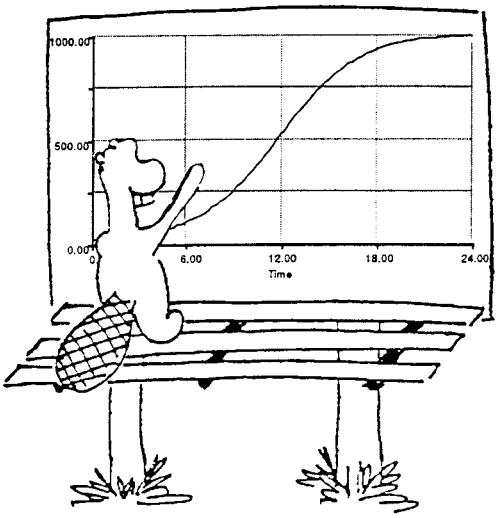
Figure 16: Vensim Equivalent of Figure 7: Simulation without a Pulse Function

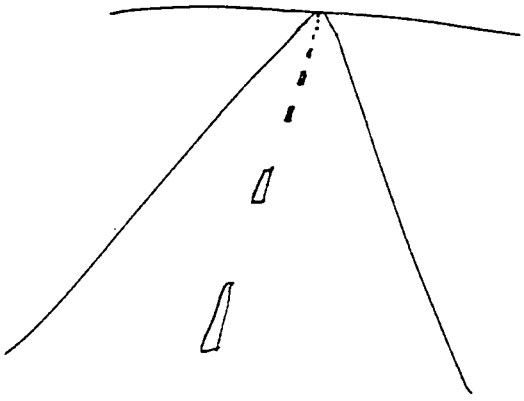


Vensim Equivalent of Figure 8: Simulation with a Pulse

Road Maps 7

A Guide to Learning System Dynamics





System Dynamics in Education Project

Road Maps 7

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

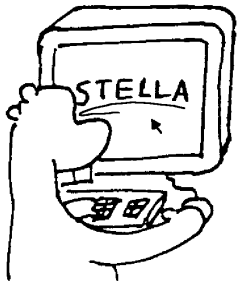
January 13, 1997
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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Seven!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. This chapter is the seventh in the Road Maps series. Road Maps One through Three gives a broad introduction to the field of system dynamics, focusing on understanding the structure and behavior of systems through positive and negative feedback loops. Road Maps Four introduces generic (or transferable) structures, and gives a taste of policy analysis using the Fish Banks model. Road Maps Five introduces the use of delays in computer models, explains the transferable structures that cause S-shaped growth, looks at the spread of an epidemic, and starts exploring model validity. Exercises in Road Maps Six model economic supply and demand, explore the generic structure producing oscillations and S-shaped growth, and for the first time let you create your own system dynamics models.

Road Maps Seven presents some unexpected behaviors that can occur in higher-order positive feedback loops and includes the first in a series of papers that address common mistakes and misunderstandings in modeling. To increase modeling proficiency, you will build more models through *Modeling Exercises: Section 2*. We also return to the topic of graphical integration by showing the process of "reverse" graphical integration. The chapter concludes with a paper by Jay Forrester on the relationship of systems thinking and soft operations research to system dynamics.

Topics Covered in Road Maps Seven

Structure and Behavior in System Dynamics

- *Unexpected Behaviors in Higher-Order Positive Feedback Loops*
  (D-4455-1) by Aaron Ashford

Improving Modeling Skills

- *Mistakes and Misunderstandings: Examining Dimensional Inconsistency*
  (D-4452-1) by Michael Shayne Gary

- *Modeling Exercises: Section 2* (D-4451-1)
by Joseph Whelan

Graphical Integration Exercises

- *Graphical Integration Exercises Part 4: Reverse Graphical Integration* (D-4603) by Laughton Stanley

The Methodology of System Dynamics

- *System dynamics, systems thinking, and soft OR* (D-4405-1)
by Jay W. Forrester

Things You'll Need for Road Maps Seven

Modeling Software

In order to complete Road Maps Seven and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on

Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

How to Use Road Maps Seven

Road Maps Seven explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Many of the chapters conclude with a prominent paper from the literature in the system dynamics field.

We present the fundamental concepts of system dynamics as *System Principles* in Road Maps. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times.

Each time a principle is revised in Road Maps, you will build upon your previous understanding of the principle by learning something new about it. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

As part of the spiral learning approach that we use in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains a number of series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic in system dynamics or the developing of a particular skill. The series start out with a simple paper, and progress to further develop the idea in subsequent chapters.

Now let's get started!

Structure and Behavior in System Dynamics

Positive feedback loops are usually associated with their characteristic behavior of exponential growth. However, when a positive feedback loop is of a higher order (contains more than 1 level), it can exhibit a variety of other behaviors.

- Unexpected Behaviors in Higher-Order Positive Feedback Loops[1]

by Aaron Ashford

This paper discusses possible types of behavior in first to fifth-order positive feedback loops. Only the first-order loop always shows exponential growth (unless the initial value of the stock is equal to zero). Depending on the order of the loop as well as on the initial values of all the levels, a positive feedback loop can exhibit not only exponential growth, but also asymptotic growth, and damped, sustained, or expanding oscillations. The paper offers several independent explorations and provides solutions to them.[2]

Please read *Unexpected Behaviors in Higher-Order Positive Feedback Loops* now.

---

[1] Aaron Ashford, 1995. *Unexpected Behaviors in Higher-Order Positive Feedback Loops* (D-4455-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 8, 21 pp.

[2] Please note that if you want to simulate the positive feedback loops with STELLA, you should make sure that all the flows in your model are biflows and that all the stocks can become negative.

After reading *Unexpected Behaviors...*

A positive feedback loop is able to show a surprising variety of behaviors. The initial values of the levels determine what behavior the loop will exhibit. However, the behavior modes created by such specific initial values are very unstable—even a slight change in one of the initial values will cause the loop to destabilize to exponential growth. Exponential growth is the most common behavior of positive feedback loops. We now review this idea as a new system principle.



System Principle #17:

Higher-order, positive-feedback loops usually show exponential behavior.

Positive feedback loops of nth order usually exhibit simple exponential growth (ignoring possible initial transients).

Look back to Figure 4 on page 8 of *Unexpected Behaviors in Higher-Order Positive Feedback Loops*. The figure shows the most common behavior of positive feedback loops. Exponential growth is the only stable behavior of positive loops. All other behaviors are unstable—a very slight perturbation of the initial values of the levels will cause the behavior to destabilize to exponential growth. Figures 6, 9, and 12 show examples of such destabilization.

In most real-world systems, the initial values are such that the positive feedback loops will generate exponential growth.

Improving Modeling Skills

While modeling, you may find yourself consistently making the same mistakes, or misunderstanding certain concepts. However, you should not be discouraged. Making a mistake and then examining it can be a very good way to learn system principles and thus good modeling practices.

***- Mistakes and Misunderstandings: Examining Dimensional Inconsistency*[3]**

by Michael Shayne Gary

This is the first in a series of papers to be included in Road Maps that will examine common mistakes and misunderstandings in system dynamics modeling. Using a simple thermostat model, this first paper analyzes the common mistake of dimensional inconsistency in modeling. It looks at where and why a dimensional inconsistency mistake could be made, as well as ways to avoid mistakes in the future.

Please read *Mistakes and Misunderstandings* now.

After reading *Mistakes and Misunderstandings...*

The paper addressed common mistakes such as designating stocks and flows with inconsistent dimensions or creating stocks that cannot accumulate. You should keep a note of common mistakes you make and ask yourself why it happened. Did you not understand the concept? Or was it a mistake in applying the concept to the structure of the model? There will be more mistakes and misunderstandings in future Road Maps.

Dimensional equality is one of the most basic and powerful ways to check against incorrect formulations of models. To underscore the importance of checking for dimensional consistency, we now review dimensional equality as a system principle that we introduced in Road Maps Four.

---

[3] Michael Shayne Gary, 1992. *Mistakes and Misunderstandings: Examining Dimensional Inconsistency* (D-4452-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, January 1992, revised September 1995, 6 pp.



System Principle #9:

Every equation must have dimensional equality.

In any equation, every term must be measured in the same dimensions. "One cannot add apples and oranges." This is true within converters, and it is true within level and rate equations—as is seen *in Mistakes and Misunderstandings: Examining Dimensional Inconsistency*.

House_Temperature(t+dt) = House_Temperature(t) + Heat_Production(t)*dt.

Dimensionally this equation becomes:

[Degrees Celsius] = [Degrees Celsius] + [BTUs/min]*[min]

= [Degrees Celsius] + [BTUs],

which is clearly "apples and oranges." Dimensional inequality between terms indicates a faulty equation formulation.

We now introduce a new system principle about conversion coefficients.



System Principle #18:

Conversion coefficients are identifiable within real systems.

Conversion coefficients should always have a clear, real meaning. They are not inserted just to balance equations.

The model shown in Figure 2 on page 6 of *Mistakes and Misunderstandings: Examining Dimensional Inconsistency* contains a "Conversion Factor." The conversion factor, or coefficient, specifies how much heat must be generated by the furnace to increase the temperature in the house by one degree. This way, it transforms the units of "Heat," calories, into the units of "Temperature," degrees Celsius or degrees Fahrenheit, and provides dimensional equality. The conversion factor thus has a real physical meaning and balances the equations dimensionally.

Taking a given system and modeling it from scratch is not an easy thing to do, and all of us are bound to make mistakes. Now that you have some good modeling experience under your belt, it might be helpful to go back and analyze your

mistakes. See if you consistently make the same types of mistakes and ask yourself how you can fix them. Mistakes can be great things if we are willing to learn from them!

The next exercises test your model-building and mental simulation skills. This is your chance to apply the skills and knowledge you have acquired so far in Road Maps to build your own model from scratch.

- ***Modeling Exercises: Section 2***[4]

by Joseph Whelan

This is the second in a continuing series of papers in Road Maps to provide you with exercises to develop your modeling skills. The first exercise in this paper presents an urban growth and stagnation scenario. Exploration questions lead you through the modeling process, but the model will be largely your creation. The second exercise presents a simple model for practicing mental simulation. You should think about and attempt each exercise before looking at the answers.

Please read and do the exercises in *Modeling Exercises: Section 2* now.

After reading and doing the exercises in *Modeling Exercises: Section 2*...

Using the modeling process outlined by the author as a guideline for modeling often helps to avoid confusion and mistakes. Also, mentally simulating a model before running it on the computer will help you to catch your mistakes. More importantly, mental simulation will help you to better understand the relationship between the structure and behavior of systems.

Modeling Exercises: Section 2 also explained the use of multipliers and time constants in system dynamics models. These concepts are very important and will be useful in your future modeling.

Please read the following system principles to review some of the important ideas introduced in the paper you just read.

---

[4] Joseph Whelan, 1994. *Modeling Exercises: Section 2* (D-4451-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 1994, 35 pp. Revised by Lucia Breierova, January 1997.



System Principle #11:

Levels completely describe the system condition.

There must be a level for each quantity needed to describe the condition of the system, and the value of each level must be specified at the start of a simulation.

The solution to the Urban Dynamics Exercise in *Modeling Exercises: Section 2* contains two levels**, Business Structures** and **Population**. Both levels are necessary to describe the system condition.

Because the system condition is computed at every step of the simulation and depends on the previous values, it must be known at the start of simulation. Thus the initial values of both Business Structures and Population must be determined.

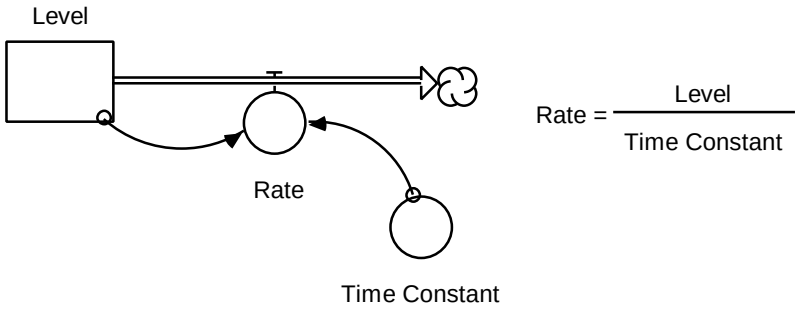


System Principle #19:

Time constant of a first-order loop relates a level to a rate.

Starting on page 26 of *Modeling Exercises: Section 2*, there is a thorough discussion of the effects of time constants on first-order loops. Here is a brief summary:

The exponential time constant of a first-order loop is the reciprocal of the multiplier that defines the rate in terms of the level. It relates a level to the rate that affects it.



The Rate is equal to the Level divided by the Time Constant, or to the Level multiplied by the reciprocal of the Time Constant (the fraction 1/Time Constant).

Graphical Integration Exercises

Previous chapters of Road Maps introduced the Graphical Integration Exercises series. So far, the papers in the series have focused on the process of integration—how to determine the value of the stock if you know the behavior of the flow. However, the reverse process is often useful in dealing with real world systems. Most information usually comes in terms of stocks, but the behavior of their flows can also be important. Therefore, the following paper explains the method of reverse graphical information—estimating the behavior of the stock's net flow from the graph of the stock.

- Graphical Integration Exercises Part 4: Reverse Graphical Integration[5] by Laughton Stanley

This paper covers three methods for drawing the graph of a net flow based on the graph of the stock. It explains how to use intervals to estimate the net flow, and how to quantify the net flow behavior by drawing tangent lines on the stock graph. The exercises included at the end of the paper give you some experience with the technique of reverse graphical integration. Solutions to the exercises are also provided.

Please read and do *Graphical Integration...* now.

After reading *Graphical Integration...*

The paper examined several ways for determining the behavior of a stock's net flow from the graph of the stock. It also clarified the general relationship between the behavior of a stock and its net flow: if the stock is increasing, the net flow is positive; if the stock is decreasing, the net flow is negative. We hope that you can now use the knowledge and techniques you have acquired so far in the graphical integration series to estimate the value of the stock if you know the behavior of the flow, or to do the reverse process. Such insights are often helpful in understanding the dynamic behavior of the system.

---

[5] Laughton Stanley, 1996. *Graphical Integration Exercises Part 4: Reverse Graphical Integration* (D-4603), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 20, 24 pp.

The following system principle reviews an important concept introduced in the paper you just read.



System Principle #20:

Rates are not instantaneously measurable.

No rate of flow can be measured instantaneously. A rate is a change over time. Without an observation over a time interval, a rate cannot be measured.

Look back to page 6 of *Graphical Integration Exercises Part 4: Reverse Graphical Integration*. The Introduction describes several examples in which the rate is determined from the accumulation of the level over a period of time. The rate is determined as the change in the value of the level over time. There is no way to measure the instantaneous value of a rate.

The Methodology of System Dynamics

System dynamics, systems thinking, and soft operations research (soft OR) are all methods for studying complex systems. The relation and differences between them can often be unclear to someone who has not had much experience in the field. In the following article that appeared in the *System Dynamics Review*, Prof. Forrester offers his point of view.

***- System dynamics, systems thinking, and soft OR*[6]**

by Jay W. Forrester

The following article describes the relation of systems thinking and soft operations research to system dynamics. Unlike system dynamics that uses explicit models and simulations of dynamic behavior, systems thinking and soft OR lack such a rigorous foundation. However, they can still contribute useful insights from the real-world system and help in the conceptualization phase of building a system dynamics model.

[6] Jay W. Forrester, 1994. "System dynamics, systems thinking, and soft OR" (D-4405-1), *System Dynamics Review* Vol. 10, nos. 2-3 (Summer-Fall 1994), pp. 245-256.

Please read *System dynamics, systems thinking, and soft OR* now.

After reading *System dynamics, systems thinking, and soft OR*...

Professor Forrester outlined six steps through which system dynamics moves from a problem in a system to an improvement. He suggested that in some of them soft OR and systems thinking are closely related to system dynamics. His critical opinion of systems thinking and soft OR comes from the fact that they are not based on a quantitative structure such as the stock and flow diagrams in system dynamics.

Finishing off Road Maps Seven

Road Maps Seven presented some surprising behaviors that may occur in higher-order positive feedback loops. We introduced a new series of papers in Road Maps, Mistakes and Misunderstandings. Its first paper looked at dimensional inconsistency. The second paper in the Modeling Exercises series gave you a chance to create a model on urban growth and stagnation as well as to improve your mental simulation skills. We also explained the method of "reverse" graphical integration, or graphical differentiation. The final reading in Road Maps Seven discussed how system dynamics relates to systems thinking and soft OR.

In Road Maps Seven, you learned about four more system principles:
a) that higher-order positive feedback loops usually show exponential growth;
b) that conversion coefficients are identifiable within real world systems;
c) that the time constant of a first-order loop relates the level to the rate; and
d) that rates are not instantaneously measurable.

Key Terms and Concepts:

closed boundary
conversion coefficient
dimensional inconsistency
higher-order positive feedback loops
multiplier
reverse graphical integration
soft operations research
time constant



Unexpected Behaviors in Higher-Order Positive Feedback Loops

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Aaron C. Ashford
May 8, 1995
Vensim Examples added October 2001

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ABSTRACT

Positive feedback loops typically show the behavior of exponential growth. However, simple unbounded growth is not the only behavior a single loop is able to exhibit. Other possible behaviors include: exponential decay, damped oscillation, sustained oscillation, and expanding oscillation. Each of these additional behavior modes is unstable, and each will turn to exponential growth if perturbed out of the stable mode. This paper demonstrates and explores these unexpected behaviors in simple positive feedback loops. The loops used for experimentation are all elementary, with direct positive influences and no minor loops. The loops vary from first to fourth order.

1. INTRODUCTION

We naturally expect positive feedback to grow exponentially. It is the behavior we experience daily. Our savings accounts gather interest, as do our debts from loans, and typically at an alarming rate. The world population is exploding about us, and the AIDS epidemic quickly spreads within it. The level of technology doubles every few years. Unchecked nuclear explosions power the sun, which gives us the energy we live by. In all of these examples, the rule is simple: the more you've got, the faster you get it. This behavior is a part of any system whose rate of advancement is directly dependent on the level of its current state—what is commonly called positive feedback. Thus we expect loop structures that have positive influences between its connections to quickly grow without bound—exponentially.

However, simple unbounded growth is not the only behavior a single loop is able to exhibit. Other possible behaviors include: exponential decay, damped oscillation, sustained oscillation, and expanding oscillation. These are behaviors that we expect to see in negative feedback loops; they are signs of stability. As we shall see, in positive feedback loops each of these behavior modes is unstable, and each will turn to exponential growth if perturbed out of the uncharacteristic mode.

To help gain a better understanding of this special kind of behavior, we will experiment with simple loops only. Each level in a loop will have a direct positive influence on the rate of the next level. There will be no other inputs to the rates. In addition, our investigation will start with a second-order loop, and we will increase complexity one level at a time. To explore the behavior of each loop, we will change the initial conditions of the level variables, but we will not alter the model structure. Although changing the structure leads to a wealth of interesting investigations, there will be no changes to the rate equations. I recommend that the reader, while proceeding through the paper, build each model independently and experiment with it freely.

2. UNEXPECTED BEHAVIORS

2.1 First-Order Loop

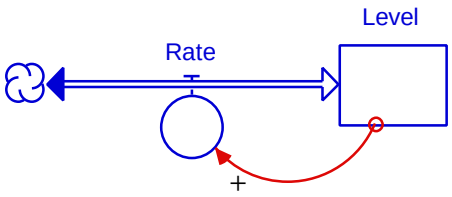


Figure 1: A first-order positive feedback loop

We are familiar with the simple first-order positive feedback loop (Figure 1). In the most simple case Rate = Level. In that case, if Level is positive, and thus Rate is positive, then the rate increases the level, and the level in turn increases the rate, further increasing the level, and so on. This process yields exponential growth, the result of standard positive feedback. Likewise, if the level and the rate start off negative, then the rate decreases the level, making the level more negative, making the rate more negative, which further decreases the level, and so on—again we have positive feedback, now in the direction towards very negative numbers.

In both cases there is the reinforcing pattern. In a specific model the loop may have many additional factors affecting it, depending on the system being modeled (birth fraction, interest rate, and learning ability are a few possibilities). However, it will always exhibit the behavior of exponential growth. The same, however, cannot be said of higher-order positive feedback loops.

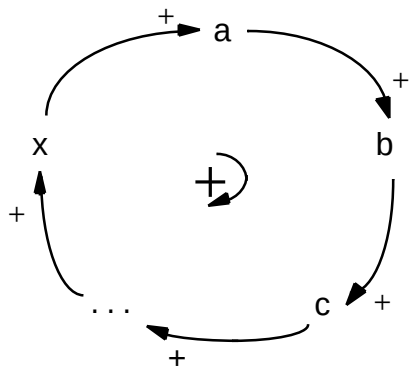


Figure 2: Positive Feedback

In higher-order positive feedback loops, exponential growth is not always exhibited, and unexpected behavior modes do appear. This may seem surprising, especially after looking at a causal loop representation of a positive feedback loop (Figure 2). If a increases b and b increases c and ... and x increases a, then shouldn't everything increase (or decrease to very negative numbers) together? Shouldn't everything grow together, faster and faster, exponentially? Usually, yes, but not always.

For example, look again at the first-order model above. If the level is initially 0, then it will stay forever 0. This is because the rate is also 0, as specified in the rate equation. Thus no change takes place. Even though the model is in equilibrium, it is in an unstable balance. If the value of the level is changed the slightest bit in either direction, then the behavior will slip into exponential growth. This is what is meant by an unstable equilibrium. The higher-order models we will see have similarly unstable behavior modes.

2.2 Second-Order Loop

We now examine the simplest second-order positive feedback loop possible (Figure 3). The rate of level a, da, is equal to the value of level b. Likewise, the rate of level b, db, is equal to the value of level a. (Note: the system equations for all models can be found in Appendix B.)

Once again, the model structure will not be altered. We will only change the initial values of a and b.

First note that when a = b = 0, the rate of change for both level a and b is 0; thus the system is in equilibrium, much the same as with the first-order model. Now let a(0) = b(0) = 1 (this notation simply means that we are setting the initial condition, when time=0, of both levels to 1). We have moved the system out of equilibrium. It can be intuitively seen now to explode exponentially—standard positive feedback. Level a increases at a rate equal to level b. Because initially a=b, b increases at the same rate. In fact, a and b push each other identically; they behave as one, growing faster and faster together.

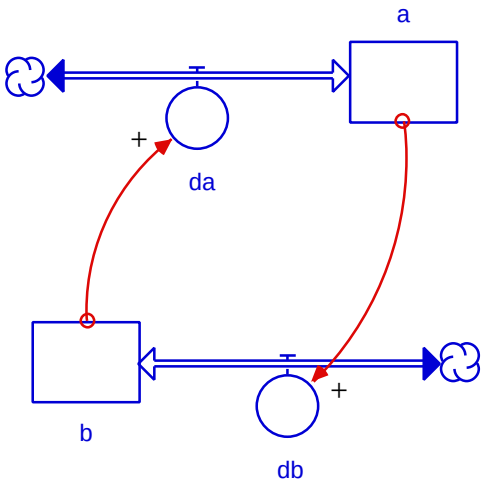


Figure 3: Second-order positive feedback loop

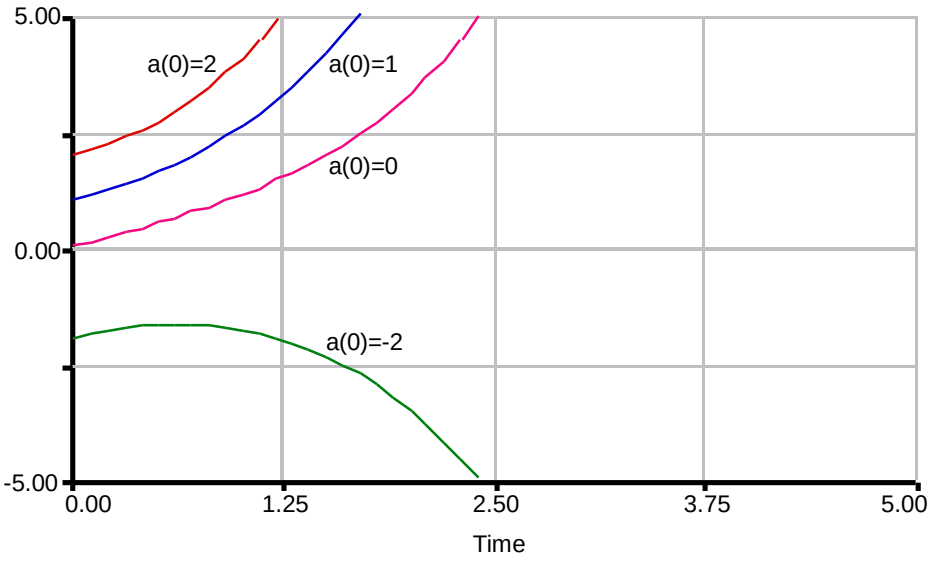


Figure 4: Exponential behavior from four different initial conditions (second-order positive feedback loop). b(0) = 1 in all cases.

Leave b(0) = 1 and try a(0) = 2, then a(0) = 0. In both cases the level that is initially greater stays greater, and the levels again escape to positive infinity. Next, let a(0) = -2. This time a is sufficiently negative to reverse the direction of escape. a consistently decreases b by more than b increases a; eventually b drops below 0, and both a and b plummet. Figure 4 shows the time series of a for the above four cases.

All of these show the expected exponential behavior, some up and some down. But there is a balance between them. Let a(0) = -1. Now b is decreased by a at the same rate a is increased by b. The two ride in a perfect balance, asymptotically approaching 0 (Figure 5). This behavior would occur for any initial conditions a(0) = -b(0). This behavior is usually seen in first-order negative feedback, which shows an approach to stable equilibrium. However, this is **not** a stable behavior mode. The slightest change in either variable destabilizes the approach and returns both to an exponential growth. This is seen in Figure 6 where we let a(0) = -0.99, a 1% disturbance.

Although exponential growth is the dominant behavior, it is possible to elicit asymptotic approach to (unstable) equilibrium in a purely positive second-order loop.

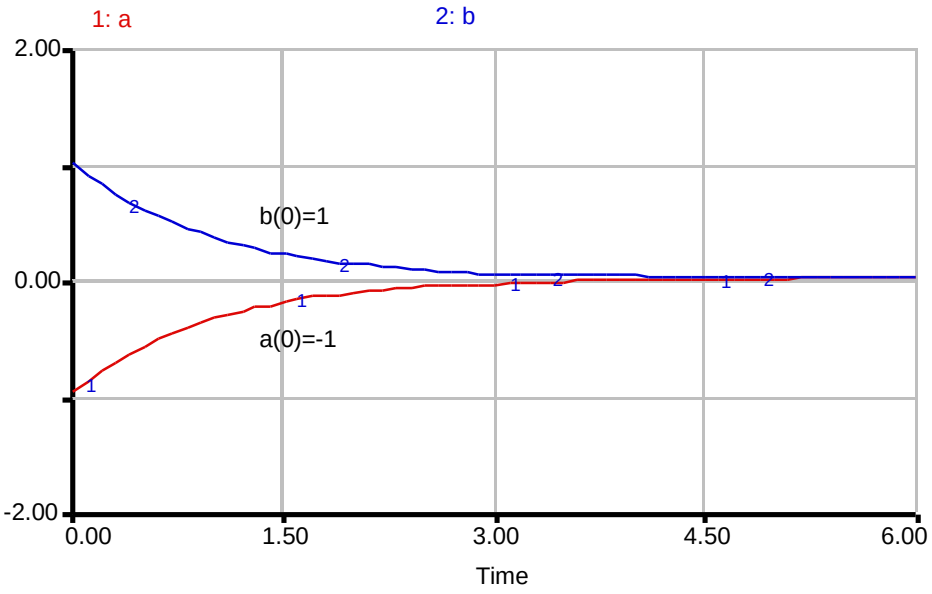


Figure 5: Convergent behavior from initial conditions a(0) = -1, b(0) = 1 (second-order positive feedback loop).

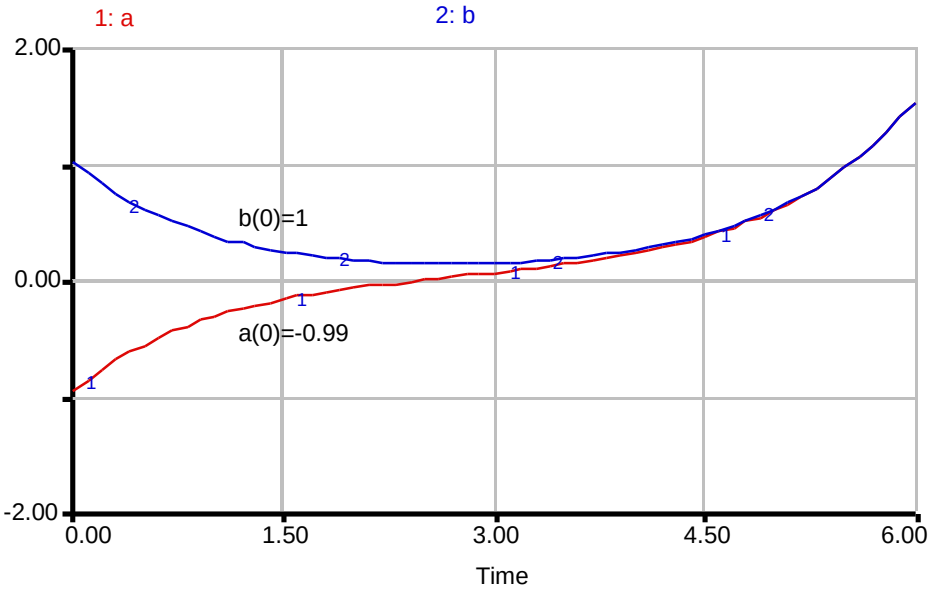


Figure 6: Unstable convergence perturbed to exponential growth (second-order positive feedback loop).

Independent Exploration:

Make a comparative scatter graph of a vs. b, and try many initial values of a and b. This is done in the Graph Pad Dialog box. First, select Scatter under Graph Type. Then double click a and b under Allowable to be Selected, as normally done. To plot

many initial values on the same graph, you must first check Comparative before identifying the Selected variables. What pattern emerges?

Introduce time constants as an additional input to the rate equations (i.e. da = b/ta, db = a/tb, where ta and tb are the time constants). These time constants are present in the loops already described above; they are simply equal to 1 and so have no effect. How does changing the time constants affect the system? How about negative time constants (and can you imagine what such a thing might mean)?

2.3 Third-Order Loop

The third-order system being examined is, like the second-order system, the simplest possible. Each level feeds the rate of the next with no other input (Figure 7).

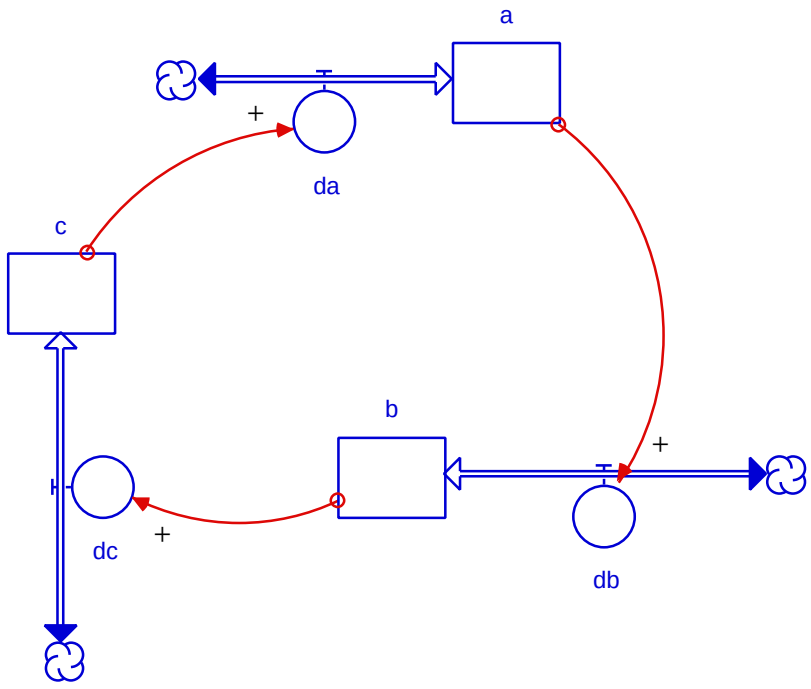


Figure 7: Third-order positive feedback loop

This system can also produce exponential growth from many initial conditions (E.g. a(0) = b(0) = c(0) = 1), but we will jump directly to other behavior modes. Let

a(0) = 1, b(0) = -1, and c(0) = 0. This produces the surprising behavior in Figure 8. Here we see, instead of exponential growth or even decay, damped periodic oscillations!

Here, as in the second-order system, we have found a balanced state. The levels, in this balance, can not decide whether to escape up or down, and so they do neither. But again, this apparently stable approach to equilibrium—for when a=b=c=0 the system is in equilibrium—is not truly stable at all. Change one value only slightly, by a scant 1/1000, and the balance collapses as in Figure 9.

Where does this periodic behavior come from? There is no negative feedback to drive it directly. There are no exogenous inputs. There are, however, three delays. These are not delays in the traditional sense, where an intervening level may change a step increase inflow into a smoothed outflow (using additional negative feedback structure). The delays are present inherently within the levels. To see this, notice what happens when a rate suddenly changes. If we are looking only at the level changed by this rate, the surge is not immediately felt—at least not strongly. After some time passes by, we may notice that the level is changing in a new way, due to a new rate value. However, the information passing through the rate and then the level still has this inherent delay. It takes time for qualitative behavior to propagate through.

Thus, while b is increasing from the start, the effect does not have a strong effect on a until approximately time = 2, in the previous example. The delays—and the balance of the levels around 0—incite the oscillations. The oscillations occur in the third-order system and not the second-order system because the higher order system has more room for dynamic behavior; there are not enough delays present in the second-order system. So what might we see in systems of even higher orders?

Independent Exploration:

Again, create comparative scatter graphs, this time of a vs. b, a vs. c, and b vs. c. In this way you can begin to see a three dimensional picture of the behavior (in the space of a, b, and c). The three graphs give you perspectives that we might call front, side, and top. What do you see now, and how does it compare to what you saw in the second-order system?

The balance for convergent behavior in the third-order system occurs when a(0) + b(0) + c(0) = 0. Can you discover why?

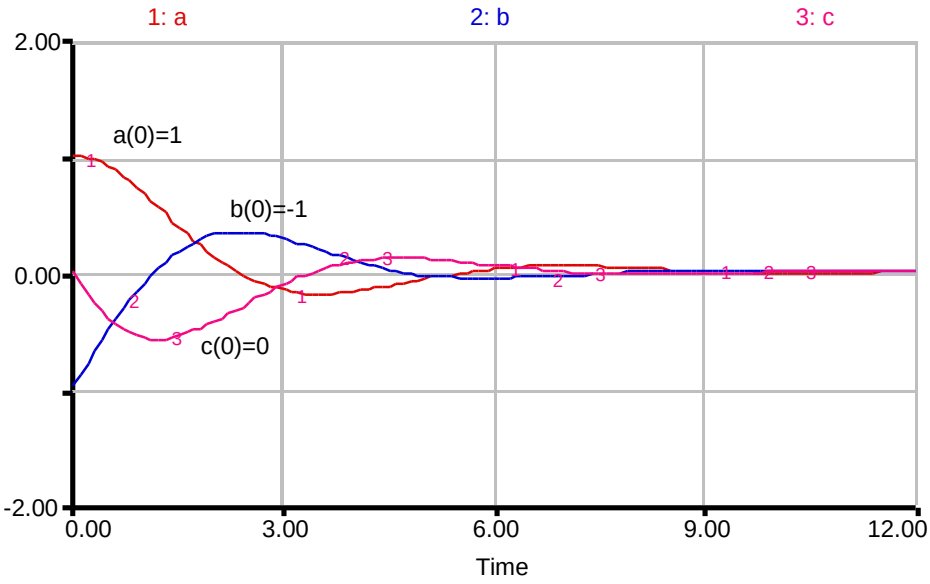


Figure 8: Convergent periodic oscillations (third-order positive feedback loop).

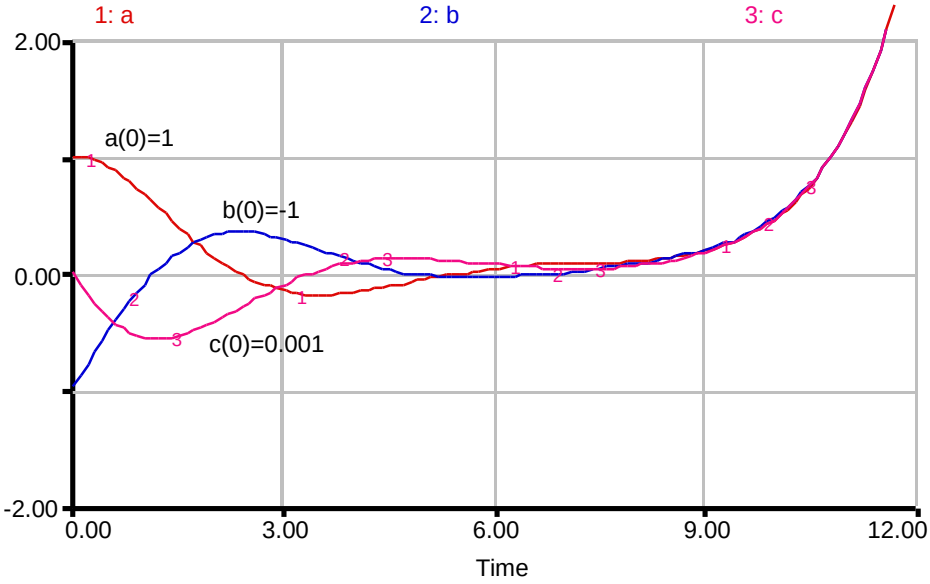


Figure 9: Convergent periodic oscillations destabilizing to exponential growth (third-order positive feedback loop)

2.4 Fourth-Order Loop

The fourth-order system is identical to the previous systems except for that it adds one more level to the chain (Figure 10).

What surprising behavior can we obtain within the freedom of four level variables? As it happens, three distinct modes appear. The expected divergent exponential (a(0) = b(0) = c(0) = d(0) = 1) and convergent decay (a(0) = c(0)=1, b(0) = d(0) = -1) we have seen already, but—as unbelievable as it may seem—we can now also have sustained oscillations! With only positive feedback between the individual variables—and with the initial conditions properly balanced—the delays between distant levels is great enough to create sustained oscillations. In Figure 11 a(0) = 1, b(0) = -1, and c(0) = d(0) = 0. (You may get expanding oscillations when you simulate this. If so, it is due only to DT error, and it can be rectified by using a different integration method or a smaller DT. I used DT=0.01; Runge-Kutta at DT=0.1 works just as well.)

As is the case in the other systems, this peculiar behavior mode is not stable. A 1/1000 perturbation—changing c(0)=0.001—creates the destabilization in Figure 12.

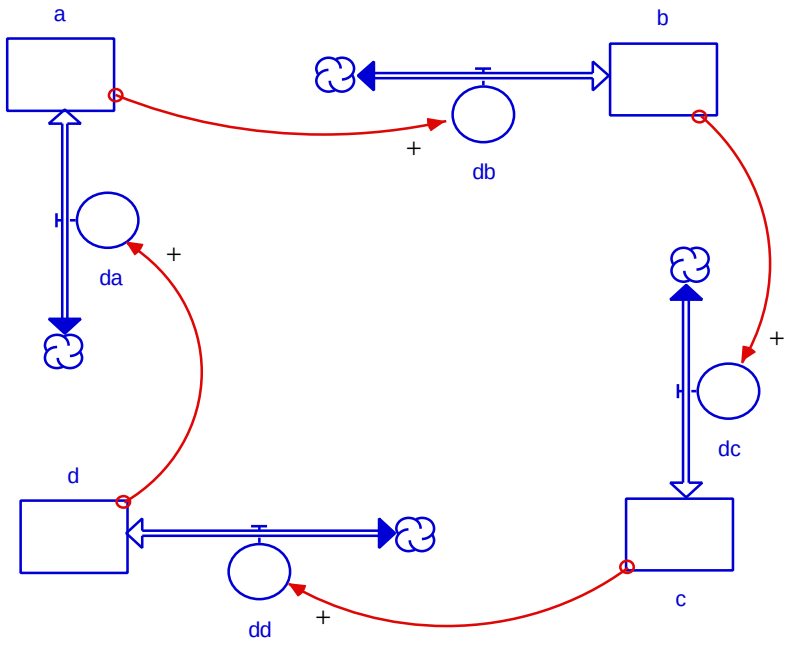


Figure 10: Fourth-order positive feedback loop

Fourth-order feedback has the flexibility and inherent delay to foster sustained oscillations, as well as convergence to an unstable equilibrium. This system might be viewed as taking a two-level oscillating system and "hiding" it within the second-order system seen earlier; their behaviors together form the behavior we see here.

Independent Exploration:

Look closely at the time-series for each of the feedback loops examined. In the second and third-order systems, the convergent behavior mode died out when it was perturbed into destabilization (given a little push, it blew up). The damped oscillations turned into (nearly) pure exponential growth. After destabilization (as in Figure 12), do the sustained oscillations disappear in the fourth-order system as well? (You may find the geometries constructed from the scatter graphs useful in exploring this question.)

The fifth-order system offers still greater behavior. Explore and see what you can find. Under special initial conditions, it will show damped oscillations, but I'll warn you, they are quite difficult to find (it took me half a year)!

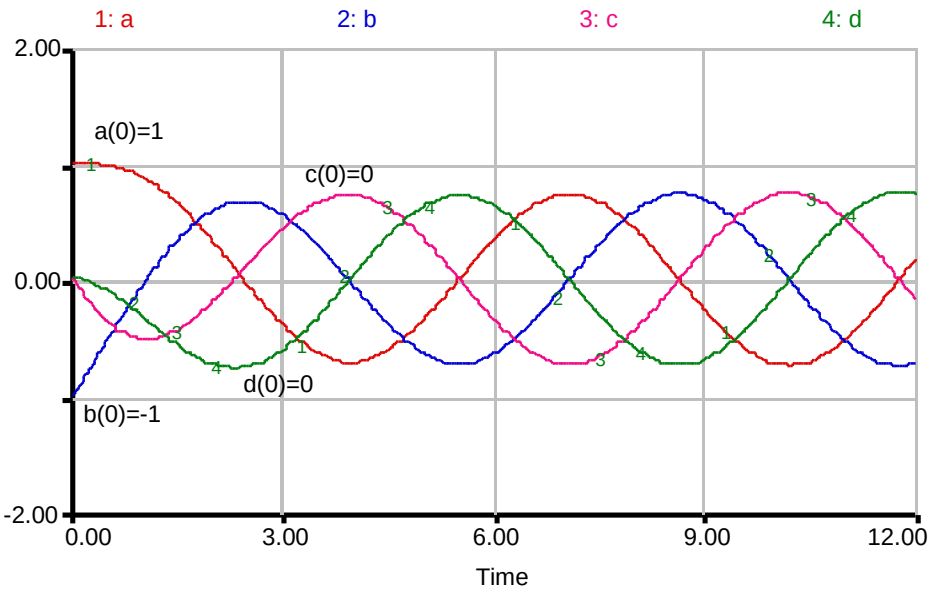


Figure 11: Sustained oscillations within a fourth-order positive feedback loop

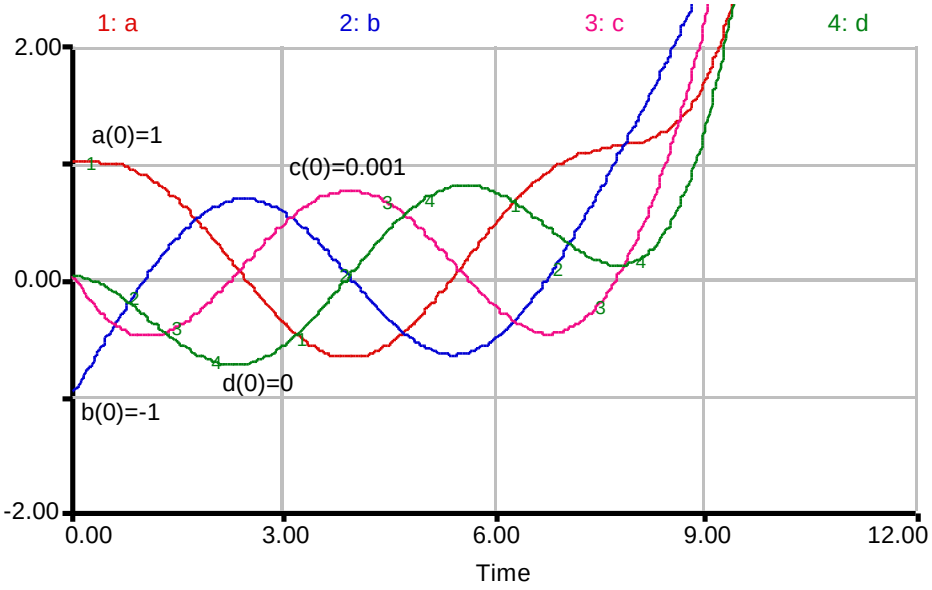


Figure 12: Sustained oscillations destabilizing to exponential growth (fourth-order positive feedback loop)

3. SUMMARY

Although positive feedback loops are usually associated with exponential growth, that is not the only behavior possible within them. Convergence and oscillation can also be found. We looked at four different positive feedback loops.

The first order loop only behaves exponentially. All initial conditions will reinforce the system state, and push it out to infinity (be it up or down). The only exception is the degenerate behavior at the origin. There it stays fixed at one point, but note that this special behavior, like in the other systems, is unstable. Exponential growth is still the stable behavior.

The second order loop showed that exponential growth was not the only possibility for positive loops. While in perfect balance, the two levels settled to unstable equilibrium. Notice that the fixed behavior of the origin is still present (and unstable). The higher—order positive feedback loops contain many repeated behaviors.

The third order loop brought us an interesting twist: oscillations! As in the second order loop, the levels had to be in a special balance, but this time their decent to equilibrium was done with a rhythmic dance.

The fourth order loop, the final loop examined, went a step further than the third and produced stable oscillations. This result clearly demonstrated the surprising character of positive feedback. A feedback loop that has, at each individual step, only positive influences can produce behavior that is undeniably negative in appearance.

We have found great behavioral diversity even within the simplest of positive loops. The truly stable behavior in each case is exponential growth. It is stable in that small perturbations do not change the behavior greatly, and yet it is unstable in that its behavior looks most unsettled. On the other hand, the other behaviors we have seen are unstable because small perturbations do drastically change their behavior, even though they all give the appearance of stability. We can almost always find unusual unstable behavior disguised as what truly would be stable in other models.

As it happens, the behaviors we have seen here demonstrate all the possibilities that models of this simple nature can exhibit. Remember that most models of the real world are not nearly so constrained or simplified as what we have seen in this paper. Most positive feedback loops in real world models will show (for the most part) only

exponential growth. But still keep in mind that unexpected behaviors are possible, even likely, and they are bound to be quite complex.

4. APPENDIX A: SOME SOLUTIONS TO EXPLORATIONS

4.1 Second-order System

Make a comparative scatter graph of a vs. b, and try many initial values of a and b; what pattern emerges?

This geometric pattern (shown in Figure 13.) is called a saddle. To see why it is given this name, imagine letting a marble roll off an actual horse saddle. It moves along the surface, falling down the sides where one's legs would normally go. If you were able to place the marble just right, you could balance it in the center of the saddle, and it would not fall in either direction. If you were even more careful, you could place the marble near the top of the saddle, still centered left and right, and it would roll down to the center, but not fall to either side. If the saddle were sticky, then the marble would also slow to a stop at the center, just as we have in our saddle here. The two orbits with initial conditions (-3, 3) and (2, -2) are in a balanced path along the saddle. They exponentially approach the origin, while orbits to either side quickly zoom off to infinity.

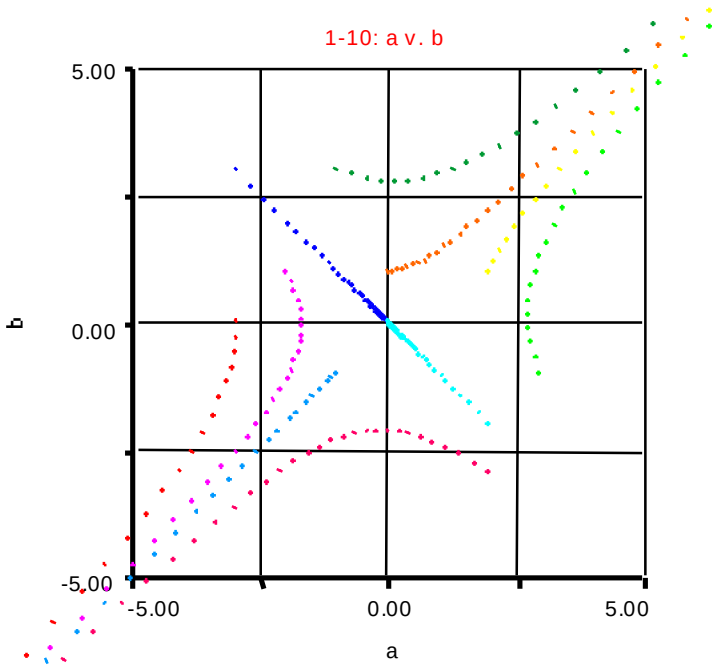


Figure 13: Paths on a saddle from 10 different initial conditions

4.2 Third-order System

What do you see in the new scatter plots, and how does it compare to what you saw in the second-order system?

This pattern is significantly more complicated to look at than the saddle. The problem is, it is a three-dimensional pattern. Figure 14 shows what it looks like from one perspective. The best way for you to see it is probably to watch it as the simulation runs, flipping between perspectives.

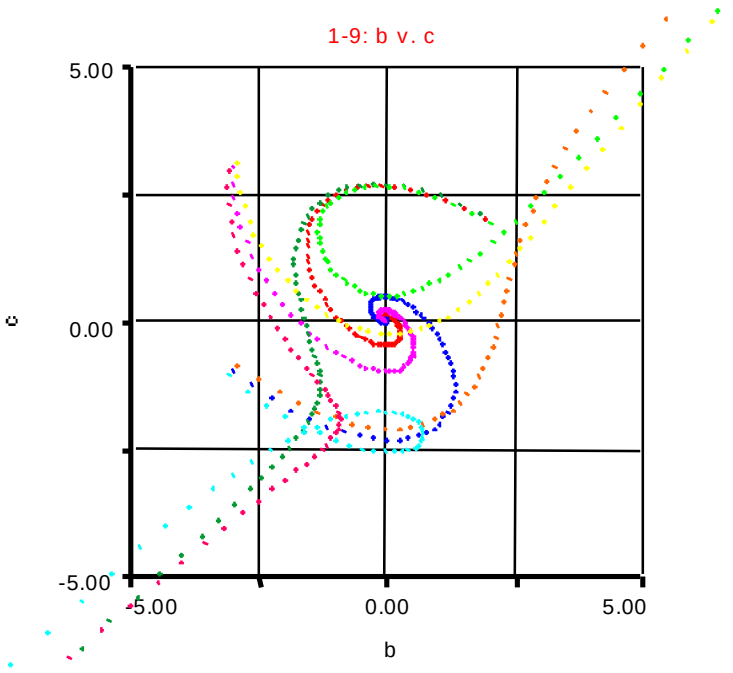


Figure 14: Paths in the third-order system; 3 paths are decaying; 6 are expanding

In the center you can see the decaying oscillations, they form the spiral pattern. Shooting off to the upper right and lower left are orbits perturbed into exponential growth. The spiral lies in a plane, and the exponential jets point perpendicular to this plane. If a point starts on the plane, it will stay on it, spiraling in to the origin. But if the point is even slightly off the plane, then it will be pulled away into the jet on its side of the plane.

If you could look at the plane edge on, the paths would look much like the paths seen in the saddle, only intertwining. This spiraling geometry is a kind of extension of the saddle. Now you might think of it as a pair vortices put end to end. Water rushes down a whirlpool or shoots up in a spout. The saddle is a little like a vertical cross section of the vortices. Perhaps a simpler way to look at it is just as the combination of decaying oscillations (spirals) and exponential growth (jets), each in its own direction.

4.3 Fourth-order System

After destabilization, do the sustained oscillations disappear in the fourth-order system as well?

No, the oscillations continue, even while rocketing off to infinity. The way to picture this is to use the geometries just shown for the second and third-order systems. The pattern now is even more complicated, and now it is in **four** dimensions! For this reason, I won't try to show it to you here, although I applaud all those who try to make sense out of the scatter graphs.

The way to think about it is to picture the orbits as traveling around a cylinder. At the plane cutting through the cylinder and the origin, the orbits go round and round in a circle. This is the sustained oscillations. Above and below this plane, the orbit fly away, but they continue to spiral around the cylinder. So, even though they become more and more drowned out by the exponential growth, the oscillations do not disappear.

The way to check it is to see what happens to the differences between the level variables. Plot a time series of a-b, for example. This should oscillate even while exploding exponentially.

4.4 Fifth-order System

Can you find the initial condition (for the fifth-order system) that results in decayed oscillations?

Here it is. Try it out, and look at nearby points too.

a=1	c=cos(2p/5)	e= -cos(p/5)
b= -cos(p/5)	d=cos(2p/5)	

5. APPENDIX B: MODEL EQUATIONS

5.1 First-Order Loop

a(t) = a(t - dt) + (da) * dt
INIT a = 1
da = a

5.2 Second-Order Loop

a(t) = a(t - dt) + (da) * dt
INIT a = 1
da = b

b(t) = b(t - dt) + (db) * dt
INIT b = -1
db = a

5.3 Third-Order Loop

a(t) = a(t - dt) + (da) * dt
INIT a = 1
da = c

b(t) = b(t - dt) + (db) * dt
INIT b = -1
db = a

c(t) = c(t - dt) + (dc) * dt
INIT c = 0
dc = b

5.4 Fourth-Order Loop

a(t) = a(t - dt) + (da) * dt
INIT a = 1
da = d

b(t) = b(t - dt) + (db) * dt
INIT b = -1
db = a

c(t) = c(t - dt) + (dc) * dt
INIT c = 0
dc = b

d(t) = d(t - dt) + (dd) * dt
INIT d = 0
dd = c

5.5 Fifth-Order Loop

a(t) = a(t - dt) + (da) * dt
INIT a = 1
da = e

b(t) = b(t - dt) + (db) * dt
INIT b = -cos(p/5)
db = a

c(t) = c(t - dt) + (dc) * dt
INIT c = cos(2p/5)
dc = b

d(t) = d(t - dt) + (dd) * dt
INIT d = cos(2p/5)
dd = c

e(t) = e(t - dt) + (de) * dt
INIT e = -cos(p/5)
de = d

Vensim Examples:

Unexpected Behaviors in Higher-Order Positive Feedback Loops

Aaron Diamond

October 2001

3.1 First-Order Loop

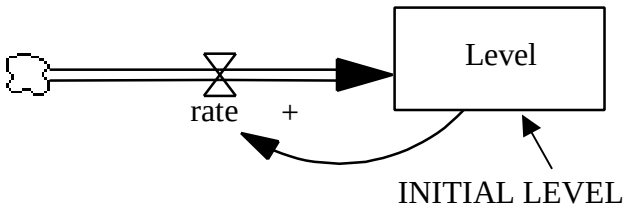


Figure 15: Vensim Equivalent of Figure 1: A first-order positive feedback loop

Documentation for first-order positive feedback loop

(1) FINAL TIME = 6
Units: none
The final time for the simulation.

(2) INITIAL LEVEL=1
Units: **undefined**

(3) INITIAL TIME = 0
Units: none
The initial time for the simulation.

(4) Level= INTEG (Rate, INITIAL LEVEL)
Units: **undefined**

(5) rate=Level
Units: **undefined**

(6) SAVEPER = TIME STEP
Units: none
The frequency with which output is stored.

(7) TIME STEP = 0.0625
Units: none
The time step for the simulation.

3.2 Second-Order Loop

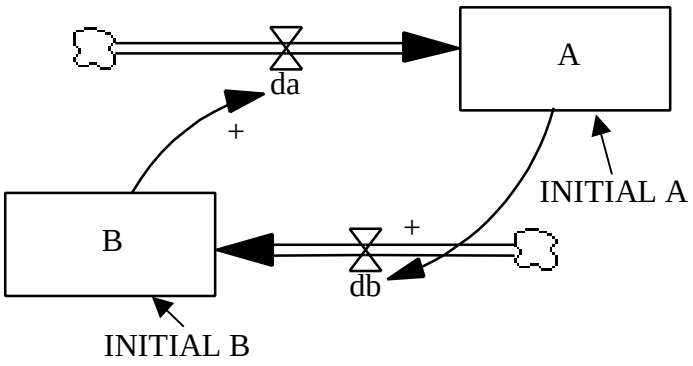


Figure 16: Vensim Equivalent of Figure 3: Second-order positive feedback loop

Documentation for second-order positive feedback loop

(1) A= INTEG (da, INITIAL A)
Units: **undefined**

(2) B= INTEG (db, INITIAL B)
Units: **undefined**

(3) da=B

Units: **undefined**

(4) db=A

Units: **undefined**

(5) FINAL TIME = 6

Units: none

The final time for the simulation.

(6) INITIAL A=1

Units: **undefined**

(7) INITIAL B=1

Units: **undefined**

(8) INITIAL TIME = 0

Units: none

The initial time for the simulation.

(9) SAVEPER = TIME STEP

Units: none

The frequency with which output is stored.

(10) TIME STEP = 0.0625

Units: none

The time step for the simulation.

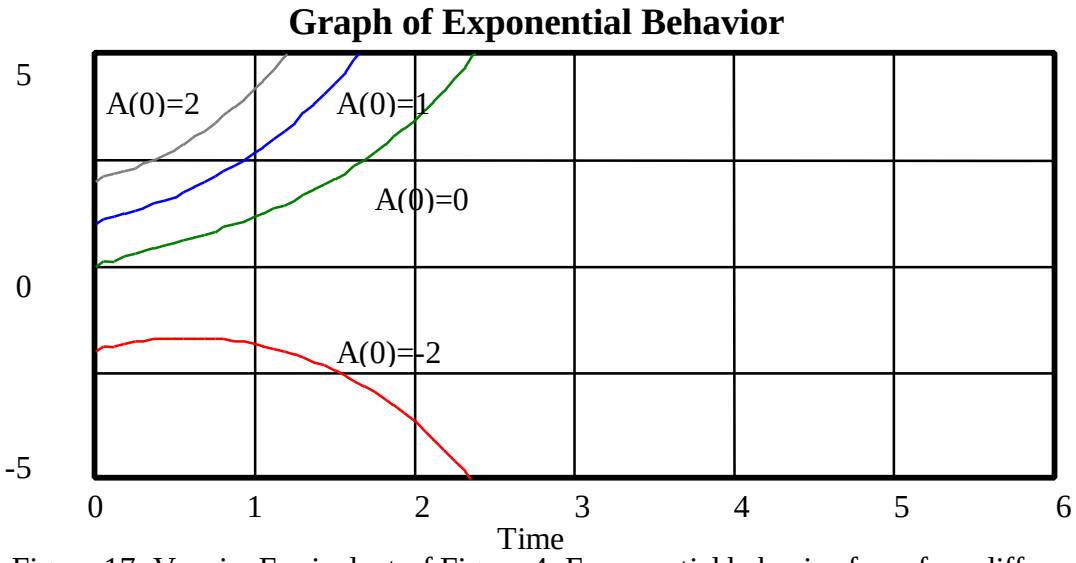


Figure 17: Vensim Equivalent of Figure 4: Exponential behavior from four different initial conditions (second-order positive feedback loop). B(0)=1 in all cases.

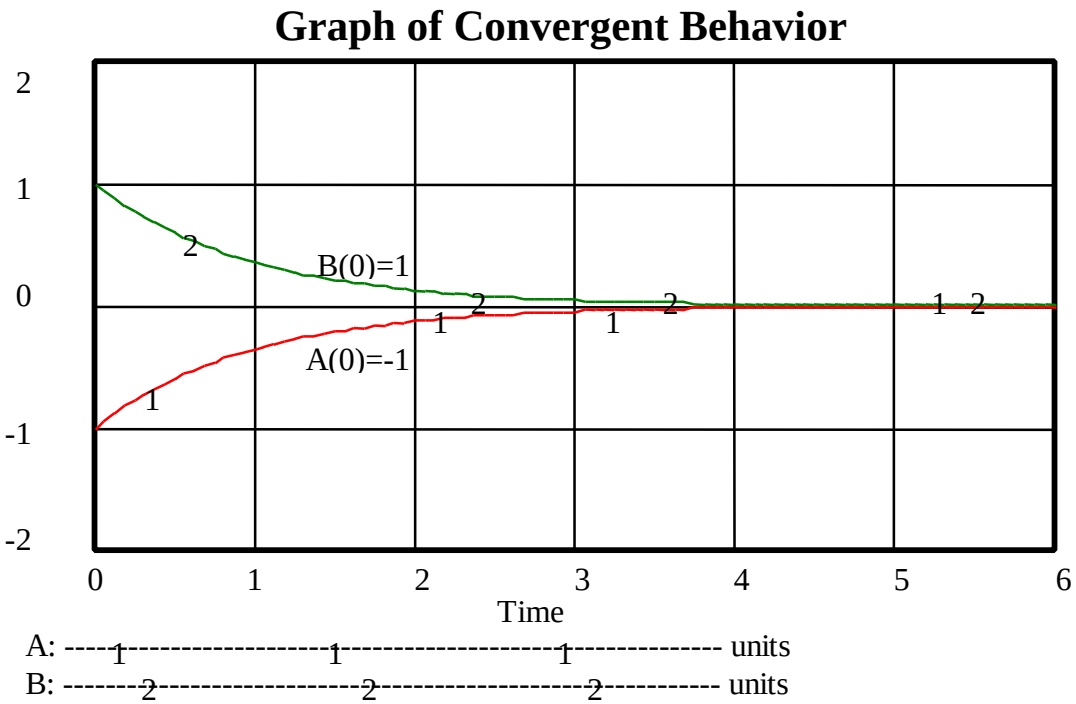
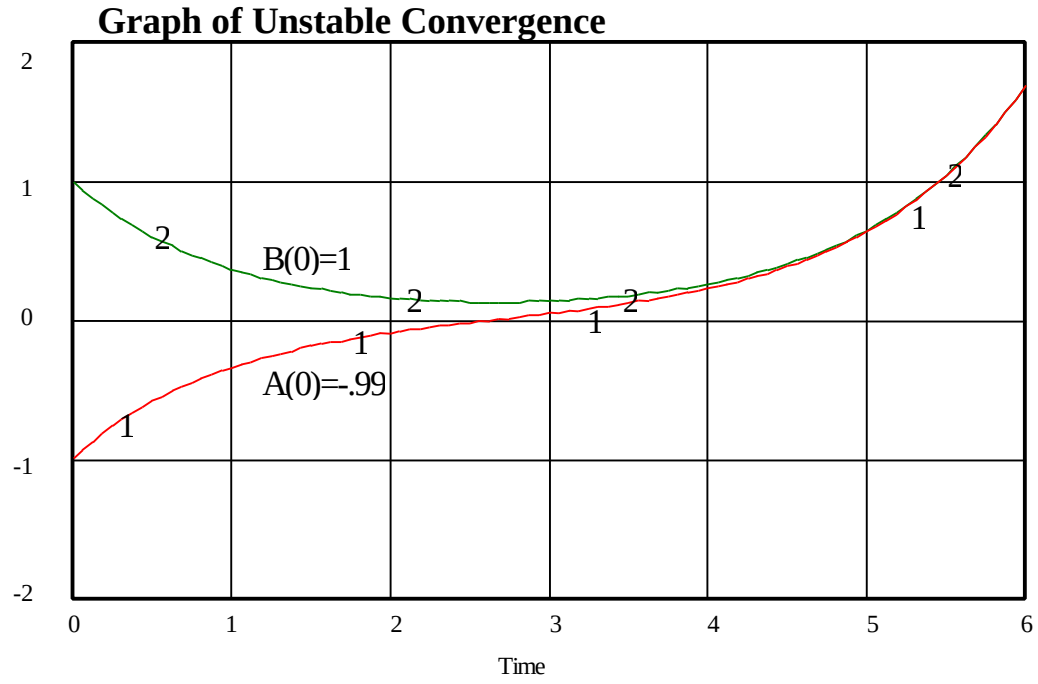


Figure 18: Vensim Equivalent of Figure 5:Convergent behavior from initial conditions A(0)=-1, B(0)=1 (second order positive loop).



A: -----1----------1----------1---------- units

B: --------2----------2----------2---------- units

Figure 19: Vensim Equivalent of Figure 6: Unstable convergence perturbed to exponential growth (second-order positive feedback loop).

3.3 Third-Order Loop

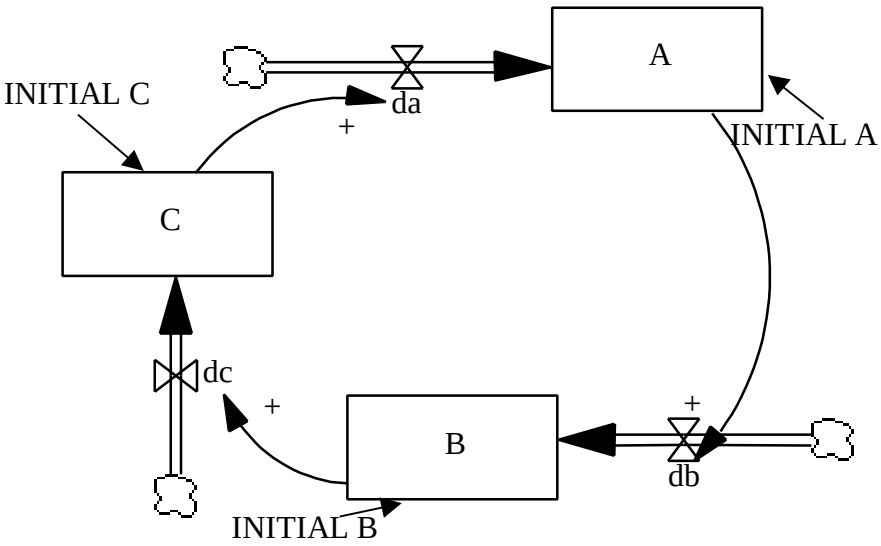


Figure 20: Vensim Equivalent of Figure 7: Third-order positive feedback loop.

Documentation for third-order positive feedback loop

(01) A= INTEG (da, INITIAL A)
Units: **undefined**

(02) B= INTEG (db, INITIAL B)
Units: **undefined**

(03) C= INTEG (dc, INITIAL C)
Units: **undefined**

(04) da=C
Units: **undefined**

(05) db=A
Units: **undefined**

(06) dc=B
Units: **undefined**

(07) FINAL TIME = 12
Units: none
The final time for the simulation.

(8) INITIAL A=1
Units: **undefined**

(9) INITIAL B=-1
Units: **undefined**

(10) INITIAL C=0
Units: **undefined**

(11) INITIAL TIME = 0
Units: none
The initial time for the simulation.

(12) SAVEPER =TIME STEP
Units: none
The frequency with which output is stored.

(13) TIME STEP = 0.0625
Units: none
The time step for the simulation.

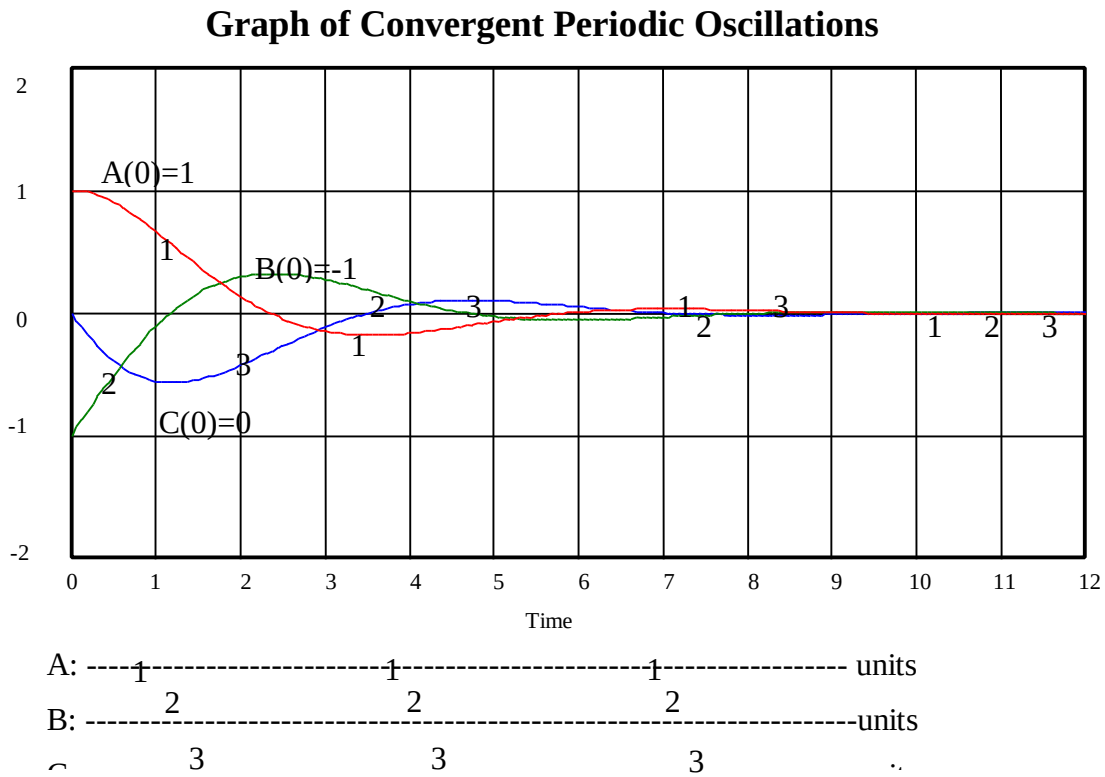


Figure 21: Vensim Equivalent of Figure 8: Convergent periodic oscillations (third-order positive feedback loop).

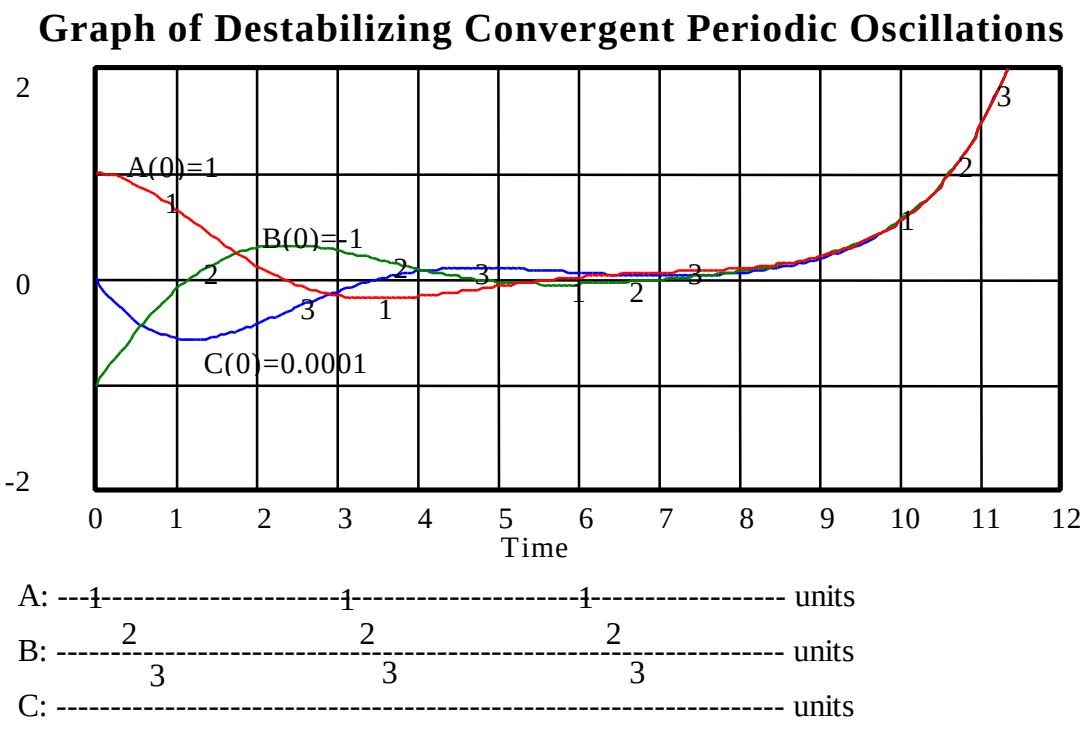


Figure 22: Vensim Equivalent of Figure 9: Convergent periodic oscillations destabilizing to exponential growth (third-order positive feedback loop).

3.4 Fourth-Order Loop

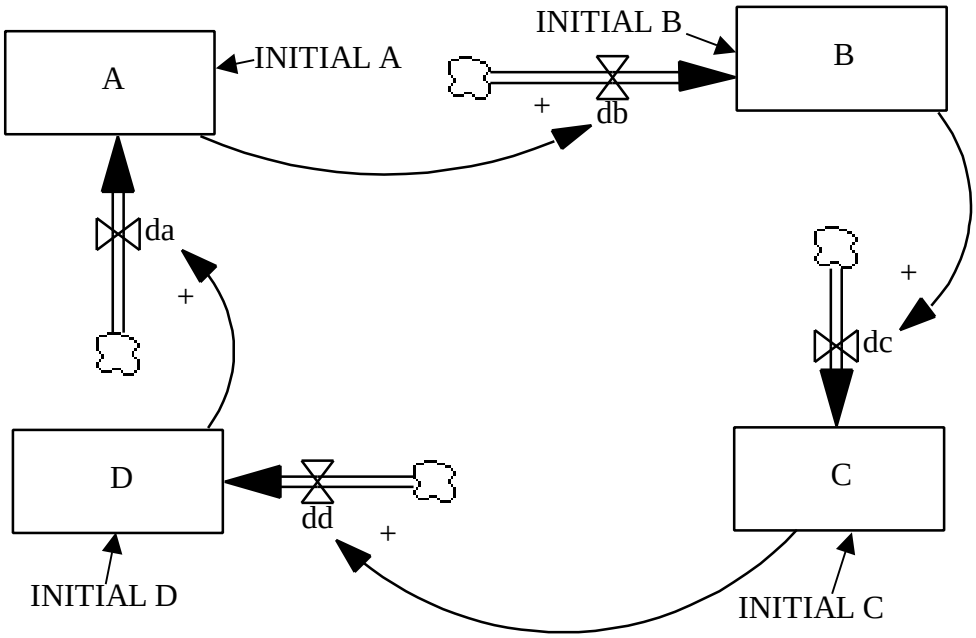


Figure 23: Vensim Equivalent of Figure 10: Fourth-order positive feedback.

Documentation for fourth order feedback loop

(01) A= INTEG (da, INITIAL A)
Units: **undefined**

(02) B= INTEG (db, INITIAL B)
Units: **undefined**

(03) C= INTEG (dc, INITIAL C)
Units: **undefined**

(04) D= INTEG (dd, INITIAL D)
Units: **undefined**

(05) da=D
Units: **undefined**

(06) db=A
Units: **undefined**

(07) dc=B
Units: **undefined**

(08) dd=C
Units: **undefined**

(09) FINAL TIME = 12
Units: none
The final time for the simulation.

(10) INITIAL A=1
Units: **undefined**

(11) INITIAL B=-1
Units: **undefined**

(12) INITIAL C=0
Units: **undefined**

(13) INITIAL D=0
Units: **undefined**

(14) INITIAL TIME = 0
Units: none
The initial time for the simulation.

(15) SAVEPER = TIME STEP

Units: none

The frequency with which output is stored.

(16) TIME STEP = 0.015625

Units: none

The time step for the simulation.

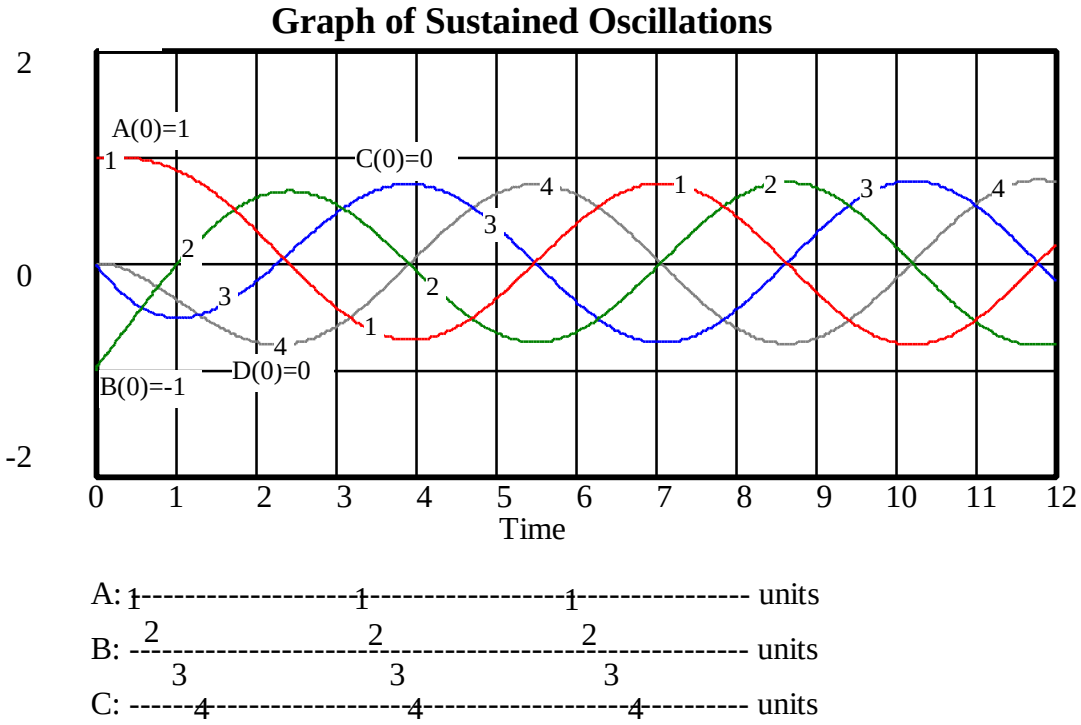
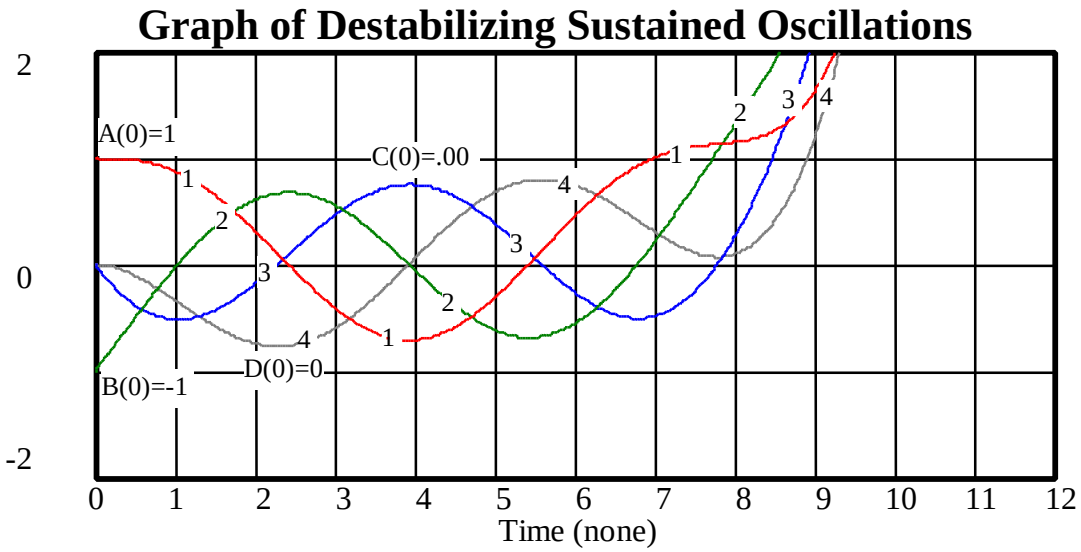


Figure 24: Vensim Equivalent of Figure 11: Sustained oscillations within a fourth-order positive feedback loop.



A: ----------1----------1----------1---------- units
B: ----------2----------2----------2---------- units
C: ----------4----------4----------4---------- units

Figure 25: Vensim Equivalent of Figure 12: Sustained oscillations destabilizing to exponential growth (fourth-order positive feedback loop).

Mistakes and Misunderstandings: Examining Dimensional Inconsistency

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

Initiated by
Michael Shayne Gary[1]
January 1992

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[1] The paper in its present form owes itself to the contributions of Marc Glick, Alice Oh, and Mark Choudhari. Revised September, 1995.

1. THE SYSTEM

Let us consider a familiar physical system, the thermostat-controlled furnace in a house. The mechanics of this system are simple; the furnace turns on or off depending on the temperature inside the house. Let us assume that the thermostat is set at 75°F. The furnace remains off as long as the temperature in the house stays above 75°F. When heat escapes the house through windows, walls, and doorways, the temperature in the house falls below 75°F, and the furnace turns on. After a while, the temperature reaches 75°F again, then the furnace shuts off until reactivated by another temperature drop. In short, the difference between the temperature within the house and the temperature set on the thermostat represents a temperature gap which controls whether the furnace is on or off.

2. A FIRST ATTEMPT TO MODEL THE SYSTEM

Given the information we have about the system, let us begin with the model in Figure 1:

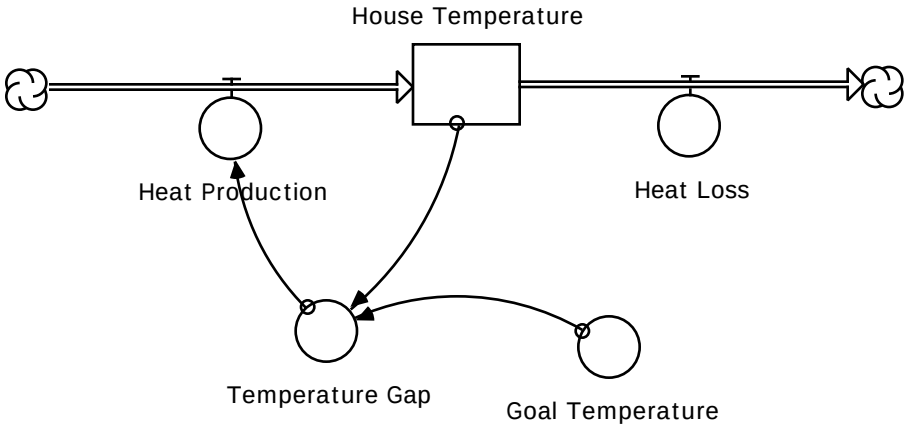


Figure 1: Model of a thermostat-controlled furnace in a house

3. MISTAKES AND MISUNDERSTANDINGS

A few questions often serve to identify mistakes made when modeling. Experience and patience teach the right questions to ask.

STUDENT: So, *Heat Production* is a rate that flows into a stock of *House Temperature*.

TEACHER: If that is the case, then what are the units of *House Temperature*?

STUDENT: Degrees, of course, are the units of temperature. Degrees Celsius or Degrees Fahrenheit.

TEACHER: Well, then what are the units of *Heat Production*?

STUDENT: BTU's per unit time or Calories per unit time; those are the standard units of *Heat Production*.

The dimensions of the flow, *Heat Production* are:

Heat / Time --- Calories per minute or BTU's per minute

The dimensions of the stock, House Temperature are:

Degrees --- Degrees Celsius or Degrees Fahrenheit

At this point, it should be clear that a mistake has been made. The units for the stock are not consistent with the units of the flow. Remember that a stock is something that accumulates over time, and the flows into and out of the stock are defined in stock units per time. Using *House Temperature* as a stock and *Heat Production* as an inflow is incorrect. *Heat Production* must flow into a stock of BTU's or Calories, not degrees. *Heat Production*, in units of BTU's per unit time or Calories per unit time, cannot be directly summed over time into units of Degrees. There must be a conversion from BTU's or Calories to Degrees. The dimensions must be consistent for the equations to be correct, or the model is not valid.

Now, we move to another, but related, question. Could temperature be used as a stock?

No, stocks must correspond to a quantity that can accumulate in the real world. Temperature is not an accumulation. In the physical world, degrees do not accumulate into an amount of temperature. It is easy to confuse temperature for a stock when just beginning to model.

To understand why temperature is not a stock, let's do a few thought experiments.

Experiment 1: Put a kettle with 1 inch of water on a kitchen range, and time how long it takes to boil. Now, do the same experiment with 4 inches of water. Would it take more, less, or the same amount of time to boil? It is obvious that it would take more time to boil. In both cases, the water in the kettle boils at 100°C. The difference in the time to boil different amounts of water is not because we are heating the kettle to different temperatures. The larger volume of water needs more time than the smaller volume of water to achieve the same boiling temperature of 100°C because we need more heat to raise a larger amount of water to the boiling temperature.

We are not adding degrees of temperature to the water in the kettle, we are adding heat. If the kitchen range were simply adding degrees to the water, it would take the same amount of time to raise the different quantities of water to the boiling temperature. This shows that temperature does not accumulate as a stock should.

Experiment 2: Imagine a small electric room heater in a small room, and an identical heater in a large warehouse. We turn both the heaters on and measure the temperature in the small room and the large warehouse after a few hours. Which place will have the higher temperature? The small room will be warmer than the large warehouse. Why? If the electric heaters were merely adding degrees of temperature to the system, both the small room and the large warehouse should be at the same temperature. But they are not! This is because the heaters are adding heat and not temperature. It is heat that is accumulating in the room and not temperature. The large warehouse has a greater volume of air that is being heated than the small room. It takes longer to heat a larger volume of air than a small volume of air to a particular temperature.

The addition of heat results in the increase in temperature. However, the rate at which temperature is increasing depends also on the amount of object that we are heating as we saw in Experiment 1. The temperature of a larger quantity will increase at a slower rate than a smaller quantity being heated by the same heat source. Also, the rate at which temperature increases depends on the physical properties of the object being heated. For example, the time it takes to heat a pan of oil to some temperature is not equal to the time it takes to heat the same amount of water to the same temperature. In the next section, you will see that these properties are lumped together in the form of a conversion factor that is used to translate from heat to temperature.

We have seen that we cannot directly add a degree to the temperature of a system. To change the temperature one must add heat to the system. The final result of this distinction is that the property of an object, identified as temperature, does not constitute an accumulation. One cannot change temperature by just adding or taking away degrees. However, one can add heat to a system, which would subsequently result in a temperature change. Therefore, heat should be the stock in the system because heat is the quantity that can flow and accumulate.

4. OVERCOMING OUR MISTAKES AND MISUNDERSTANDINGS

In Figure 2 is the corrected formulation of the thermostat-controlled furnace model, with the information given so far.

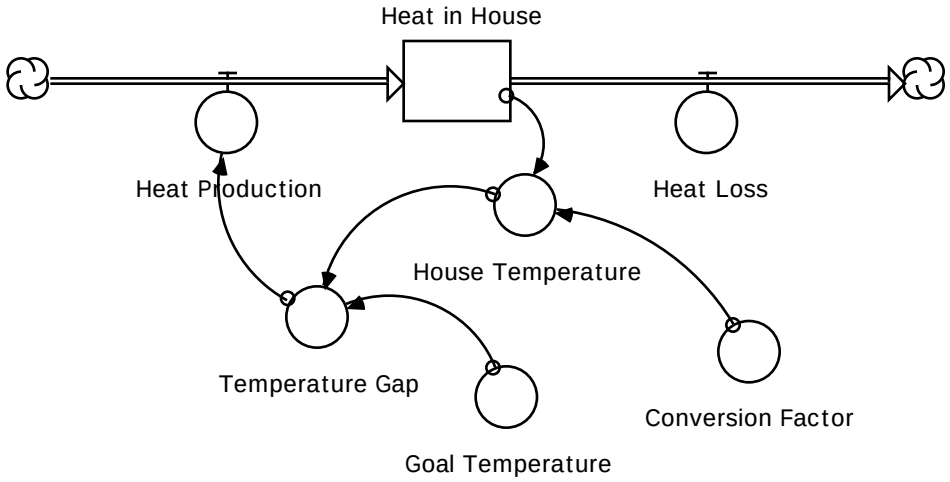


Figure 2: Corrected model of a thermostat-controlled furnace in a house

The change to the model defines the physical situation more accurately. The furnace produces heat, which fills the house with heat. By changing the amount of heat in the house, one can change the temperature in the house. Then one can calculate the temperature of the house from the total heat in the house using the conversion factor.[2] The temperature gap drives the heat production, just as described for the model in Figure 1.

5. KEY LESSONS

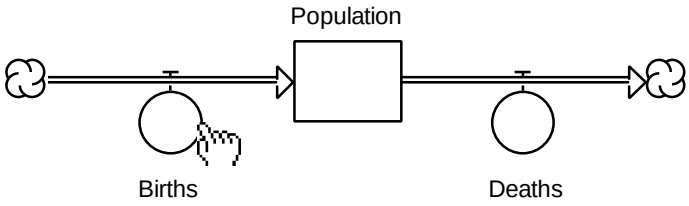
1. All equations must be defined in units that maintain dimensional consistency. Do not let an unclear mental model lead you into defining relationships that are dimensionally impossible.
2. Stocks must correspond to quantities that can accumulate in the real world. In choosing the stocks for your model, think of the quantities that are accumulating and/or draining in the system.

---

[2] For the more curious mind: The conversion factor is based upon physical properties specific to the system. These properties are the volume of the system, the specific heat capacity of the system, and the density of the system. If you are interested, you can find a more detailed treatment of these properties in any high school Physics text book.

Modeling Exercises

Section 2



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Joseph G. Whelan
August, 1994

Vensim Examples added October 2001

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1. ABSTRACT

This paper is the second in a continuing series of papers designed to enhance the reader's modeling skills. Modeling Exercises Section 2 contains two separate modeling exercises. In the first exercise the reader will build a system dynamics model based on a detailed description of an urban system. The reader will run the urban model under varying conditions. The second exercise asks the reader to perform mental simulations on a pre-built model, and to estimate the model behavior under different conditions. The reader is expected to have prior experience with system dynamics, model building, and mental simulation. Before beginning this paper, please review Modeling Exercises Section 1, which outlines the process of model building. [1]

---

[1] Joseph G. Whelan, 1994. *Modeling Exercises Section 1* (D-4421), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May, 1994, 37 pp.

Mental simulation is explained in the following papers:

Joseph G. Whelan, 1995. *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback* (D-4487), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 27, 15 pp.

Helen Zhu, 1996. *Beginner Modeling Exercises Section 3: Mental Simulation of Simple Negative Feedback* (D-4536), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, January 28, 23 pp.

Alan Coronado, 1996. *Beginner Modeling Exercises Section 4: Mental Simulation: Adding Constant Flows* (D-4546), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 27 pp.

Laughton Stanley and Helen Zhu, 1996. *Beginner Modeling Exercises Section 5: Mental Simulation of Combining Feedbacks in First Order Systems* (D-4593), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 28 pp.

2. INTRODUCTION

Modeling is a crucial element of system dynamics. People rely on mental models daily to make decisions without even questioning the model being used. Creating and simulating models using STELLA or other modeling software[2] allows the modeler to solidify his mental models. The modeler thus observes the behavior of complex systems where mental models are inadequate. Both mental and computer models can give wrong answers. A computer will give an answer with precision even if that answer is based on an inadequate model. A computer will not say "A population cannot be negative," or "The number of sick people cannot be greater than the total population." The modeler must be the one to verify the quality of a model. This paper and future Modeling Exercises in Road Maps will enhance the reader's modeling skills.

[2] Powersim or Vensim can be used if the reader can make interpretations from the material in this paper.

3. EXERCISE 1: URBAN DYNAMICS[3]

In this exercise the reader will build a simple model to explain growth and stagnation in an urban area. Read the material in Section 3.1, then answer the questions in Section 3.2. Section 3.3 further describes the system, and Section 3.4 gives instructions to build a STELLA model to fit the description. The model should then be used to answer the questions given in Section 3.5.

3.1 Background of the Urban System

A city's life cycle is characterized by a period of economic growth followed by a period of transition toward equilibrium. During the growth period, the city appears to be economically healthy: business activity is expanding and unemployment is low. The "Population" and the number of "Business Structures" grow quickly. During the transition period, conditions become less desirable and pressures arise which impede further growth. The "Construction" rate of "Business Structures" becomes smaller, and so does the rate of "In Migration" into the city. In equilibrium, when the "Population" and the number of "Business Structures" stop growing, the city suffers from problems such as high unemployment.

3.2 Preliminary Exploration of the Urban System[4]

Please answer the following questions before continuing.

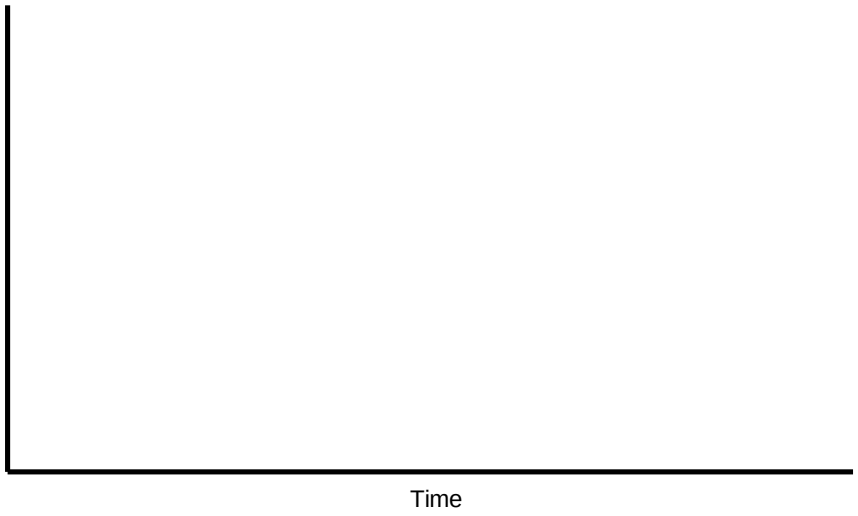
1. What is the problem the model will address? What is the reason for building the model? What are the key variables in the system?

---

[3] Note: The following exercise was based on an exercise prepared by John Sterman. It has been revised for use in Road Maps. The model structure and description were adapted from the population-business (POPBSN) model in Introduction to Urban Dynamics.
John Sterman, 1981. Principles Of Systems I, 15.852/872, Fall 1981, Assignment #3: Urban Modeling Assignment (D-3296-1), System Dynamics Group, Massachusetts Institute of Technology.
Alfeld, L. E., and Graham, A. K., 1976. Introduction to Urban Dynamics. Cambridge, MA: Productivity Press. The POPBSN model is developed in Chapter 5 of the book.

[4] Suggested answers to this question are included in Section 5.

Sketch the expected behavior of all the key variables over an appropriate time horizon on the blank graph provided.



3.3 General Description of the Urban System

The availability of "Jobs" and promise of higher incomes are prime motivations for "In Migration" into an urban area. People tend to move to areas where employment opportunities are favorable. Television news, newspaper stories, advertisements, word-of-mouth accounts, and industrial recruiting all tend to create perceptions of higher wages and greater opportunities in areas where "Labor Availability" is low. On the other hand, in areas where "Labor Availability" is high, job conditions are perceived to be relatively unfavorable.[5] People tend to migrate to cities that, at a given time, are perceived to offer more job opportunities.

When people move into or out of an area, they add to or subtract from the number of people in the area's "Labor Force." Businesses cannot ignore the "Labor Availability" in their location and expansion decisions. Readily available labor allows businesses greater

flexibility in choosing employees and shortens the time necessary to fill open positions. Moreover, high "Labor Availability" tends to decrease wage competition for labor among businesses.

3.4 Detailed Instructions for Building the Urban Growth Model

The urban growth model that the reader is asked to build in Section 3.5 should focus on the dynamics of population and business growth within a fixed area of a city. The model should contain two sectors: the "Business Structures" sector and the "Population" sector.

3.4.1 The "Business Structures" Sector

The "Business Structures" sector consists of a level called "Business Structures" with initial value of 1000. The number of "Business Structures" increases through a rate of "Construction" and decreases through a rate of "Demolition."

The rate of "Demolition" is determined by the number of existing "Business Structures" and the "Average Structure Lifetime" of 50 years.

The rate of "Construction" can be modeled as a product of "Business Structures" and several factors:[6]

? A "Construction Fraction" represents "Construction" as 2% of "Business Structures" under normal conditions—when there is ample land, and labor availability matches the available jobs. The "Construction Fraction" thus has a value of 0.02.

? A "Land Availability Multiplier" is a nonlinear function of the "Land Fraction Occupied." "Land Fraction Occupied" is defined as the number of "Business Structures"

---

[5] "Labor Availability," as it will be defined in the urban growth model, is the ratio of the "Labor Force" in the urban area to the total number of "Jobs." When the "Labor Force" is greater than the number of "Jobs" ("Labor Availability" is greater than 1), not all the workers in the area can find "Jobs," and the area experiences unemployment. Thus, "Labor Availability" represents the concept of unemployment from the point of view of the businesses.

multiplied by the average "Land per Structure" (1 acre per structure), and divided by the total "Land Area" of 5000 acres. When almost all "Land Area" is available, the "Land Availability Multiplier," and thus "Construction," is quite low because there are very few businesses around to provide capital. As more of the "Land Area" is occupied, the effect on "Construction" increases. Eventually, the "Land Area" is used up, and the "Land Availability Multiplier" and "Construction" drop again, reaching zero when all the "Land Area" is occupied. In equilibrium, "Land Availability Multiplier" is equal to 1.

? A "Labor Availability Multiplier" is a nonlinear function of "Labor Availability." "Labor Availability" is the ratio of "Labor Force" to "Jobs." "Labor Force" is the product of the "Population" and the "Labor Participation Fraction" of 0.35, or 35%. "Jobs" is the product of "Business Structures" and 20 "Jobs per Structure." When "Labor Availability" is greater than 1 ("Labor Force" is greater than "Jobs"), businesses are more likely to construct new "Business Structures." When "Labor Availability" falls below 1, the "Labor Availability Multiplier" and "Construction" fall. In equilibrium, "Labor Availability Multiplier" is equal to 1.

3.4.2 The "Population" Sector

The "Population" sector consists of a level called "Population." The initial value of "Population" is 50000 people. "Population" increases through the rates of "Births" and "In Migration," and decreases through the rates of "Deaths" and "Out Migration."

The rate of "Births" is a constant percentage of the "Population." Approximately 1.5 people are born per 100 people per year.

The rate of "Deaths" is also a constant percentage of the "Population." The "Average Lifetime" of a person is approximately 67 years.

The rate of "Out Migration" equals 8% of the existing "Population."

The rate of "In Migration" is also a product of "Population" and several factors:

---

[6] In system dynamics, rates are often defined as a level multiplied by one or more factors. These factors can be aggregated into the format of a "normal" value and one or more multipliers. The normal represents the ratio of the rate to the level under standard conditions of equilibrium, while the multipliers all vary around the value of 1 to include the effects of varying conditions on the system. It is considered good practice if the multipliers are all equal to 1 in equilibrium. A later section of Road Maps will discuss how this can be achieved.

The "In Migration Normal" of 8% is the fraction of the "Population" that immigrates into the city each year under normal conditions.

The "Job Attractiveness Multiplier" depends on "Labor Availability." When there are many available jobs ("Labor Availability" is less than 1 because "Labor Force" is smaller than "Jobs"), the "Job Attractiveness Multiplier" increases to values greater than 1. People are inclined to move to the city. When available jobs are scarce ("Labor Availability" is greater than 1 because "Labor Force" is greater than "Jobs"), the "Job Attractiveness Multiplier" decreases to values between 0 and 1. People tend not to immigrate to the city.

3.5 Exploration of the Urban Growth Model[7]

2. State the initial working hypothesis as to why "Business Structures" and "Population" initially grow and eventually become stagnant. Use the description above for help with the answer.

3. Using the instructions from Section 3.4, map out in STELLA the two interacting sectors of the "Business Structures" and the "Population."

4. Define the elements in the model and enter the model equations correctly into the computer. Make sure to indicate clearly any numerical assumptions not given above (e.g., values assumed in nonlinear "graphical functions"). Try to make the model clear so that others can understand its structure easily. Then simulate the model. Did the model produce a behavior similar to that predicted earlier?

5. What causes the growth of "Population" and "Business Structures" of an urban area during the early years of its development? Use the structure of the model to answer this question. How does the answer relate to the answer to question 2?

[7] Suggested answers to the following questions are included in Section 5.

6. What is the behavior of "Labor Availability" during the life cycle of an urban area? What causes the behavior?

7. How does the finite "Land Area" come to limit growth? Will all the available "Land Area" be filled up in equilibrium?

Use the model to show the effects of annexing surrounding land to increase the "Land Area" to 7500 acres. Initialize the model in equilibrium.[8] Use the STEP function in STELLA to increase the "Land Area" in year 10. How does the behavior change? Will all the "Land Area" be filled up in equilibrium?

8. Does the assumption of a fixed "Land Area" invalidate the model? Most cities can and have expanded from their original areas. How does such expansion influence the results given by the model? For example, what would be the likely consequences of expanding the "Land Area" within which the city is allowed to grow? Simulate the model with a sequence of such expansions, say every 20 years. (The reader might want to refer back to the analysis in question 7.)

[8] To initialize the model in equilibrium, first find the equilibrium values of the two levels, "Business Structures" and "Population," from the base run of the model. Then enter the equilibrium values as initial values of the respective levels. The model should now remain in equilibrium during the entire simulation.

4. EXERCISE 2: MENTAL SIMULATION

Mental simulation is a key tool of system dynamics. It is often tempting to run a model immediately after building it on the computer. It is equally tempting to trust the results from the first simulation run and assume that the modeler has adequately explored the system. In order for system dynamics modeling to be a useful tool, the modeler must be able to mentally simulate models. Mental simulation forces the modeler to thoroughly understand the reasons behind the model's behavior, and helps find problems of computer models.

In the following exercise, the reader will use the model shown in Figure 1 to perform mental simulation.

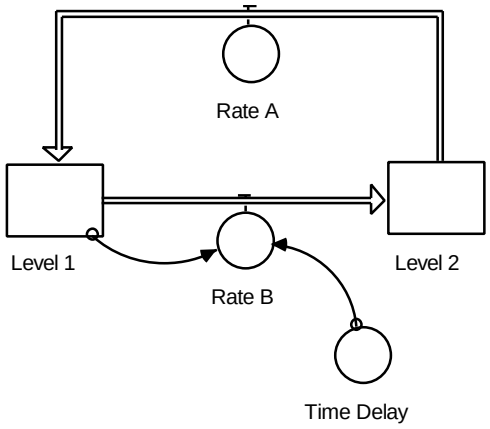


Figure 1: A simple model used for practicing mental simulation skills

The model in Figure 1 is analogous to the physical system shown in Figure 2.

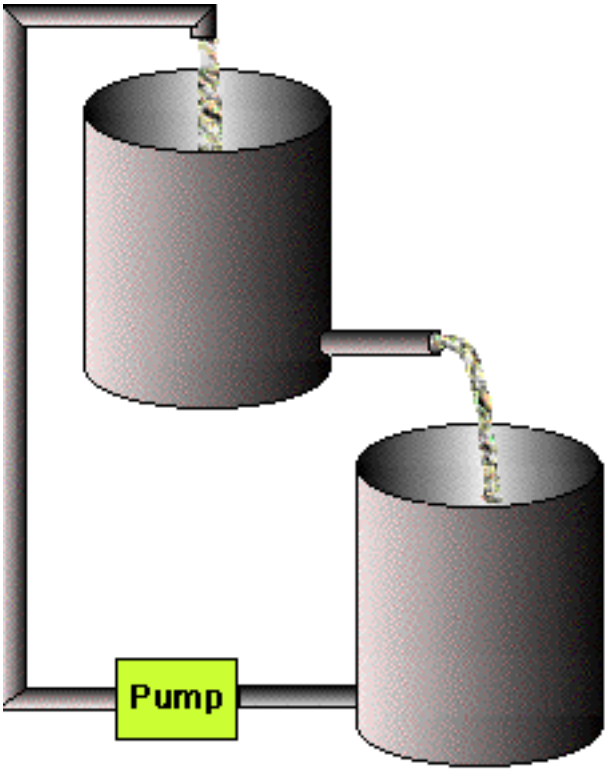


Figure 2: The physical equivalent

In the physical system, the level of water in the containers corresponds to “Level 1” and “Level 2.” The pump corresponds to a constant “Rate A.” “Rate B” represents the spigot on the first container that flows at a rate proportional to the level of fluid in the upper container (“Level 1”).

The model is set up with the following conditions:

? “Level 1” is initially 0 gallons
? “Level 2” is initially 4 gallons
? The “Time Delay” is equal to 2 minutes
? “Rate A” is equal to 1 gallon per minute
? “Rate B,” in gallons per minute, is defined by:

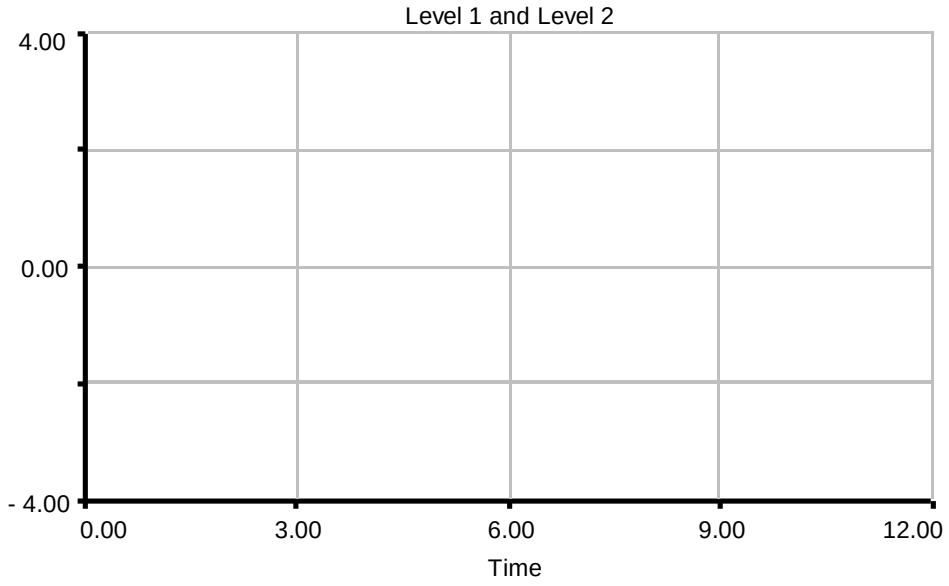
$$\textbf{Rate B} = \frac{\textbf{Level 1}}{\textbf{Time Delay}}$$

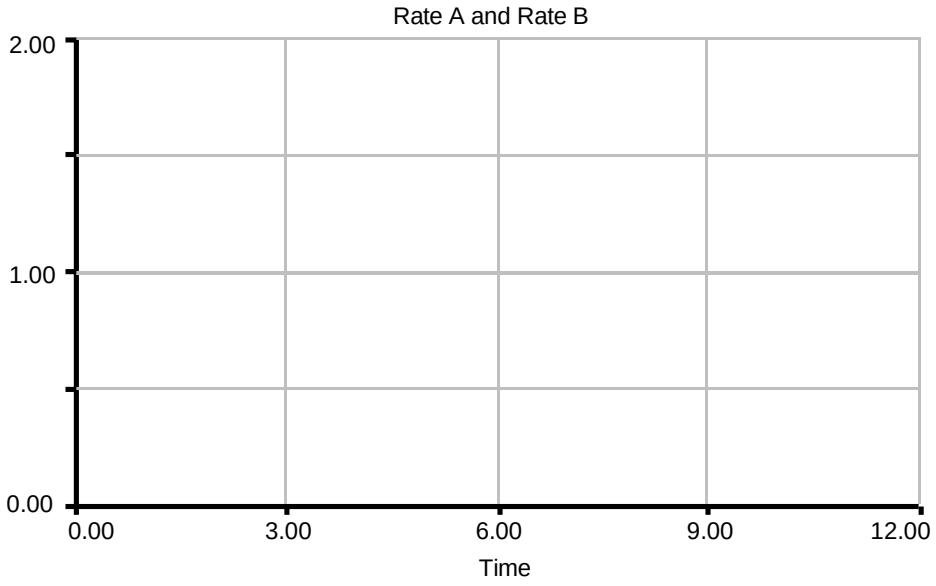
4.1 Exploration of the Mental Simulation Model

In the following exercises the reader will mentally simulate the model under three different conditions. Include as much information relating to the behavior of the model as possible. Only simple calculations are necessary. Some things that the reader might want to include are:

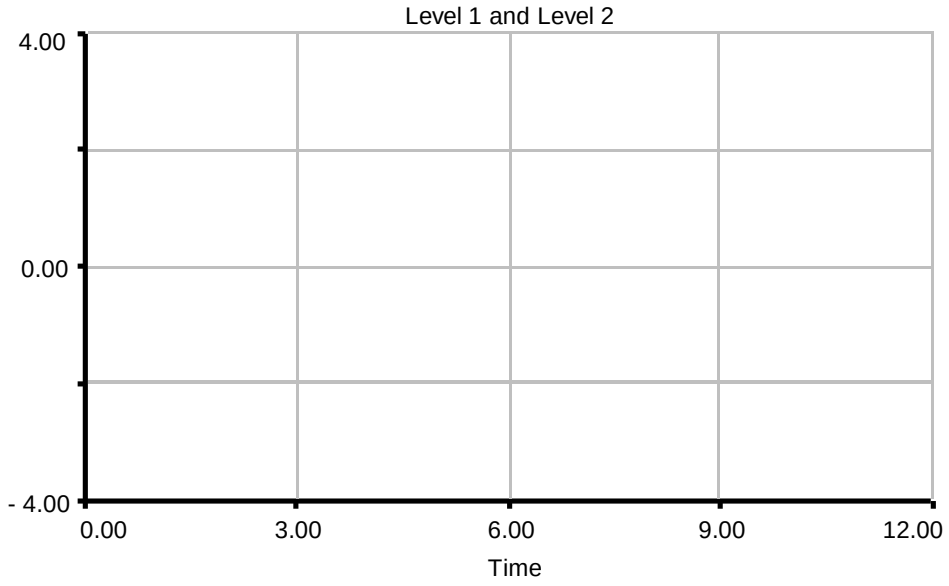
- ? initial and final value for “Level 1” and “Level 2”
- ? initial slope of “Level 1” and “Level 2”
- ? time constant of exponential behavior
- ? time-scale over which behavior occurs

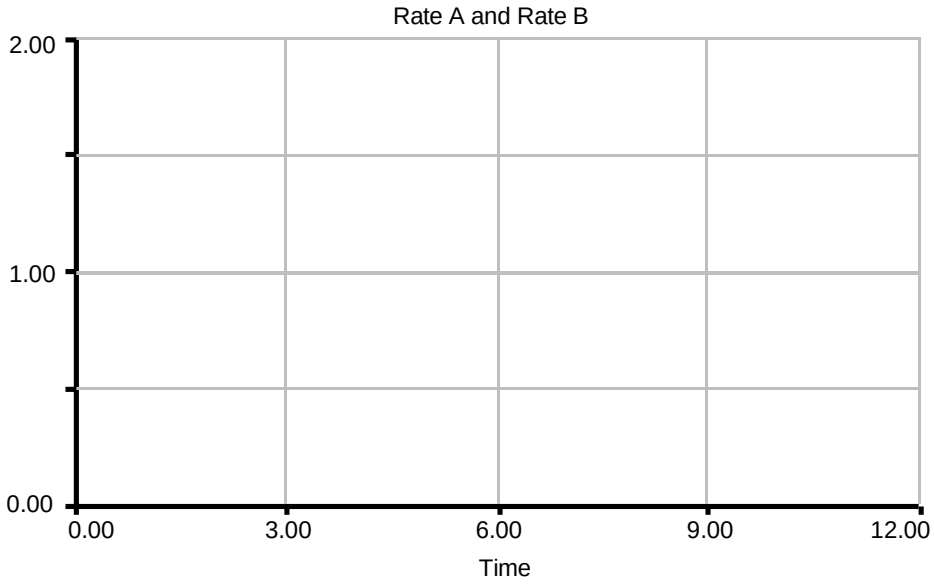
1. Mentally simulate the model with the initial conditions described in the previous section ("Level 1" = 0; "Level 2" = 4). Sketch the behavior of all model components on the blank graphs. Include as much information as possible.



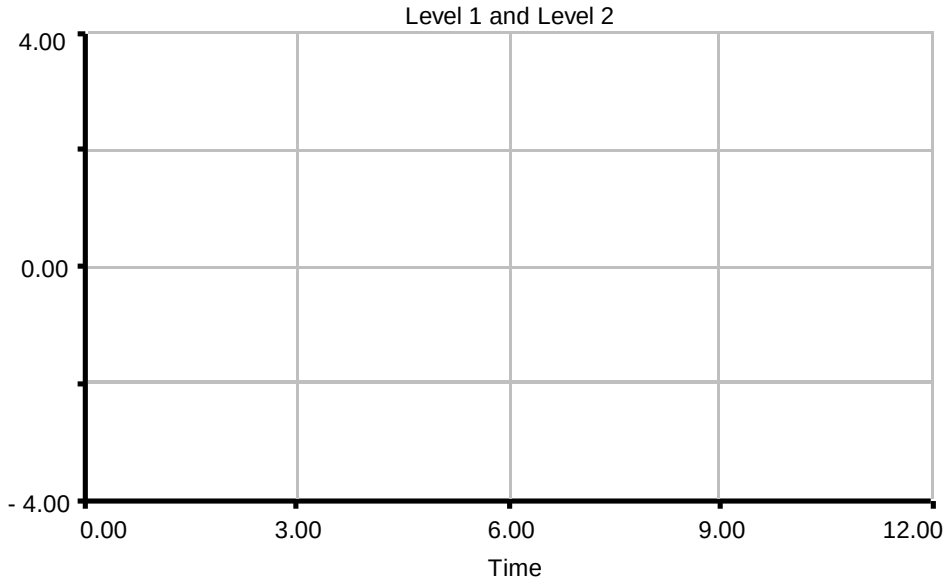


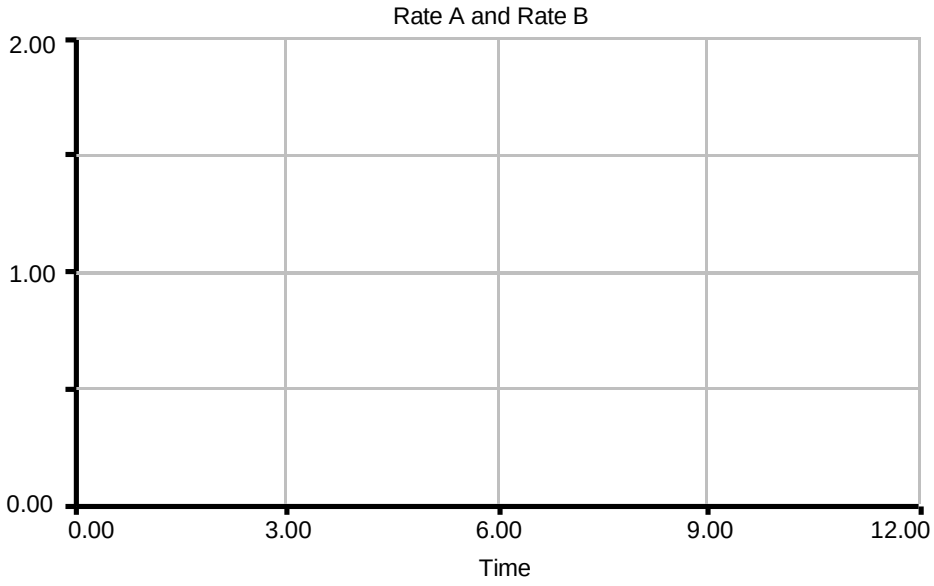
2. Mentally simulate the same scenario ("Level 1" = 0; "Level 2" = 4). This time, change the value of "Time Delay" to 4.





3. Now, assume that the “Time Delay” is equal to 2 again, but “Level 1” is initialized at 4 and “Level 2” at 0. Sketch what happens.





5. SOLUTION TO URBAN DYNAMICS EXERCISE

The following is one possible set of solutions to the questions posed in the exercise on urban growth.

1. Problem Statement: The rapid early growth of "Population" and "Business Structures" of an urban system ultimately gives way to a subsequent stagnant equilibrium characterized by a tight, static job market. After the initial growth, the total number of job opportunities per worker declines slowly, as "Business Structures" and "Population" move toward equilibrium. The reason for building the urban growth model is to explain the characteristic behavior of S-shaped growth that the system exhibits. The key variables of the system are "Business Structures," "Population," and "Labor Availability." The time horizon over which the behavior occurs is approximately 60 to 80 years. Figure 3 shows a graph of the expected behavior of the key variables. The sketch shows the transition to equilibrium with high "Labor Availability" (unemployment) that was described in Section 3.1.

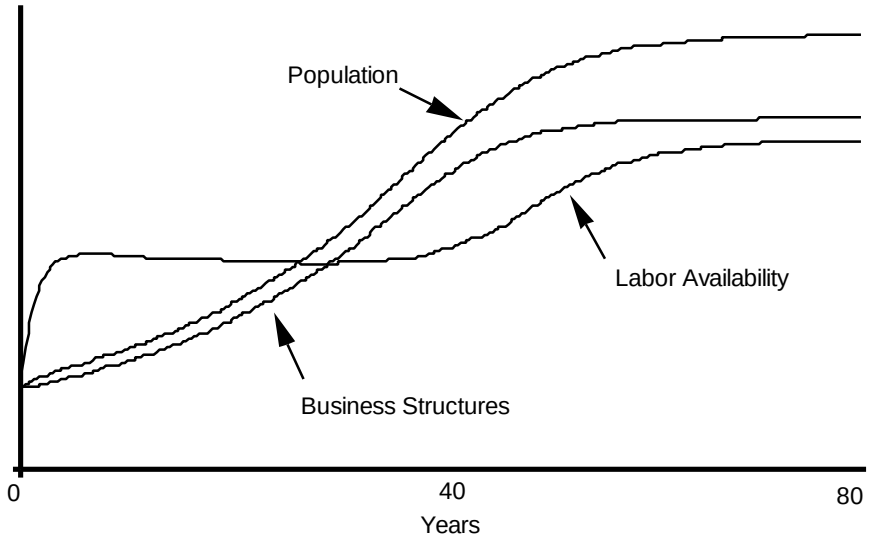


Figure 3: System behavior

2. The “Construction” of “Business Structures” is encouraged when “Land Fraction Occupied” is low, but becomes constrained as the “Land Area” fills up. People are attracted to the city at first when the “Job Attractiveness Multiplier” is high. As “Population” increases, “Labor Availability” increases to keep pace with business expansion, thereby encouraging further development. The success of previous investments also attracts more development. Eventually, the increased “Land Fraction Occupied” decreases the “Land Availability Multiplier,” and thus causes “Construction” to fall. The employment situation deteriorates because business “Construction” no longer keeps pace with demand for jobs. The city stagnates with a job shortage and most of the “Land Area” is occupied. The job shortage offsets the inherent attractiveness of the area to potential migrants.

3. Figure 4 shows a STELLA model for the urban growth system. Detailed model equations are included in Section 7.1 of the Appendix.

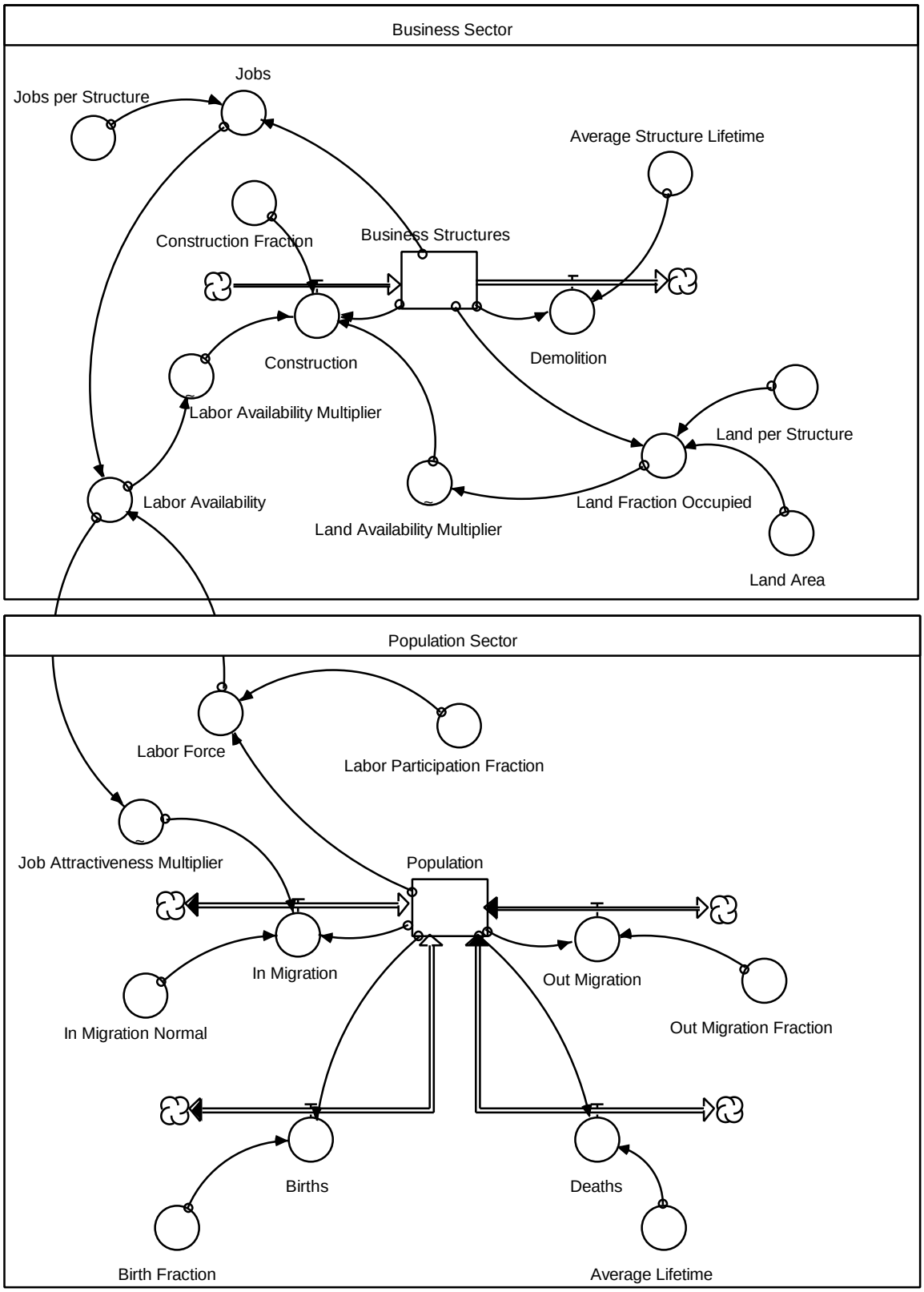


Figure 4: STELLA model of urban growth and stagnation

4. The model is a simple implementation of the description given in Section 3.4. The "Business Structures" sector consists of one stock, "Business Structures," which is filled by "Construction" and drained by "Demolition."

"Demolition" is defined by the familiar average lifetime formulation: the number of existing "Business Structures" divided by the "Average Structure Lifetime."

"Construction" is defined by the number of existing "Business Structures" multiplied by a "Construction Fraction" and two multipliers, the "Land Availability Multiplier" and the "Labor Availability Multiplier."

The "Land Availability Multiplier" incorporates the effect of limited development "Land Area" on the "Construction" rate. It is determined by the "Land Fraction Occupied," which is the amount of land occupied by existing "Business Structures" divided by the total "Land Area" available for commercial development. For low values of "Land Fraction Occupied," increasing business development will lead to higher values for the "Land Availability Multiplier." The high values occur because of the more advanced infrastructure associated with denser development. As the "Land Fraction Occupied" increases further, more desirable land is used up and land prices begin to rise, so "Construction" decreases. Eventually, when all the "Land Area" is used up and the "Land Fraction Occupied" is 1, "Construction" must fall to zero. The graphical function for the "Land Availability Multiplier" is shown in Figure 5.

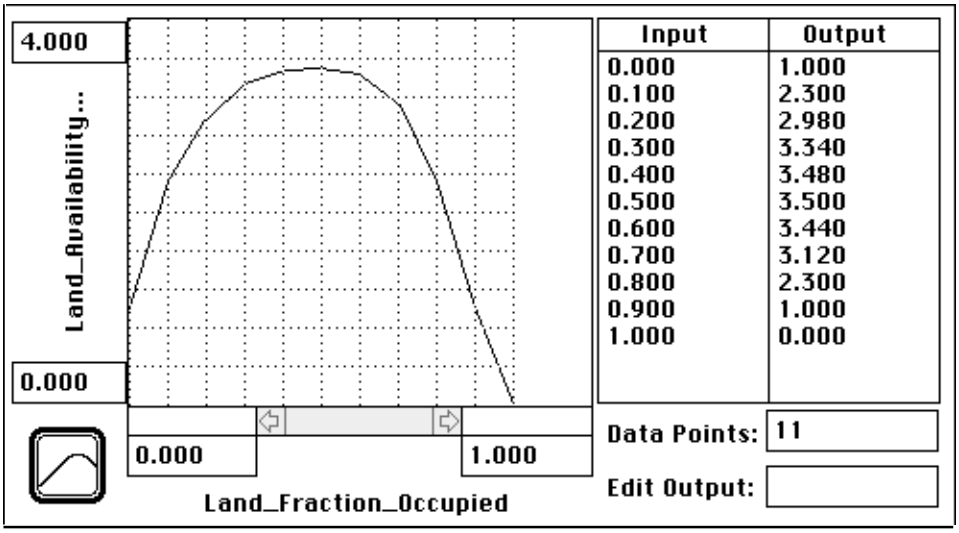


Figure 5: "Land Availability Multiplier" as a function of "Land Fraction Occupied"

The "Labor Availability Multiplier" incorporates the effect of varying human resources on the "Construction" of new "Business Structures." It is determined by the value of "Labor Availability," the ratio of "Labor Force" to "Jobs." When "Labor Availability" is low, there is little incentive to construct new "Business Structures" because employees are difficult to find. The "Labor Availability Multiplier" does not go all the way to zero because some "Construction" will still occur based on the investors' assumption that the creation of "Jobs" will cause "In Migration." As "Labor Availability" increases, so does the incentive to construct new "Business Structures." Eventually, the "Labor Force" exceeds the number of "Jobs" to a great enough extent that the effects of high unemployment outweigh the advantages of high "Labor Availability," so the "Labor Availability Multiplier" stops increasing and levels off. The graphical function for the "Labor Availability Multiplier" is shown in Figure 6.

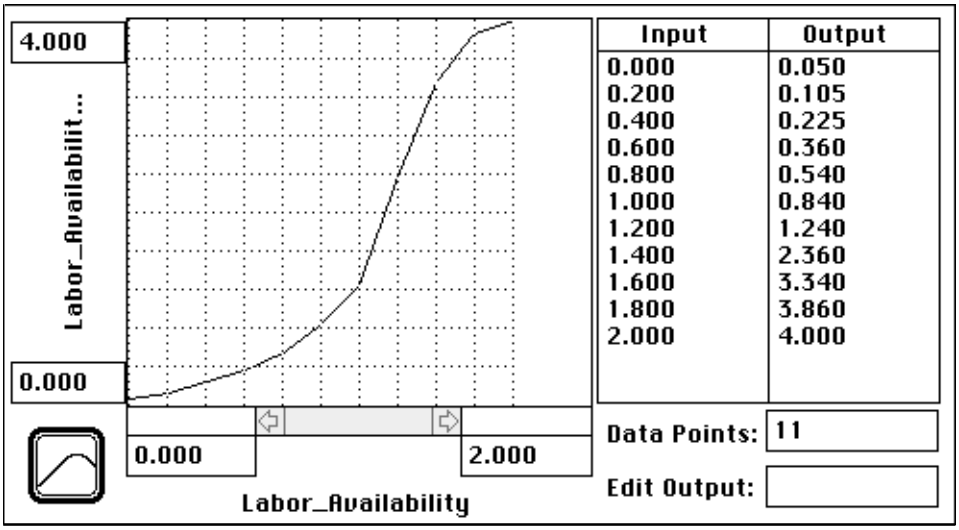


Figure 6: "Labor Availability Multiplier" as a function of "Labor Availability"

The "Population" sector contains the "Population" stock that is affected by "Births," "Deaths," "In Migration," and "Out Migration." "Births" are equal to 1.5% of the "Population" per year. "Deaths" are equal to the "Population" divided by the

"Average Lifetime" of 67 years. "Out Migration" is assumed to be a constant 8% of the "Population" per year. "In Migration" is defined as the "Population" multiplied by the "In Migration Normal" of 8% per year and a "Job Attractiveness Multiplier."

The "Job Attractiveness Multiplier" shows the effect of the "Labor Availability" on the number of people migrating into the urban area. When "Labor Availability" is low, the job market is very favorable to the working part of the "Population." The number of "Jobs" exceeds the "Labor Force," so good "Jobs" are relatively easy to come by. Naturally, people are attracted to move into the urban area, so the "Job Attractiveness Multiplier" is greater than 1. The curve is almost flat at the far left because it is assumed that when the conditions are already extremely favorable, small changes one way or the other will have little effect. As the "Labor Availability" increases, the employment conditions begin to decrease, so the tendency to migrate into the area declines. Eventually, when the conditions are extremely poor, the "Job Attractiveness Multiplier" falls to nearly zero. At this point, there is almost no "In Migration." The graphical function of "Job Attractiveness Multiplier" is shown in Figure 7.

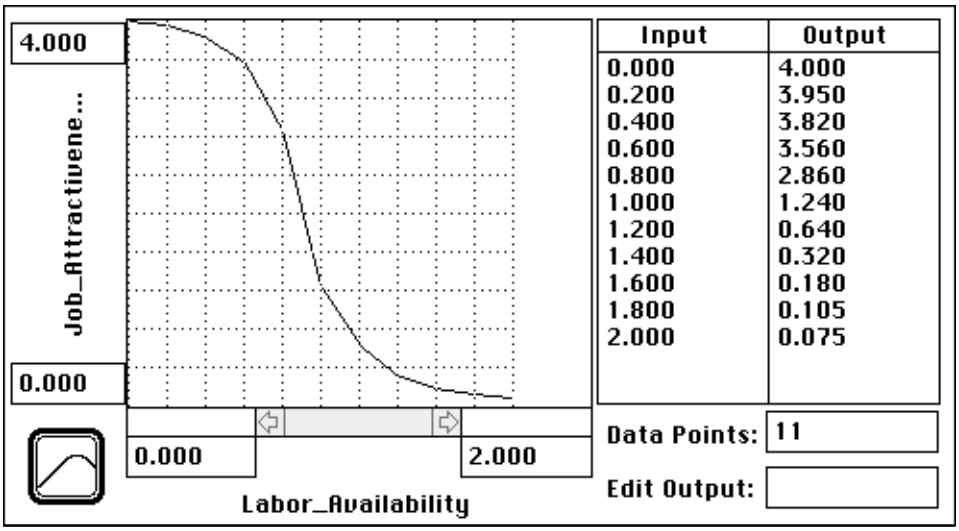


Figure 7: "Job Attractiveness Multiplier" as a function of "Labor Availability"

Figure 8 shows the base run of the urban growth model.

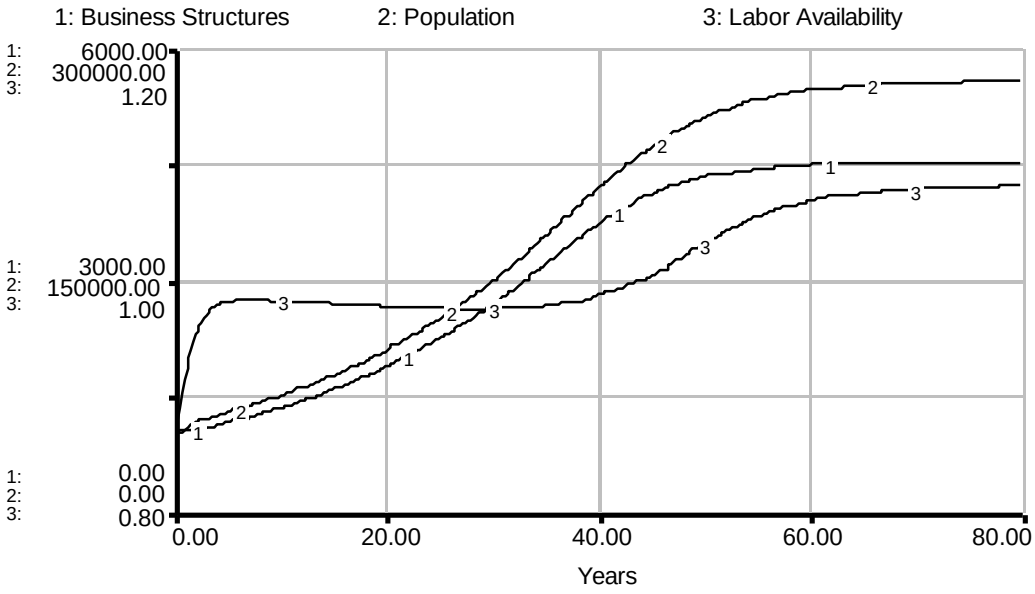


Figure 8: Base run of the Urban Growth Model

5. In the first portion of the model simulation, the city is attractive to residents and to people planning to move to the city because "Jobs" are readily available ("Labor Availability" is less than 1). The increasing "Population" and the success of previous investments demonstrate the potential for further growth and the profitability of "Construction" of "Business Structures." The development of the urban infrastructure caused by business expansion permits further development to occur. The continued availability of "Land Area" during the growth stage allows business "Construction" to keep pace with the increasing "Population" so people continue to be attracted by the job opportunities in the area. The hypothesis stated in the answer to question 2 correctly identifies the reasons for the initial growth.

6. The quick initial increase in "Labor Availability" is due to the initial values of "Population" and "Business Structures" that are "out of balance" with one another. After the initial growth phase of "Business Structures" and "Population," during which "Labor Availability" declines slightly (in other words, job availability improves slightly), "Jobs" become less available to the average job-seeker ("Labor Availability" starts increasing). The lower availability of "Jobs" occurs because "Jobs" begin rising slower than "Population" once the "Land Area" constraint slows business "Construction." The increase in "Labor Availability" (corresponding to a decline in job availability) continues

until a sufficiently large unemployment rate is generated to discourage further "In Migration." As noted earlier, in equilibrium, a job shortage must exist in the system in order to offset the inherent attractiveness of the area.

7. The constraint of a fixed "Land Area" limits growth by making additional "Construction" of "Business Structures" less and less economically justifiable. The model reaches equilibrium when the "Land Availability Multiplier" equals 1 and thus reduces "Construction" until it is equal to "Demolition." Because "Construction" must be greater than zero, the "Land Fraction Occupied" must be less than 1 in equilibrium. In the base run, 90% of the "Land Area" is occupied in equilibrium.

Figure 9 shows the effect of annexing 50% additional "Land Area" to allow further urban expansion. This is a common policy used to counter the effects of urban stagnation. The model was initialized in equilibrium with the final equilibrium values from the base run. In year 10, the available "Land Area" was raised from 5000 to 7500 acres. Figure 9 shows that expanding the "Land Area" available creates a new expansion of "Business Structures" and "Population." The expansion causes employment conditions to improve, but the relief is only temporary and the same problem of high "Labor Availability" in equilibrium exists. Again only 90% of the "Land Area" is occupied in equilibrium.

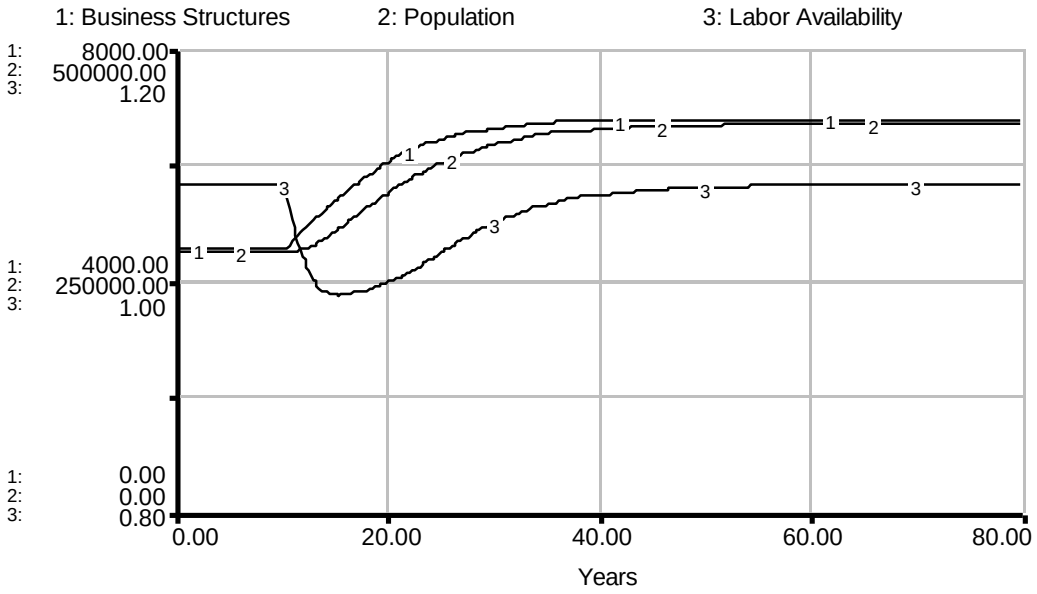


Figure 9: Equilibrium for 50% step increase in available "Land Area"

8. Assuming a fixed “Land Area” simplifies real life. Cities do expand their geographical boundaries, sometimes dramatically (consider Los Angeles and Mexico City). When total “Land Area” available is expanded in the simple two-level model, “Business Structures” and “Population” grow to fill the larger “Land Area.” However, the system still equilibrates with high “Labor Availability” (i.e., unemployment). If a series of expansions of the “Land Area” available is tested, each expansion will stimulate a further growth spurt. Each growth spurt will lower “Labor Availability” in the short term, due to increased business “Construction.” However, as each growth period gives way to a new equilibrium, “Labor Availability” will rise again to its previous equilibrium value. Therefore, expanding the city's geographic limits will lead to a temporary improvement in the availability of “Jobs,” but the improvement will eventually be reversed as the new “Land Area” is filled. The high unemployment equilibrium can only be avoided if the city continues to grow indefinitely. Because no city can expand indefinitely, analyzing growth within a fixed “Land Area” becomes a useful approach to studying the types of pressures that can limit growth.

6. SOLUTION TO MENTAL SIMULATION EXERCISE

6.1 A Note on Time Constants

An understanding of time constants is useful in mental simulation of models. A time constant is a number that relates a flow to the stock it affects. The definition of a time constant is the reciprocal of the number which, multiplied by the level, defines the rate. The model in Figure 11 shows one of the simplest examples of a time constant.

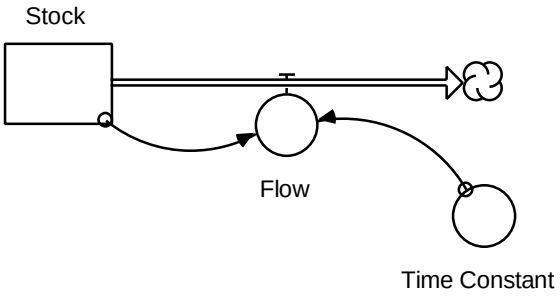


Figure 11: A simple model using a time constant

The rate equation in the model in Figure 11 is:

$$\textbf{Flow} = \frac{\textbf{Stock}}{\textbf{Time Constant}}$$

The model contains one negative feedback loop. The "Time Constant" affects the "Flow" that drains the "Stock." The structure gives the following behavior for the "Stock":

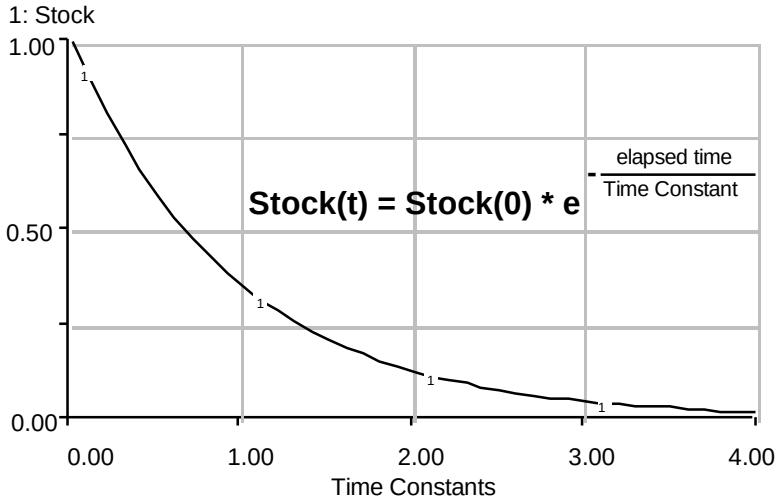


Figure 12: Behavior of a single negative loop[9]

6.2 Mental Simulation Exercise Solution

1. Initial model run: Figures 13 and 14 show the behavior of the model. Both levels converge asymptotically to an equilibrium valuethat can be found by setting the net rate at each level equal to zero. Equilibrium occurs when "Rate A" equals "Rate B." When "Level 1"/"Time Constant" equals 1, the system will be in equilibrium, and the equilibrium value for "Level 1" is 2. "Level 2" is affected by the same rates as "Level 1," but in the opposite direction. As a result, any change in "Level 1" is mirrored by "Level 2." Because "Level 1" increased by 2, "Level 2" decreased by 2 to a final value of 2.

The slope of the graph of a level is simply the net rate entering the level. At the beginning of the simulation, "Level 1" is empty, so "Rate B" is 0. Therefore the only non-zero rate is "Rate A" which equals 1. The initial slope of "Level 1" is +1 and the initial slope of "Level 2" is –1.

The rest of the behavior can be determined using the "Time Constant" of the system, which is 2. The shape of the curve can be determined accurately with the chart and description outlined above. In Figure 13, the first three time constants are labeled.

[9] Mathematically inclined readers may realize that the negative sign in the exponent causes asymptotic growth. If the exponent is positive, growth is exponential.

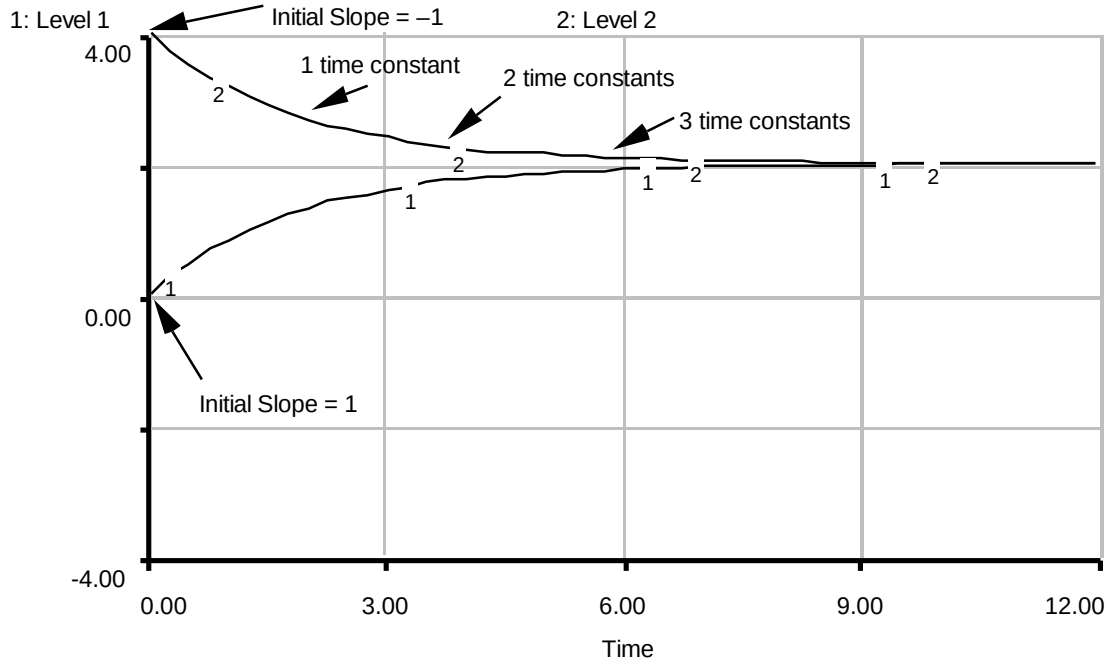


Figure 13: First run behavior of "Level 1" and "Level 2"

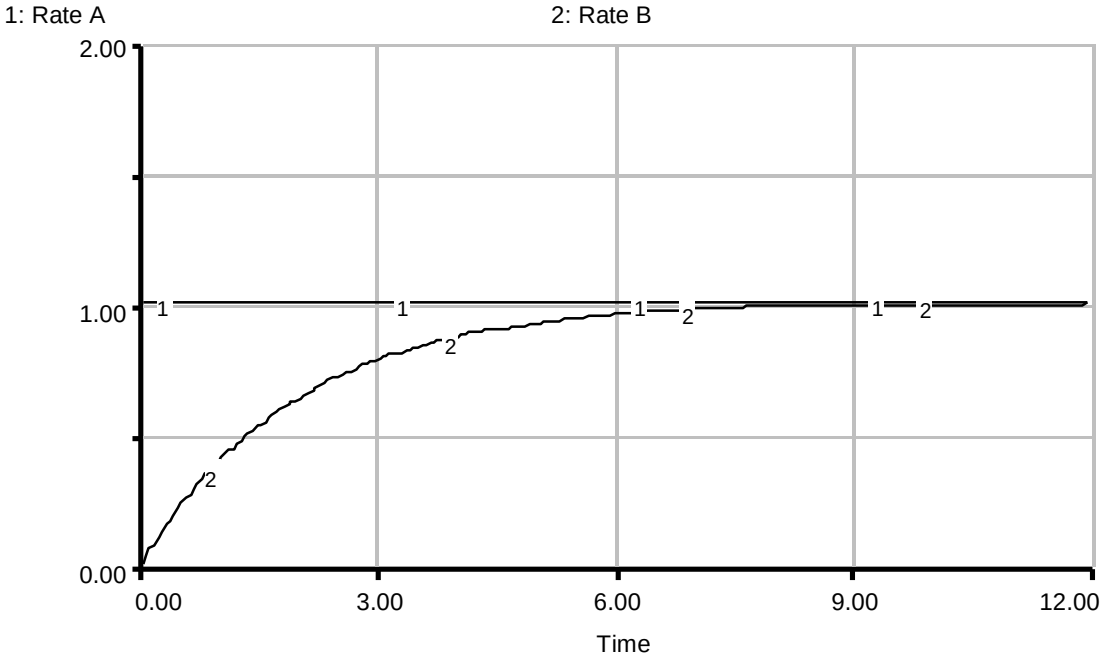


Figure 14: First run behavior of "Rate A" and "Rate B"

2. Changing the "Time Delay": Figures 15 and 16 show the behavior of the second run. For the second run, the "Time Delay" was changed from 2 to 4. As a result, the equilibrium value for "Level 1" moved from 2 to 4. In addition, the approach to equilibrium was half the speed because the time constant was twice as large. As before, the change in "Level 2" mirrored "Level 1," so "Level 2" went from 4 to 0.

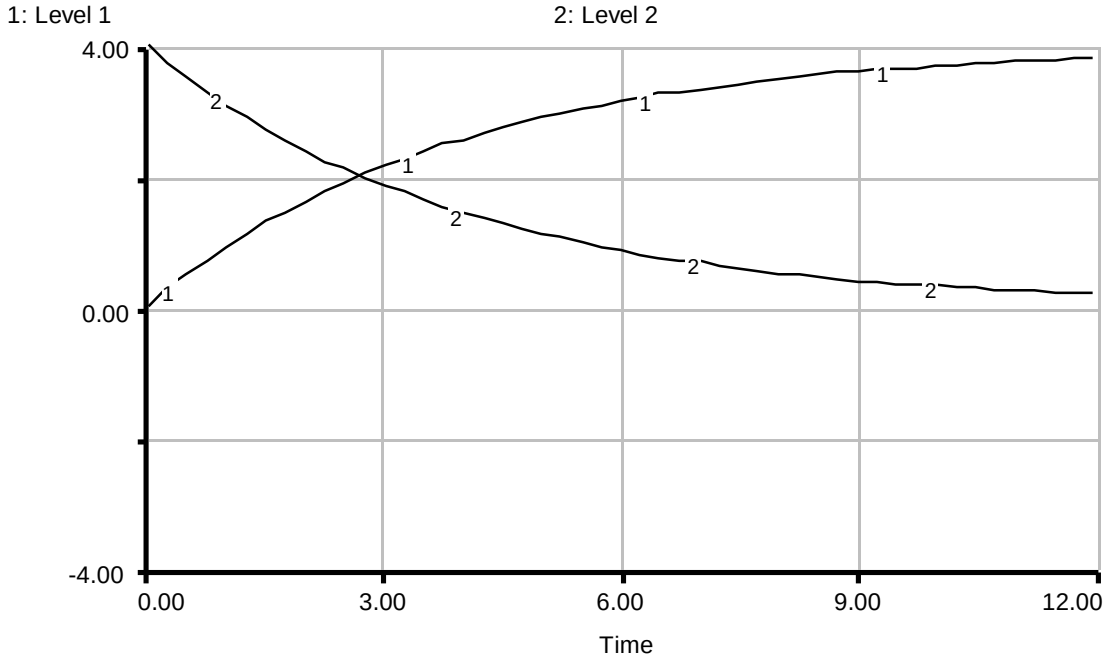


Figure 15: Second run behavior of "Level 1" and "Level 2"

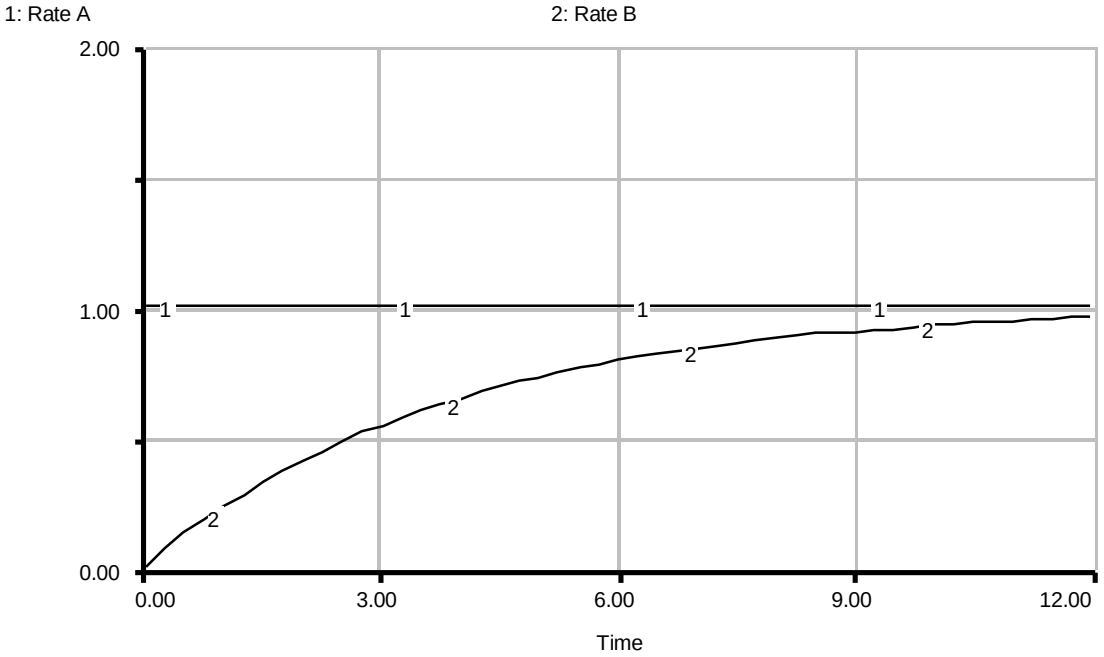


Figure 16: Second run behavior of "Rate A" and "Rate B"

3. Changing initial conditions: Figures 17 and 18 show the behavior of the model when the initial value of "Level 1" is changed from 0 to 4, and "Level 2" is initialized at 0. For the third model run, the time constant and equilibrium value for "Level 1" are the same as in the first run. However, because the initial value of "Level 1" is different, the model takes a different path to equilibrium. The first noticeable changes are the initial slopes for the levels. Because "Rate B" is not initially 0, but rather 2, the initial slopes of "Level 1" and "Level 2" are –1 and +1 respectively (the reverse of the first run). Again, the behavior of "Level 2" mirrors that of "Level 1," so "Level 2" increases from 0 to 2.

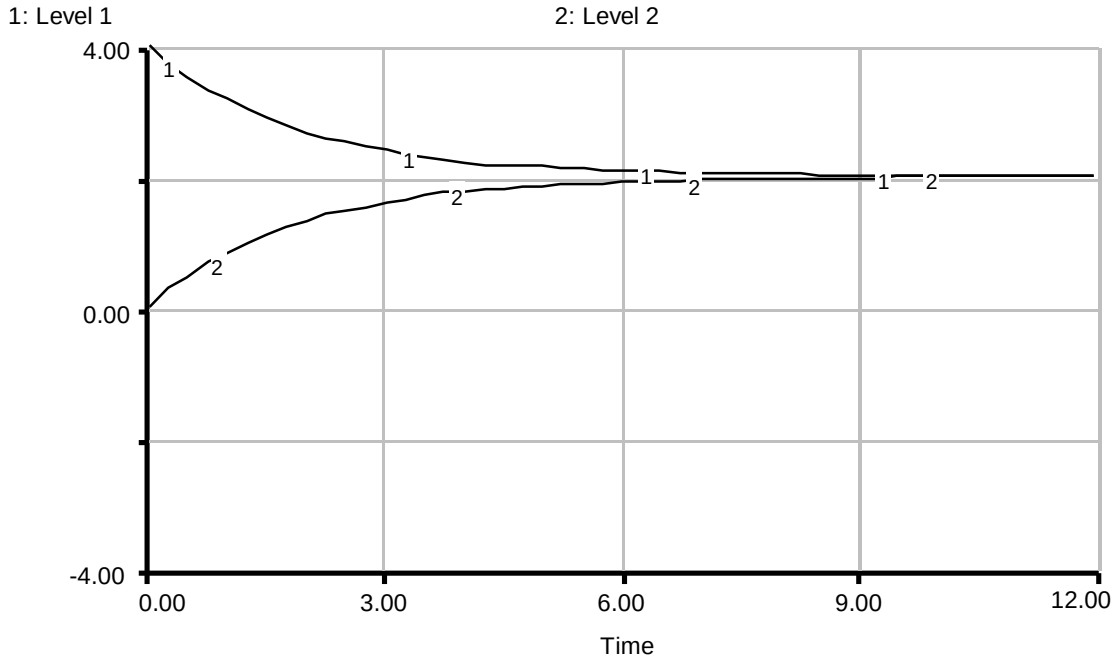


Figure 17: Third run behavior of "Level 1" and "Level 2"

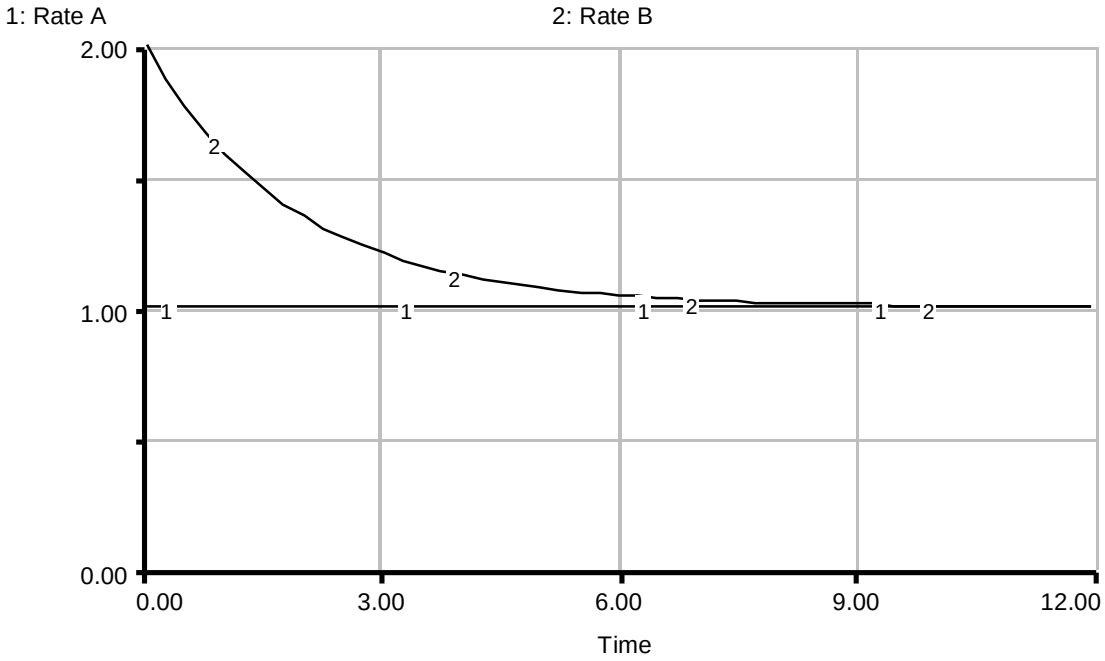


Figure 18: Third run behavior of "Rate A" and "Rate B"

7. APPENDIX

7.1 Equations for Urban Growth Model

Business Sector

□ **Business_Structures(t)** = Business_Structures(t - dt) +
(Construction - Demolition) * dt
INIT Business_Structures = 1000
DOCUMENT: Number of business structures in the city.
Units: structures

☞ INFLOWS:
Construction = Business_Structures * Construction_Fraction *
Labor_Availability_Multiplier *
Land_Availability_Multiplier
DOCUMENT: Rate of construction of business structures. It is affected by the number of already existing business structures, a normal construction fraction, and the availability of land and labor.
Units: structures/year

☞ OUTFLOWS:
Demolition = Business_Structures/Average_Structure_Lifetime
DOCUMENT: Rate of demolition of business structures.
Units: structures/year

○ **Average_Structure_Lifetime** = 50
DOCUMENT: Average lifetime of a business structure.
Units: years

○ **Construction_Fraction**= 0.02
DOCUMENT: Normal rate of construction of business structure per existing business structure.
Units: 1/year

○ **Jobs** = Business_Structures * Jobs_per_Structure
DOCUMENT: Number of jobs provided by the existing business structures. It is the product of the number of business structures and the average number of jobs per structure.
Units: jobs

○ **Jobs_per_Structure** = 20
DOCUMENT: The number of jobs provided by each structure.
Units: jobs/structure

○ **Labor_Availability** = Labor_Force/Jobs

DOCUMENT: The ratio between the labor force and the number of available jobs.
Units: people/job

○ **Land_Area** = 5000
DOCUMENT: Total land area available for commercial development.
Units: acres

○ **Land_Fraction_Occupied** = Business_Structures * Land_per_Structure/Land_Area
DOCUMENT: The fraction of the commercial land that has already been developed.
Units: dimensionless

○ **Land_per_Structure** = 1
DOCUMENT: The amount of land required by each business structure.
Units: acres/structure

⊘ **Labor_Availability_Multiplier** = GRAPH(Labor_Availability)
(0.00, 0.05), (0.2, 0.105), (0.4, 0.225), (0.6, 0.36), (0.8, 0.54), (1.00, 0.84), (1.20, 1.24), (1.40, 2.36), (1.60, 3.34), (1.80, 3.86), (2.00, 4.00)
DOCUMENT: The multiplier shows the effect of Labor Availability on Construction. When Labor Availability is greater than 1, businesses are more likely to construct new structures.
Units: dimensionless

⊘ **Land_Availability_Multiplier** = GRAPH(Land_Fraction_Occupied)
(0.00, 1.00), (0.1, 2.30), (0.2, 2.98), (0.3, 3.34), (0.4, 3.48), (0.5, 3.50), (0.6, 3.44), (0.7, 3.12), (0.8, 2.30), (0.9, 1.00), (1, 0.00)
DOCUMENT: The multiplier shows the effect of availability of land on business construction. When nearly all the land is available, Construction is slow because there are very few businesses around to provide capital. As more of the Land Area is occupied, the effect on Construction increases. As the available Land Area is used up, Construction drops until it is zero at maximum capacity.
Units: dimensionless

Population Sector

□ **Population(t)** = Population(t-dt)+(In_Migration + Births - Out_Migration - Deaths)*dt
INIT Population = 50000
DOCUMENT: The number of people living in the urban area.
Units: people

INFLOWS:
In_Migration = Population * In_Migration_Normal * Job_Attractiveness_Multiplier

DOCUMENT: The number of people who move into the urban area each year. It is affected by the current population, a normal fraction of in-migration, and the availability of jobs.
units: people/year

♂Births = Population * Birth_Fraction
DOCUMENT: The number of people born in the area per year.
Units: people/year

♂OUTFLOWS:
Out_Migration = Population * Out_Migration_Fraction
DOCUMENT: The number of people who leave the urban area each year.
Units: people/year

♂Deaths = Population / Average_Lifetime
DOCUMENT: Number of people who die each year.

Units: people/year

○**Average_Lifetime** = 66.7
DOCUMENT: The average lifetime of a person living in the urban area is approximately 67 years.
Units: years

○**Birth_Fraction** = 0.015
DOCUMENT: The fraction of the population that reproduces each year.
Units: 1/year

○**In_Migration_Normal** = 0.08
DOCUMENT: The fraction of the population that immigrates each year under normal conditions.
Units: 1/year

○**Labor_Force** = Population * Labor_Participation_Fraction
DOCUMENT: The number of people who are eligible to work. It is a constant fraction of the population.
Units: people

○**Labor_Participation_Fraction** = 0.35
DOCUMENT: The fraction of the total population that is willing and able to work.
Units: dimensionless

○**Out_Migration_Fraction** = 0.08
DOCUMENT: The fraction of the population that emigrates each year.
Units: 1/year

⊘ **Job_Attractiveness_Multiplier** = GRAPH(Labor_Availability)
(0.00, 4.00), (0.2, 3.95), (0.4, 3.82), (0.6, 3.56), (0.8, 2.86), (1.00, 1.24), (1.20, 0.64), (1.40, 0.32), (1.60, 0.18), (1.80, 0.105), (2.00, 0.075)
DOCUMENT: The multiplier shows the effect of Labor Availability on immigration. When there are many available jobs (Labor Availability is less than 1), people are inclined to move to the city. When there are not enough available jobs (Labor Availability is greater than 1), people tend not to immigrate to the urban area.
Units: dimensionless

7.2 Equations for Mental Simulation Model

□ **Level_1(t)** = Level_1(t - dt) + (Rate_A - Rate_B) * dt
INIT Level_1 = 4
Units: gallons

☞ INFLOWS:
Rate_A = 1
Units: gallons/minute

☞ OUTFLOWS:
Rate_B = Level_1/Time_Delay
Units: gallons/minute

□ **Level_2(t)** = Level_2(t - dt) + (Rate_B - Rate_A) * dt
INIT Level_2 = 0
Units: gallons

☞ INFLOWS:
Rate_B = Level_1/Time_Delay
Units: gallons/minute

☞ OUTFLOWS:
Rate_A = 1
Units: gallons/minute

○ **Time_Delay** = 2
Units: minutes

Vensim Examples:

Modeling Exercises

Section 2

Aaron Diamond & Nathaniel Choge
October 2001

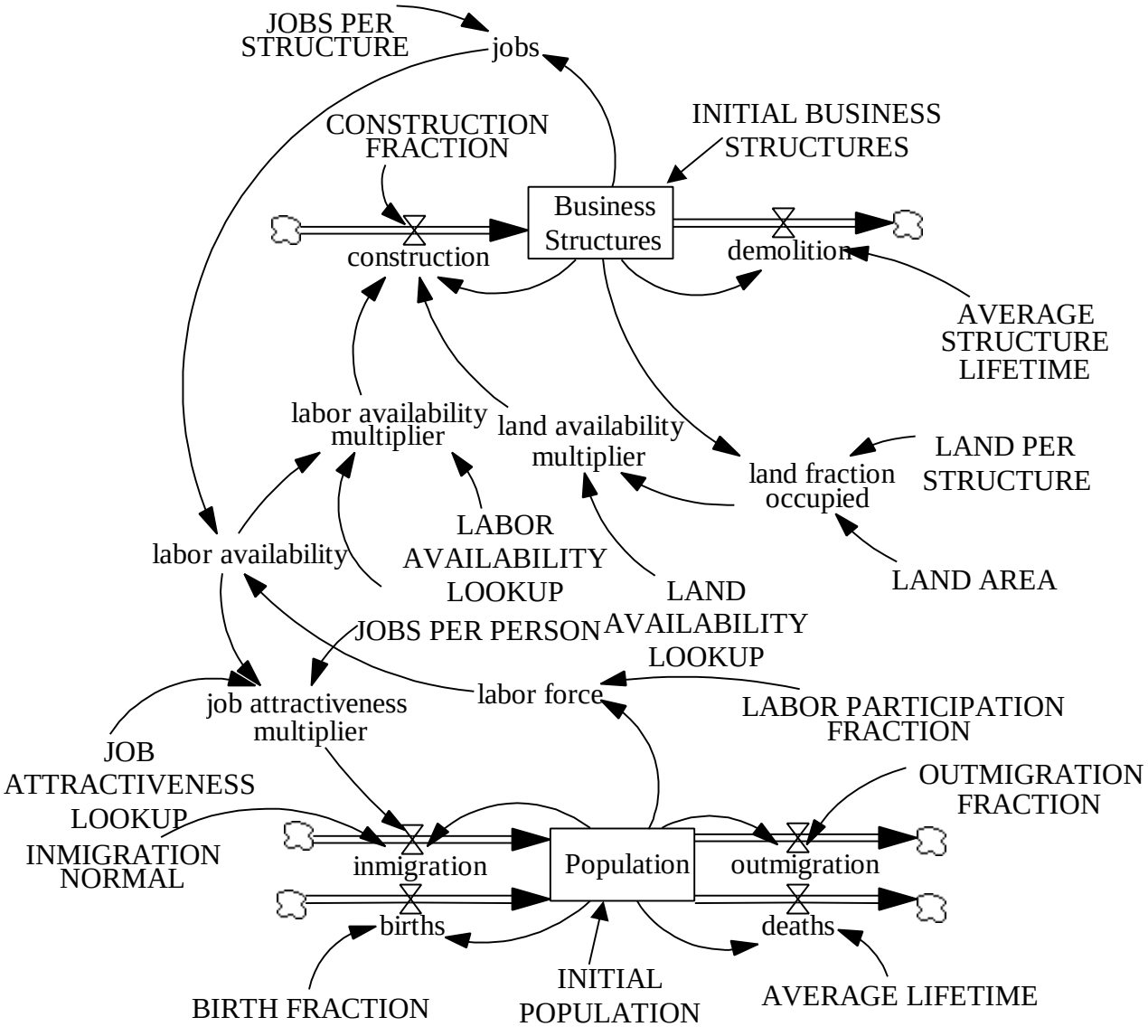


Figure 19: Vensim equivalent of Figure 4: model of urban growth and stagnation

Documentation for the Urban Growth Model:

(01) AVERAGE LIFETIME=66.7
Units: year
The average lifetime of a person living in the urban area is approximately 66.7 years.

(02) AVERAGE STRUCTURE LIFETIME=50
Units: year
Average lifetime of a business structure.

(03) BIRTH FRACTION=0.015
Units: 1/year
The fraction of the Population that reproduces each year.

(04) births=Population*BIRTH FRACTION
Units: people/year
The number of people born in the area per year.

(05) Business Structures= INTEG (construction-demolition, INITIAL BUSINESS STRUCTURES)
Units: structures
Number of business structures in the city.

(06) construction=Business Structures*CONSTRUCTION FRACTION*labor availability multiplier*land availability multiplier
Units: structures/year
Rate of construction of business structures. It is affected by the number of already existing business structures, a normal construction fraction, and the availability of land and labor.

(07) CONSTRUCTION FRACTION=0.02
Units: 1/year
Normal rate of construction of business structure per existing business structure.

(08) deaths=Population/AVERAGE LIFETIME
Units: people/year
Number of people who die each year.

(09) demolition=Business Structures/AVERAGE STRUCTURE LIFETIME
Units: structures/year
Rate of demolition of business structures.

(10) FINAL TIME = 80
Units: year
The final time for the simulation.

(11) INITIAL BUSINESS STRUCTURES=1000
Units: structures

(12) INITIAL POPULATION=50000
Units: people

(13) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(14) inmigration=Population*INMIGRATION NORMAL*job attractiveness multiplier
Units: people/year
The number of people who move into the urban area each year. It is affected by the current Population, a normal fraction of in-migration, and the availability of jobs.

(15) INMIGRATION NORMAL=0.08
Units: 1/year
The number of pepole who are eligible to work. It is a constant fraction of the population.

(16) JOB ATTRACTIVENESS LOOKUP=([(0,0)(10,10)],(0,4),(0.2,3.95),(0.4,3.82),(0.6,3.56),(0.8,2.86),(1,1.24),(1.2,0.64),(1.4,0.32),(1.6,0.18),(1.8,0.105),(2,0.075))
Units: dmnl

(17) job attractiveness multiplier=JOB ATTRACTIVENESS LOOKUP (labor availability/JOBS PER PERSON)
Units: dmnl
The multiplier shows the effect of labor availability on inmigration. When there are many available jobs (labor availability is less than 1), people are inclined to move to the city. When there are not enough available jobs (labor availability is greater than 1), people tend not to immigrate to the urban area.

(18) jobs=Business Structures*JOBS PER STRUCTURE
Units: jobs
Number of jobs provided by the existing structures. It is the product of the number of business structures and the average number of jobs per structure.

(19) JOBS PER PERSON=1
Units: people/jobs

(20) JOBS PER STRUCTURE=20

Units: jobs/structures
The number of jobs provided by each structure.

(21) labor availability=labor force/jobs
Units: people/jobs
The ratio between the labor force and the number of available jobs.

(22) LABOR AVAILABILITY LOOKUP([(0,0)-(10,10)],(0,0.05), (0.2,0.105), (0.4,0.225),(0.6,0.36),(0.8,0.54),(1,0.84),(1.2,1.24),(1.4,2.36),(1.6,3.34),(1.8,3.86),(2,4))
Units: dmnl

(23) labor availability multiplier=LABOR AVAILABILITY LOOKUP (labor availability/JOBS PER PERSON)
Units: dmnl
The multiplier shows the effect of labor availability on Construction. When labor availability is greater than 1, businesses are more likely to construct new structures.

(24) labor force=Population*LABOR PARTICIPATION FRACTION
Units: people
The number of people who are eligible to work. It is a constant fraction of the population.

(25) LABOR PARTICIPATION FRACTION=0.35
Units: dmnl
The fraction of the total Population that is willing and able to work.

(26) LAND AREA=5000

Units: acres
The total land area available for commercial development.

(27) LAND AVAILABILITY LOOKUP([(0,0)-(10,10)],(0,1),(0.1,2.3),(0.2,2.98),(0.3,3.34),(0.4,3.48),(0.5,3.5),(0.6,3.44),(0.7,3.12),(0.8,2.3),(0.9,1),(1,0))
Units: dmnl
The multiplier shows the effect of availability of land on business construction. When nearly all the land is available, construction is slow because there are very few businesses
around to provide capital. As more of the LAND AREA is occupied, the effect on construction increases. As the available LAND AREA is used up, construction drops until it is zero at maximum capacity.

(28) land availability multiplier=LAND AVAILABILITY LOOKUP (land fraction occupied)
Units: dmnl

(29) land fraction occupied=Business Structures*LAND PER STRUCTURE/LAND AREA
Units: dmnl
The fraction of the commercial land that has already been
developed.

(30) LAND PER STRUCTURE=1
Units: acres/structures
The amount of land required by each business structure.

(31) outmigration=Population*OUTMIGRATION FRACTION
Units: people/year
The number of people who leave the urban area each year.

(32) OUTMIGRATION FRACTION=0.08

Units: 1/year

The fraction of the Population that emigrates each year.

(33) Population= INTEG (births+inmigration-deaths-outmigration, INITIAL POPULATION)

Units: people

The number of people living in the urban area.

(34) SAVEPER = TIME STEP

Units: year

The frequency with which output is stored.

(35) TIME STEP = 0.0625

Units: year

The time step for the simulation.

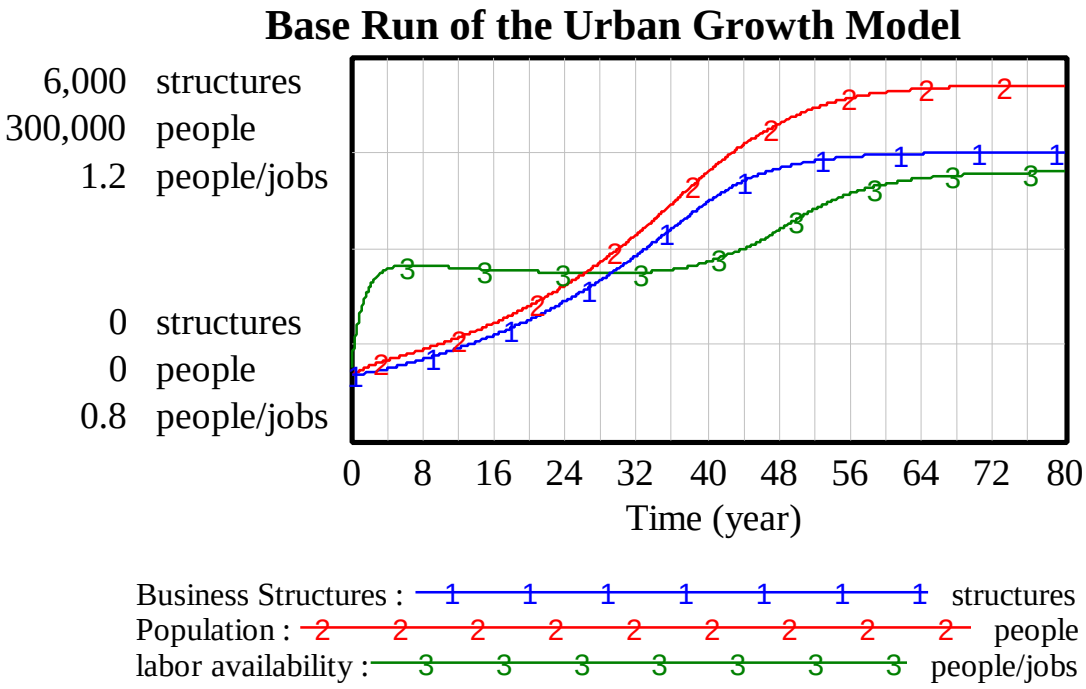


Figure 20: Vensim equivalent of Figure 8: Base run of the Urban Growth Model

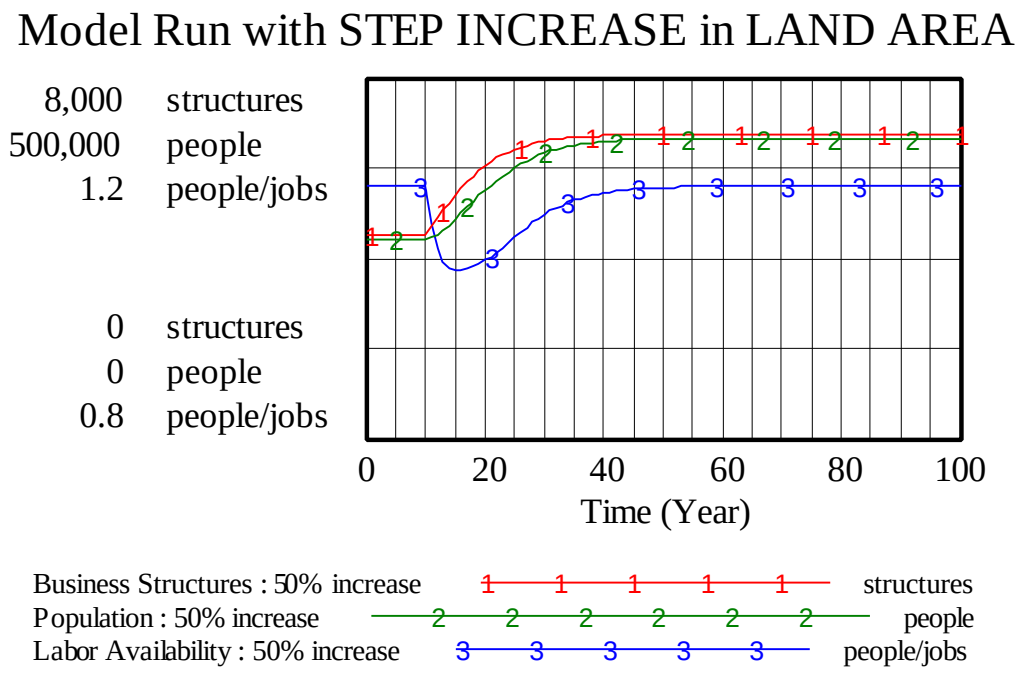


Figure 21: Vensim correction of Figure 9: Equilibrium for 50% step increase in available "LAND AREA"

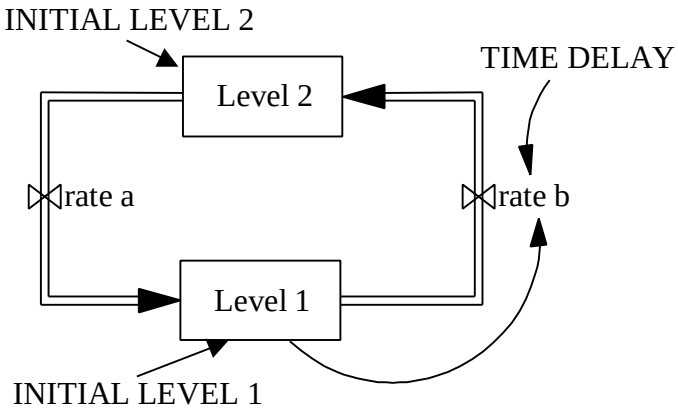


Figure 22: Vensim Equivalent of Figure 1: A simple model used for practicing mental simulation skills.

Documentation for Mental Simulation Model:

(01) FINAL TIME = 12
Units: minutes
The final time for the simulation.

(02) INITIAL LEVEL 1=4
Units: gallons

(03) INITIAL LEVEL 2=0
Units: gallons

(04) INITIAL TIME = 0
Units: minutes
The initial time for the simulation.

(05) Level 1= INTEG (rate A-rate B,INITIAL LEVEL 1)
Units: gallons

(06) Level 2= INTEG (+rate B-rate A,INITIAL LEVEL 2)
Units: gallons

(07) rate a=1
Units: gallons/minutes

(08) rate b=Level 1/TIME DELAY
Units: gallons/minutes

(09) SAVEPER = TIME STEP

Units: minutes

The frequency with which output is stored.

(10) TIME DELAY=2

Units: minutes

(11) TIME STEP = 0.0625

Units: minutes

The time step for the simulation.

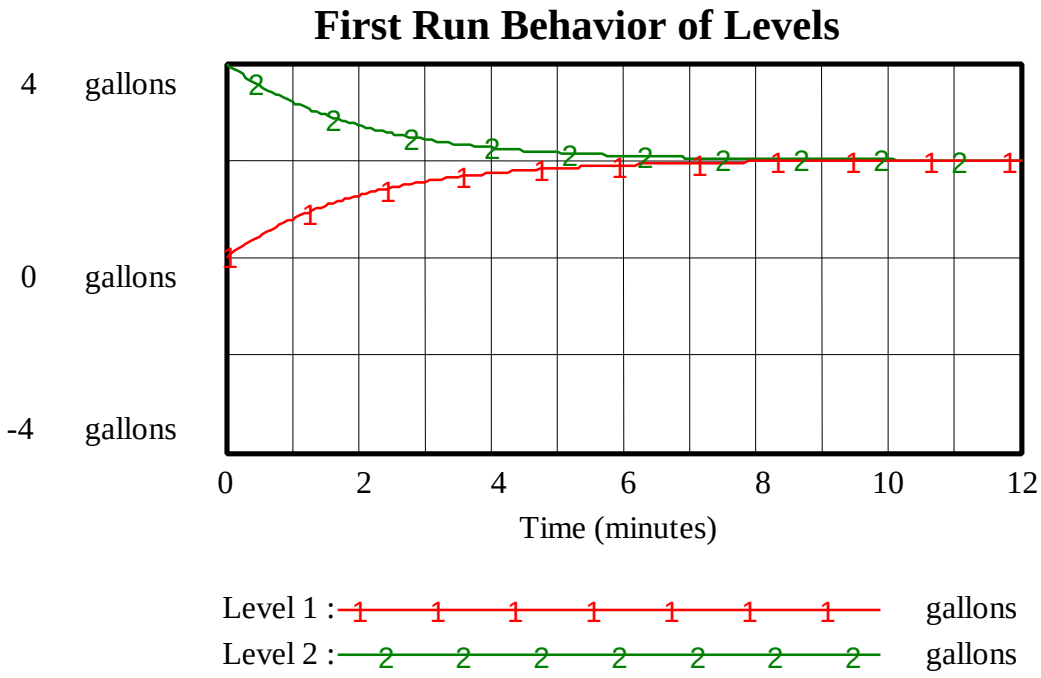


Figure 23: Vensim equivalent of Figure 13: First run behavior of "Level 1 and Level 2" (Level 1 is initially 0 and level 2 is initially 4 for the first and second runs)



Figure 24: Vensim equivalent of Figure 14: First run behavior of “rate a” and “rate b”



Figure 25: Vensim equivalent of Figure 15: Second run behavior of “Level 1” and “Level 2”

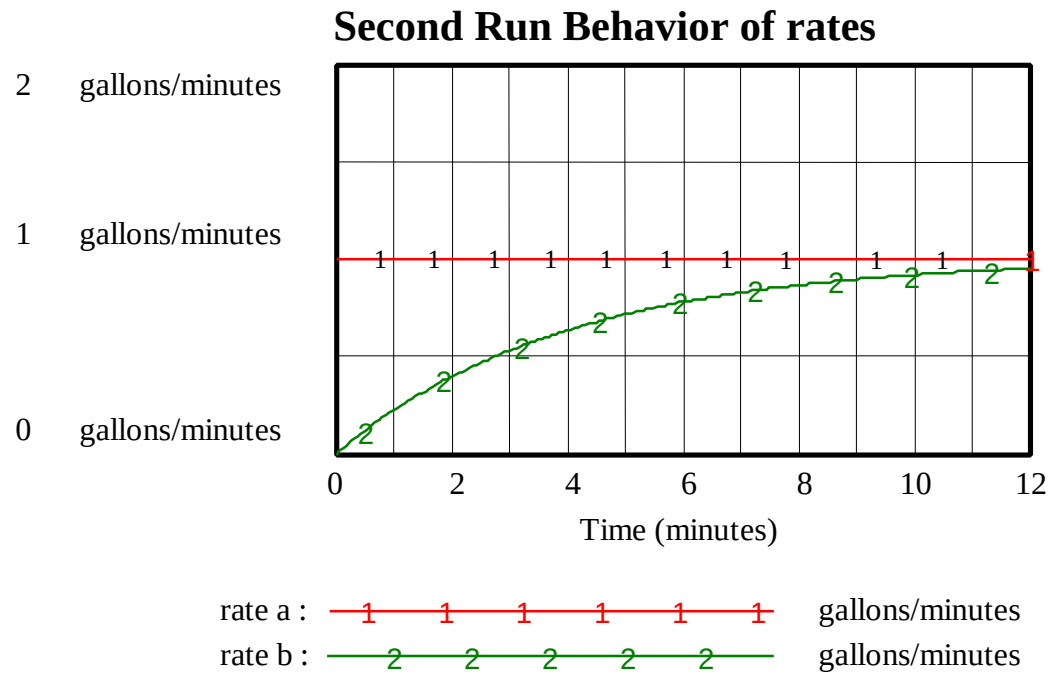


Figure 26: Vensim equivalent of Figure 16: Second run behavior of “rate a” and “rate b”

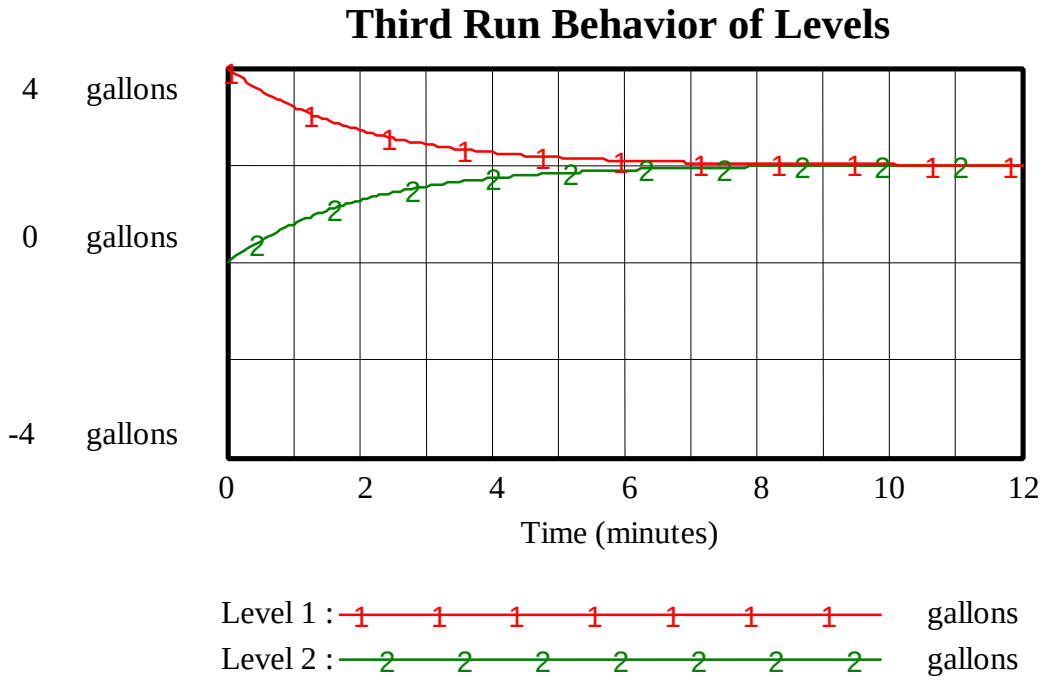


Figure 27: Vensim equivalent of Figure 17: Third run behavior of “Level 1” and “Level 2”

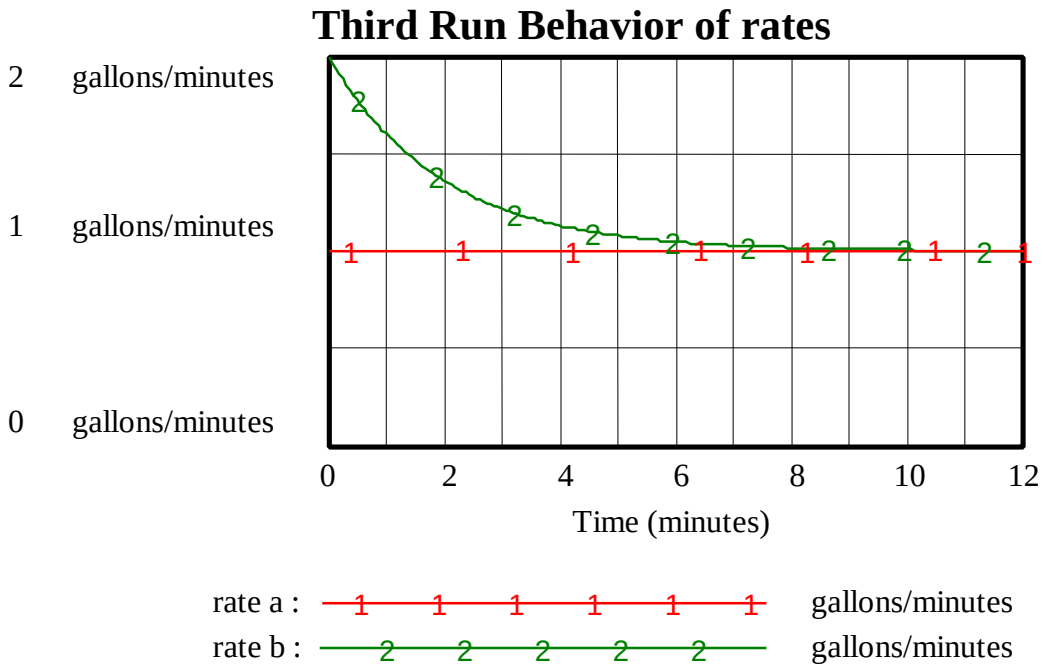


Figure 28: Vensim equivalent of Figure 18: Third run behavior of “rate a” and “rate b”

Graphical Integration Exercises
Part Four:
Reverse Graphical Integration

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Laughton Stanley
August 20, 1996

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1. ABSTRACT

Reverse graphical integration is a method which allows one to quickly estimate the values of a stock's net flow based on a graph of the stock's behavior over time. This technique is useful in cases where a stock's behavior is known and one would like to find the associated flow behavior. This paper covers two simple ways to determine information about a net flow which can be used to draw its graph. The reader should be familiar with the concepts covered in *Graphical Integration Exercises Part Three: Combined Flows*.[1]

[1]Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Three: Combined Flows* (D-4596), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, March 14, 32 pp.

2. INTRODUCTION

When one deals with real systems, the data which describes model variables comes in the form of stocks. The height of a high tide, the number of customers, the size of a city, the balance of a bank account, and the national debt are all stocks. Even values which are thought of as rates come in the form of stock measurements over time. The rate of snowfall is determined by checking the amount of snow which has fallen over time. A typist's speed can be determined by looking at the number of words which have been typed over time. Rates are not instantaneously measurable; all information about rates comes to us through stock values.

Determining a rate from real world stock values is helpful when writing rate equations for a model. Being able to sketch a graph of a flow from a graph of a stock can make working with stock data more informative.

The first three papers in this series discussed how to look at inflows and outflows to determine the behavior of a stock. This paper covers methods for doing the reverse: graphically determining flow information from a stock graph.

3. REVERSE GRAPHICAL INTEGRATION

3.1 Overview

The method of determining rate information from stocks, which this paper shall refer to as *reverse graphical integration*, is also known in calculus as the graphical form of *differentiation*. Although reverse graphical integration may be done with mathematical equations or with calculations on a computer, for the purposes of understanding a system's flows, this level of accuracy is time consuming and unnecessary. A graph pad and a few guidelines are all that one needs to effectively determine a stock's net flow.

The amount of information about a stock's inflows and outflows which can be determined from the stock is limited. When several flows are combined together to change the value of a stock, information about the characteristics of the individual flows is lost. For this reason, reverse graphical integration will give the behavior of the stock's net flow,[2] but it will not identify the specific inflows and outflows which may be associated with the system. For example, one could determine that the rate of the world

---

[2] For more information on net flows, see *Graphical Integration Exercises Part Three: Combined Flows* (D-4596).

population's growth is increasing from the world population stock's behavior but one could not determine whether this is the result of a higher birth rate in China or in Mexico.

Reverse graphical integration takes the graph of a stock's behavior over time and produces a graph of the net flow associated with the stock. An easy way to think of this is to divide the stock behavior graph into several vertical sections. Within each of these sections one can calculate the slope of the stock. These slope values provide a rough sketch of the net flow of the stock. This method is the basis of all methods of reverse graphical integration. For more accuracy, all that one must do is divide the graph into smaller vertical sections.[3] However, this is cumbersome. In practice it is easier to divide the graph into sections to get the general shape. Then, if it is necessary, one can calculate the exact net flow at certain points to add accuracy to the net flow graph. If the points picked cover the behavior well, then connecting the dots to produce the graph is a simple operation. Though this may sound confusing, it is not. The following example demonstrates how simple slope calculations may be used to estimate the net flow.

3.2 Example 1: Using Intervals with Linear Stock Behaviors

The Compton widget factory makes widgets and ships them to distributors around the nation. Figure 1 shows a graph of their widget inventory over the past three quarters:

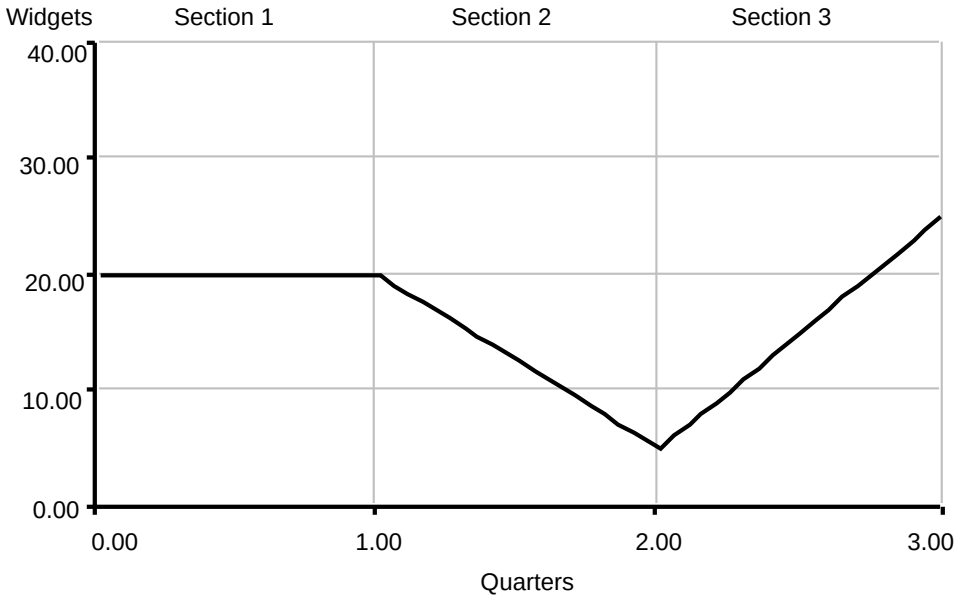


Figure 1: Widget inventory behavior over three months

The widget inventory graph above is the graph of a stock. To aid in determining the graph of the net flow into the stock, the stock graph has been divided into three

[3] Calculus achieves perfect accuracy by dividing the graph into infinitely many sections.

equally spaced sections. In Section 1, the value of the stock remains constant. This indicates that the difference between the initial and final stock values within Section 1 is zero. Hence we say that the value of the net flow over the first quarter is zero even though the actual inflows and outflows may have been in the hundreds of widgets per quarter. All that the graph shows is that the sum of the inflows and sum of the outflows of widgets were equal.

Section 2 is slightly different from Section 1. The stock of widgets decreases linearly from 20 widgets to 5 widgets. This is a difference of negative 15 widgets (final widgets minus initial widgets), thus the net flow over Section 2 is negative 15 widgets per quarter. In Section 3 the inventory changes again, from 5 widgets to 25 widgets. This is a change of 20 widgets per quarter. The results of these measurements can then be plotted on a graph.

The change in stock value over time is also known as the slope.[4] Slope is a quantity which is measured by the change in the value of the vertical scale divided by the change in the value of the horizontal scale for any two given points on a graph. The net flow graph shows the slope of the stock over time, so by definition the change in the stock per unit time is the "net flow" as well as the slope.

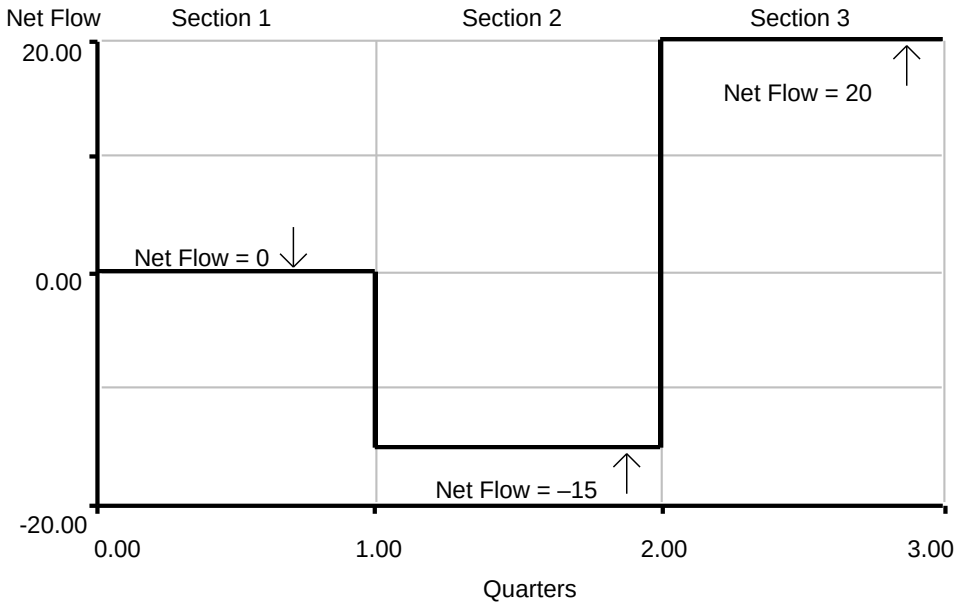


Figure 2: Net Flow of Widgets

[4] For more information on slopes, see: Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571), Systems Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, March 25, 25 pp.

The results of the reverse graphical integration of the widget inventory are shown in Figure 2. Several inferences, delineated in Table 1, can be made from the results. Remember:

Net Flow = Inflow - Outflow.

When the value of a stock is increasing, the inflow must be greater than the outflow, so the net flow will be positive. Likewise, when the value of the stock is decreasing, the outflow will be greater than the inflow so the net flow of the stock will be negative.

Table 1: Summary of net flow behavior for a linearly changing stock

Stock Behavior	Net Flow Behavior
Constant	Over a period where the value of a stock does not change, such as Section 1, the net flow will have a value of zero.
Increasing Linearly	Over a section where the value of the stock increases, as in Section 3, the net flow will be constant and positive.
Decreasing Linearly	Over a section where the value of the stock decreases, as in Section 2, the net flow will be constant and negative.

3.3 Example 2: Using Intervals for More Complicated Behaviors

Shreve's Gator Farm buys and breeds alligators. Inventory records produced the graph in Figure 3.

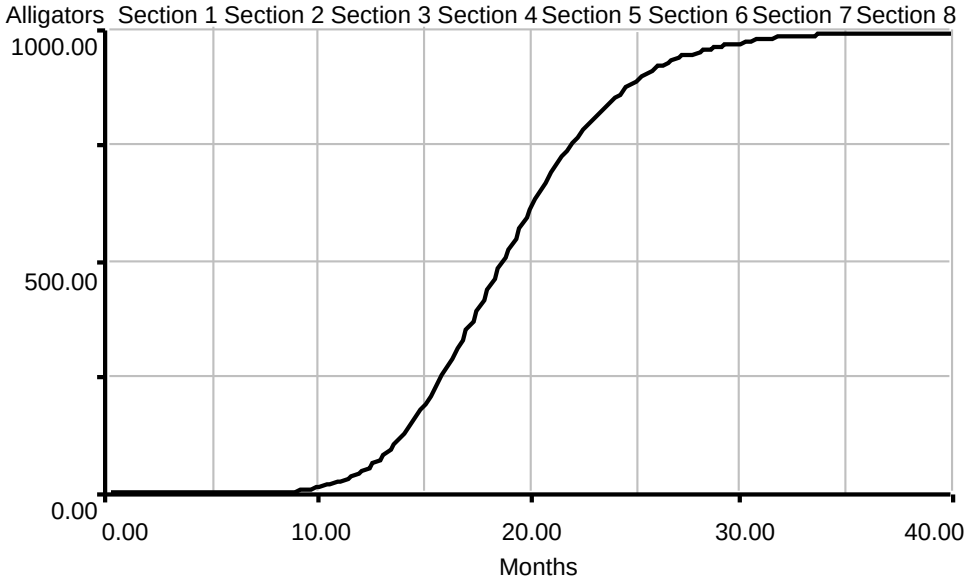


Figure 3: Alligator Inventory

The above graph of Alligator inventory shows S-shaped growth. It has been divided into smaller sections than before because of the greater complexity of the

behavior. When faced with a complicated graph, a good technique is to simply break it up into small pieces.

To graph this net flow, assume that the stock makes approximately a straight line within each section. Graphing the flow then becomes a matter of calculating the slopes and plotting them. The calculations proceed as follows:

By definition,

$$Slope = \frac{F - I}{T}$$

where I is the *initial* value of the stock at the beginning of the section, F is the *final* value of the stock at the end of the section, and T is the length of the section. In this example, T is 5 months for each section.

In Section 1, the value of the stock for the whole section is 10 alligators,[5] so the slope is:

$$\frac{F - I}{T} = \frac{10 - 10}{5} = 0 \text{ alligators/month.}$$

In Section 2, the value of the stock at the beginning of the section is 10 alligators, and the value at the end of the section is 24 alligators. The slope is:

$$\frac{F - I}{T} = \frac{24 - 10}{5} = 2.8 \text{ alligators/month.}$$

The rest of the calculations proceed in the same manner and are summarized in Table 2.

[5] The initial value of the stock is 10 alligators.

Table 2: Summary of the slope calculations for alligator inventory

Section	Change in Stock (alligators)[6]	Change in Time (months)	Slope (alligators/month)
Section 1	0	5	0
Section 2	14	5	2.8
Section 3	180	5	36
Section 4	418	5	83.6
Section 5	278	5	55.6
Section 6	83	5	16.6
Section 7	16	5	3.2
Section 8	1	5	.2

The slope information can be plotted on a graph to reveal the flow behavior as in Figure 4.

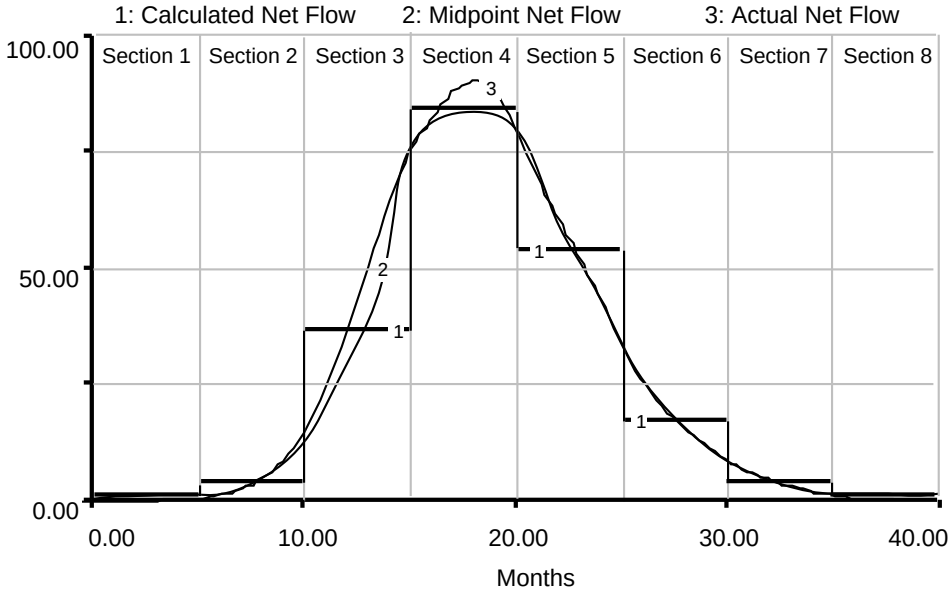


Figure 4: Net flow behavior of Alligator Inventory

Figure 4 shows the net flow behavior from the calculations in the table along with the actual net flow behavior. Drawing a curve which connects the midpoints of the line segments usually makes a good approximation of the actual net flow. The section calculations are helpful, but do not give actual points on the graph. Tangent lines, covered in the next section, are needed to find these specific points.

[6] The numbers in this table are more accurate than one would be able to pull directly off of the Stella graph because they were taken from a Stella table.

It should be noted that sections do not need to be evenly spaced. As long as T is adjusted for the width of each section, the flow will be scaled correctly. Varying the spacing can save calculations in some cases. If the stock is fairly linear over a certain part of the graph, then using one section will be as effective as using multiple sections over that part. Over parts of the graph that curve, more sections should be used because the slope is changing quickly. The greater number of sections will add more definition to the net flow in this area.

3.4 Example 3: Using Tangent Lines

When it is necessary to know the maximum value of the flow to a greater accuracy, tangent lines[7] may be used. Take the graph of the length of a work week in a company with a variable work week policy in Figure 5.

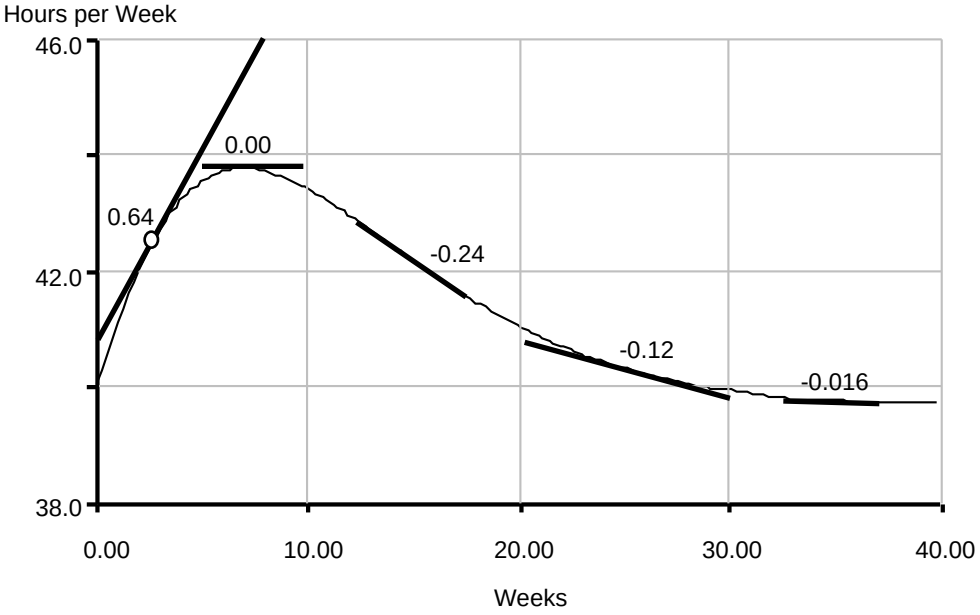


Figure 5: Length of Work Week

The graph shows both the length of the average work week and a series of lines drawn at various points on the curve. These are called tangent lines because they are parallel to the stock line at the point where the two touch. Essentially these lines have the same slope as the stock line at that point. This slope is also the value of the net flow at that point in time.

There is no real trick to drawing tangent lines. One should simply draw a straight line which is parallel to the graph at the point where the graph and the line connect. As

[7] For more information on tangent lines, see *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571).

long as the tangent touches the graph at the point where one is calculating the slope and the tangent line does not cross over the graph line, the tangent line is probably a good one. In some cases the tangent line may intersect the stock at a later or earlier point. Such secondary intersections should simply be ignored. Finding the slope of a tangent line is the same calculation covered in Example 2. Since the slope along the entire tangent line is constant, simply pick two points on the tangent line to use for reference. In Figure 5, the first tangent line has been extended to the sides of the graph for convenience. In the case of this line, it begins with a height of 40.9 hours/week at the 0 week mark. It leaves the edge of the graph at a height of 46 hours/week and a distance of 8 weeks. Recall that

$$Slope = \frac{F - I}{T}.$$

In this case, T is 8 weeks because it is the horizontal distance covered between the two reference points. As before, F is the final height, or 46 hours/week, and I is the first height, or 40.9 hours/week. The resulting slope is

$$\frac{F - I}{T} = \frac{46 - 40.9}{8} = \frac{5.1}{8} = 0.6375$$

which we can round to 0.64. The other tangent lines are calculated in the same manner. Table 4 gives times and net flows calculated using these tangent lines.

Table 3: Summary of tangent line slopes

Time	Net Flow[8]
2 Weeks	0.64 Hours/(Week*Week)
7 Weeks	0.00 Hours/(Week*Week)
14 Weeks	-0.24 Hours/(Week*Week)
25 Weeks	-0.12 Hours/(Week*Week)
35 Weeks	-0.016 Hours/(Week*Week)

These points can then be used in conjunction with the previous sectioning technique to produce a more accurate graph of the net flow. Instead of drawing a series of steps, the points given by the tangent lines can be used as references so that the graph may be rounded to a more realistic shape. The results of this process and the previously described section method are shown in Figure 6.

[8] Since the length of the work week is measured in hours per week, the change in this value is measured in Hours per Week per Week, or, as shown above, Hours/(Week*Week).

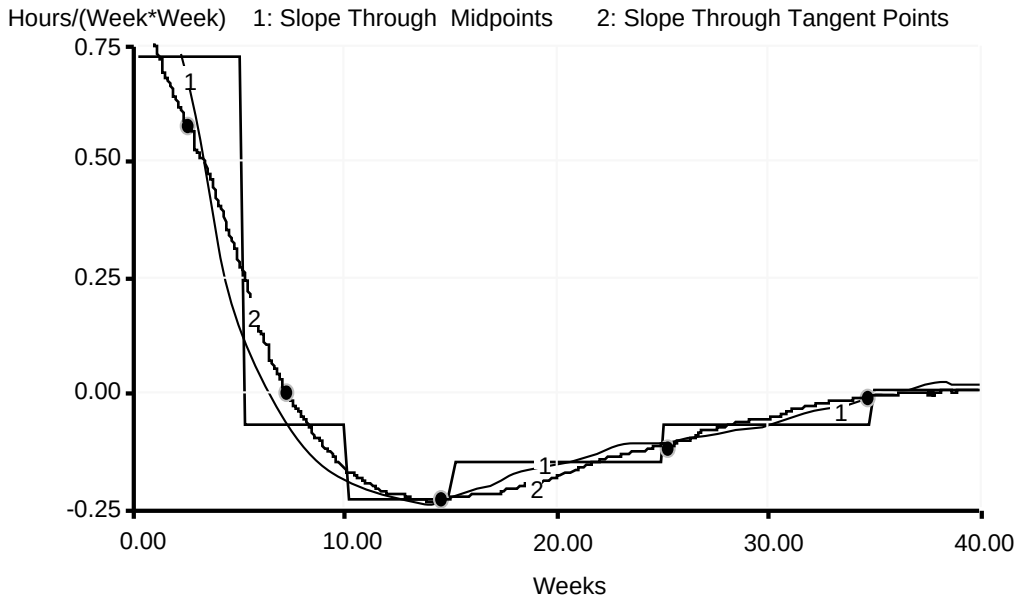


Figure 6: Net flow using tangent lines and sectioning

It can be seen that the section based method for determining the slope, while not as exact as the tangent method, can still work well. Using tangent lines to determine the slope at certain points, represented by the black circles, adds the extra points needed to adjust the curve more accurately. An intuition for the behaviors that a graph may exhibit is also helpful in producing a good graph of the net flow.

4. CONCEPT REVIEW

1. Reverse graphical integration takes a stock behavior and gives the net flow associated with that behavior.
2. Reverse graphical integration can determine the sum of the stock's inflows and outflows but it cannot determine the actual inflows and outflows.
3. When a stock is increasing, its net flow is positive; when a stock is decreasing its net flow is negative.
4. Split the graph into sections and calculate the slope within each section to get an idea of the shape and dimension of the net slope.
5. Draw tangent lines to determine specific values of a net flow and add accuracy to a sketch.

5. CONCLUSION

The techniques demonstrated here do a good job of sketching the net flow of any stock. When modeling, keep these in mind and make use of them when looking at stock

behaviors. Knowing what the net flow of a stock looks like can save time and trouble when de-bugging a model in progress or understanding a new one. With practice, one can develop an intuition for the general shapes of stocks and their net flows.

6. EXERCISES

6.1 Exercise 1: Joe's Bank Account

Joe has been keeping an eye on his bank account. He is interested in finding out the reasons for the changes in its behavior. His balance for the last 40 days is shown in Figure 7.

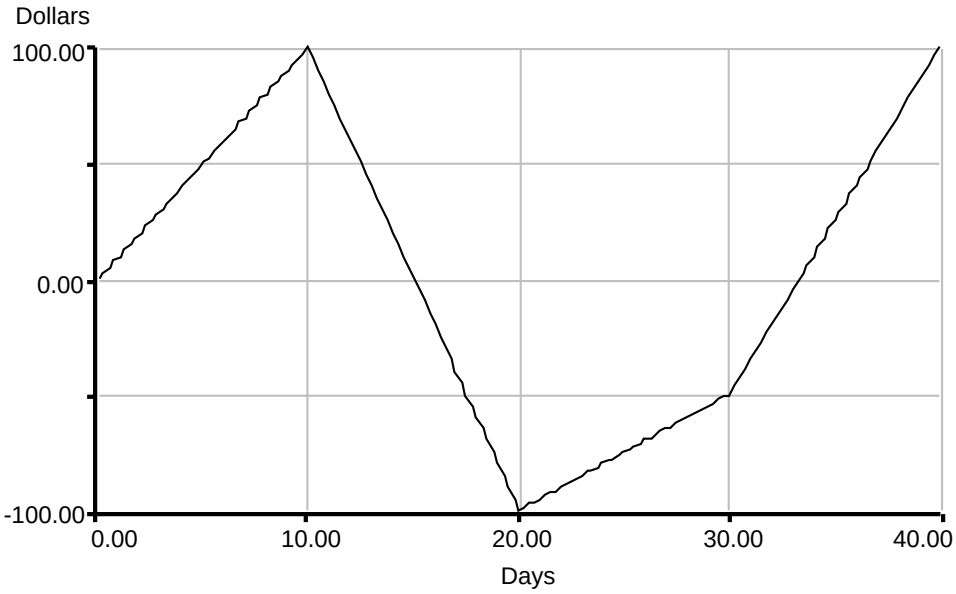
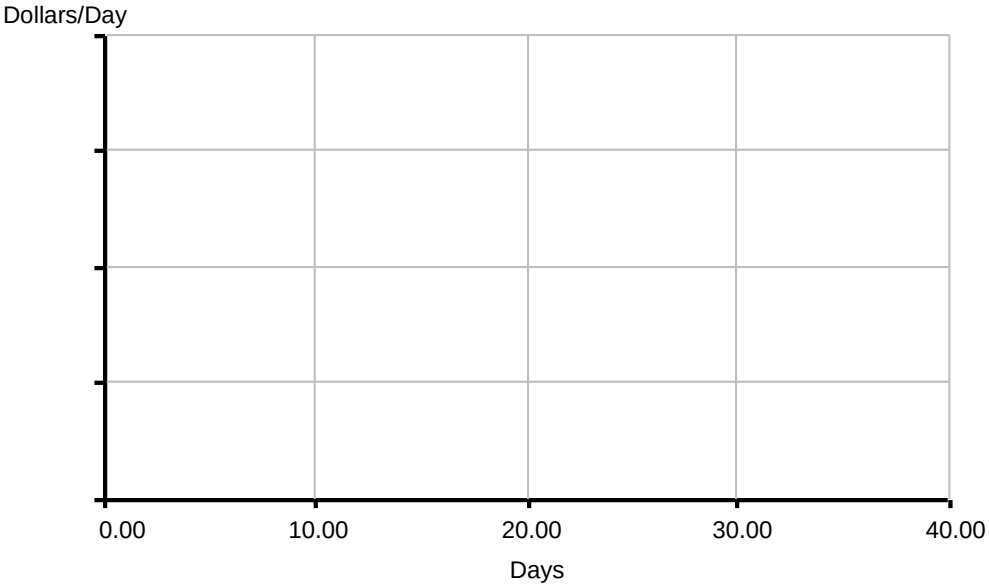


Figure 7: Joe's bank account balance

Use reverse graphical integration to draw the net flow associated with his bank balance on the graph pad provided. Remember that answers may always be checked by graphically integrating the net flow to confirm that the result is the stock behavior.



6.2 Exercise 2: Golf Course Development

Figure 11 is a graph of the development of the golf course from Beginner Modeling Exercises 5.[9]

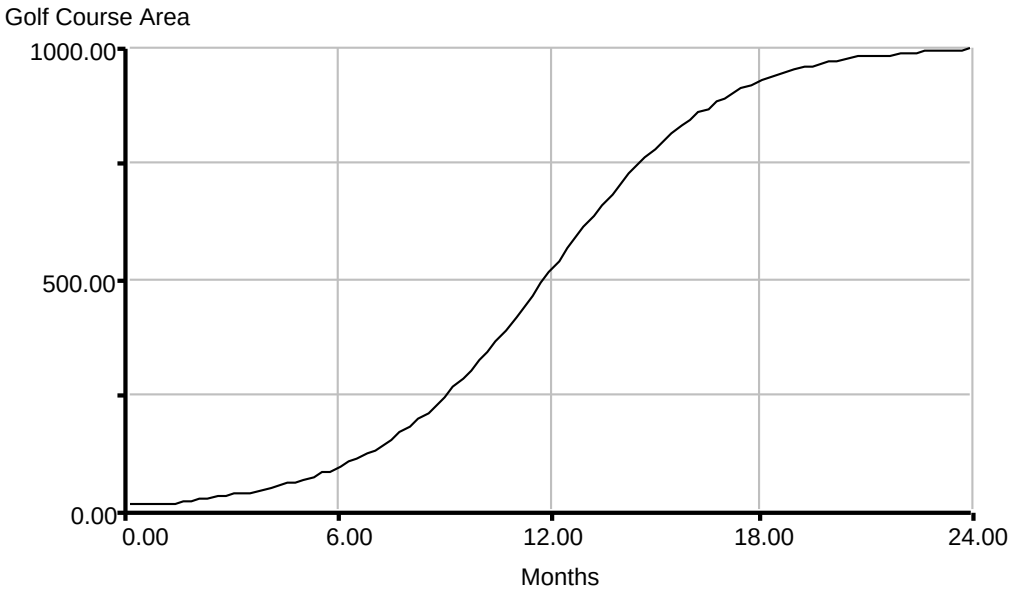
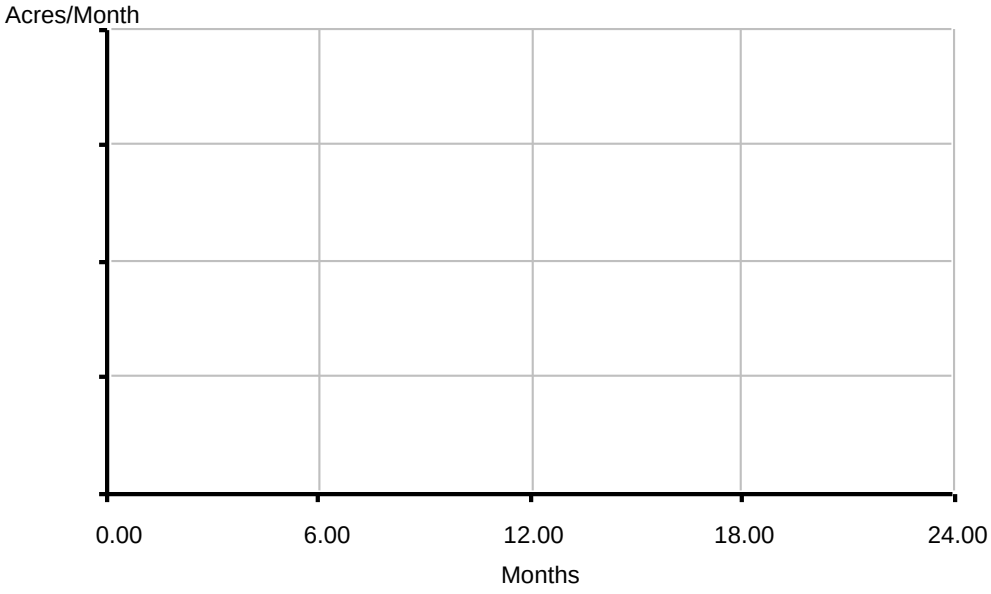


Figure 8: Golf course development

Draw a graph of the rate at which the course was developed on the graph pad provided.



[9]Laughton Stanley and Helen Zhu, 1996. *Beginner Modeling Exercises 5: Mental Simulation of Combining Feedback in First Order Systems* (D-4593), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 1996.

6.3 Exercise 3: Overshoot and Collapse

One of the generic behaviors of systems is overshoot and collapse, as seen in the fish banks model, or with the introduction of a popular product such as an Atari system. Figure 9 is a graph of an overshoot and collapse model of a population of deer caused by the removal of natural predators.

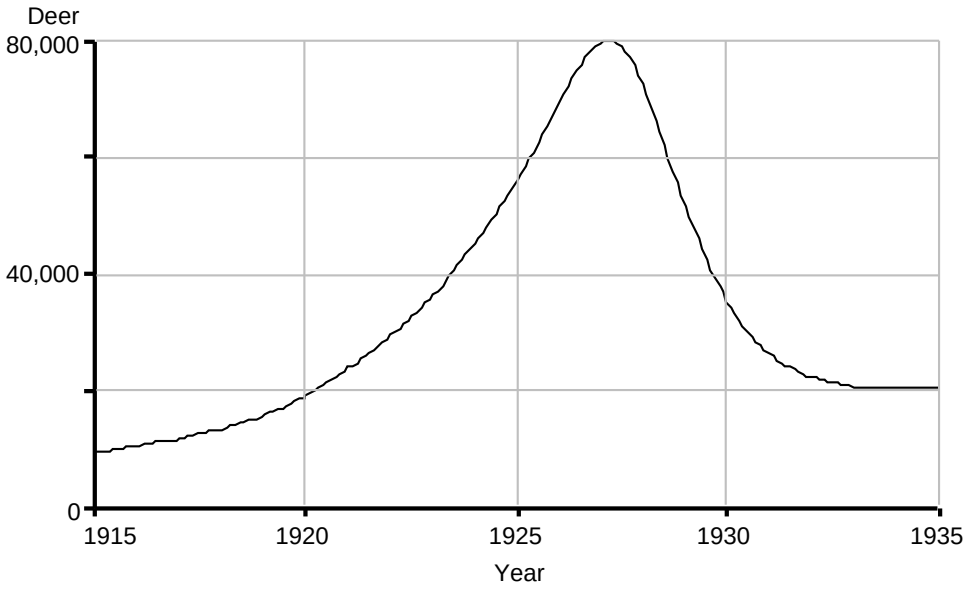
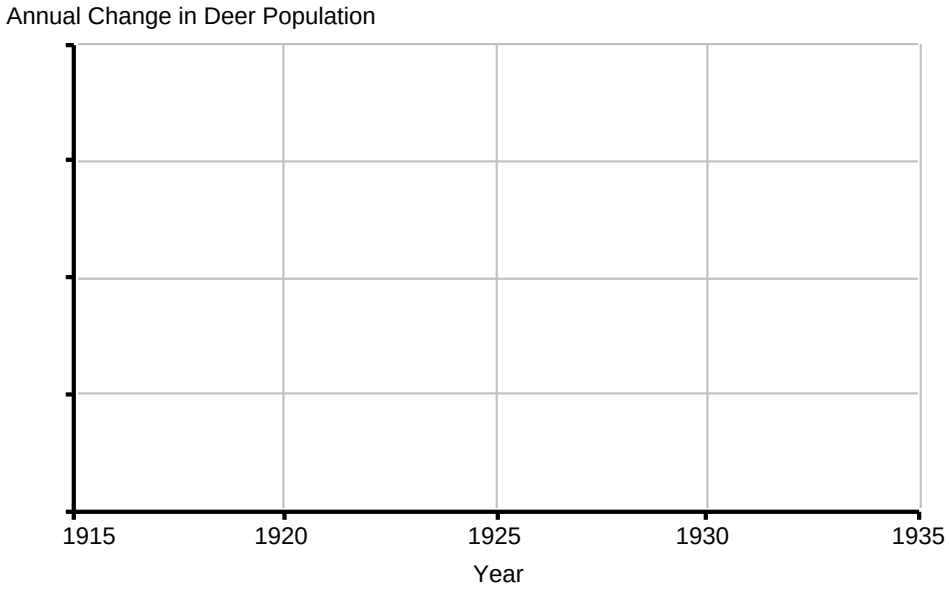


Figure 9: Overshoot and collapse in a deer population model

Draw a graph of the rate at which the deer population changed on the blank graph pad provided.



7. EXERCISE SOLUTIONS

7.1 Solutions to Exercise 1: Joe's Bank Account

The stock behavior of Joe's balance is made up of linear increases and decreases. Because slope is constant along a straight line, it is easiest to divide up the graph into sections at the points where the lines change. This occurs every 10 days. Within each section, all that is needed is a simple slope calculation. For example, the slope of the third section, between 20 and 30 days, would be calculated as follows:

$$Slope = \frac{F - I}{T} = \frac{(-50) - (-100)}{10} = \frac{50}{10} = 5 \text{ Dollars/Day.}$$

The slope between 30 and 40 days is calculated in the same manner:

$$Slope = \frac{F - I}{T} = \frac{100 - (-50)}{10} = \frac{150}{10} = 15 \text{ Dollars/Day.}$$

The resulting graph is shown in Figure 10.

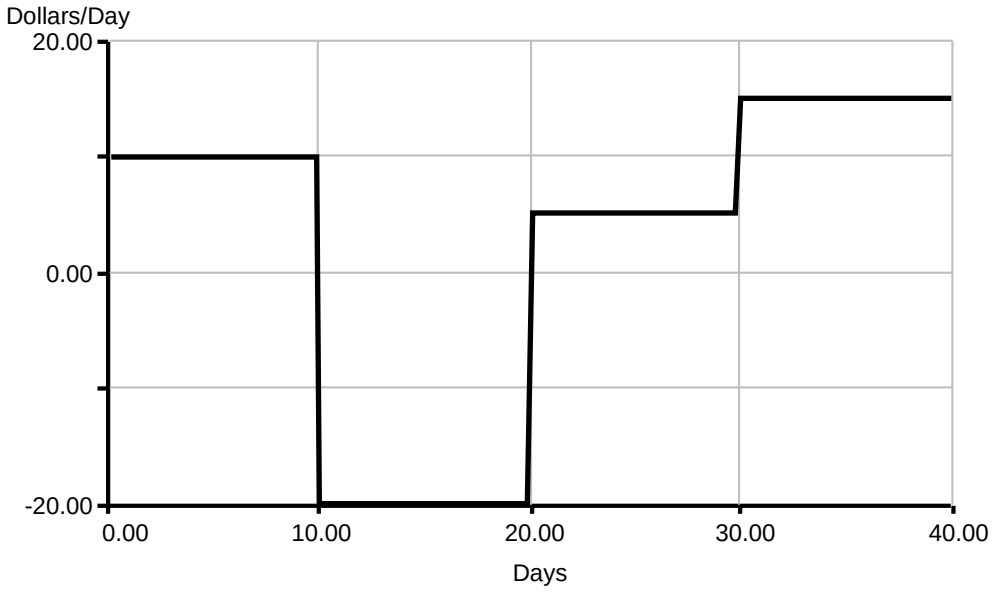


Figure 10: Net Flow of Joe's Bank Account

7.2 Solutions to Exercise 2: Golf Course Development

First, divide the graph into sections as shown in Figure 11. It is important to have enough sections to describe the behavior, but not so many as to over-complicate the calculation process.

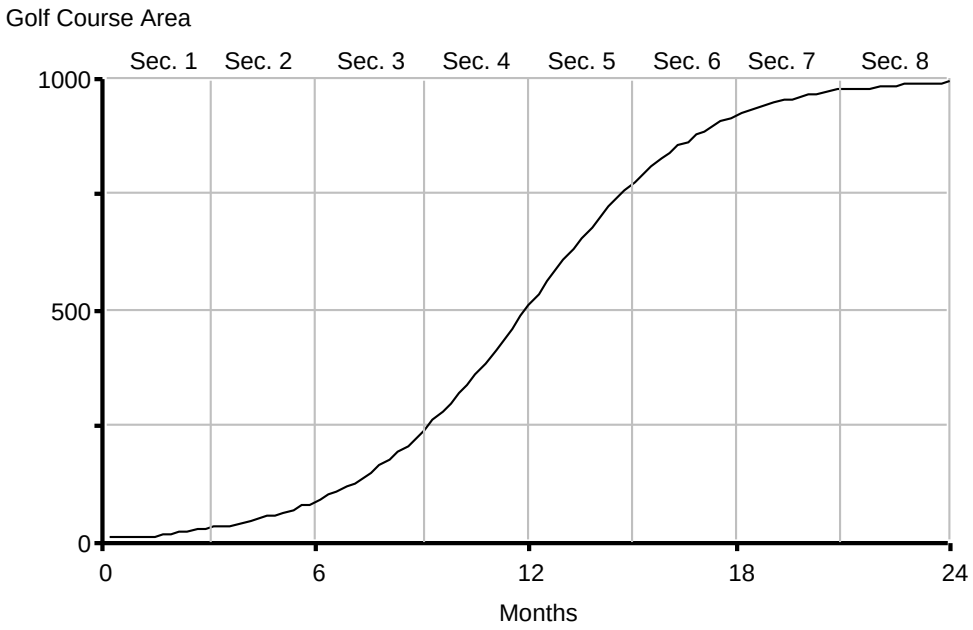


Figure 11: Division of golf course development into sections

Next, calculate the slopes within these sections, shown in Table 4, as in the previous examples.

Table 4: Section based slope calculations for golf course development

Section	Initial Value (acres)	Final Value (acres)	T (months)	Slope (acres/month)
1	10	30	3	7
2	30	90	3	20
3	90	240	3	50
4	240	510	3	90
5	510	780	3	90
6	780	920	3	47
7	920	980	3	20
8	980	990	3	3

Next, pick several points of interest on the graph and draw tangent lines, as in Figure 12. It is usually not necessary to draw as many tangent lines as sections because the tangent lines can be drawn so as to concentrate around the more interesting parts of the graph.

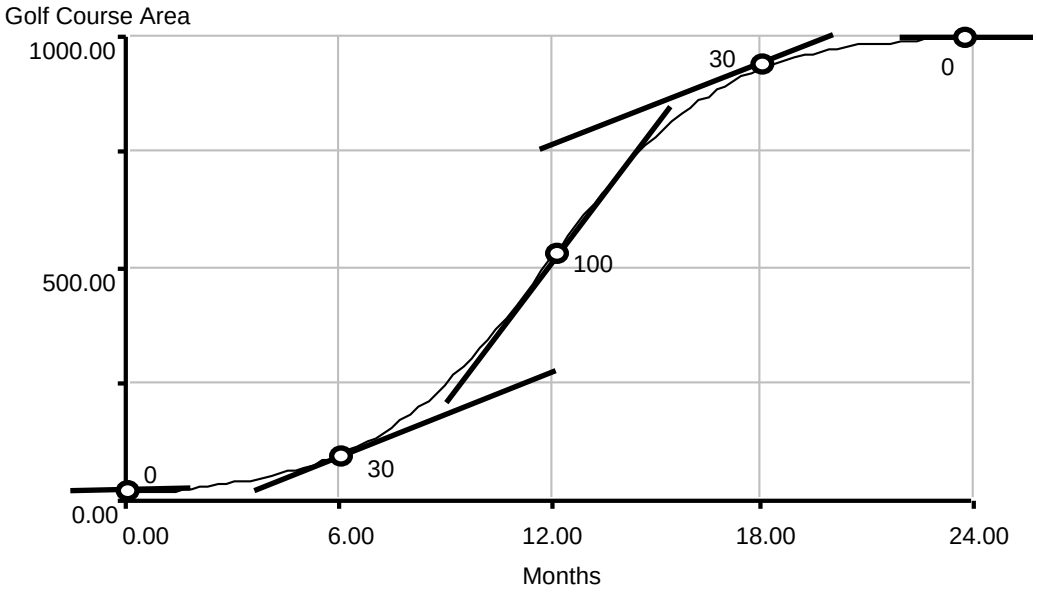


Figure 12: Tangent lines for golf course development

Calculate the slopes of these tangent lines. For example, the fourth tangent line crosses the 12 and 18 month points on the horizontal axis. At the 18 month mark, its value, F, is 940 acres. The value at 12 months, I, is 750 acres. This is over a time period, T, of 6 months. The slope is

$$\frac{F-I}{T} = \frac{190}{6} = 31.\overline{6} \approx 30 \text{ acres/month.}$$

We can round the answer to the nearest 10 because this type of measurement of slope is not good enough to warrant more precision and multiples of 10 are easier numbers to work with. The other calculations are done exactly the same way. When the sectioning results are combined with the slope numbers, a scaled graph can be produced, as shown in Figure 13.

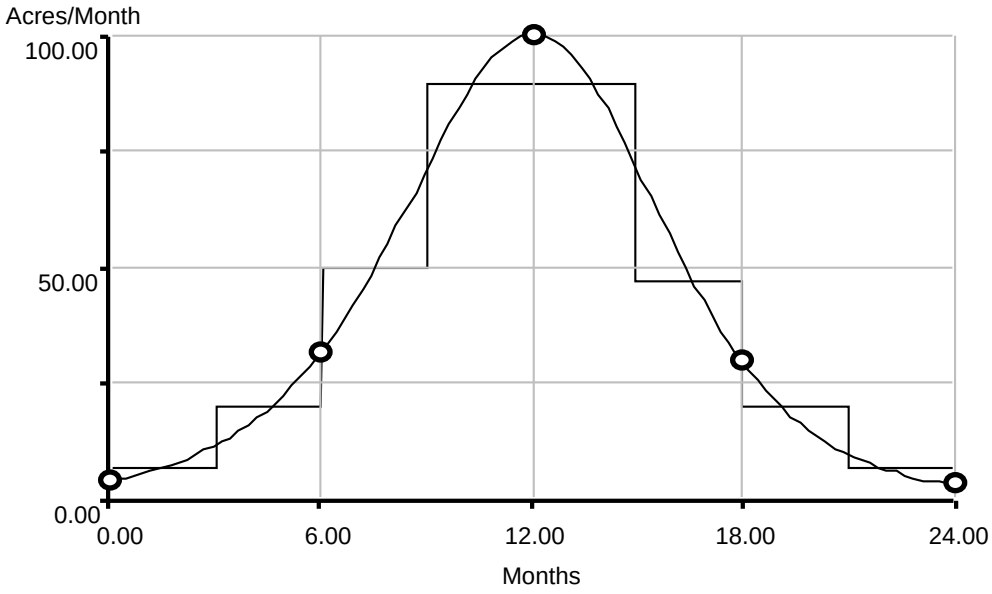


Figure 13: Rate of golf course development fit to section midpoints and tangent line points.

It should be noted that tangent lines have a limited range of accuracy. In this case, it is difficult to be closer than 5 units to the actual values. For instance, even though the tangent slope calculations showed that the slope at the beginning and end was close to zero, the actual slope turned out to be about 3 in each case.

7.3 Solutions to Exercise 3: Overshoot and Collapse

Begin by sectioning the graph and calculating the slopes, as shown in Figure 14 and Table 5.

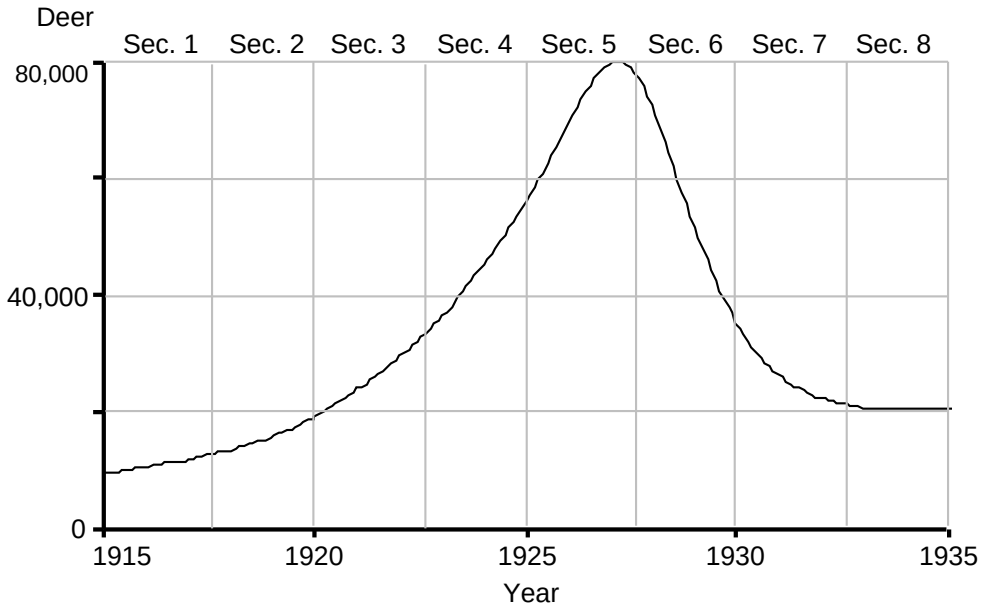


Figure 14: Overshoot and collapse of fishing divided into sections

Table 5: Slopes of the sections of the overshoot and collapse of fishing

Section	Initial Value (Deer)	Final Value (Deer)	T (Years)	Slope (Deer / Year)
1	9200	12,300	2.5	1240
2	12,300	19,100	2.5	2720
3	19,100	33,100	2.5	5600
4	33,100	57,300	2.5	9680
5	57,300	77,600	2.5	8120
6	77,600	34,700	2.5	-17,160
7	34,700	21,500	2.5	-5280
8	21,500	20,600	2.5	-360

One may then draw tangent lines in order to better define the graph, as shown in Figure 15.

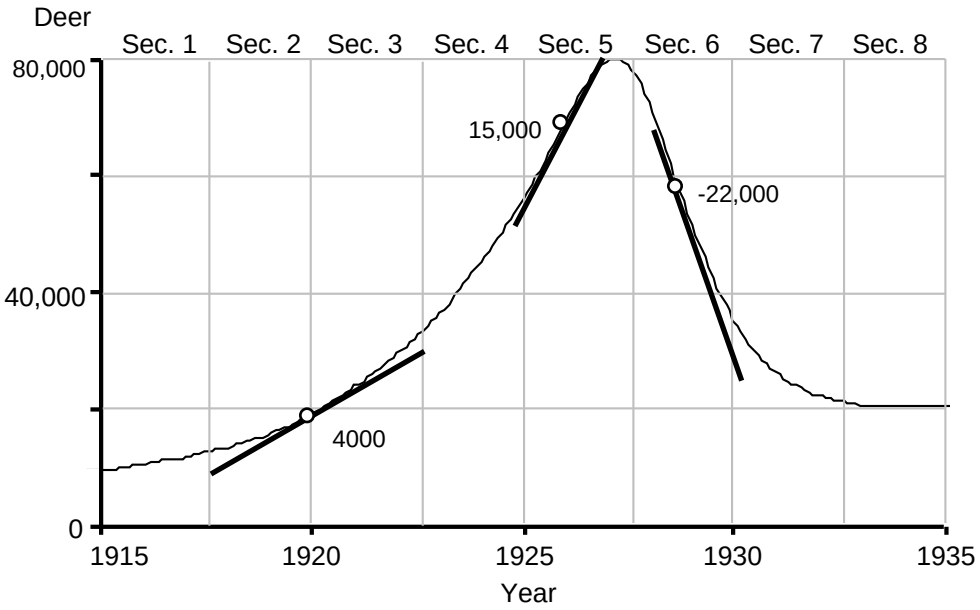


Figure 15: Overshoot and collapse of the deer population with tangent lines

The slopes of these tangent lines may be calculated in the same way as those in Exercise 2. When one combines the sectioning results with the slope information from the tangent lines, one can create a graph such as Figure 16.

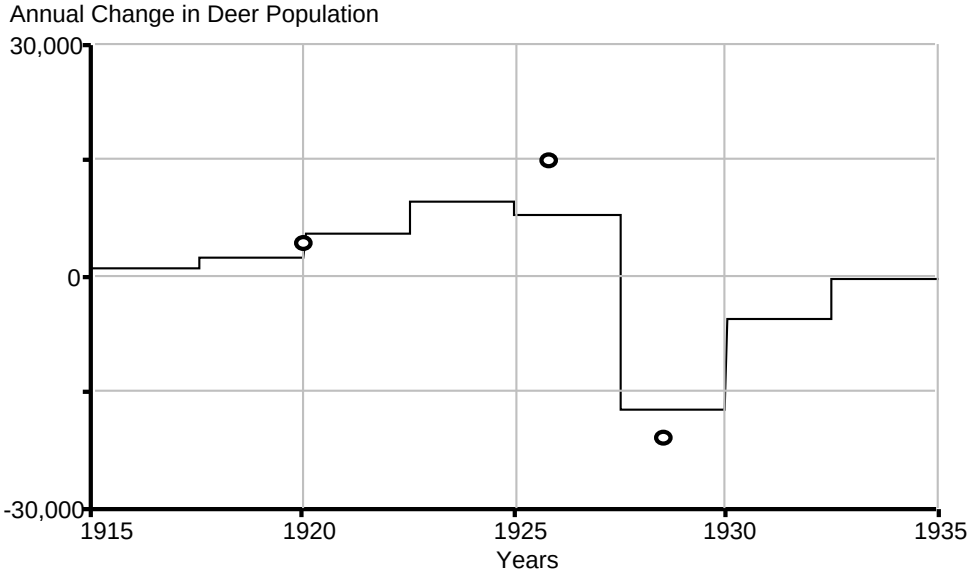


Figure 16: Annual change in deer population with section net slopes and tangent line points

The a net flow line may then be drawn by stretching the shape of the midpoints of the section net slopes to include the tangent line points. The result is shown in Figure 17.

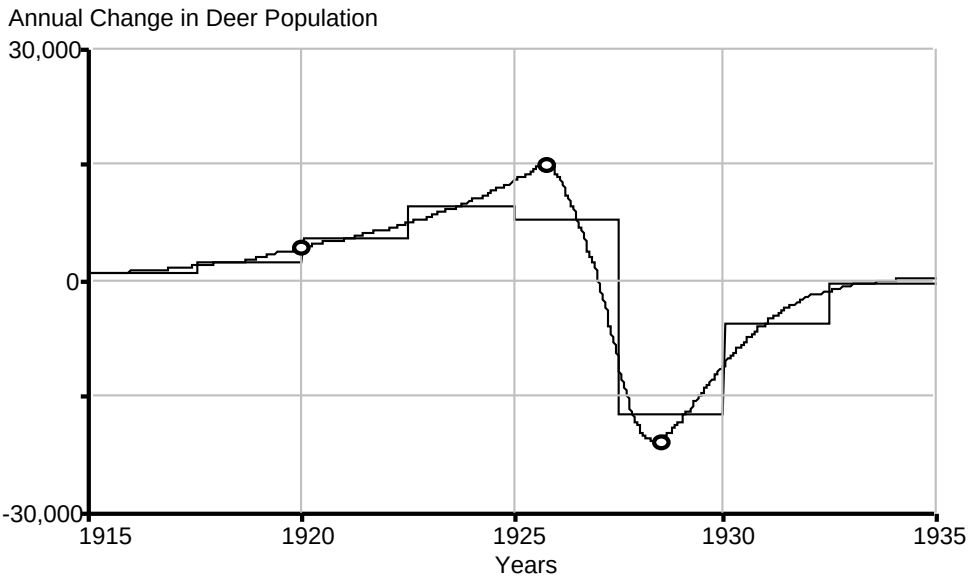
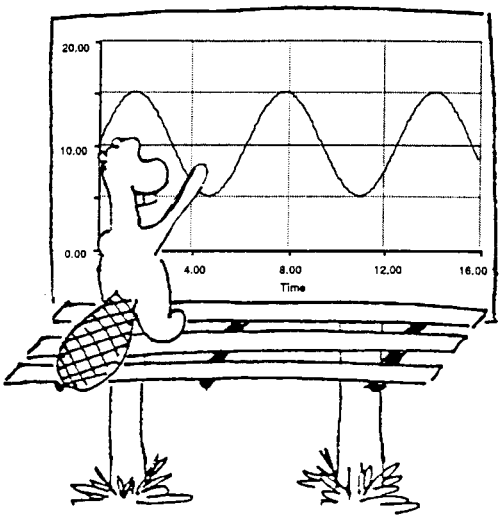


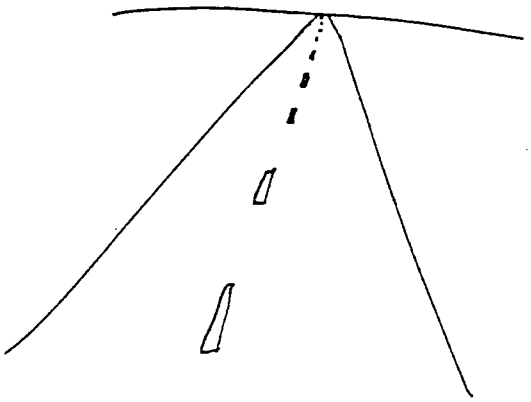
Figure 17: Annual change in deer population fit to section midpoints and tangent line points

It is rare that the net flow of a stock appears more complex than the stock itself, however this is one of those occasions. The net flow of overshoot and collapse is a very complicated behavior and one should not expect to be able to draw it easily. Note that the stock increases as long as the net flow is above zero. Once it drops below zero, the stock begins to decrease in value. Thus the high point of the stock is not in 1926, the high point of the net flow, but in 1927, the year in which the net flow reaches zero.

Road Maps 8

A Guide to Learning System Dynamics





System Dynamics in Education Project

Road Maps 8

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

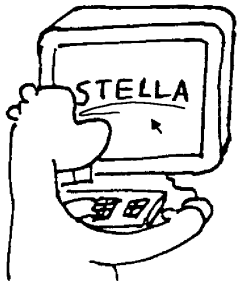
July 1, 1997
Latest Revision March 18, 2001

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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Eight!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. This chapter is the eighth in the Road Maps series. Road Maps One through Three give a broad introduction to the field of system dynamics, focusing on understanding the structure and behavior of systems through positive and negative feedback loops. Road Maps Four through Six introduce several generic structures, give a taste of policy analysis and model validity, and provide opportunities for independent modeling.

Road Maps Seven presents some unexpected behaviors that can occur in higher-order positive feedback loops. Also, Road Maps Seven contains the first paper explaining common mistakes and misunderstandings in system dynamics modeling. The chapter offers more independent modeling exercises, and explains "reverse" graphical integration. Road Maps Seven concludes with a paper on the relationship of systems thinking and soft operations research to system dynamics.

In Road Maps Eight we begin a series of papers on the model-building process with an analysis of the conceptualization stage of building a system dynamics model. The second mistakes and misunderstandings paper warns against an incorrect use of generic structures and emphasizes the real nature of stocks and flows. Road Maps Eight continues with a detailed explanation of the structural causes of sustained oscillation, and presents the first in a series of papers on sensitivity analysis. The chapter ends with a transcription of a speech by Prof. Jay Forrester given at the Systems Thinking and Dynamics Modeling Conference for K-12 Education in June 1994.

Topics Covered in Road Maps Eight

Improving Modeling Skills

- *Building a System Dynamics Model Part 1: Conceptualization* (D-4597)
by Stephanie Albin

- *Mistakes and Misunderstandings: Use of Generic Structures and Reality of Stocks and Flows* (D-4646)
by Lucia Breierova

Oscillation

- *Oscillating Systems II: Sustained Oscillation* (D-4602)
by Kevin Agatstein

Sensitivity Analysis

- *An Introduction to Sensitivity Analysis* (D-4526)
by Lucia Breierova and Mark Choudhari

System Dynamics in Education

- *Learning through System Dynamics as Preparation for the 21st Century* (D-4434-1)
by Jay W. Forrester

Things You'll Need for Road Maps Eight

Modeling Software

In order to complete Road Maps Eight and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is

located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

Books

You will need the following book for Road Maps Eight. The book was previously required in Road Maps.

Goodman, Michael R., 1974. Study Notes in System Dynamics.
Portland, Oregon: Productivity Press, 388 p.

If you have any problems in getting this book, contact Productivity Press (see Appendix).

How to Use Road Maps Eight

Road Maps Eight explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Many of the chapters conclude with a prominent paper from the literature in the field of system dynamics.

As part of the spiral learning approach used in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains several series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic or skill in system dynamics. The series start with a simple paper, and progress to further develop the idea in subsequent chapters.

Road Maps present the fundamental concepts of system dynamics as *System Principles*. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revisited in Road Maps, you will build upon your previous understanding of the principle by learning something new about it. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

Now let's get started!

Improving Modeling Skills

Building a system dynamics model is not an easy task. Most modelers follow a similar set of rules and steps when building a model. The four stages of the model-building process are *conceptualization*, *formulation*, *testing*, and *implementation*, which will be examined one by one in individual papers. The following paper examines the first stage, conceptualization.

- Building a System Dynamics Model Part 1: Conceptualization[1]

by Stephanie Albin

This paper provides an in-depth explanation of the conceptualization stage of building a system dynamics model. The paper provides a written description of a Heroin-Crime system and then uses the system to demonstrate the four steps used in conceptualization: determining the model purpose, model boundary, shape of the reference modes, and basic mechanisms. The paper then offers an exercise in conceptualization and suggests a solution. You will need Study Notes in System Dynamics by Michael R. Goodman to complete the exercise.[2]

Please read *Building a System Dynamics Model Part 1* now.

After reading *Building a System Dynamics Model Part 1...*

Conceptualization is a crucial stage in building a system dynamics model. During this stage, the modeler must define the purpose of the model, set its boundary, and determine the reference modes and the basic mechanisms. Without completing such a rigorous set of steps, the model-building process may be more difficult because crucial pieces of information may be overlooked. Later chapters of Road Maps will describe the remaining three stages of the model-building process.

The previous paper introduces an important system principle on the concept of closed boundary. Later chapters of Road Maps will return to this system principle.

---

[1] Stephanie Albin, 1997. *Building a System Dynamics Model: Conceptualization* (D-4597), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology June 30, 36 pp.

[2] Michael R. Goodman, 1974. Study Notes in System Dynamics, Portland, Oregon: Productivity Press, 388 pp.



System Principle #21:

Every system has a closed boundary.

In creating a model of a real system, any interaction which is essential to the behavior mode being investigated must be included inside the system boundary. If our model is to generate the same behavior as the real system, then the system structure that is responsible for that behavior must be included inside the model. The behavior and its generator are endogenous to the closed system.

Reread section 4.2 of *Building a System Dynamics Model Part I: Conceptualization.* The section explains the concept of model boundary and shows how a modeler sets it during the conceptualization stage. In defining the model boundary, a modeler selects all the components necessary to create the behavior mode under investigation.

- Mistakes and Misunderstandings: Use of Generic Structures and Reality of Stocks and Flows[3]

by Lucia Breierova

This is the second in a series of papers in Road Maps that examine common mistakes and misunderstandings in system dynamics modeling. This paper explains a mistake made by using an unsuitable generic structure to model a system. It also reminds the reader that stocks represent real-world accumulations and flows change the stocks over time.

Please read *Mistakes and Misunderstandings* now.

After reading *Mistakes and Misunderstandings...*

When building a model, keep in mind that stocks in a model must represent accumulations within the system, and that flows represent changes in the stocks. In addition, remember that many structures can produce similar behaviors. Thus, generic structures should not be used blindly without first thinking about the

[3] Lucia Breierova, 1996. *Mistakes and Misunderstandings: Use of Generic Structures and Reality of Stocks and Flows* (D-4646), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, December 18, 10 pp.

structure of the specific system being modeled. Future chapters of Road Maps will point out other mistakes and misunderstandings.

Oscillation

Road Maps Six introduced the behavior of oscillation through *Generic Structures in Oscillating Systems I.*[4] This paper discussed the generic structure that produces oscillation and showed how the same structure can be applied to a variety of systems. The following paper provides an in-depth explanation of the causes of sustained oscillation.

***- Oscillating Systems 2: Sustained Oscillation*[5]**

by Kevin Agatstein

This paper offers a detailed analysis of the structural causes of oscillation. It starts with description of a first-order system and shows why such a system cannot oscillate. Two second-order systems, an Academic Performance Model, and a Cleanliness of a College Dorm Room Model, are then discussed. For each system, the discussion starts with a stock-and-flow model of the system and follows with a brief explanation of the feedback relationships. A step-by-step explanation of one cycle of the model behavior is then offered, focusing on structural reasons of the oscillation.

Please read *Oscillating Systems 2: Sustained Oscillation* now.

After reading *Oscillating Systems 2...*

A system must meet several conditions in order to oscillate. It must be a second or higher-order feedback loop. The system also must have a momentum associated with it that causes it to undershoot and overshoot the goal value, thus producing oscillation. Oscillation is a complex behavior mode and later chapters of Road Maps will return to it.

---

[4] Celeste V. Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 17, 24 pp.

[5] Kevin Agatstein, 1997. *Oscillating Systems 2: Sustained Oscillation* (D-4602), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 15, 31 pp.

The paper you just read emphasizes an important system principle that was already presented in previous chapters of Road Maps.



System Principle #11:

Levels completely describe the system condition.

Before the start of a simulation, a set of initial conditions must be specified for all the levels.

Look at the Academic Performance Model in Figure 6 on page 13 of *Oscillating Systems II: Sustained Oscillation*. The system contains two levels: "Current Grades" and "Hours of Weekly Studying." Before running a simulation, the initial values of both levels must be specified. For example, the initial value of "Current Grades" is 3.0 GPA units, and the initial value of "Hours of Weekly Studying" is 21 hours in a week. Without knowing the initial values, the simulation cannot be run. The simulation must begin from a specified condition of the system, reflecting the state of the system at the initial point in time.

Oscillating Systems 2: Sustained Oscillation also presents a new system principle on information links. Please read it carefully. Later chapters of Road Maps will review the system principle several times.



System Principle #22:

Information links connect levels to rates.

Information links, or connectors, link levels to the control of rates. Through information links, values of level variables go to the rate equations, determining the rates of flow.

In the Cleanliness of a College Dorm Room Model, shown in Figure 8 on page 19 of *Oscillating Systems II: Sustained Oscillation,* the level "Daily Complaints of my Roommate" controls the rate of "picking up laundry." The connector between the two variables is an information link. Through the information link, the value of "Daily Complaints of my Roommate" goes to the rate equation determining the value of the "picking up laundry" rate.

Sensitivity Analysis

When building system dynamics models, you may have wondered how to choose specific values of some parameters in a model. How would the behavior demonstrated by a model change if you decided to increase or decrease a parameter? What if the simulation started with a different initial value of a stock? What value should you choose for a parameter if you are not completely sure about its value from the real world system? The next section introduces the concept of sensitivity analysis that aims to answer these and many other questions.

- An Introduction to Sensitivity Analysis[6]

by Lucia Breierova and Mark Choudhari

The following paper introduces sensitivity analysis, a powerful tool in model building and evaluation. The paper presents two models, a lemonade stand model and an epidemics model, and then conducts sensitivity analysis on model parameters to see how behavior is affected when parameter values change. You are then encouraged to experiment on your own with another suggested model. It is

[6] Lucia Breierova and Mark Choudhari, 1996. *An Introduction to Sensitivity Analysis* (D-4526), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 6, 40 pp.

recommended that you build all the presented models and that you run the tests along with the paper to increase your understanding of the model's dynamic behavior as well as of sensitivity analysis.

Please read *An Introduction to Sensitivity Analysis* now and do the suggested exercises on your computer.

After reading *An Introduction to Sensitivity Analysis...*

Sensitivity analysis is very important in building a valid system dynamics model. This paper shows that system dynamics models are usually insensitive to many parameter changes: even though the detailed behavior seems to be affected, the fundamental behavior mode stays the same. Sensitivity analysis, however, can help to identify high leverage points by finding parameters whose specific values can greatly influence the mode of behavior. Sensitivity analysis can also increase confidence in a model and reduce the uncertainties of some parameter values. It is often the structure of a system, more than the parameter values, that primarily determines the system behavior.

Please read the following system principle on conversion coefficients that was first introduced in Road Maps Seven.



System Principle #18:

Conversion coefficients should be identifiable within real systems.

Conversion coefficients should have numerical values that can be logically deduced from observation. They are not the result of only statistical analysis.

Look back to Figure 13 of *An Introduction to Sensitivity Analysis*. The parameter called "Productivity" is a conversion factor connecting the stock of "Workers" to the flow of "Making Coffee." The value of "Productivity" can be obtained from observing the real world system, the Coffeehouse. For example, if each Coffeehouse worker has time to prepare a ten-cup pot of coffee in half an hour, then the workers' productivity is 20 cups per worker per hour.

The sensitivity analysis paper you just read also introduces a new system principle on decisions in systems. Later chapters of Road Maps will present other aspects of this system principle.



System Principle #23:

Decisions (rates) are based only on available information.

Decisions are made based on the policy statements in the rate equations.

The rate equations in a system dynamics model are policy statements that determine how "decisions" are made. For example, in the Lemonade stand model shown in Figure 1 of *An Introduction to Sensitivity Analysis,* the rate of "Making lemonade" determines how much lemonade should be made. Howard's decision rule on how much lemonade to prepare is captured in the rate equation:

Making lemonade = MAX (Expected lemonade buying + Correction in amount of lemonade, 0)

System Dynamics in Education

The next paper is Prof. Forrester's keynote address for the Systems Thinking and Dynamic Modeling Conference for K-12 Education in June 1994 at the Concord Academy in Concord, Massachusetts. In his speech, Prof. Forrester proposes three objectives that should be achieved through a system dynamics education: developing personal skills; shaping an outlook and personality to fit the 21st century; and understanding the nature of complex systems in which we work and live.

- Learning through System Dynamics as Preparation for the 21st Century[7]

by Jay W. Forrester

Of the three objectives outlined above, it is a continuing goal of Road Maps to help its readers understand the nature of complex systems that surround us. Complex systems differ from simple systems in many ways. The same decisions that can improve simple systems are often counterproductive when applied to

[7]Jay W. Forrester, 1994. *Learning through System Dynamics as Preparation for the 21st Century* (D-4434-1), Sloan School of Management, Massachusetts Institute of Technology.

complex systems. Prof. Forrester's speech presents six characteristics of complex systems; please read about them carefully, and try to think of other situations in which you have experienced similar behavior of complex systems.

Please read *Learning through System Dynamics...* now.

After reading *Learning through System Dynamics...*

Apart from developing students' personal skills and building their outlook and personality, a systems education should lead students to understand the nature of complex systems. According to Prof. Forrester, in complex systems, cause and effect are not closely related in time or space. In addition, most policies that try to improve a complex system's behavior have little leverage. If a high-leverage policy is found, it is often used incorrectly, thus worsening the initial problem. Moreover, in complex systems, problems are caused by decisions of people within that system. As our low performance causes a failure to reach our goals, we keep lowering the goals to make them more achievable. Finally, a conflict often exists between short-term and long-term goals of a complex system.

After reading Prof. Forrester's speech, you should try to look for examples of these characteristics of complex systems in your own experiences. As you read through the following chapters of Road Maps, you will often be reminded of the characteristics of complex systems in the models that you build and study.

Finishing off Road Maps Eight

Road Maps Eight presented several papers through which you can significantly improve your modeling skills. The chapter covered the conceptualization stage of the model-building process and warned you against more mistakes and misunderstandings in modeling. It then continued to explain the structural causes of sustained oscillation. In addition, Road Maps Eight introduced sensitivity analysis, a useful tool in building and evaluating a model, as well as in understanding its dynamic behavior. Finally, the chapter discussed the goals of a systems education and presented some basic characteristics of complex systems.

In Road Maps Eight, you learned three new system principles:

a) that every system should be formulated with a closed boundary that contains the forces causing the behavior being studied;

b) that information links connect levels to rates; and
c) that decisions (rates) are based only on available information.

Key Terms and Concepts:

basic mechanisms
complex systems
conceptualization
dynamic hypothesis
model boundary
parameter
period of oscillation
reference mode
sensitivity analysis
sustained oscillation
time horizon



Building a System Dynamics Model
Part 1: Conceptualization

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Stephanie Albin
June 30, 1997

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1. ABSTRACT

Building a System Dynamics Model is a series of papers written to demystify the model building process. This paper is the first in the series and explains the first stage of the model building process called *conceptualization*. The paper examines in depth the following steps of conceptualization:

1. Define the purpose of the model.
2. Define the model boundary and identify key variables.
3. Describe the behavior or draw the reference modes of the key variables.
4. Diagram the basic mechanisms, the feedback loops, of the system.

The first step, deciding on the model purpose, means focusing on a problem and narrowing down the model's audience before concretely stating the model purpose. Defining the model boundary involves selecting components necessary to generate the behavior of interest as set by the model purpose. After defining the model boundary and identifying key variables, some of the most important variables are graphed over time as a reference mode. The major types of reference modes are the hypothesized and the historical reference modes. The final step in conceptualization is deciding on the basic mechanisms (feedback loops) of the system. The basic mechanisms are commonly diagrammed using either stock-and-flow diagrams or causal-loop diagrams.

Each step is illustrated by a general description of the step. After the general description of each step, that step will be applied to a Heroin-Crime system description. Further papers in this model-building series will also use the Heroin-Crime system. Section 5 contains a conceptualization exercise. The reader will need to refer to Study Notes in System Dynamics by Michael R. Goodman,[1] which was used previously in Road Maps. Appendix A contains a Modeling Worksheet to assist the modeler in the conceptualization process.

---

[1] Michael R. Goodman, 1974. Study Notes in System Dynamics. Portland, Oregon: Productivity Press, 388 pp.

2. INTRODUCTION

In spite of a wide range of applications using system dynamics, most SD models are created in four stages. The four stages of modeling are outlined below, with the essential steps of each stage bulleted under the stage name:[2]

1. **CONCEPTUALIZATION**
    - **Define the purpose of the model**
    - **Define the model boundary and identify key variables**
    - **Describe the behavior or draw the reference modes of the key variables**
    - **Diagram the basic mechanisms, the feedback loops, of the system**
2. FORMULATION
    - Convert feedback diagrams to level and rate equations
    - Estimate and select parameter values
3. TESTING
    - Simulate the model and test the dynamic hypothesis
    - Test the model's assumptions
    - Test model behavior and sensitivity to perturbations
4. IMPLEMENTATION
    - Test the model's response to different policies
    - Translate study insights to an accessible form

This paper covers the **conceptualization stage** of modeling and provides practical advice for completing each step. Further papers in the model-building series will cover the remaining three stages of modeling. Although this paper does not cover the remaining stages, note that the four stages are recursive. After completing each stage the modeler might have to return to previous stages to incorporate new information or insights. No strict dividing lines exist between the stages.

---

[2] Jorgen Randers, 1980. Elements of the System Dynamics Method. (pp.117 - 139) Portland, OR: Productivity Press, 344 pp.

The only way to become comfortable with the modeling process and improve one's modeling is through practice. Section 3 below contains a system description that will be used for practice throughout this paper.

3. HEROIN-CRIME SYSTEM DESCRIPTION

To better understand conceptualization, this paper will apply each step of the conceptualization stage to a Heroin-Crime system.[3] Read the following excerpt describing the Heroin-Crime system; refer back to the description below as needed.

> "The next time you hear of a big drug bust and the seizure of large quantities of heroin, don't go believing that your city's streets are necessarily any safer. In fact, a new study of heroin traffic in Detroit has concluded that the tighter the heroin market, the more likely you will become a victim of a robbery or a burglary by an addict in need of a fix."[4]

Most people believe that the stronger law enforcement works to get drugs off the streets, the safer those streets become. Evidence from the Drug Abuse Council, however, challenges the prevailing belief. "If the price of a bag of street heroin increased from $7 to $9 in any given month," the Drug Abuse Council said, "the number of revenue-raising crimes which occurred at the rate of about 11,000 a month...would increase to almost 12,000 a month because of heroin alone."[5]

Heroin demand does not react immediately to a shortage of supply because of heroin's addictive nature. Heroin is a "need" and not just a desire" and heroin demand can thus be taken as a constant in the short-run. When the heroin supply and heroin demand are in equilibrium, a certain price is charged. An increase in drug busts results in a decrease in the heroin supply. Basic economics states that when supply is less than demand, the price increases. An increase in price will lead to an increase in heroin importation to the area. Assuming the increase in drug busts is temporary, the heroin supply will eventually increase to the original level and bring the price of heroin back down to normal.

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[3] Further papers in this series will continue the model-building process using the Heroin-Crime system.
[4] Excerpt from Saul Friedman. "When heroin supply cut, crime rises, says report." *Boston Globe*, April 22, 1976.
[5] Taken from Friedman, "When heroin supply cut, crime rises, says report."

4. CONCEPTUALIZATION STEPS

During the conceptualization stage, a modeler must determine the purpose of the model, the model boundary, the shape of the reference modes, and the nature of the basic mechanisms. This paper will now cover each of the four steps of conceptualization in subsections 4.1 through 4.4. Each subsection is further divided into *"Discussion,"* containing a more abstract definition of the conceptualization step, and *"Heroin-Crime System,"* containing an example of an application of the conceptualization step to the Heroin-Crime system described in section 3 of this paper.

4.1 Purpose of the Model

4.1.1 Discussion

The first step of the modeling process, deciding on the model purpose, is a two-part decision. Deciding on the model purpose means focusing on a problem and narrowing down the model's audience. By deciding on the model's purpose, a modeler makes the later choices of both components and structure feasible.

A system dynamics model is built to understand a system of forces that have created a "problem" and continue to sustain it. To have a meaningful model, there must be some underlying problem in a system that creates a need for additional knowledge and understanding of the system. The goal of the conceptualization stage is to arrive at a rough conceptual model capable of addressing the relevant problem in a system. After choosing what problem area to focus on, a modeler must gather relevant data and further define the focus of the model. Relevant data for a system dynamicist consists not only of measured statistical data, but also operating knowledge from people familiar with the system being analyzed.

The modeler should also consider a model's primary audience. A model explaining causes of acid rain would be much different if built for an 8th grade biology class, than if built for the United States Environmental Protection Agency for policy decisions. If the model's structure and behavior cannot be understood by its audience, or if it does not answer questions interesting to the audience, then the model is rendered useless.

The first step in creating a meaningful model from available data is defining the purpose of a model while keeping in mind the model's audience. The model purpose should mention some type of action or behavior over time that the model will analyze. Coming to an agreement on the purpose of the model is essential. Without a clear and strictly defined purpose it is very difficult to decide which components of the system are

important. Another concern of experienced modelers is whether it is worthwhile to build the model as defined. A very abstract purpose, such as "I am building this model about the environment to understand how it works," is likely to result in a waste of time. Such a model would probably include too many components and be too complex for any practical analysis.

The purpose of a model usually falls into one of the following categories:

- to clarify knowledge and understanding of the system
- to discover policies that will improve system behavior
- to capture mental models and serve as a communication and unifying medium.

4.1.2 Heroin-Crime System

The Heroin-Crime system description presents the modeler with the problem of increased police activity in the form of drug busts having the unintended side effect of increased crime.

Given the broad problem area in the Heroin-Crime system, the model could be built for many reasons. Assume that the modeler has just read an article about the heroin-crime system in the newspaper and would like to build a model to see if the theories presented are legitimate.

- The purpose of the model is to test whether increased drug busts increase revenue-raising crime in the short-run.

The audience for this model is the average person and not a panel of experts in drug trafficking or law enforcement. Detailed statistics are not needed. The model's audience would probably be most interested in a simple, easy-to-understand model with highly aggregated components.

4.2 Model Boundary

4.2.1 Discussion

Every feedback system has a closed boundary within which the behavior of interest is generated. When creating a system dynamics model of a feedback system, a modeler must clearly define the model boundary. The model boundary contains all components present in the final model. First, a modeler must brainstorm for all components he sees as necessary for creating a model of the system, even those of which he is unsure. The initial list shall be referred to as the initial components list.

When selecting components for the initial list, the following guidelines should help:

Components must be necessary. The modeler sets the boundary such that nothing excluded from the model is necessary to generate and properly represent the behavior of interest as set by the model purpose.[6] Also, nothing included should be unnecessary.

Components should be aggregated. Similar concepts should be aggregated if doing so does not change the nature of the problem being modeled or the model purpose. Fewer components help to avoid unnecessary complications. Do not, however, aggregate components if doing so creates a model which no longer reflects reality, for example the flows "births" and "deaths" should not usually be aggregated to "net births."

Components must be directional. All important components must have a directional name that can grow larger or smaller, for example "anger" or "happiness" instead of "mental attitude."

Second, to further specify a model boundary, the modeler must separate the initial components list into two important groups:

- endogenous—dynamic variables involved in the feedback loops of the system
- exogenous—components whose values are not directly affected by the system

It is helpful to make two columns labeled "endogenous" and "exogenous" on a sheet of paper and fill the columns after carefully examining each component on the initial components list in the context of the model purpose. There may be some components on the initial list which, after further examination, seem unnecessary in the model. Thus it is important to save the initial components list and the subset categories of endogenous and exogenous components. The modeler should now have a better understanding of the model structure. By saving the initial components list, the modeler can always, without too much additional effort, go back and reexamine some of the initial assumptions made about the importance of variables in the system or even try an alternative model structure.

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[6] Jay W. Forrester, 1980. *Conceptualization of a Model to Study Market Growth* (D-3221), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, 24 pp.

Finally, after dividing the initial list into two categories, it is useful for the modeler to re-examine the endogenous and exogenous components and specify which components are stocks and which are flows. (Note that exogenous components will usually be constants or time varying constants, and not stocks or flows.) The additional information gained will be of great use in the last step of conceptualization—deciding on the basic mechanisms. To make identification easier, remember that stocks are accumulations. They are usually something that can be visualized and measured, such as population, but can also be abstract, such as level of fear or reputation. Flows are changes in stocks. They are rates and are measured in units of the stock over time. Birth rate, death rate, and shipping rate are examples of flows.

4.2.2 Heroin-Crime System

A brainstorm of components for the Heroin-Crime system might lead to the following initial components list:

heroin stock
number of addicts
heroin importation
revenue-raising crime
money to support population's habit
number of arrests
drug busts
heroin demand
price of heroin
heroin use

Figure 1 defines a possible model boundary for the Heroin-Crime system. (Note that the "S" and "F" next to the component name mark it as a stock or a flow, respectively.)

ENDOGENOUS COMPONENTS	EXOGENOUS COMPONENTS
heroin stock (S)	heroin demand
revenue-raising crime	drug busts (F)
heroin importation (F)	heroin use (F)
price of heroin (S)	
money to support population's habit	

Figure 1: Components for Heroin-Crime System

Because the model purpose is to examine the theory that crime increases as drug busts increase, the number of addicts and number of arrests are not important to the dynamic behavior. The number of addicts and number of arrests components are thus left on the initial list and not placed in the endogenous or exogenous lists. A drug bust is defined as a seizure of drugs from the local stock of heroin. In the short-run, heroin demand is constant because the effect of rehabilitation and entrance of new addicts is minor. Heroin is addictive, therefore addicts require heroin regardless of price, so the need for a heroin fix in the short-run is constant. The component driving the behavior of interest is the number of drug busts per month. The model purpose focuses on the resultant behavior due to an increase in the drug bust component, and not on the reason for the increase in drug busts. The frequency of drug busts and the amount of drugs seized are therefore exogenous. The dynamic components important to the model purpose are the heroin stock, the price of heroin, and the amount of revenue-raising crime. Other important variables, such as the money to support the population's habit and the level of heroin importation, also affect and are affected by these three dynamic components. All components in such a feedback loop are endogenous to the system. Note that a modeler may interpret the system description in another way and divide up the components into columns differently.

It is important to remember that the model is not built during the conceptualization stage. The modeler does not use the computer. The modeler is simply trying to extract as much relevant information and understanding of the system as possible. The final system dynamics model of the Heroin-Crime system may have a few additions and deletions from the components listed in Figure 1. The components list is just a guideline and not a strict frame for the model.

4.3 Reference Modes

4.3.1 Discussion

Reference mode and behavior chart are simply other names for a plot of the behavior of key variables of a system over time. A reference mode graph has time on the horizontal axis and units of the variables on the vertical axis. The time plots of the key variables are often quite useful, both before and after the model is built. The reference mode captures mental models and historical data on paper, gives clues to appropriate model structure, and can check plausibility once the model is built. A modeler constructs reference modes to check for the existence of some phenomenon or behavior worth modeling. Experienced modelers often know which structures can produce which behavior modes, thus making the reference mode an invaluable resource during the

formulation of the stock-and-flow structure of the model.[7] Often reference modes are not drawn due to lack of information and lack of effort. They are, however, essential both as a guide and a test throughout the model-building process.

While verbal descriptions or a set of statistics about system behavior can serve the same purpose as the graphic reference mode, preference for the graphic form exists for a number of reasons. A verbal description can be lengthy, confusing, and does not carry the same visual impression a graph does. A list of numbers is meaningless until the modeler puzzles out the patterns by creating a graph. The visual appeal of a graph is helpful in communicating the system to others and a modeler can easily hold it in memory throughout the modeling process.

Beware that reference modes are not infallible. They can change throughout the modeling process as a modeler begins to understand the system better and updates his mental model. It is also important to note that on the basis of these initial reference modes, a modeler can re-examine and restate the model purpose more precisely.

The historically observed and the hypothesized reference modes are the two types of reference modes that a modeler may create during conceptualization. Before moving to the heroin-crime system, this paper will further examine reference modes with an example of each type of reference mode.

A historically observed reference mode usually exists when a modeler is given a problem and wants to generate knowledge about possible causes or solutions. If one were presented with the question of what causes varying profitability in a specific company, research might reveal the historical reference mode shown in Figure 2. Now the modeler can focus on one lead or possible cause of varying profitability—inventory fluctuations.

[7] Formulation is the second stage of modeling and will be addressed in a later paper.

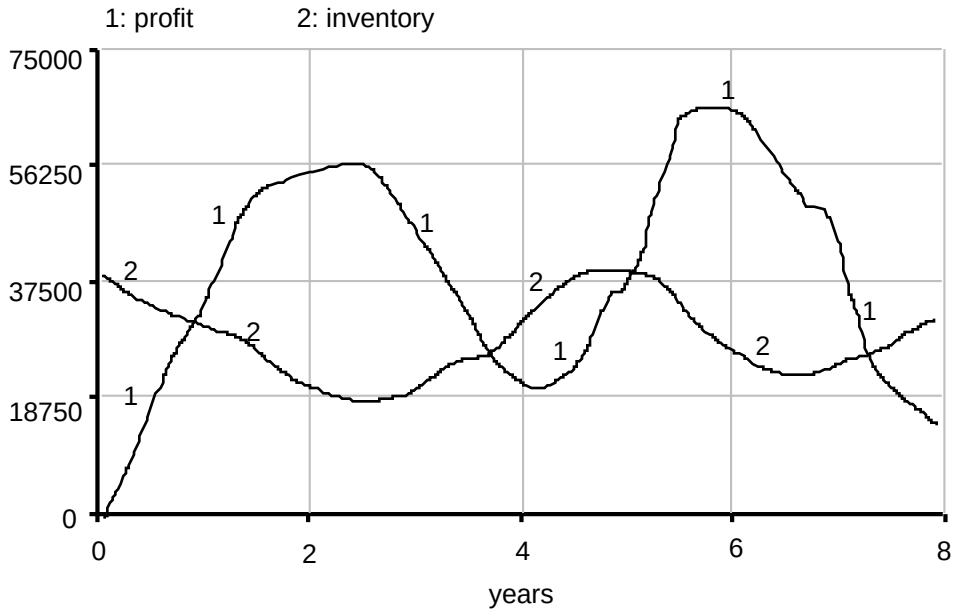


Figure 2: Historical Reference Mode for a Company Profit System

Historical reference modes use historical data. Comparing model output to the historical reference mode is particularly useful in later stages of model construction. If the model does not produce behavior similar to historical observations, it is an indication that the model might need re-work.

When no historical information is available, a modeler must create a hypothesized reference mode. The hypothesized reference mode consists of a simplified curve, typically drawn by hand, capturing the key features of the behavior pattern of the important system components. Sketching a hypothesized curve requires abstracting the interesting features of a variable from the given system details. Common hypothesized reference mode behaviors are exponential growth, exponential decay, overshoot and collapse, S-shaped growth, and damped, sustained and expanding oscillations. A hypothesized reference mode might show the future behavior once a specific policy is carried out. The reference mode in Figure 3 is an illustration of a mental model of the effects of attempting to reduce the deer population in a specific area. Half of the deer are killed over 2 months without other changes in the environment.

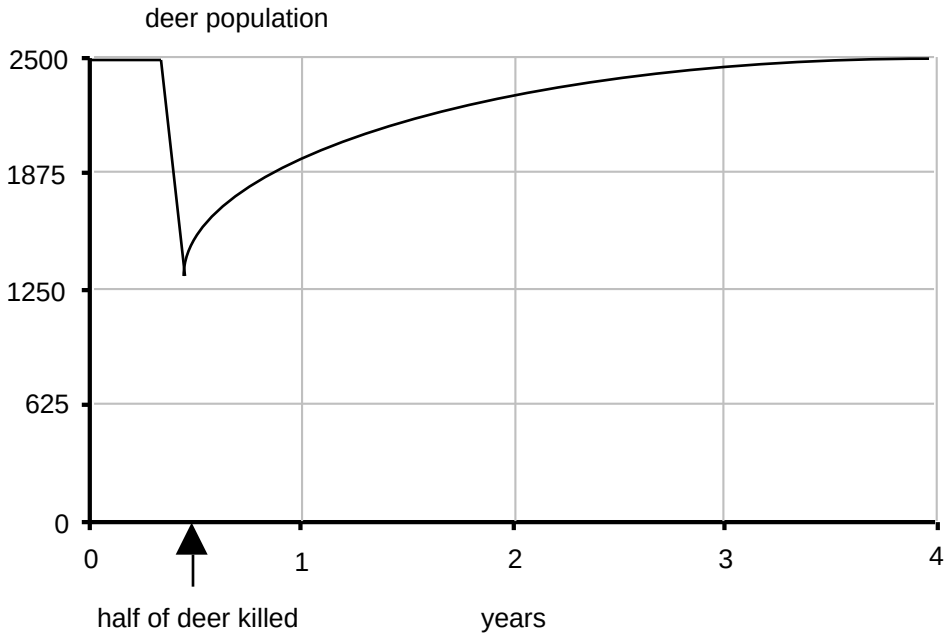


Figure 3: Hypothesized Reference Mode for Deer Population Policy System[8]

By drawing the reference modes, a modeler roughly simulates the situation in his mind. A modeler needs to think clearly about which factors influence each other. A modeler takes the most important factors, usually the stocks and flows of the system, and graphs their behavior over time. While it is not necessary to have set values for the factors, a modeler must specify the time frame and keep it constant for each reference mode component curve. One should draw reference modes on the same graph if possible, though multiple graphs may become necessary due to crowding. Start by picking one component and drawing that component's reference mode curve. Then graph the other components one at a time, relating the behavior to the curves already drawn.

The choice of time horizon is critical in drawing reference modes. If the time scale is too long or too short, it may obscure important behavior from view. Figure 4 contains an example of one component of an observed historical reference mode of an oscillating population graphed over a time horizon of 200 years. In Figure 5 the same population's reference mode is shown with a time horizon of four years. Because of the shorter time horizon, the population appears as a gently upward sloping line and not the oscillations in Figure 4. To determine which time horizon is relevant for the system being modeled, it is necessary to go back to the model purpose. A population oscillating with a period of approximately 100 years might only need a reference mode plot over 4

[8] Reference modes, particularly hypothesized reference modes, should be drawn by hand. The reference modes included in this paper were drawn using a computer drawing program *only* for the sake of clarity.

years if the model purpose focuses on a significant change over those 4 years. Reviewing the model purpose when choosing the time horizon of the reference mode is an essential step.

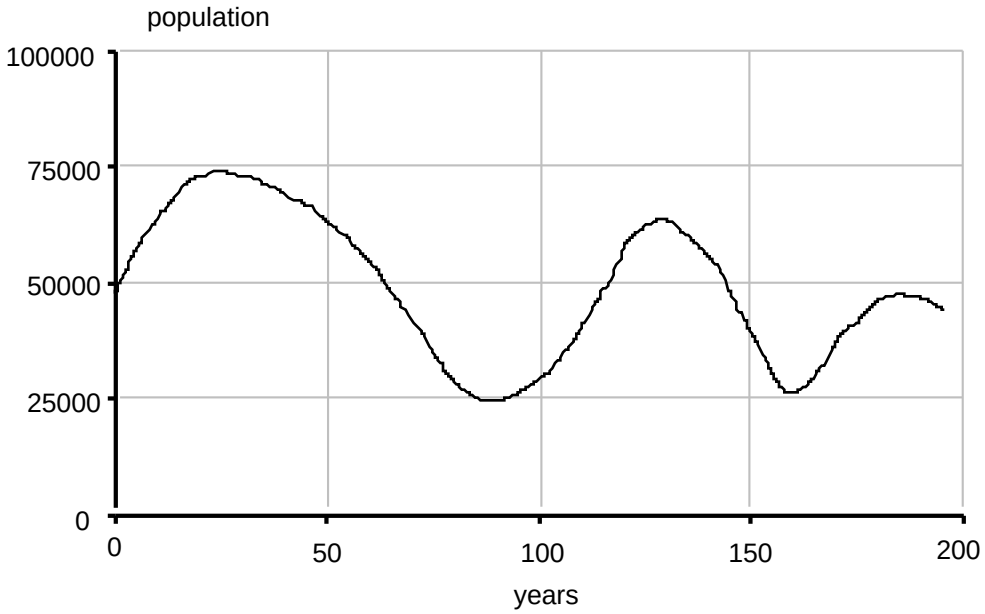


Figure 4: Historical Reference Mode of a Population over 200 Years

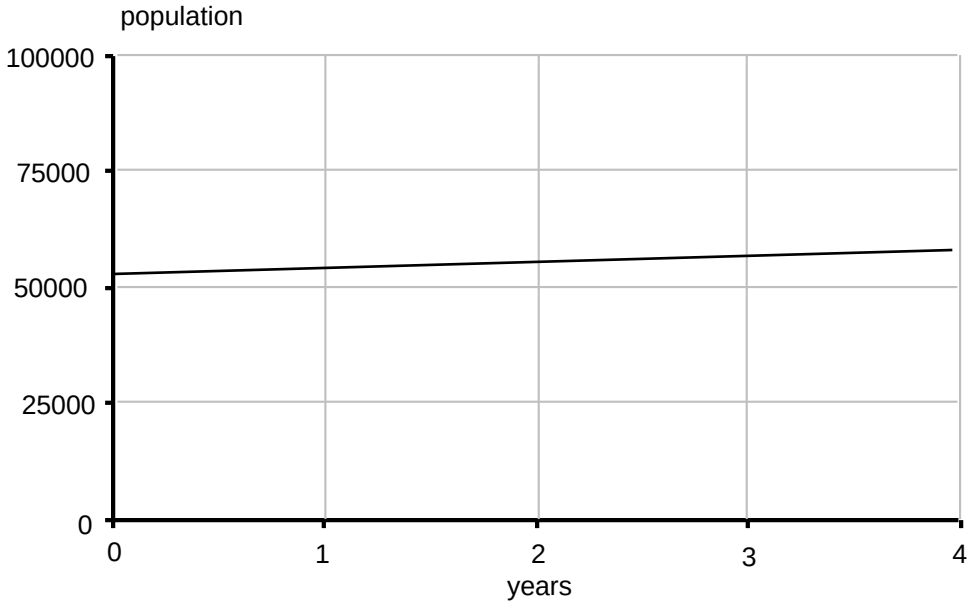


Figure 5: Historical Reference Mode of a Population over 4 Years

4.3.2 Heroin-Crime System

In the Heroin-Crime system from section 3, the reference mode could look like Figure 6. Different people will draw different reference modes because of individual mental models.

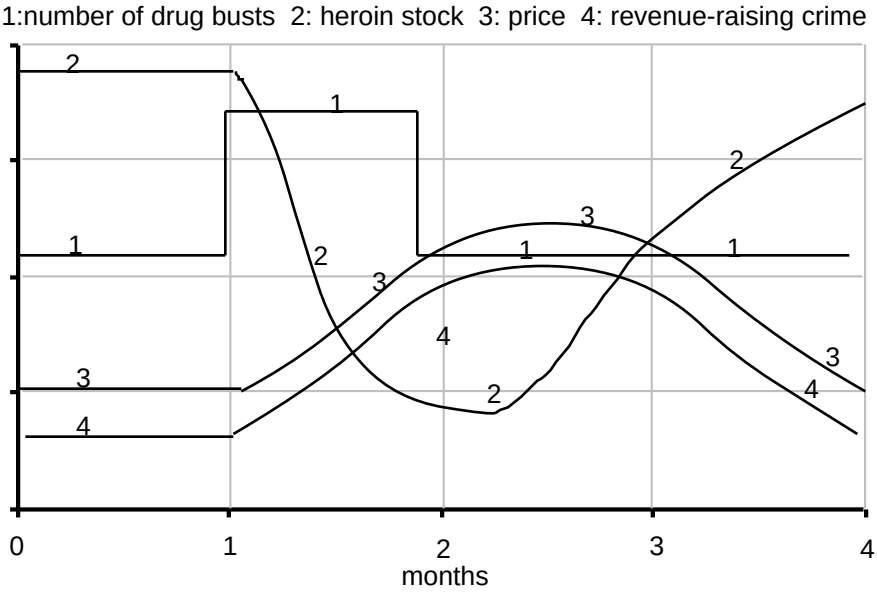


Figure 6: Reference Mode for Heroin-Crime System

To create the graph in Figure 6, the number of drug busts—the cause of the dynamics as defined by our model purpose—was graphed first. The other important variables were then graphed with respect to the drug bust curve. The number of drug busts, the component driving the important behavior isolated by our model purpose, is treated as a step input. As the number of drug busts increases, the heroin stock is depleted. With a lower heroin stock, but constant demand, the price of heroin increases. In order to pay the higher prices, many addicts commit more revenue-raising crimes, such as burglary. The higher prices (and higher profits for the sellers) will lead to a higher importation of heroin to the area. Eventually the heroin stock and price return to normal levels. The shapes of the reference mode curves are mainly hypothesized from the few historical data points given in the system description. Note that there are no numerical values on the vertical or y-axis of the graph because it is the behavior relationship among the curves that is important. Based on the quantity and quality of available data, a modeler may choose whether or not to put specific values on the vertical axis.

4.4 Basic Mechanisms

The final step in conceptualization is deciding on the basic mechanisms of the system. Specifically, the basic mechanisms are the feedback loops in the model. The basic mechanisms represent the smallest set of realistic cause-and-effect relations capable of generating the reference mode. The basic mechanisms may also be thought of as the simplest story that explains the dynamic behavior of the system.

When deciding on the basic mechanisms, a modeler must first mentally decide on a dynamic hypothesis. A dynamic hypothesis is an explanation of the reference mode behavior and should be consistent with the model purpose. A modeler uses a dynamic hypothesis to draw out and test the consequences of the feedback loops. Then a modeler creates diagrams illustrating the basic mechanisms driving the system's dynamic behavior. A model cannot be built without an understanding of the feedback loops. Having a good dynamic hypothesis and well-defined basic mechanism implies having enough information to begin formalizing the system into level and rate equations. One may then move to formulation, the next stage of the modeling process.

The field of system dynamics divides on the issue of how best to diagram the basic mechanisms. Some favor presenting the basic mechanisms in the form of causal-loop diagrams. Others prefer to begin by mapping out the stock-and-flow structure. In Road Maps we prefer the stock-and-flow representation. It is important to note that mapping out the causal-loop diagrams or the stocks and flows is not formulation. In conceptualization, both are just diagrams and the equations are not touched. Before moving on to the Heroin-Crime system, this paper will discuss both diagramming methods and give examples of each. We suggest that the reader use stock-and-flow diagrams for conceptualizing systems.

4.4.1 Discussion: Stock-and-Flow Diagrams

Modelers who choose to diagram the basic mechanisms in terms of stocks and flows often have the causal-loop relationships in the back of their minds. Stock-and-flow diagrams have a tendency to be more detailed than a causal-loop diagram representation, forcing the modeler to think more specifically about the system structure. Many simple mistakes can be avoided by diagramming the basic mechanisms with stocks and flows rather than causal loops because the relationships between components of a stock-and-flow diagram are more strictly defined than those in a causal-loop diagram. While generally more complex and more time consuming to create, stock-and-flow diagrams are more informative than causal-loop diagrams.

Figure 7 contains the basic mechanism of the company from Figure 2 in a stock-and-flow diagram. Modifications to the stock-and-flow diagram will be made when the simulation model is built and the model equations are written during the formulation stage of model building. Keep in mind that Figure 7 is just a diagram of the basic mechanisms. Certain components, such as table functions, will need to be added to create a set of equations which can be simulated. Until the model is built and simulated, the modeler will not know how reasonable the conceptualized diagram is.

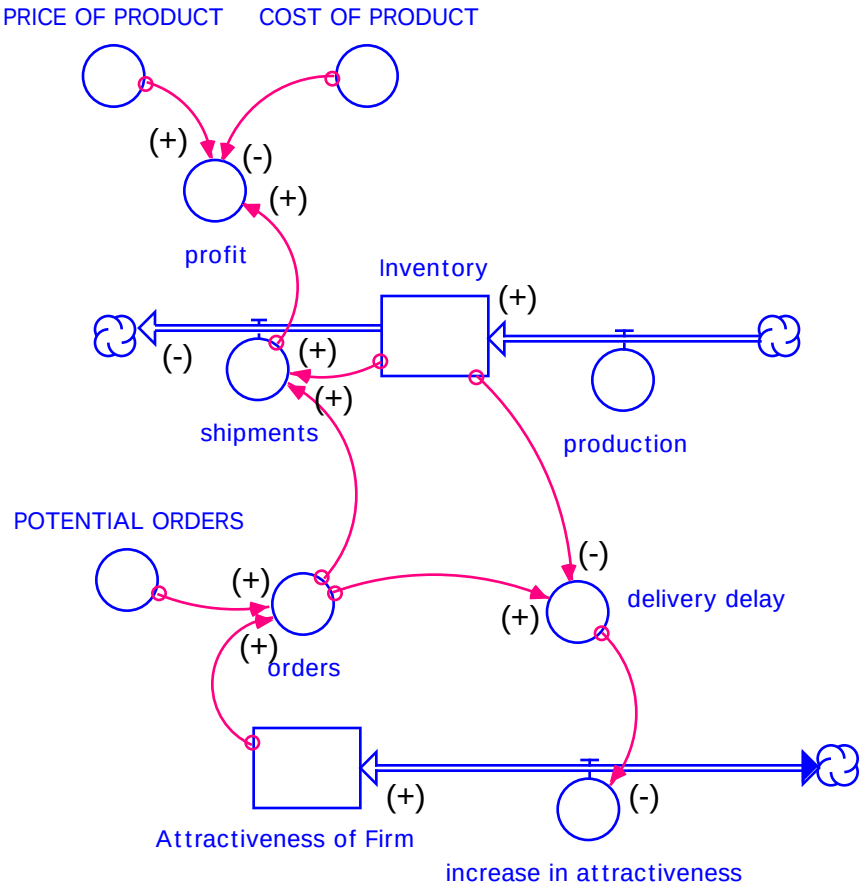


Figure 7: Stock-and-Flow Diagram of Basic Mechanism of a Company (see Fig. 2)

4.4.2 Discussion: Causal-loop Diagrams

Modelers who conceptualize the basic mechanisms of a system in terms of causal-loop diagrams usually have some idea of the stock-and-flow structure of the model in the back of their minds. It is good practice to draw boxes around the stocks in the causal loops. By definition, each causal loop must contain at least one stock. Stocks are accumulations. If a causal loop did not contain a stock, the loop dynamic would be instantaneous. No behavior over time would exist to examine.

If one ends the conceptualization stage with causal-loop diagrams, it becomes easy to overemphasize the value of causal-loop diagrams. A causal-loop diagram is not a model. One cannot conduct any type of policy analysis, locate leverage points, or tell which loop is dominant with just a causal-loop diagram. Causal-loop diagrams are, however, simple to understand and easy to use, and therefore, they are often used *after* simulation to explain to others the insights gained from the model. It is important to understand the limitations of causal loop diagrams if they are used.[9]

Look back to Figure 2, the reference mode of a company's profit and inventory levels. The cause and effect relationship between inventory and profit, leading to profit fluctuations, is captured by the basic mechanism in Figure 8.

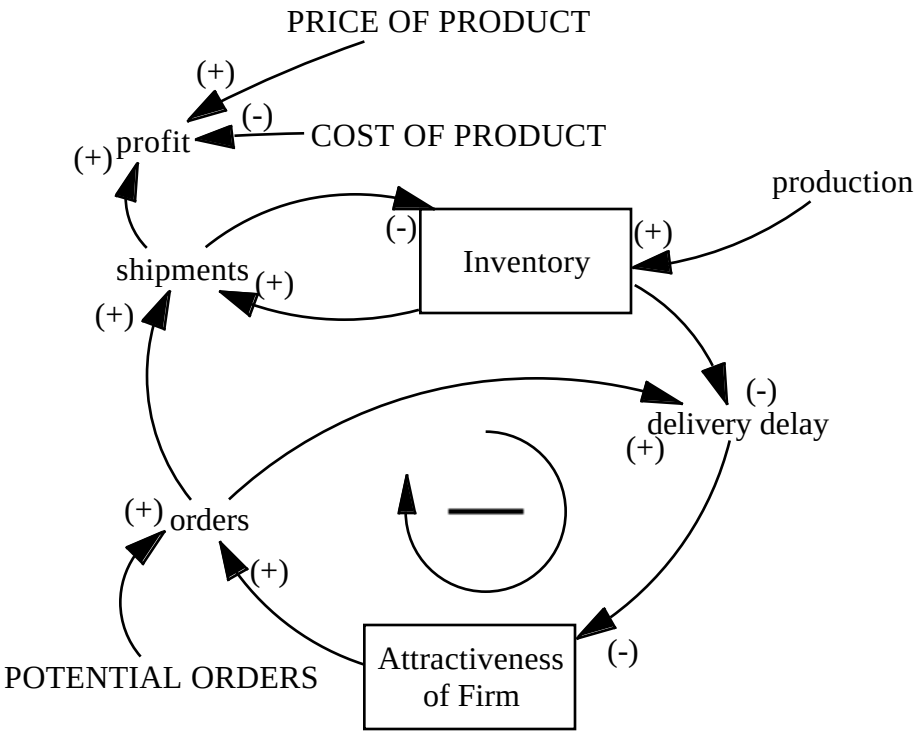


Figure 8: Causal-loop Diagram of Basic Mechanism of a Company (see Fig. 2)

Some people believe that a stock-and-flow structure is more complex than a causal-loop diagram. In some cases, however, a causal-loop diagram may seem more complex than a stock-and-flow structure. For example, Figure 9 contains the stock-and-

[9] The limitations of causal-loop diagrams are summarized by: George P. Richardson, 1976. *Problems with causal-loop diagrams* (D-3312), Sloan School of Management, Massachusetts Institute of Technology, 13 pp. (available in Road Maps 4)

flow structure of an aging chain for a company's employees. The stock-and-flow diagram is conceptually easier to understand and less complex than the causal-loop of the aging chain shown in Figure 10. Because a stock-and-flow diagram is a more specific representation of the system being modeled, we believe that initial conceptualization should be done with stock-and-flow structures.

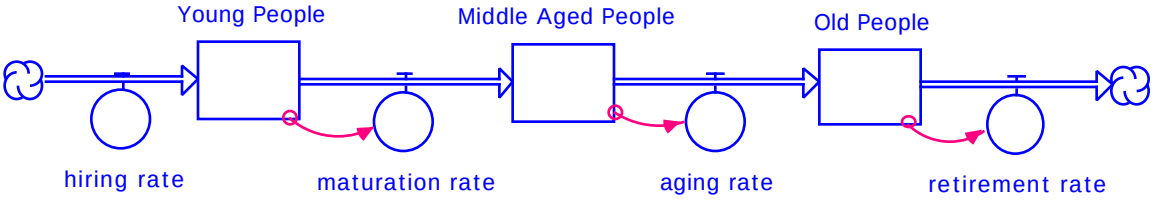


Figure 9: Stock-and-Flow Diagram of an Aging Chain

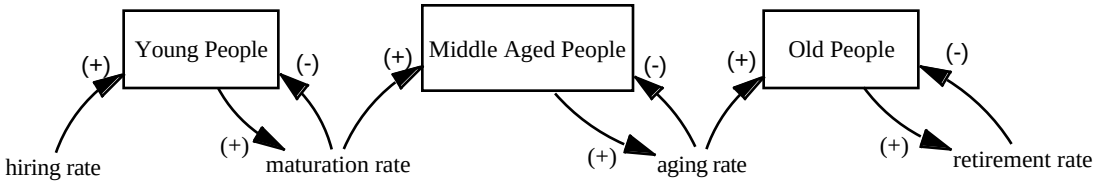


Figure 10: Causal-loop Diagram of an Aging Chain

To have a complete description of the basic mechanisms, a modeler must make certain that all feedback loops are complete and captured within the closed boundary. If not, either the basic mechanisms, the model boundary, or both must be revised. The modeling effort should be performed by choosing and linking levels and rates. Causal-loop diagrams can sometimes be useful for later communication of the model insights.

4.4.3 Heroin-Crime System

Figure 11 contains the stock-and-flow diagram representation of the basic mechanisms of the heroin-crime system. Figure 11 does not show the final model structure. It is simply a representation of the relationships between the system components that create the dynamic behavior of the reference mode. Other components, such as "demand-supply ratio" or "heroin availability factor," as well as additional information links, might be added to make the heroin-crime model plausible and more robust. Such additions will be made during the formulation stage.

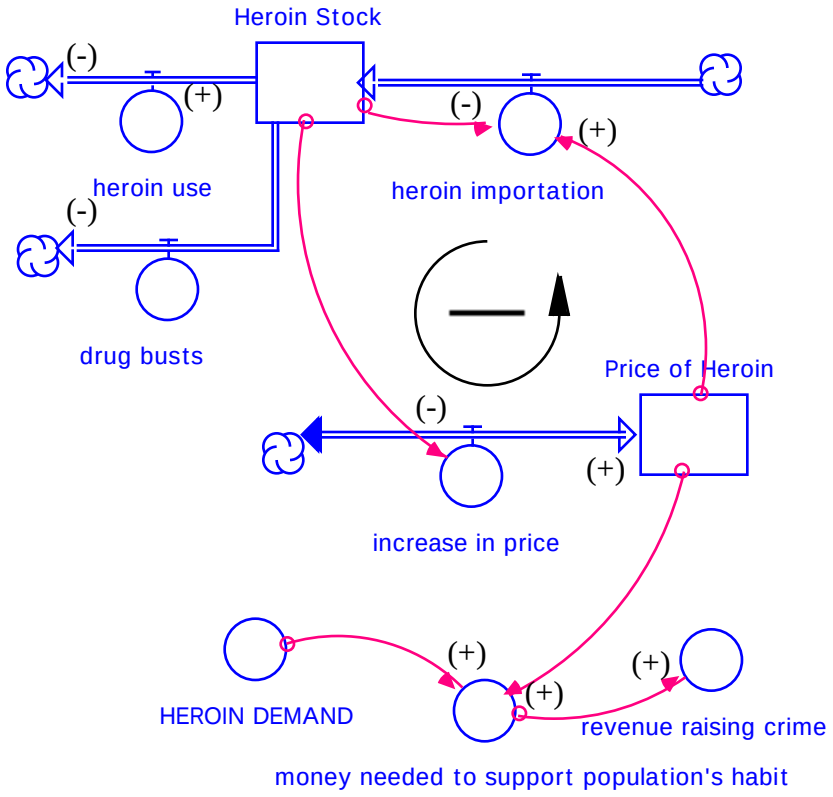


Figure 11: Stock-and-Flow Diagram of the Basic Mechanisms of the Heroin-Crime System

For the Heroin-Crime system, a causal-loop representation of the basic mechanisms might look like Figure 12.

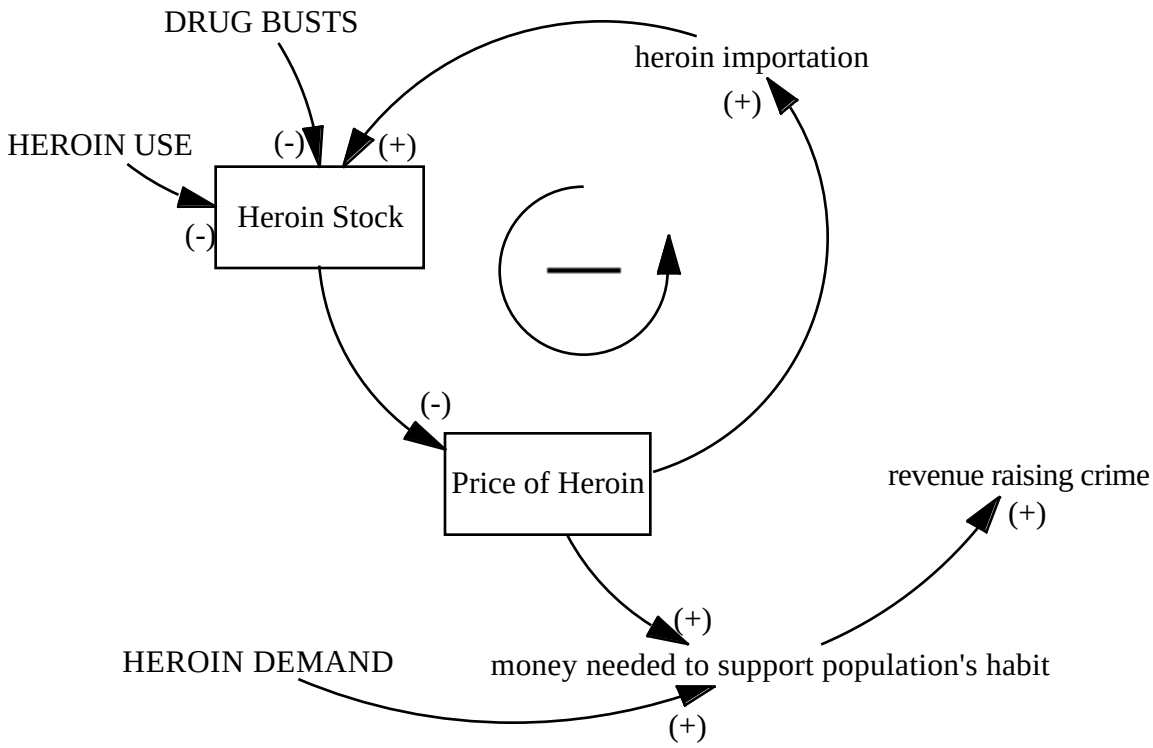


Figure 12: Causal-loop Diagram of the Basic Mechanisms of the Heroin-Crime System

In creating the diagrams of the basic mechanism, the modeler must look back to the components list for the Heroin-Crime System in Figure 1 and the reference mode in Figure 6. One can begin the diagram with the endogenous components. Heroin Stock is drawn on the paper. Depending on the level of detail the modeler would like, the modeler adds the various flows to and from the Heroin Stock that affect its level. These flows are drug busts, heroin use, and heroin importation. Because heroin importation is an endogenous component, it seems natural to analyze that component next. Heroin importation is affected in the Heroin-Crime System described primarily by the Price of Heroin. The Price of Heroin is effected by the Heroin Stock, thus completing our major negative feedback loop. The remaining endogenous and exogenous components of heroin demand, money to support population's habit, and revenue raising crime can easily be added to the diagram now that the major loop has been diagrammed.

The modeler has now reached the end of the conceptualization stage of model-building. The remaining steps will be discussed in future Road Maps papers. The modeler should now have extracted as much information as possible from a simple system description, and have reached a new level of understanding of that system.

5. CONCEPTUALIZATION EXERCISE[10]

Turn to Exercise 14 of *Study Notes in System Dynamics*.[11] Read the description of the Yellow-Fever Epidemic on pages 365 to 367. **Do not** do the exercises in the book that follow the description. Instead, assume that you want to build a system dynamics model of the Yellow-Fever system. Begin by conceptualizing the system using the steps described in this paper. The system described in the book gives much more information than needed to conceptualize the system. It is not necessary to use all of the facts given.

Now, answer the following questions. Appendix A contains a Modeling Worksheet that may be useful as a guide to answering the exercise questions.

1. What is the purpose of the model being created? Keep the answer to this question in mind while answering the remaining questions.
2. What is the model boundary? Develop an initial components list, divide it up into two categories (endogenous and exogenous) and do a preliminary identification of stocks and flows.
3. What does the reference mode look like? Draw a graph by hand.
4. Describe the basic mechanisms. Give only those essential for the model behavior.

Answers to the above four questions create a dynamic hypothesis to explain the behavior present in the Yellow-Fever Epidemic system.

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[10] The solution to the following exercise is in section 6.

[11] Michael R. Goodman, 1974. Study Notes in System Dynamics. Portland, OR: Productivity Press, 388 pp.

6. CONCEPTUALIZATION EXERCISE SOLUTION

Many different solutions are possible to the questions asked in the exercise in section 5. The following solution is only one of the possibilities.

1. The purpose of the model could be to estimate the course of yellow-fever once the virus hits a city. The information generated could be useful for city officials in developing a plan of action and immunization programs.

2. A reasonable initial components list could contain the following components:

mosquito population	infections mosquitoes
death fraction for mosquitoes	incubating mosquitoes
birth fraction for mosquitoes	vulnerable humans
sick humans	incubating or infectious humans
immunized humans (by vaccine)	immune humans
recovery rate for humans	death rate for humans
death fraction for sick humans	human population
birth fraction for humans	birth rate for humans

The initial components list is then divided up and the model boundary is defined by the table of components in Figure 13.

ENDOGENOUS COMPONENTS	EXOGENOUS COMPONENTS
infectious mosquitoes (S)	mosquito population
incubating or infectious humans (S)	birth fraction for mosquitoes
sick humans (S)	death fraction for mosquitoes
death rate for humans (F)	death fraction for sick humans
incubating mosquitoes (S)	
vulnerable humans (S)	
immune humans(S)	
recovery rate for humans (F)	

Figure 13: Model Boundary for Yellow-Fever System

Because the purpose for building the model is to trace the natural path of a yellow-fever epidemic in a city, the base model should not include immunizations. Immunizations would mask the natural course of the disease. The relevant disease in a city is urban or classical yellow-fever, not jungle yellow-fever. Both the number of immunized people and any component relating to jungle yellow-fever are thus to be excluded from the model boundary, though they exist on the initial components list.

The system description states that the mosquito population can be taken as a constant, thereby making it an exogenous component. Carrying the disease makes no difference to the mosquitoes so the death and birth fractions for all mosquitoes (normal and infectious) are constant exogenous components of the system. The death fraction for sick people is also an exogenously measured constant fraction.

The dynamic components of greatest interest when charting the course of an epidemic are the number of sick and number of vulnerable humans, and the human death and recovery rates. Other important components are the number of infectious mosquitoes, the number of infectious humans, and the number of incubating humans. All of the above dynamic variables are endogenous components of the system.

Because the model system deals with a short time horizon, the birth fraction and birth rate for people will not affect the dynamics of yellow-fever. Because all humans living in the area can be divided into one of the human population stocks (vulnerable humans, immune humans, sick humans, incubating or infectious humans), there is no need to create an additional human population stock in the model.

3. The reference mode for the most important dynamic components is shown in Figure 14. Again, note that the specific components included in the reference mode can vary depending on the model purpose. The stocks of sick humans and immune humans, the human death rate, and the stock of infectious mosquitoes were chosen for the reference mode since these four components will reflect the dynamics of both the mosquito and human population. Using the information in the system description, the first curve graphed was the number of sick people, because naturally people must be sick before they either die or become immune. Initially, before the epidemic hits, there are no sick people. After the epidemic has passed and the sick period of 7 days for humans is over, there will also be no sick people. In between these two points, the number of sick humans rises dramatically as the epidemic spreads and then reduces as people either recover or die. The reference mode for the number of sick humans was thus graphed as a bell-shaped curve. Given the fast spread of an epidemic and the data given in the reading, it seems reasonable to have a time frame of only 4 months.

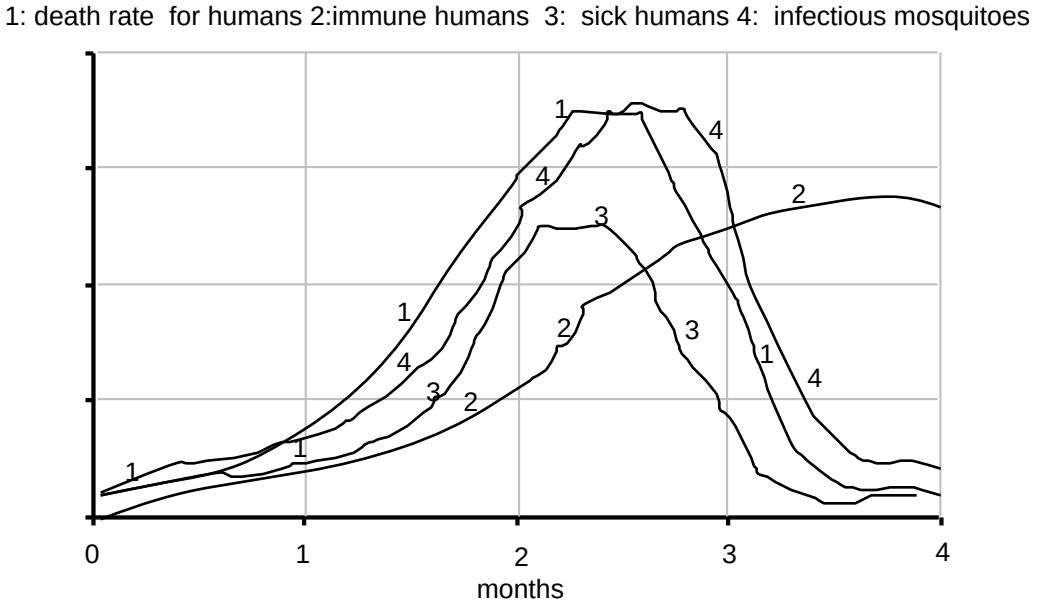


Figure 14: Reference Mode for Yellow-Fever System

The remaining important components were graphed with respect to curve of sick humans. Because the death rate is going to be some constant percentage of the number of sick humans, the reference mode for the death rate for humans is also a bell-shaped curve. The recovery rate of sick humans will also be a constant percentage of the number of sick humans, and is thus also a bell-shaped curve. In the reference mode in Figure 14, however, the modeler is graphing the stock of immune humans and not the recovery rate

of humans, which is the flow into the immune humans stock. If the flow into any stock is a bell-shaped curve, the behavior of the stock will be S-shaped.[12] The reference mode for immune humans is S-shaped. Because mosquitoes pass on yellow-fever to humans, making humans sick, and humans pass on the disease to mosquitoes, making mosquitoes infectious, it seems natural that the infectious mosquito curve also mirrors the bell-shaped behavior of the sick humans curve. To double check this assumptions, the modeler should check the expected value of the number of infectious mosquitoes before and after the epidemic hits. Given the short life-span of the mosquito, the bell-shaped reference mode of infectious mosquitoes accurately captures the assumption that there will be no infectious mosquitoes before or after the epidemic. Note that the infectious mosquitoes curve peaks slightly after the sick humans curve peaks. By examining the data in the reading, it takes humans only 3 to 6 days when bitten by an infectious mosquito to become sick. Humans are then sick for about 7 days, and then humans either die or recover. After biting an infectious human, mosquitoes go through a 12 day incubation process before becoming infectious. Once infectious, mosquitoes will only live about another 4 days. From this data, it seems reasonable to assume that it takes longer for the infectious mosquitoes population to peak because mosquitoes have a longer 12 day incubation delay as compared to the 3 to 6 day human incubation delay.

4. The simplified stock-and-flow and causal-loop diagrams for the basic mechanism driving the yellow-fever reference modes are shown in Figures 15 and 16 respectively. The dynamic hypothesis that is driving the reference mode behavior consists of two main points, stressed in the Goodman reading[13]:

1. Infectious mosquitoes pass the disease on to humans. These humans are now contagious and can, in turn, make mosquitoes into carriers of the disease. Yellow-fever is thus **self-perpetuating**, creating an important **positive** or **reinforcing loop** in the model.
2. Mosquitoes which bite infected humans first go through an incubation period before becoming infectious to vulnerable humans. Incubating mosquitoes **do not** transmit yellow-fever to humans.
3. A human bitten by an infectious mosquito goes through a disease incubation period, during which the human is infectious to a mosquito. After the

---

[12] For more explanation please see: Leslie Martin, 1996. *Exploring S-shaped growth* (D-4476), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, October 3, 40 p.
[13] Goodman, pp. 365 - 367.

incubating period, the humans become visibly sick, but cannot pass the disease on to mosquitoes. Incubating humans **do** transmit yellow-fever to mosquitoes.

4. Given that the spread of yellow-fever is a positive loop, some limiting force must exist to prevent the infection of the entire human population. One example of such a limit is a **negative feedback or balancing loop** containing the component of vulnerable humans. As the number of vulnerable humans decreases, the spread of the disease through mosquitoes and humans also decreases.

Keep in mind the model boundary defined in Figure 13 and which components were designated as stocks and which as flows. The endogenous stocks and flows are a good starting point for diagramming the basic mechanisms. Some modelers may find it easiest to start by drawing all the stocks first, and then figure out how they connect to one another, keeping in mind the positive and negative loops mentioned above.

The stock-and-flow structure in Figure 15 depicts one way of representing the basic mechanisms. Remember, the stock-and-flow diagram is not a model ready for simulation, but only a diagram of the feedback loops causing the dynamic behavior depicted in the reference modes for the system.

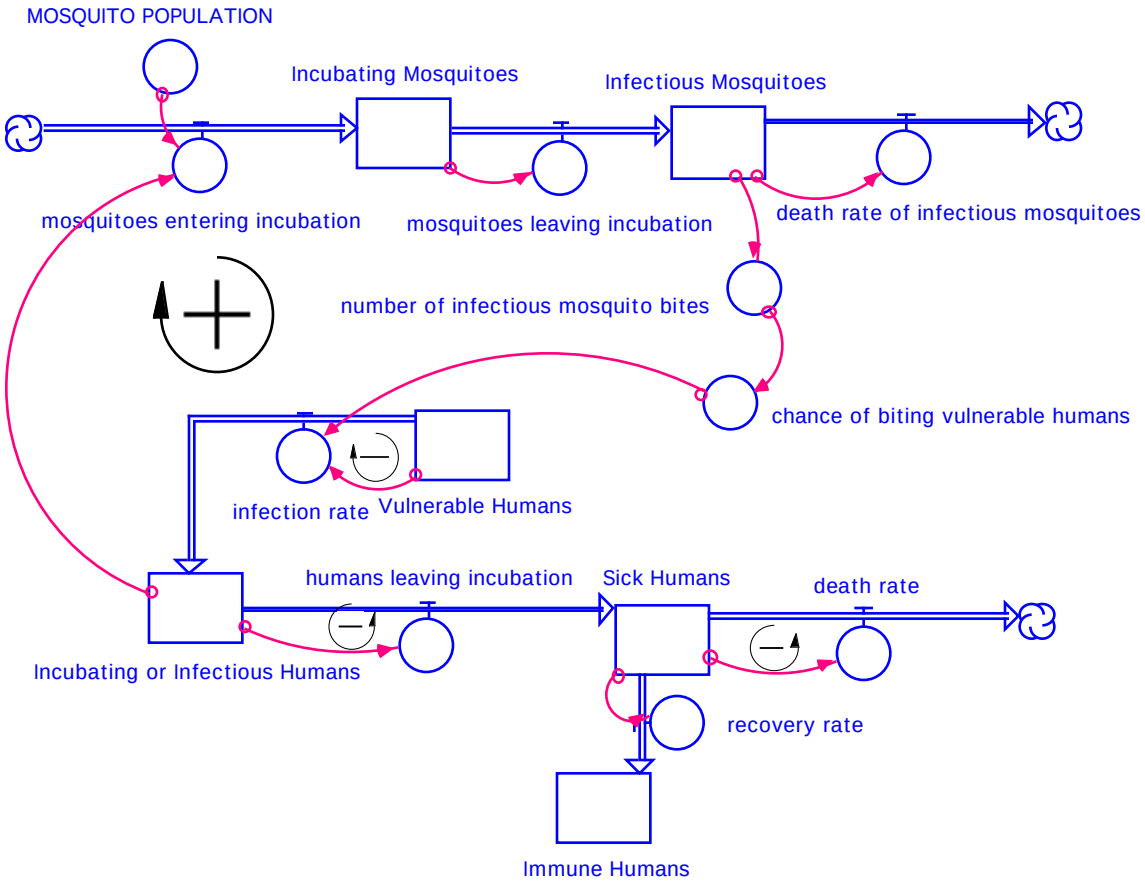


Figure 15: Stock-and-flow Diagram of the Basic Mechanisms of the Yellow-Fever System

Figure 16 contains the alternative causal-loop diagram representation of the basic mechanisms.

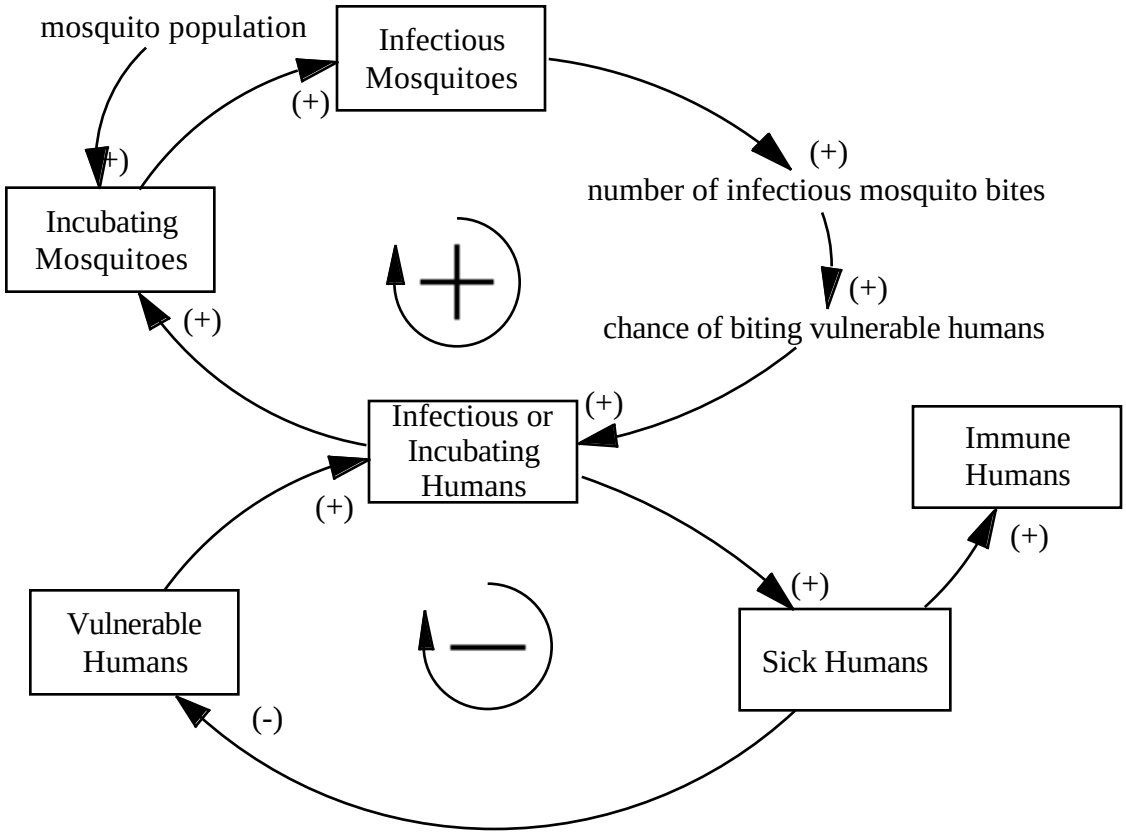


Figure 16: Causal-loop Diagram of the Basic Mechanisms of the Yellow-Fever System

7. APPENDIX A: MODELING WORKSHEET

The following modeling worksheet can be used as a guide to the conceptualization stage of model building.

Problem Area:

_____
_____
_____

Purpose of Model:

_____
_____
_____
_____

Model Audience:

_____
_____

Model Boundary Initial Component List:

_____ _____
_____ _____
_____ _____
_____ _____
_____ _____
_____ _____
_____ _____
_____ _____
_____ _____
_____ _____

Model Boundary Column Component List:

ENDOGENOUS COMPONENTS	EXOGENOUS COMPONENTS

Assumptions/Comments on Column Component List:

_____

_____

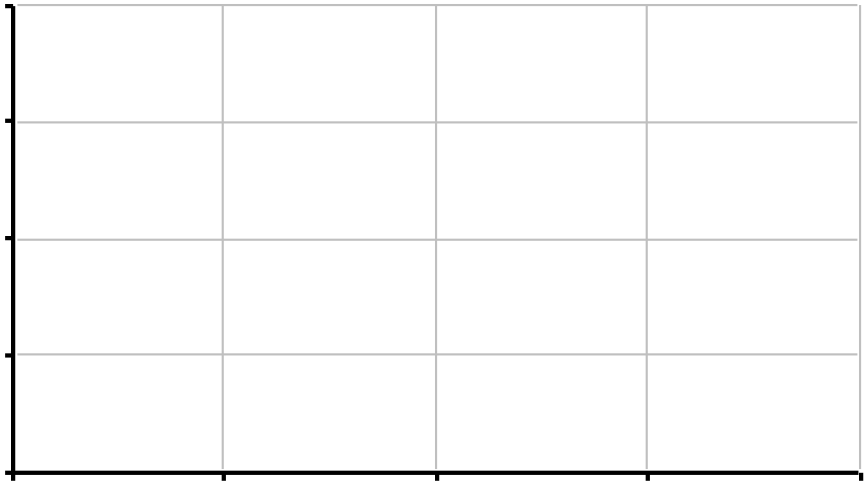
_____

_____

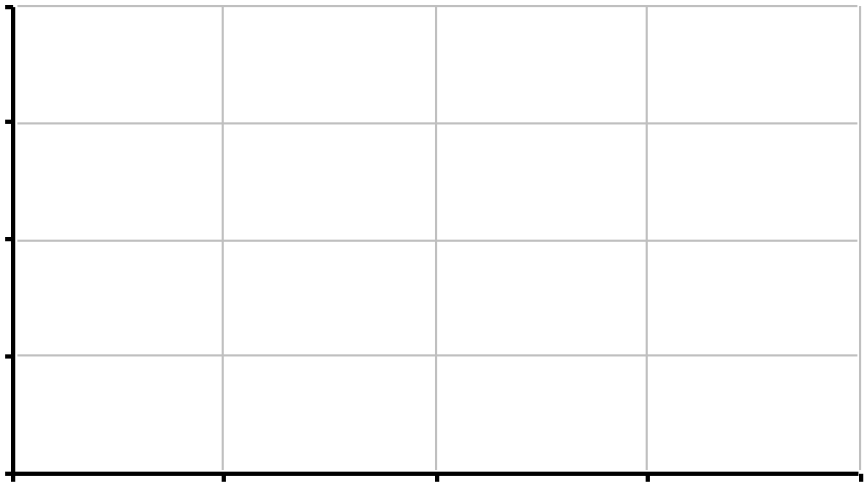
On the Column Component List above, mark each stock with the letter “S” and each flow with the letter “F.”

Reference Modes:

(Note: It is essential to fill in values for the time horizon on the horizontal axis of the reference mode graphs.)



Time



Time

Assumptions/Comments on Reference Modes:

_____

_____

_____

_____

Hypothesis for the dynamics of the model behavior:

_____
_____
_____
_____
_____
_____

Stock-and-flow Diagram of the basic mechanisms:

8. APPENDIX B: RELATED READINGS

The following is a list of readings related to the heroin-crime sample system used in this paper.

Keith Gardiner and Raymond Shreckengost, 1987. "A system dynamics model for estimating heroin imports into the United States." *System Dynamics Review*, Vol. 3, No. 1, pp. 8 - 27.

Gilbert Levin, Gary B. Hirsch, and Edwards B. Roberts, 1975. The Persistent Poppy: A Computer Aided Search for Heroin Policy. Cambridge, MA: Ballinger, 229 pp. (out of print)

Nancy Roberts et al (ed.)1983. *Heroin Addiction and its Impact on the Community.* Chapter 24 in Nancy Roberts et al (ed.). Introduction to Computer Simulations, a System Dynamics Approach. Portland, OR: Productivity Press, 562 pp.

Mistakes and Misunderstandings:

Use of Generic Structures and Reality of Stocks and Flows

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Lucia Breierova
December 18, 1996
Vensim Examples added October 2001

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1. INTRODUCTION

This paper studies two related mistakes and misunderstandings. First, it warns against the incorrect use of a generic structure. Secondly, it explains that stocks represent real-world accumulations and that flows represent changes over time in the stocks. This paper will examine two mistakes made in modeling a simple population of rabbits.

2. A FIRST ATTEMPT TO MODEL THE SYSTEM

A student decides to build a simple model in order to study how an area with limited resources affects the growth of a population. Initially, ample resources do not limit the exponential growth of the population. As the population grows, however, resources become more scarce, and the rate of growth slows. The population asymptotically approaches an equilibrium value.[1] The student expects the behavior of such a system to be S-shaped growth. Figure 1 shows a generic structure known to produce S-shaped growth.[2]

[1] The equilibrium value is known as the carrying capacity of the environment.

[2] For more information on generic structures producing S-shaped growth, please refer to: Terri Duhon and Marc Glick, 1994. Generic Structures: S-Shaped Growth I (D-4432), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 24, 30 pp.

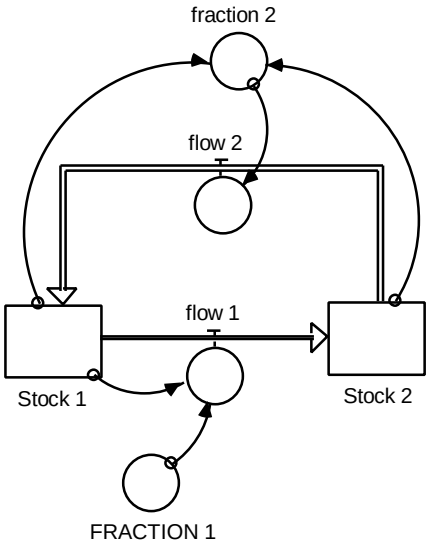


Figure 1: Generic structure producing S-shaped growth

The student applies the generic structure to model a rabbit population as shown in Figure 2. The model equations are in section 6.1 of the Appendix.

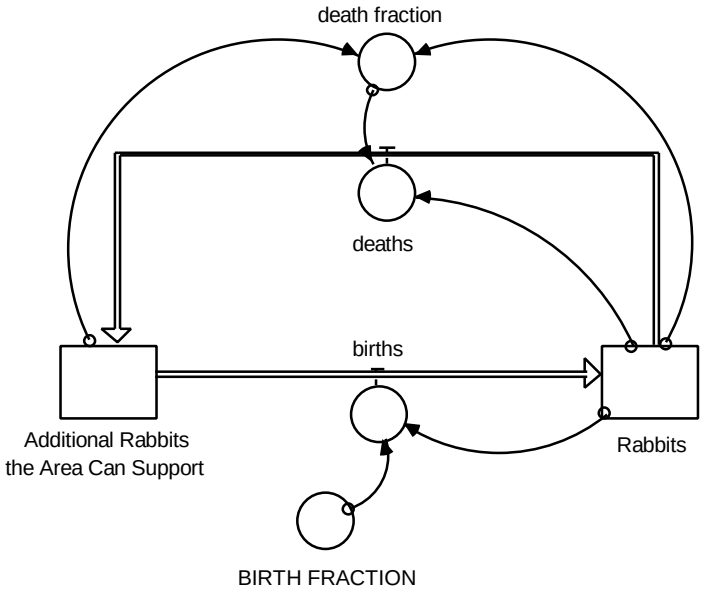


Figure 2: First model of the rabbit population

The model does indeed generate S-shaped growth of the “RABBITS” stock, as shown in Figure 3.

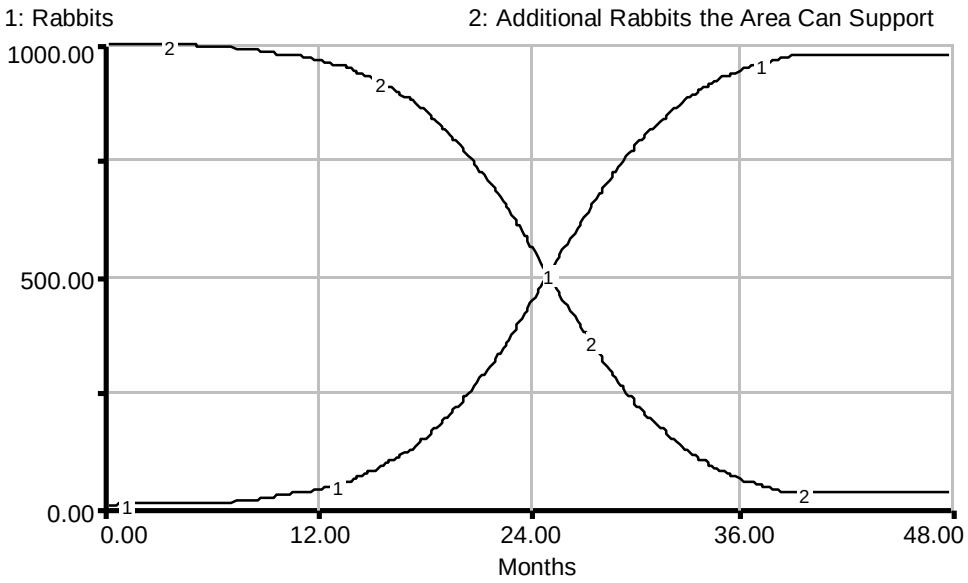


Figure 3: Stock behavior of the first model

3. MISTAKES AND MISUNDERSTANDINGS

Even though the model shown in Figure 2 generates S-shaped growth, it is not a realistic model. By simply fitting the system to a generic structure, the student did not realize that the structure does not represent the system.

One of the stocks, "Additional Rabbits the Area Can Support," is not an accumulation. The model implies that "Rabbits" are born from the stock of "Additional Rabbits the Area Can Support" through the flow of "births." Thus, hypothetically, a rabbit, before being born, exists as an additional rabbit. Likewise, "Rabbits" that die become "Additional Rabbits the Area Can Support" through the flow of "deaths."

The model in Figure 2 consequently suggests that a rabbit can exist in two states: either as a rabbit, in the stock of "Rabbits," or as an additional rabbit, in the stock of "Additional Rabbits the Area Can Support." In the real-world system, however, dead rabbits do not accumulate as additional rabbits, and additional rabbits are not reincarnated into rabbits. Such a formulation of the model is incorrect and does not represent the real system. The student must rebuild the model.

In addition, the formulation of the "death fraction" equation is implausible. With the initial values of the two stocks, the "death fraction" initially has the value of 1/2000 rabbits per rabbit per month, implying an average lifetime of a rabbit close to 167 years.

4. OVERCOMING OUR MISTAKES AND MISUNDERSTANDINGS

Figure 4 shows a corrected model of the rabbit population. Documented equations are in section 6.2 of the Appendix.

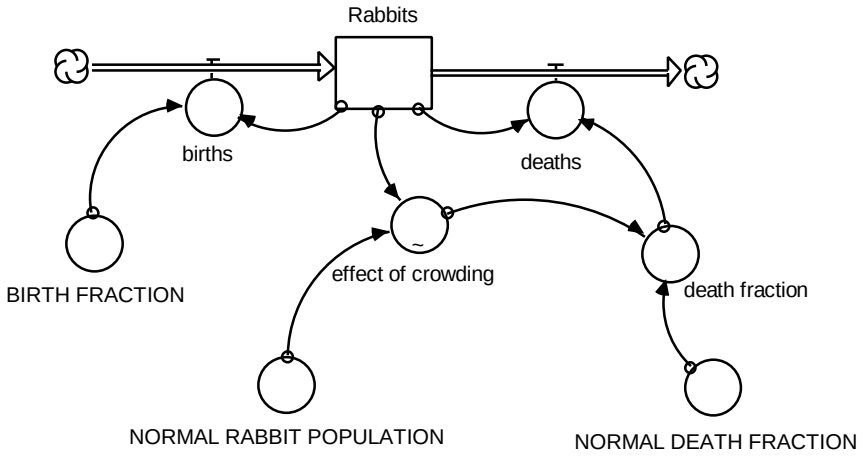


Figure 4: A corrected model of the rabbit population

In Figure 4, the total number of rabbits that the area can support is the converter "NORMAL RABBIT POPULATION." The model only contains one stock, "Rabbits," with an inflow of "births" and an outflow of "deaths." The "death fraction" is the product of a "NORMAL DEATH FRACTION" (set to be one third of the "BIRTH FRACTION") and the "effect of crowding." The "effect of crowding," shown in Figure 5, is a nonlinear function of the ratio of "Rabbits" to "NORMAL RABBIT POPULATION."[3]

[3] Further papers in the Mistakes and Misunderstandings series will explain how graph functions such as the "effect of crowding" should be constructed.

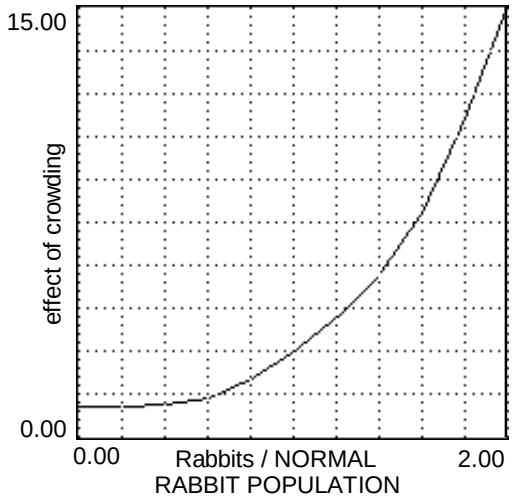


Figure 5: The "effect of crowding" graph function

The "effect of crowding" has a value of 1 when the population of "Rabbits" is low. The "effect of crowding" then increases to the value of 3 when the number of "Rabbits" is equal to the "NORMAL RABBIT POPULATION" (the ratio of "Rabbits" to the "NORMAL RABBIT POPULATION" is equal to 1).[4] Furthermore, the "effect of crowding" continues to increase if the ratio of "Rabbits" to "NORMAL RABBIT POPULATION" becomes greater than 1. Thus, when the ratio of "Rabbits" to "NORMAL RABBIT POPULATION" is 1, all resources are being used. The "effect of crowding" table function outputs a value of 3, and the "death fraction" is equal to the "BIRTH FRACTION." (Remember that "BIRTH FRACTION" is equal to three times "NORMAL DEATH FRACTION.") "Births" are then equal to "deaths," and the system stabilizes at equilibrium with 1000 "Rabbits."

[4] The value of 3 was determined by dividing the "BIRTH FRACTION" by the "NORMAL DEATH FRACTION."

The corrected model produces S-shaped growth, as shown in Figure 6.

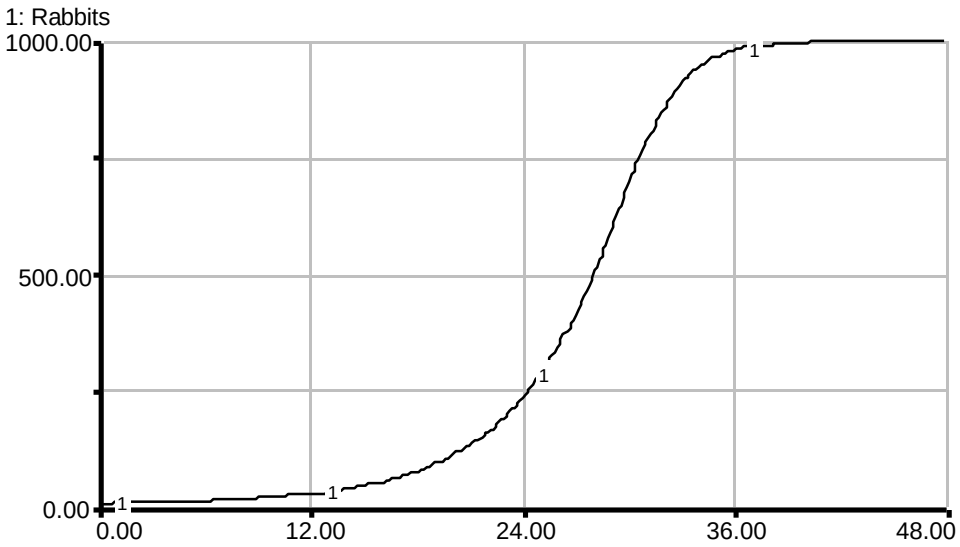


Figure 6: Behavior of the corrected model of rabbit population

The student did not make the modeling mistake because of a lack of knowledge about the system. Instead, the mistake resulted from an incorrect use of a generic structure. By simply applying the generic structure to the specific system, the student did not realize that one of the stocks used was not a real-world accumulation. Generic structures are useful, but a modeler should not adopt them blindly without considering the particular system being modeled.

The structure that was incorrectly used in the first rabbit population model can, however, be used in a variety of other situations. It could, for example, be used to model a market for a specific product, or the spread of epidemics. Nevertheless, it is not suitable to model the growth of a population within a restricted area.

The generic structure from Figure 1 contains two stocks that represent accumulations of the same units but in two different states. For example, in a model of the spread of epidemics (generating S-shaped growth), the two stocks would both represent people. One of the stocks would be "Healthy People," and the other stock would be "Sick People." The two flows, "infection rate" and "recovery rate" would then represent the

rate of change of the people from the state of being healthy to the state of being sick, or vice versa. Figure 7 shows a possible epidemics model.[5]

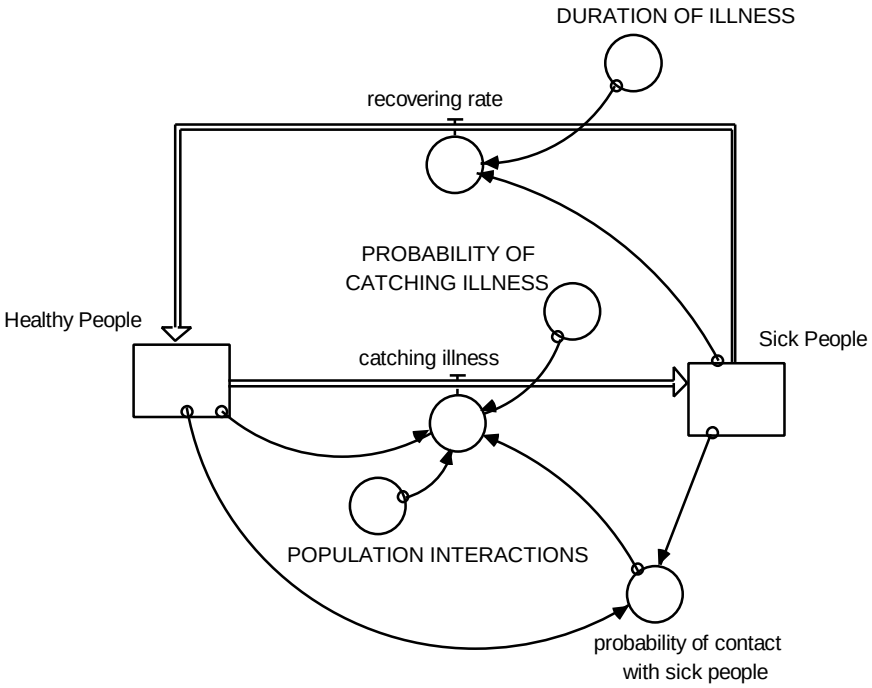


Figure 7: Epidemics model

On the other hand, rabbits in a population cannot exist in the two states of being either a rabbit or an "additional rabbit." Rabbits can be either living or dead. Dead rabbits, however, cannot become living rabbits again.

5. KEY LESSONS

When modeling, the modeler must keep in mind that stocks are accumulations within the system. Flows move real quantities to and from stocks, causing them to change over time.

More importantly, when modeling a system that exhibits a common behavior, the modeler should avoid incorrectly applying the corresponding generic structure. Not every

[5] The epidemics model is explained in more detail in Duhon and Glick, 1994.

system producing S-shaped growth, for example, can be modeled using every structure producing S-shaped growth. Instead, the modeler should first identify the stocks and flows in the system, then formulate a model that best represents the real-world system.

6. APPENDIX: MODEL DOCUMENTATION

6.1 First (Incorrect) Rabbit Population Model

☐**Additional_Rabbits_The_Area_Can_Support(t)** =
Additional_Rabbits_The_Area_Can_Support(t - dt) + (deaths - births) * dt
INIT Additional_Rabbits_The_Area_Can_Support = 998
DOCUMENT: Additional number of rabbits that the area can support. It is equal to the maximum number of rabbits the area can support (1000) minus the current number of rabbits.
Units: Rabbits

☞INFLOWS:
deaths = Rabbits * death_fraction
DOCUMENT: Number of rabbits dying every month.
Units: Rabbits/month

☞OUTFLOWS:
births = Rabbits * BIRTH_FRACTION
DOCUMENT: Number of rabbits born every month.
Units: Rabbits/month

☐**Rabbits(t)** = Rabbits(t - dt) + (births - deaths) * dt
INIT Rabbits = 2
DOCUMENT: Total number of rabbits in the population.
Units: Rabbits

☞INFLOWS:
births = Rabbits * BIRTH_FRACTION
DOCUMENT: Number of rabbits born every month.
Units: Rabbits/month

☞OUTFLOWS:
deaths = Rabbits * death_fraction
DOCUMENT: Number of rabbits dying every month.
Units: Rabbits/month

○**BIRTH_FRACTION** = 0.25
DOCUMENT: Number of rabbits born per rabbit per month.
Units: 1/month

○**death_fraction** = 0.25*Rabbits/(Rabbits+Additional_Rabbits_The_Area_Can_Support)
DOCUMENT: Number of rabbits dying per rabbit per month. It is equal to 25% of the ratio of rabbits to the maximum rabbit population.
Units: 1/month

6.2 Corrected Rabbit Population Model

□**Rabbits(t)** = Rabbits(t - dt) + (births - deaths) * dt
INIT Rabbits = 2
DOCUMENT: Total number of rabbits in the population.
Units: Rabbits

☞INFLOWS:
births = Rabbits * BIRTH_FRACTION
DOCUMENT: Number of rabbits born each month.
Units: Rabbits/month

☞OUTFLOWS:
deaths = Rabbits * death_fraction
DOCUMENT: Number of rabbits dying each month.
Units: Rabbits/month

○**BIRTH_FRACTION** = 0.3
DOCUMENT: Number of rabbits born per rabbit per month.
Units: 1/month

○**death_fraction** = NORMAL_DEATH_FRACTION * effect_of_crowding
DOCUMENT: Number of rabbits dying per rabbit per month.
Units: 1/month

○**NORMAL_DEATH_FRACTION** = 0.1
DOCUMENT: Number of rabbits dying per rabbit per month when there is no shortage of resources.
Units: 1/month

○**NORMAL_RABBIT_POPULATION** = 1000
DOCUMENT: Number of rabbits that the area can support under normal conditions.
Units: rabbits

⊘ **effect_of_crowding** = GRAPH(Rabbits/NORMAL_RABBIT_POPULATION)
(0.00, 1.00), (0.2, 1.00), (0.4, 1.10), (0.6, 1.30), (0.8, 2.00), (1.00, 3.00), (1.20, 4.20), (1.40, 5.62), (1.60, 7.80), (1.80, 11.1), (2.00, 15.0)
DOCUMENT: Effect of the ratio of rabbits to the normal number of rabbits on the death fraction.
Units: dimensionless

Vensim Examples: Mistakes and Misunderstandings:

Use of Generic Structures and Reality of Stocks and Flows

By Aaron Diamond

October 2001

2. Rabbit Population

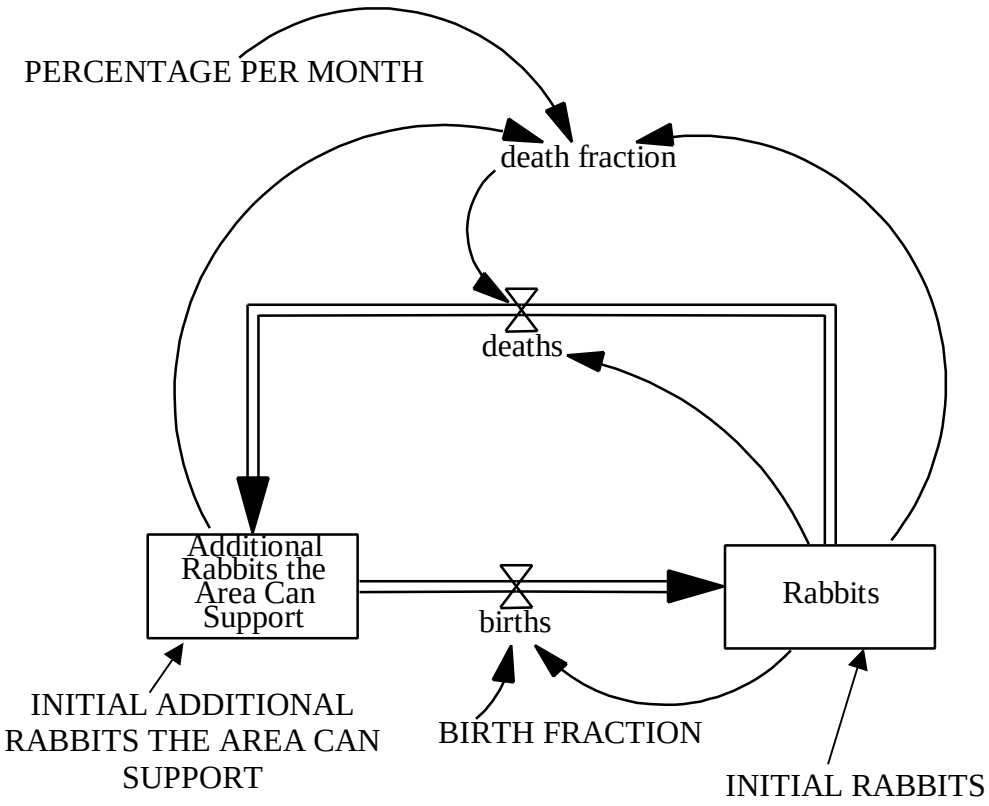


Figure 8: Vensim Equivalent of Figure 2: First model of the rabbit population

Documentation for First model of the rabbit population

(01) Additional Rabbits the Area Can Support= INTEG (+deaths-births, INTIAL ADDITIONAL RABBITS THE AREA CAN SUPPORT)
Units: rabbits
Additional number of the rabbits that the area can support. It is equal to the maximum number of rabbits the area can support (1000) minus the current number of rabbits.

(02) BIRTH FRACTION=0.25
Units: 1/Month
Number of rabbits born per rabbit per month.

(03) births=Rabbits*BIRTH FRACTION
Units: rabbits/Month
Number of rabbits born every month.

(04) death fraction=PERCENTAGE PER MONTH*Rabbits/(Rabbits+Additional Rabbits the Area Can Support)
Units: 1/Month
Number of rabbits dying per rabbit per month. It is equal to 25% of the ratio of rabbits to the maximum rabbit population.

(05) deaths=Rabbits*death fraction
Units: rabbits/Month
Number of rabbits dying every month.

(06) FINAL TIME = 48
Units: Month
The final time for the simulation.

(07) INITIAL ADDITIONAL RABBITS THE AREA CAN SUPPORT= 998
Units: rabbits

(08) INITIAL RABBITS= 2
Units: rabbits

(09) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(10) PERCENTAGE PER MONTH= 0.25
Units: 1/Month
25% of the ratio of rabbits to maximum rabbit population is used
to calculate the death fraction.

(11) Rabbits= INTEG (births-deaths, INTIAL ADDITIONAL RABBITS)
Units: rabbits
There are initially two rabbits in the population.

(12) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(13) TIME STEP = 0.0625
Units: Month
The time step for the simulation.

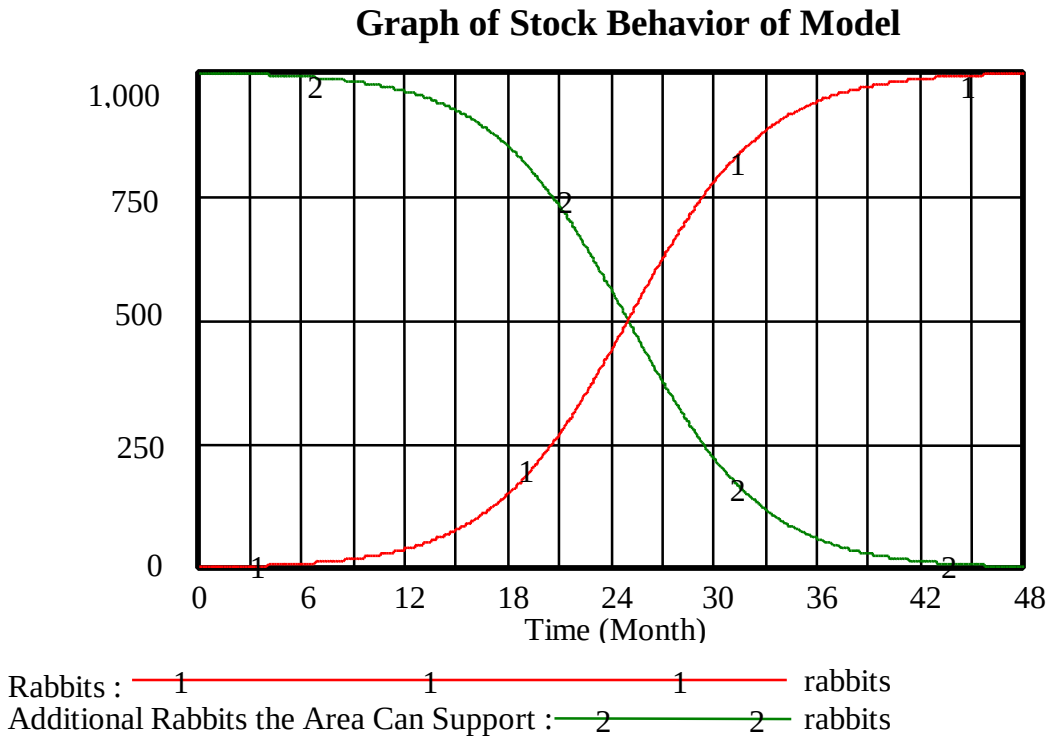


Figure 9: Vensim Equivalent of Figure 3: Stock behavior of first model

4. Overcoming Our Mistakes and Misunderstandings

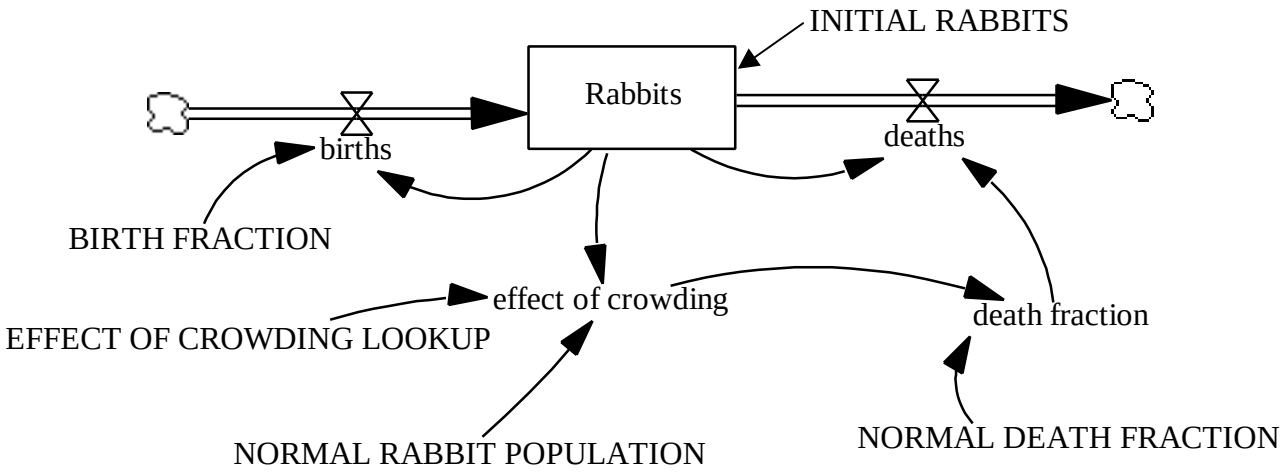


Figure 10: Vensim Equivalent of Figure 4: A corrected model of the rabbit population

Documentation for corrected model of rabbit population

(01) BIRTH FRACTION=0.3
Units: 1/Month
Number of rabbits born per rabbit per month.

(02) births=Rabbits*BIRTH FRACTION
Units: rabbits/Month
Number of rabbits born each month.

(03) death fraction=NORMAL DEATH FRACTION*effect of crowding
Units: 1/Month
Number of rabbits dying per rabbit per month.

(04) deaths=Rabbits*death fraction
Units: rabbits/Month
Number of rabbits dying each month.

(05) effect of crowding= EFFECT OF CROWDING LOOKUP (Rabbits/NORMAL RABBIT POPULATION)
Units: dmnl
Effect of the ratio of rabbits to the normal number of rabbits
on the death fraction.

(06) EFFECT OF CROWDING LOOKUP=([(0,0)(10,20)], (0,1), (0.2,1), (0.4,1.1), (0.6,1.3),(0.8,2),(1,3),(1.2,4.2),(1.4,5.62),(1.6,7.8),(1.8,11.1),(2,15))
Units: dmnl

(07) FINAL TIME = 48
Units: Month
The final time for the simulation.

(08) INITIAL RABBITS=2
Units= rabbits

(09) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(10) NORMAL DEATH FRACTION=0.1
Units: 1/Month
Number of rabbits dying per rabbit per month when there is no
shortage of resources.

(11) NORMAL RABBIT POPULATION=1000

Units: rabbits

Number of rabbits that the area can support under normal conditions.

(12) Rabbits= INTEG (births-deaths, INTIAL RABBITS)

Units: rabbits

Total number of rabbits in the population.

(13) SAVEPER = TIME STEP

Units: Month

The frequency with which output is stored.

(14) TIME STEP = 0.0625

Units: Month

The time step for the simulation.

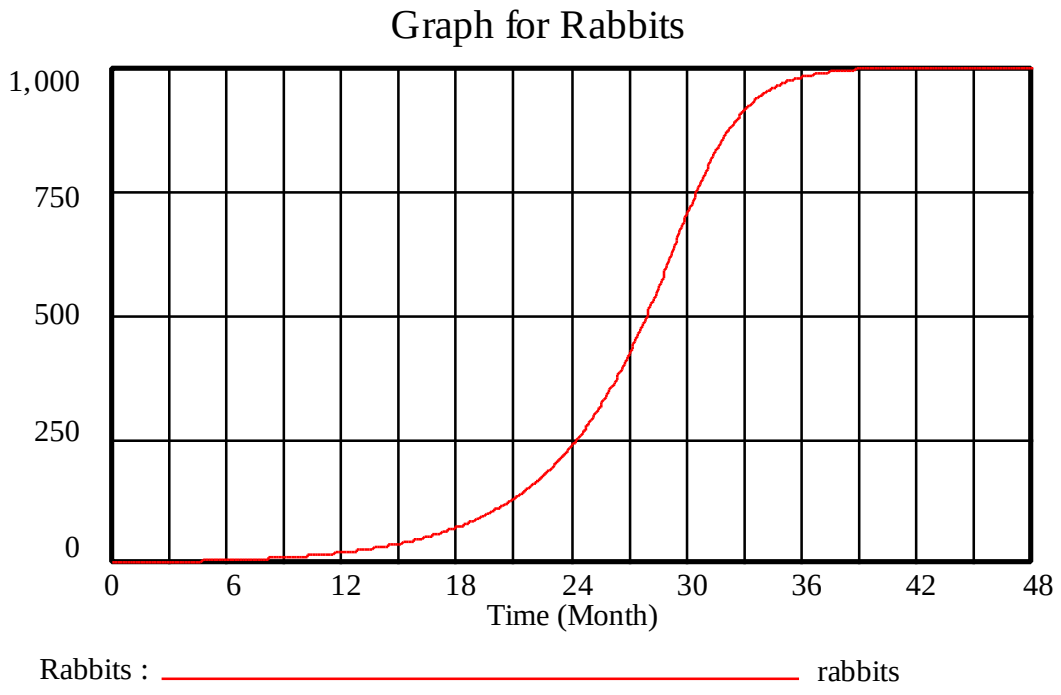
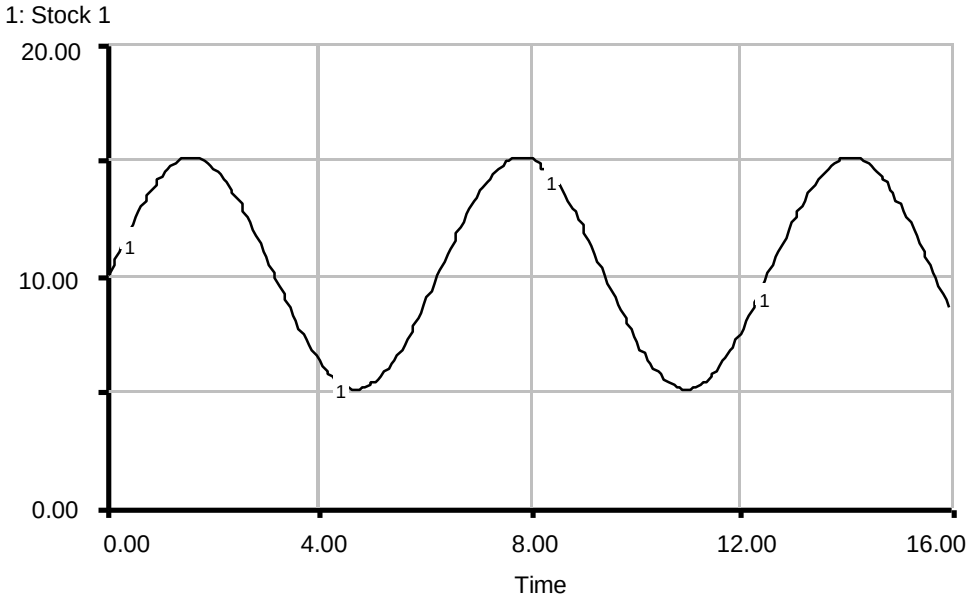


Figure 11: Vensim equivalent of Figure 6: Behavior of the corrected model of Rabbit population

Oscillating Systems II:

Sustained Oscillation



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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May 15, 1997
Vensim Examples added October 2001

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1. Abstract

Oscillating Systems II: Sustained Oscillation is the second paper in a series dedicated to understanding oscillation. Please read *Generic Structures in Oscillating Systems I,*[1] before continuing with this paper. This paper assumes knowledge of STELLA[2] software, as well as simple system dynamics structures such as positive and negative feedback, exponential growth, S-shaped growth, and oscillation.

Oscillating Systems II: Sustained Oscillations will examine the structural features that allow for sustained oscillation. First, this paper will analyze a simple first-order system that cannot oscillate in order to develop structural criteria for oscillation. Then, by studying two different models, an Academic Performance Model of a college student and the Cleanliness of a College Dorm Room Model, the causes of oscillation will be analyzed.

[1]Celeste V. Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 17, 25 p.

[2] STELLA is a registered trademark of High Performance Systems, Inc.

2. Introduction

Oscillating systems are abundant in nature: a person's sleep-wake patterns, the number of solar sunspots, the national economy, the pendulum on a grandfather clock, to name but a few. While the average person observes oscillating systems throughout his life, understanding why the systems exhibit the oscillating behavior mode is a significant intellectual undertaking. However, once one understands why a particular system oscillates, he can transfer that knowledge to many other oscillating systems. The purpose of this paper is to explore in great detail two simple oscillating systems, and from them develop an intimate understanding of oscillation and its causes.

3. Sustained Oscillation

Oscillation refers to a behavior in which the values of the stocks vary around some average value in a repeating pattern.[3] Figure 1 plots the behavior of a stock exhibiting oscillation over the course of 12 time units.

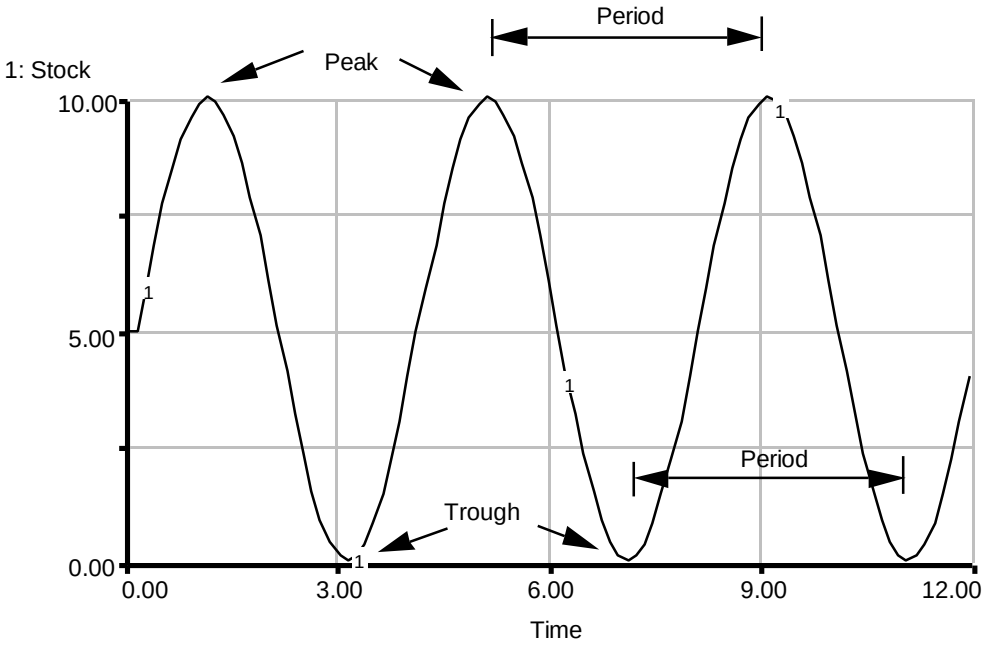


Figure 1: An example of sustained oscillation

Figure 1 is an example of a specific type of oscillation, called **sustained oscillation**. The stock varies in a cyclical pattern around 5.00 units. The behavior mode is called sustained oscillation because each cycle of the oscillation is identical to the previous one. It is easy to recognize sustained oscillation because the maximum value of the stock, known as a "peak," is the same each cycle. Also, the minimum value of the stock, known as a "trough," is the same. (The peaks and troughs are labeled on Figure 1.) The time taken to complete one cycle of the oscillation is called a "period." The period is the time to go from one peak to the next peak, or from one trough to the next trough. Again, because the oscillation is sustained, all the periods are the same length of time. Later papers in the oscillation series will study other types of oscillation, where the height of the peak becomes greater each period, or the height of the peak becomes smaller each period. These two behavior modes are called expanding oscillation and dampened oscillation, respectively.[4]

Generic Structures in Oscillating Systems I[5] explored two systems that demonstrate sustained oscillation: the frictionless pendulum and the Inventory-Employment system. Many more systems exhibit sustained oscillation. This paper will look at two more systems, the academic performance of a college student, and the cleanliness of a college dorm room. Before looking at those two systems, however, it is useful to look at a simpler one, that of a rabbit population in a field without predators. As explained in *Generic Structures in Oscillating Systems I*, first-order systems cannot generate oscillatory behavior. The following section will determine why first-order systems do not oscillate.

---

[3] If the term oscillation is unfamiliar, please review Chung, D-4426-1.

[4] For those who are more interested in mathematics, one can determine whether the oscillation is sustained if the stock's behavior can be curve fit to the function:

Stock = A sin (? *time+?)

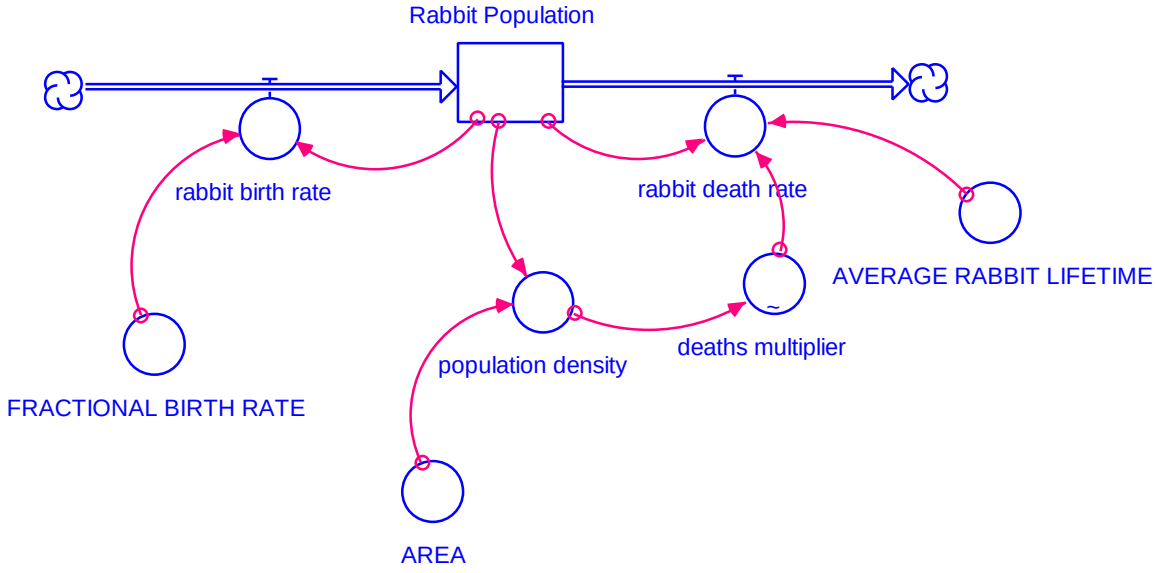
where A, ? , and ? are constant.

[5] Chung, D-4426-1.

4. First-order Systems

4.1 The Rabbit Population Model[6]

The model of "rabbits in a field" is a classic model often used to introduce system dynamics modeling students to the behavior of S-shaped growth. The only stock in the model is the "Rabbit Population" in a given-sized field. As one familiar with population models would predict, the "Rabbit Population" grows exponentially at first. The simple positive feedback loop of rabbit births leading to more rabbits, which in turn leads to more rabbit births, causes the exponential growth. Eventually, the "Rabbit Population" will begin to approach the "carrying capacity"[7] of the field. The behavior of "Rabbit Population" then switches over to an asymptotic approach to the carrying capacity of the field. One possible model of the system is shown in Figure 2.[8] See Appendix 9.1 for model equations and documentation.



[6] This section of the paper is based on: Nathaniel J. Mass and Peter M. Senge, 1975. *Understanding Oscillations in Simple Systems* (D-2045-2), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 24, 29 p.

[7] "Carrying capacity" is an ecological term referring to the number of a species a given environment can sustain.

[8] The Rabbit Population Model was developed by Terri Duhon and Marc Glick, 1995 *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 24, 30 p.

Figure 2: Model of a rabbit population in a field

In Figure 2 the "Rabbit Population" increases as a result of the positive feedback loop between "rabbit birth rate" and "Rabbit Population." The increase in "Rabbit Population" in turn increases the "population density," which increases the "deaths multiplier." The "deaths multiplier" table function reflects that as the "population density" increases, greater competition develops among the rabbits for food and water. The increase in the "deaths multiplier" causes the rabbit death loop, a negative feedback loop, to dominate. This model thus generates S-shaped growth,[9] which Figure 3 illustrates.

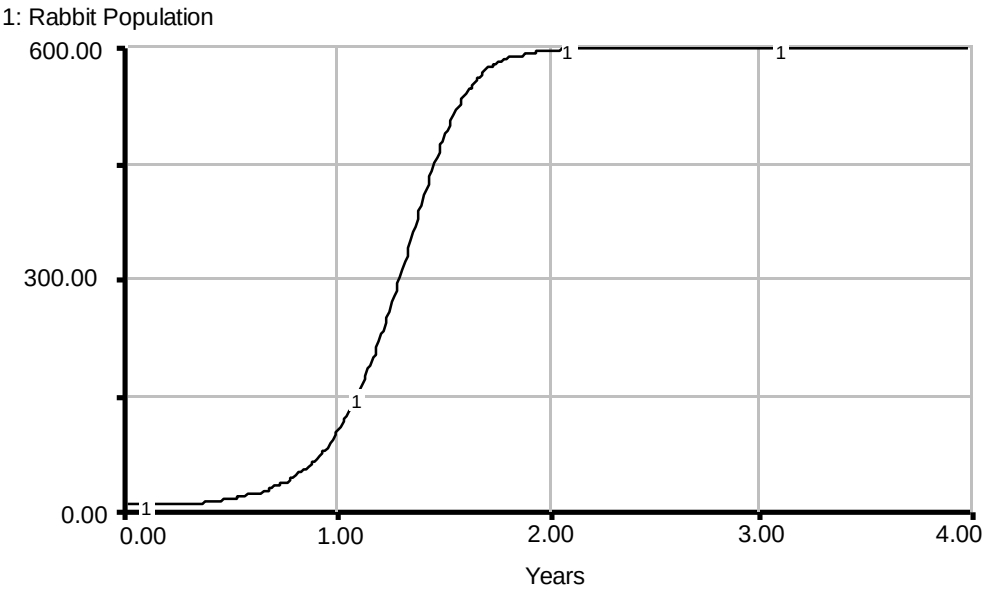


Figure 3: Graph of the behavior of "Rabbit Population" in a field

4.2 Why a first-order system cannot oscillate

When asked what type of behavior the simplified "Rabbit Population," presented in Figure 2, can exhibit, many students do not respond that it will generate S-shaped growth as Figure 3 illustrates. Instead, the students erroneously argue that the "Rabbit Population" rises until it overshoots the carrying capacity of the resource-limited environment, and then oscillates about the carrying capacity until equilibrium is

established. Analysis of the model shows it is impossible for the "Rabbit Population" to overshoot the carrying capacity. To prove that overshoot is impossible, incorrectly assume that the rabbit population did in fact overshoot the carrying capacity of the field. The overshoot and subsequent decline in the "Rabbit Population" incorrectly predicted by the students is shown in Figure 4.

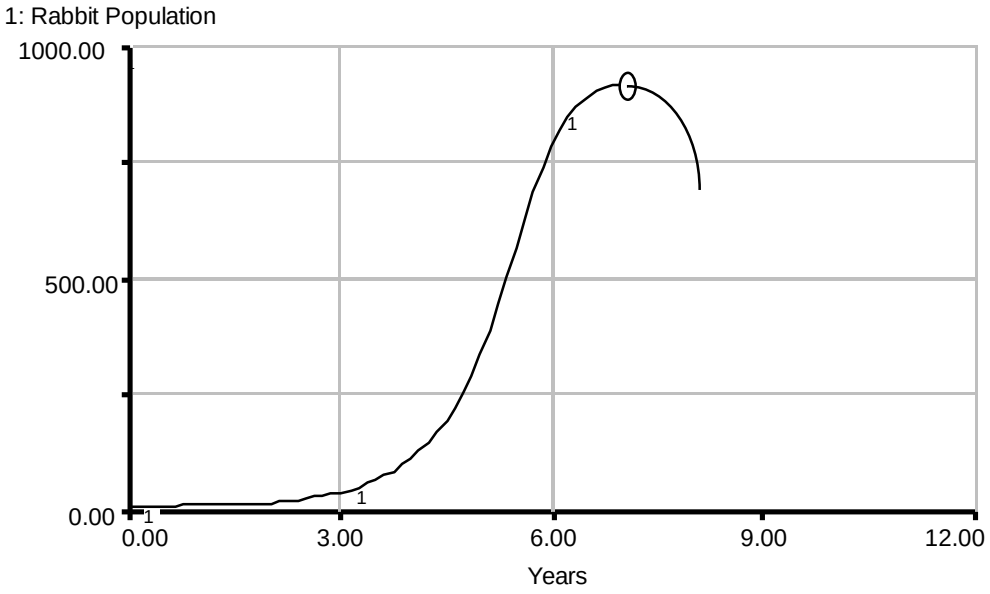


Figure 4: Erroneous prediction of the behavior of the Rabbit Population Model

On the initial rise, the "rabbit birth rate" exceeds the "rabbit death rate," and the "Rabbit Population" grows. As the limited space and resources are used, the effect of crowding becomes significant, and the "rabbit death rate" gradually approaches the "rabbit birth rate." Thus, the "Rabbit Population" is growing more and more slowly. If the overshoot of the carrying capacity is to occur, the "rabbit birth rate" and "rabbit death rate" must be equal momentarily. The point when the flows are equal is the very peak of the population curve, and is indicated by an oval on the graph in Figure 4. However, if the "Rabbit Population" is in temporary equilibrium, nothing can move it away from equilibrium. The "rabbit birth rate" and the "rabbit death rate," the only two flows to and from the "Rabbit Population" stock, only vary if the "Rabbit Population" varies. All the

[9] If S-shaped growth is unfamiliar please review Duhon and Glick, D-4432.

other terms in the rate equations are constant.[10] Moreover, the "Rabbit Population" stock can only change if the flows into it, the "rabbit birth rate" and "rabbit death rate," are not equal. Once the "rabbit birth rate" and "rabbit death rate" are equal, which is the case at the oval on Figure 4, nothing can alter the value of the stock. Thus, nothing can alter the value of the two flows in the system. Because the balance cannot be tipped, the system is locked into equilibrium. The system exhibits the S-shaped growth behavior shown in Figure 3—NOT the overshoot of equilibrium displayed in Figure 4.

If the "Rabbit Population" system is to deviate from its equilibrium point as represented by the oval on Figure 4, another variable must continue to change even if the "Rabbit Population" is momentarily unchanging. The additional variable can be a changing food or water supply, a predator population, or some other environmental factor. However, if the variable is to drive the system out of equilibrium, the variable must not be a function only of the "Rabbit Population." For example, if an auxiliary variable affecting the "rabbit birth rate" was solely a function of "Rabbit Population," it would not be able to disrupt the temporary equilibrium and generate oscillation.[11] Therefore, another stock-and-flow structure must exist to change the "rabbit birth rate" or "rabbit death rate" to make the system oscillate. Thus, the "Rabbit Population" example of a first-order, non-oscillating system shows that a second stock is required for oscillation.

Now, by looking at two second-order systems (systems with two stocks each), this paper will show two or more stocks are required for oscillation. The first system is a college student's academic performance.

5. Academic Performance Model

The first example of an oscillating system this paper will study is Nan's Grade Point Average (GPA) over the course of a year. Nan is a college student who hopes to devote time to her studies as well as to athletics and her social life. As a result she has a desired goal of graduating with a respectable 3.5 GPA out of a possible 4.0. When Nan's grades fall below 3.5, she increases the amount of studying she does each evening. When

[10] See equations in Appendix 9.1.

her grades exceed the desired 3.5, she decreases her amount of daily studying, using the extra time for other activities.[12] Figure 5 shows one way in which the system can be modeled. See Appendix 9.2 for model equations and documentation.

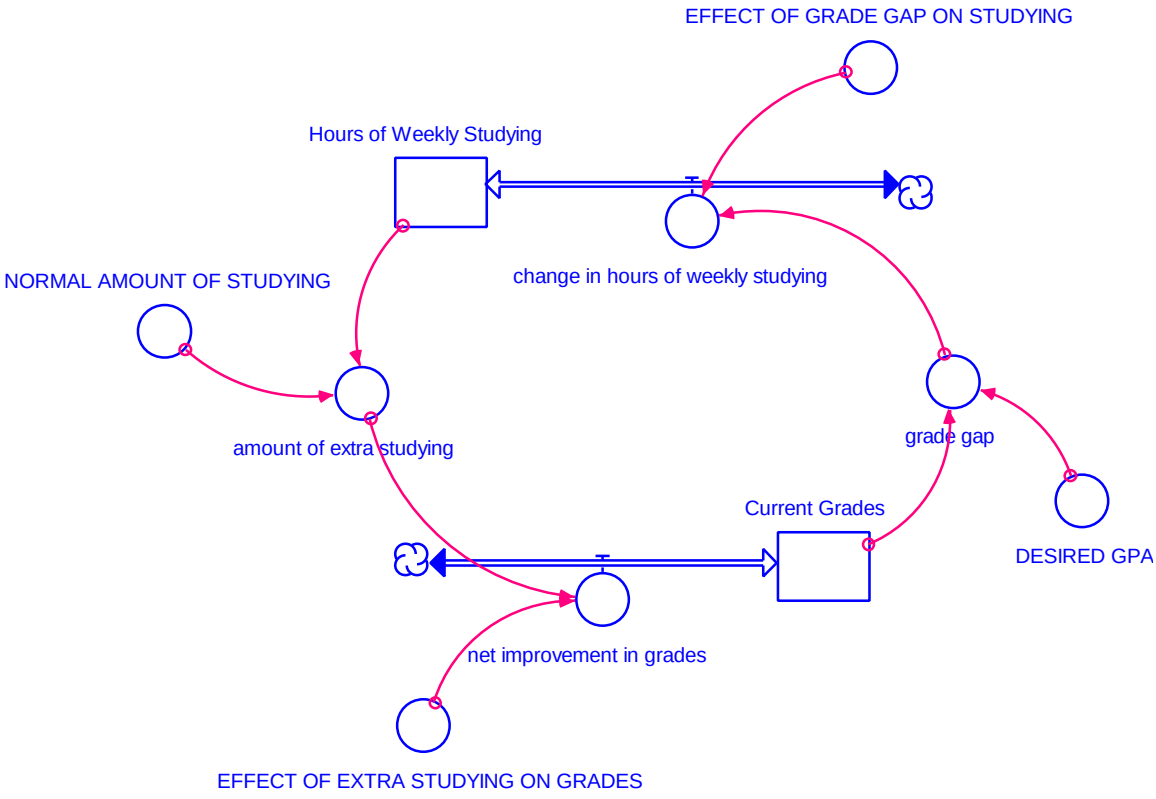


Figure 5: Model of Nan's grades and studying

Nan's "Current Grades" and her "Hours of Weekly Studying" are the two stocks in the model. The stocks are initialized to reflect how Nan began her junior year of college. She was studying her normal amount of 3 hours a day, just like she did the year before. Nan got sick the first day of classes, however, and missed the first lecture in each of her classes. Thus, initially, her grades were only at a 3.0, as opposed to her normal 3.5. Thus, the initial condition of the "Current Grades" stock is 3.0. Remember, in any model,

---

[11] It may, however, change the equilibrium point or the time taken to reach the equilibrium.
[12] A number of other dynamics affect Nan's GPA, besides the ones examined in this paper. However, the model presented here provides a reasonable explanation of Nan's academic performance, as well as serves as an introduction to sustained oscillation.

all stocks must have initial conditions, not only because the mathematics dictates it, but because the stocks define the state of the system. The state of the system must be defined initially.

Before describing the model's behavior, it is useful to examine the feedback relationships in the model. Figure 5 shows that as "Current Grades" go down, the "grade gap" goes up. As the "grade gap" goes up, the "change in hours of weekly studying" goes up. Because "change in hours of weekly studying" is an inflow to the "Hours of Weekly Studying" stock, the stock begins to fill more quickly. As "Hours of Weekly Studying" increases, the "amount of extra studying" also increases. As Nan puts in more and more extra studying, her "net improvement in grades" begins to rise, increasing her "Current Grades" over time. Thus, as Nan's "Current Grades" fall, she puts in more studying, and her "Current Grades" eventually rise. The only feedback loop in the model is a negative feedback loop. When the model is simulated, it should demonstrate a behavior characteristic of a negative feedback loop.

It is also important to notice how each stock affects the flow into the other stock. As explained by the Rabbit Population Model in the previous section of the paper, two stocks are required for oscillatory behavior. Nan's "Current Grades" influence her "change in hours of weekly studying," and Nan's "Hours of Weekly Studying" affect her "net improvement in grades." Structures similar to the Academic Performance Model appeared in *Generic Structures in Oscillating Systems I.*[13] Thus, it is not surprising that the system oscillates. The oscillation is shown in Figure 6.

[13] Chung, D-4426-1.

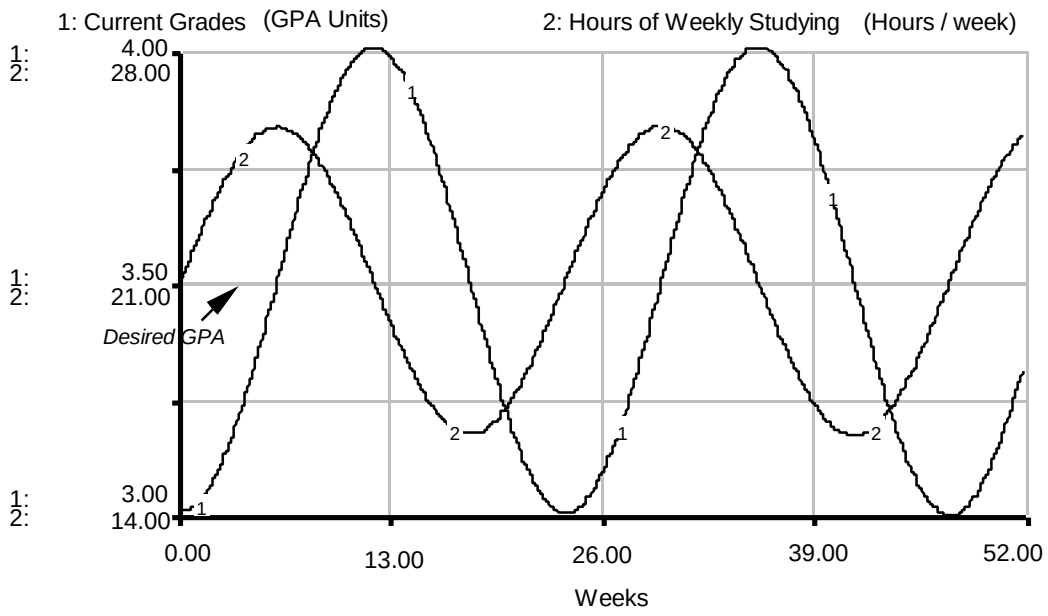


Figure 6: Behavior of the Academic Performance Model

Note that the oscillation is sustained because the height of all the peaks is the same, the height of all the troughs is the same, and the period for each cycle of the oscillation is the same.[14]

5.1 Detailed Model Behavior Analysis

The next step in studying the system is to explain why the system demonstrates the behavior mode of sustained oscillation. The analysis of the model's behavior will focus on one period of the system's oscillation. By studying one period, the whole behavior pattern can be understood, because each subsequent period is identical to the first one. Figure 7 shows the graph of the stocks and their respective flows of the model during the first period.

[14] For running this model it is recommended that you use the "Runge-Kutta 4" method of integration. To do this in Stella® select RK4 under the "Time Specs..." window that can be opened from the "Run" pull down menu. If the solution interval (DT) is set small enough relative to the time constants of the model, it does not make a great deal of difference what method of integration you use. The graphs look nicer, however, when RK4 is used because the oscillation repeats itself more identically than with Euler's Method.

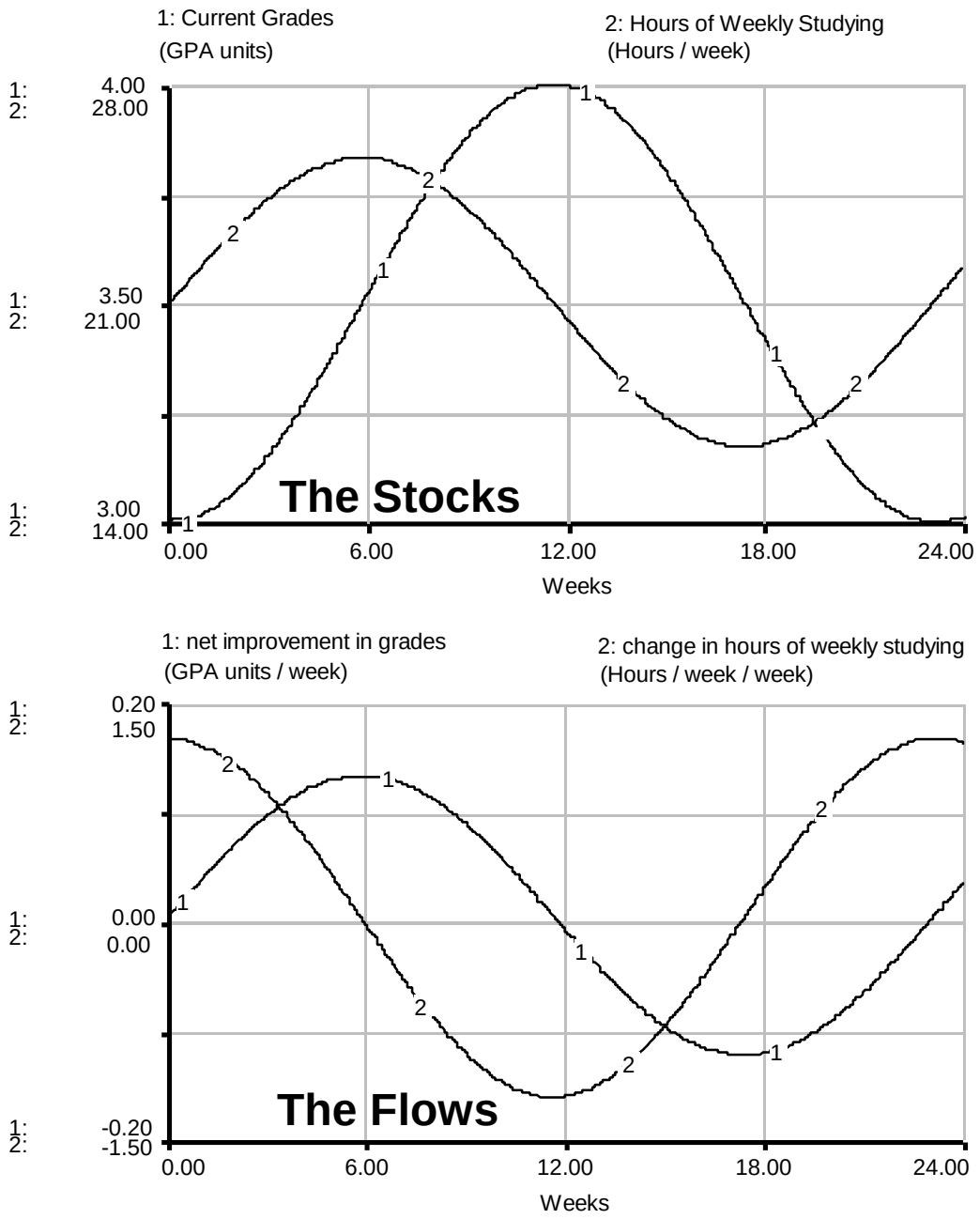


Figure 7: The first period of the Academic Performance Model

Nan starts out her academic year with a 3.0 GPA, as shown in curve 1 of "The Stocks" graph of Figure 7. Three point zero (3.0) is obviously below her desired 3.5. Thus, because her low "Current Grades" make the "grade gap" large, Nan starts increasing the amount of studying she does each day, as shown on the graph of her "Hours of Weekly Studying." At time 0, "Hours of Weekly Studying" is increasing very rapidly,

and thus the graph of the "change in hours of weekly studying" is at its highest point. As time passes, Nan's "Current Grades" begin to improve because of her extra studying. As her "Current Grades" increase, the gap between her "DESIRED GPA" and her "Current Grades" shrinks. Nan continues to increase her "Hours of Weekly Studying" because she is still below her "DESIRED GPA." However, she increases her amount of studying by less and less as the gap between her "Current Grades" and her "DESIRED GPA" shrinks. By 6 weeks, when she finally reaches her "DESIRED GPA," she is no longer increasing her "Hours of Weekly Studying." Notice that at week 6, her "change in hours of weekly studying" is zero hours in a week per week.

So far, the system does not demonstrate anything particularly new. Nan's grades were below what she wanted them to be, so she increased her studying, a great deal at first, then less and less, until she eventually closed the gap and reached her goal. When she finally reached her goal, she stopped increasing her amount of studying. The feedback relationship described is an example of a negative feedback loop. The system oscillates because at week 6, when her "Current Grades" are the desired 3.5, all the studying Nan put in up until that point then causes her "Current Grades" to rise past the goal of 3.5. Remember, by week 6 Nan has been studying 26 hours a week, so she is going to do well on her quizzes and exams for the next few weeks. In fact, her "net improvement in grades," which is the flow into the "Current Grades" stock, is at its highest. Her "net improvement in grades" is at its highest because Nan at week 6 has been over the last six weeks continually increasing her weekly amount of studying. Thus, by week 6, she is putting in the most hours a week in studying, almost 26 hours weekly! All the studying means that her "Current Grades" are rising most rapidly when she is at the goal.

At week 6, when Nan reaches her academic goal, she is no longer increasing her "Hours of Weekly Studying." Thus, exactly at week 6 the "change in hours of weekly studying" is zero. Had Nan's academic performance been a first-order system, the negative feedback loop would have reached its goal and permanent equilibrium would have been established. To Nan's delight, however, her "Current Grades" continue to rise past her goal. While she is happy about her continually rising grades, Nan decides to cut back on her amount of time studying so that she is free to do other things she enjoys.

Immediately after week 6, Nan's "change in hours of weekly studying" becomes negative. Her "Current Grades," however, still continue to rise from all the studying she had done before then. Her "Current Grades" do rise more and more slowly though because the rise is no longer reinforced by an increase in her "Hours of Weekly Studying." Thus, the "net improvement in grades" is positive, but begins to head back down towards zero GPA units per week. As Nan's "Current Grades" continue to rise, Nan feels more and more like using her time elsewhere, and her "Hours of Weekly Studying" continue to decrease.

As everyone knows, if a student keeps cutting back on studying, her grades will eventually fall. The same holds true for Nan as well. At week 12, Nan's "Current Grades" have reached 4.0, and she is back down to studying 21 hours a week. Remember, Nan's "Current Grades" have been growing, even though she has been cutting back on her amount of studying because of the extra work done initially. Her "Current Grades," however, have been growing slower and slower since week 6 when she was studying the maximum amount. Nan is unbelievably happy—she has a great GPA, and is studying a mere 21 hours a week.

After week 12, things begin to turn sour. For the previous six weeks Nan has been cutting back on her studying, so her "Current Grades" are beginning to fall. Because Nan's GPA is still above her goal she continues reducing her "Hours of Weekly Studying." The reduction in her "Hours of Weekly Studying" continues for the entire time between week 12 and week 18. She does, however, reduce her studying less and less each week as her GPA falls closer and closer to her goal.

By week 18, Nan's "Current Grades" have fallen from their peak at 4.0 and have reached her "DESIRED GPA" of 3.5. If the system had been a first-order system, like the Rabbit Population system looked at in section 4, once the system reached its goal equilibrium would be established. Unfortunately for Nan, her academic performance is not a first-order system. Nan's "Current Grades" continue to fall after week 18, and thus fall below her goal. Nan's "Current Grades" fall below the goal because since week 6, when her GPA was high, she was studying less than average. After week 18, when Nan's "Current Grades" fall below her goal, she increases her "Hours of Weekly Studying." However, the increase in studying takes time to increase her "Current Grades" the desired

amount. Remember, studying does not directly increase her "Current Grades." Instead, studying increases her "net improvement in grades," the flow into her "Current Grades" stock. The increase in her "net improvement in grades" does increase her grades over time.

Immediately after week 18, Nan's GPA will continue to fall no matter what Nan does. Eventually, the studying she does will reverse the system, and her "Current Grades" will rise again. In fact, just as when her "Current Grades" were low at week 0, they will rise again. The time between Nan's studying and the improvement in her "Current Grades" will cause her "Current Grades" to overshoot her goal and perpetuate the oscillation.

5.2 Debrief of The Academic Performance Model

The Academic Performance Model demonstrates a great deal about oscillating systems. As expected from studying the Rabbit Population Model, an oscillating system requires two stocks. The second stock allows for the first stock to continue to change even if the first stock was temporarily in its equilibrium position. Specifically, in Nan's Academic Performance model, the "Hours of Weekly Studying" change even if Nan's "Current Grades" are at the goal value of 3.5 (Nan's "DESIRED GPA"). Nan's "Current Grades" keep increasing because Nan's "net improvement in grades" is a function of her "Hours of Weekly Studying," NOT of her "Current Grades." Studying Figure 7 closely will show that the net flow into or out of a stock is greatest when the stock is in its goal position.[15]

Another thing to notice about oscillating systems from Nan's Academic Performance Model is that oscillating systems have some momentum associated with them. In a first-order negative feedback loop, the system takes action when a gap exists between the state of the system and the goal of the system. The system thus approaches the goal asymptotically. The asymptotic approach can be seen in the Rabbit Population Model studied earlier. For instance, assume that a hundred rabbits escape from a

[15] Later papers in the oscillation series will expose the reader to systems in which sustained oscillation is not exhibited, and dampened and expanding oscillation will occur.

laboratory and try to live in the field that was originally in equilibrium. Because the number of rabbits in the field is suddenly greater than the carrying capacity, "rabbit death rate" increases, and the number of rabbits approaches the carrying capacity of the environment. Equilibrium is re-established. In Nan's Academic Performance model, when Nan's "Current Grades" are below her goal, she cannot immediately raise her GPA. Instead, she must increase her amount of studying that will in turn, after a few weeks, raise her "Current Grades" to her desired level. Unfortunately for Nan, until her "Current Grades" actually reach the "DESIRED GPA" she does not know to stop increasing her "Hours of Weekly Studying." Thus she overstudies, which allows her grades to overshoot and undershoot her goal GPA, and thus the oscillatory behavior of the model is produced.

Building and experimenting with the model will yield tremendous insights into oscillation. Try changing the value of the constants and initial conditions of the stocks. As always when simulating a model, try to understand why changing a certain variable alters the model's behavior.

6. Cleanliness of a College Dorm Room Model

Imagine a college dorm room with one very messy roommate (me) who always drops his dirty clothes on the floor. My very neat roommate always complains about the untidiness of the room, especially when my dirty clothes spill over to his side of the room. When the clothes begin to untidy his side of the room, he complains, and I pick up some of the clothes. His complaining then subsides as the room becomes tidier. When the complaining subsides, I stop picking up the clothing because I am a slob. As the amount of laundry on the floor begins to reaccumulate, the number of complaints registered by my roommate each day increases. The complaints increase my willingness to pick up laundry, which decreases the amount of clothes on the floor. The increase in the cleanliness of the room of course reduces his complaining. A model of the system is shown in Figure 8. For equations and documentation see Appendix, Section 9.3.

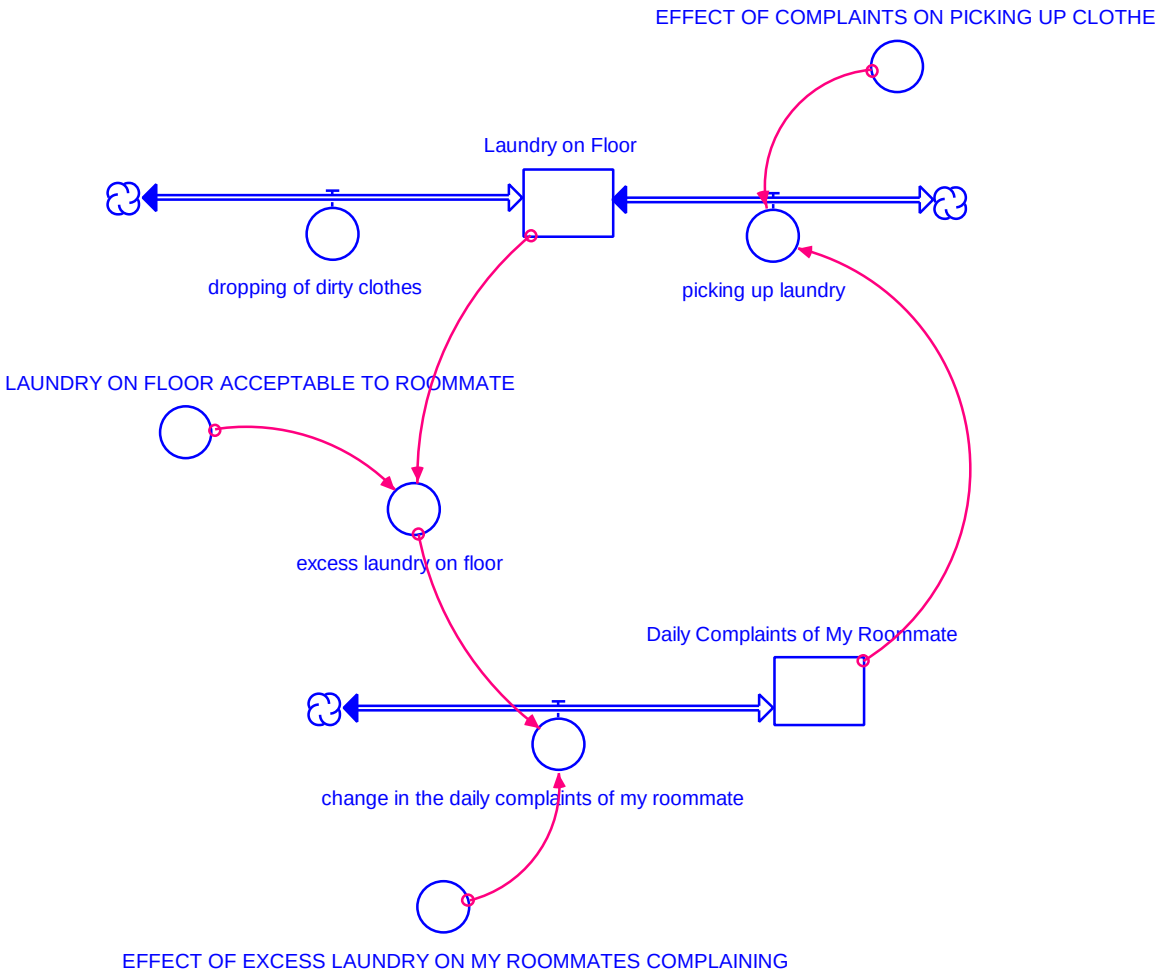


Figure 8: Cleanliness of a College Dorm Room Model

The model, like all other models, makes many assumptions. The model assumes:

- that I always drop all 5 articles of clothes that I wear in a day on the floor,
- that my roommate never picks up my clothes himself, and
- that my roommate can tolerate 3 articles of clothing on the floor without becoming upset and increasing his complaining.

The model in Figure 8 looks extremely similar to Nan's Academic Performance Model, as well as the two models presented in *Generic Structures in Oscillating Systems I*. The model has two stocks, with the value of one stock affecting the flow into the other stock. Also, the model is composed of one negative feedback loop. Assume that the

"Laundry on Floor" increases and thus the "excess laundry on floor" increases. As the amount of excess clothing on the floor rises, my roommate's unhappiness increases, and thus the "change in the daily complaints of my roommate," the flow into the "Daily Complaints of My Roommate" stock, also increases. As the flow into a stock increases, the value of the stock increases as well. Thus the "Daily Complaints of My Roommate" increases. The increased complaining causes the "picking up laundry" flow to increase. Because "picking up laundry" is an outflow, the "Laundry on Floor" decreases. Thus, the model is one negative feedback loop—as the amount of laundry increases, my roommate complains more, which causes me to pick up more clothes. The amount of laundry on the floor thus decreases.

Because the system is a second-order negative feedback loop, one expects the system to oscillate. Figure 9 shows that sustained oscillation is the behavior mode demonstrated by the system when initially three articles of clothing were on my floor, and my roommate was complaining to me three times a day.

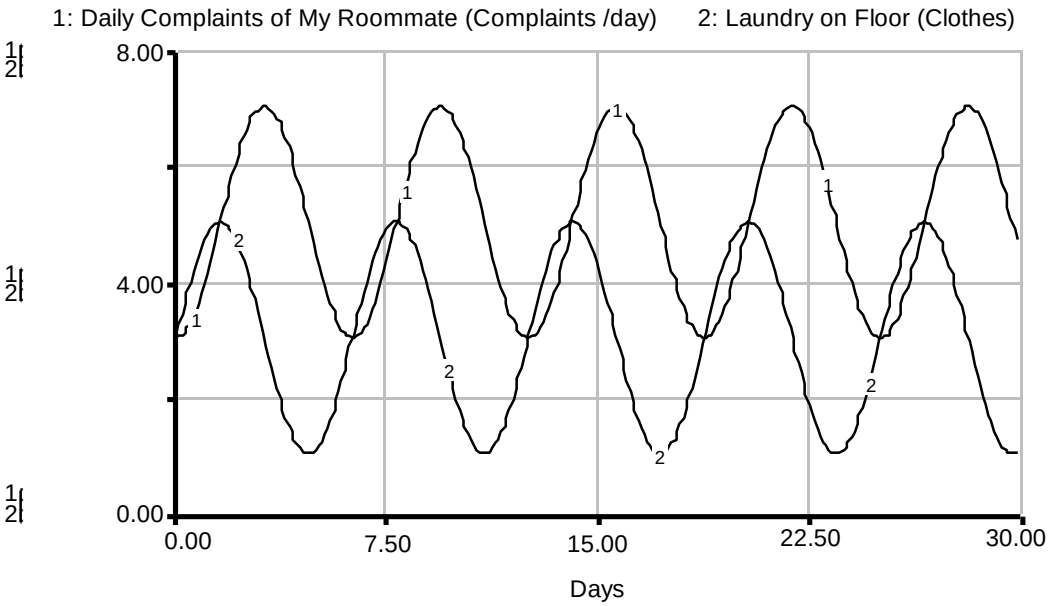


Figure 9: Behavior of the Cleanliness of a College Dorm Room Model

Just as expected, the system oscillates around 3 articles of clothing on my floor, which is the goal of the system. My roommate's complaints oscillate around five per day,

which is the number of complaints per day required to keep me picking up as many clothes as I drop. The initial conditions of the simulation reflect what happened at the start of February, 1996. Originally, 3 articles of clothes untidied the floor and my roommate was complaining 3 times a day. Three times a day was less than he normally complains because he just received a very generous financial aid package from MIT.

It is important to notice that the oscillation is sustained because all the peaks for each stock are the same height and all the troughs for each stock are the same height. To explain why the oscillation occurs, one can study the first few days of the system and recognize that any subsequent behavior is simply a repetition of the initial behavior.

6.1 Detailed Model Behavior Analysis

Figure 10 shows the behavior of the stocks of the system, "Laundry on my Floor" and "Daily Complaints of My Roommate," for the first week. Figure 10 also shows the behavior of the "change in the daily complaints of my roommate" and the "picking up laundry" flows. "Dropping dirty clothes," the inflow to "Laundry on Floor," is constant at 5 clothes per day.

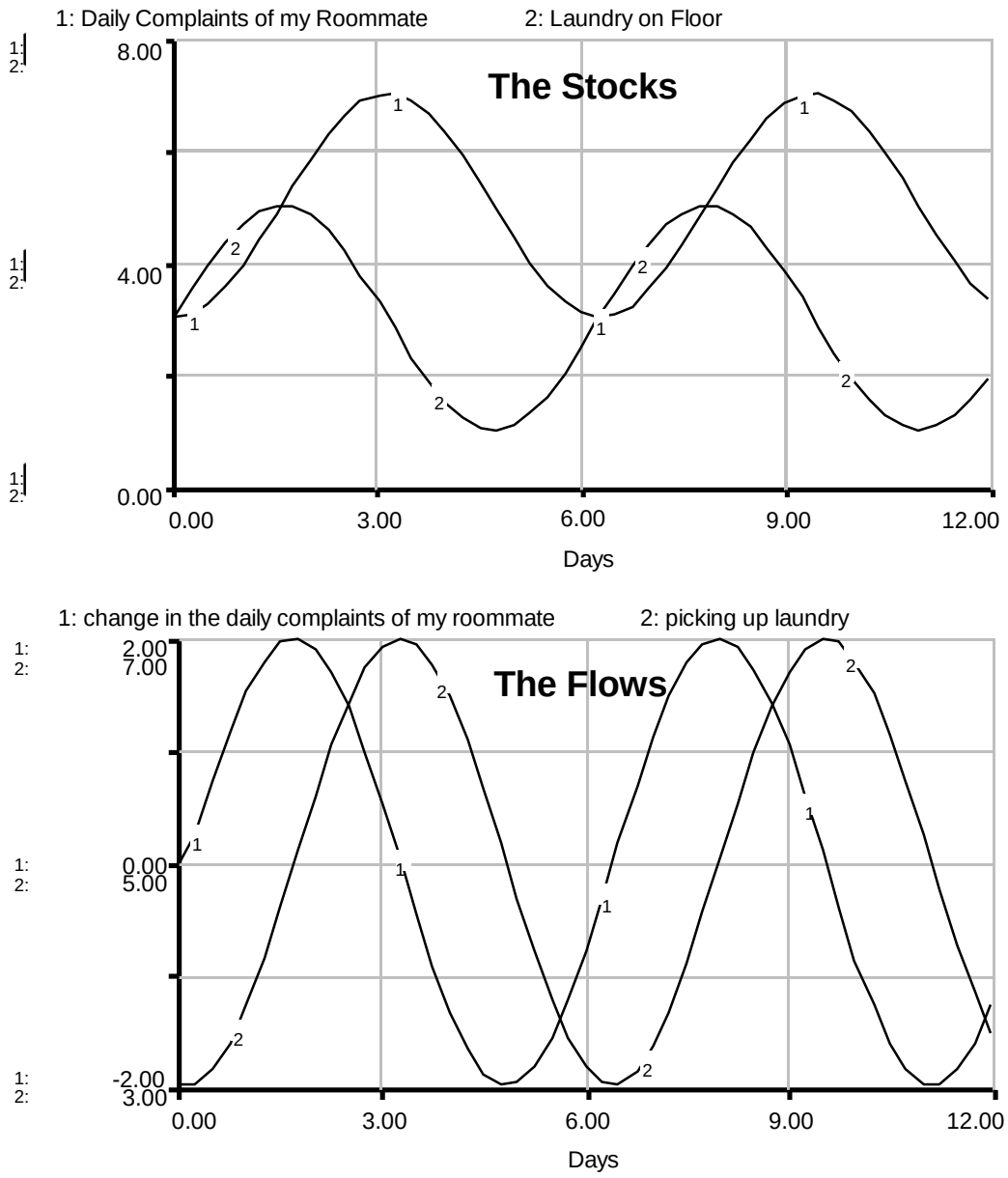


Figure 10: Graph of the behavior of the stocks and flows of the Cleanliness of a College Dorm Room Model for the first 12 days

For simplicity's sake, the analysis of the model will begin at day 1.5. Because the behavior of the model repeats itself, it does not matter where the analysis begins. At day 1.5 the "Laundry on Floor," which is shown as curve 2 on "The Stocks" graph of Figure 10, is at its maximum of 5 articles of clothing. It is not surprising that after day 1.5, when the "Laundry on Floor" is at its maximum, I am picking up 5 articles of clothing a day (as

shown on curve 2 of "The Flows" graph of Figure 10). Remember, every day I drop 5 articles of clothing on the floor, so when I am picking up 5, the stock is in temporary equilibrium. Even though the stock of "Laundry on Floor" is temporarily in equilibrium (the inflow equal to the outflow), the number of articles of clothing on the floor is greater than the 3 articles my roommate considers tolerable. Thus, my roommate continues to increase his number of daily complaints. The increase in complaints is shown on Figure 10 where curve 1 on "The Flows" graph is at positive 2.00 (Complaints/day)/day.

After day 1.5 my roommate's complaints drive me to pick up more laundry off my floor. Because picking up laundry is the outflow from the "Laundry on Floor" stock, the amount of "Laundry on Floor" decreases. My roommate, however, continues to increase his complaining because more than 3 articles of clothing are on the floor. A little after day 3, I have reduced the amount of laundry on the floor to 3 articles. Thus, at a little after day 3, my roommate stops increasing his amount of complaining. However, I am still hearing him complain 7 times a day. Remember, he has only stopped increasing his amount of complaining, he has NOT stopped complaining. What is interesting is that my roommate can normally tolerate 3 articles of clothing on the floor. Because he is so mad at me, however, it takes him time to change his daily number of complaints. Thus, the amount of laundry on the floor drops below 3 articles of clothing. The seven complaints a day, the peak of his daily complaints, cause me to continue to pick up more clothes than I drop. The amount of "Laundry on Floor" continues to fall. The decrease of "Laundry on Floor" pleases my roommate. He begins softening up on me by reducing his daily complaints. He is still, at least right after day 3, complaining quite a bit, and I continue to pick up more clothes than I drop. As my roommate's number of daily complaints falls, so does my picking up of my laundry.

By around day 5 the "Laundry on my Floor" is at a minimum of 1 article of clothing. Between slightly after day 3 and day 4.5, I have been decreasing the amount of laundry I have been picking up off the floor because my roommate has begun to reduce his complaining. By day 4.5, I am only picking up 5 articles of clothing a day, which means that the amount of "Laundry on my Floor" is temporarily in equilibrium again. Because only 1 article of clothing is on the floor, which is far below the 3 articles my roommate

finds acceptable, he continues to reduce his amount of complaining. My roommate's lack of complaining causes me to continue to reduce my picking up of laundry, and the clothes begin to reaccumulate on my floor. Eventually, by day 6.0 (actually, slightly after day 6.0) the amount of "Laundry on Floor" reaches 3 articles and my roommate stops decreasing his amount of complaining. Because he is complaining so little, I continue not to pick up as many clothes off the floor as I drop, and my laundry accumulates. The accumulation continues until my roommate's complaints reach a high enough level to force me to pick up more clothes than I drop. As Figure 10 shows, the oscillation perpetuates indefinitely.

6.2 Debrief of the Cleanliness of a College Dorm Room Model

Similar to the Academic Performance Model, the Cleanliness of a College Dorm Room Model shows an oscillatory pattern.[16] One could expect the behavior mode because the model meets the requirement of being second-order (two stocks), with the flows into one stock determined by the value of the other stock. For example, the flow "picking up laundry" is a function of the "Daily Complaints of My Roommate" stock. Also, the flow "change in the daily complaints of my roommate" is a function of the stock of "Laundry on Floor."

Furthermore, one would also expect the system to oscillate because it is a negative feedback loop. The momentum in the system is that my roommate gets into a good or bad mood and cannot change his amount of complaining immediately. Thus, when my roommate is not satisfied with the state of the system, he can start to get more annoyed and increase his complaining. It takes him about a day to change his amount of complaining. Because of his inability to change his mood immediately, when finally the amount of "Laundry on Floor" is acceptable to him, he is already so mad at me that it takes him time to stop complaining. The time for my roommate to change his mood causes me to overshoot the equilibrium and continue to pick clothes up faster than I drop them. The same holds true in the reverse. When the "Laundry on Floor" stock is small and then reaccumulates so that it reaches 3 articles (the amount acceptable to my roommate), it immediately overshoots because my roommate is complaining very little. It

takes time for his mood to change sufficiently to complain enough to keep the laundry on the floor at his acceptable level. Thus, the momentum in the system always causes the system to overshoot and undershoot the equilibrium, and thus oscillation occurs.

Again, it is important to build and experiment with the model to gain a strong intuition as to why the sustained oscillating behavior exists. Try changing the "EFFECT OF ROOMMATES COMPLAINTS ON PICKING UP CLOTHES" variable in particular. This variable represents how much of an effect my roommate's complaining has on my picking up my clothes off the floor.

7. Why Second-order Systems Can Oscillate

The three systems studied in this paper illustrate some of the structural requirements of sustained oscillation. The requirements are:

? The system must be a negative feedback loop.

? The system must be at least second-order (have two stocks).

Negative feedback loops always attempt to close the gap between some desired state of the system and the actual state of the system. For example, the negative feedback loop showed in the Rabbit Population Model over time closed the gap between the "Rabbit Population" and the carrying capacity of the field. Also, the negative feedback loop in the Cleanliness of a College Dorm Room Model attempted over time to close the gap between the amount of laundry on the floor and the amount of laundry my roommate considered acceptable. Earlier papers in Road Maps showed how first-order (one stock) negative feedback loops generate asymptotic growth, also known as goal-seeking behavior.[17] The goal-seeking behavior occurs because the system realizes that a gap exists between the desired state and actual state, and adjusts the rates of the system. The system then immediately recompares the actual state of system to the goal, and readjusts the rates

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[16] It also shows how much of a slob and a rotten roommate I am.

[17] Stephanie Albin, 1996. *Generic Structures: Negative Feedback Loops* (D-4475-1), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 28 pp.

accordingly. For example, if the initial "Rabbit Population" was larger than the carrying capacity, goal-seeking behavior would result. The "rabbit death rate" would increase, and thus the "Rabbit Population" would shrink, which then would decrease the "rabbit death rate."

Now, for comparison, consider the model of Nan's academic performance. When Nan's "Current Grades" are too low, she cannot simply improve her "Current Grades" a proportional amount immediately. Thus, the second-order system is different from the first-order Rabbit Population Model. In the Rabbit Population Model, should the number of rabbits exceed the carrying capacity, more rabbits would die than would be born. The stock-and-flow structure of the Academic Performance Model, shown in Figure 5, contains no direct link between "grade gap" and "net improvement in grades." Instead, the corrective action the system takes is to increase Nan's "Hours of Weekly Studying." The increase of studying raises her "Current Grades," but not in the same fashion as in a first-order feedback loop. The oscillation occurs because Nan keeps increasing her studying until she reaches her goal. She overshoots her "DESIRED GPA" because she overstudied. When Nan's "Current Grades" stock is at its desired value, all the studying she put in up to then increased her "Hours of Weekly Studying" stock to almost 28 hours a week. Nan's large amount of studying means that her "net improvement in grades," the flow into the "Current Grades" stock, is large. Thus, Nan overshoots the goal of the system (Nan's "DESIRED GPA") and the system is able to oscillate. Nan later undershoots her "DESIRED GPA" for the same reason—because of the momentum in the system when one stock is at its goal value, the other stock drives it out of equilibrium. As learned from studying the Rabbit Population Model, a second stock allows for oscillation. The second stock can continue to change the system even if one of the stocks is at its goal value.

8. Conclusion

Through studying the first-order system of a rabbit population in a field, it was determined that for a system to oscillate, the system must be a negative feedback loop with at least two stocks. Also, the flows into one stock must be a function of the other

stock in the model. If each flow was only a function of the stock it fed into or out of, the system could not move from equilibrium once it was established. For example, the "Rabbit Population" was stable when it reached its maximum in the first model of this paper because the "rabbit birth rate" and "rabbit death rate" were only functions of the "Rabbit Population." Once the "Rabbit Population" reached the carrying capacity of the environment, nothing could move the system from equilibrium. In Nan's Academic Performance Model, however, when Nan's "Current Grades" were in equilibrium (equal to her "DESIRED GPA,") the amount of studying she had done previously caused her "Current Grades" to move away from equilibrium. Lastly, the notion that all models must have a set of initial conditions for the stocks was reiterated.

Future papers in Road Maps will expose the reader to more complex types of oscillation, both expanding and dampened. *Oscillating Systems II: Sustained Oscillation* outlines the basic causes of sustained oscillation. The reader's understanding will grow as he experiments with the models presented in this paper, as well as develops oscillating models from scratch. The reader is encouraged to build the models presented here and study how their behavior changes as the various parameters and initial conditions of the stocks are altered. It is important to understand the mechanism of sustained oscillation before studying more complex oscillatory behaviors.

9. Appendix

The following are the equations and the documentation of the three models presented in this paper:

9.1 First-order Rabbit Population Model

Rabbit_Population(t) = Rabbit_Population(t - dt) + (rabbit_birth_rate - rabbit_death_rate) * dt
INIT Rabbit_Population = 2
DOCUMENT: The number of rabbits in the field.
UNITS: Rabbits

INFLOWS:

rabbit_birth_rate = Rabbit_Population * FRACTIONAL_BIRTH_RATE
DOCUMENT: The number of rabbits born each year.
UNITS: Rabbits / year

OUTFLOWS:
rabbit_death_rate = (Rabbit_Population / AVERAGE_RABBIT_LIFETIME) * deaths_multiplier
DOCUMENT: The number of rabbits dying each year.
UNITS: Rabbits / year

AREA = 1
DOCUMENT: The size of the field the rabbits are living in.
UNITS: Acres

AVERAGE_RABBIT_LIFETIME = 4
DOCUMENT: The average life span of a rabbit living in an uncrowded environment.
UNITS: Years

FRACTIONAL_BIRTH_RATE = 5.0
DOCUMENT: The number of rabbits born each year into the population per rabbit in the population.
UNITS: Fraction / year

population_density = Rabbit_Population / AREA
DOCUMENT: The number of rabbits per acre of field.
UNITS: Rabbits / acre

deaths_multiplier = GRAPH(population_density)
(0.00, 1.00), (100, 2.50), (200, 5.00), (300, 7.75), (400, 10.75), (500, 15.25), (600, 20.25), (700, 26.50), (800, 34.75), (900, 43.00), (1000, 50.00)
DOCUMENT: Deaths multiplier is a multiplication factor which depends upon POPULATION DENSITY. It converts density to a factor that affects the number of rabbit deaths (i.e., it makes the rabbit death rate a function of POPULATION DENSITY). The deaths multiplier reflects that the more dense the rabbit population is, the more competition there is for food and water, so the more rabbits die.
UNITS: dimensionless

9.2 Academic Performance Model

Current_Grades(t) = Current_Grades(t - dt) + (net_improvement_in_grades) * dt
INIT Current_Grades = 3.0
DOCUMENT: Nan's academic performance at any given time.

UNITS: GPA units

INFLOWS:
net_improvement_in_grades = amount_of_extra_studying *
EFFECT_OF_EXTRA_STUDYING_ON_GRADES
DOCUMENT: This is the change in Nan's grades as a result of studying more or less than her normal amount.
UNITS: GPA units / week

Hours_of_Weekly_Studying(t) = Hours_of_Weekly_Studying(t - dt) +
(change_in_hours_of_weekly_studying) * dt
INIT Hours_of_Weekly_Studying = 21
DOCUMENT: The number of hours Nan spends per week studying. The initial value of 21 hours per week corresponds to 3 hours per night.
UNITS: hours / week

INFLOWS:
change_in_hours_of_weekly_studying = grade_gap *
EFFECT_OF_GRADE_GAP_ON_STUDYING
DOCUMENT: The number of hours by which Nan increases or decreases her amount of studying based on her current academic performance.
UNITS: (hours / week) / week

amount_of_extra_studying = Hours_of_Weekly_Studying -
NORMAL_AMOUNT_OF_STUDYING
DOCUMENT: The number of hours in a week Nan spends studying above her normal amount.
UNITS: hours / week

DESIRED_GPA = 3.5
DOCUMENT: The GPA Nan wants.
UNITS: GPA units

EFFECT_OF_EXTRA_STUDYING_ON_GRADES = 0.0285
DOCUMENT: The amount by which Nan's academic performance is increased one extra hour.
UNITS: GPA units / hour

EFFECT_OF_GRADE_GAP_ON_STUDYING = 2.45
DOCUMENT: The amount by which Nan increases her weekly studying when she is below her desired GPA by 1.0 points.
UNITS: [(Hours / week) / week] / GPA units

grade_gap = DESIRED_GPA - Current_Grades
DOCUMENT: The difference between Nan's desired and actual GPA.

UNITS: GPA units

NORMAL_AMOUNT_OF_STUDYING = 21
DOCUMENT: The number of hours Nan normally spends studying in a week.
UNITS: hours / week

9.3 Cleanliness of a College Dorm room Model

Daily_Complaints_of_My_Roommate(t) = Daily_Complaints_of_My_Roommate(t - dt) + (change_in_the_daily_complaints_of_my_roommate) * dt
INIT Daily_Complaints_of_My_Roommate = 3
DOCUMENT: The number of complaints my roommate registers with me each day about the cleanliness of the room.
UNITS: Complaints / day

INFLOWS:
change_in_the_daily_complaints_of_my_roommate = EFFECT_OF_EXCESS_LAUNDRY_ON_MY_ROOMMATES_COMPLAINING * excess_laundry_on_floor
DOCUMENT: This flow represents how my roommate's amount of complaining changes over time. It is a function of the amount of excess laundry on the floor.
UNITS: (Complaints / day) / day

Laundry_on_Floor(t) = Laundry_on_Floor(t - dt) + (dropping_dirty_clothes - picking_up_laundry) * dt
INIT Laundry_on_Floor = 3
DOCUMENT: The number of articles of clothing on my dormitory floor.
UNITS: Clothes

INFLOWS:
dropping_dirty_clothes = 5
DOCUMENT: The number of dirty clothes I drop on my floor every day. The model assumes that my roommate's complaining does not stop me from dropping all my clothes on the floor, it only changes how many I pick up.
UNITS: Clothes / day

OUTFLOWS:
picking_up_laundry = EFFECT_OF_COMPLAINTS_ON_PICKING_UP_CLOTHES * Daily_Complaints_of_My_Roommate
DOCUMENT: The number of clothes I pick up each day. It is a function of how many complaints my roommate registers with me.
UNITS: Clothes / day

EFFECT_OF_EXCESS_LAUNDRY_ON_MY_ROOMMATES_COMPLAINING = 1
DOCUMENT: This constant reflects how my roommate increases his complaining based on the addition of one more article of clothing to the floor.
UNITS: ((Complaints / day) / day) / Clothes

EFFECT_OF_ COMPLAINTS_ON_PICKING_UP_CLOTHES = 1
DOCUMENT: This variable is the number of extra clothes I will pick up each day if my roommate increases his complaining by one complaint per day.
UNITS: (Clothes / day) / (Complaint / day), or more simply, Clothes / Complaints

excess_laundry_on_floor = Laundry_on_Floor - LAUNDRY_ON_FLOOR_ACCEPTABLE_TO_ROOMMATE
DOCUMENT: This variable is the difference between the number of articles of clothing on my floor and the number of articles acceptable to my roommate.
UNITS: Clothes

LAUNDRY_ON_FLOOR_ACCEPTABLE_TO_ROOMMATE = 3
DOCUMENT: This is the number of clothes on the floor my roommate finds acceptable (because they don't spill over onto his side of the room).
UNITS: Clothes

Vensim Examples: Oscillating Systems II: Sustained Oscillation

By Aaron Diamond

March 2000

4.1 The Rabbit Population Model

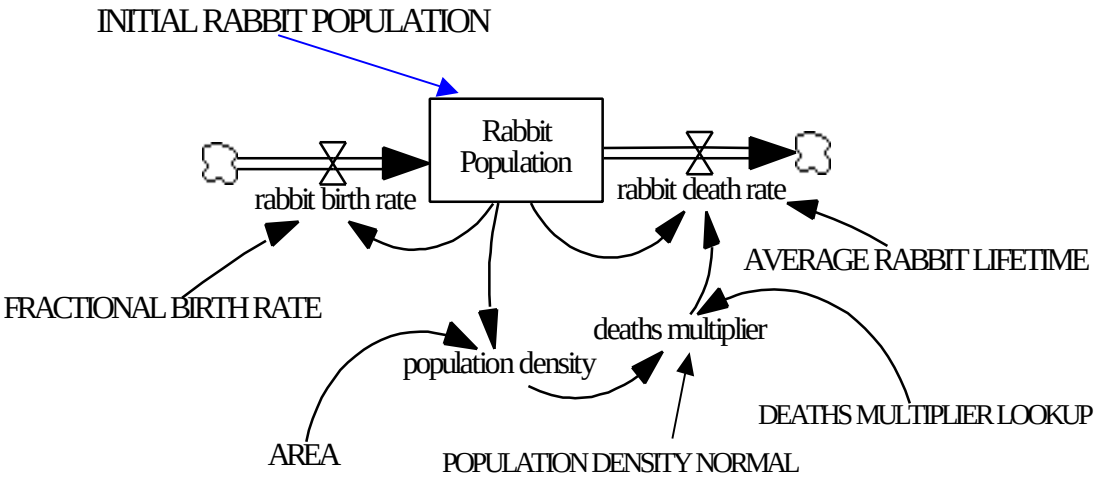


Figure 11: Vensim Equivalent of Figure 2: Model of a rabbit population in a field

Documentation for rabbit population model

(01) AREA=1

Units: acre

The size of the field the rabbits are living in.

(02) AVERAGE RABBIT LIFETIME=4

Units: year

The average life span of a rabbit living in an uncrowded environment.

(03) deaths multiplier=DEATHS MULTIPLIER LOOKUP (population density/POPULATION DENSITY NORMAL)

Units: dmnl

Deaths multiplier is a multiplication factor which depends upon

POPULATION DENSITY. It converts density to a factor that affects the number of rabbit deaths (i.e., it makes the rabbit death rate a function ofPOPULATION DENSITY). The deaths multiplier reflects that the more dense the rabbit population is, the more competition there is for food and water, so the more rabbits die.

(04) DEATHS MULTIPLIER LOOKUP= ([(0,0),(10,60)], (0,1), (100,2.5), (200,5), (300,7.75),(400,10.75),(500,15.25),(600,20.25),(700,26.5),(800,34.75),(900,43), (1000,50))
Units: dmnl

(05) FINAL TIME = 4
Units: year
The final time for the simulation.

(06) FRACTIONAL BIRTH RATE=5
Units: 1/year
The number of rabbits born each year into the population per rabbit in the population.

(07) INITIAL RABBIT POPULATION=2
Units: rabbits

(08) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(09) population density=Rabbit Population/AREA
Units: rabbits/acre
The number of rabbits per acre of field.

(10) POPULATION DENSITY NORMAL=1
Units: rabbits/acre

(11) rabbit birth rate=Rabbit Population*FRACTIONAL BIRTH RATE
Units: rabbits/year
The number of rabbits born each year.

(12) rabbit death rate=(Rabbit Population/AVERAGE RABBIT LIFETIME)*deaths multiplier
Units: rabbits/year
The number of rabbits dying each year.

(13) Rabbit Population= INTEG (rabbit birth rate-rabbit death rate, INITIAL RABBIT POPULATION)
Units: rabbits
The number of rabbits in the field.

(14) SAVEPER =TIME STEP
Units: year
The frequency with which output is stored.

(15) TIME STEP = 0.0625
Units: year
The time step for the simulation.

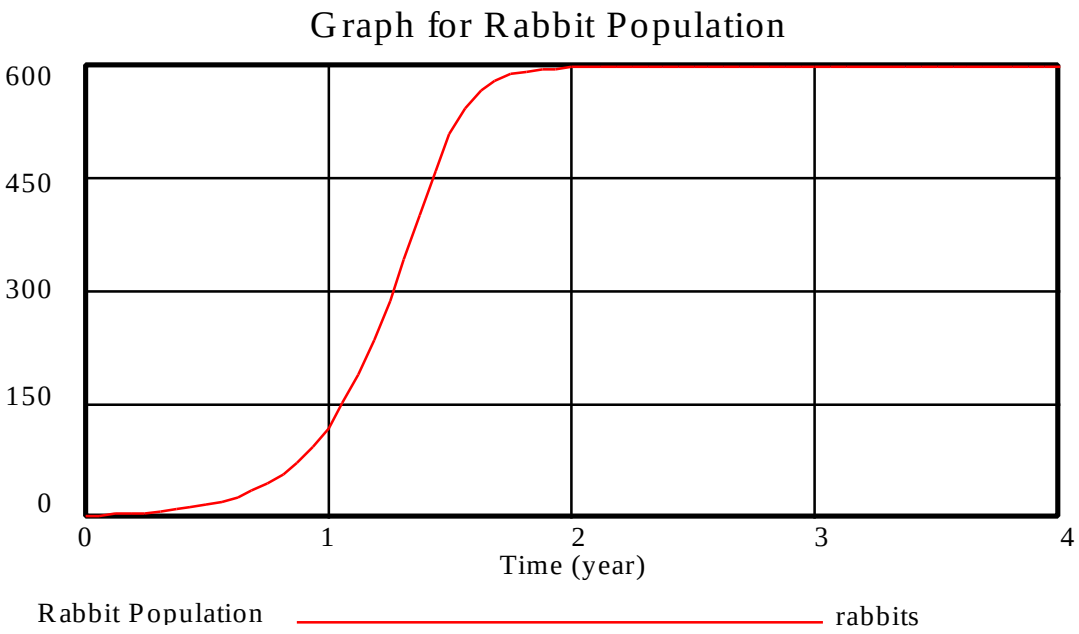


Figure 12: Vensim Equivalent of Figure 3: Graph of the behavior of "Rabbit Population" in a field

5. Academic Performance Model

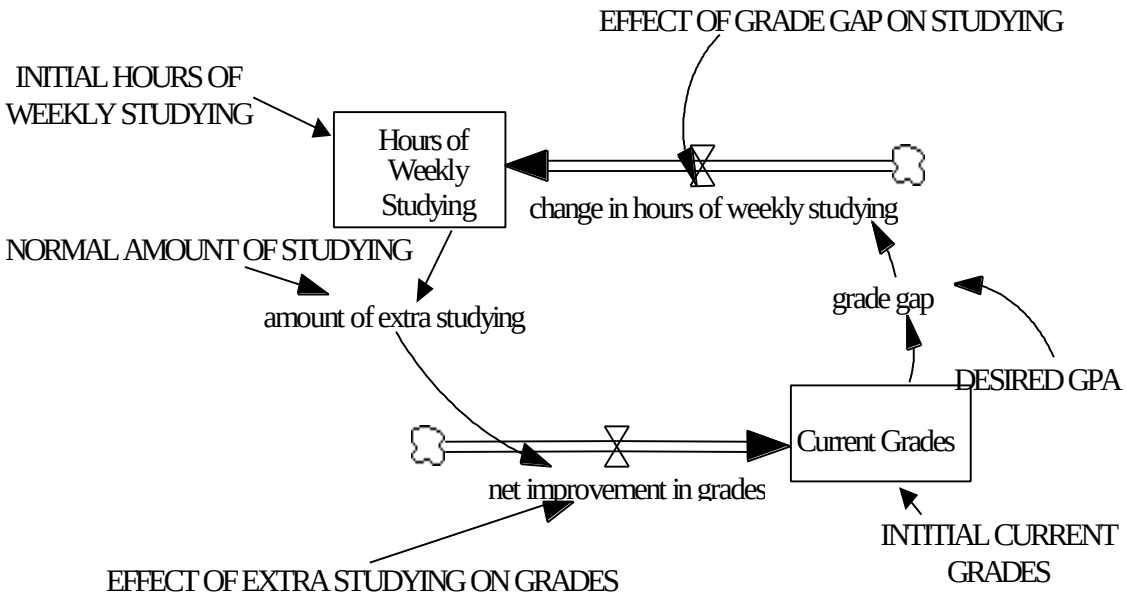


Figure 13: Vensim Equivalent of Figure 5: Model of Nan's grades and studying

Documentation for Academic Performance Model

(01) amount of extra studying=Hours of Weekly Studying-NORMAL AMOUNT OF STUDYING
Units: hours/week
The number of hours in a week Nan spends studying above her normal amount.

(02) change in hours of weekly studying=grade gap*EFFECT OF GRADE GAP ON STUDYING
Units: (hours/week)/week
The number of hours by which Nan increases or decreases her amount of studying based on her current academic performance.

(03) Current Grades= INTEG (net improvement in grades, INITIAL CURRENT GRADES)
Units: GPA units
Nan's academic performance at any given time.

(04) DESIRED GPA=3.5
Units: GPA units
The GPA Nan wants.

(05) EFFECT OF EXTRA STUDYING ON GRADES=0.0285
Units: (GPA units)/hours
The amount by which Nan's academic performance is increased one extra hour.

(06) EFFECT OF GRADE GAP ON STUDYING=2.45
Units: (hours)/(week*week*GPA units)
The amount by which Nan increases her weekly studying when she

is below her desired GPA by 1.0 points.

(07) FINAL TIME = 24
Units: week
The final time for the simulation.

(08) grade gap=DESIRED GPA-Current Grades
Units: GPA units
The difference between Nan's desired and actual GPA.

(09) Hours of Weekly Studying= INTEG (change in hours of weekly studying, INITIAL HOURS OF WEEKLY STUDYING)
Units: hours/week
The number of hours Nan spends per week. The initial value of 21
hours per week corresponds corresponds to 3 hours per week.

(10) INITIAL CURRENT GRADES=3
Units: GPA units

(11) INITIAL HOURS OF WEEKLY STUDYING=21
Units: hours

(12) INITIAL TIME = 0
Units: week
The initial time for the simulation.

(13) net improvement in grades=amount of extra studying*EFFECT OF EXTRA STUDYING ON GRADES
Units: (GPA units)/week
This is the change in Nan's grades as a result of studying more

or less than her normal amount.

(14) NORMAL AMOUNT OF STUDYING=21

Units: hours/week

The number of hours Nan normally spends studying in a week.

(15) SAVEPER = TIME STEP

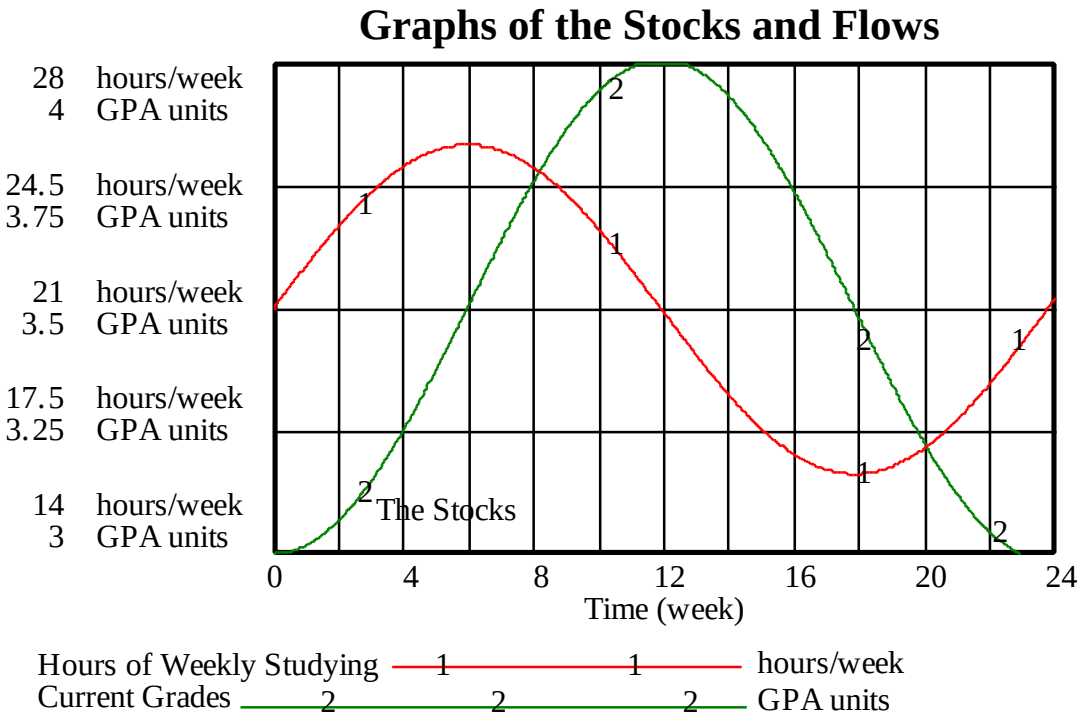
Units: week

The frequency with which output is stored.

(16) TIME STEP = 0.0625

Units: week

The time step for the simulation.



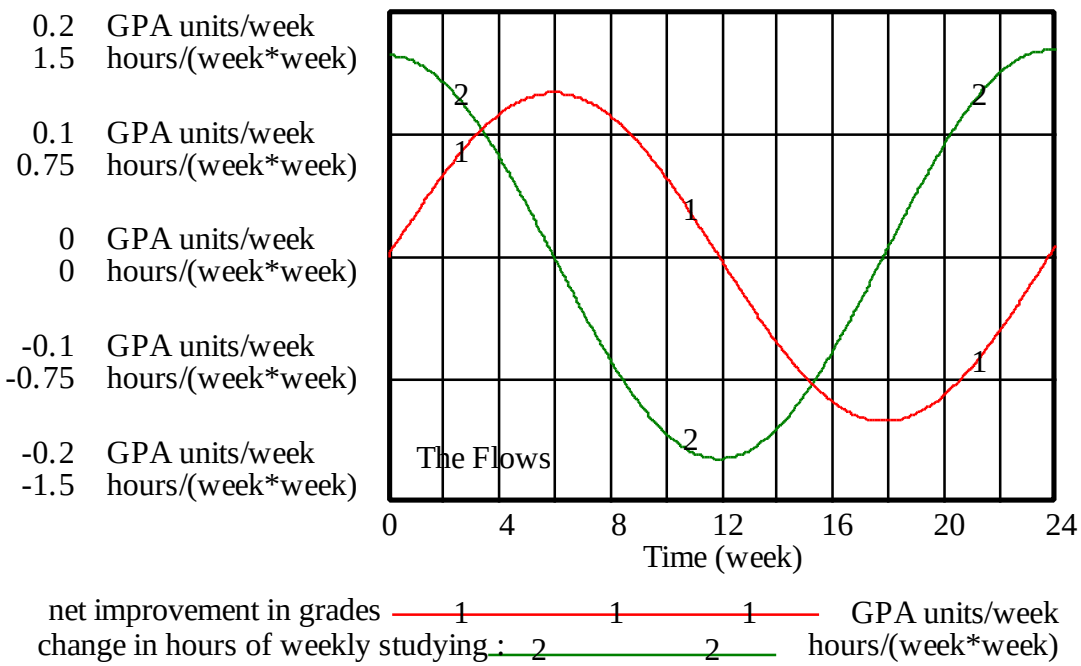


Figure 14: Vensim Equivalent of Figure 7: The first period of the Academic Performance Model

6. Cleanliness of a College Dorm Room Model

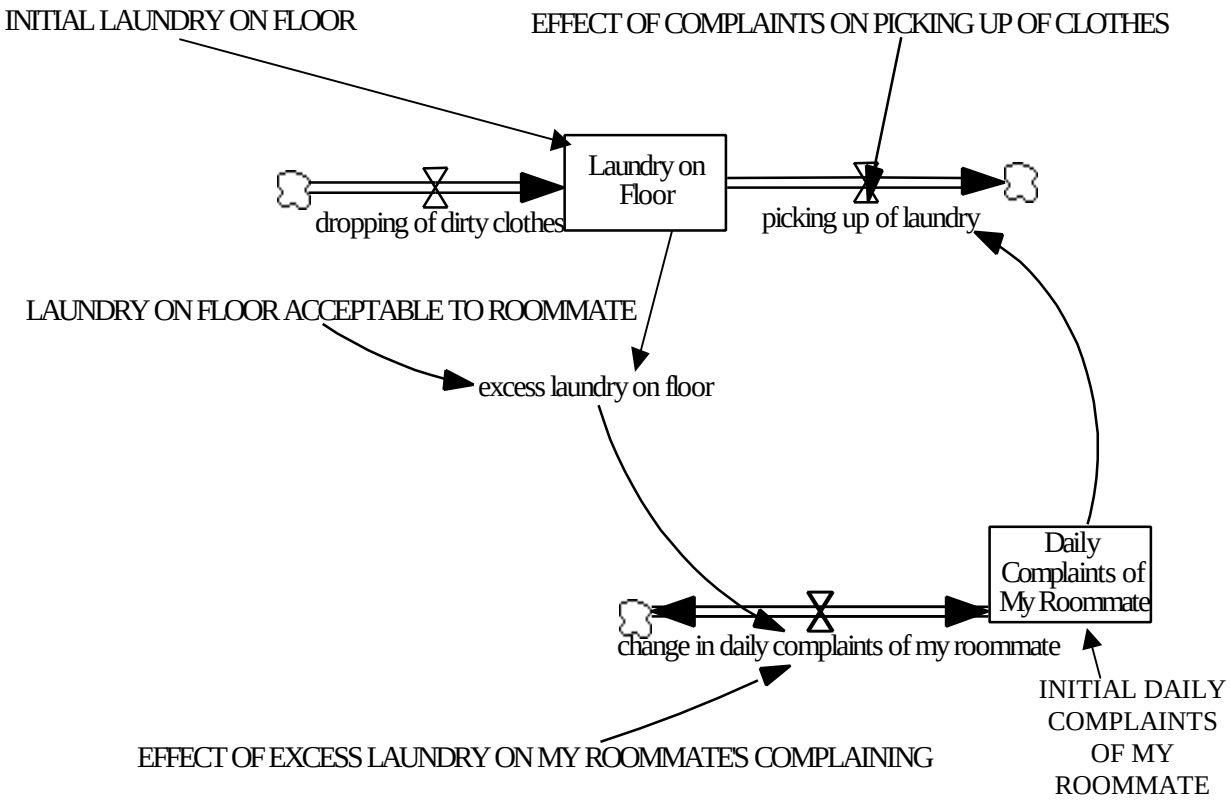


Figure 15: Vensim Equivalent of Figure 8: Cleanliness of College Dorm Room Model

Documentation for Cleanliness of a College Dorm Room Model

(01) change in daily complaints of my roommate=EFFECT OF EXCESS LAUNDRY ON MY ROOMMATE'S COMPLAINING*excess laundry on floor
Units: (complaints/day)/day
This flow represents how my roommate's amount of complaining changes over time. It is a function of the amount of excess laundry on the floor.

(02) Daily Complaints of My Roommate= INTEG (change in daily complaints of my roommate, INITIAL DAILY COMPLAINTS OF MY ROOMMATE)
Units: complaints/day
The number of complaints my roommate registers with me each day about the cleanliness of the room.

(03) dropping of dirty clothes=5
Units: clothes/day
The number of dirty clothes I drop on my floor every day. The model assumes that my roommate's complaining does not stop me from dropping all my clothes on the floor, it only changes how many I pick up.

(04) EFFECT OF COMPLAINTS ON PICKING UP OF CLOTHES=1
Units: clothes/complaints
This variable is the number of extra clothes I will pick up each day if my roommate increases his complaining by one complaint per day.

(05) EFFECT OF EXCESS LAUNDRY ON MY ROOMMATE'S COMPLAINING=1
Units: ((complaints/day)/day)/clothes
This constant reflects how my roommate increases his complaining based on the addition of one more article of clothing to the floor.

(06) excess laundry on floor=Laundry on Floor-LAUNDRY ON FLOOR ACCEPTABLE TO ROOMMATE
Units: clothes
This variable is the difference between the number of articles of clothing on my floor and the number of articles acceptable to my roommate.

(07) FINAL TIME = 30
Units: day
The final time for the simulation.

(08) INITIAL DAILY COMPLAINTS OF MY ROOMMATE=3
Units: complaints/day

(09) INITIAL LAUNDRY ON FLOOR=3
Units: clothes

(10) INITIAL TIME = 0
Units: day
The initial time for the simulation.

(11) Laundry on Floor= INTEG (dropping of dirty clothes-picking up of laundry, INITIAL LAUNDRY ON FLOOR)
Units: clothes
The number of articles of clothing on my dormitory floor.

(12) LAUNDRY ON FLOOR ACCEPTABLE TO ROOMMATE=3
Units: clothes
This is the number of clothes on the floor my roommate finds acceptable (because they don't spill over onto his side of the room).

(13) picking up of laundry=EFFECT OF COMPLAINTS ON PICKING UP OF CLOTHES*Daily Complaints of My Roommate
Units: clothes/day
The number of clothes I pick up each day. It is a function of how many complaints my rommate registers with me.

(14) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.

(15) TIME STEP = 0.0625

Units: day

The time step for the simulation.

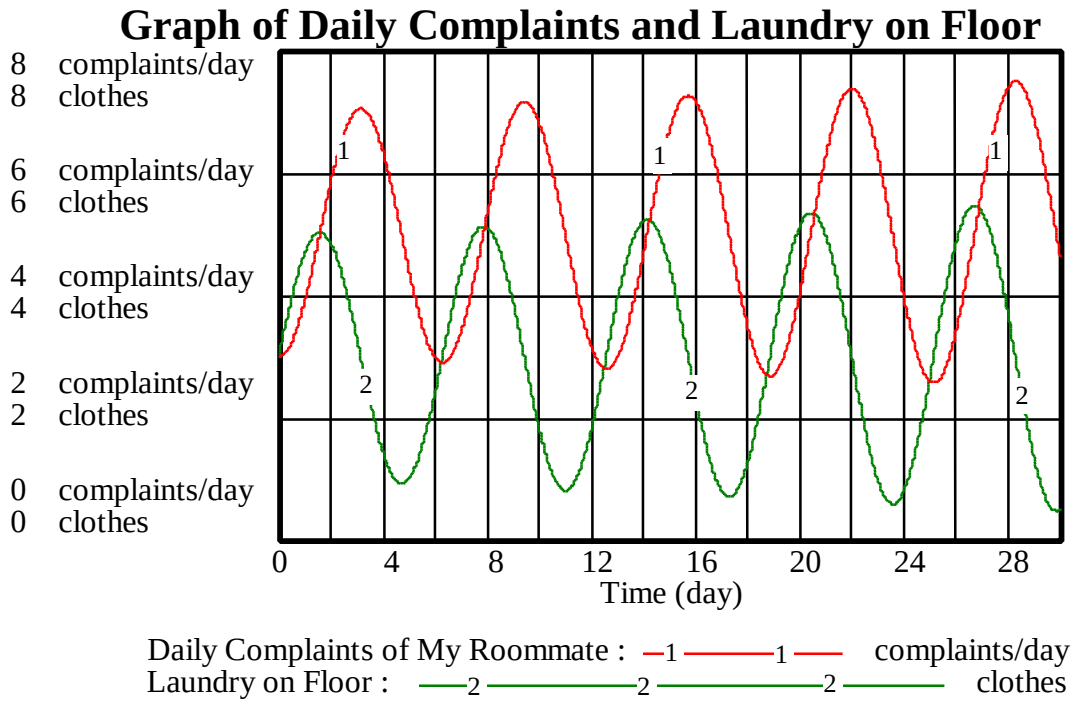
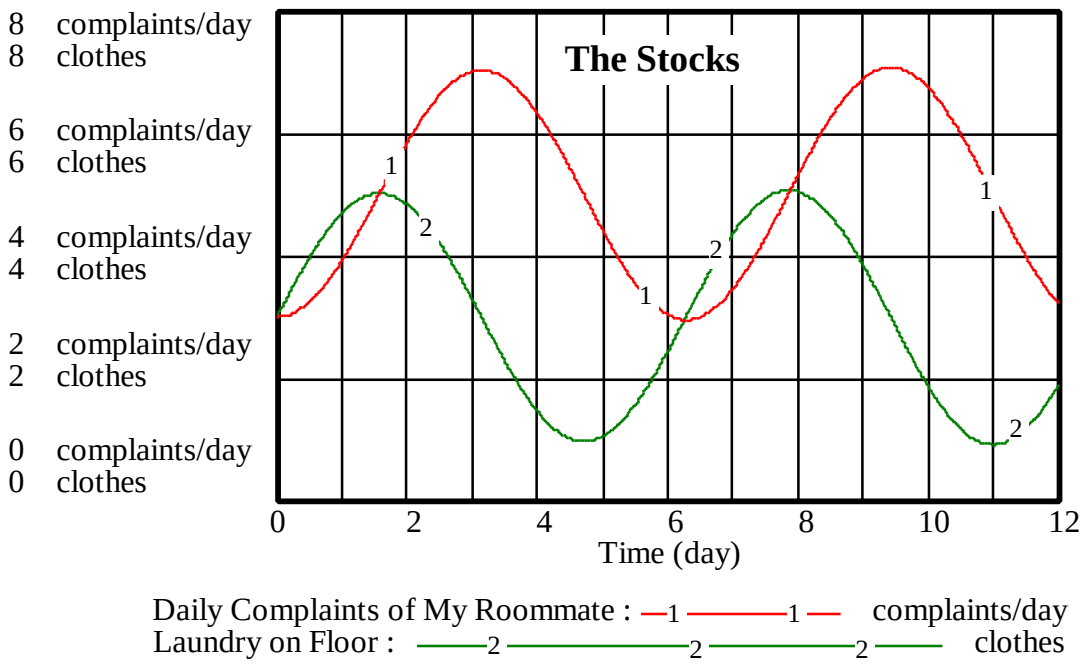


Figure 16: Vensim Equivalent of Figure 9: Behavior of the Cleanliness of a College Dorm Room Model



Graphs of the Stocks and Flows

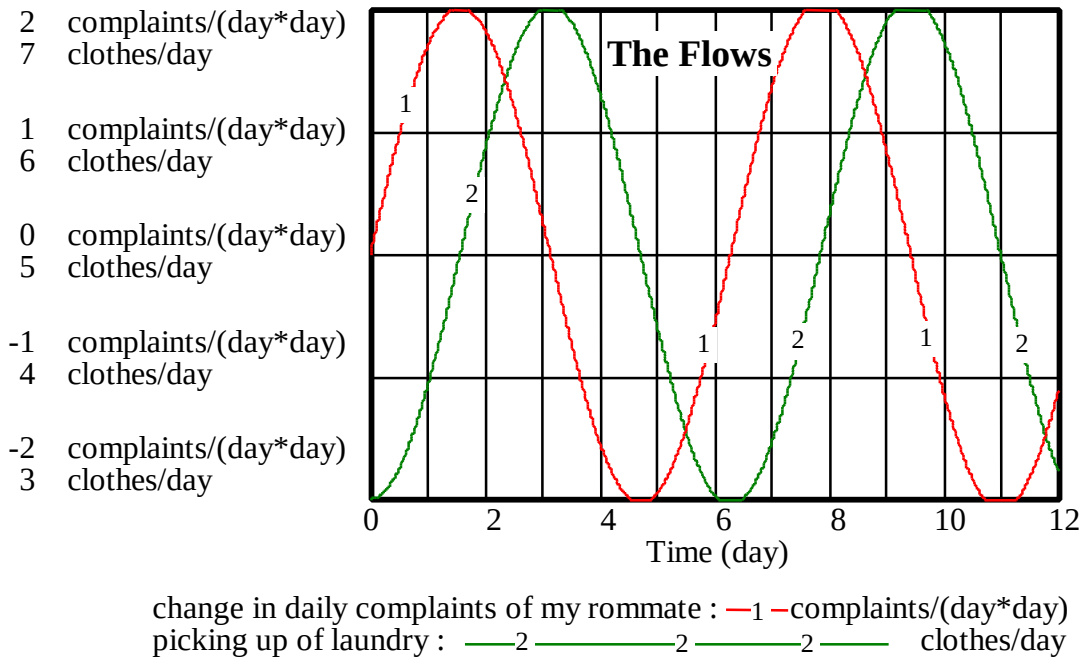


Figure 17: Vensim Equivalent of Figure 10: Graph of the behavior of the stocks and flows of the Cleanliness of a College Dorm Room Model for the first 12 days

An Introduction to Sensitivity Analysis

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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Vensim Examples added October 2001

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Abstract

This paper is an introduction to a series of papers on sensitivity analysis. It contains three exploratory exercises demonstrating the effects of various parameter and initial value changes on system behavior.

The paper assumes that the reader is able to build and understand a multiple-level model, and has experience with the sensitivity feature in the STELLA software.[1] We encourage the reader to build all the models and to run the simulations described.

[1] For a review of the sensitivity feature in STELLA, please refer to your STELLA user manual or to *Road Maps 3: A Guide to Learning System Dynamics* (D-4503-3), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, p. 12-13. STELLA is a registered trademark of High Performance Systems, Inc.

Introduction

Sensitivity analysis is used to determine how "sensitive" a model is to changes in the value of the parameters of the model and to changes in the structure of the model. In this paper, we focus on parameter sensitivity. Parameter sensitivity is usually performed as a series of tests in which the modeler sets different parameter values to see how a change in the parameter causes a change in the dynamic behavior of the stocks. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model building as well as in model evaluation.

Sensitivity analysis helps to build confidence in the model by studying the uncertainties that are often associated with parameters in models. Many parameters in system dynamics models represent quantities that are very difficult, or even impossible to measure to a great deal of accuracy in the real world. Also, some parameter values change in the real world. Therefore, when building a system dynamics model, the modeler is usually at least somewhat uncertain about the parameter values he chooses and must use estimates. Sensitivity analysis allows him to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid. If the tests reveal that the model is insensitive, then it may be possible to use an estimate rather than a value with greater precision. Sensitivity analysis can also indicate which parameter values are reasonable to use in the model. If the model behaves as expected from real world observations, it gives some indication that the parameter values reflect, at least in part, the "real world."

Sensitivity tests help the modeler to understand dynamics of a system. Experimenting with a wide range of values can offer insights into behavior of a system in extreme situations. Discovering that the system behavior greatly changes for a change in a parameter value can identify a leverage point in the model—a parameter whose specific value can significantly influence the behavior mode of the system.[2]

[2] In this paper, the term "behavior mode" refers to the general kind of behavior, such as exponential growth, asymptotic growth, S-shaped growth, or oscillation.

Exploratory Exercises

In this section we look at two models and explore how sensitive they are to changes in parameters and initial values of stocks. The first exploration shows that parameter changes produce some change in the appearance of behavior of the system, but they do not change the behavior mode. The second exploration demonstrates that changes in different parameters create different types of changes in the behavior of the system.

In the first exploration, we conduct sensitivity analysis on all the constant parameters in the model. However, in a large model, such an extensive treatment of sensitivity analysis is often impossible.[3] The modeler must pick the parameters he expects to have most influence on the behavior, or the ones that he is most uncertain about, and only use those in the sensitivity analysis. We will see in the second exploration that an examination of the structure of the model can indicate, without running the sensitivity tests, what kind of effect changes in some parameters would have. An explanation of how to choose the parameters for testing will be offered in later papers in the sensitivity analysis series.

Exploration 1: Lemonade Stand

In the first exploration, let's look at a lemonade stand located on a college campus. As usual, we are particularly interested in the behavior of the stock, the number of cups of lemonade that are ready to be sold to customers. The stand is open eight hours every day. Howard, the owner, is the only person working in the stand.

We encourage you to try to build the model and then compare it with the model suggested in Figure 1. The model documentation and equations can be found in section 7.1 of the Appendix.[4]

[3] Some system dynamics software, such as Vensim, can make automatic sensitivity tests of all parameters. Vensim is a registered trademark of Ventana Systems, Inc.

[4] The Lemonade stand model is based on: John Sterman, 1988. *Formulation of a simple business model* (D-4148), 15.874, System Dynamics for Business Policy, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

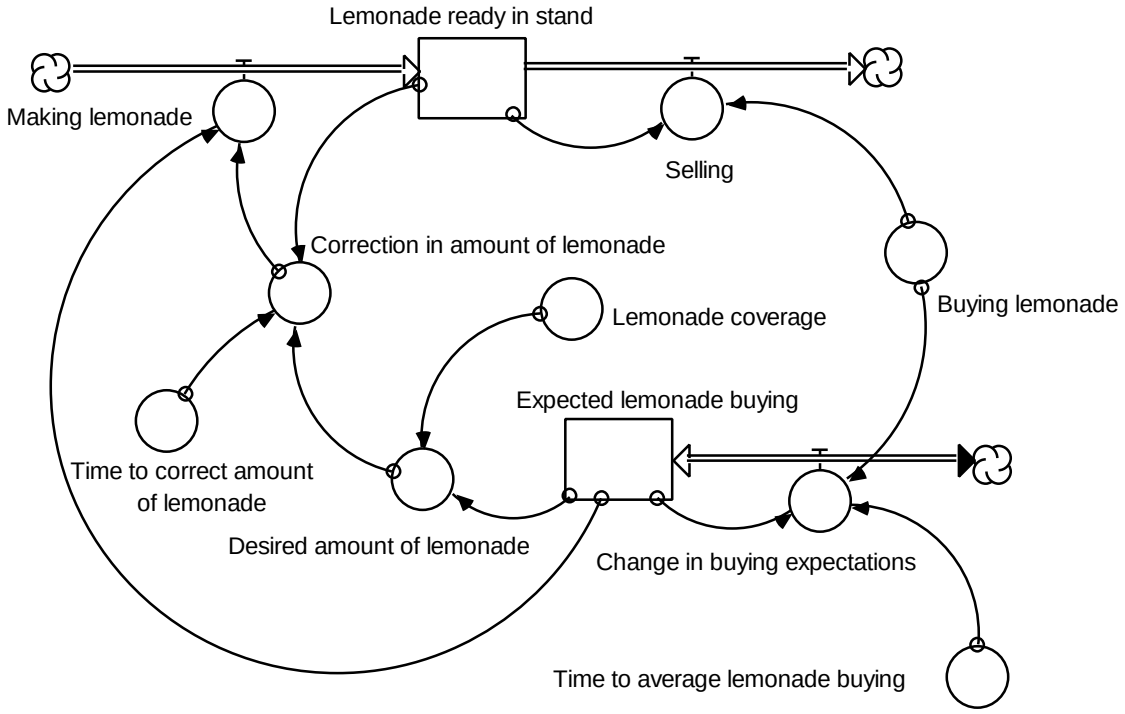


Figure 1: Model of the Lemonade stand

The stock, called "Lemonade ready in stand," is increased by an inflow called "Making lemonade," and decreased by the outflow, "Selling." "Selling" (cups per hour) is equal to "Buying lemonade," an exogenous parameter (not present in a feedback loop of the system). Howard has found out that he sells approximately 20 cups of lemonade every hour. However, he cannot sell more than what he has. Therefore, if "Buying lemonade," or the demand for lemonade, at any given time is greater than the amount of "Lemonade ready in stand," he can only sell the smaller one of these amounts, that is, the amount he has ready.

Based on his experience, Howard expects to sell a certain number of cups every hour. If the demand for lemonade changes, it usually takes him approximately an hour to recognize the shift in sales as opposed to just random fluctuations. This 1 hour time constant is the "Time to average lemonade buying."

Howard also wants to have lemonade prepared for some time in advance. A parameter called "Lemonade coverage" is the number of hours worth of lemonade he wants to have ready. For example, a "Lemonade coverage" of 2 hours means that if he stops making lemonade at four o'clock, he will be able to sell all the remaining lemonade by six o'clock, provided that the demand for lemonade does not change during this time. Therefore, "Lemonade coverage" determines his "Desired amount of lemonade."

Howard then compares the desired amount to the actual amount of "Lemonade ready in the stand." He needs some time to compensate for this difference, the "Time to correct amount of lemonade." The difference between the desired and the actual amount, divided by the "Time to correct amount of lemonade," is his hourly "Correction in amount of lemonade." He then adds the correction to the amount of lemonade he expects to sell, and makes a corresponding number of cups of lemonade. This number is the hourly

inflow to the stock. Of course, "Making lemonade" cannot be negative. So, if the correction is negative (if the desired amount is smaller than the actual amount), and if the sum of the correction and his expectations is still negative, he will not make any lemonade until the sum of the correction and his expectations becomes positive again.

Let's now look at the behavior of the system when started in equilibrium. It remains in equilibrium during the entire eight hours of the simulation. The value of "Lemonade ready in stand" is 40 cups, as one can quickly check by multiplying the 2 hour "Lemonade coverage" by the constant value of "Buying lemonade" of 20 cups per hour. Figure 2 shows the base run.

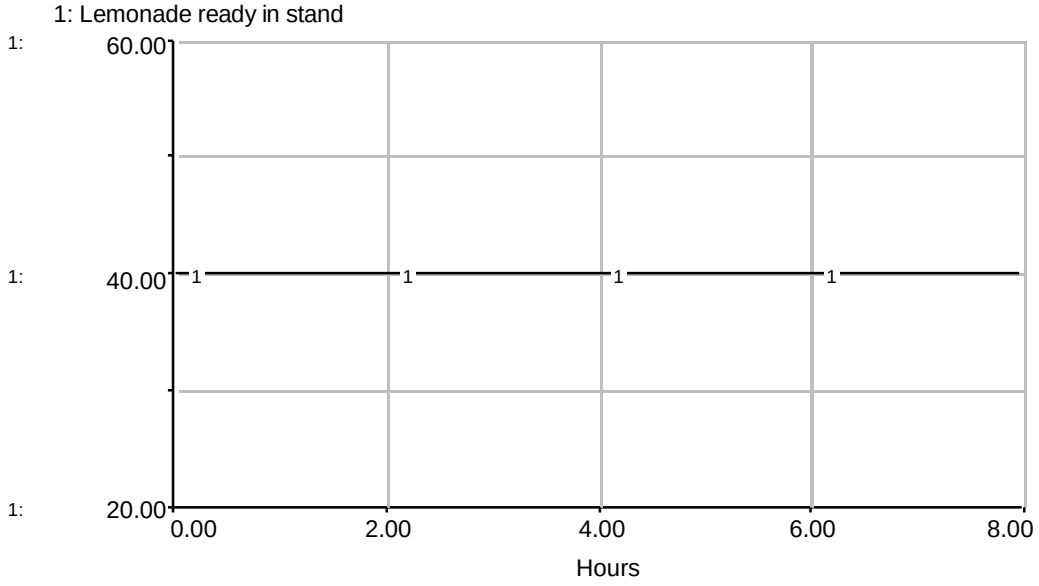


Figure 2: Base run of the Lemonade Stand model

Now let's assume that the demand for lemonade changes. At time 1 hour, we introduce a step increase of 5 cups in "Buying lemonade." Figure 3 shows the resulting behavior of "Lemonade ready" and "Buying lemonade."

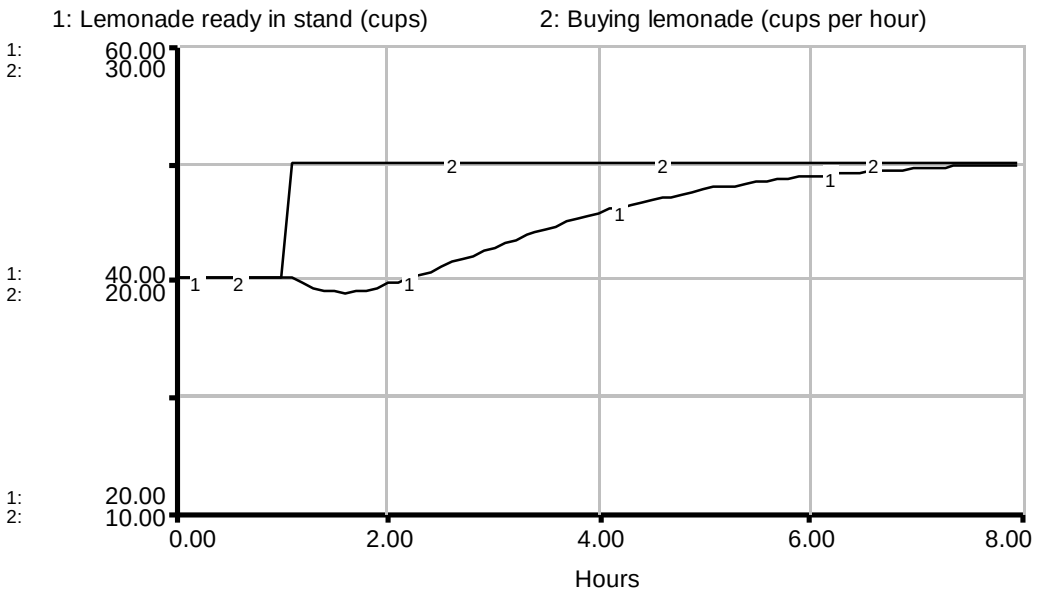


Figure 3: The system responding to a step increase in Buying lemonade

The system starts off in equilibrium, but only remains in equilibrium during the first hour. Immediately after the change in demand for lemonade, the amount of "Lemonade ready in stand" decreases slightly because "Selling" increases together with "Buying lemonade." In the meantime, it takes Howard some time to perceive the change and then to adjust his expectations, so he will not be able to react immediately by making more lemonade. It is the difference between the "Desired amount of lemonade" and the actual amount of "Lemonade ready in stand" that prompts him to make more lemonade so that the stock can start increasing and approach its new equilibrium value. To obtain the equilibrium value of 50 cups, one only needs to multiply the new value of "Buying lemonade," 25 cups per hour, by the "Lemonade coverage," 2 hours.

Exercise

Using the change in "Buying," we can now experiment with the model to see how the behavior resulting from disturbance changes under different parameter settings.

When building a model, we are often uncertain about the exact value of a parameter. However, it is important to be able to pick reasonable values for the parameters. This means that the values chosen should stay in a plausible range that could occur in the real world system. For example, in the Lemonade stand model it would not be reasonable to expect the "Time to correct amount of lemonade" to be just a few minutes. Howard needs a longer time to perceive the change in the stock, and then to correct the amount of lemonade so that it is ready to be sold. On the other hand, a value of 5 or 10 hours to correct the amount of lemonade would prevent him from responding to a change in his selling rate. The stock of lemonade might deplete quickly, resulting in a loss of customers, or overproduction. However, values of one or two hours seem quite reasonable: he will have enough time to find all the necessary ingredients, mix them together, and prepare the lemonade to be sold. The results of the sensitivity tests will

indicate whether the conclusions drawn from the model can be supported even without being certain about the exact values of the parameters.[5]

Three parameters and the initial value of the stock "Lemonade ready in the stand" can be used to explore the sensitivity of this model. The three parameters are "Time to average lemonade buying," "Time to correct amount of lemonade," and "Lemonade coverage."

It would probably be difficult to find out exactly how long it takes Howard to realize that "Buying lemonade" has changed. We can try to use several values for the "Time to average lemonade buying" to see if and how much the behavior of the system changes when the time is shorter or longer. Three values that seem reasonable are 30 minutes, 1 hour, and 1 hour and a half. The comparative run is shown in Figure 4.

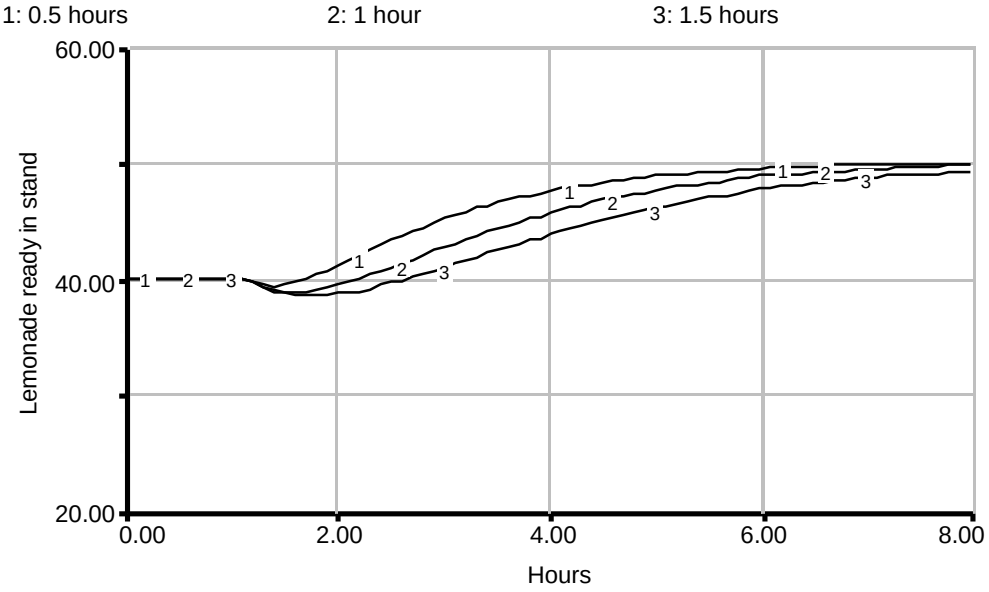


Figure 4: The effect of changes in "Time to average lemonade buying"

Although the three curves do not look exactly the same, these parameter changes do not affect the general mode of behavior of the system. All three curves show a small decrease in the stock right after the step increase and then a slow approach to the new equilibrium value of 50 cups. The similarity of the results shows that Howard does not have to be certain how long it would take him to perceive a change in his "Selling" rate to be able to estimate the overall behavior of his stock of lemonade.

The curves indicate that the faster Howard adjusts his expectations, the faster the stock of lemonade will approach equilibrium. The initial decrease in the stock is greater for larger values of this parameter. If it takes Howard longer to perceive a decline in the stock, the stock declines for a longer time and to a lower value.

[5] The parameter values used in this paper are of course not the only reasonable values, and they were only picked as examples. We encourage you to use other values if you feel that they would represent the situation more realistically.

"Time to correct amount of lemonade" is another parameter about whose value Howard is uncertain. Three values that seem plausible are one hour, one hour and a half, and two hours. Figure 5 shows the comparative run.[6]

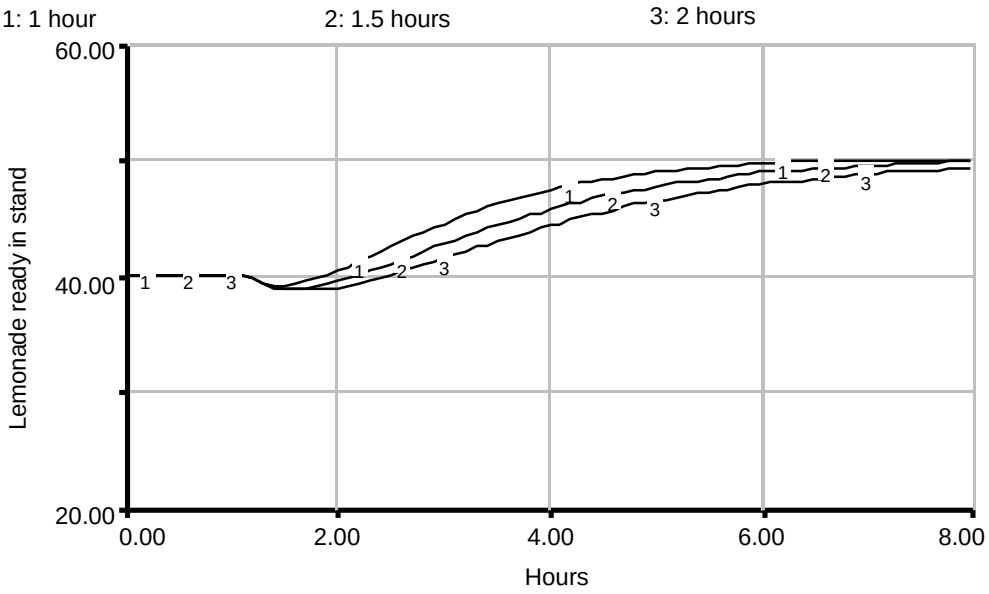


Figure 5: The effect of changes in "Time to correct amount of lemonade"

Again, notice that the general type of behavior remains the same for these parameter changes, but each curve is slightly different. As before, this means that Howard only needs to know an approximate range of values for this parameter to be confident in the behavior simulated by the model. The effect of changing the value of "Time to correct amount of lemonade" is similar to the "Time to average lemonade buying." A lower value of "Time to correct amount of lemonade" makes the stock reach the equilibrium value faster because the "Correction in amount of lemonade" is greater, so Howard prepares the lemonade faster. Also, an increase in the value of this parameter makes the initial decrease in the stock larger. If it takes Howard a longer time to correct the amount of "Lemonade ready," he will not react quickly enough by starting to make more lemonade, so he will not prevent his stock of lemonade from decreasing.

Next, let's look at changes in "Lemonade coverage." It is Howard himself who decides how many hours worth of lemonade he wants to keep ready at all times. He always wants to have a sufficient amount of lemonade ready, but he also wants to be sure that the lemonade he sells is always fresh. He can therefore run tests with several values of "Lemonade coverage" to see how the behavior of his stock of "Lemonade ready" would change if he decided to have smaller or larger "Lemonade coverage." Some values that he can try for this parameter are one and a half hours, two hours, and two and a half hours. Figure 6 shows the comparative run.

[6] Note that the sensitivity feature in STELLA automatically resets the value of the previous parameter to its base run value when you select another parameter for the sensitivity test.

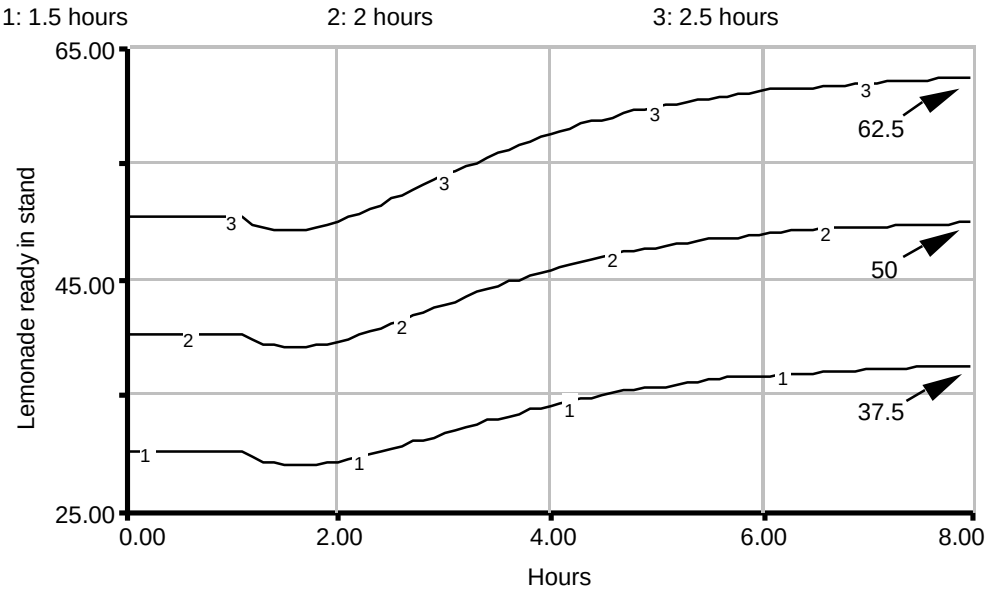


Figure 6: The effect of changes in Lemonade coverage

The effect of a change in "Lemonade coverage" is different from changes in the two previous parameters.[7] It is important to notice, however, that even though the behavior of the three curves is not exactly the same the general behavior is again unchanged. The curves start out in equilibrium with different initial values. Immediately after the step increase, there is always a slight decrease in the stock, followed by a slow asymptotic approach to equilibrium. Contrary to the changes in the first two parameters that we examined, changing "Lemonade coverage" does not influence the time it takes the stock to approach equilibrium. Instead, it changes the equilibrium value of the amount of "Lemonade ready in stand." The equilibrium values can be obtained by multiplying "Buying lemonade" by the respective values of "Lemonade coverage." Therefore, if Howard changes his "Lemonade coverage," only the equilibrium value of the stock of "Lemonade ready" will be affected. Because "Lemonade coverage" compensates for possible variations and fluctuations in demand for lemonade, a higher value of "Lemonade coverage" protects Howard from running out of lemonade. Therefore, to make sure that he always has enough "Lemonade ready" for the customers, Howard should not choose a very small value for "Lemonade coverage."

Finally, we can make changes in the initial value of "Lemonade ready in stand." This value is set when Howard prepares lemonade before opening the stand every morning. He could have initial values of 30, 40, or 50 cups ready, as shown in Figure 7.

[7] Please note that to start this simulation in equilibrium, you should set the initial value of "Lemonade ready in stand" to "Buying lemonade * Lemonade coverage."

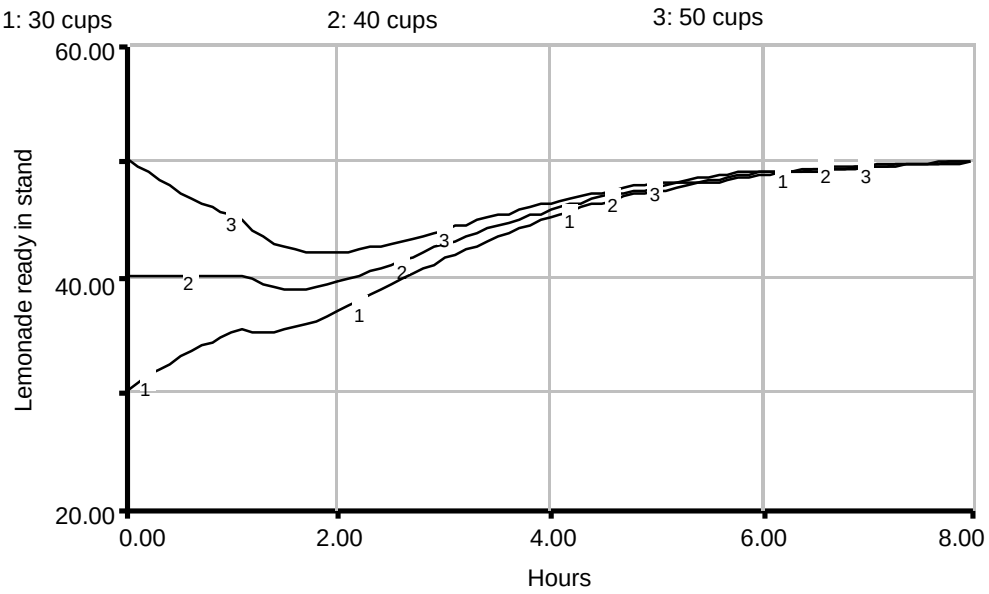


Figure 7: The effect of changes in the initial value of "Lemonade ready in stand"

Notice again that even though the behavior of the curves looks different from the ones in the previous simulations of this model the general behavior has not changed. Even when Howard starts out with a larger or smaller amount of lemonade, the behavior of the stock of Lemonade ready does not change greatly. When the initial amount of Lemonade ready is lower than what the equilibrium value should be, that is less than 40 cups, the stock will be increasing during the first hour, as in curve 1. When the initial amount is larger than 40 cups, the stock will decrease for the first hour, as in curve 3. However, after the step increase, all the curves immediately decline slightly, and then start slowly increasing to approach the same equilibrium value of 50 cups nearly at the same time.

Debrief

As expected, changing the value of parameters in the model does make some difference in the behavior observed. Also, the sensitivity tests indicate that some parameter changes result in "greater," or more significant, changes than others. For example, compare Figures 5 and 6. In Figure 5 the changes in "Time to correct amount of lemonade" produce little difference in the behavior, while in Figure 6 the curves show the same behavior, but at different values of the stock. This measure of more significant changes is studied through sensitivity analysis. In all cases, however, it is the structure of the system that primarily determines the behavior mode. In general, but with exceptions, parameter values, when altered individually, only have a small influence on behavior.

Exploration 2: Epidemics

In the second exploration we look at an epidemics model. The model was already used in a previous chapter in Road Maps, so it is possible that you have already built it.[8]

[8] For more information about the model, see: Terri Duhon and Marc Glick, 1994. *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics in Education Project, System Dynamics Group, Sloan

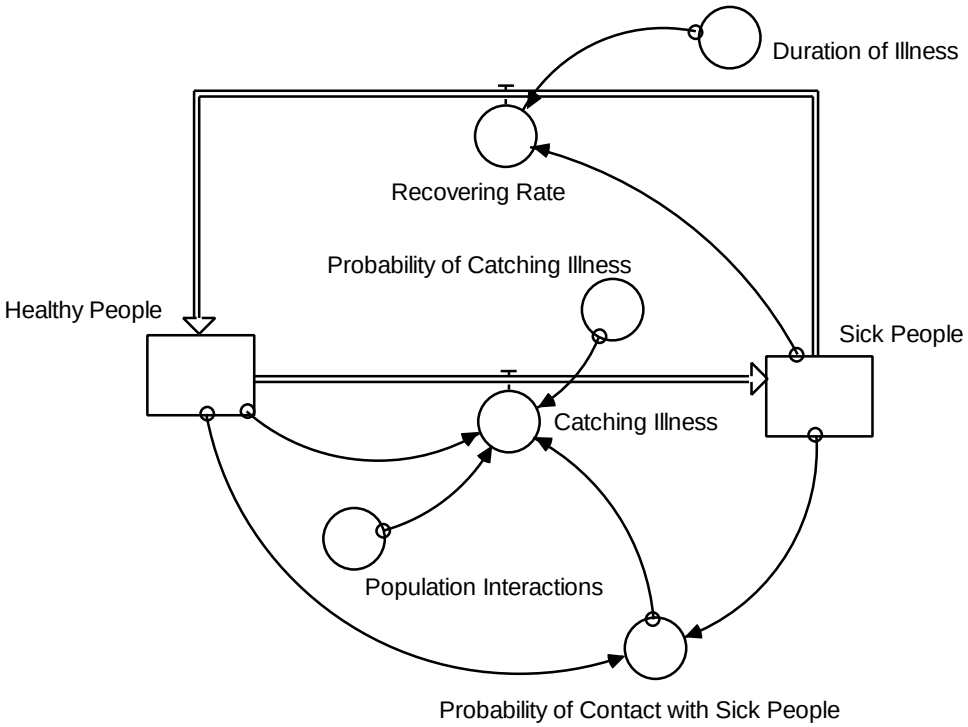


Figure 8: The Epidemics Model

The model shown in Figure 8 has two stocks, "Healthy People" and "Sick People." The total population is a constant 100 people in all the simulations, so the initial value of "Healthy People" is:

Initial number of Healthy People = 100-Sick People

In the base run, the initial value of "Sick People" is 1. The flow of "Catching Illness" converts the "Healthy People" to "Sick People." It is affected by three variables: "Probability of Contact with Sick People," "Probability of Catching Illness," and "Population Interactions." The "Probability of Contact with Sick People" is the ratio of "Sick People" to the total population of 100 people. Therefore, the more "Sick People" there are in the population, the greater the probability of coming into contact with a sick person. When a healthy person comes into contact with a sick person, there is a certain probability that the healthy person will catch the illness. This is the "Probability of Catching Illness." The value of "Population Interactions" determines the total number of interactions or contacts that any one person has with other people in the population during a month. The "Recovering Rate" converts "Sick People" back to "Healthy People," with a time constant called "Duration of Illness." See section 7.2 of the Appendix for the model documentation and equations.

Figure 9 shows the base run of the model, using the parameter values indicated in the Appendix.

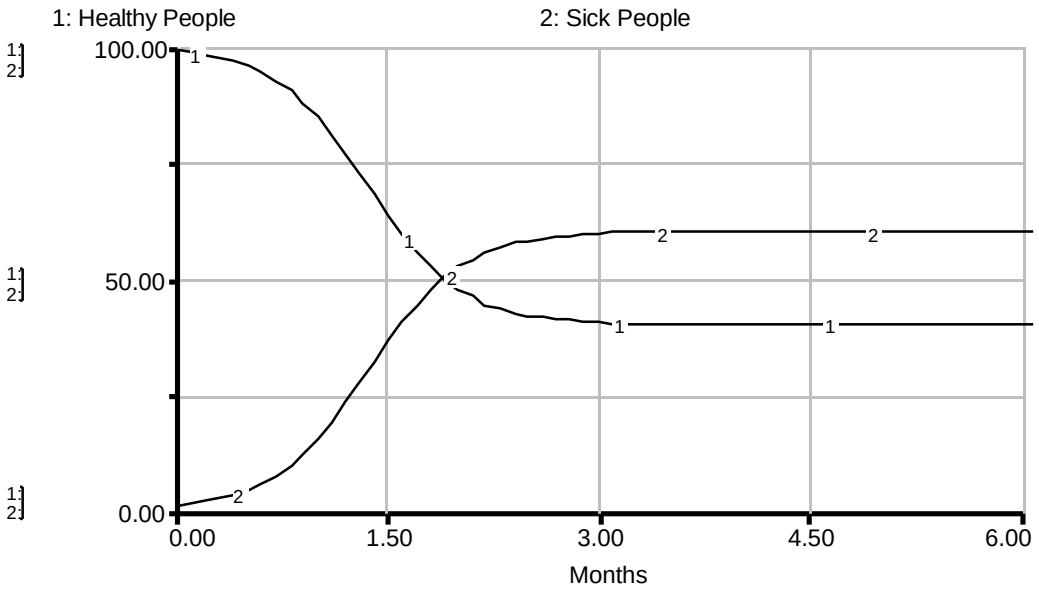


Figure 9: Base run of the Epidemics Model

Both stocks exhibit S-shaped growth. The number of "Sick People" at first increases exponentially, but then shifts to asymptotic growth to approach the equilibrium value of 60 "Sick People." The number of "Healthy People," on the other hand, decreases, at first exponentially and then asymptotically to the equilibrium value of 40 "Healthy People."

Exercise

The initial value of "Sick People," as well as three parameters—"Duration of Illness," "Population Interactions," and "Probability of Catching Illness"—can be used to evaluate the sensitivity of this model.

In this exercise, we use some parameter values or initial values that correspond to extreme cases. Such tests are very helpful in better understanding the dynamics of the system. For example, the extreme values of the initial number of "Sick People" would be 0 or 100 people. An extreme value of "Duration of Illness" would be a very large value, say one million months, illustrating the case of a terminal disease with no recovery. When using such extreme values, however, it is important to make sure that they stay in the range of values that are possible in the real-world system (e.g., do not use a negative initial value for a stock that cannot be negative). Sensitivity tests using extreme values are performed mostly to learn about the dynamics of the system.

The initial value of "Sick People" represents the number of people who are sick when the simulation begins. The stock can, for example, start with no sick people, one sick person, 20, 50, or even 100 sick people. The behavior of both stocks, Healthy People and Sick People, is shown in Figure 10.

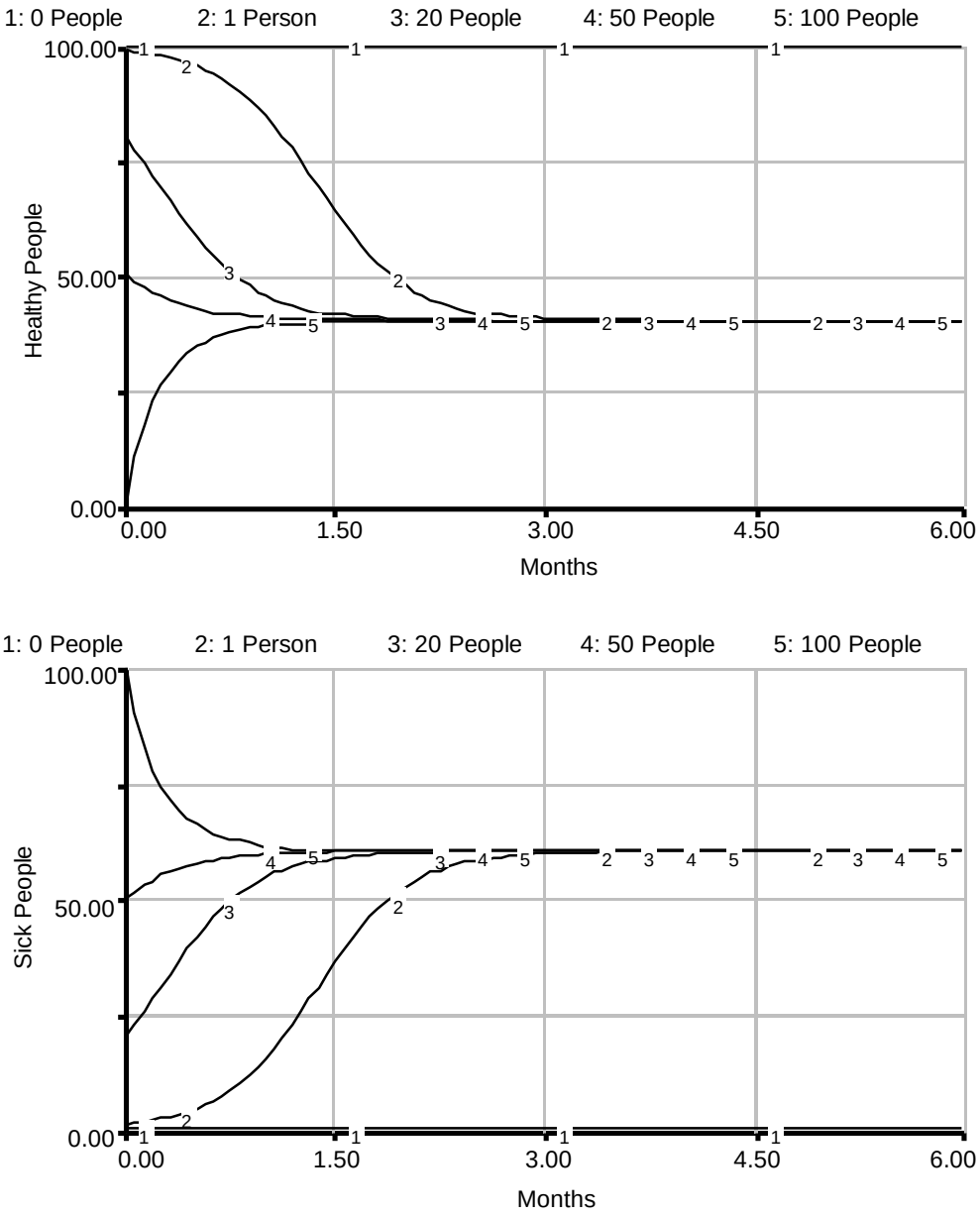


Figure 10: The effect of changes in the initial value of Sick People

Let's look at the individual curves. Curve 1 represents the case of zero "Sick People." During the entire simulation, all the hundred people in the population stay healthy and no one becomes sick. If there are no "Sick People," the "Probability of Contact with Sick People" and therefore the flow "Catching Illness" are both zero. Thus, it is obvious that there must be at least one sick person to obtain the characteristic S-shaped growth behavior. Curve 2 corresponds to the case of one sick person initially. The presence of only one sick person makes a great difference in the behavior of the system. The one person starts spreading the disease to more people. Just like in the base run, the two stocks follow the S-shaped growth pattern, and stabilize at the base run equilibrium values. Curves 3 and 4 behave similarly to curve 2, but approach their respective equilibrium values faster because their initial values are closer to equilibrium.

In curve 5, the behavior of the stocks is reversed—the number of "Healthy People" increases to its equilibrium value of 40, while the number of "Sick People" decreases to 60. As long as there is initially at least one sick person present in the population the equilibrium value does not depend on the initial number of "Sick People."

The parameter "Duration of Illness" determines how long it takes a sick person to become healthy again. Try using the values of 3 days, two weeks, a month, six months, and an extremely large value, such as a million months, as shown in Figure 11.[9]

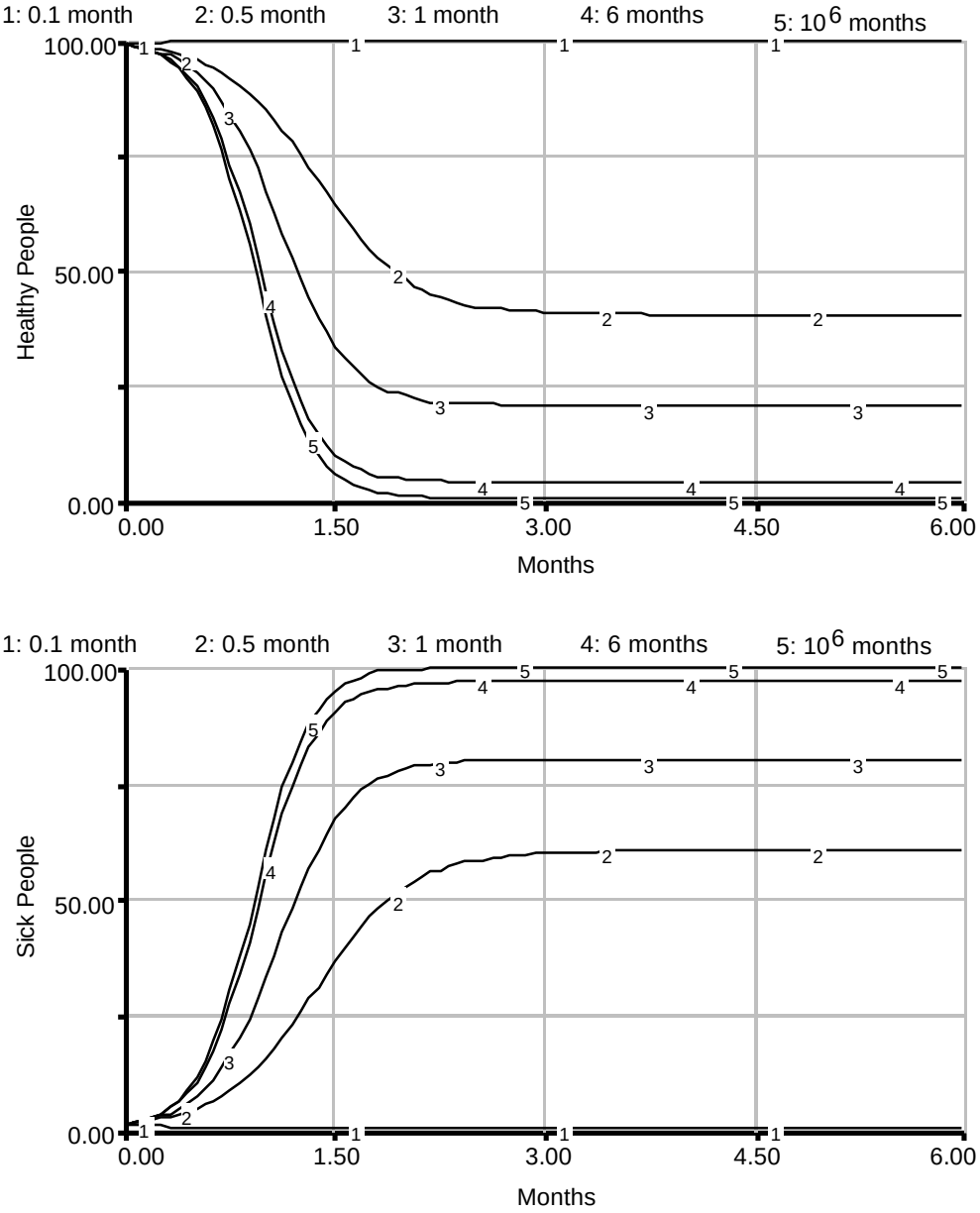
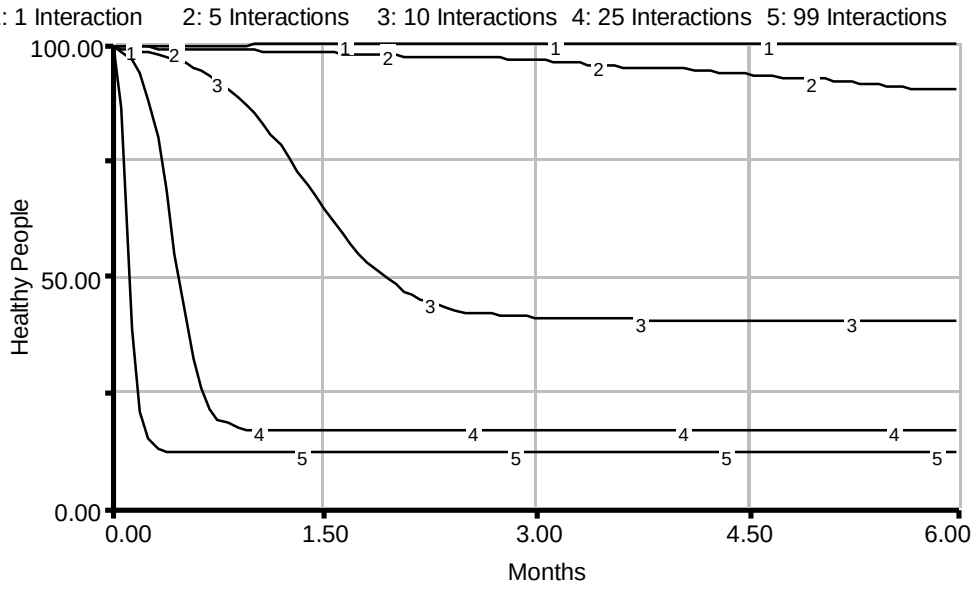


Figure 11: The effect of changes in "Duration of Illness"

[9] Note that you should set the time interval of the simulation to 0.1 or less to test for the value of "Duration of Illness" of 0.1.

Let's again analyze the individual curves. Curve 1 represents a case of a very short disease that only lasts a few days. The only sick person becomes healthy before spreading the disease to anyone else, and all the "Healthy People" stay healthy. Curve 2 corresponds to the base run. Notice from comparing curves 2, 3, 4, and 5, that increasing the "Duration of Illness" results in higher equilibrium values of "Sick People" and lower equilibrium values of "Healthy People." The reason for this behavior is that when the disease is longer, each sick person can spread it to more people. In particular, curve 5 shows that when the disease is extremely long, or terminal (no recovery), all 100 people become sick. The longer the "Duration of Illness," the sooner the stocks approach equilibrium. This result should be expected—the longer the disease, the more people get sick and have a longer time to spread the disease further and faster, so equilibrium occurs sooner.

The parameter "Population Interactions" determines how many contacts any person has with other people during a unit of time. The value of a parameter such as "Population Interactions" is quite uncertain—it can be different for each person and can even vary over time. Therefore, the value chosen for the model can only be an estimate of the average of the actual value. A sensitivity test that uses different values of "Population Interactions" indicates how the behavior of the system is affected when its value changes. Depending on the nature of the population, the number of "Population Interactions" can vary significantly. A person can, for example, meet only one other person each month, 5 people, 10 people, 25 people, or even all the rest of the population—99 people a month. Figure 12 shows the behavior of both stocks.



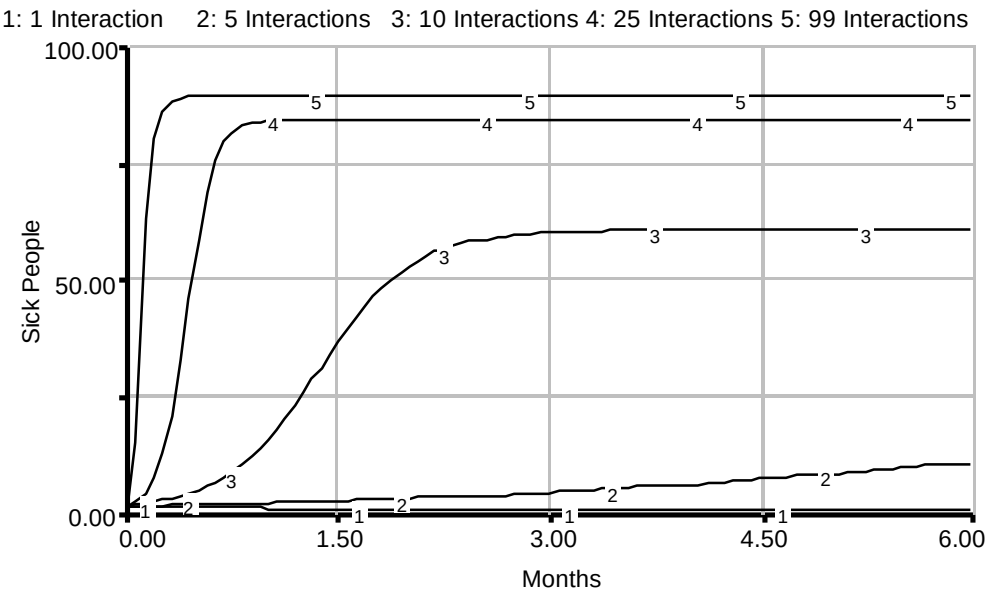


Figure 12: The effect of changes in "Population Interactions"

If there are few interactions in the population, the only sick person becomes healthy and no one else becomes sick, as in curve 1. By comparing curves 2, 3, 4, and 5, notice that increasing the number of interactions increases the equilibrium value of "Sick People" and decreases the equilibrium value of "Healthy People." Therefore, if you want to study the spread of an epidemic in a specific population, and if you are interested in the equilibrium number of people who will be affected, you will have to have an idea of the value of "Population Interactions" in that population. Also, an increase in "Population Interactions" makes the stocks reach their equilibrium values faster—it increases the value of "Catching Illness," and thus "Healthy People" become sick sooner.

Before looking at the effects of changing "Probability of Catching Illness," turn back to Figure 8 showing the Epidemics model. Notice that both "Population Interactions" and "Probability of Catching Illness" enter the rate of "Catching Illness" in a similar way. The rate equation is:

Catching Illness = Healthy People * Probability of Contact with Sick People * Population Interactions * Probability of Catching Illness

Both "Population Interactions" and "Probability of Catching Illness" are linear factors of the rate equation—they both simply multiply "Healthy People" and "Probability of Contact with Sick People." One would therefore expect that their effect on the system behavior should be similar, and that they should produce the same kind of changes in the behavior when their value changes. We encourage you to run sensitivity tests with different values of "Probability of Catching Illness." Because this value is also very uncertain in the real world, try values of 0.05, 0.25, 0.5, 0.75, and 1, for example. The graphs you obtain should resemble the ones in Figure 12 above.

Debrief

Sensitivity analysis again showed that changing the value of parameters makes some difference in the behavior of the model, while the general behavior mode is relatively insensitive to parameter changes. Some parameter changes affect the behavior to a larger extent than others. Changes in some parameters affect the equilibrium values, such as "Lemonade coverage" from the Lemonade stand model. Other parameters affect the time necessary to approach equilibrium—for example "Time to correct amount of lemonade" from the Lemonade stand model, or the initial value of "Sick People" from the Epidemics model. Others again have influence over both—"Population Interactions" for example. When two parameters have similar roles in the model, they usually affect the behavior in a similar way, such as "Population Interactions" and "Probability of Catching Illness." Using extreme values in sensitivity tests is an excellent tool for gaining a deeper understanding of the behavior generated by the structure of the model as well as for testing the assumptions about the model.

Independent Exploration: Coffeehouse

We now return to Howard, the owner of the lemonade stand on a college campus. Howard realized that it could be more profitable for him to sell coffee because students tend to drink more coffee than lemonade, and they drink it at any time of the day and night. Therefore, he closed his lemonade stand and opened a 24-hour Coffeehouse. Howard bases the Coffeehouse model on the model he used in his lemonade stand to model the number of cups of "Coffee ready." We will run the simulation over a period of two days, or 48 hours.

Figure 13 shows the full model. The model documentation and equations are in section 7.3 of the Appendix.[10]

[10] The Coffeehouse model is based on: John Sterman, 1988. *Understanding Business Fluctuations—The Causes of Oscillations* (D-4149), 15.874, System Dynamics for Business Policy, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

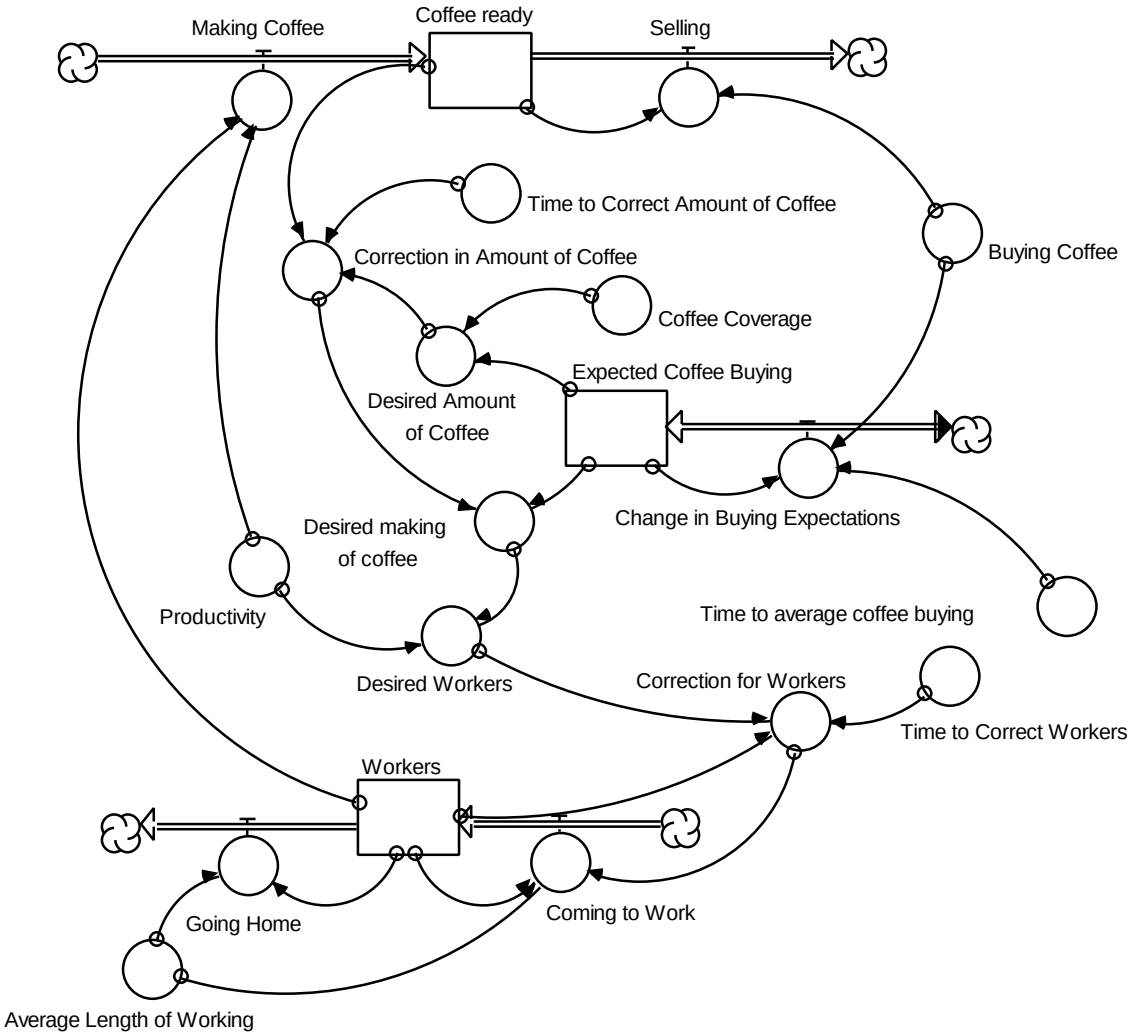


Figure 13: The Coffeehouse Model

Because Howard cannot work 24 hours a day all by himself, he needs to hire student workers to work in the Coffeehouse. To represent the new situation, he adds a sector to his model for the number of workers. He employs several workers, but not all of them are at work at the same time. The stock of "Workers" in the model measures only the number of workers who work in the Coffeehouse at a specific time.

Each worker is able to prepare a certain number of cups every hour, determined by the parameter called "Productivity." In the base run, "Productivity" is 20 cups per worker per hour. When Howard divides the "Desired making of coffee" by "Productivity," he obtains the "Desired Workers," or the number of workers necessary to prepare the desired amount of coffee.

Howard then compares the "Desired Workers" to the actual number of "Workers." The difference between these two values, divided by a time constant called "Time to Correct Workers," gives his hourly "Correction for Workers." The "Time to Correct Workers," 3 hours in the base run, is the time constant that he needs to compensate for the difference between the desired and actual number of workers. It is the time in which he wants to call them on the phone and have them come to work.

Howard then adds the Correction to the number of workers who leave the Coffeehouse every hour. He has to make sure that a corresponding number of workers also come to work at the Coffeehouse each hour—this is the inflow called "Coming to Work" that increases the number of "Workers." The outflow, "Going Home," decreases the number of present "Workers." "Going Home" is the number of "Workers" divided by 4 hours, the "Average Length of Working." Finally, Howard multiplies the number of "Workers" by their "Productivity" to obtain the flow of "Making Coffee."

Without any outside disturbance, the system starts out and remains in equilibrium at 40 cups (Coffee coverage * Buying coffee) during the 48 hours of simulation. Figure 14 shows the base run of the model, using the parameter values from section 6.3 of the Appendix.

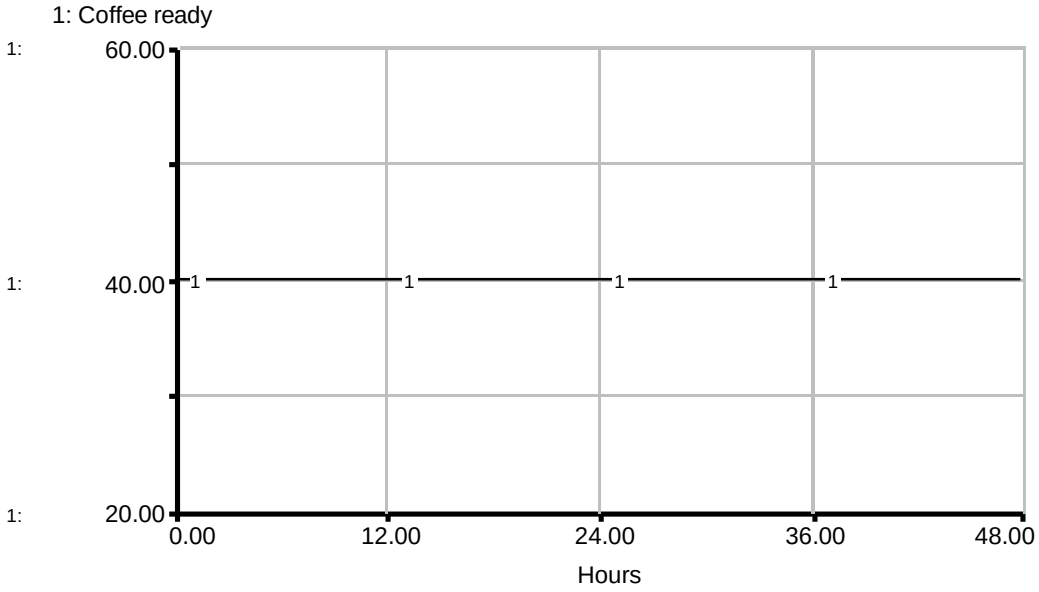


Figure 14: Base run of the Coffeehouse model

Let's assume again that the demand for coffee suddenly changes. The system starts off in equilibrium, with 40 cups of coffee ready. At hour 3, we introduce a step increase of 5 cups in the buying of coffee. Figure 15 shows the resulting behavior of "Coffee ready" and "Buying Coffee."

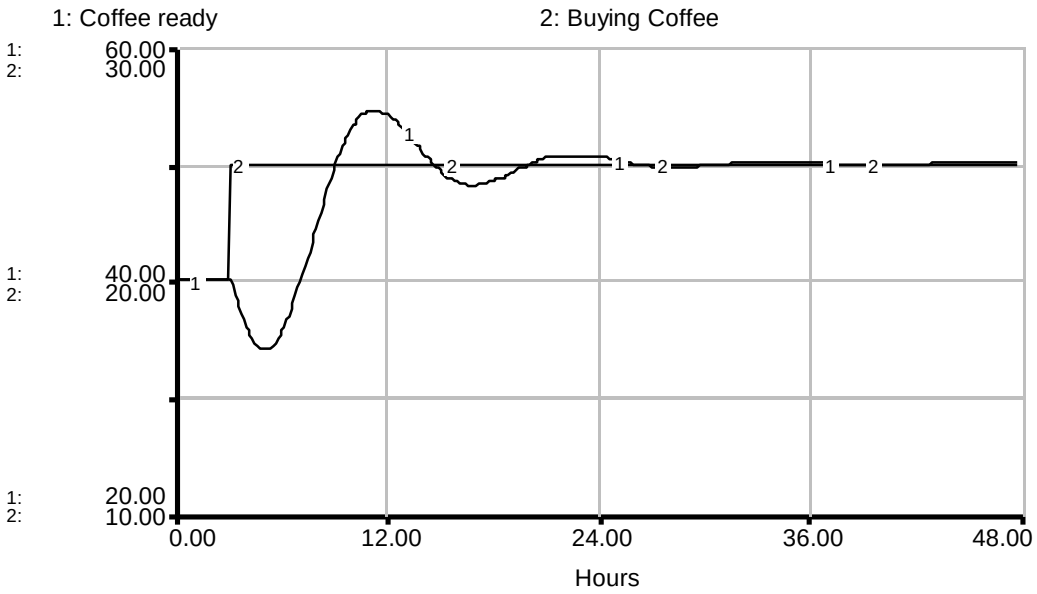


Figure 15: The system responding to a step increase in Buying Coffee

The additional structure added to the Lemonade stand model significantly changes the behavior of the system—the stock of coffee now exhibits "damped oscillations."

After the increase in buying, "Coffee ready" decreases because "Selling" steps up together with demand. However, "Making Coffee" has not changed yet. It first takes Howard a certain time to perceive the change in buying as opposed to random noise and to find out how much coffee the workers should be making. He then determines how many more workers should be working in the Coffeehouse, and calls them to make them come to work. However, because "Desired making of coffee" is high, Howard keeps calling more workers than the equilibrium number. As they come to work, the stock of coffee starts increasing, reaches its new equilibrium value of 50 cups, [11] overshoots it, and continues to increase until the number of workers decreases again to its equilibrium value. The oscillations continue, but they become smaller and smaller, until both stocks eventually approach their equilibrium values.[12]

Choosing Parameters for Sensitivity Analysis

Howard is interested in the behavior of "Coffee ready." What parameters and initial values should he use in a sensitivity analysis of the Coffeehouse model?

Performing Sensitivity Tests

Now take the parameters and initial values that you determined in section 4.1 one by one. For each of them, choose three values that seem reasonable to you and that you

[11] As in the first exploration, the equilibrium value can be obtained by multiplying "Coffee Coverage" (2 hours) by "Buying Coffee" (25 cups per hour).

[12] For a more detailed explanation of the causes of oscillations, see: Kevin Agatstein, 1996. *Oscillating Systems II: Sustained Oscillation* (D-4602), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

believe would offer insights about the behavior of the system. BEFORE running the sensitivity tests, think about the parameter as part of the structure of the model. Compare it to the parameters from the Lemonade stand and Epidemics models, and try to guess the effect of changing it on the behavior of "Coffee ready." Why would Howard want to know the effect? Then simulate the model and compare the graphs to your predictions. If a behavior seems surprising at first, try to explain it through the structure of the model.

Conclusion

Specific parameter values can change the appearance of the graphs representing the behavior of the system. But significant changes in behavior do not occur for all parameters. System dynamics models are in general insensitive to many parameter changes. It is the structure of the system, and not the parameter values, that has most influence on the behavior of the system.

Sensitivity analysis is an important tool in the model building process. By showing that the system does not react greatly to a change in a parameter value, it reduces the modeler's uncertainty in the behavior. In addition, it gives an opportunity for a better understanding of the dynamic behavior of the system.

We encourage you to experiment with the three models from this paper (as well as any other models that you have built) on your own. For example, try to change several parameters at the same time, observe the behavior produced, and compare it to the conclusions in this paper. Can you suggest any parameter values that would produce the "optimal," or most desirable behavior? The use of sensitivity analysis in such policy analysis will be explored in a later paper in this series.

Suggested Solutions to Section 4

Solution to Section 4.1

The initial value of "Coffee ready," as well as six parameters—"Time to average coffee buying," "Time to correct amount of coffee," "Coffee coverage," "Time to correct workers," "Productivity," and "Average length of working"—can be used to explore the sensitivity of this model.[13]

Solution to Section 4.2

The initial value of "Coffee ready" represents the number of cups ready in the Coffeehouse before the beginning of the simulation. It is usually set to "Coffee coverage * Buying coffee," but Howard may want to find out what would happen if he prepared more or less coffee at the beginning of the simulation. Three possible initial values are 30, 40, and 50 cups. Figure 17 shows the comparative run.

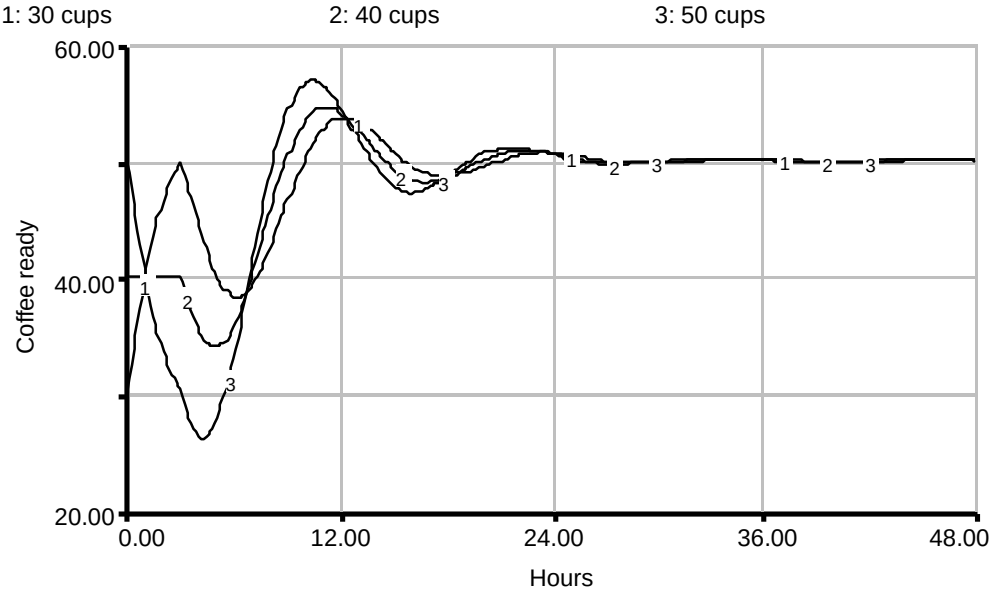


Figure 16: The effect of changes in the initial value of "Coffee ready"

As with the initial value of "Lemonade ready" in the Lemonade stand model, changing the initial value of "Coffee ready" creates differences in the behavior of the three curves, but does not affect the behavior mode of the system. Therefore, in the long run it is not very important how much coffee Howard initially makes. A change in the initial value of "Coffee ready" changes the magnitude of oscillations, but the curves approach equilibrium at nearly the same time. Notice that except for the initial value of 40 cups, the simulation does not start in equilibrium. Also, a higher initial value leads to a lower drop in "Coffee ready." A larger initial amount of coffee means initially fewer workers

[13] In the Coffeehouse model, there are three stocks whose behavior can be of interest: Coffee ready, Expected Coffee Buying, and Workers. In this paper, we usually show only the behavior of Coffee ready. In some cases, however, the behavior of the other stocks will also be necessary to understand the behavior of Coffee ready. We encourage you to perform similar runs with the other stocks in all the sensitivity tests.

necessary at the Coffeehouse. As the stock decreases and undershoots the equilibrium value, it takes longer for the additional workers to be called and put to work, so the stock declines to a lower value.

The "Time to average coffee buying" is the equivalent of the "Time to average Lemonade buying" in the first exploration, so their effects on the behavior should be similar. The value of "Time to average coffee buying" is again uncertain, and Howard is not sure how the behavior of "Coffee ready" changes when it takes the workers less or more time to recognize a change in "Buying Coffee." This time, let's use values of 1, 2 and 3 hours, as shown in Figure 17.

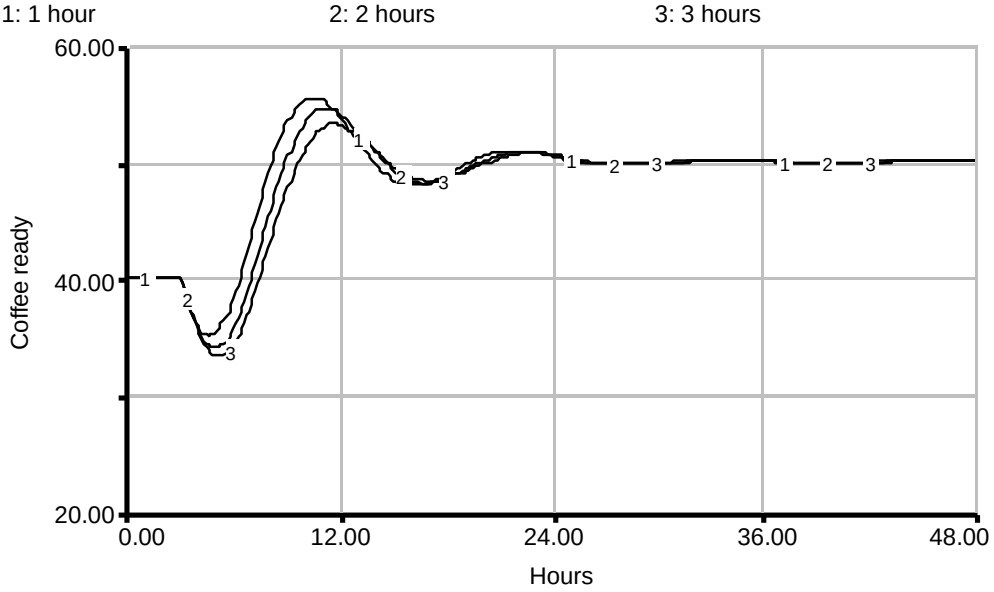


Figure 17: The effects of changes in "Time to average coffee buying"

Notice again that even though the three curves are not identical, the general type of behavior—damped oscillations—is the same in all three curves. More accuracy is not needed for this parameter because the behavior is not very sensitive to a change in it. An increase in the "Time to average coffee buying" increases the initial decline of the stock, but makes the overshoot smaller. When the "Time to average coffee buying" is long, it takes the stock longer to reach equilibrium.

As in the first exploration, the "Time to correct amount of coffee" indicates how long it takes to change the amount of coffee as a result of a sudden change in buying. The value of this parameter is also quite uncertain, so Howard wants to know how precisely he needs to know it to support the conclusions of his model. Three possible values, 0.5 hour, 1 hour, and 1.5 hours are shown in Figure 18.

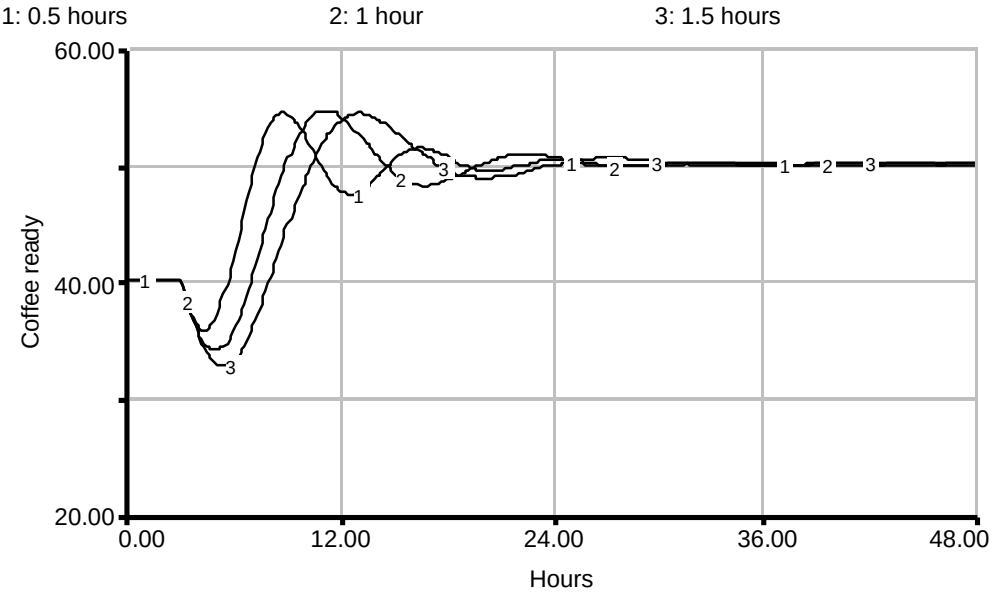


Figure 18: The effect of changes in the "Time to correct amount of coffee"

Again, although the three curves are not exactly the same, they all show the same type of behavior. Howard can now be more confident that it is not a specific value of "Time to correct amount of coffee" that creates the system behavior. Increasing the "Time to correct amount of coffee" increases the initial decline of the stock because the workers are not able to respond quickly enough and start preparing more coffee. A longer "Time to correct amount of coffee" also increases the time it takes the stock to reach equilibrium.

The parameter "Coffee Coverage" determines how many hours worth of coffee Howard would like the workers to keep ready in the Coffeehouse. It has the same role as the "Lemonade coverage" in the first exploration. As with lemonade, Howard wants to make sure that he always has a sufficient amount of "Coffee ready," while it is still fresh. He would like to know what would be the optimal value to set for "Coffee Coverage." Three possible values that he could test for are 1, 2, and 3 hours, as in Figure 19.

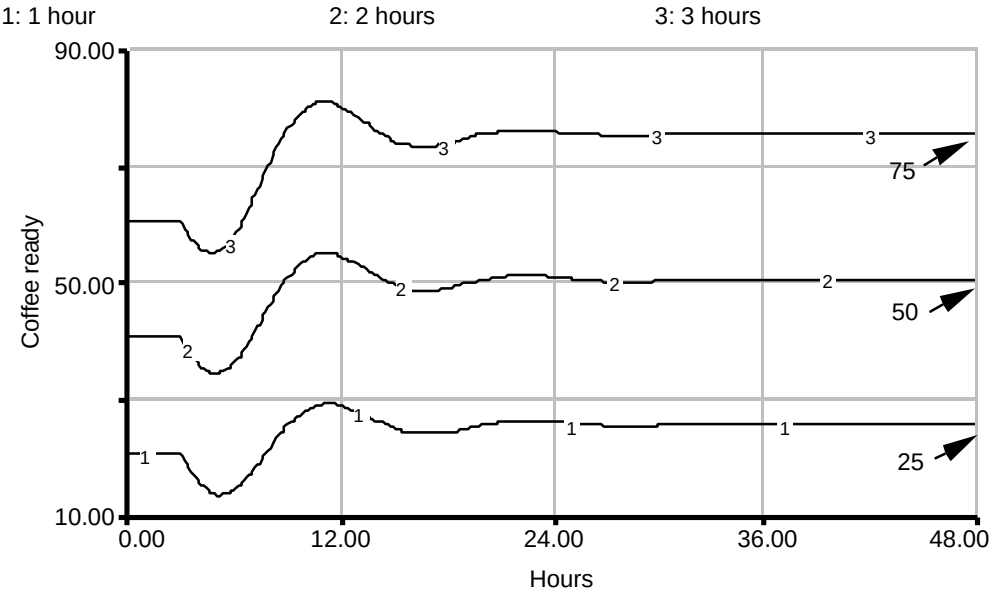


Figure 19: The effect of changes in "Coffee coverage"

As with "Lemonade coverage," changes in "Coffee coverage" only affect the initial and equilibrium values of the stock, but do not affect the time it takes to reach equilibrium.[14]

The "Time to correct workers" indicates how long it takes Howard to make more workers come to work. Because the workers in the Coffeehouse are students, they are usually not available immediately, and it can take Howard up to a few hours to make the necessary number of workers come to work. But he is not completely certain about the exact time required to make them come over, so he wants to try out different values to see how the behavior of the system changes. He knows that it usually takes him somewhere between 2 and 4 hours to call them up and make sure they come. However, he also wants to know whether the behavior would change in any way if the workers could come to work when he needed them—that is, if "Time to correct workers" was close to zero. He therefore tries one very low value, such as 0.25 hours, along with two more reasonable values of 2 and 4 hours, as in Figure 20.[15]

[14] To start the simulation in equilibrium, again set the initial value of "Coffee ready" to "Coffee coverage * Buying coffee."

[15] If you try a low value, make sure that it is at least twice the value of the solution interval (DT) you are using, to avoid a "DT error."

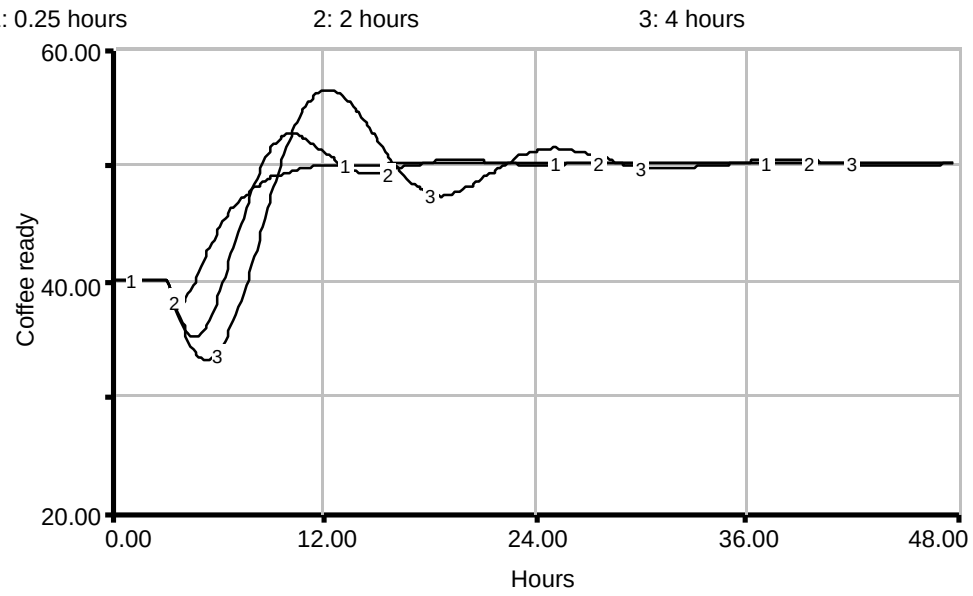


Figure 20: The effect of changes in "Time to correct workers"

A comparison of the curves again indicates that, except for very low values of "Time to correct workers," the general type of behavior remains the same. Notice how curve 1 resembles the behavior observed in all runs of the Lemonade stand model from section 3.1. In the Lemonade stand, there was no "Time to correct workers" because Howard was the only worker and was always immediately available. If the Coffeehouse workers were able to come to work immediately, the structure and thus the behavior of the Coffeehouse model would be the same as that of the Lemonade stand model. It is therefore the additional structure in the "Workers" sector that creates a time lag between the decision to make coffee, the time when the workers are available, and the time when the coffee is made; thus, it causes the Coffeehouse model to exhibit oscillation. However, a situation in which the workers would be available immediately is unlikely to happen, and the higher values of "Time to correct workers" are probably more reasonable. Howard thus does not need to know exactly how long it takes him to find workers who are available to come to work. The higher the value of "Time to correct workers," the greater the initial decrease of the stock, and the higher the magnitude of oscillations. Also, increasing the value of Time to correct workers increases the time it takes the stock to approach the same equilibrium value of 50 cups of coffee.

"Productivity" measures the amount of coffee that one worker can prepare in one hour. Because the workers also have other responsibilities in the coffeehouse, they cannot spend the entire hour preparing coffee. Howard does not know exactly how productive his workers are, so he tries three possible values of 10, 20, and 30 cups per worker per hour, as in Figure 21.

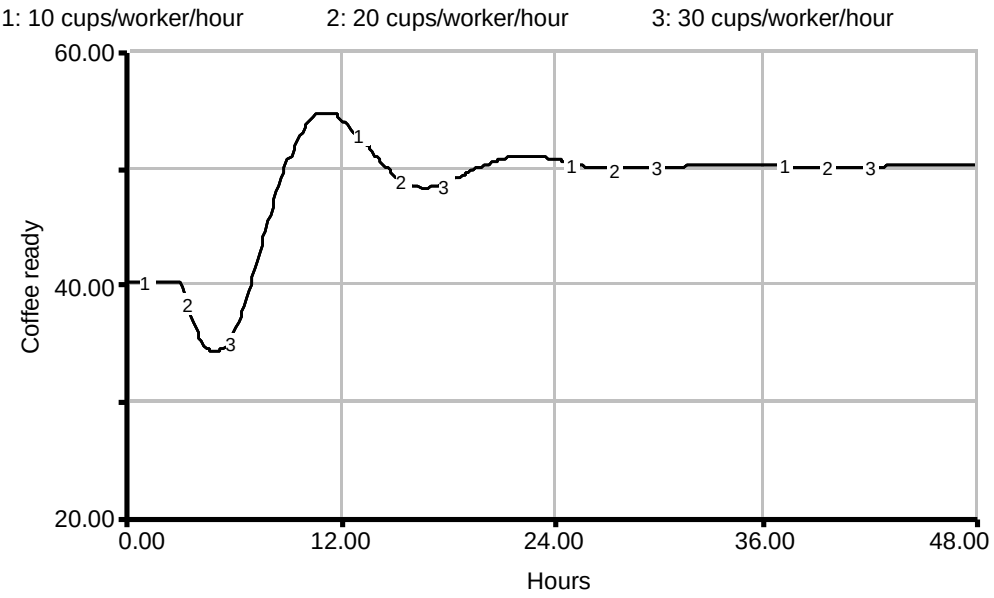


Figure 21: The effect of changing "Productivity"

Changing the value of "Productivity" does not have any effect on the amount of "Coffee ready." To explain this result, let's look at the behavior of the stock of "Workers" for the same three parameter values of "Productivity," shown in Figure 22.

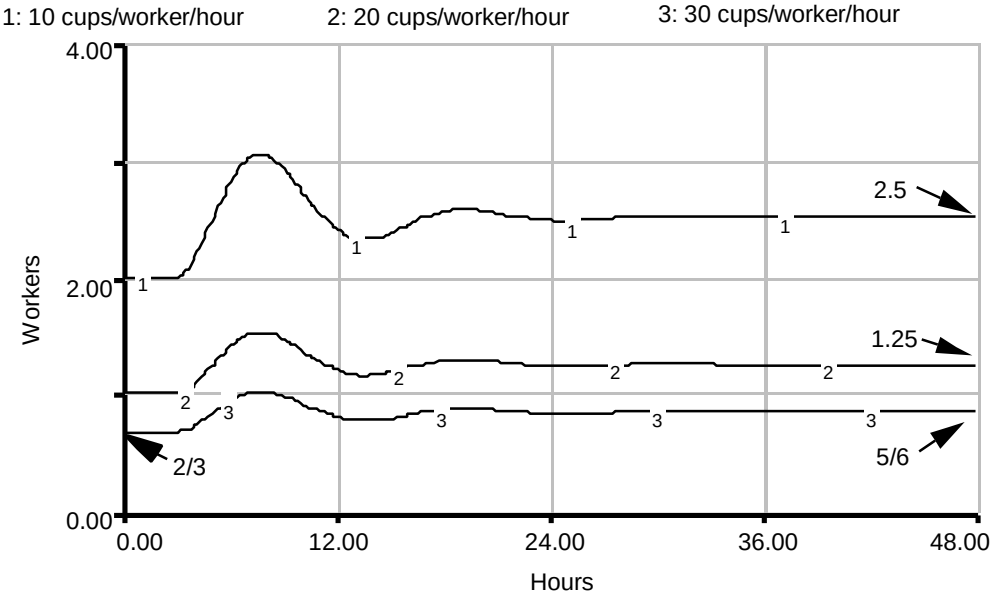


Figure 22: The effect of changing "Productivity" on the number of "Workers"

A change in "Productivity" creates a simultaneous change in the initial number of "Workers." Let us first compare the behavior of the three curves before the increase in buying. In curve 1, where "Productivity" is low, more workers have to come to the Coffeehouse to maintain the desired amount of coffee. Their number can be obtained by dividing the value of "Making Coffee" by the value of "Productivity." For equilibrium, the number of cups made every hour must equal the number of cups sold every hour, i.e., initially 20 cups per hour. Dividing 20 cups per hour by 10 cups per worker per hour

gives 2 "Workers," which is the initial value in curve 1. Similarly, in curve 2, the initial number of "Workers" is 1. Finally, in curve 3, the initial number of "Workers" is 2/3.[16] After the step increase in "Buying Coffee," the three curves show similar behavior. The oscillations occur at the same time, but their magnitude decreases as "Productivity" increases because with a higher "Productivity," the workers can react better to sudden changes in buying. The curves approach their equilibrium values at the same time—curve 1 approaches the value of 2.5 workers, curve 2 approaches 1.25 workers, and for curve 3 the equilibrium value is 5/6 of a worker.[17] Therefore, it is the number of "Workers" that compensates for the changes in "Productivity" so that the amount of "Coffee ready" remains unchanged.[18]

The "Average length of working" indicates the average number of hours a worker spends in work. Howard can easily find out this value, but he would like to know whether there would be any differences if his workers stayed in the Coffeehouse for a longer time. Possible values for this parameter that would be convenient for him and for the workers are, for example, 2, 4, and 6 hours, represented in Figure 23.

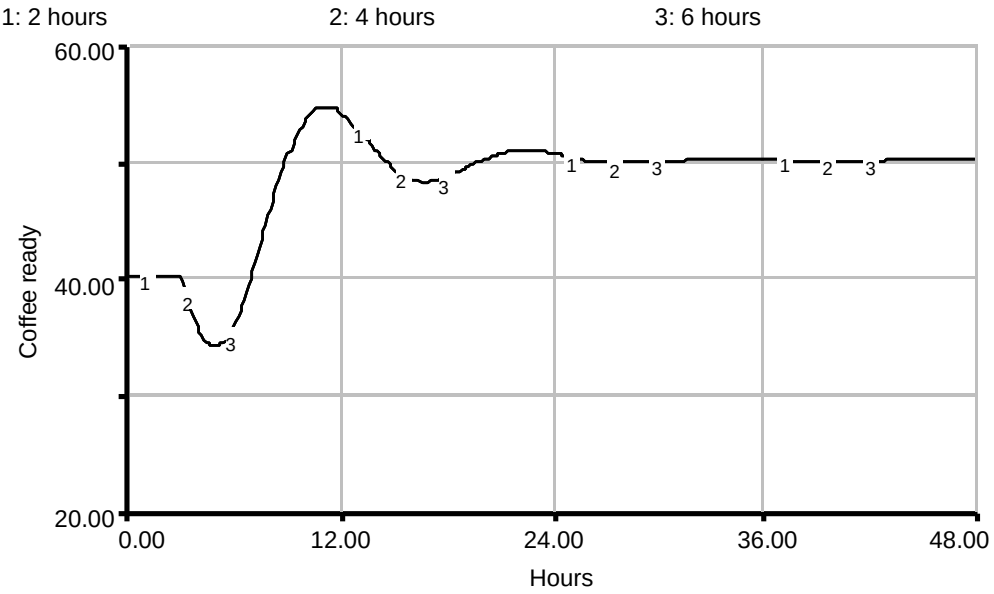


Figure 23: The effect of changes in "Average length of working"

A change in the value of "Average length of working" does not influence the amount of "Coffee ready." Let us again look at the behavior of "Workers" for an explanation. It is shown in Figure 24.

[16] Of course, in the real world, there could not be 2/3 of a worker working in the Coffeehouse. But a model is just a representation of the real world. In this case, when our primary purpose is to test the model for sensitivity, we do not have to be concerned about this problem. The manager would have to round up the number of workers to the next whole number.

[17] Again, the manager should just round up the equilibrium values to the next whole number.

[18] However, if we were to model the profits of the Coffeehouse, "Productivity" would certainly make a large effect.

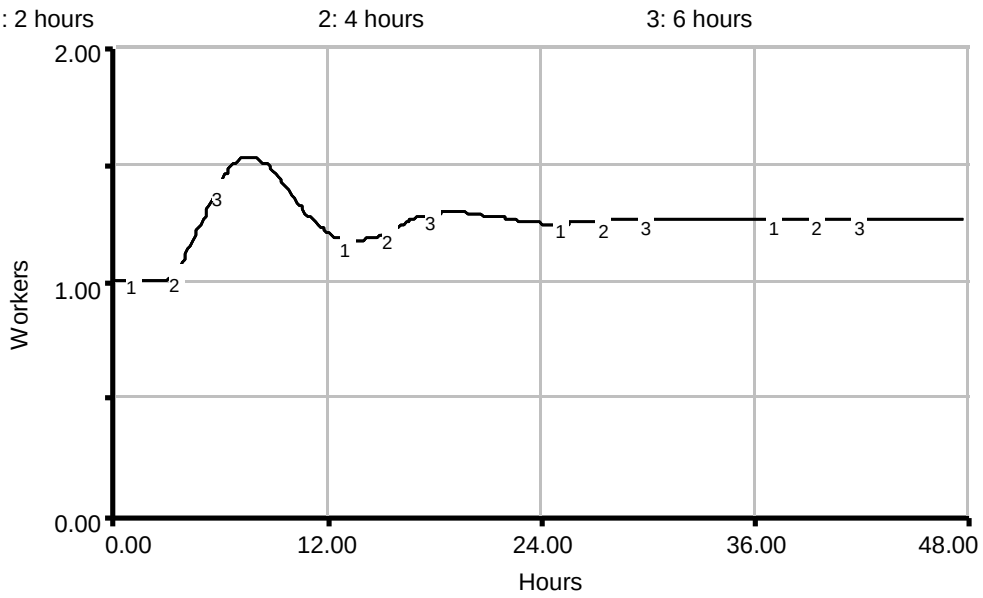


Figure 24: The effect of changes in "Average length of employment" on the number of "Workers"

Unlike "Productivity," changes in the "Average length of working" do not influence the number of "Workers" presently working in the Coffeehouse.[19] The reason is that "Average length of working" affects both flows into the stock of "Workers." Increasing the value of Average length of employment will simultaneously increase the value of "Coming to Work" and "Going Home," so the net flow into "Workers" will not change. In addition, when the number of "Workers" does not change, the amount of "Coffee ready" will not change either (assuming that all the other variables in the model remain unchanged).

Debrief

The exercise confirmed the conclusions established in the two explorations: changing parameter values makes a difference to the behavior of the model, but the overall behavior mode stays the same. A change in behavior corresponds to a change in structure. Also, some parameters cause a more significant or different change in the behavior than others. For example, compare Figures 19, 20, and 21. Changes in three different parameters ("Coffee coverage," "Time to correct workers," and "Productivity") can produce different changes in the behavior of the stock of "Coffee ready," or no change at all, as in the case of "Productivity."

[19] However, for large values of Average length of employment, starting at approximately 14 hours, the behavior starts to change—the oscillations become larger as Average length of employment increases. You should still keep in mind that the values in the sensitivity tests should always stay in a range that is possible in the real world. Fourteen hours does not seem to be a reasonable value for the number of hours that a worker spends in work.

Appendix: Model Documentation

Before simulating the following three models, please make sure that stocks can become negative and that flows that can change direction are set to be bi-directional. This can help to identify errors in the formulation of the model. For directions, please consult your STELLA user manual.

Lemonade Stand Model

Expected_lemonade_buying(t) = Expected_lemonade_buying(t - dt) + (Change_in_buying_expectations) * dt
INIT Expected_lemonade_buying = Buying_lemonade
DOCUMENT: The hourly demand for lemonade that Howard expects.
Units: cups/hour

INFLOWS:
Change_in_buying_expectations = (Buying_lemonade - Expected_lemonade_buying) / Time_to_average_lemonade_buying
DOCUMENT: The rate at which Howard's expectations about demand for lemonade change.
Units: (cups/hour)/hour

Lemonade_ready_in_stand(t) = Lemonade_ready_in_stand(t - dt) + (Making_lemonade - Selling) * dt
INIT Lemonade_ready_in_stand = Buying_lemonade * Lemonade_coverage
DOCUMENT: The number of cups of lemonade that Howard has ready in the stand.
Units: cups

INFLOWS:
Making_lemonade = MAX(Expected_lemonade_buying + Correction_in_amount_of_lemonade, 0)
DOCUMENT: The number of cups of lemonade that Howard prepares each hour.
Units: cups/hour

OUTFLOWS:
Selling = MIN(Buying_lemonade, Lemonade_ready_in_stand/DT)
DOCUMENT: The number of cups of lemonade that Howard sells each hour. Usually, it is equal to the demand for lemonade, Buying_lemonade. However, if the demand is higher than the lemonade he has ready, he will sell all the lemonade quickly, which is equal to Lemonade_ready_in_stand/DT.
Units: cups/hour

Buying_lemonade = 20 + STEP(5,1)
DOCUMENT: The hourly demand for lemonade.
Units: cups/hour

Correction_in_amount_of_lemonade = (Desired_amount_of_lemonade - Lemonade_

ready_in_stand) / Time_to_correct_amount_of_lemonade
DOCUMENT: The number of cups of lemonade that Howard prepares every hour as a result of a difference between the desired and the actual amount of lemonade.
Units: cups/hour

Desired_amount_of_lemonade = Lemonade_coverage * Expected_lemonade_buying
DOCUMENT: The number of cups of lemonade that Howard would like to have at the stand. It is equal to the demand for lemonade he expects times the Lemonade coverage.
Units: cups

Lemonade_coverage = 2
DOCUMENT: Lemonade coverage determines the number of hours worth of lemonade that Howard wants to keep ready at the stand at all times.
Units: hours

Time_to_average_lemonade_buying = 1
DOCUMENT: The time it takes Howard to recognize a permanent change in demand for lemonade from random fluctuations.
Units: hours

Time_to_correct_amount_of_lemonade = 1.5
DOCUMENT: The time in which Howard attempts to correct a difference between the desired and the actual amount of lemonade.
Units: hours

Epidemics Model

Healthy_People(t) = Healthy_People(t - dt) + (Recovering_Rate - Catching_Illness) * dt
INIT Healthy_People = 100 - Sick_People
DOCUMENT: The number of healthy people in the population.
Units: people

INFLOWS:
Recovering_Rate = Sick_People/Duration_of_Illness
DOCUMENT: The rate at which people recover from the illness.
Units: people/month

OUTFLOWS:
Catching_Illness = Healthy_People * Probability_of_Contact_with_Sick_People *
Population_Interactions * Probability_of_Catching_Illness
DOCUMENT: The rate at which people catch the illness.
Units: people/month

Sick_People(t) = Sick_People(t - dt) + (Catching_Illness - Recovering_Rate) * dt
INIT Sick_People = 1
DOCUMENT: The number of sick people in the population.

Units: people

INFLOWS:
Catching_Illness = Healthy_People * Probability_of_Contact_with_Sick_People * Population_Interactions * Probability_of_Catching_Illness
DOCUMENT: The rate at which people catch the illness.
Units: people/month

OUTFLOWS:
Recovering_Rate = Sick_People/Duration_of_Illness
DOCUMENT: The rate at which people recover from the illness.
Units: people/month

Duration_of_Illness = .5
DOCUMENT: The duration of the illness.
Units: months

Population_Interactions = 10
DOCUMENT: The number of interactions, or contacts, that a person has with other people during a month.
Units: (people/people)/month

Probability_of_Catching_Illness = .5
DOCUMENT: The probability that a person will become infected when in contact with a sick person.
Units: dimensionless

Probability_of_Contact_with_Sick_People = Sick_People / (Sick_People + Healthy_People)
DOCUMENT: The probability that a person will meet a sick person. It is equal to the ratio of the number of sick people to the total population.
Units: dimensionless

Coffeehouse Model

Coffee_ready(t) = Coffee_ready(t - dt) + (Making_Coffee - Selling) * dt
INIT Coffee_ready = Coffee_Coverage * Buying_Coffee
DOCUMENT: The number of cups of coffee ready in the Coffeehouse.
Units: cups

INFLOWS:
Making_Coffee = Workers * Productivity
DOCUMENT: The number of cups of coffee that the Coffeehouse workers prepare each hour. It is equal to the number of workers times their average productivity.
Units: cups/hour

OUTFLOWS:
Selling = MIN(Buying_Coffee, Coffee_ready/DT)
DOCUMENT: The number of cups of coffee sold in the Coffeehouse every hour. Usually, it is equal to the demand for coffee, Buying_Coffee. However, if the demand is higher then the coffee that is ready, the workers will sell all the coffee quickly, which is equal to Coffee_ready/DT.
Units: cups/hour

Expected_Coffee_Buying(t) = Expected_Coffee_Buying(t - dt) + (Change_in_Buying_Expectations) * dt
INIT Expected_Coffee_Buying = Buying_Coffee
DOCUMENT: The hourly demand for coffee that the workers expect.
Units: cups/hour

INFLOWS:
Change_in_Buying_Expectations = (Buying_Coffee-Expected_Coffee_Buying) / Time_to_average_coffee_buying
DOCUMENT: The rate at which the workers' expectations about demand for coffee change.
Units: (cups/hour)/hour

Workers(t) = Workers(t - dt) + (Coming_to_Work - Going_Home) * dt
INIT Workers = Desired_Workers
DOCUMENT: The number of workers currently working at the Coffeehouse.
Units: workers

INFLOWS:
Coming_to_Work = Correction_for_Workers + (Workers/Average_Length_of_Working)
DOCUMENT: The rate at which workers come to work.
Units: workers/hour

OUTFLOWS:
Going_Home = Workers/Average_Length_of_Working
DOCUMENT: The rate at which the workers leave the Coffeehouse to go home and study.
Units: workers/hour

Average_Length_of_Working = 4
DOCUMENT: The number of hours that a worker spends in the Coffeehouse.
Units: hours

Buying_Coffee = 20 + STEP(5,3)
DOCUMENT: The hourly demand for coffee.
Units: cups/hour

Coffee_Coverage = 2
DOCUMENT: Coffee coverage determines the number of hours worth of coffee that Howard wants the workers to keep at the Coffeehouse at all times.
Units: hours

Correction_for_Workers = (Desired_Workers-Workers)/Time_to_Correct_Workers
DOCUMENT: The number of workers who come to the Coffeehouse as a result of a difference between the desired and the actual number of workers.
Units: workers/hour

Correction_in_Amount_of_Coffee = (Desired_Amount_of_Coffee - Coffee_ready) /
Time_to_Correct_Amount_of_Coffee
DOCUMENT: The number of cups of coffee that the workers prepare every hour as a result of a difference between the desired and actual amount of coffee.
Units: cups/hour

Desired_Amount_of_Coffee = Coffee_Coverage * Expected_Coffee_Buying
DOCUMENT: The number of cups of coffee that the workers would like to have at the Coffeehouse. It is equal to the demand for coffee that they expect times the Coffee coverage.
Units: cups

Desired_making_of_coffee = Correction_in_Amount_of_Coffee + Expected_Coffee_
Buying
DOCUMENT: The rate at which the workers would like to make coffee.
Units: cups/hour

Desired_Workers = Desired_making_of_coffee/Productivity
DOCUMENT: The number of workers that Howard wants to be working at the Coffeehouse.
Units: workers

Productivity = 20
DOCUMENT: The number of cups of coffee that a worker makes in an hour.
Units: cups/(worker/hour)

Time_to_average_coffee_buying = 2
DOCUMENT: The time it takes the workers to recognize a permanent change in demand for coffee from random fluctuations.
Units: hours

Time_to_Correct_Amount_of_Coffee = 1
DOCUMENT: The time in which the workers attempt to correct a difference between the desired and the actual amount of coffee.
Units: hours

Time_to_Correct_Workers = 3
DOCUMENT: The time in which Howard wants to make more workers come to work.
Units: hours

Vensim Examples:
An Introduction to Sensitivity Analysis

By Aaron Diamond and Nathaniel Choge
October 2001

3.1: Lemonade Stand

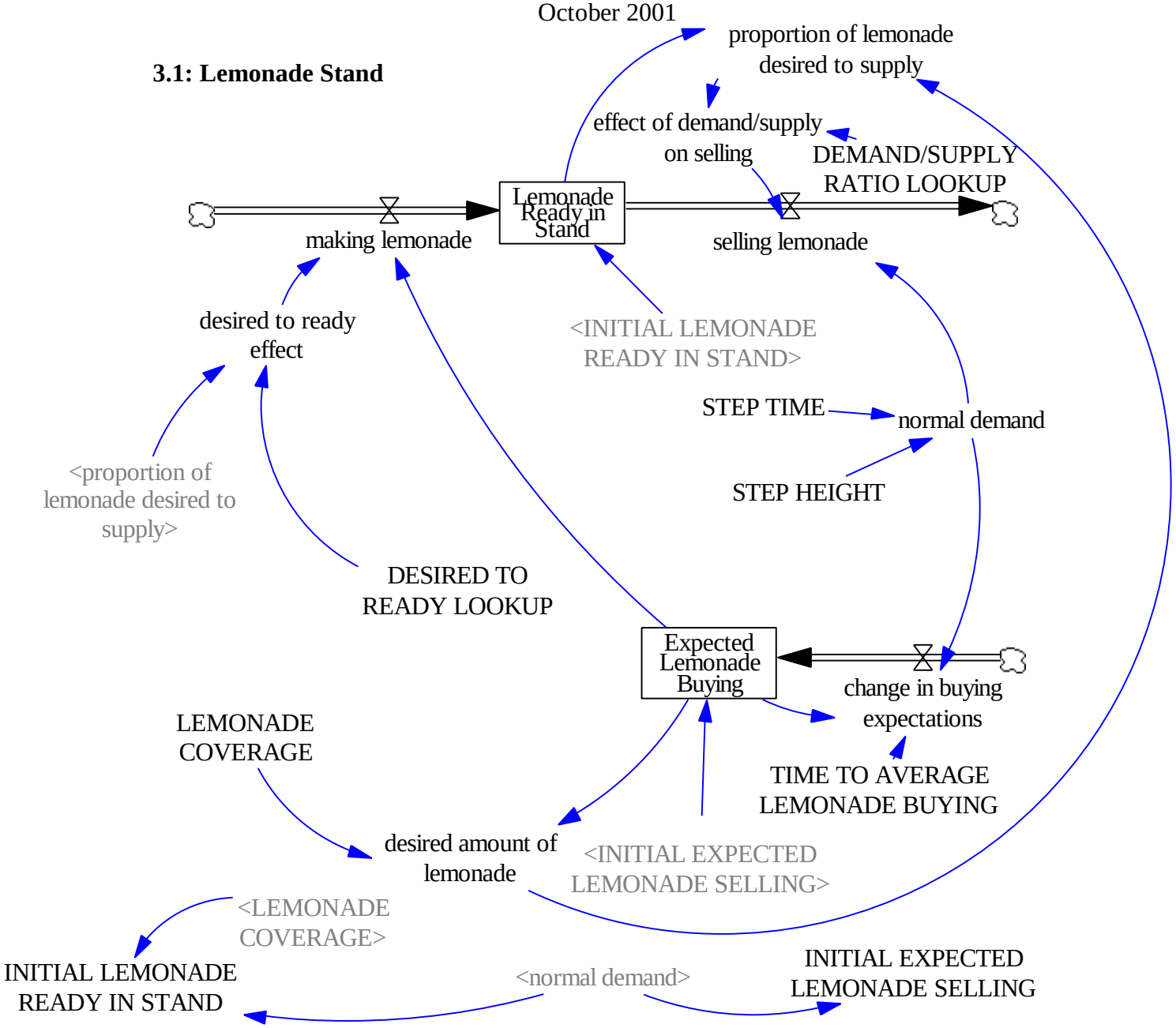


Figure 25: Vensim Equivalent of Figure 1: Model of the Lemonade stand
NOTE: Differences between the Vensim and Stella Lemonade Stand Model are described below Documentation.

Documentation for Lemonade Stand Model:

(01) change in buying expectations=(normal demand-Expected Lemonade Buying)/TIME TO AVERAGE LEMONADE BUYING
Units: (cups/hour)/hour
The rate at which Howard's expectations about demand for lemonade change.

(02) "DEMAND/SUPPLY RATIO LOOKUP"(
[(0,0)-(10,10)],(0.33,3),(0.5,2),(1,1),(2,0.5),(3,0.33),(5,0.2))
Units: dmnl

(03) desired amount of lemonade=LEMONADE COVERAGE*Expected Lemonade Buying
Units: cups
The number of cups of lemonade that Howard would loke to have at the stand. It is equal to the demand for lemonade he expects times the LEMONADE COVERAGE.

(04) desired to ready effect=DESIRED TO READY LOOKUP(proportion of lemonade desired to supply)
Units: dmnl

(05) DESIRED TO READY LOOKUP(
[(0,0)-(4,3)],(0,0.05),(1,1),(2,2),(2.5,2.4),(3,2.8),(3.5,3),(4,3))
Units: dmnl

(06) "effect of demand/supply on selling"=
"DEMAND/SUPPLY RATIO LOOKUP"(proportion of lemonade desired to supply)
Units: dmnl

(07) Expected Lemonade Buying= INTEG (change in buying expectations, INITIAL EXPECTED LEMONADE SELLING)
Units: cups/hour
The hourly demand for lemonade that Howard expects.

(08) FINAL TIME = 8
Units: hour
The final time for the simulation.

(09) INITIAL EXPECTED LEMONADE SELLING=normal demand
Units: cups/hour

(10) INITIAL LEMONADE READY IN STAND=LEMONADE COVERAGE*normal demand

Units: cups

(11) INITIAL TIME = 0
Units: hour
The initial time for the simulation.

(12) LEMONADE COVERAGE=2
Units: hour
LEMONADE COVERAGE determines the number of hours worth of lemonade that Howard wants to keep ready at the stand at all
times.

(13) Lemonade Ready in Stand= INTEG (+making lemonade-selling lemonade, INITIAL LEMONADE READY IN STAND)
Units: cups
The number of cups of lemonade that Howard has ready in the stand.

(14) making lemonade=desired to ready effect*Expected Lemonade Buying
Units: cups/hour
The number of cups of lemonade that Howard prepares each hour.

(15) normal demand=20+STEP(STEP HEIGHT,STEP TIME)
Units: cups/hour
The hourly demand for lemonade.

(16) proportion of lemonade desired to supply=desired amount of lemonade/Lemonade Ready in Stand
Units: dmnl
Calculates the proportion of demand for lemonade (units cups/hour) to supply of lemonade (divided by TIME TO SELL CUP OF LEMONADE to achieve units cups/hour).

(17) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.

(18) selling lemonade="effect of demand/supply on selling"*normal demand
Units: cups/hour

(19) STEP HEIGHT=5
Units: dmnl

(20) STEP TIME=1
Units: dmnl

(21) TIME STEP = 0.0625
Units: hour
The time step for the simulation.

(22) TIME TO AVERAGE LEMONADE BUYING=1
Units: hour
The time it takes Howard to recognize a permanent change in demand for lemonade from random fluctuations.

Differences Between the Vensim and Stella Lemonade Stand Model:

1. Below the large model is a smaller model that shows the calculation of the INITIAL stock variables. The model is split to avoid confusion and emphasize the dynamic parts of the system.
2. The variable "buying lemonade" in the Stella version has been changed to "normal demand" in the Vensim paper to more explicitly show that this variable captures the demand for lemonade. The "STEP HEIGHT" and "STEP TIME" are constants that are seperately entered into the "normal demand" equation and are the inputs to the STEP function in the "buying lemonade" equation.
3. To avoid poor modeling practices, the Vensim model does not use MIN/MAX functions.
    a. **making lemonade-**Instead of inputting the equation "MAX(Expected Lemonade Buying + correction in amount of lemonade, 0)," we have constructed an effect variable, called "desired to ready effect" that takes the proportion of the "lemonade desired" to the "Lemonade Ready in Stand" (called "proportion of lemonade desired to supply") and outputs the result of the "DESIRED TO READY LOOKUP". The flow "making lemonade" is computed by multiplying the effect by "Expected Lemonade Buying". The lookup is constructed so that when the output of "proportion of lemonade desired to supply" is multiplied by "Expected Lemonade Buying," the result is equivalent to adding "Expected Lemonade Buying" and "correction in amount of lemonade". The lookup curve is linear until about 2.5 times the current level of lemonade is desired. We assume that the stand is not able to more than triple their production rate at any time, so the curve approaches 3, and for lemonade demands larger than about 3.5 times the current level of lemonade, the stand will only be able to make lemonade at 3 times current expectations, and the lookup is function remains flat at 3.The lookup is shown below:



Figure 26: "CORRECTION/EXPECTED BUYING LOOKUP"

b. **selling lemonade-** Instead of inputting the equation "MIN(NORMAL DEMAND, Lemonade Ready in Stand/DT), we compute the ratio of the "desired amount of lemonade" to the "Lemonade Ready in Stand" to obtain a value of demand/supply (called "proportion of lemonade desired to supply"). We know that if the demand is greater than the supply of lemonade (i.e., if the proportion is greater than 1) then the sales of lemonade will slow, since much of the stand's efforts will be toward making lemonade. If the proportion is smaller than 1, however, then the stand's efforts will be toward selling lemonade, and their "selling" rate will increase to a maximum value. The "DEMAND/SUPPLY RATIO LOOKUP" captures this effect, and is shown below:

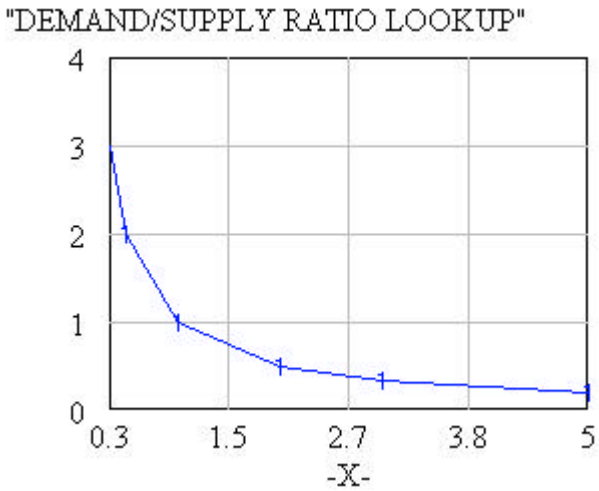


Figure 27: DEMAND/SUPPLY RATIO LOOKUP

The appropriate output of the lookup is computed by the "effect of demand/supply on selling" variable, which is multiplied by the "NORMAL DEMAND" to compute "selling."

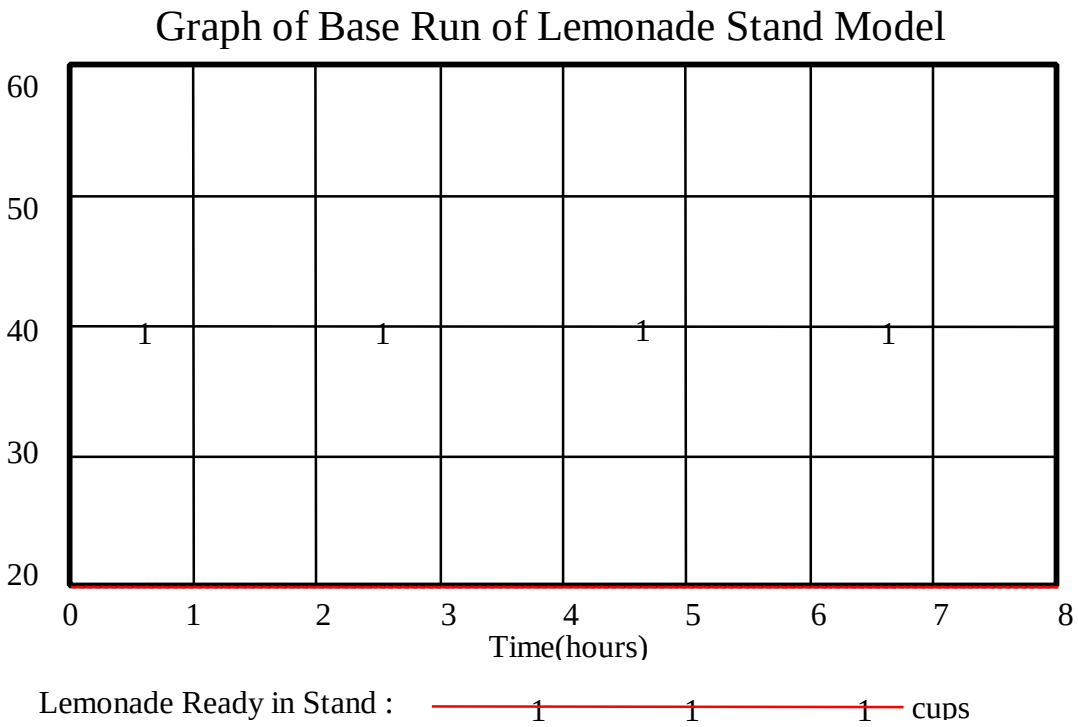


Figure 28: Vensim Equivalent of Figure 2: Base run of the Lemonade Stand model

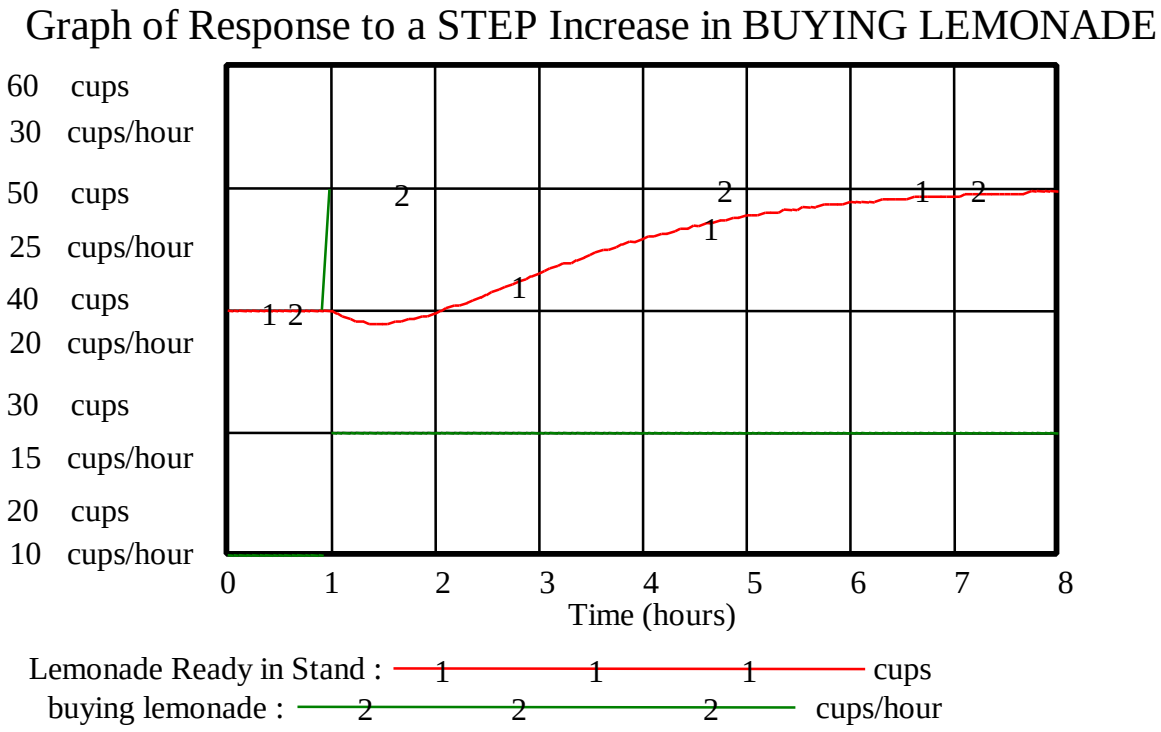


Figure 29: Vensim Equivalent of Figure 3: The system responding to a step increase in "buying lemonade."

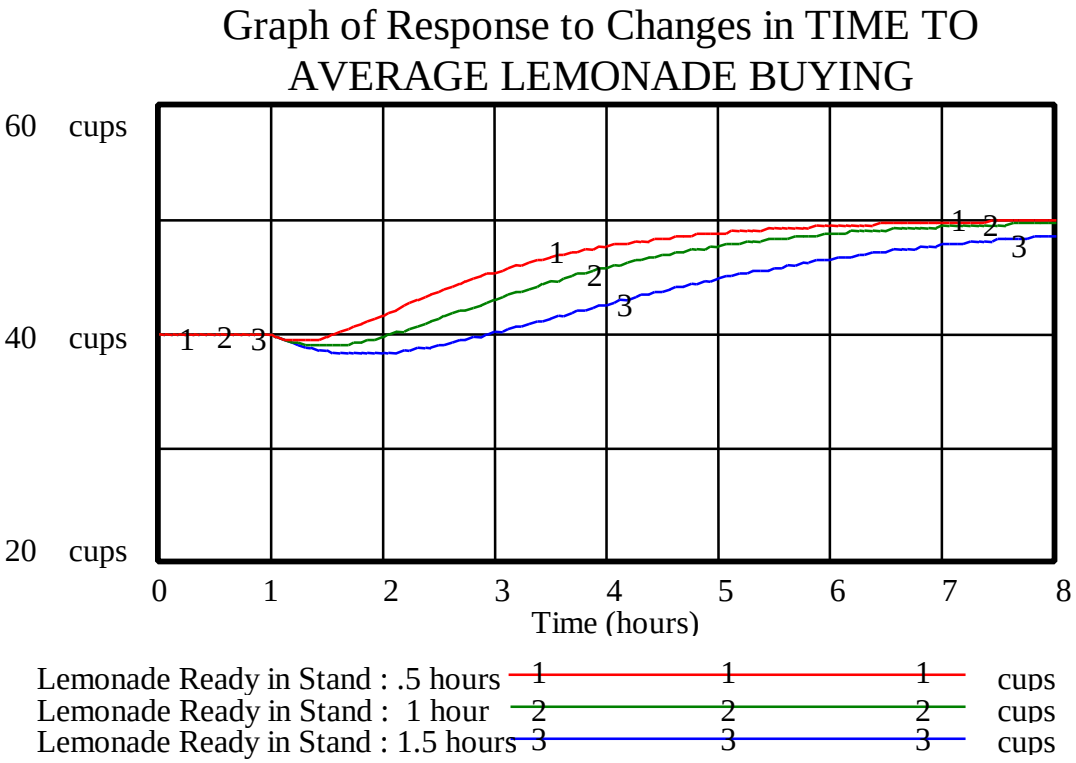


Figure 30: Vensim Equivalent of Figure 4: The effect of changes in "TIME TO AVERAGE LEMONADE BUYING"

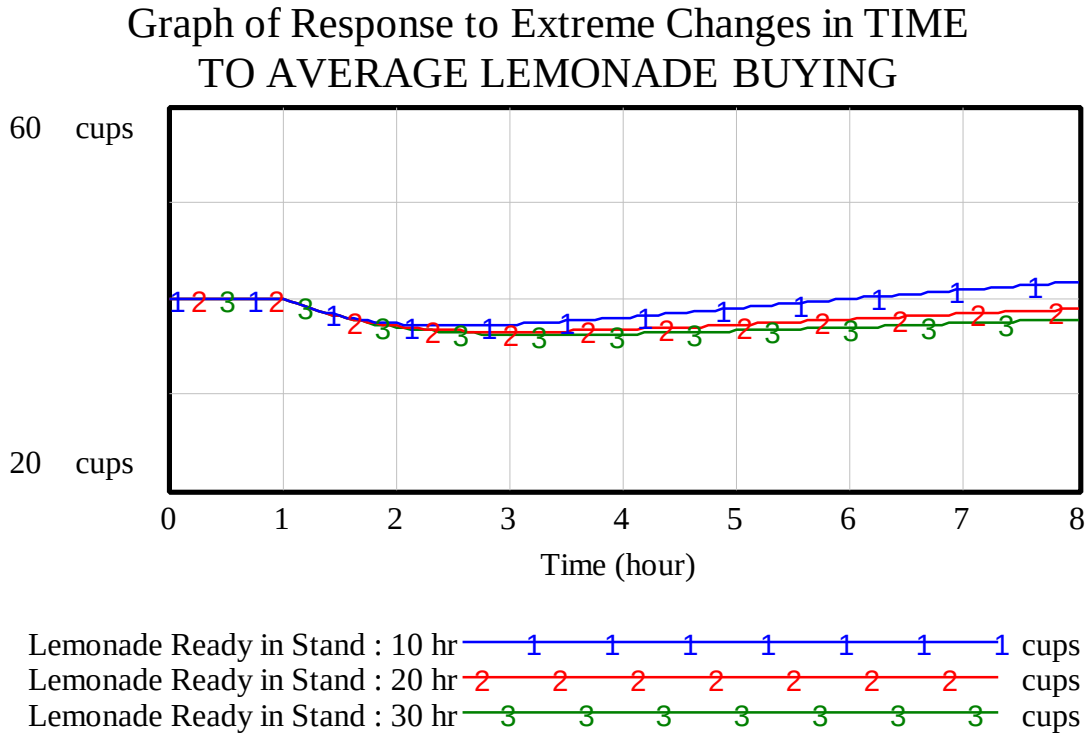


Figure 31: Extra graph. Effect of extreme changes to "TIME TO AVERAGE LEMONADE BUYING"

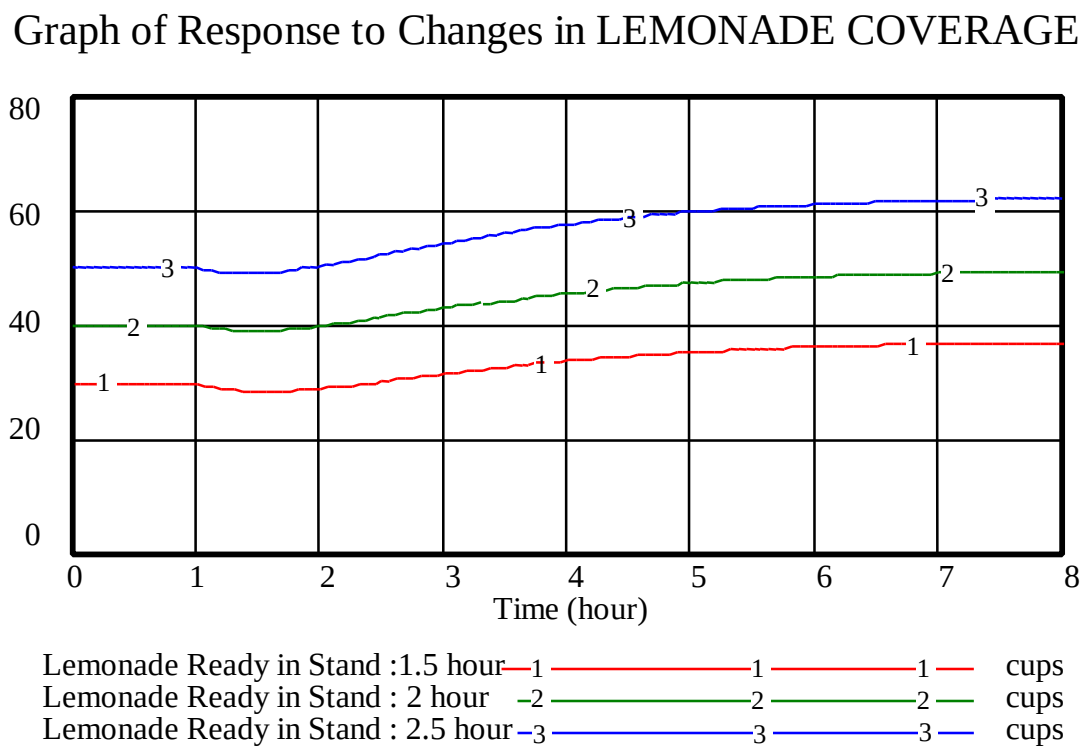


Figure 32: Vensim Equivalent of Figure 6: The effect of changes in 'LEMONADE COVERAGE"

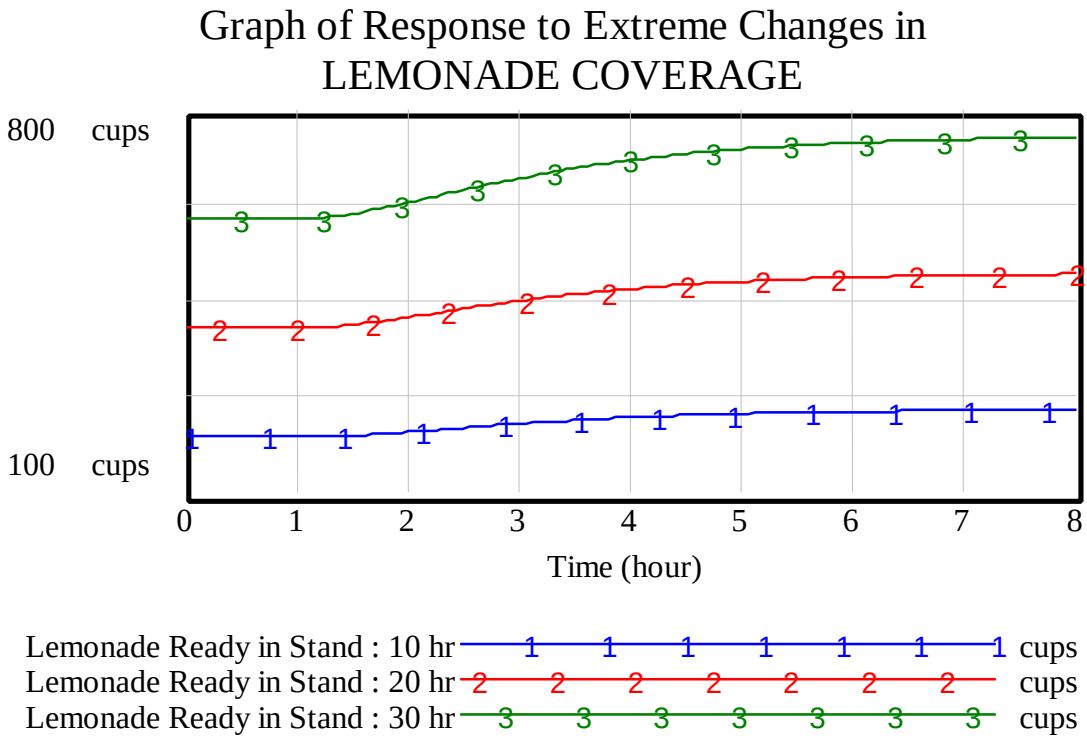


Figure 33: Extra Graph. The effect of extreme changes in "LEMONADE COVERAGE"

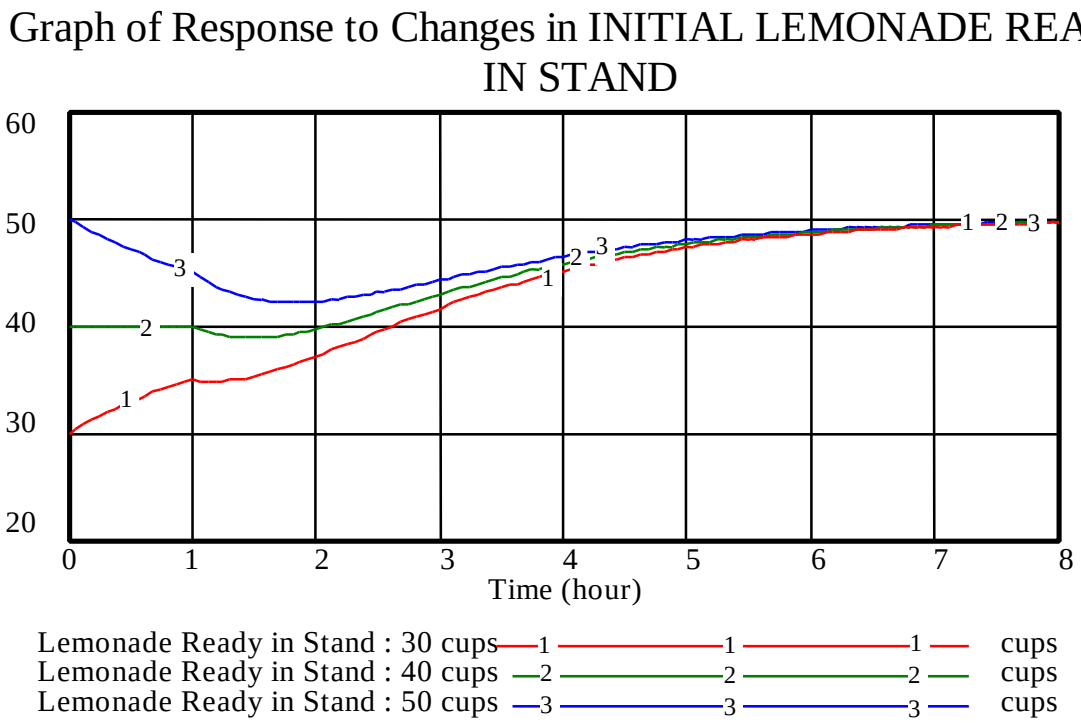


Figure 34: Vensim Equivalent of Figure 7: The effect of changes in “INITIAL LEMONADE READY IN STAND”

3.2 Exploration 2: Epidemics

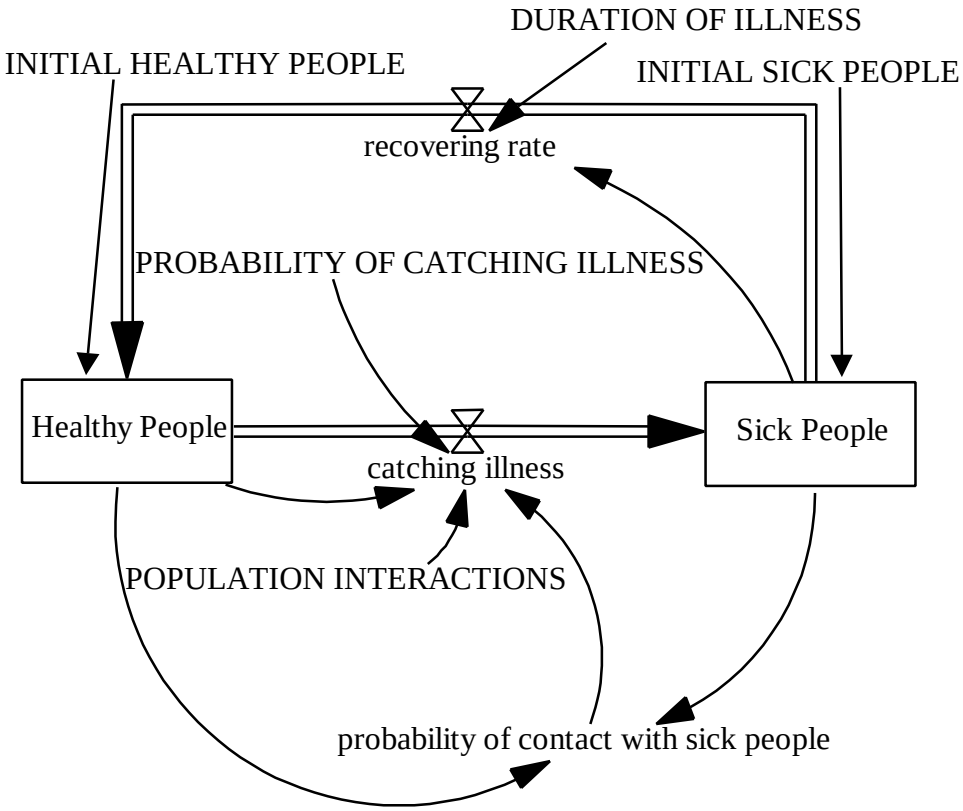


Figure 35: Vensim Equivalent of Figure 8: The Epidemics Model

Documentation for Epidemics Model

(01) catching illness=Healthy People*PROBABILITY OF CATCHING ILLNESS *probability of contact with sick people*POPULATION INTERACTIONS
Units: people/Month
The rate at which people catch the illness.

(02) DURATION OF ILLNESS=0.5
Units: Month
The duration of the illness.

(03) FINAL TIME =6
Units: Month
The final time for the simulation.

(04) Healthy People= INTEG (recovering rate-catching illness, INITIAL HEALTHY PEOPLE)
Units: people
The number of healthy people in the population.

(05) INITIAL HEALTHY PEOPLE=99
Units: people

(06) INITIAL SICK PEOPLE=1
Units: people

(07) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(08) POPULATION INTERACTIONS=10
Units: (people/people)/Month
The number of interactions, or contacts, that a person has with other people during a month.

(09) PROBABILITY OF CATCHING ILLNESS=0.5
Units: dmnl
The probability that a person will become infected when in contact with a sick person.

(10) probability of contact with sick people=Sick People/(Sick People+Healthy People)
Units: dmnl
The probability that a person will meet a sick person. It is equal to the ratio of the number of sick people to the total population.

(11) recovering rate=Sick People/DURATION OF ILLNESS
Units: people/Month
The rate at which people recover from the illness.

(12) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(13) Sick People= INTEG (catching illness-recovering rate, INITIAL SICK PEOPLE)
Units: people
The number of sick people in the population.

(14) TIME STEP = .0625
Units: Month
The time step for the simulation.

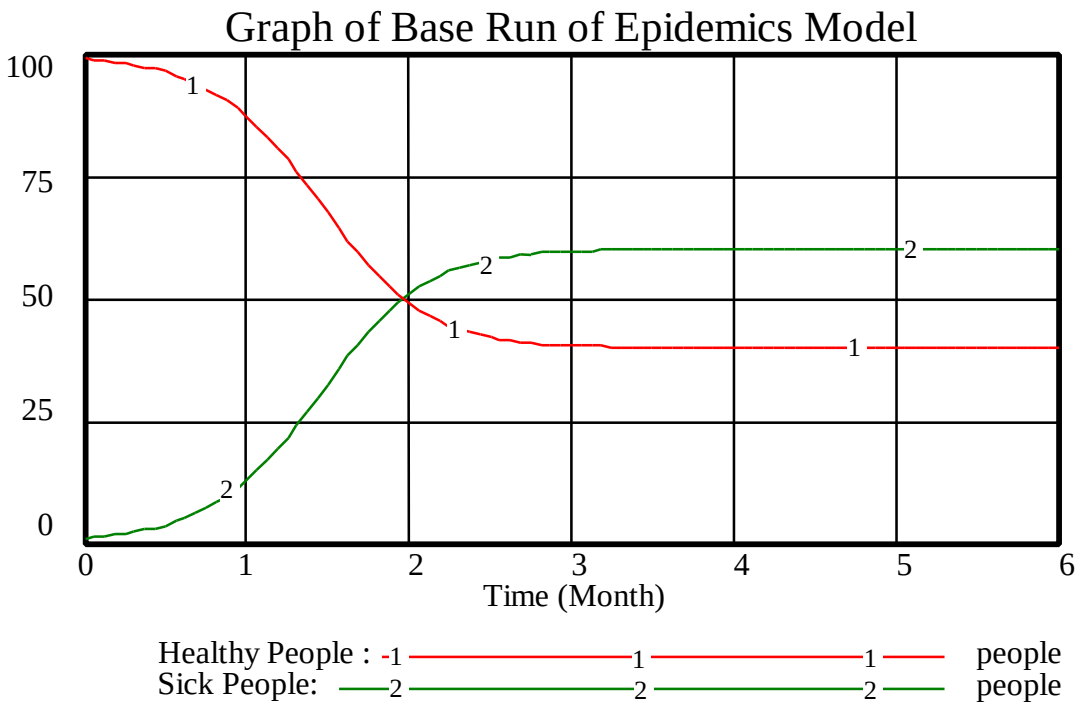


Figure 36: Vensim Equivalent of Figure 9: Base run of the Epidemics Model

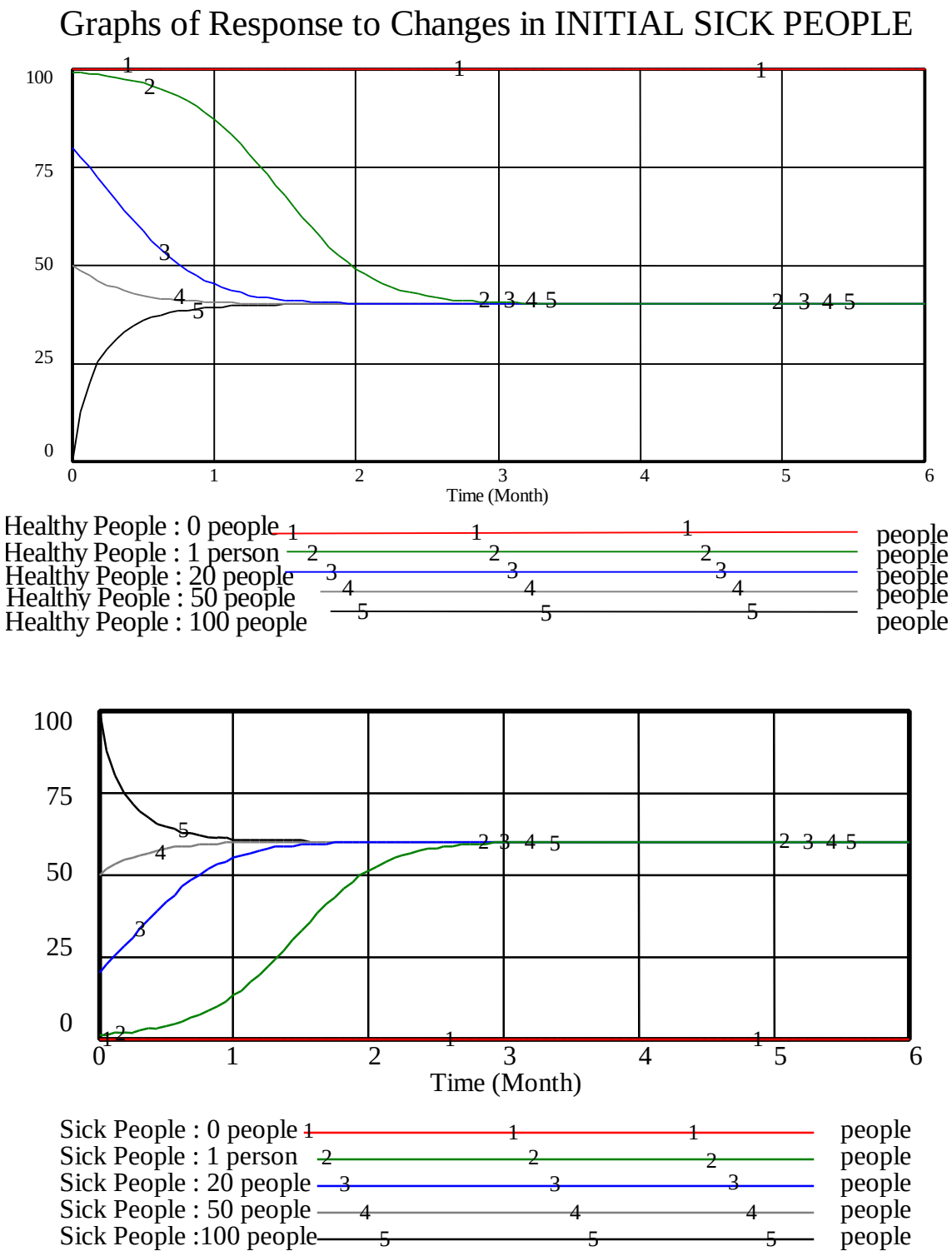
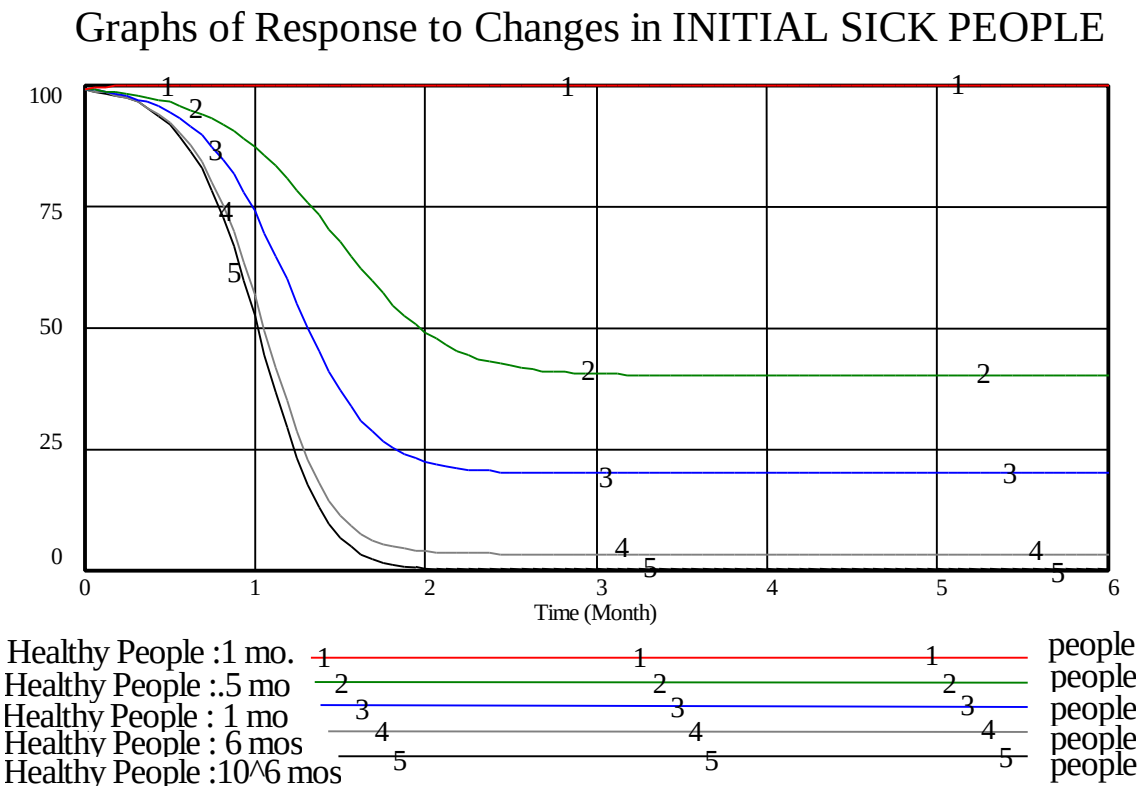


Figure 37: Vensim equivalent of Figure 10: The effect of changes in the initial value of Sick People



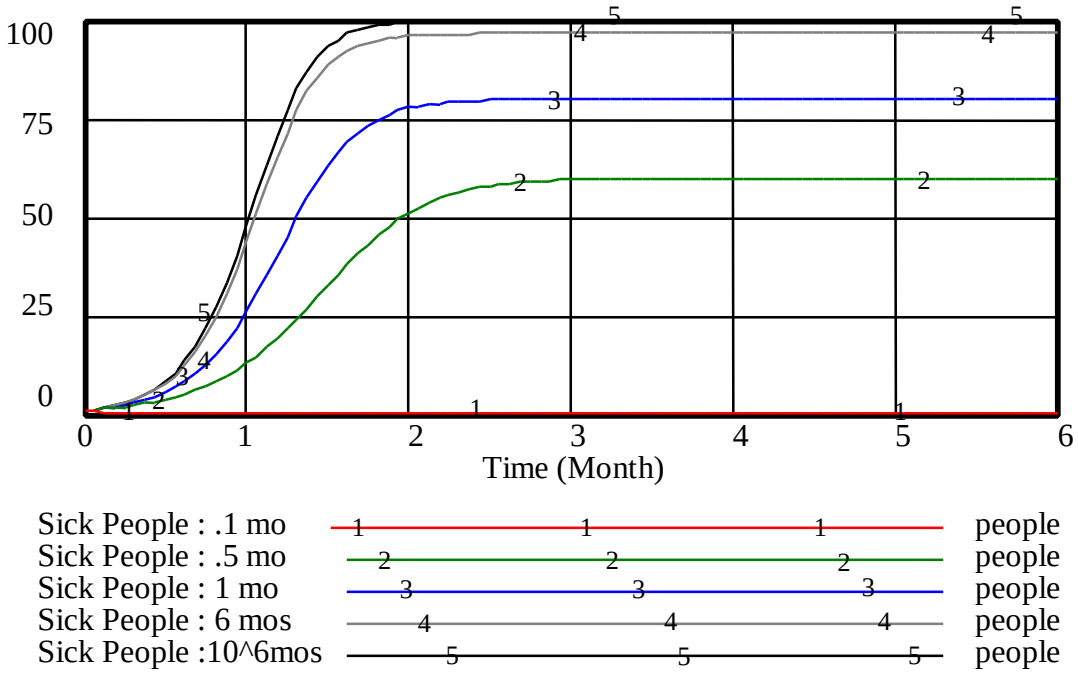


Figure 38: Vensim equivalent of Figure 11: The effect of changes in "DURATION OF ILLNESS"

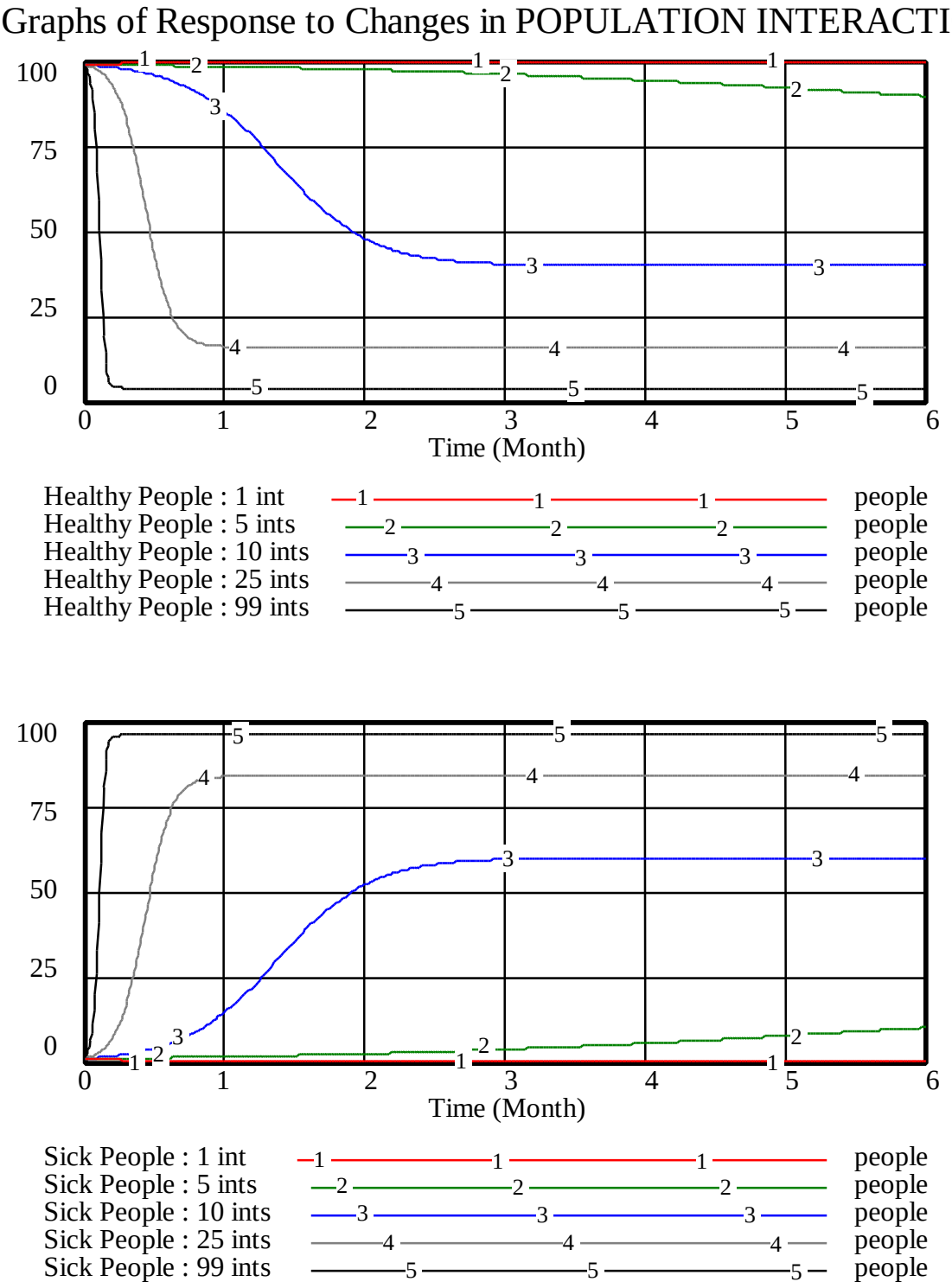


Figure 39: Vensim Equivalent to Figure 12: The effect of changes in "POPULATION INTERACTIONS'

4. Independent Exploration: Coffeehouse

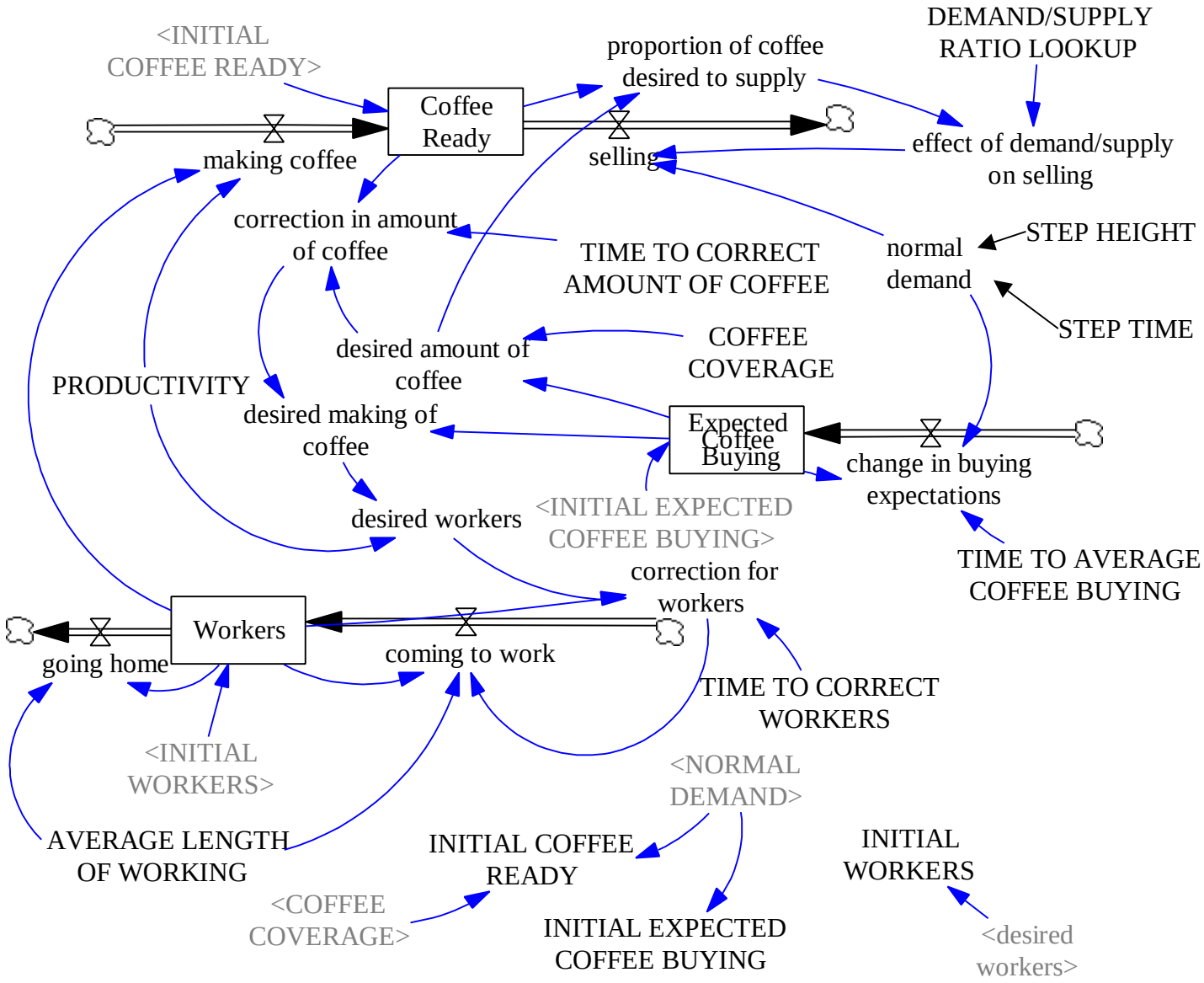


Figure 40: Vensim equivalent of Figure 13: The Coffeehouse Model
NOTE: Differences between the Vensim and Stella Coffeehouse Model are described below Documentation.

Documentation for the Coffeehouse Model

(01) AVERAGE LENGTH OF WORKING=4
Units: hour
The number of hours that a worker spends in the Coffeehouse.

(02) change in buying expectations=(NORMAL DEMAND-Expected Coffee Buying)/TIME TO AVERAGE COFFEE BUYING
Units: (cups/hour)/hour
The rate at which the workers' expectations about demand for coffee change.

(03) COFFEE COVERAGE=2
Units: hour
Coffee coverage determines the number of hours worth of coffee that Howard wnats the workers to keep at the Coffeehouse at all times.

(04) Coffee Ready= INTEG (making coffee-selling, INITIAL COFFEE READY)
Units: cups
The number of cups of coffee ready in the Coffeehouse.

(05) coming to work=
correction for workers+(Workers/AVERAGE LENGTH OF WORKING)
Units: workers/hour
The rate at which workers come to work.

(06) correction for workers=
(desired workers-Workers)/TIME TO CORRECT WORKERS
Units: workers/hour
The number of workers who come to the Coffeehouse as a result of a difference between the desired and the actual number of workers.

(07) correction in amount of coffee=(desired amount of coffee-Coffee Ready)/TIME TO CORRECT AMOUNT OF COFFEE
Units: cups/hour
The number of cups of coffee that the workers prepare every hour as a result of a difference between that desired and actual amount of coffee.

(08) DEMAND/SUPPLY RATIO LOOKUP([(0,0)-(10,10)],(0.33,3),(0.5,2),(1,1),(2,0.5),(3,0.33),(5,0.2))
Units: dmnl

(09) desired amount of coffee=COFFEE COVERAGE*Expected Coffee Buying
Units: cups
The number of cups of coffee that the workers would like to have at the Coffeehouse. It is equal to the demand for coffee that they expect times the COFFEE COVERAGE.

(10) desired making of coffee=correction in amount of coffee+Expected Coffee Buying
Units: cups/hour
The rate at which the workers would like to make coffee.

(11) desired workers=desired making of coffee/PRODUCTIVITY
Units: workers
The number of workers that Howard wants to be working at the Coffeehouse.

(12) "effect of demand/supply on selling"=
"DEMAND/SUPPLY RATIO LOOKUP"(proportion of coffee desired to supply)
Units: dmnl

(13) Expected Coffee Buying= INTEG (change in buying expectations, INITIAL EXPECTED COFFEE BUYING)
Units: cups/hour
The hourly demand for coffee that the workers expect.

(14) FINAL TIME = 48
Units: hour
The final time for the simulation.

(15) going home=Workers/AVERAGE LENGTH OF WORKING
Units: workers/hour
The rate at which the workers leave the Coffeehouse to go home and study.

(16) INITIAL COFFEE READY=COFFEE COVERAGE*NORMAL DEMAND
Units: cups

(17) INITIAL EXPECTED COFFEE BUYING=NORMAL DEMAND
Units: cups/hour

(18) INITIAL TIME = 0
Units: hour
The initial time for the simulation.

(19) INITIAL WORKERS=desired workers
Units: workers

(20) making coffee=Workers*PRODUCTIVITY
Units: cups/hour
The number of cups of coffee that the coffhouse workers prepare each hour. It is equal to the number of workers times their average productivity.

(21) NORMAL DEMAND=20+STEP(STEP HEIGHT,STEP TIME)
Units: cups/hour
The hourly demand for coffee.

(22) PRODUCTIVITY=20
Units: (cups/hour)/workers
The number of cups of coffee that a worker makes in an hour.

(23) proportion of coffee desired to supply=desired amount of coffee/Coffee Ready
Units: dmnl

(24) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.

(25) selling=NORMAL DEMAND*"effect of demand/supply on selling"
Units: cups/hour

(26) STEP HEIGHT=5
Units: dmnl

(27) STEP TIME=3
Units: dmnl

(28) TIME STEP = 0.125
Units: hour
The time step for the simulation.

(29) TIME TO AVERAGE COFFEE BUYING=2
Units: hour
The time it takes the workers to recognize a permanent change in demand for coffee from random fluctuations.

(30) TIME TO CORRECT AMOUNT OF COFFEE=1
Units: hour
The time in which the workers attempt to correct a difference between the desired and the actual amount of coffee.

(31) TIME TO CORRECT WORKERS=3
Units: hour
The time in which Howard wants to make more workers come to work.

(32) Workers= INTEG (coming to work-going home,
INITIAL WORKERS)
Units: workers
Thenumber of workers currently working at the Coffeehouse.

Differences Between the Vensim and Stella Coffeehouse Model:

1. Below the large model is a smaller model that shows the calculation of the INITIAL stock variables. The model is split to avoid confusion and emphasize the dynamic parts of the system.
2. The variable "Buying Coffee" in the Stella version has been changed to "normal demand" in the Vensim paper to more explicitly show that this constant captures the demand for coffee. The "STEP HEIGHT" and "STEP TIME" are constants that are seperately entered into the "normal demand" equation.
3. To avoid poor modeling practices, the Vensim model does not use MIN/MAX functions.

selling lemonade- Instead of inputting the equation "MIN(NORMAL DEMAND, Coffee Ready/DT), we compute the ratio of the "desired amount of coffee" to the "Coffee Ready" to obtain a value of demand/supply (called "proportion of coffee desired to supply"). We know that if the demand is greater than the supply of coffee (i.e., if the proportion is greater than 1) then the sales of coffee will slow, since much of the Coffeehouse's efforts will be toward making coffee. If the proportion is smaller than 1, however, then the coffeehouse's efforts will be toward selling coffee, and their "selling" rate will increase to a maximum value. The "DEMAND/SUPPLY RATIO LOOKUP" captures this effect, and is shown below:

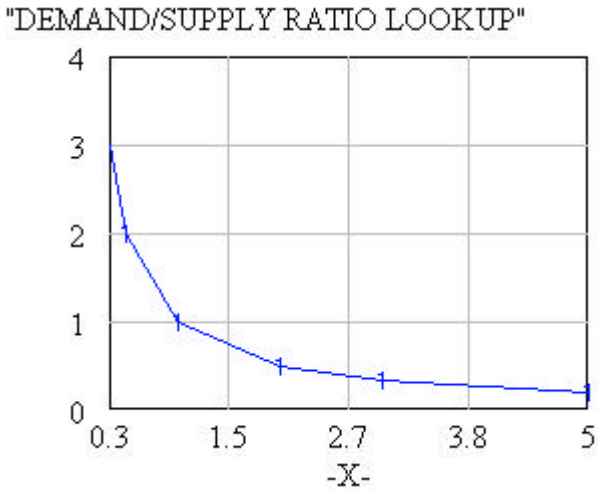


Figure 41: DEMAND?SUPPLY RATIO LOOKUP

The appropriate output of the lookup is computed by the “effect of demand/supply on selling” variable, which is multiplied by the “NORMAL DEMAND” to compute “selling.”

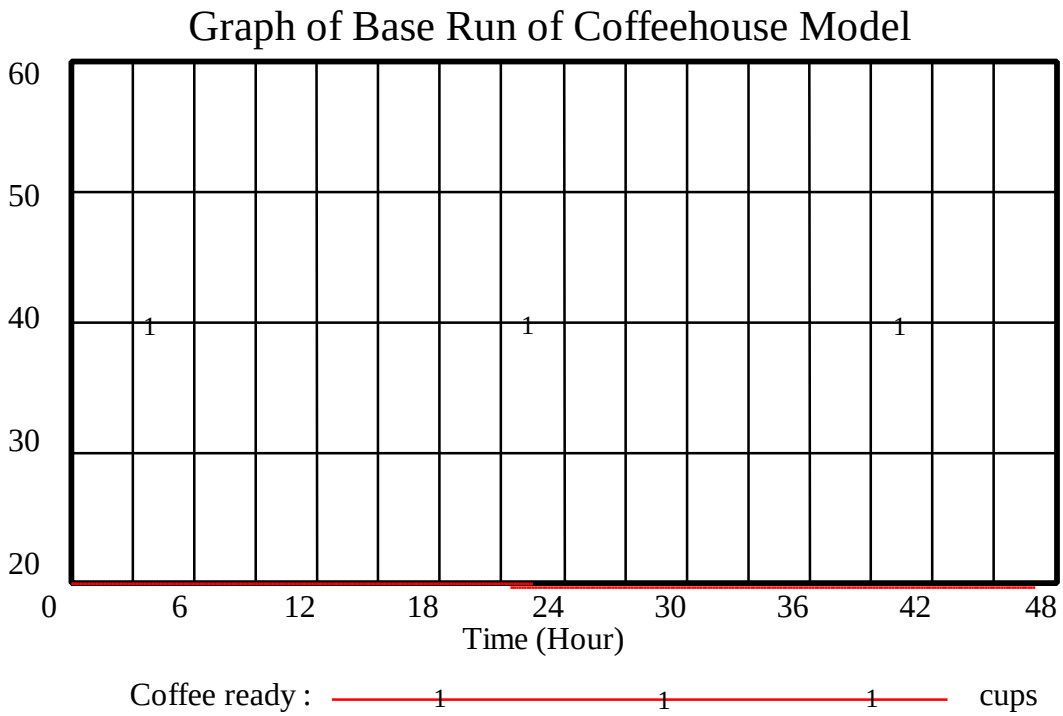


Figure 42: Vensim equivalent of Figure 14: Base run of the Coffeehouse model

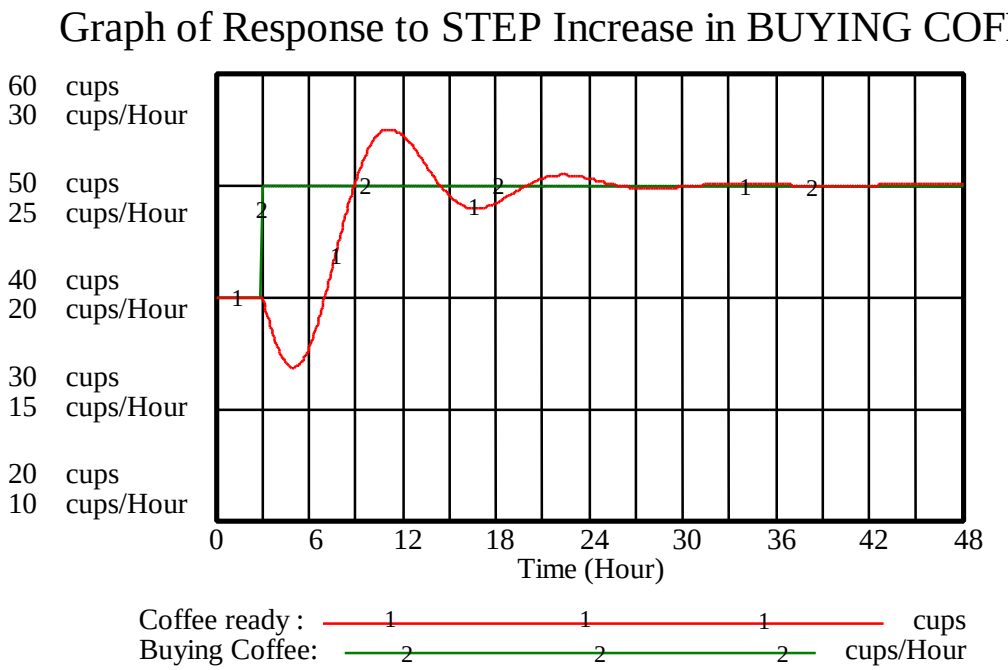


Figure 43: Vensim equivalent of Figure 15: The system responding to a step increase in "BUYING COFFEE"

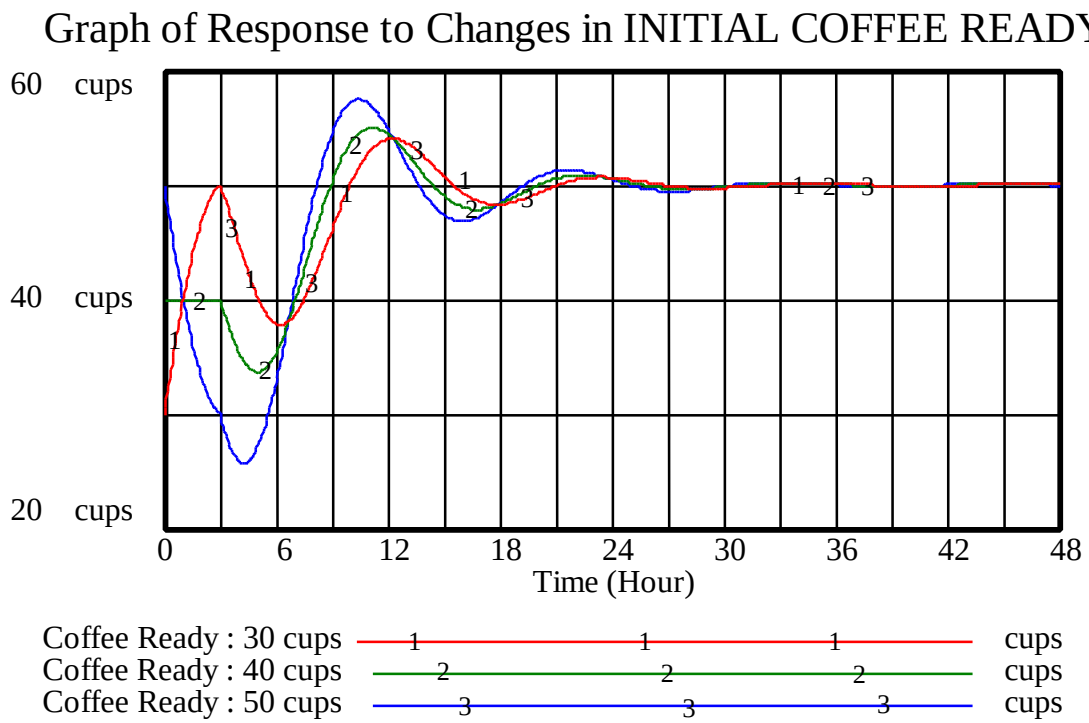


Figure 44: Vensim equivalent of Figure 16: The effect of changes in "INITIAL COFFEE READY"

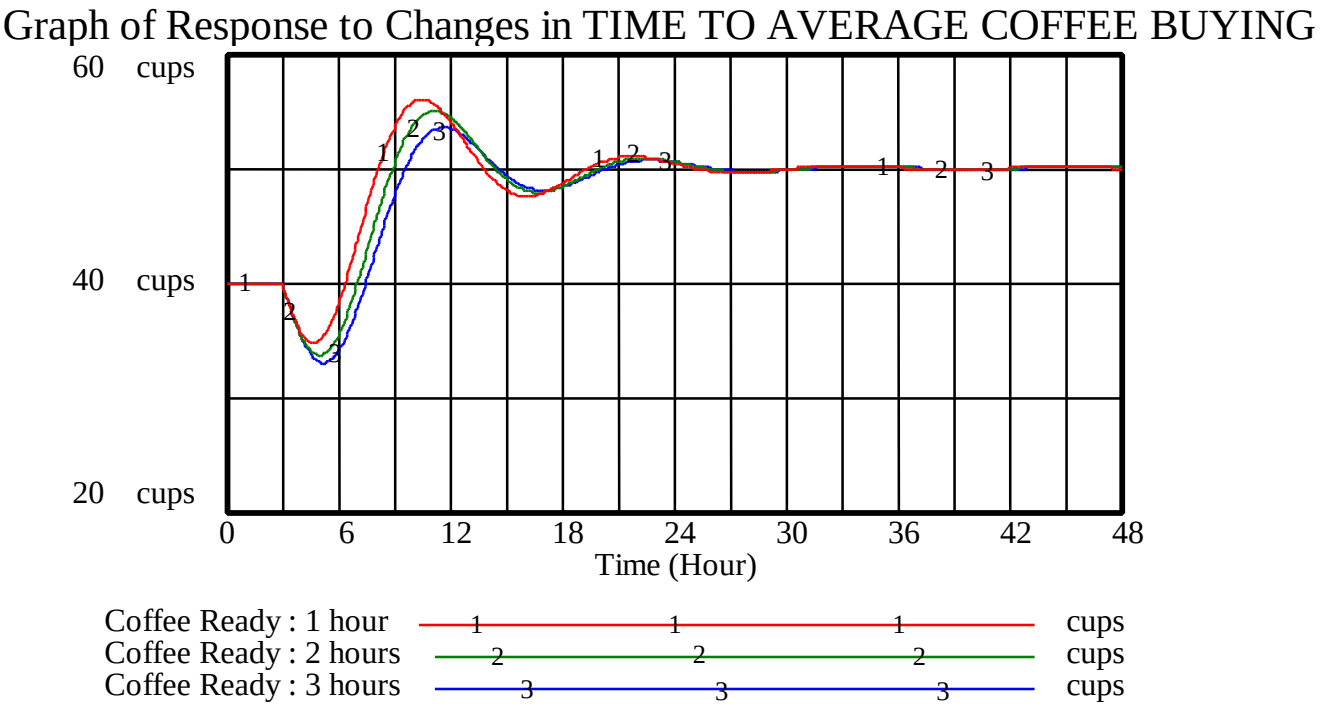


Figure 45: Vensim equivalent of Figure 17: The effects of changes in "TIME TO AVERAGE COFFEE BUYING"

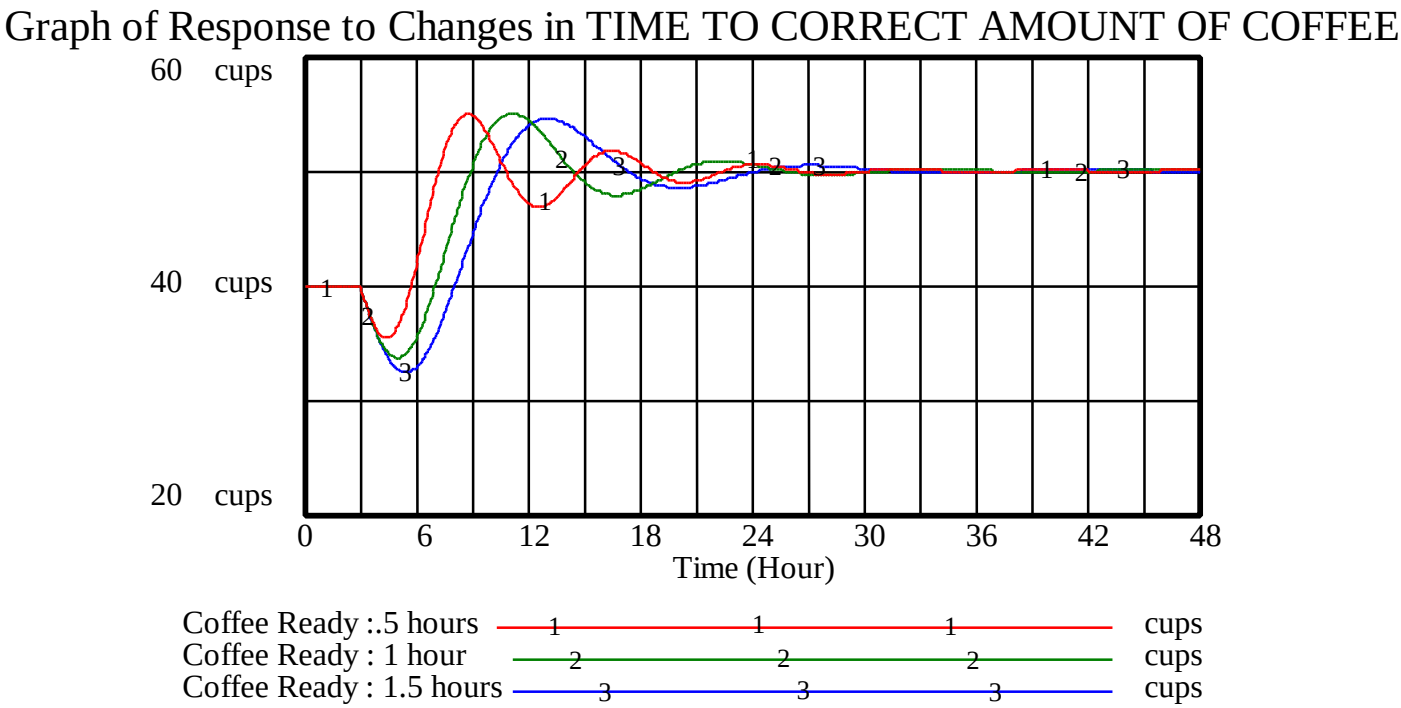


Figure 46: Vensim equivalent of Figure 18: The effect of changes in the "TIME TO CORRECT AMOUNT OF COFFEE"

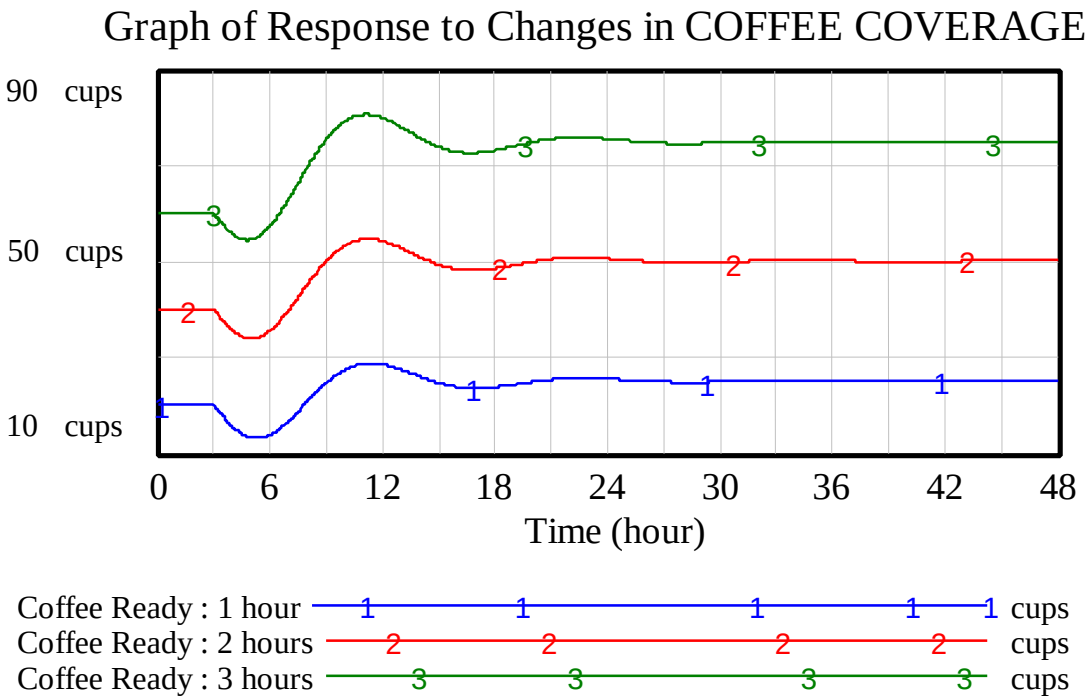


Figure 47: Vensim equivalent of Figure 19: The effect of changes in "COFFEE COVERAGE"

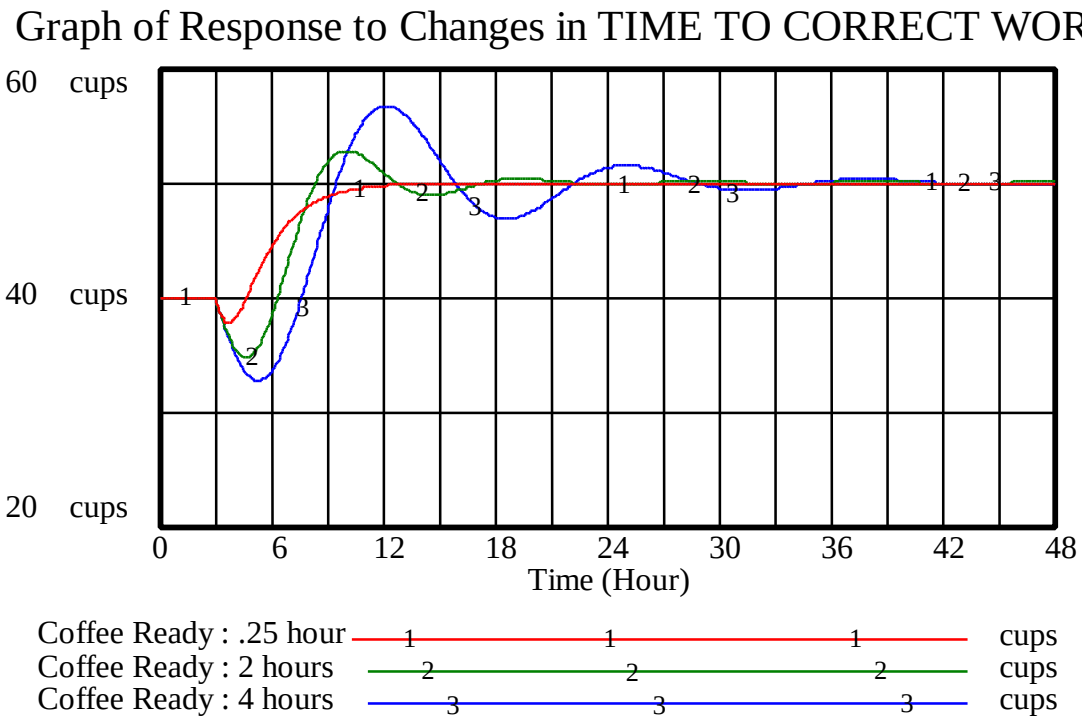


Figure 48: Vensim equivalent of Figure 20: The effect of changes in "TIME TO CORRECT WORKERS"

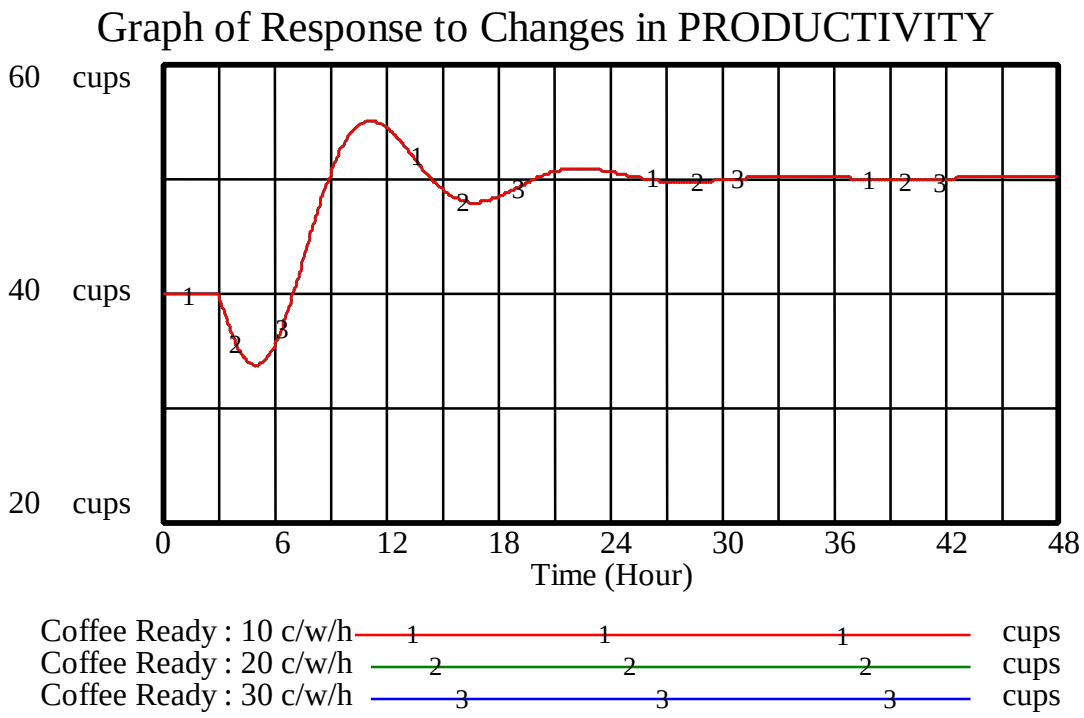


Figure 49: Vensim equivalent of Figure 21: The effect of changing "PRODUCTIVITY"

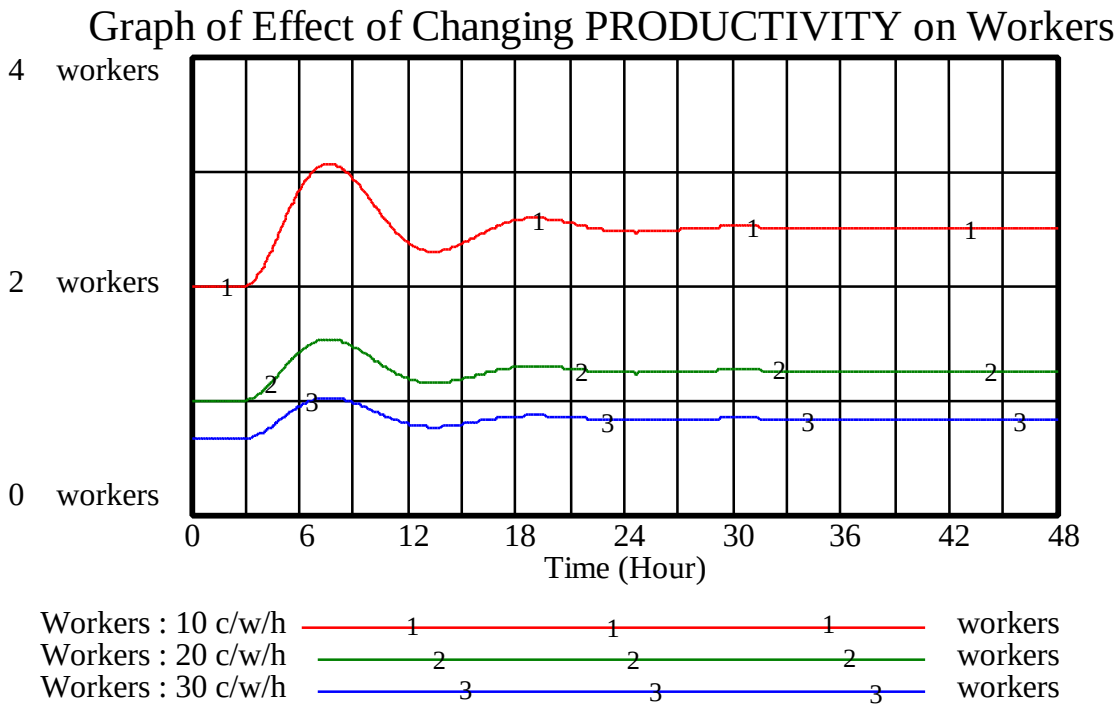


Figure 50: Vensim equivalent of Figure 22: The effect of changing "PRODUCTIVITY" on the number of "Workers"

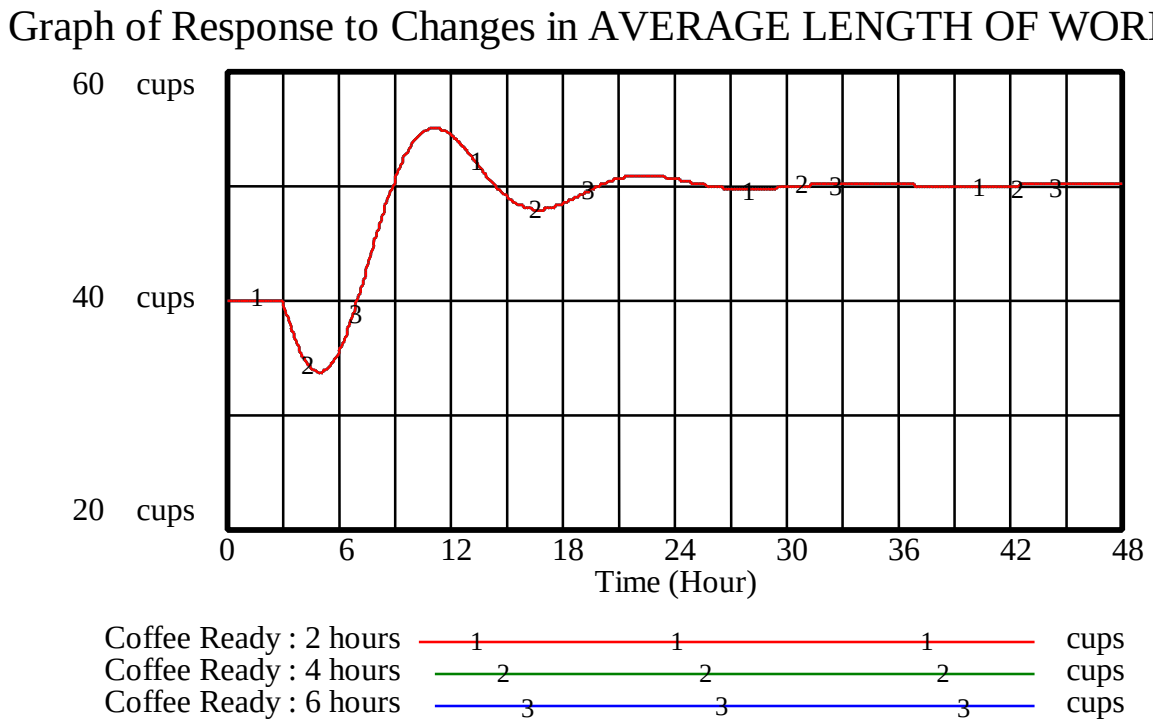


Figure 51: Vensim equivalent of Figure 23: The effect of changes in "AVERAGE LENGTH OF WORKING"

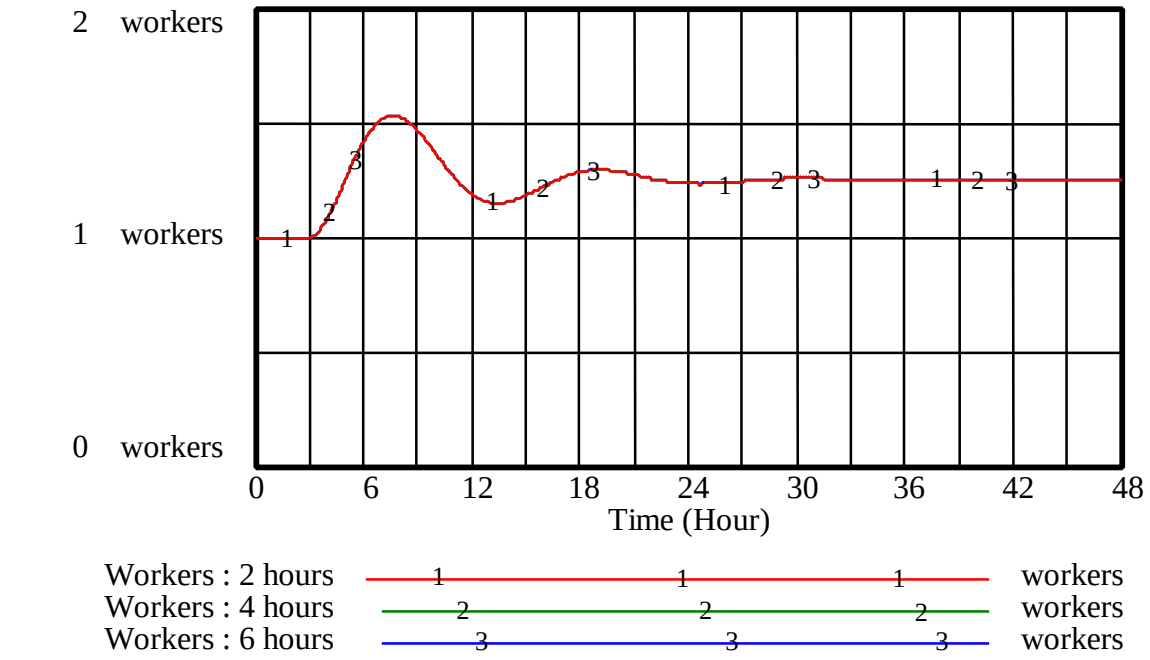


Figure 52: Vensim equivalent of Figure 24: The effect of changes in "AVERAGE LENGTH OF WORKING" on the number of "Workers"

Learning through System Dynamics as Preparation for the 21st Century

by
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Keynote Address
for
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Learning through System Dynamics as Preparation for the 21st Century

by
Jay W. Forrester

This conference emphasizes systems thinking and system dynamics modeling. But what should such activities accomplish? We do not expect most students to spend their lives in front of a computer building system dynamics models. What should be the outcome of a systems education beyond the subjects in this week's program?

I believe we should give students a more effective way of interpreting the world around them. They should gain a greater and well-founded confidence for managing their lives and the situations they encounter.

The objectives of a system dynamics education might be grouped under three headings:

1. Developing personal skills,
2. Shaping an outlook and personality to fit the 21st century, and
3. Understanding the nature of systems in which we work and live.

DEVELOPING PERSONAL SKILLS

A system dynamics education should sharpen clarity of thought and provide a basis for improved communication. It should build courage for holding unconventional opinions. It should instill a personal philosophy that is consistent with the complex world in which we live.

Basis for Clear Thought and Communication

The ordinary spoken and written language allows a person to hide behind ambiguous, incomplete, and even illogical statements. Language, within itself, does not impose a discipline for clarity and consistency. By contrast, computer modeling requires clear, rigorous statements.

In ordinary discussion, a general statement like, "How people respond

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depends on the situation," might be accepted. But, if this were to become an input for a model, one would be forced to specify which people, what response, dependence on what specific aspect of the situation, and what precise action would to be taken under various conditions.

Students must struggle to achieve the precision of expression required to go from language to explicit statements in a simulation model. Even a process as simple as filling a bathtub with water, or describing the cooling of a cup of coffee, can be surprisingly demanding. Such clarity is not achieved after only a few exercises. Learning precision in thinking requires years of reinforcement.

Translating from descriptive language to model language is only half of the story. One can then make the reverse translation. From a simulation model, reverse translation to descriptive language yields clear statements that embody the precision that came from building and using the model.

I experienced the power of reverse translation from a system dynamics model after publication of my Urban Dynamics book dealing with the growth and stagnation of cities (Forrester, 1969). The book achieved such visibility that I would be invited to conferences on urban problems held anywhere in the world. At such meetings, I had a unique power and influence derived from being able to talk for 20 minutes without contradicting myself. Not contradicting oneself might seem an ordinary competence. But others could not hope for comparable clarity because of incompleteness and inconsistency in their thinking about complex situations. Furthermore, they could not draw the correct dynamic consequences for the future implied by the assumptions they were making. To know the behavior that follows from assumptions about parts of a system can be achieved only through modeling and computer simulation.

In my situation at the conferences on cities, I knew the assumptions that went into the *Urban Dynamics* model. I knew the behavior that resulted from those assumptions. Also, I knew how the behavior would change if one adopted different political policies for guiding the evolution of a city. Within the framework of the model, I could be entirely consistent in everything that I said. Of course, one can be internally consistent and still be wrong or irrelevant. So, beyond consistency, it was necessary that the model also overlay and connect with the issues of interest to others.

Students should come out of a systems education convinced that a much better understanding is possible in the present puzzling behavior of personal, social, economic, and business situations. They should realize that any debate about policies for the future can be clarified and made more meaningful if someone

will make the underlying assumptions explicit and show which assumptions lead to behavior that best fits the knowledge we have of the real world.

Students in kindergarten through 12th grade should have the repeated experience of using modeling to resolve debates, misunderstandings, and differences of opinion. One discovers that the most intense disagreements usually arise, not because of differences about underlying assumptions, but from different and incorrect intuitive solutions for the behavior implied by the assumptions. In building a system dynamics model, one starts from the structure and the decision-making rules in a system. Usually there is little debate about structure and the major considerations in decisions. When a model has been constructed from the accepted structure and policies, the behavior will often be unexpected. As the reasons for that behavior become understood, I have often seen extreme differences of opinion converge into agreement.

Students should see modeling and an understanding of systems as a way to reduce social and political conflict.

Building Courage

A strong background in modeling should show students that conventionally accepted opinions about social and economic policies are often actually the causes of our most serious problems. If they realize that popular opinions are not necessarily correct, they should develop courage to think more deeply, look beyond the immediate situation, and stand against majority opinion that is ill founded and short sighted.

Working with models should not only enhance skill in making precise statements, but also bolster the courage to do so. Very often people take refuge in statements that are so general, so incomplete, and so superficial that they cannot be proved wrong. On the other hand, such vague statements can not be effective.

Making precise statements opens one to being wrong. By a precise statement I mean one that is unambiguous. A precise statement has a unique meaning; it is clear. However, a precise statement is not necessarily accurate or correct. Precise statements are necessary for clear communication. If such statements are wrong, that will be more quickly discovered if communication is clear. In model building, students will many times have the experience of making assertions that model simulations demonstrate to be incorrect. Students should develop the courage to be precise, even if wrong, in the process of learning and improving understanding.

Personal Philosophy

Experience in computer simulation should change the way students respond to the world around them.

From simulation models, students should appreciate the complexity of social and economic systems, whether those systems be at the level of families, communities, corporations, nations, or international relationships. They should have seen many times the counterintuitive nature of such systems. They should understand that "obvious" solutions to problems are not always correct, and that apparently correct actions are often the causes of the very problems that are being addressed.

The *Urban Dynamics* book illustrates how well-meaning actions can worsen conditions that the actions are intended to alleviate. The book showed how most popular governmental policies all lay somewhere between neutral and highly detrimental, either from the viewpoint of the city as an institution, or from the viewpoint of unemployed low-income residents. The most powerful influence on a city was shown to be the policy governing building of low-income housing. The United States has followed a policy that makes urban poverty worse. As a city ages, it becomes imbalanced. As industrial structures grow older, they are used in ways that employ fewer people. However, as housing ages, it drifts to lower rents and higher population densities. Building low-income housing accelerates the rate of decay. The "obvious" policy of building low-cost housing occupies land that could have been used for job-creating business structures while at the same time the housing attracts still more people who need jobs. The apparently humanitarian policy of building more housing actually creates poverty by pulling people into areas of declining economic opportunity.

We can hope that students will develop caution about jumping to premature conclusions and will search for a wider range of alternatives.

Even if individual students do not construct models in later life, they should expect that system dynamics models will be constructed by those who are proposing changes in economic and social policies. Moreover, in the 21st century, citizens should expect that such models will be made available for public inspection. From their K through 12th grade experiences they will know that they can read, understand, and evaluate such models. More and more, computer models will be used as the basis for determining social and economic policies. In order to participate, the public will need to know the nature of such models, to evaluate the assumptions in models, and to feel comfortable in pushing the proponents of policy models to reveal their assumptions and to justify their conclusions.

Seeing Interrelatedness

Interrelationships in systems are far more interesting and important than separate details. The interrelationships reveal how the feedback loops are organized that produce behavior. Students with a strong background in systems modeling should be sensitized to the importance of how the world is organized. They should want to search for interconnecting structure that gives meaning to the parts.

One sees the significance of modeling in a discussion I had with a student who had graduated from MIT several years before. I asked him what his system dynamics study had done for him. His answer: "It gives me an entirely different way of reading the newspapers." He meant that he sees the relationships between different things that are happening today, he understands the relationships between today's news and what happened last week and last year, and he reads between the lines to know what must have been part of the story but was not reported.

"Renaissance Man," Unifying Knowledge, Mobility

The 21st century will exhibit rapid changes in societies. We already see turmoil in the former Soviet Union and in Africa. In the past century, change came from new technologies. In the next century I believe change will be driven mostly by population growth, crowding, environmental degradation, pollution, and shortages of food, water, and resources. In other words, societies will be continually reshaped and, as a consequence, the roles of individuals will continually change. Today's students should be prepared for unexpected change.

Education must reverse the trends of the last century toward more and more specialization. A specialization interest can start early in life and lead to a professional training in college that will often become obsolete within an individual's working career. Education should provide a foundation that gives a student mobility to shift with changing demands and opportunities.

System dynamics provides a foundation underlying all subjects. When that foundation is mastered, an individual will have mobility to move from field to field. An MIT undergraduate in electrical engineering demonstrated such mobility. He studied system dynamics during his junior and senior years. When he continued for a master of science degree in electrical engineering, he did his thesis on the way the body handles insulin and glucose in various aspects of diabetes. That may not sound like electrical engineering, but about 10% of such students move on to careers in medicine. He immediately developed a working-colleague relationship with doctors in Boston's research clinic for diabetes because for the first time they were able to put together their fragments of medical knowledge into

a meaningful system (Foster, 1970). But he did not intend to go into medicine. He next worked with me in extending the *Urban Dynamics* model. For a year, he led discussions with a group from Boston's black community to incorporate many aspects of education into the model. Later he went to work with a corporation. He could move from one setting to another because his fundamental understanding of systems allowed him to provide a dynamic organizing framework to any activity.

A person with an understanding of systems sees the common elements in diverse settings rather than focusing on differences. For example, communities may have identical basic structures but behave quite differently because of different policies that are followed at crucial places. Systems with the same structure show the same range of behaviors. For example, a simple two-level model for a swinging pendulum can be relabeled and it becomes oscillating employment and inventories at the core of economic business cycles.

I had an opportunity to illustrate transferability of structure in a discussion with several medical doctors and pharmacologists. They had described an experimental treatment that a research doctor was trying. Nothing had been said about the results when I suggested, "Let me try something here. If I were to judge that treatment by the behavior we saw in the *Urban Dynamics* model of growth and decline in cities, I would predict that treatment would cause atrophy of the pancreas." And they responded, "You are right, that is exactly what happens." We expect that students should develop ability to transfer understanding of dynamic structures among very different fields.

Transferability of structure and behavior should create a bridge between science and the humanities. Feedback-loop structures are common to both. An understanding of systems creates a common language. Science, economics, and human behavior rest on the same kinds of dynamic structures.

I see a reversal of the trend toward specialization. As the underlying unity between fields becomes teachable, we can move back toward that concept of the "Renaissance Man," who has broad intellectual interests and is accomplished in areas of both the arts and the sciences.

OUTLOOK AND PERSONALITY

A systems education should give students confidence that they can shape their own futures. A systems education should help mold a personality that looks for causes and solutions. Working with systems should reveal the strengths and weaknesses of mental models and show how mental models and computer models can reinforce one another.

Confidence in Creating the Future

Many of the stresses in modern life arise because people feel buffeted by forces they neither understand nor know how to control. Such sense of helplessness can be traced to not understanding the systems of which we are a part. Events that seem capricious when viewed locally are often understandable when seen from a broader systems perspective.

I hope that a system dynamics thread in K through 12 education would leave individuals willing and able to appreciate the nature of complexity. They should want to look beyond their immediate setting in search of the fundamental causes of problems. They should develop optimism about understanding those problems of society that earlier generations have found so baffling. Inflation, wars, unfavorable balance of trade, and destruction of the environment have persisted for hundreds of years without public understanding of the causes. Such problems are too serious to be left to the self-appointed experts; the public must acquire the insights that permit participation in debates of such importance.

Such better understanding comes in small steps. I am reminded of the story told by a television producer who was taking video pictures in a group of parents, teachers, and students at a school where the systems approach is making excellent progress. The producer turned to a junior high school boy and asked, “What have these systems studies meant to you?” His immediate answer: “I am much better able to deal with my mother.”

Such ability to deal better with one’s environment starts with even very simple systems. One of our MIT doctoral students in system dynamics went to work for the Department of Energy. Two years later he told me he was amazed by the amount of influence he could have on governmental thinking with a two-level simulation model. Even such a simple system is often beyond what people in important policy positions are taking into account.

Authoritarian vs. Innovative Personality

A systems education should mold the personality of students by enhancing innovative tendencies in children and counteracting the forces in society that convert an innovative personality into an authoritarian one. I am here using authoritarian and innovative personalities in the sense described by Everett Hagen in his book, *On the Theory of Social Change* (Hagen, 1962) Hagen contrasts two opposite extremes of personality.

The authoritarian personality fits into a rigid hierarchy. Life is capricious. One does as ordered by those of higher status. There are no reasons for such orders. Capricious orders fit the old army saying borrowed from Tennyson, "Yours not to reason why, yours but to do and die." The reward for yielding to higher authority comes from the individual having authority over someone of lower rank. The pure authoritarian personality expects no reasons for why things happen and has no will to search for reasons.

By contrast, the innovative personality believes there are reasons for why things happen. Even if the reasons are unknown, there is still the assumption that reasons exist. Also, it is worth looking for the reasons because, if one understands, then one can probably change and improve what is happening. The innovative personality looks for causes and works toward beneficial advances.

I believe that babies are born as innovative personalities. They want to explore, to understand, and to see how things work and how to master their environments. But our social processes work to stamp out exploration and questioning. The child is continually confronted with, "Do as you are told," or "Stop asking questions and just mind me," or "Study this because it is good for you." Repeated restraint of innovative inclinations gradually forces personalities into the authoritarian mold.

A system dynamics modeling curriculum, by letting students formulate the structure and policies causing behavior under study, will help preserve and rebuild the innovative outlook. Simulation emphasizes reasons for consequences. To be innovative, one must be willing to make mistakes while searching for reasons and improvement. Computer simulation modeling is a repeating process of trial and error. One learns that progress is made through exploration and by learning from mistakes. An authoritarian personality fears mistakes and does not try the unknown. An innovative personality knows that mistakes are stepping stones to better understanding.

Mental Models and Computer Models

Students should learn that all decisions are made on the basis of models. Most models are in our heads. Mental models are not true and accurate images of our surroundings, but are only sets of assumptions and observations gained from experience.

Mental models control nearly all social and economic activities. Mental models have great strengths, but also serious weaknesses. From a systems education, students should learn how mental models can be useful and when they

are unreliable. Furthermore, they should appreciate how computer simulation models can compensate for weaknesses in mental models.

Mental models contain a vast wealth of information that is available no where else. Mental models contain information about the structure and policies in systems. By structure I mean the elements in a system and the connections between the elements—who has what information, who is connected to whom, and, what decisions are made and where. By policies I mean the rules that govern decision making—what factors influence decisions, what is a particular decision point trying to accomplish, and what goals are sought. At this detailed level of structure and policies, mental models are rich and reasonably reliable sources of information.

However, mental models have serious shortcomings. Partly, the weaknesses in mental models arise from incompleteness, and internal contradictions. But more serious is our mental inability to draw correct dynamic conclusions from the structural and policy information in our mental models.

System dynamics computer simulation goes a long way toward compensating for deficiencies in mental models. In model building, one must remedy incompleteness and internal contradictions before the system dynamics software will even allow simulation. After a logically complete model has been created, one can be certain that the computer is correctly simulating the system based on the assumptions that were incorporated in the model. It is in simulation, or determining consequences of the structural and policy assumptions, that mental models are unreliable, but computer models are completely dependable.

Students should also realize that there are no possible proofs of the validity of any models, whether they are mental or computer models. Models are to be judged by their comparative usefulness. Assumptions about structure and policies should be compared with any available information. Computer simulation results should be compared with behavior in the real system being represented. Discrepancies lead to improving both mental and computer models.

A two-way street runs between mental models and computer models. Mental models contribute much of the input for computer models. Creating a computer model requires that the mental models be clarified, unified, and extended. From the computer simulations come new insights about behavior that give new meaning to mental models. Mental models will continue to be the basis for most decisions, but those mental models can be made more relevant and more useful by interacting with computer models.

UNDERSTANDING THE NATURE OF SYSTEMS

We live in a network of complex systems. Yet few people realize the extent to which those systems control human actions. In fact, few people realize the extent to which complex systems actively mislead people into making counterproductive decisions. Students, after a 12-year encounter with systems, should be on guard against the deceptive nature of systems.

Cause and Effect Not Closely Related in Time or Space

Most understandable experiences teach us that cause and effect are closely related in time and space. However, the idea that the cause of a symptom must lie nearby and must have occurred shortly before the symptom is true only in simple systems. In the more realistic complex systems, causes may be far removed in both timing and location from their observed effects.

From earliest childhood we learn that cause and effect are closely associated. If one touches a hot stove, the hand is burned here and now. When one stumbles over a threshold, the cause is immediately seen as not picking the foot high enough, and the resulting fall is immediate. All simple feedback processes that we fully understand reinforce the same lesson of close association of cause and effect. However, those lessons are aggressively misleading in more complex systems.

In systems composed of many interacting feedback loops and long time delays, causes of an observed symptom may come from an entirely different part of the system and lie far back in time.

To make matters even more misleading, such systems present the kind of evidence that one has been conditioned to expect. There will be apparent causes that meet the test of being closely associated in time and in location. However, those apparent causes are usually coincident symptoms arising from the distant cause. People are thereby drawn to actions that are not relevant to the problem at hand.

Comments such as I have just made about cause and effect carry little conviction from being stated in a lecture. Only after a student has repeatedly worked with models that demonstrate such behavior, and has had time to observe the same kinds of behavior in real life, will the idea be internalized and become part of normal thinking.

Low-Leverage Policies

Complex systems differ from simple systems in another way. In simple systems, the policies to yield better results are obvious and they work. To avoid burning your fingers on a hot stove, you keep away from the stove. But in complex systems, the apparently influential policies often have very little effect.

When I talk to a group of business executives I ask how many have ever had the experience of facing a serious problem, devising policies to correct the situation, and five years later find there has been no improvement. Most will hold up their hands. Perhaps you have experienced the same in education. The quality of education has been severely criticized, many educators have tried remedies, and often there is little change.

In complex systems, there are many interconnecting feedback loops. A new policy, which is intended to solve a problem, causes reactions in other parts of the system that counteract the new policy. In education that reaction may come from administrators, from school boards, from parents who do not want new experimental ideas tried on their children, or from budget pressures.

I believe that a very high percentage, say 98%, of the policies in a system have very little leverage to create change. They do not matter. However, most of the heated debates in communities, companies, and governments are about policies that are not influential. Such debates are a waste of time and energy. Debates about low-leverage policies divert attention from the few policies that could lead to improvement.

Students must have experience working with models of complex systems to appreciate how often proposed policies fail to produce results.

High Leverage Policies, Often Wrongly Applied

Fortunately, a few high-leverage policies exist that can alter the behavior of a system. However, high-leverage policies lay another trap for the unwary. One occasionally finds a person who is working with a high-leverage policy. However, I estimate that more than 90% of the time that person is pushing the high-leverage policy in the opposite direction relative to what that person wants to accomplish. In complicated systems, intuition provides no reliable guide even to the direction that a high-leverage policy should be changed.

I have several times had the experience of going into a company with a serious difficulty where intended policies were causing the problem. We are talking here of a highly visible problem. It might be low profitability, or falling

market share, or severe instability with the company working overtime one year and having half the people laid off two years later. One carries on extensive interviews to determine the policies (decision-making rules) that people are using in different positions in the company. People justify their policies as intended to solve the major problem. One then puts the expressed policies into a system dynamics simulation model and finds that the model generates the same difficulty that the company is experiencing. In other words, the policies that people know they are following are the cause of their trouble. Local interpretation of symptoms leads to local actions that combine to produce detrimental results. This is a treacherous situation. If people believe their actions will reduce the problem, but do not know those actions are making it worse, then as matters become worse there is growing incentive to take the presumed corrections that are actually causing further decline.

One sees this spiral of system deterioration at all levels in society. Individuals in a family in serious psychiatric difficulty know they are in trouble, each wants to do something to help, yet everything that everyone does makes matters worse. In the *Urban Dynamics* model, we saw that governmental policies about low-cost housing do not improve cities but cause more decay. In the same way, we might suspect that our national foreign trade policies lead to importing goods made by low-skilled labor while our own low-skilled population lose the jobs that could provide an employment and training ladder to higher skills.

I do not know of any way to determine which are high-leverage policies and which direction to apply them except to do a systems dynamics simulation of the situation. Students should have many experiences working with models that reveal the multitude of policies having little effect, that allow them to search for high-leverage policies, and that show them the danger of intuitively judging even the direction of effect of high-leverage policies. Students should come out of a systems education with an appreciation for how mental models alone can lead one astray in multiple-loop systems. They should demand that important issues be modeled, and that the models be made available to the public. They should have confidence that they can read and evaluate such models. Models then become a powerful and explicit means of communication.

We Cause Our Own Problems

The often quoted line from the comic strips, “We have met the enemy, and he is us,” has more than a grain of truth. Usually, problems exhibited by a social system are caused by the people in that system. However, people naturally tend to blame others. When Detroit was losing market share to Japanese automobiles, executives of American companies blamed Japan for dumping at low prices, when the real cause was Detroit’s own declining quality. Parents blame schools for low

competence of students, when perhaps the deficiency arises more from preschool home life and failure in parental guidance. A company is more inclined to blame falling sales on unfair competition or fickle consumers than on its own poor products and service.

In preparation for the 21st century, a systems education should condition students to look for the source of their troubles first in their own actions before blaming others.

Drift to Low Performance, Collapse of Goals

One component of any feedback loop is the goal toward which the feedback process is striving. In simple models, goals are usually given as constants, for example, the goal of a pendulum is to seek the vertical as it swings from one side to the other. The goal of an inventory manager may be to maintain a given level of inventory, The goal that determines the amount of sleep we get is to maintain a certain degree of restfulness. But in a more complete representation of systems, the goals themselves are properly shown as variables. We may be striving toward a certain goal, but, failing to reach the goal, we may readjust our goal to something that seems more achievable.

There is a strong tendency for goals of all kinds—personal, community, corporate, or national—to drift downward. Pressures tend to cause performance to fall short of goals. But failing to meet goals is uncomfortable. The response is often to let the goals adjust downward toward the actual performance. As goals fall, the incentives for high achievement decline. Performance continues to fall short of the new lower goals and the downward spiral continues.

Falling goals will in time lead to crisis, but by then recovery may be impossible. One sees erosion of goals in attitudes toward the national deficit. Thirty years ago, the present size of the national deficit would have been unthinkable. But as the deficit rose, people came to accept each new rise and adjusted to the higher deficit. Eventually such goal erosion can lead to disaster. Successful people, successful corporations, and successful countries have leadership or deeply held beliefs that stop such goal erosion.

Students should be exposed to the dynamics of goal collapse in models and have an opportunity to relate the process to their own lives. Goal collapse, that is, becoming accustomed to and accepting falling standards, may be the greatest threat to the future of individuals and countries.

Long-Term vs Short-Term Goals

A fundamental conflict exists between short-term and long-term goals. Students should observe this conflict between the present and the future in system dynamics models and then relate the lessons to their own lives. Actions that yield immediate rewards almost always exact punishment in the long run, and vice versa. Quick gratification is the enemy of future well-being. It is hard to find exceptions where actions with an immediate reward do not extract a price in the more distant future.

A person who steals may benefit immediately, but usually suffers later. A person who works all night to finish an important task pays by being inefficient for the next several days. Taking mind-altering drugs may give an immediate sense of well being at the expense of future ill health or poverty. Borrowing on credit cards allows an immediate increase in standard of living but the consequence in the longer term is a lower standard of living while paying back the loan and interest. Under pressure from voters, the U.S. Congress is borrowing money to provide ever-increasing goodies to constituents, with the probable future consequence that government becomes insolvent and may not be able to provide basic public services. Over a much longer time horizon, improved public health and modern agriculture raised the standard of living and reduced death rates, resulting now in the threat of an unsustainable population explosion.

Conversely, accepting a short-term disadvantage can often yield rewards in the longer-term. For example, saving now, rather than spending all one's income, can increase the future standard of living. A company that foregoes higher dividends and increased executive salaries can invest in research on new products and increase future income.

The conflict between short-term and long-term goals bears directly on what should be considered ethical and humanitarian. Humanitarian impulses are usually based on short-term considerations but often lead to worsening the situation in the more distant future. Food aid to starving populations seems humanitarian in the short run, but may well encourage population growth and greater starvation of even more people in the future.

Students should study the fundamental conflicts between short-term and long-term goals in the context of system dynamics models and have the opportunity to relate the lessons to their families, communities, and nation.

ACHIEVING THE BENEFITS OF A SYSTEMS EDUCATION

A systems thinking and systems modeling curriculum will not automatically yield the lifetime insights and personal guidance that I have been discussing. A student might easily go through the motions of working with models without gaining the understanding that is potentially available.

Experience and Participation

Students will not internalize their understanding of systems merely from being told. Nor will discussion and debate be effective. Coming to an understanding of systems must be a participative activity. Learning about systems in not a spectator sport, such learning comes from active involvement. One does not learn to ride a bicycle or play basketball from lectures alone; one must practice. A person learns from experience. Computer modeling allows an accelerated vicarious experience.

The Deeper Lessons

A student can work with computer simulation models without realizing the deeper lessons that should be absorbed. Most learning for the 21st century that I have discussed can be missed by students unless the right guidance is provided. Students must create their own models and learn from trial and error. They must be led toward models that can teach the lessons that I have been discussing. Even with models that contain the lessons, students can miss the most important implications, so they should be encouraged to see the deeper consequences of what they are doing. They should relate what they are learning to systems they already know in families, community, and school.

Systems Thinking vs System Dynamics

This conference is advertised as “Systems Thinking and Dynamic Modeling.” Consider those two activities in the context of learning for the 21st century. I understand and define the two terms, systems thinking, and dynamic modeling, to mean quite different activities.

Systems thinking appears to be thinking about systems, talking about the characteristics of systems, acknowledging that systems are important, discussing some of the insights from system archetypes, and relating the experiences people have with systems. Systems thinking is lecturing about systems as I am doing tonight. Systems thinking can be a door opener and a source of incentive to go deeper into the study of systems. But I believe that systems thinking has almost no chance of instilling the lessons that I have described. Systems thinking will

change very few of the mental models that students will use in their future decision making. Systems thinking is not more than five percent of a systems education.

On the other hand, system dynamics modeling is learning by doing. It is learning through being surprised by the mistakes one makes. System dynamics modeling is a participative activity in which one learns by trial and error and practice. I believe that immersion in such active learning can change mental models.

Revision of Road Maps

Many of you are already familiar with the Road Maps series being written by undergraduate students at MIT with my guidance. Road Maps are documents intended as a self-study guide to learning about systems. First drafts of four chapters have been available through the Creative Learning Exchange.[1]

However, in thinking about lessons that students should learn, it became clear that intended insights about systems could be missed. Work with systems was there, but implications had not been stressed. We are now revising the early drafts of Road Maps to be more explicit about fundamental principals of systems and to call attention to the general characteristics of systems that should be observed. The series is also being extended to more comprehensive chapters.

On Teaching Systems

I believe that confining student learning to systems thinking and to discussion about systems will convey very little understanding of the nature and behavior of the systems within which we live.

To appreciate the nature of systems, students must have extensive personal experience in working with systems. This means creating system dynamics models on a computer, simulating their behavior, exploring how the models respond to changes in structure and policies, and comparing model behavior to the real systems being represented. Such active modeling should extend at least throughout the several years of middle school and high school. As early as possible, schools should move away from canned models that have been previously prepared for student use. Instead, students should create models, examine their short comings, and learn from discovering improvements.

---

[1] Creative Learning Exchange, Lees Stuntz, Executive director, 1 Keefe Road, Acton, MA, 01720, tel: 508-287-0070, fax: 508-287-0080.

Students should gain experience in modeling systems in which they have a personal interest. Such systems can be drawn from family and community situations. Items from the newspapers should be converted to formal models to reveal student understanding of current events, to detect omissions and contradictions in the news items, and to provide practice in moving in both directions between mental and computer models. History and literature likewise provide material that can be made more explicit and understandable through modeling.

Throughout student work with models, more should be learned than just the details of the models themselves. Beneath such models are the underlying principles of systems (Forrester, 1968). Beyond such models are the kinds of learning discussed in this talk. Students probably will not see such general and transferable insights merely from exposure to models. The larger and more enduring lessons must be pointed out. Students should work with examples of the broader implications. Such active use of the insights will thereby become part of their thinking and the way they look at the world around them.

For Further Reading

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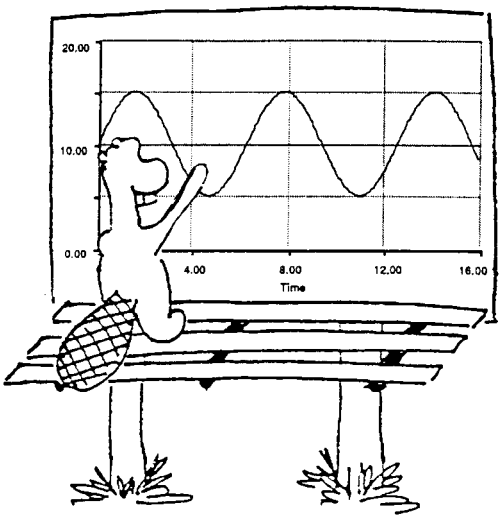
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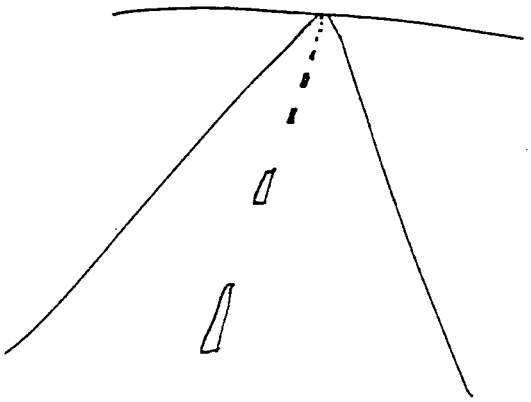
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Road Maps 9

A Guide to Learning System Dynamics





System Dynamics in Education Project

Road Maps 9

System Dynamics in Education Project
System Dynamics Group
Sloan School of Management
Massachusetts Institute of Technology

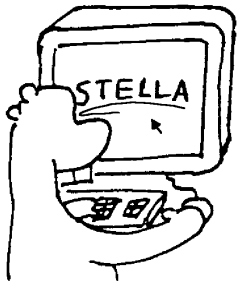
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Compiled under the direction of Professor Jay W. Forrester

Welcome to Road Maps Nine!



Road Maps is a self-study guide to learning the principles and practice of system dynamics. This chapter is the ninth in the Road Maps series. Road Maps One through Three give a broad introduction to the field of system dynamics, focusing on understanding the structure and behavior of systems through positive and negative feedback loops. Road Maps Four through Six introduce several generic structures, give a taste of policy analysis and model validity, and provide opportunities for independent modeling. Road Maps Seven and Eight explain some common mistakes and misunderstandings in system dynamics modeling, offer exercises for improving modeling skills, analyze higher-order positive feedback loops as well as oscillating systems, and introduce sensitivity analysis.

Road Maps Nine improves your understanding of simple economic systems by modeling the use of credit cards. Another mistakes and misunderstandings paper illustrates how to correctly formulate robust table functions. Road Maps Nine then presents an important generic structure producing the behavior known as "overshoot and collapse." The chapter also explains how to perform graphical integration qualitatively, without any calculations. The chapter concludes with a paper by Professor John Sterman about using various types of computer models.

Topics Covered in Road Maps Nine

Improving Modeling Skills

- *The Credit Card Model* (D-4683)
  by Manas Ratha

- *Mistakes and Misunderstandings: Table Functions* (D-4653)
  by Leslie A. Martin

Generic Structures

- *Generic Structures: Overshoot and Collapse* (D-4480)
by Lucia Breierova

Graphical Integration Exercises

- *Graphical Integration Exercises Part 5: Qualitative Graphical Integration* (D-4675)
by Manas Ratha and Helen Zhu

Computer Models

- *A Skeptic's Guide to Computer Models* (D-4101-1)
by John D. Sterman

Things You'll Need for Road Maps Nine

Modeling Software

In order to complete Road Maps Nine and subsequent Road Maps, you will need to have access to modeling software. The Road Maps guides and most papers included in Road Maps were written with the use of STELLA II for the Macintosh. STELLA II is currently available for both the Macintosh and the Windows platforms. If you have any questions about STELLA, contact High Performance Systems (see Appendix). Ask about prices for educational use.

Vensim, Powersim, and DYNAMO are other software programs designed for building system dynamics models. Vensim is produced by Ventana Systems, which offers a free introductory version of its software, Vensim PLE, that can be downloaded off the World Wide Web. See the Appendix for more information about obtaining Vensim and Powersim.

Notice written June, 2000:

We have written a guide on how to use Vensim modeling software for each section of the Road Maps series that involves computer modeling. Each guide is located in the back of the exercise document. When Chapters 1-9 of the Road Maps series were written, STELLA software was the most common beginner modeling program available. Now you may choose from a number of system dynamics modeling software packages. If you would like more information on

Vensim, please go to http://www.vensim.com. A free version called Vensim PLE is located there.

For more detailed information on using Vensim software in the Road Maps series, please refer to the paper titled: "Vensim Guide (D-4856)" in the Appendix section at the end of Road Maps.

From now on as additional papers for the Road Maps series are written, the Vensim software will be used exclusively for modeling exercises.

A Computer

To run the latest version of STELLA, STELLA 5.0, on a Macintosh, you will need an Apple Macintosh computer (68020 processor or higher) with at least 8 MB of RAM, a 12 MB hard disk and System 7.1 or higher. To run STELLA 5.0 for Windows you will need an IBM PC-compatible computer with a 486-class processor running Windows 3.1 or greater. You will need at least 8 MB RAM, a hard disk with a least 16 MB of free space. Previous versions of STELLA have similar requirements.

In either case, if you plan on continuing to model, it may be a good idea to have access to a computer with more memory, hard disk space and a faster processor.

How to Use Road Maps Nine

Road Maps Nine explores several topics in system dynamics through selected readings and exercises. Before each reading or exercise is a short description of the reading and its most important ideas. After each reading or exercise, we highlight the main ideas before moving on.

Each chapter in Road Maps contains readings that introduce and strengthen some of the basic concepts of system dynamics. Other readings focus on practicing the acquired skills through various exercises or simulation games. Many of the chapters conclude with a prominent paper from the literature in the field of system dynamics.

As part of the spiral learning approach used in Road Maps, many concepts will be briefly introduced early on and then explained later in greater detail. Road Maps contains several series of papers that are spread out over successive chapters. Each of these series focuses on a specific topic or skill in system dynamics. The

series start with a simple paper, and progress to further develop the idea in subsequent chapters.

Road Maps present the fundamental concepts of system dynamics as *System Principles*. These principles are enclosed in boxes that highlight them from the rest of the text to emphasize their importance. The progression of system principles in Road Maps allows you to revisit each principle several times. Each time a principle is revisited in Road Maps, you will build upon your previous understanding of the principle by learning something new about it. The system principles are the core of Road Maps around which the readings, exercises, and papers are built.

Now let's get started!

Improving Modeling Skills

More and more often, people use their credit cards when making purchases. Using a credit card is convenient because the user can spend money without having to carry cash, or even without having the amount of money necessary to make the purchase in his checking account. Unfortunately, the money borrowed on a credit card must be repaid, and, if it is not repaid on time, the credit card company that issued the card charges interest at a high rate. The following paper builds a simple model of using a credit card and examines the consequences of borrowing too much money on the borrower's quality of life.

- The Credit Card Model[1]

by Manas Ratha

This paper describes a simple system of purchasing goods with a credit card. It then leads the reader through the steps of building a system dynamics model: conceptualization, formulation of the model equations, editing the model, and evaluating the behavior of the model. In addition, the paper also explains the economic implications of using a credit card on the quality of life of the card user. The reader should build and simulate the model along with the paper.

[1] Manas Ratha, 1997. *The Credit Card Model* (D-4683), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 17, 29 pp.

Please read *The Credit Card Model* now and do the suggested exercises on your computer.

After reading *The Credit Card Model*...

The paper you just read provides an example of building a system dynamics model from scratch, starting from a system description. The paper leads the reader through conceptualization of the system, formulation and editing of the model equations, and finally, through simulation and evaluation of the model. Apart from improving modeling skills, the paper also shows some possible effects of overusing a credit card. Even though in the short run the quality of life of the card user improves, an unpaid balance is likely to decrease the user's quality of life in the long run. Thus, the system demonstrates a behavior characteristic of many complex systems: "short-term benefit, long-term cost."

The previous paper also reviews an important system principle that has already been introduced in Road Maps.



System Principle #20:

Rates are not instantaneously measurable.

No rate can, in principle, control another rate without an intervening level variable (as rates cannot be measured instantaneously and must be averaged by using a level for integration). Two rates may be directly connected in some models as a short cut, but the occasion is rare and should be avoided by inexperienced modelers.

Look back at section 5.3 of *The Credit Card Model*. For simplicity, the flow of "payments" is first set equal to the flow of "interest charges," as shown in Figure 4. Because it is impossible to measure the flow of "interest charges" instantaneously, the modeler instead introduces a variable called "interest on balance." Both flows, "payments" and "interest charges," are then set equal to "interest on balance." Good modeling practice does not set a flow equal to another flow.

We will come back to this system principle again in later chapters of Road Maps.

***- Mistakes and Misunderstandings: Table Functions*[2]**

by Leslie A. Martin

This is the third in a series of papers in Road Maps that examine common mistakes and misunderstandings in system dynamics modeling. Using a simple population model, this paper explains that table functions should have dimensionless inputs and outputs and shows how to correctly formulate robust table functions.

Please read *Mistakes and Misunderstandings* now.
After reading *Mistakes and Misunderstandings*...

Table functions are an excellent tool for representing nonlinear relationships between model variables. For a table function to be dimensionally consistent, its input as well as its output should be nondimensional. In addition, table functions should be robust: they should cover the entire ranges of reasonable parameter values. As the next reading shows, many models of complex systems rely heavily on a correct formulation of table functions.

Generic Structures

Previous chapters of Road Maps presented several generic structures that are known for the behavior they produce. Such generic structures include S-shaped growth and oscillation structures. Many systems that generate another common type of behavior, overshoot and collapse, share the same basic structure, which is presented in the next paper.

***- Generic Structures: Overshoot and Collapse*[3]**

by Lucia Breierova

This paper begins by modeling the structure and showing the behavior of two systems, a system of wells drilling petroleum on a field and a deer population system. Both models produce the type of behavior known as overshoot and collapse. Then, the generic structure that generates overshoot and collapse is

---

[2] Leslie A. Martin, 1997. *Mistakes and Misunderstandings: Table Functions* (D-4653), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 15, 13 pp.

[3] Lucia Breierova, 1997. *Generic Structures: Overshoot and Collapse* (D-4480), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, July 21, 40 pp.

presented, with special attention paid to the nonlinear table functions contained in the structure. The paper then offers two exercises in which the reader is asked to use the generic structure to model two different systems.

Please read *Generic Structures: Overshoot and Collapse* now and do the suggested computer exercises.

After reading *Generic Structures...*

Systems producing the behavior of overshoot and collapse contain a second-order negative feedback loop linking a nonrenewable resource to the growth of another stock. As the stock of interest grows, it consumes more and more of the resource that is incapable of regeneration. The decline of the amount of the resource affects the stock's growth, slowing down the growth until the stock is no longer able to grow because of the depletion of the resource. The stock starts declining and eventually collapses.

The paper you just read reviews an important system principles that you have already seen several times in Road Maps.



System Principle #1:

The feedback loop is the basic structural element of systems.

The more complex systems are assemblies of interacting feedback loops.

Consider the model of the Mayan population, shown in Figure 14 of *Generic Structures: Overshoot and Collapse*. In addition to the simple positive loop of "Population" and "births" and the negative loop of "Population" and "deaths," the Mayan population system contains several other feedback loops.

The loop relating the "State Of Environment" and "environment regeneration" is a positive feedback loop: the higher the value of "State Of Environment," the lower the "regeneration time," so the higher the rate of "environment regeneration," which further increases the value of "State of Environment."

The system, however, also contains another negative feedback loop, relating the "density" of the "Population" to the rate of "deaths" through the "damage to environment": the higher the "Population," the higher the "density," so the more "damage to environment" occurs. More "damage to environment" decreases the "State Of Environment," which in turn causes the "death fraction" and thus the "death rate" to increase, resulting in a stabilizing decrease of the "Population."

The structure of the Mayan population system is thus composed of several feedback loops that interact with each other to produce the dynamic behavior of overshoot and collapse.

The models presented in the previous paper also demonstrate another system principle, which you will see reviewed several times in later chapters of Road Maps. Remember from System Principle #22, introduced in Road Maps 8, that information links connect levels to the control of rates. Through information links, values of level variables go to the rate equations, determining the rates of flow. In the petroleum extraction model in *Generic Structures: Overshoot and Collapse*, information links determine the rate of "closing wells" directly from the level of "Wells" and indirectly from the level of "Petroleum Reserves."



System Principle #24:

Auxiliary variables lie only in the information links.

An auxiliary variable, or converter, is a subdivision of a rate equation. It allows a model to be disaggregated into easier to understand equation statements.

Look back at Figure 2 of *Generic Structures: Overshoot and Collapse* showing the model of petroleum extraction. The flow of "closing wells" is defined as:

closing wells = Wells * closing fraction (1)

The "closing fraction" is an auxiliary variable, the product of the "NORMAL CLOSING FRACTION" and the "effect of extraction on closing fraction" multiplier. Therefore, without using the auxiliary variable, one could also write the rate equation as:

closing wells = Wells * NORMAL CLOSING FRACTION * effect of extraction on closing fraction (2)

Equation (2), however, is more complicated and less understandable than equation (1), which contains the auxiliary variable "closing fraction." Thus, auxiliary variables help in the formulation of easily understandable rate equations.

Graphical Integration Exercises

Previous chapters of Road Maps presented several papers in which graphical integration was explained as a mathematical process used to determine the behavior of a stock from the graph of the behavior of flows into or out of the stock. The next paper in the Graphical Integration Exercises series treats graphical integration in a more intuitive way and explains how to estimate stock behavior without knowing the precise values of flows and without doing any calculations.

***- Graphical Integration Exercises Part 5: Qualitative Graphical Integration*[4]**

[4] Manas Ratha and Helen Zhu, 1997. *Graphical Integration Exercises Part 5: Qualitative Graphical Integration* (D-4675), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 19, 33 pp.

by Manas Ratha and Helen Zhu

This paper explains how to determine, in general, a stock's behavior by looking at the graphs of the inflows and outflows from the stock, without performing any calculations. The paper focuses on linear flows (flows that can be graphed as straight lines) and explores seven possible types of flows. For each type of flow, the associated stock behavior is presented and explained. Examples and exercises with solutions are also offered to give the reader an opportunity to practice the skills acquired from the paper.

Please read *Graphical Integration Exercises* now and do the suggested exercises.

After reading *Graphical Integration Exercises...*

The paper you just read classifies linear flows into seven categories and describes the associated stock behavior. The sign of the flow determines whether the stock remains constant (if the flow is zero), grows (if the flow is positive), or declines (if the flow is negative) over time. The magnitude of the flow determines whether the stock changes at a constant, increasing, or decreasing rate. Qualitative graphical integration is important because it helps a modeler to intuitively understand the relationship between the behavior of a stock and the flows into or out of the stock.

Graphical Integration Exercises Part Five reviews an important system principle that was introduced in previous chapters of Road Maps:



System Principle #4:

Levels are accumulations (integrations).

Levels change smoothly but not instantaneously—there are no discontinuities, no jumps.

Look back at Example 3: Graphical Integration of Combined Flows in *Graphical Integration Exercises Part Five: Qualitative Graphical Integration.* The behavior of the net flow in the example contains two sudden, step changes. The value of the stock affected by the flow, however, cannot change instantaneously. Instead, when the slope of the stock graph changes as a result of the sudden change in the net flow, the value of the stock changes gradually, as shown in Figure 14.

Computer Models

Building computer models is an important part of system dynamics. System dynamics models, however, are not the only type of computer models. Various modeling techniques are now being used in an attempt to understand and forecast the behavior of complex systems. The following paper by Professor John Sterman explains the need for using computer models and examines the advantages and limitations of several types of models.

***- A Skeptic's Guide to Computer Models*[5]**

by John D. Sterman

This reading offers a detailed description of computer modeling and of the advantages and disadvantages of models. It takes a close look at some important modeling techniques (optimization, simulation, and econometrics), focusing on their assumptions and the possible implications of their application in foresight and policy making.

Please read *A Skeptic's Guide to Computer Models*... now.

[5]John D. Sterman, 1991. *A Skeptic's Guide to Computer Models* (D-4101-1), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

After reading *A Skeptic's Guide to Computer Models...*

The paper you just read emphasizes the advantage of using computer models over the use of mental models. Computer models, however, are not perfect, and must always be subjected to careful analysis before their conclusions can be used in policy making.

Several system principles about models can be drawn from the analysis in *A Skeptic's Guide to Computer Models*. We will return to them in later chapters of Road Maps. The first one explains the concept of abstract models.



System Principle #25:

Mathematical simulation models belong to the broad class of abstract models.

A model is a substitute for an object or a system. Some models are physical, such as a toy airplane or an architectural scale model. We are familiar with these. Some models are abstract. These abstract models include mental images, literary descriptions, behavior rules for games, and legal codes.

Mathematical simulation models also belong to the broad class of abstract models. Because computer modeling has become so widespread in recent years, it is important to understand the assumptions and applications of various modeling techniques.

The second system principle drawn from *A Skeptic's Guide to Computer Models* discusses model validity.



System Principle #26:
Model validity is a relative matter.

The usefulness of a mathematical simulation model should be judged in comparison with the mental image or other abstract model that would be used instead. No model is a perfect representation of a real object. A model is successful if it opens the road to improving the accuracy with which we can represent reality.

Finishing off Road Maps Nine

Road Maps Nine led you through building a simple system dynamics model of credit card spending and showed you how to formulate dimensionless and robust table functions. Your newly-acquired knowledge about table functions was then used in exercises in which you modeled systems exhibiting the overshoot and collapse behavior mode, using a simple generic structure. You also practiced your graphical integration skills in a more intuitive, noncomputational way. Finally, you improved your understanding of several types computer models and of models in general.

In Road Maps Nine, you were introduced to three new system principles:
a) that auxiliary variables lie only in the information links;
b) that mathematical simulation models belong to the class of abstract models; and
c) that model validity is a relative matter.

Key Terms and Concepts:

dimensionless inputs and outputs
econometric models
magnitude of a flow
model validity
multipliers
nonrenewable resource
normal values
optimization models
overshoot and collapse
quantitative vs. qualitative graphical integration
reference points
robustness
short-term benefit, long-term cost
simulation models
table function
value of a flow



The Credit Card Model

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Manas Ratha
June 16, 1997
Vensim Examples added October 2001

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ABSTRACT

Credit cards are one of many ways through which people borrow money. With a credit card, a person can make purchases without using cash. The credit card company pays for the purchase, but the card user has to repay the money borrowed to the card company at a later time. In addition to the amount of the purchase, the card user also has to pay interest on the loan. Interest on a loan is calculated as a fixed fraction of the loan amount and is charged for the time period of the loan.

Interest payments reduce the amount of money the user of a credit card can spend on regular purchases. This paper presents a model that studies the way interest payments affect the quality of the credit card user's life by reducing the amount of money he can spend on other purchases.

The reader is assumed to have some system dynamics modeling experience, and should build the model along with the paper.

1. INTRODUCTION

In recent years, the use of credit cards and other forms of credit has increased tremendously. The increase in borrowing has, in many cases, led to personal financial problems such as personal bankruptcies and bad credit histories. Having a bad credit history makes it more difficult and expensive for a person to make a loan later. Credit problems arise because people borrow too much money without realizing the cost of borrowing. In addition to having to repay the loan, the borrower also has to pay interest on the loan. Interest is the cost of borrowing money and is calculated as a fixed fraction of the total loan amount. Interest payments reduce the amount of money the borrower can spend on other expenses.

A common use of credit is through the use of credit cards. Consider a young man named Joe Flogg who recently graduated from college and found a new job. Five months after he started the job, Joe applied for a credit card, which he received a month later. This paper studies the way Joe uses his credit card. Using a credit card affects the quality of his life. Joe follows some simple policies while using his credit card. He calculates how much he can spend on the card and how much money he will pay back to the credit card company according to his policies. His spending and repayment policies are described later in the paper.

The article in Appendix 9.1 describes some recent problems that have emerged because of the excessive use of credit cards. The problem being modeled is the effect of the overuse of credit cards on the quality of life of credit card users. The paper leads the reader through the process of converting a written description of the problem into a system dynamics model. It is recommended that the reader build the model while reading the paper. The reader should also read "Building a System Dynamics model Part 1: Conceptualization"[1] before continuing to read this paper.

---

[1] Stephanie Albin, 1997. *Building a System Dynamics Model Part 1: Conceptualization* (D-4597), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, June 30, 36 pp.

2. USING A CREDIT CARD

Credit cards are issued by credit card companies. Joe's credit card, for example was issued by "International Express," a premier credit card company. When Joe makes a purchase using his credit card, International Express pays for his purchase. Thus, Joe owes International Express the amount of the purchase. The "Balance Payable" is the total amount of money Joe owes International Express. At the end of each month, International Express sends Joe a statement listing all purchases he charged to his card during the month and the "Balance Payable" from the previous month. The new "Balance Payable" is the sum of the new purchases charged this month, the "Balance Payable" from the previous month, and the interest charges on the previous month's "Balance Payable." Joe then pays a part or the entire amount of the new "Balance Payable." The unpaid part of the current "Balance Payable" and the interest on the unpaid part of the current "Balance Payable" will be added to next month's purchases to give next month's "Balance Payable." This model assumes that International Express starts charging interest on the amount of a purchase the moment Joe charges the purchase to his card. Read Appendix 9.2 to learn how credit card companies really charge interest on the "Balance Payable." As mentioned in the Introduction, interest is calculated as a fixed fraction, called "INTEREST RATE," of the "Balance Payable" for the time period that the "Balance Payable" is unpaid.

Joe uses his credit card for convenience, to avoid carrying large amounts of cash. When Joe does not have cash to pay for his purchases, he uses his credit card to pay for the purchases, planning to pay the credit card bill in the future. Therefore, to make the purchase, Joe is using money he does not have at the present time. He is taking a loan from International Express and will repay the loan at a later time. Therefore, he is using his future income today.

3. CONCEPTUALIZING THE USE OF A CREDIT CARD

The first step in building a model is identifying the problem to be analyzed. Appendix 9.1 describes some problems caused by the extensive use of credit cards by

people. The purpose of this paper is to model and understand the reasons and the ways using a credit card can affect the quality of Joe Flogg's life. This model will address primarily two issues. The first issue is the amount of Joe's debt, that is, his "Balance Payable." The second issue is the effect of the use of the credit card on the quality of Joe's life. Joe uses his card mostly for small expenses, such as eating at restaurants and going to movies. All benefits from his credit purchases are enjoyed today, and no purchases have a long-term effect on Joe's life. Purchases with a long-term effect would include furniture, a computer, or anything that can be used for a long period of time. Hence, the quality of Joe's life is defined by his current spending ability, that is, the total amount of money he can spend at the present moment, and not by purchases made in previous time periods.

The next step is to determine the model boundary and identify key variables. The model boundary depends on the issues that the model studies. The model boundary is set by deciding whether or not a certain variable is important in determining the system's behavior. The model boundary can be determined while studying the variables that affect the system.

The debt accumulated because of the use of the credit card is the amount Joe has to pay to International Express. The debt is called "Balance Payable." "Balance Payable" is a stock because it is the accumulation of all the money Joe has borrowed from International Express, plus the interest charged on the borrowed money, less the money he has paid back to International Express. "Balance Payable" increases when Joe charges a purchase to his credit card because Joe is borrowing more money from International Express. The flow of purchases charged to the credit card is called "credit card purchases." The interest charged on "Balance Payable" also increases the "Balance Payable." The inflow of interest is called "interest charges." "Balance Payable" is reduced by "payments" Joe makes to International Express. The "payments" reduce the amount of money Joe owes to International Express. The stock of "Balance Payable" and the flows into and out of the stock are shown in Figure 1. Figure 1 forms the core of the model.

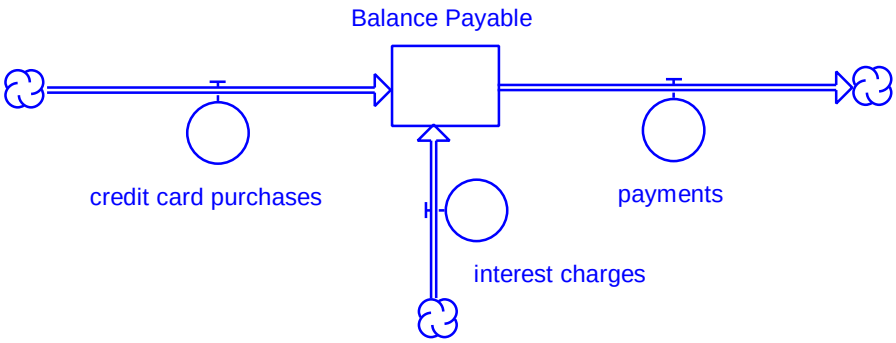


Figure 1: The stock and flows in the credit card system

Discussing the system and studying the stock and flows helps to identify other important variables in the system.

To spend any money at all, Joe must have a source of income. The source of income for most people is their weekly or monthly paycheck. Joe's paycheck is the amount of money he receives each month for the work he does at his job. Assume that Joe's paycheck is constant over the time period that the system is studied. Therefore, the model contains a constant called "PAYCHECK." Over a period of a few years, inflation will reduce the real buying power of Joe's paycheck amount. However, the effect of the inflation rate[2] is very small in the time period considered and is not important in the model in this paper. Thus, the inflation rate is outside the model boundary. This model also assumes that Joe's pay does not increase, and that he does not receive any bonuses.

The amount of money Joe can borrow on his credit card is limited. The maximum amount of money Joe can borrow is called the "CREDIT LIMIT." International Express limits the amount Joe can borrow so that he does not overcharge on his credit card. Because Joe's income is fixed, he may be unable to repay his debts if he borrows too much money. The "CREDIT LIMIT" also helps International Express limit losses in case Joe defaults, that is, if Joe does not repay the debt. If Joe defaults, the maximum amount of money International Express can lose is the "CREDIT LIMIT." For Joe, the "CREDIT LIMIT" is constant.

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[2] "Inflation rate" is the rate of increase of price levels in a certain region, usually measured for a country. Price levels in an economy are measured by the Consumer Price Index (CPI). Hence, the inflation rate is the rate of change of the CPI.

Different people have different spending habits and different ways of paying their bills. The spending and repayment policies people use can be formulated in many ways.

Joe has two sources of money to spend: the "PAYCHECK" and the credit available on his credit card. Joe's expenses include purchases, which determine the quality of his life, and "payments" to International Express. The "available credit" at any time is the maximum amount Joe can spend on his credit card at that time. The "available credit" is the difference between the maximum balance allowed, the "CREDIT LIMIT," and the current "Balance Payable." Joe spends a fixed fraction of the "available credit," called "SPENDING FRACTION," each month.

Each month, International Express sends Joe a statement, showing the "Balance Payable" and his credit purchases for that month. Joe does not, however, have to pay the entire balance each month. He is required to make a certain minimum payment, and the unpaid amount can be paid later. Joe has to pay an amount equal to or less than the "Balance Payable," but more than or equal to the required minimum payment. Because the "interest charges" are the expense on the debt, Joe pays only the "interest charges" each month. Therefore, "payments" equal "interest charges."

The model also studies the quality of Joe's life. The "quality of life" has been defined as the total value of purchases Joe can make at any time. The total value of the purchases equals the sum of the amount Joe charges to his credit card, the "credit card purchases" and the amount of his "PAYCHECK" he spends on "cash purchases." "Cash purchases" equal the amount left from the "PAYCHECK" after making the "payments." Joe cannot spend the entire "PAYCHECK" on "cash purchases," except when the "payments" are zero. "Payments" are zero only when the "Balance Payable" is equal to zero, that is, when Joe is not using the credit card.

4. FORMULATION OF THE MODEL

Section 4 describes the system and the variables. The equations of the variables must be formulated from the written description. The reader should understand the process of converting the words into mathematical terms. The italicized words in this

paper are helpful in finding the relationships between variables. For example, ë*of*' implies multiplication, ë*per*' means division, and ë*sum of*' means addition. The descriptions also help to find the units of the variables. '*Amount of money*' is measured in dollars and '*amount spent each month*' is in dollars per month.

The only stock in the system is the "Balance Payable." "Balance Payable" is the *amount of money* Joe owes to International Express. "Balance Payable" is increased by "credit card purchases" and "interest charges" and reduced by "payments" made to International Express.

Balance_Payable(t) = Balance_Payable(t - dt) + (credit_card_purchases + interest_charges - payments) * dt
INITIAL VALUE = 0
DOCUMENT: The "Balance Payable" is the amount of money Joe owes to International Express.
UNITS: Dollars

4.1 Inflow to "Balance Payable": "credit card purchases"

The "credit card purchases" are the purchases Joe makes on his credit card each month. The "credit card purchases" flow is a fixed fraction, the "SPENDING FRACTION," *of* the "available credit." The "available credit" is the maximum amount of money Joe can charge on his card and is equal to the *difference* between the "CREDIT LIMIT" and the "Balance Payable." Therefore, two factors affect the "credit card purchases" flow: the "available credit" and the "SPENDING FRACTION." The "available credit" is also affected by two factors: the "CREDIT LIMIT" and the "Balance Payable." Therefore, the systems diagram looks like Figure 2.

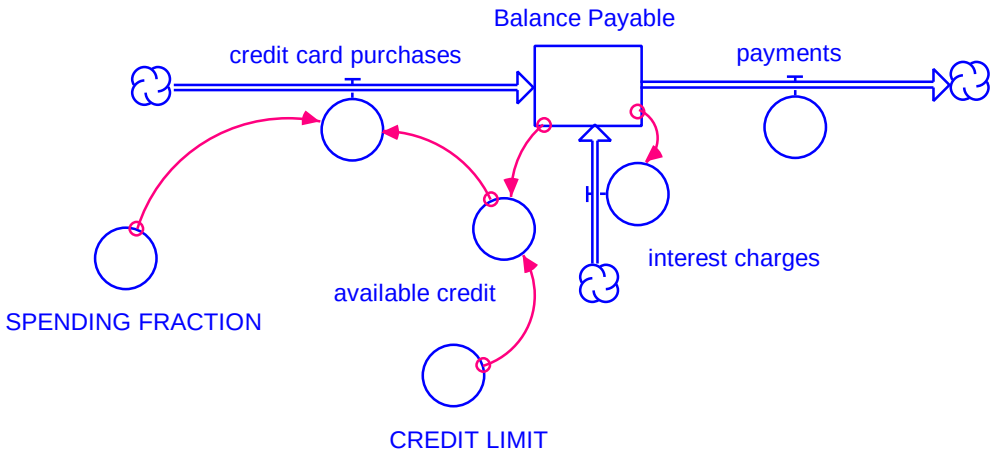


Figure 2: Inflow of "credit card purchases"

The "credit card purchases" represent the *amount of credit purchases made each month.* "Credit card purchases" is a fraction, the "SPENDING FRACTION," *of* the "available credit."

credit card purchases = SPENDING FRACTION * available credit

DOCUMENT: "Credit card purchases" is the amount Joe charges on his credit card each month, and is a fixed fraction, the "SPENDING FRACTION" of the "available credit."

UNITS: Dollars / Month

The "SPENDING FRACTION" is the fraction *of* the "available credit" Joe spends on "credit card purchases" each month. The "SPENDING FRACTION" is constant and can have any value between zero and one. A value of zero means that Joe never uses his credit card because he is not making any "credit card purchases." A value of one means that Joe spends all the "available credit" in the first month. Assume Joe spends ten percent, or 0.1 *of* the "available credit" *each month*.

SPENDING FRACTION = 0.1

DOCUMENT: The "SPENDING FRACTION" is the fraction of the "available credit" Joe spends on "credit card purchases" each month.

UNITS: 1 / Month

The "available credit" is the maximum *amount* Joe can spend on his credit card at any given time. The "available credit" is the *difference* between the maximum amount Joe is allowed to borrow, the "CREDIT LIMIT," and the "Balance Payable." The definition gives the following equation:

available credit = CREDIT LIMIT - Balance Payable
DOCUMENT: "Available credit" is the difference between the "CREDIT LIMIT" and "Balance Payable." The "available credit" is the maximum amount of money Joe can charge to his credit card at the time.
UNITS: Dollars

The "CREDIT LIMIT" is the maximum *amount* Joe can accumulate on his credit card. The "CREDIT LIMIT" is constant and is the maximum value the "Balance Payable" can have. Let Joe have a credit limit of $6000. Let time zero be the time when Joe started his job. Joe received his credit card six months after he started his new job. Thus, "CREDIT LIMIT" is zero for the first six months, and $6000 after six months. A STEP function can be used to model the "CREDIT LIMIT."[3]

CREDIT LIMIT = STEP (6000, 6)
DOCUMENT: The maximum amount Joe can charge on his credit card. It is the maximum value of the "Balance Payable."
UNITS: Dollars

4.2 Inflow to "Balance Payable": "interest charges"

Another inflow to the "Balance Payable" stock is the flow of "interest charges." As defined earlier, the "interest charges" on credit are computed as a fixed fraction, the

[3] The amount of the "CREDIT LIMIT" ($6000) and the time when Joe gets the credit card (six months) can be introduced as two separate constants. The "CREDT LIMIT" can then be modeled as a step function of the two constants instead of just the numbers. Introducing the constants makes changing the values of the constants more convenient.

"INTEREST RATE," *of* the "Balance Payable." The two factors affecting "interest charges" are the "INTEREST RATE" and the "Balance Payable," as shown in Figure 3.

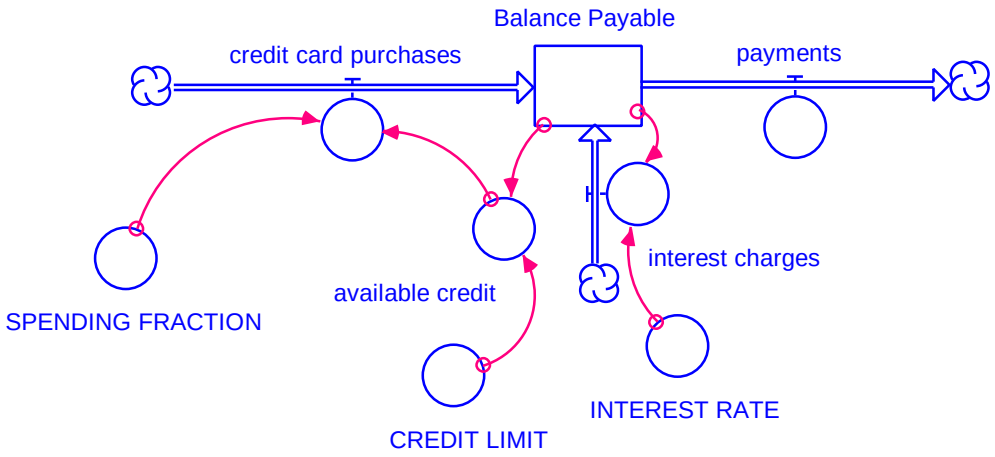


Figure 3: Inflow of "interest charges"

The "interest charges" are a fraction, the "INTEREST RATE," *of* the "Balance Payable." "Interest charges" are the *amount of money charged each month* on the loan.

interest charges = INTEREST RATE * Balance Payable

DOCUMENT: "interest charges" is the interest charged each month on the "Balance Payable."

UNITS: Dollars / Month

The "INTEREST RATE" is *constant*. International Express charges interest at 18% *per year*, or approximately 1.5% *per month.*[4]

INTEREST RATE = 0.015

DOCUMENT: The fraction of the "Balance Payable" charged as interest per month.

UNITS: 1 / Month

4.3 Outflow from "Balance Payable": "payments"

The only outflow from "Balance Payable" is the flow of "payments" made to International Express.

Joe pays only the "interest charges" on the "Balance Payable." Thus, the "payments" outflow is exactly equal to the inflow of "interest charges." Therefore, "payments" can be modeled as in Figure 4.

[4] Eighteen percent interest each year should equal 18? 12 = 1.5% per month. However, because interest is compounded, the monthly interest rate will be less than 1.5%. Read Appendix 9.3 to understand how interest on a credit card balance is calculated.

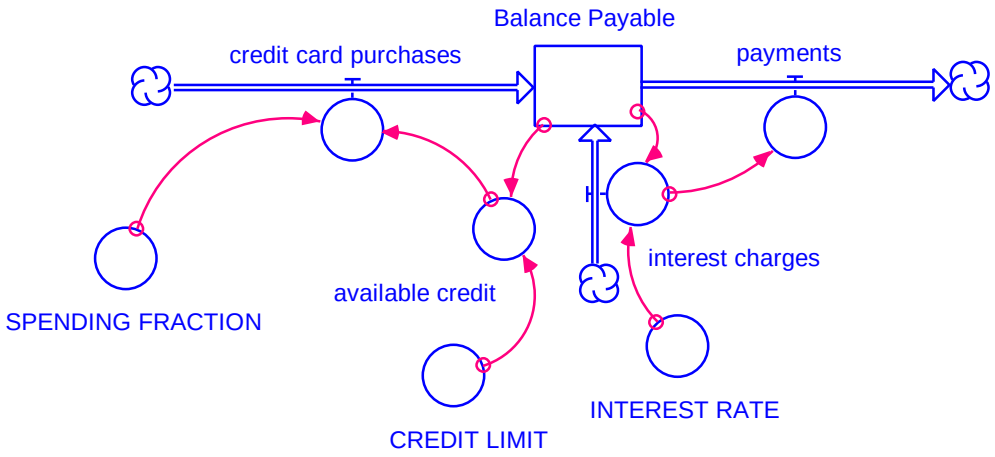


Figure 4: Outflow of "payments"

payments = interest charges

DOCUMENT: The "payments" is the money Joe pays to International Express.

UNITS: Dollars / Month

Flows cannot, however, be measured instantaneously. It is incorrect to set a flow equal to another flow. A new variable called "interest on balance," which is equal to the "interest charges," can be introduced. Thus, "interest charges" and "payments" both equal "interest on balance." The model changes as shown in Figure 5.

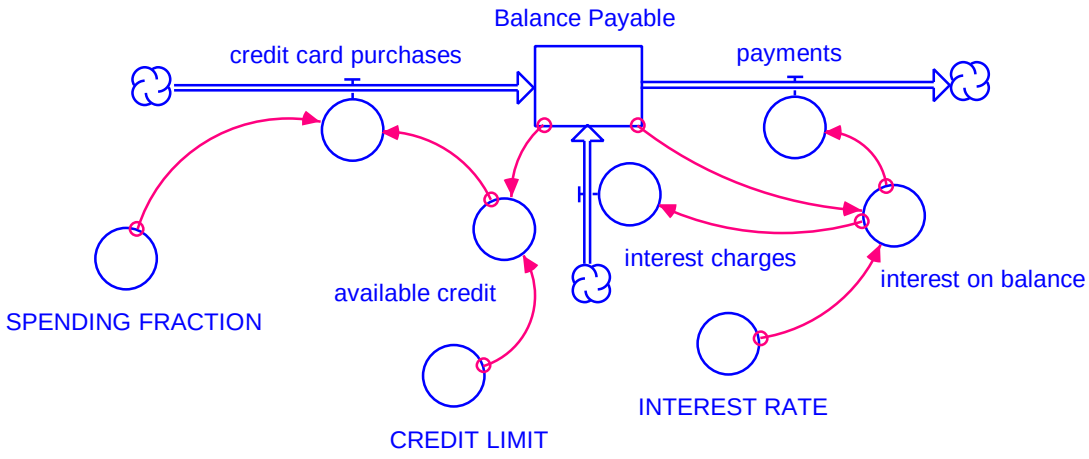


Figure 5: interest on balance

interest on balance = Balance Payable * INTEREST RATE

DOCUMENT: The interest charged on Joe's "Balance Payable" by International Express.

UNITS: dollars / Month

The equations for "interest charges" and "payments" also change to:

interest charges = interest on balance

payments = interest on balance

Introducing the variable "interest on balance" eliminates the problem of setting two flows equal to each other in the model. In principle, rates cannot control other rates. As shown, each rate comes from the same level and constant. Good modeling practice dictates that if easily avoidable, two flows should not be set equal to one another.

4.4 Other variables

Joe pays his credit card bills with money from his "PAYCHECK." By assumption, Joe spends the money remaining after paying the credit card bill on purchases. Purchases made with money from his "PAYCHECK" are called "cash purchases."

The "PAYCHECK," the amount Joe earns each month, is *constant*. The model does not consider raises in salary over time. At his present job, Joe earns $2000 *per month*.

PAYCHECK = 2000

DOCUMENT: The "PAYCHECK" is the amount of money Joe earns each month.

UNITS: Dollars / Month

The "cash purchases" equal the *amount left from* the "PAYCHECK" after paying the "interest charges," that is, after making the "payments." Because the "payments" are equal to the "interest on balance," "cash purchases," as shown in Figure 6, is the *difference* between the "PAYCHECK" and "interest on balance."

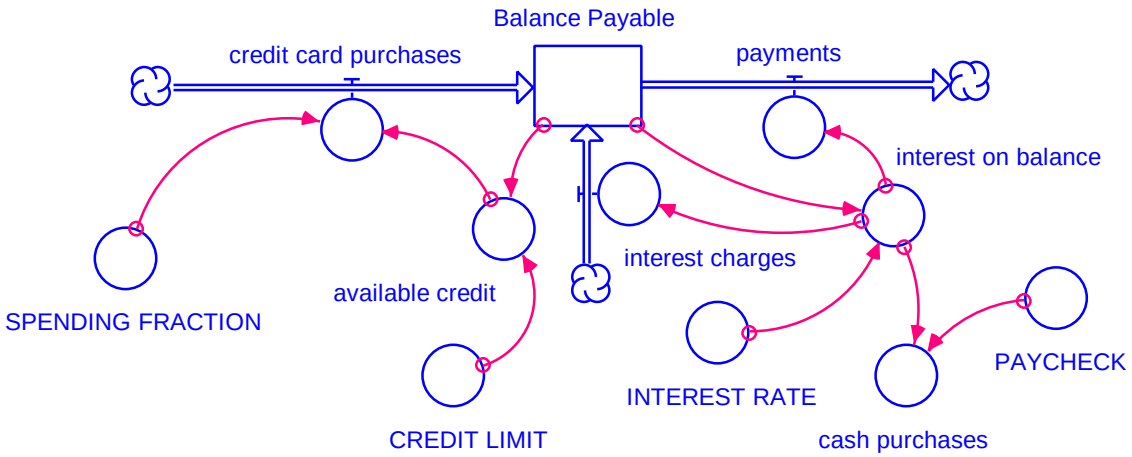


Figure 6: "cash purchases"

cash purchases = PAYCHECK - interest on balance

DOCUMENT: The purchases made each month with the money left from the "PAYCHECK" after making the "payments."

UNITS: Dollars / Month

By definition, the quality of Joe's life at any time is the total amount of money he can spend at that time. Joe can make cash purchases and credit purchases; hence, Joe's "quality of life," as shown in Figure 7 is the *sum* of "cash purchases" and "credit card purchases."

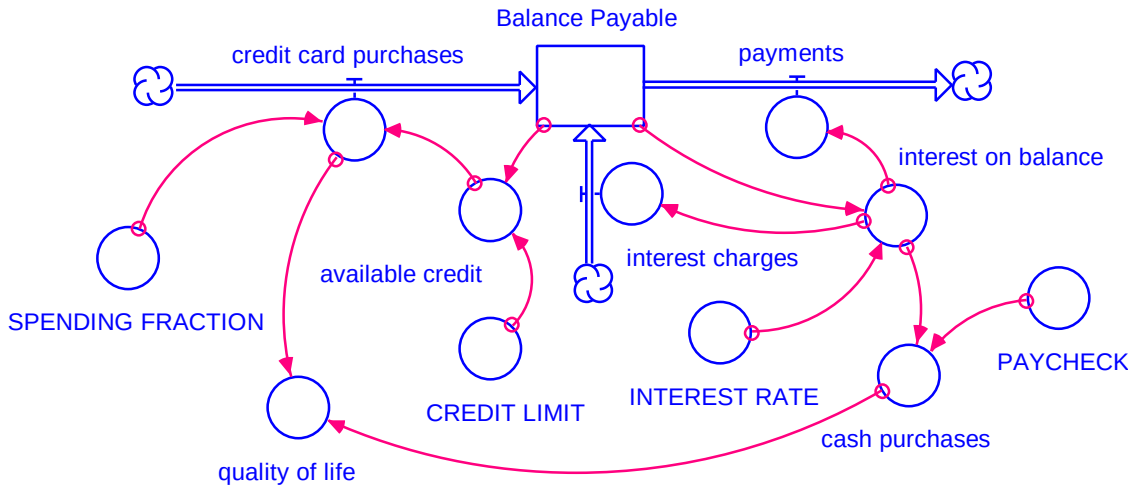


Figure 7: Model of the credit card system

quality of life = cash purchases + credit card purchases

DOCUMENT: The "quality of life" is the total amount of money Joe can spend at any time.

UNITS: Dollars / Month

The stock, flows, constants, and variables in the system have been modeled. Figure 7 shows the complete model of the system. The reader should not simulate the model yet.

The model is now complete but has not been edited. Editing is the process of deleting any part of the model that is not important in the system. Be sure to save the model in Figure 7 before making any changes.

5. EDITING THE MODEL

After building the model of the entire system, it is important to edit the model. Often, parts of the system are repeated, insignificant, or can be modeled in an easier way. Editing the model makes the model concise by simplification and more useful by removing insignificant parts.

In Figure 7, the "interest charges" inflow and the "payments" outflow are equal. Therefore, the "interest charges" inflow and "payments" outflow change the "Balance Payable" by equal amounts but in opposite directions. Thus, the combined effect of "interest charges" and "payments" on the "Balance Payable" is zero. Therefore, both "interest charges" and "payments" flows can be deleted to simplify the model. Deleting the flows should not affect the behavior of the model.

Editing a model may affect the equations of variables in a model. In Figure 7, only the "Balance Payable" is directly affected by the "payments" and "interest charges" flows.

The equation for the "Balance Payable" becomes:

Balance_Payable(t) = Balance_Payable(t - dt) + (credit_card_purchases) * dt
INIT Balance_Payable = 0

The final, edited model of Joe using a credit card is shown in Figure 8:

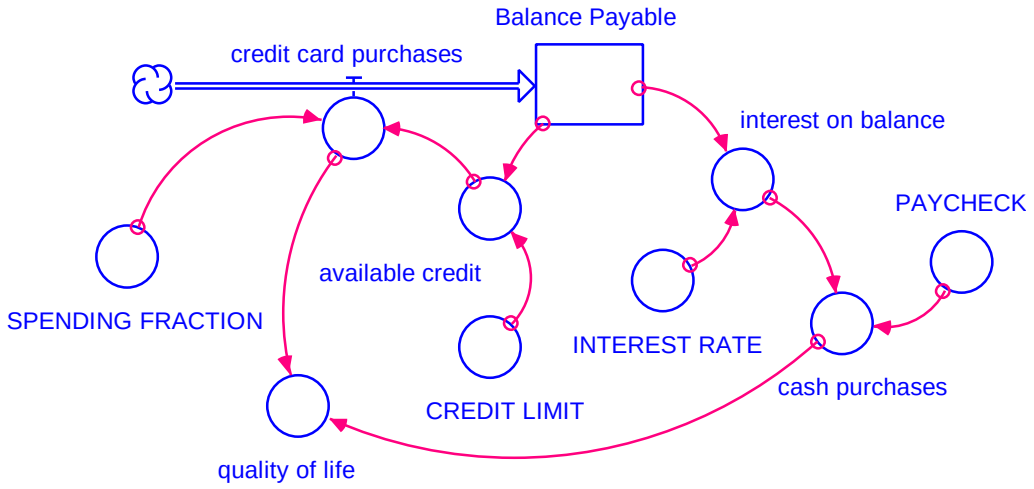


Figure 8: deleting "interest charges" and "payments"

Because "interest on balance" represents the interest charged on the "Balance Payable," the interest charges can be studied directly by observing the behavior of "interest on balance" over time.

After deleting the flows of "interest charges" and "payments" and changing the equation of "Balance Payable," the model looks like Figure 8. Save the model in Figure 8 as a new file. The model in Figure 7 will be required later. The documented equations for the model in Figure 8 are in the Appendix 9.3. Do not run the model yet.

The behavior of the model in Figure 8 should be the same as the behavior of the model in Figure 7. The only way to make sure that the behavior of the model has not been changed by the editing is to compare the behavior of the variables over time. Do not run the models at this time. Some questions about the models have to be answered first.

6. EVALUATING THE MODEL

The key variables in the model are "Balance Payable," "quality of life," and "interest payments." Read the equations for the variables and look at the feedback loops that affect them. Try to estimate the behavior of the variables and plot the expected behavior over time. Appendix 9.4 explains the process of mentally simulating the model to estimate the behavior of the system.

Now run both the model in Figure 7 and the edited model in Figure 8 over a time horizon of 72 months. A period of 72 months is probably a long enough time period to observe the long-term dynamics of the system. Change the time period and rerun the models to observe the behavior of the models over different time intervals. Compare the behavior of the models to make sure they demonstrate the same behavior. Clearly, editing the model did not change the behavior of the variables.

Analyzing a model gives insight to the feedback loops that drive the behavior of the system. Understanding the system's behavior makes it possible to formulate policies to improve the performance of the system.

The main purpose of the model is to study the quality of Joe's life, represented by the variable "quality of life." Look at the graph for "quality of life" and "PAYCHECK," shown in Figure 9.

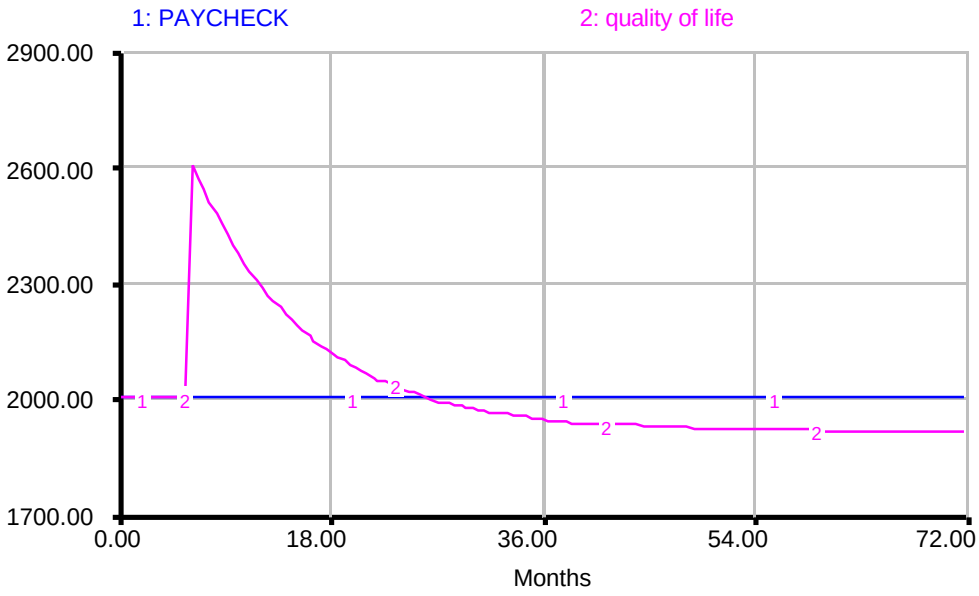


Figure 9: Behavior of "quality of life"

The quality of Joe's life equals his "PAYCHECK" when he does not have a credit card. When Joe does not use a credit card, he does not have to pay any interest because he owes no money to any credit card company. Therefore, Joe can spend his entire "PAYCHECK" on "cash purchases," and "quality of life" is equal to the "PAYCHECK."

The graph in Figure 9, therefore, shows the quality of Joe's life under two situations. The "PAYCHECK" curve shows the quality of Joe's life if Joe never uses a credit card. The "quality of life" curve shows the quality of Joe's life when he uses a credit card.

The "quality of life" is constant and equal to the "PAYCHECK" until Joe receives the credit card. Immediately after receiving the card, Joe's "quality of life" increases sharply. Joe now has a large amount of "available credit" to spend in addition to the "PAYCHECK." When he uses his credit card to make purchases, his "quality of life" increases because he can spend more than his "PAYCHECK." As Joe makes more "credit

card purchases," his "Balance Payable" increases. Therefore, the "available credit" decreases, and so do the "credit card purchases." The behavior of "credit card purchases" is driven by a negative feedback loop. As the "credit card purchases" decrease, Joe's "quality of life" also decreases. When Joe has spent an amount equal to the "CREDIT LIMIT," he cannot charge any more purchases to his credit card. Joe's "quality of life" is now equal to his "cash purchases." The "cash purchases" have an equilibrium value of about $1910 per month, as can be seen from a graph or table of the "quality of life." The equilibrium value of "cash purchases" can also be calculated by setting "credit card purchases" equal to zero and solving the equations for the variables to get the equilibrium values of "interest on balance," "cash purchases," "credit card purchases," and "quality of life." The "interest on balance" is $90 per month. The equilibrium value of "quality of life," $1910 per month, is lower than the value of the "PAYCHECK."

7. CONCLUSION

The quality of Joe's life rises when he starts using his new credit card. Over the next few months, however, the quality of his life gradually declines and reaches an equilibrium value. The equilibrium value is lower than the quality of Joe's life before he started using the credit card. The quality of Joe's life stays lower than the "PAYCHECK" as long as he follows his current policy of paying only the interest charges on the "Balance Payable." Therefore, using the credit card improves the quality of Joe's life for a short period of time, but reduces the quality of his life for a long period.

Joe also has a huge debt, the "Balance Payable," that he will have to repay to International Express. To repay the loan, Joe's payments will have to be larger than the "interest on balance," so his "cash purchases" will decline further. Therefore, repaying the loan will further reduce the quality of Joe's life.

The model presented in this paper is a classic example of the "short-term benefit, long-term cost" behavior of complex systems. Borrowing on credit cards allows an immediate increase in Joe's "quality of life" for a short period of time. The consequence in the longer term is a lower quality of life while repaying the loan and interest.

The system presented in this paper shows that a short-term benefit can prove to be expensive in the long-term. The trade-off in short-term and long-term results occurs not only in the system of Joe using his credit card, but in many other systems. Often, policies generate excellent results quickly, but if implemented for a long time, they prove to be detrimental. The opposite is also true. Most strategies which are beneficial in the long-term usually have negative effects in the short-term. Before taking actions or implementing a policy, the long-term effects of the policy should be considered.

8. APPENDIX

8.1 Credit Card Problems

An article in the September 25, 1996 issue of *The Wall Street Journal* carried an article titled "Banks' Marketing Blitz Yields Rash of Defaults."[5] The article describes the effect of the increased use of credit cards on the lives of many credit card users.

In 1995, credit card companies mailed out 2.7 billion pre-approved credit card applications. Credit card delinquencies soared in 1995, reaching a ten year high. Three point six percent of all credit card accounts were delinquent and personal bankruptcies increased by 23% compared to 1994. Many people interviewed for the article said that they charged too much to their credit cards because they had large credit limits, but later were unable to pay even the minimum required payments. In some cases, the minimum required payments were greater than the persons' income.

Clearly, the increasing personal bankruptcies and problems people face in paying credit card bills are a reason for concern. This paper studies the cause of large interest payments that make it more difficult for people to repay their credit card bills because the amounts to be paid back to the card company increase over time.

8.2 Credit Card Billing

Credit card companies send a statement to every card user once a month. The user has one month to pay the bill. During the one month allowed to pay the bill, no interest is charged on the card purchases made during the period covered by the statement. Therefore, if the card user always pays the entire amount on the statement within one month, the "Balance Payable" will be zero for the next billing period, and no interest will ever be charged. For example, the statement listing all the credit card purchases for the month of January will be mailed to Joe on about February 15. Joe is allowed until March

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[5] Laurie Hays, *"Banks' Marketing Blitz Yields Rash of Defaults."* The Wall Street Journal, Wednesday 25, 1996, page B1.

15 to pay the amount on the statement. Assume that at the beginning of January, Joe had a "Balance Payable" equal to $0.00. On February 15, for example, he receives the statement for $600.00. If Joe pays the $600.00 before March 15, he will not be charged any interest. If Joe continues the practice of paying the entire amount on the statement within a month of receiving the statement, he will never have to pay any interest. Therefore, it is possible to use a credit card and have a standard of living exactly equal to the paycheck.

In this paper, it is assumed that Joe does not have to pay an annual fee for his credit card. Many credit card companies require card users to pay an annual fee to use their credit card. In recent years, however, many credit card companies have canceled the annual fee to encourage people to use their card. It can, therefore, be assumed that Joe does not pay an annual fee on his card.

Another benefit of getting a credit card is the advertisement and discount coupons credit card companies usually send with the card. Coupons often give the card user frequent flier miles, airline discount coupons and other discount and gift coupons. If used, the coupons can save the card user a lot of money. There are also other services provided by credit cards, such as buyer replacement options, cash withdrawal, and other special deals which have been ignored in this paper.

Therefore, it is beneficial to own credit cards, but credit cards should not be abused, and the suggested policy of always paying the entire balance should be followed.

8.3 Equations: The Credit Card Model in Figure 8

Balance_Payable(t) = Balance_Payable(t - dt) + (credit_card_purchases) * dt
INIT Balance_Payable = 0
DOCUMENT: The "Balance Payable" is the amount of money owed to the credit card company by the card user.
UNITS: Dollars

INFLOWS:
credit_card_purchases = available_credit*SPENDING_FRACTION
DOCUMENT: DOCUMENT: "credit card spending" is the total amount charged on credit cards, and is a fixed fraction, the "SPENDING FRACTION" of the "available credit."
UNITS: Dollars / Month

available_credit = CREDIT_LIMIT-Balance_Payable
DOCUMENT: DOCUMENT: It is equal to the difference between the "CREDIT LIMIT" and "Balance Payable." The available credit is the maximum amount of money the card user can charge on the card at the time.
UNITS: Dollars

cash_purchases = PAYCHECK-interest_on_balance
DOCUMENT: DOCUMENT: The purchases made each month with the money left from the "PAYCHECK" after paying the "interest charges."
UNITS: Dollars / Month

CREDIT_LIMIT = STEP(6000,6)
DOCUMENT: DOCUMENT: The maximum amount Joe can charge on his credit card. It is the maximum value of the "Balance Payable."
UNITS: Dollars

interest_on_balance = Balance_Payable*INTEREST_RATE
DOCUMENT: The interest charged on Joe's "Balance Payable" by International Express.
UNITS: dollars / Month

INTEREST_RATE = 0.015
DOCUMENT: DOCUMENT: The fraction of the "Balance Payable" charged as interest per month.
UNITS: I / Month

PAYCHECK = 2000
DOCUMENT: DOCUMENT: The "PAYCHECK" is the money earned each month.
UNITS: Dollars / Month

quality_of_life = credit_card_purchases+cash_purchases
DOCUMENT: DOCUMENT: The quality of life is how much the card user purchases at any time.
UNITS: Dollars / Month

SPENDING_FRACTION = 0.1
DOCUMENT: DOCUMENT: The "SPENDING FRACTION" is the fraction of the "available credit" the user spends on credit purchases each month.
UNITS: 1 / Month

8.4 Estimating Model Behavior

The flow of "credit card purchases" is highest when "available credit" is highest. The "available credit" is highest when "Balance Payable" equals zero, that is at six months.

"Credit card purchases" increase the "Balance Payable," which reduces the "available credit," which in turn reduces the "credit card purchases." Lower "credit card purchases" still increase the "Balance Payable" but by a smaller amount than in the previous cycle because the amount of "credit card purchases" has declined. Therefore, the "credit card purchases" flow is constantly decreasing and exhibits asymptotic decay. The "Balance Payable" is the accumulation of the "credit card purchases" flow; therefore, "Balance Payable" increases and approaches a maximum value asymptotically. "Balance Payable" increases fastest as soon as Joe receives his credit card, and the "credit card purchases" are largest. The maximum value of "Balance Payable" is the "CREDIT LIMIT." Therefore, the "Balance Payable" asymptotically approaches the "CREDIT LIMIT," while "credit card purchases" asymptotically approach a value of zero dollars per month.

Estimating the behavior of "Balance Payable" and "credit card purchases" is not difficult. A first-order negative feedback loop[6] drives the behavior of the "Balance Payable" and "credit card purchases." The behavior of "quality of life" and "cash purchases" is more difficult to estimate.

Looking at the model and the equations, it is clear that:

? as "Balance Payable" increases over time, "interest on balance" increase;

? as "interest on balance" increase, "cash purchases" decrease.

Therefore, "cash purchases" and "credit card purchases" decline asymptotically over time. Hence, the "quality of life" also decreases.

"Interest on balance" is a fixed fraction of the "Balance Payable." Therefore, "interest on balance" also exhibits asymptotic growth. Therefore, "cash purchases" decline asymptotically and approach the value equal to "PAYCHECK" minus the equilibrium value of "interest on balance."

The "interest on balance" is highest when the "Balance Payable" is at the highest value, that is, equal to the "CREDIT LIMIT." Therefore, the maximum "interest on

---

[6] For an explanation of first-order negative feedback loops, read: Helen Zhu, 1996. *Beginner Modeling Exercises: Mental Simulation of Simple Negative Feedback* (D-4536), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, March 25, 1996, 23 pp.

balance" is equal to the "CREDIT LIMIT" times the "INTEREST RATE." Using the equations, the equilibrium value of "cash purchases" can be calculated.

After estimating the behavior of the models from Figure 7 and Figure 8, the reader should run the models and compare his graphs with the graphs of the behavior of variables after running the models.

8.5 Interest charges on credit card balances

Credit card companies calculate the interest rate in two steps. The annual interest rate (18% in the system this paper studied) is called the Annual Percentage Rate (APR). The APR is calculated by adding the U.S. prime rate and the variable rate. The U.S. prime rate is determined by the financial markets, and the credit card companies have no control over the prime rate value. The variable rate is set by the company. In May 1997, at the time of writing this paper, the prime rate was about nine percent, and most credit card companies charged a variable rate of about nine percent. Therefore, the APR for most credit cards was about 18% in May 1997[7].

Interest charges on loans are usually compounded. The formula for compound interest is:

Final Amount = Initial Amount * (1 + interest rate)^Time periods

Irrespective of whether the "annual interest rate" or the "monthly interest rate" is used, the "Final Amount" will be the same for a given "Initial Amount" and "interest rate."

Consider a time period of 1 year or 12 months.

Using the "annual interest rate," "Time" is equal to 1. Using the "monthly interest rate," "Time" is equal to 12. Let the "annual interest rate" = 18% = 18 ? 100 = 0.18.

Irrespective of whether the "annual interest rate" or the "monthly interest rate" is used in the calculations, the "Final Amount" and "Initial Amount" will be the same. Two

[7] Credit card companies usually offer a wide range of credit cards for different users. Different credit cards offered by the same company will have different APRs. For most cards, the APR is about 18%. For credit cards with very large credit limits and cards that offer many options and special services and features, the interest rate may be higher than 18%.

equations can be formulated. The time period when the monthly interest rate is used willbe "12," because there are 12 months in a year.

Equation 1: Final Amount = Initial Amount * (1 + annual interest rate)^1

Equation 2: Final Amount = Initial Amount * (1 + monthly interest rate)^12

Therefore, (1 + annual interest rate)^1 = (1 + monthly interest rate)^12

(1 + 0.18) = (1 + monthly interest rate)^12

Hence, the monthly interest rate is 0.01388, or 1.39%

8.6 Other forms of Credit and their expenses

Credit cards are only one of many forms of credit commonly used. Mortgages, bank loans, corporate and government bonds are other common forms of borrowing money. Various types of credit are different from each other, and are used for various purposes. Interest, however, has to be paid on every kind of loan. Due to the interest payments, the money that has to be paid back to the lender is more than the money borrowed.

Many countries have a budget deficit for many years. The government spends more money than it collects. The difference between the expenses and revenues is paid by taking loans from other nations and organizations and by issuing bonds. Bonds are financial instruments used to borrow money in financial markets. The amounts borrowed by countries can be large. Soon, the countries have to spend large sums of money just to service the debt, that is, pay the interest on the debt. As the amount of the debt increases, it becomes more difficult to repay the loan, and the interest payments become larger. Therefore, governments face the same problem as people who use credit cards carelessly, but on a much larger magnitude. [8]

[8] Under certain economic conditions, such as rapid growth, it may be more beneficial to the economy if the government borrows money and runs a deficit rather than balances the budget. Most countries that run a deficit, however, do not face these special conditions.

Vensim Examples :The Credit Card Model

By Aaron Diamond

October 2001

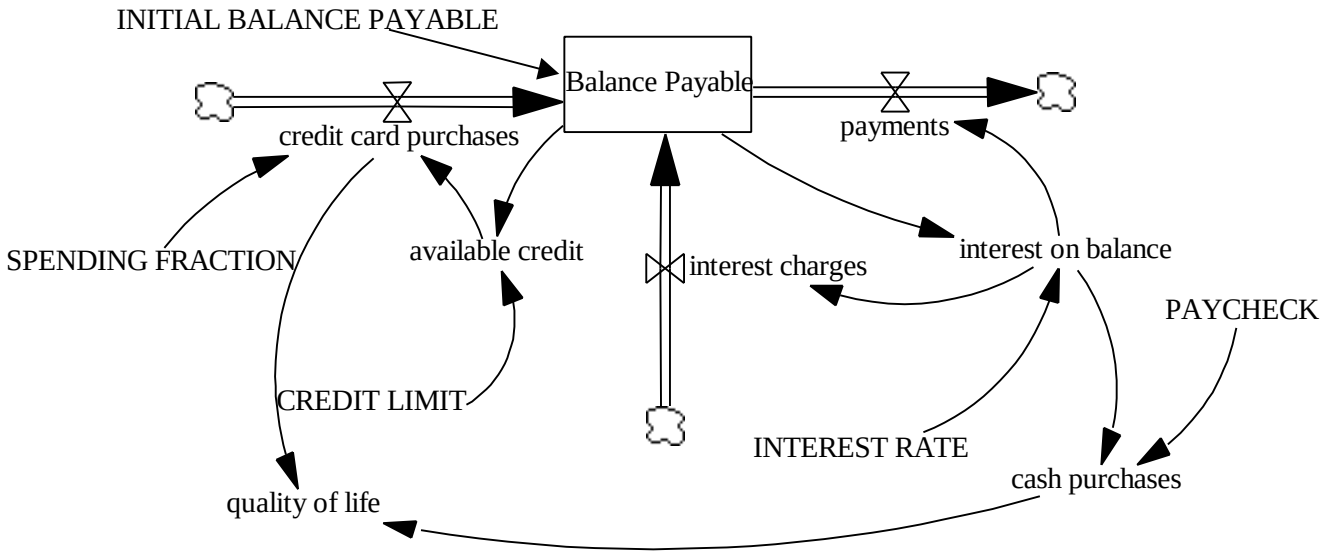


Figure 10: Vensim Equivalent of figure 7: Model of the credit card system.

Documentation for the credit card model

(01) available credit=CREDIT LIMIT-Balance Payable
Units: dollars
"Available credit" is the difference between the "CREDIT LIMIT" and the "Balance Payable". The "available credit" is the maximum amount of money Joe can charge to his credit card at the time.

(02) Balance Payable= INTEG (credit card purchases+interest charges-payments, INITIAL BALANCE PAYABLE)
Units: dollars
The "Balance Payable" is the amount of money Joe owes to International Express

(03) cash purchases=PAYCHECK-interest on balance
Units: dollars/Month
The purchases made each month with the money left from the "PAYCHECK" after paying the "interest charges."

(04) credit card purchases=SPENDING FRACTION*available credit
Units: dollars/Month
"Credit card purchases" is the amount Joe charges on his credit card each month, and is a fixed fraction, the "SPENDING FRACTION" of the "available credit".

(05) CREDIT LIMIT=STEP (6000,6)
Units: dollars
The maximum amount Joe can charge on his credit card. It is the maximum value of the "Balance Payable."

(06) FINAL TIME = 72
Units: Month
The final time for the simulation.

(07) INITIAL BALANCE PAYABLE=0
Units: dollars

(08) INITIAL TIME = 0
Units: Month
The initial time for the simulation.

(09) interest charges=interest on balance
Units: dollars/Month

"Interest charges" is the interest charged each month on the "Balance Payable."

(10) interest on balance=Balance Payable*INTEREST RATE
Units: dollars/Month
The interest charged on Joe's "Balance Payable" by International Express.

(11) INTEREST RATE=0.015
Units: 1/Month
The fraction of the "Balance Payable" charged as interest per month.

(12) PAYCHECK=2000
Units: dollars/Month
The "PAYCHECK" is the money earned each month.

(13) payments=interest on balance
Units: dollars/Month

(14) quality of life=cash purchases+credit card purchases
Units: dollars/Month
The quality of life is how much the card user purchases at any time.

(15) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.

(16) SPENDING FRACTION=0.1

Units: 1/Month

The "SPENDING FRACTION" is the fraction of the "available credit

"Joe spends on "credit card purchases each month.

(17) TIME STEP = 0.0625

Units: Month

The time step for the simulation.

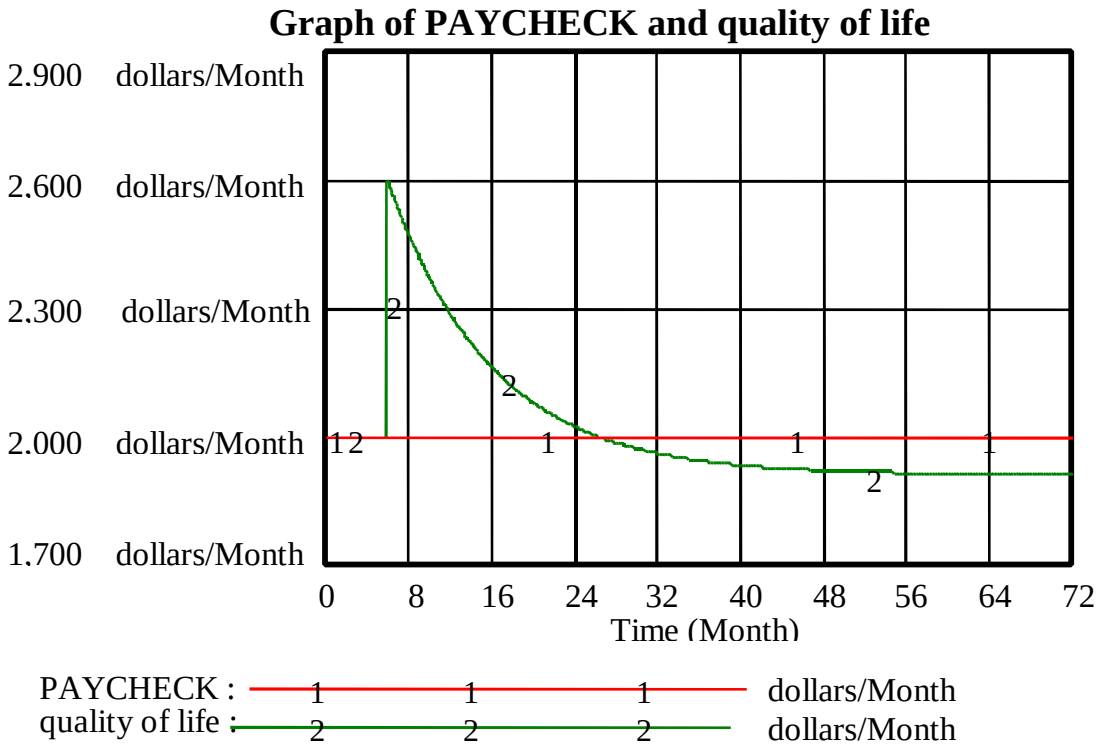
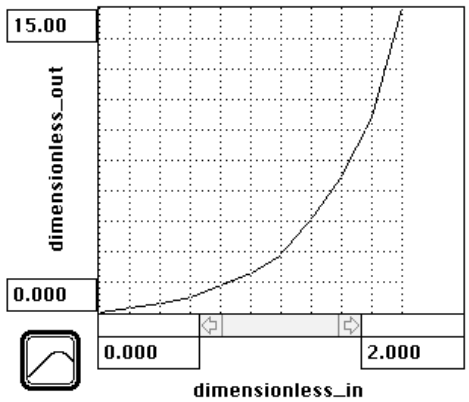


Figure 11: Vensim Equivalent of Figure 9: Behavior of “quality of life.”

Mistakes and Misunderstandings:

Table Functions



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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by
Leslie A. Martin
July 15, 1997
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1. THE SYSTEM

"A population model? Oh sure, I know how to make population models. They're all over Road Maps..."

I sat down at my desk and stared at the information I had gathered to build my model. Aardvarks. Stout, pig-like animals up to six feet long (hmm, longer than I am tall) with a long snout, rabbit-like ears, and short legs. Coat varies from glossy black and full to sandy yellow and scant. The aardvark is native to Africa. Today approximately one million aardvarks roam the savannah south of the Sahara. The name aardvark is Afrikaans for "earth pig."

Photo of aardvark removed due to copyright restrictions.

Figure 1: An aardvark

My population model needed to describe the spread of the aardvark population. A few aardvarks searching for food discover a lush savannah. They settle down and, as seasons pass, reproduce and grow in number. Typically, a young aardvark is born to each middle-aged female aardvark once a year. After several years, though, the savannah is no longer a paradise for aardvarks. Many aardvarks compete for the same food, the same shelters. Due to limited natural resources, aardvarks die at a faster rate. Eventually, the population in the savannah stabilizes.

I realized that the key to my population model would lie in correctly representing the constraint of the environment on the aardvark system. The death fraction is a

nonlinear function of the aardvark population. A small increase in population for a population that is small (when compared to the maximum population the environment can support) only slightly changes the death fraction. A small increase in population for a population that is large (with respect to the maximum population), however, causes a large increase in the death fraction. Therefore the relationship between population and death fraction is nonlinear. Table functions are excellent tools for modeling nonlinear relations.

Table functions are graphical relationships, usually nonlinear, between two variables. Table functions graph a table of data points. They allow a modeler to easily specify relationships that are not amenable to algebraic description. Table functions often aggregate several real processes into one relationship. In general, table functions are used for the representation of nonlinear variables. The effect of the constraint of the environment, which is negligible when relatively few aardvarks live in the savannah and overwhelming when many aardvarks roam around, is a nonlinear variable.

2. A First Attempt to Model the System

In my first attempt to model the aardvark system, I created the model in Figure 2. Aardvarks are the total number of aardvarks currently living in the savannah. The number of aardvarks is increased by births and decreased by deaths. In the model the birth fraction is constant while the death fraction changes with the current number of aardvarks.

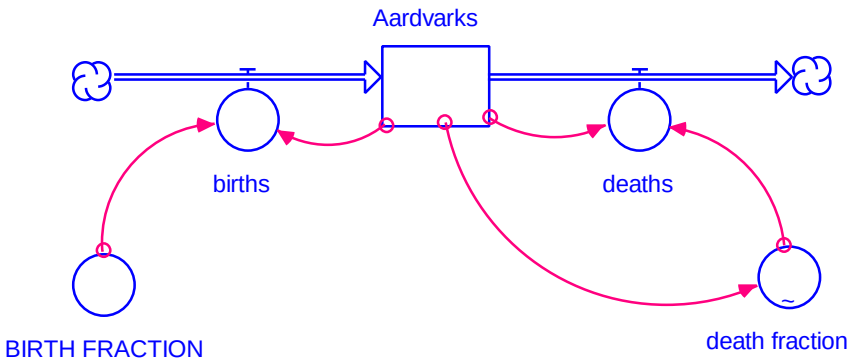


Figure 2: My first aardvark population model

Female aardvarks give birth to one baby each year. Estimating that about half of the aardvark population is female, and more than half of those females are able to reproduce (not too young, not too old) then approximately one third of the aardvarks are able to give birth each year. So I entered an approximate "BIRTH FRACTION" of 0.3 aardvarks/aardvark/year (which is equivalent to 0.3 per year).

I then tackled the "death fraction". I plotted the "death fraction" as a function of the current number of "Aardvarks" in the table function shown in Figure 3. My first step in obtaining the curve was to identify the values of the "death fraction" for its extreme cases, the limits of the curve. I know that aardvarks have an average lifespan of 10 years. So initially the "death fraction" is 0.1 per year.[1] Very large aardvark populations, however, are constrained by the environment. I referred to an encyclopedia to find out that, at most, 500 aardvarks live in a 1000 acre savannah. So when a 1000 acre savannah holds 500 aardvarks, the population should be at equilibrium... which means that the number of aardvark deaths must equal the number of births. Therefore, when the population of aardvarks reaches 500 animals, the "death fraction" should equal the "BIRTH FRACTION", 0.3 per year. My next step in obtaining the curve was to estimate the overall shape of the curve connecting the extreme points. The curve between 0.1 and 0.3, between the two extremes, increases slowly for relatively small populations when natural resources abound and then increases sharply for large populations as the natural resources become scarce.

[1] The death fraction is the reciprocal of the time constant. Here, the time constant is the average lifespan of an aardvark, the time for an aardvark to die, 10 years.

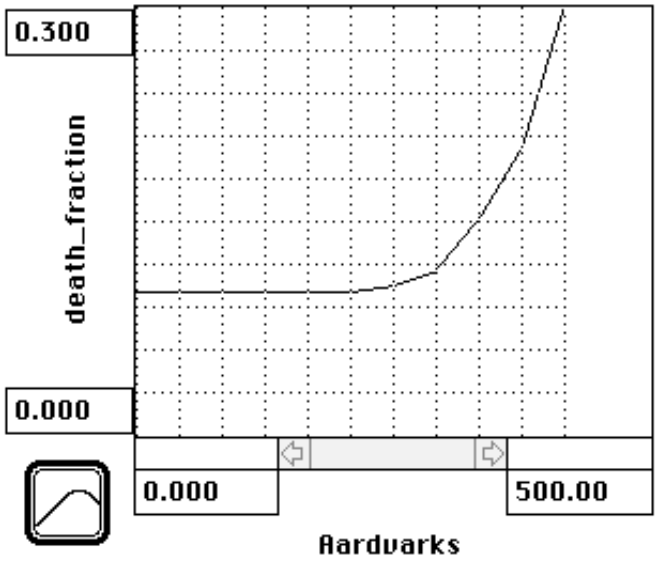


Figure 3: The table function for my first model

With my table function complete, I ran the model and analyzed the graphs I obtained. Figure 4 depicts the behavior of the aardvark population model: S-shaped growth. Proud of my modeling skills, I submitted my work.

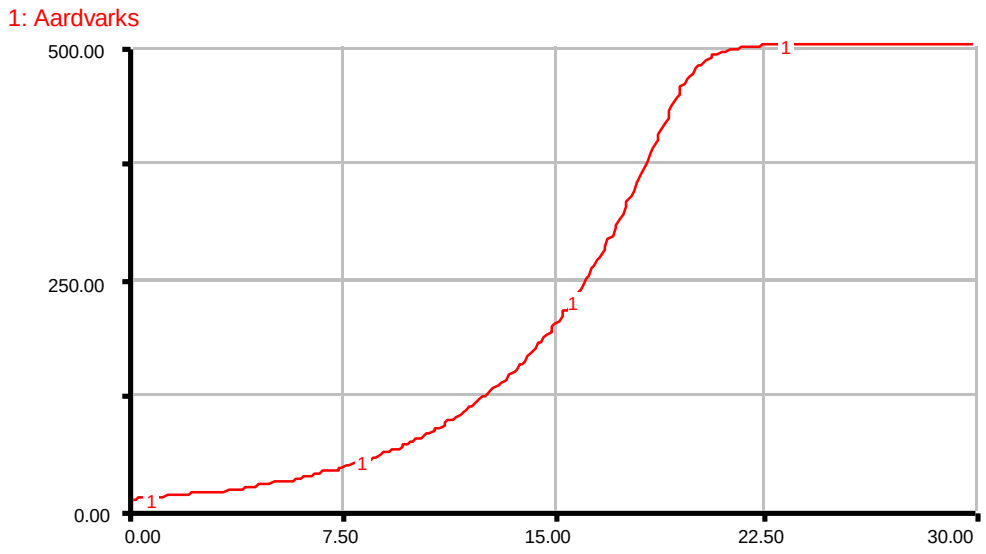


Figure 4: Behavior of the aardvark population

3. FIRST MISTAKE AND MISUNDERSTANDING

One week later the professor returned our papers. I eagerly flipped the paper over and my jaw dropped as I couldn't even recognize my work, buried beneath a bloody trail of red ink. I quickly hid it in my notebook and did not look at it again until late that afternoon, when I had reached the privacy of my own room. Bold letters along the front page blared "inputs and outputs to a table function must be dimensionless." I did not see why dimensionless inputs and outputs were such a big deal. My model produced the right type of behavioróso where was the problem? Convinced that my model had been unfairly criticized I took it over to my friend Mathildaís house to complain about the injustices of this world. Mathilda looked at my model and then looked back up at me, surprised. She asked me to run through the loops and explain what my units were. I showed her that the "births" flow is the product of the number of "Aardvarks" and a fraction per year, which combine to give a flow of aardvarks/year. Likewise, the "deaths" flow is the product of the number of "Aardvarks" and another fraction per year, which also combine to give a flow of aardvarks/year. Then I suddenly realized my mistake. The "death fraction" as I had modeled it was not in units of 1/year but in units of aardvarks, because the current number of aardvarks drives the table function. I wanted my table function to output units of 1/year, but I had given it an input of aardvarks. Table functions do not mysteriously change units; whatever units go into the table function will come right out. I groaned; I had submitted a model that violated dimensional consistency. I started to feel that I might have deserved all the red editorial ink.

4. A SECOND ATTEMPT TO MODEL THE SYSTEM

Mathilda went to get her laptop and we sat down at her desk to look at my model. She explained that my model needed normals and multipliers. A normal is a reference point that is not changed by the behavior of the system. My model implicitly contained two reference points. I had hidden two critical parameters in my table function: a maximum aardvark population and an initial death fraction. The maximum population of

aardvarks in a 1000 acre savannah, a “normal” number of aardvarks, is 500 aardvarks. The maximum population works as a constraint on the system representing the limited natural resources. I decided to call this ratio of “Aardvarks” to the maximum aardvark population “fraction occupied.” The variable “fraction occupied” denotes what fraction of land and natural resources are currently taken up by aardvarks. The extent of crowding in a savannah does not depend on an *absolute* number of aardvarks but depends on a *relative* number of aardvarksóthe current number of aardvarks compared to the maximum number that the natural resources of the savannah are able to sustain. The “normal” death fraction is the death fraction that would occur under “normal” conditions: when the aardvark population grows freely without being constrained by the environment. The “normal” death fraction is 0.1 per year. As the environmental conditions change, the death fraction changes relative to the fraction of aardvarks that would die under “normal” conditions.

Mathilda and I decided to add the normal values to my model. We drew up a new model of the aardvark population that is depicted in Figure 5.

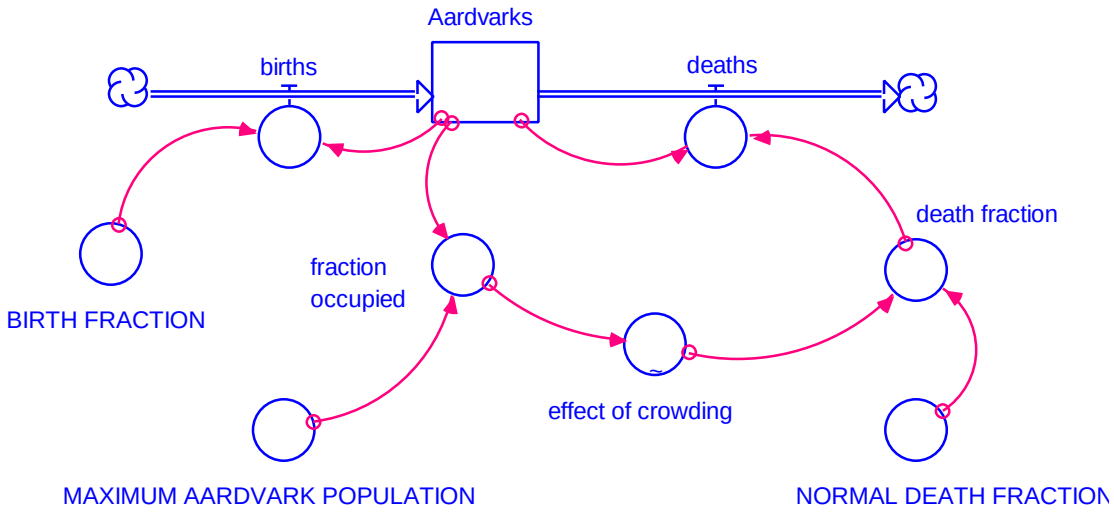


Figure 5: My second model with normals and a multiplier

In the new model both of the normal values are explicit: they have been pulled outside of the table function. The normal values are constants; they do not change as the aardvark population evolves. The changing effect of the number of aardvarks on the death

rate is captured by a multiplier: the "effect of crowding." The "effect of crowding" is a multiplier because it multiplies the "NORMAL DEATH FRACTION" with a weight that represents the changing conditions of the system, the fraction of land and resources occupied by the aardvarks. The "fraction occupied" is calculated by taking the ratio of "Aardvarks" to the "MAXIMUM AARDVARK POPULATION." Insignificant at first, the "effect of crowding" multiplier increases as the fraction of land and resources occupied increases. The multiplier then combines with the "NORMAL DEATH FRACTION" to form a "death fraction" that increases as crowding becomes severe. Under normal conditions, when natural resources are abundant and the population can grow without being constrained by the environment, the "effect of crowding" multiplier must be 1.0 so that the "death fraction" equals the "NORMAL DEATH FRACTION." The curve of the table function in the "effect of crowding" provides the nonlinear relationship between the relative number of "Aardvarks" and the extent to which the "death fraction" will change beyond the "NORMAL DEATH FRACTION."

I drew the table function in Figure 6 by first considering how the behavior of the aardvark system changes under extreme conditions. When just a few "Aardvarks" live in the savannah, the fraction of land and resources occupied is minimal (the fraction is close to 0), and crowding is negligible. The multiplier must then be 1.0 so that the "death fraction" will equal the "NORMAL DEATH FRACTION." When the 1000 acre savannah is populated with 500 aardvarks, then the number of "Aardvarks" equals the "MAXIMUM AARDVARK POPULATION", the "fraction occupied" is 1.0, the "effect of crowding" is 3.0, and the system has reached its equilibrium. The "death fraction" equals three times the "NORMAL DEATH FRACTION," which equals the "BIRTH FRACTION" of 0.3 per year. In between the extremes, I know that the multiplier increases slowly when there are few aardvarks and increases drastically when the population is close to reaching its limit. Therefore the slope of the curve is small—almost flat—at first and then steepening as the population increases rapidly.

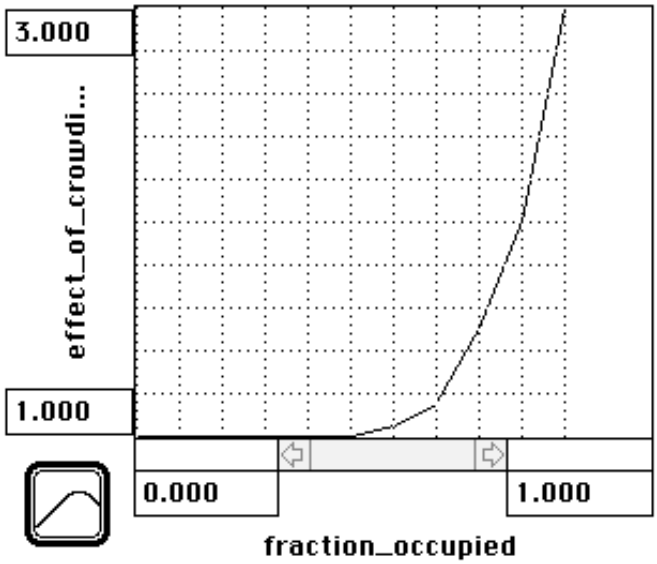


Figure 6: Table function with dimensionless inputs and outputs

Notice that in Figure 6 the input to the table function, the "fraction occupied," is the ratio of "Aardvarks" to the "MAXIMUM AARDVARK POPULATION," and is *dimensionless*. The input is not an absolute number of aardvarks, but the current number of aardvarks relative to a normal. The output is therefore also dimensionless. The output effect of crowding multiplies the "NORMAL DEATH FRACTION" (units of 1/year) with a dimensionless constant to form a "death fraction" that also has units of 1/year. The model is now dimensionally consistent. I ran the model and again obtained the same S-shaped growth as shown in Figure 4. I realized that reasonable behavior alone did not prove that I had a reasonable model. Sleepy, I printed out my model, slipped it into my backpack, and decided to ask my professor to please take another look at it.

5. SECOND MISTAKE AND MISUNDERSTANDING

A few days later my professor handed me back my model a second time. He smiled. "Well, this model is definitely much better than your first, but it still isnít robust," he said. I did not understand what he meant. "Your exercise for tonight," he explained, ìis to find out what it means for a model to be robust. I have one hint for you: try changing the birth fraction from 0.3 to 0.4 per year and think carefully about the results you obtain." After class I went straight over to Mathilda's house. We brought the model

up onto the screen and changed the "BIRTH FRACTION" from 0.3 to 0.4 per year. Figure 7 shows the results of our simulation.

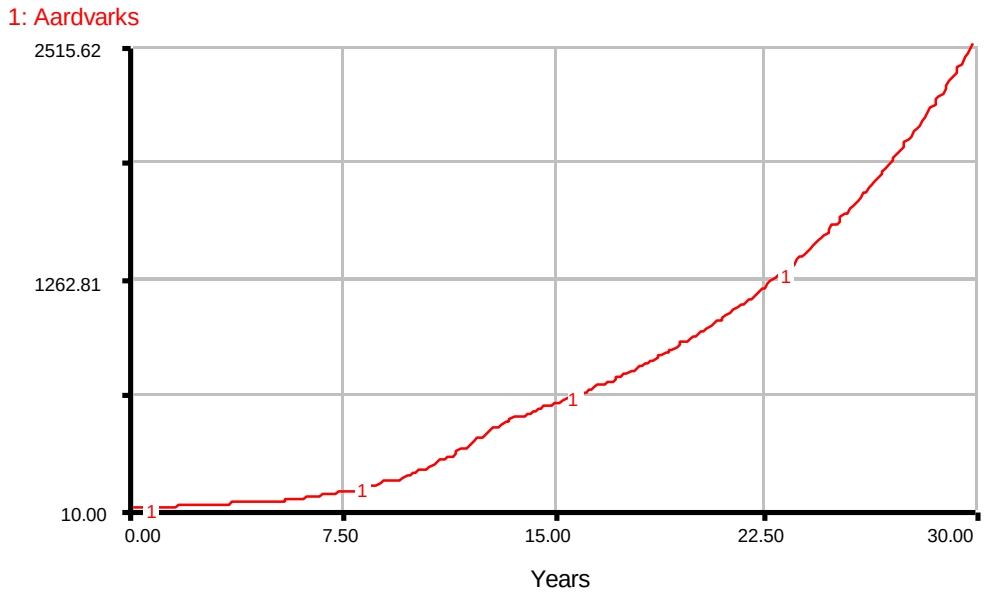


Figure 7: Behavior generated with a "BIRTH FRACTION" of 0.4 per year

6. OVERCOMING OUR MISTAKES AND MISUNDERSTANDINGS

Why did the model exhibit exponential growth? I was puzzled until I remembered that I had carefully crafted my table function to reach equilibrium when the "death fraction" caught up to the "BIRTH FRACTION," assuming a "BIRTH FRACTION" of 0.3 per year. When the number of aardvarks reaches the size of the maximum aardvark population, the "death fraction" reaches a maximum of 0.3 per year. But the "BIRTH FRACTION" of 0.4 is larger than the "death fraction" so the population continues to grow. The "death fraction" is limited by my choice of the endpoints of the table function, so it remains at 0.3 per year. Crowding therefore only slows down—but does not stop—the growth of the population. My awkward formulation of the table function, not the underlying dynamics of the system, generated an alternate, unrealistic form of behavior for the aardvark population. When my professor said that my model was not robust, he meant that the behavior that my model generated was not realistic for the entire range of

possible, realistic parameter values. The approximated aardvark birth fraction of 0.3 per year has a certain degree of uncertainty; an aardvark birth fraction could just as easily be 0.2 or 0.4 per year, depending on the specific system being modeled. My model was not robust because, although it worked fine for a “BIRTH FRACTION” of 0.3 per year, it broke down for a just-as-likely “BIRTH FRACTION” of 0.4 per year. My table function was only valid for very specific parametersóchange the parameters and my model goes from stabilizing the number of aardvarks to unrealistically growing the population exponentially. A strong dependence on a choice of parameters can mean different things. On one hand, I could have just discovered a critical leverage point within my system. On the other hand, my model couldn’t and didn’t simply suffer from not being robust.

I decided to expand my table function in order to make my model robust. Study the new table function in Figure 8.

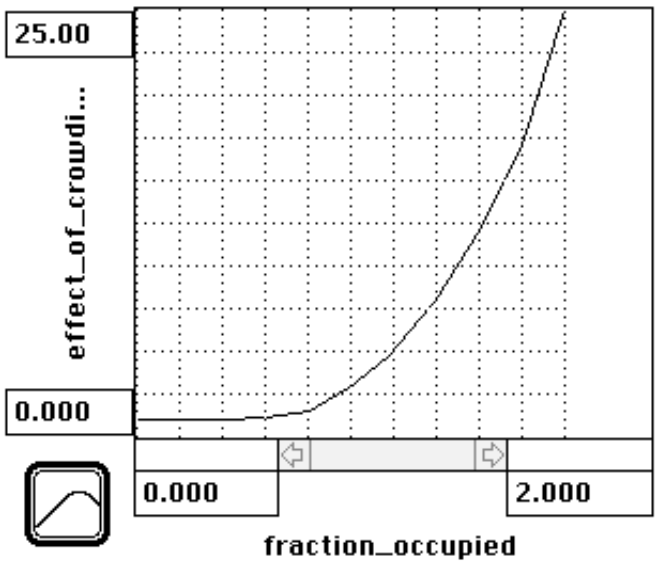


Figure 8: A robust, dimensionless table function

Now, if the aardvark population rises above the MAXIMUM AARDVARK POPULATION (if the “fraction occupied” is greater than 1.0) then the “effect of crowding” drastically increases the “death fraction” and the aardvark population stabilizes. The table function is ready to adjust the crowding multiplier accordingly for a whole range

of realistic parameter values. I ran the model again with a "BIRTH FRACTION" of 0.4 per year and obtained the graph in Figure 9.

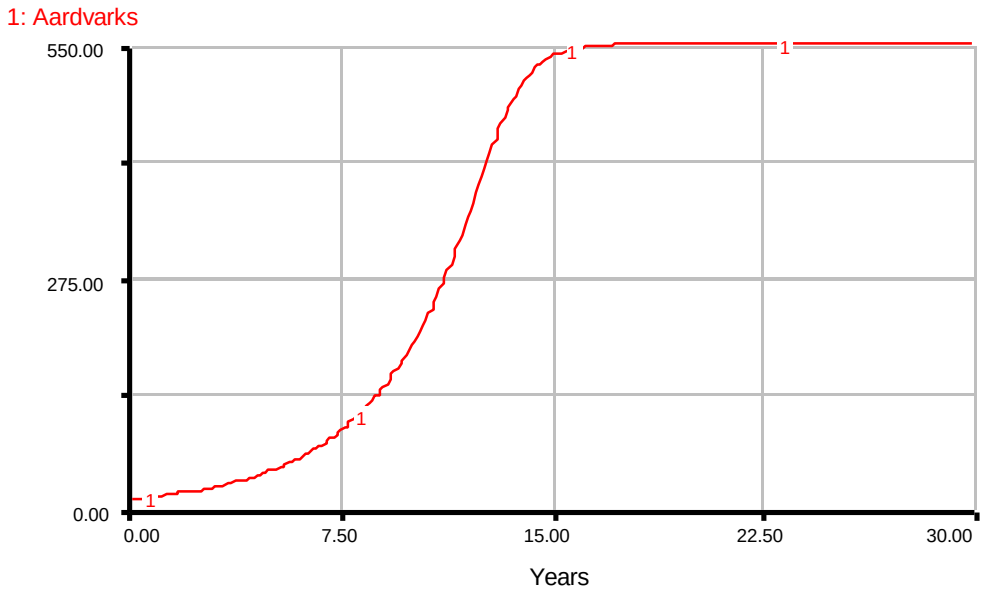


Figure 9: The corrected model run with a "BIRTH FRACTION" of 0.4 per year

I printed out another copy of my model with the new table function and a set of runs with different parameter values. All of the runs showed S-shaped growth. The next morning I handed my model to the professor for a third time. He checked the papers carefully, congratulated me for my perseverance, and then complimented me on a job—finally—well done.

7. KEY LESSONS

Table functions are useful for incorporating nonlinear relations into models. They allow modelers to easily specify nonlinear relationships between variables visually rather than algebraically. To be effective, however, table functions must be dimensionally consistent and robust. Table functions should have dimensionless inputs and outputs. Modelers nondimensionalize inputs with the use of normals. Normals are constants that act as a reference point for dynamic behavior. Multipliers, on the other hand, reflect the changing conditions within the system. To be robust, table functions must also extend to

cover the entire range of reasonable parameters values. The behavior of most real world systems is often insensitive to changes in parameters. For a model to be robust, the behavior that it generates must be as insensitive to changes in parameters as is the behavior of the system being modeled.

8. APPENDIX: EQUATIONS FOR THE CORRECTED MODEL

□ **Aardvarks(t)** = Aardvarks(t - dt) + (births - deaths) * dt
INIT Aardvarks = 10
DOCUMENT: the number of aardvarks in the current population
Units: aardvarks

☞ INFLOWS:
Births = BIRTH_FRACTION*Aardvarks
DOCUMENT: the number of baby aardvarks born each year
Units: aardvarks/year

☞ OUTFLOWS:
Deaths = death_fraction*Aardvarks
DOCUMENT: the number of aardvarks that die each year
Units: aardvarks/year

○ **BIRTH_FRACTION** = 0.3
DOCUMENT: the fraction of aardvarks each year that give birth
Units: aardvarks/aardvark/year or 1/year

○ **death_fraction** = effect_of_crowding*NORMAL_DEATH_FRACTION
DOCUMENT: the fraction of the population of aardvarks that dies each year.
Units: aardvarks/aardvark/year or 1/year

○ **fraction_occupied** = Aardvarks/MAXIMUM_AARDVARK_POPULATION
DOCUMENT: the fraction of the population of aardvarks that dies each year.
Units: aardvarks/aardvark/year or 1/year

○ **MAXIMUM_AARDVARK_POPULATION** = 500
DOCUMENT: the maximum number of aardvarks that can comfortably live in a 1000 acre savannah
Units: aardvarks

○ **NORMAL_DEATH_FRACTION** = 0.1
DOCUMENT: the fraction of the population of aardvarks that die each year when natural resources are abundant

Units: 1/year

⊘ **effect_of_crowding** = GRAPH(fraction_occupied)
(0.00, 1.00), (0.2, 1.00), (0.4, 1.00), (0.6, 1.05), (0.8, 1.50), (1.00, 3.00), (1.20, 5.00), (1.40, 8.00), (1.60, 12.0), (1.80, 17.0), (2.00, 25.0)
DOCUMENT: When few aardvarks roam the savannah, crowding has little effect on the death fraction. As the fraction of land and resources occupied by the aardvarks increases, the effect of crowding increases, increasing the death fraction.
Units: dimensionless

Vensim Examples:

Mistakes and Misunderstandings: Table Functions

By Aaron Diamond

January 2000

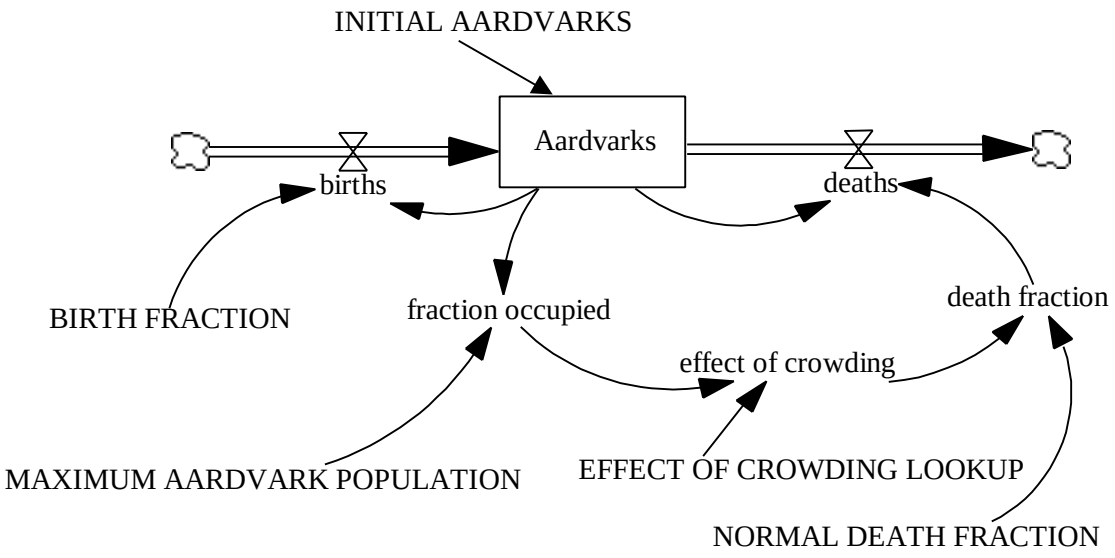


Figure 10: Vensim Equivalent of Figure 5: My second model with normals and a multiplier

Documentation for Aardvark model:

(01) Aardvarks= INTEG (births-deaths, INITIAL AARDVARKS)
Units: aardvarks
The number of aardvarks in the current population.

(02) BIRTH FRACTION=0.3
Units: 1/year
The fraction of aardvarks each year that give birth.

(03) births=BIRTH FRACTION*Aardvarks
Units: aardvarks/year
The number of baby aardvarks born each year.

(04) death fraction=effect of crowding*NORMAL DEATH FRACTION
Units: 1/year
The fraction of the population of aardvarks that dies each year.

(05) deaths=death fraction*Aardvarks
Units: aardvarks/year
The number of aardvarks that die each year.

(06) effect of crowding= EFFECT OF CROWDING LOOKUP (fraction occupied)
Units: dmnl
When a few aardvarks roam the savannah, crowding has little effect on the death fraction. As the fraction of land and resources occupied by the aardvarks increases, the effect of crowding increases, increasing the death fraction.

(07) EFFECT OF CROWDING LOOKUP=([(0,0)(10,40)],(0,1),(0.2,1), (0.4,1), (0.6,1.05),(0.8,1.5),(1,3),(1.2,5),(1.4,8),(1.6,12),(1.8,17),(2,25))
Units: dmnl

(08) FINAL TIME = 30
Units: year
The final time for the simulation.

(09) fraction occupied=Aardvarks/MAXIMUM AARDVARK POPULATION
Units: dmnl
The number of Aardvarks relative to the maximum number of aardvarks that can comfortably live in the 1000 acre savannah.

(10) INITIAL AARDVARKS=10
Units: aardvarks

(11) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(12) MAXIMUM AARDVARK POPULATION=500
Units: aardvarks
The maximum number of aardvarks that can comfortably live in a 1000 acre savannah.

(13) NORMAL DEATH FRACTION=0.1
Units: 1/year
The fraction of the population of aardvarks that die each year when natural resources are abundant.

(14) SAVEPER = TIME STEP

Units: year

The frequency with which output is stored.

(15) TIME STEP = 0.0625

Units: year

The time step for the simulation.

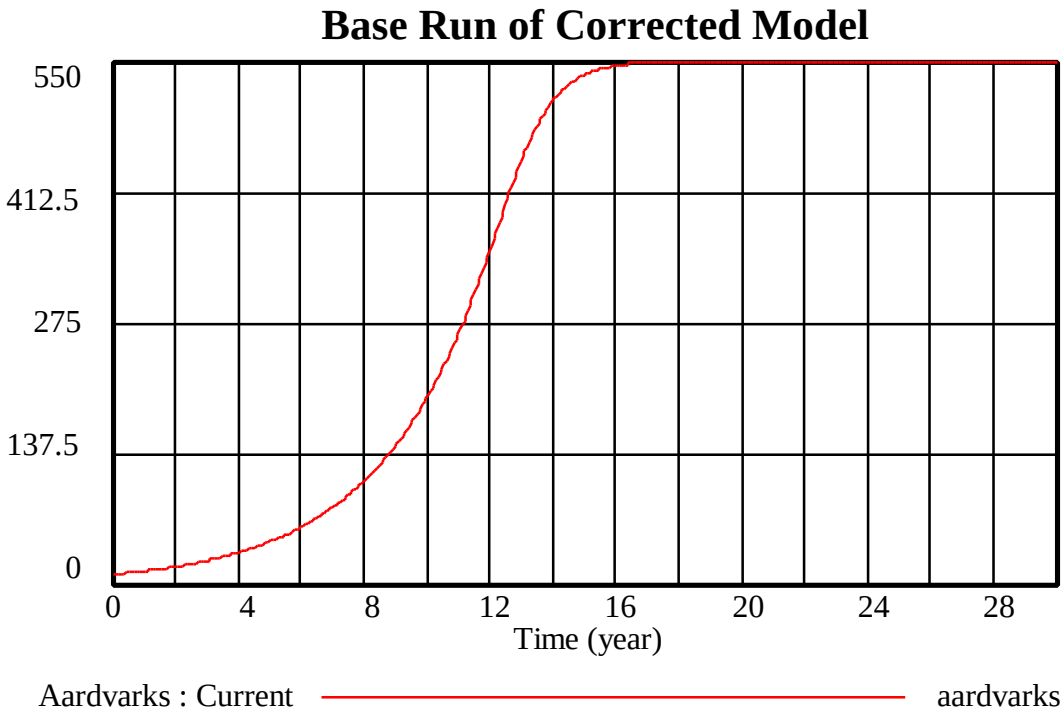
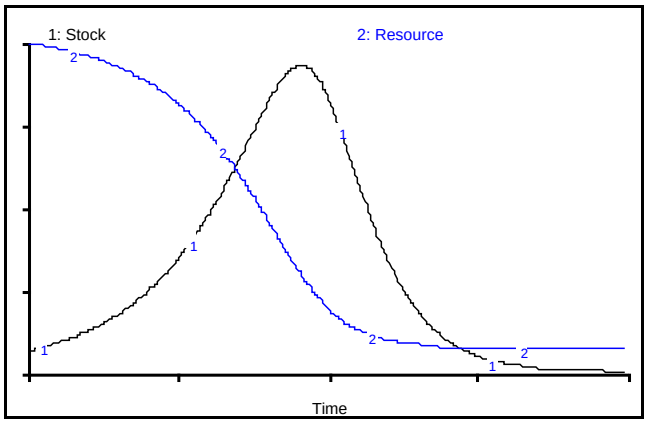


Figure 11: Vensim Equivalent of Figure 9: The corrected model run with a “Birth Fraction” of .4 per year

Generic Structures:
Overshoot and Collapse



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
Lucia Breierova
July 21, 1997

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1. INTRODUCTION

"Goldfield, Arizona was a busy bustling little town perched atop a small hill between the mighty Superstition Mountains to the east and the Goldfield Mountains to the west. The first gold-strike was made in 1892, and it took just a little under a year before the town came to life. One rich strike after another supported the town and approximately 4000 people who lived in the district for five rich years. The town boasted 3 saloons, a boarding house, general store, blacksmith shop, brewery, meat market and a school house. Just when it looked like the town would outgrow Mesa,[1] the vein faulted, the grade of ore dropped and the town died a slow painful death."[2]

Goldfield, Arizona is today one of the many ghost towns that litter the Western United States. The history of this town is an example of the behavior of overshoot and collapse. When gold was discovered in the nearby hills, the town sprung up as miners flocked in, eager in their rush for gold. The population grew rapidly as new miners arrived, and stores, saloons, and schools followed. Meanwhile, however, the miners were extracting the gold from the hills. After a few years, gold became harder and harder to find. When the miners came home with empty pans, they decided to move on, to follow the trail of gold. They abandoned their small town and, within months, Goldfield, Arizona lay deserted, a ghost town.

Overshoot and collapse is a behavior in which a stock, in the case of Goldfield, the population of the town, relies upon a depletable resource, gold. When the gold is abundant, the population grows quickly. As the population grows, however, it depletes the supply of gold. When gold becomes scarce, the population drops rapidly as the miners leave to try their luck elsewhere.

This paper introduces a structure that produces the behavior of overshoot and collapse and examines other behaviors that the structure can generate. Several examples illustrate the study of systems containing the structure. The exercises at the end of the paper provide the reader with an opportunity to see how knowledge about structure and behavior can be transferred between different systems.

---

[1] Mesa is a city in Mariscopa county, south-central Arizona. Its site was settled in 1878 by Mormons. (From the Encyclopædia Britannica.)

2. OVERSHOOT AND COLLAPSE: THE BEHAVIOR

Overshoot and collapse is a common behavior of complex systems. Figure 1 shows its characteristic behavior.

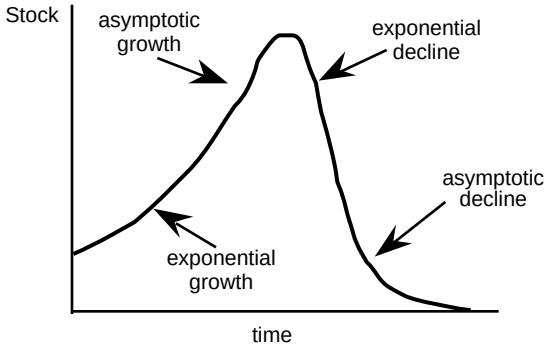


Figure 1: Overshoot and collapse

At first, the system's behavior seems satisfactory—the stock of interest, such as the rapidly growing population of Goldfield, Arizona, grows exponentially. Exponential growth, however, is not sustainable. The increasing stock soon starts to cause the depletion of a necessary and often nonrenewable resource, such as gold. The stock's growth slows down. Then, after some time, the declining resource causes the growth to stop completely. Because of the lack of the resource, the stock starts decreasing and quickly collapses—as when all the gold miners abandon Goldfield.

The following examples present two systems that exhibit the behavior of overshoot and collapse. Each example first examines the structure of the system and then explains how the structure produces the observed behavior.

2.1 Example 1: Petroleum Extraction

The first example examines the number of wells in a petroleum field. Companies drill wells to tap into the petroleum reserves under the ground. As more wells extract petroleum, the reserves start to decline, so the drilling of new wells slows down. In addition, the lower level of petroleum reserves reduces the amount of petroleum that each well can extract. As the extraction per well falls, some of the wells stop extracting and close. Eventually, the recoverable reserves are depleted, no more extraction is possible, and all wells close.

Figure 2 shows a possible model of the system; documented equations are in section 7.1 of the Appendix.

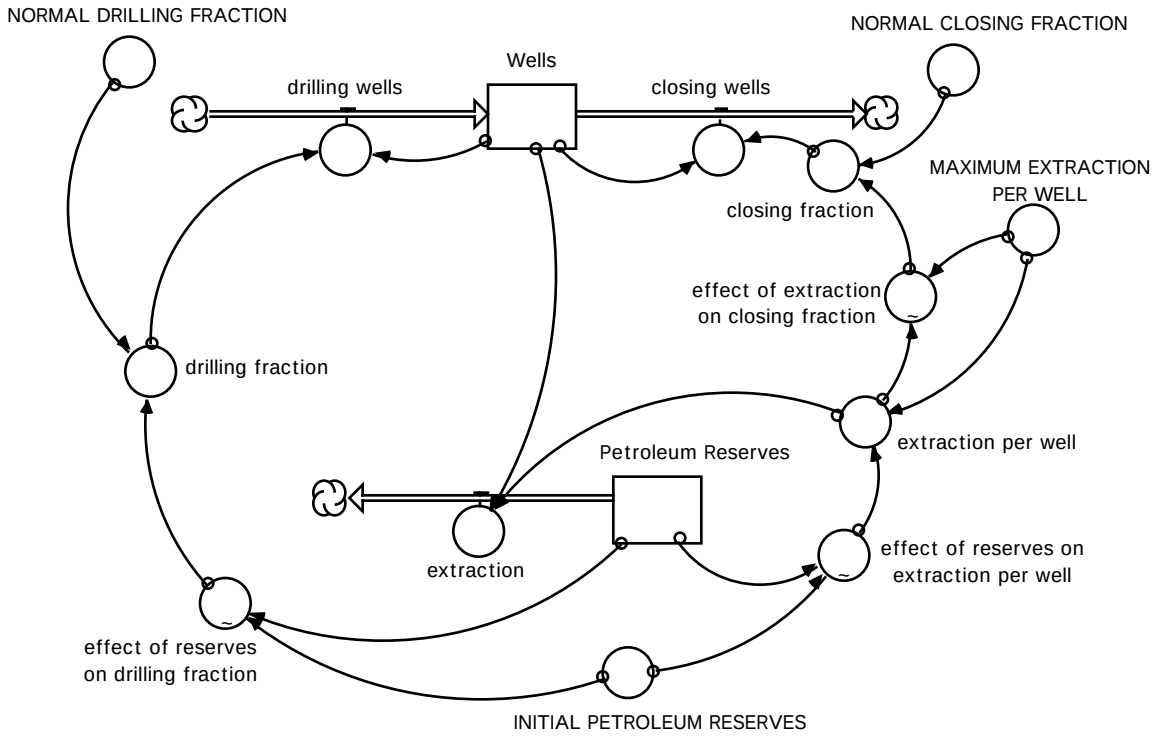


Figure 2: Model of petroleum extraction

The number of "Wells" on a field increases by an inflow of "drilling wells" dependent on the number of "Wells" and on a variable "drilling fraction." The "drilling fraction" is the product of a "NORMAL DRILLING FRACTION" and a multiplier called "effect of reserves on drilling fraction." The multiplier expresses a positive relationship between the fraction of "Petroleum Reserves" remaining and the "drilling fraction."[3] The fraction of "Petroleum Reserves" remaining is the ratio of the current level of "Petroleum Reserves" to the "INITIAL PETROLEUM RESERVES." A decrease in the ratio means that less "Petroleum Reserves" remain, so less petroleum can be extracted, and the incentive to drill new wells is lower. Hence, the "drilling fraction" decreases.

The stock of "Wells" also has an outflow, "closing wells," which is the product of a "NORMAL CLOSING FRACTION" and an "effect of extraction on closing fraction." The "effect of extraction on closing fraction" is a nonlinear function showing the effect of "extraction per well," as compared to "MAXIMUM EXTRACTION PER WELL," on the "closing fraction." A decrease in the ratio of "extraction per well" to "MAXIMUM EXTRACTION PER WELL" means that extraction is becoming more difficult, and some owners choose to close the wells. The "closing fraction" therefore increases. The actual "extraction per well" is the product of "MAXIMUM EXTRACTION PER WELL" and

[3] A positive relationship between two variables means that the variables move in the same direction.

an “effect of reserves on extraction per well” multiplier. The multiplier reflects a positive relationship between petroleum extraction and reserves. As the fraction of “Petroleum Reserves” remaining decreases, extraction becomes more difficult, so the “extraction per well” decreases.

Finally, because petroleum is a nonrenewable resource, the stock of “Petroleum Reserves” has no inflow. The outflow from “Petroleum Reserves” is the total amount of “extraction” per year, which is the product of “extraction per well” and the number of “Wells.”

Figure 3 shows the behavior generated by the petroleum extraction model.

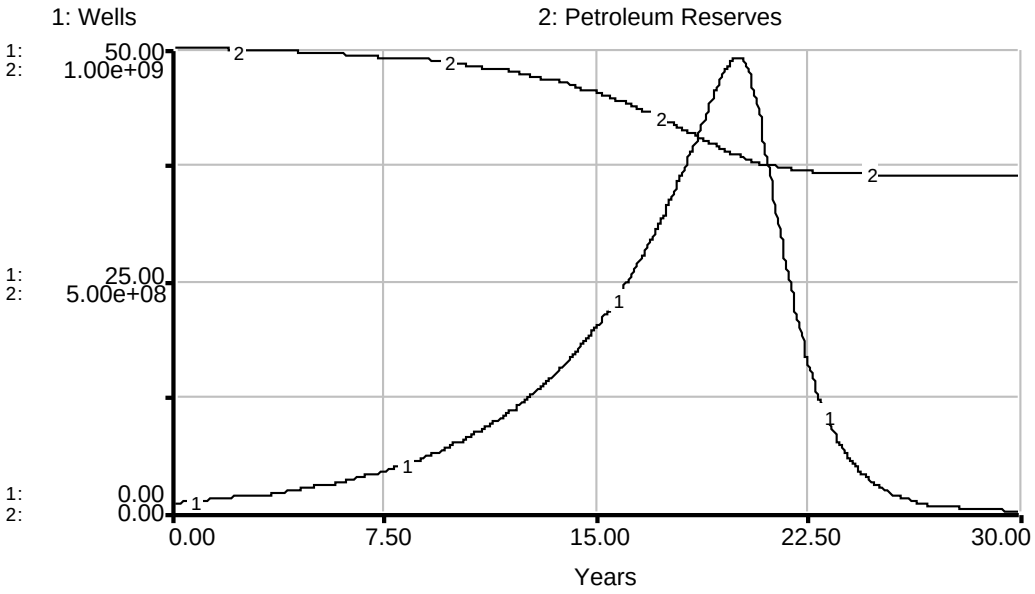


Figure 3: Behavior of “Wells” and “Petroleum Reserves”

Initially, the number of “Wells” is low but growing exponentially, starting the depletion of “Petroleum Reserves.” As “Petroleum Reserves” fall, extraction becomes more difficult. Hence, “extraction per well” decreases, causing an increase in the “closing fraction” through the “effect of extraction on closing fraction.” In addition, the declining “Petroleum Reserves” cause a decrease in the “drilling fraction,” further slowing down the growth of “Wells.” Eventually, the “closing fraction” becomes larger than the “drilling fraction,” and the outflow of “closing wells” exceeds the inflow of “drilling wells.” The number of “Wells” starts declining. Because the number of “Wells” is still large, “Petroleum Reserves” continue to decline. The number of “Wells” decreases rapidly. As long as any “Wells” remain, the decline of “Petroleum Reserves” continues. Finally, after 25% of the “Petroleum Reserves” have been depleted, extraction

becomes nearly impossible.[4] So the "Wells" close even though 75% of the initial amount of "Petroleum Reserves" still remains.

2.2 Example 2: Deer Population

The second example examines the behavior of a deer population with a limited amount of vegetation available. Figure 4 shows a model of the system. Documented equations are in section 7.2 of the Appendix.

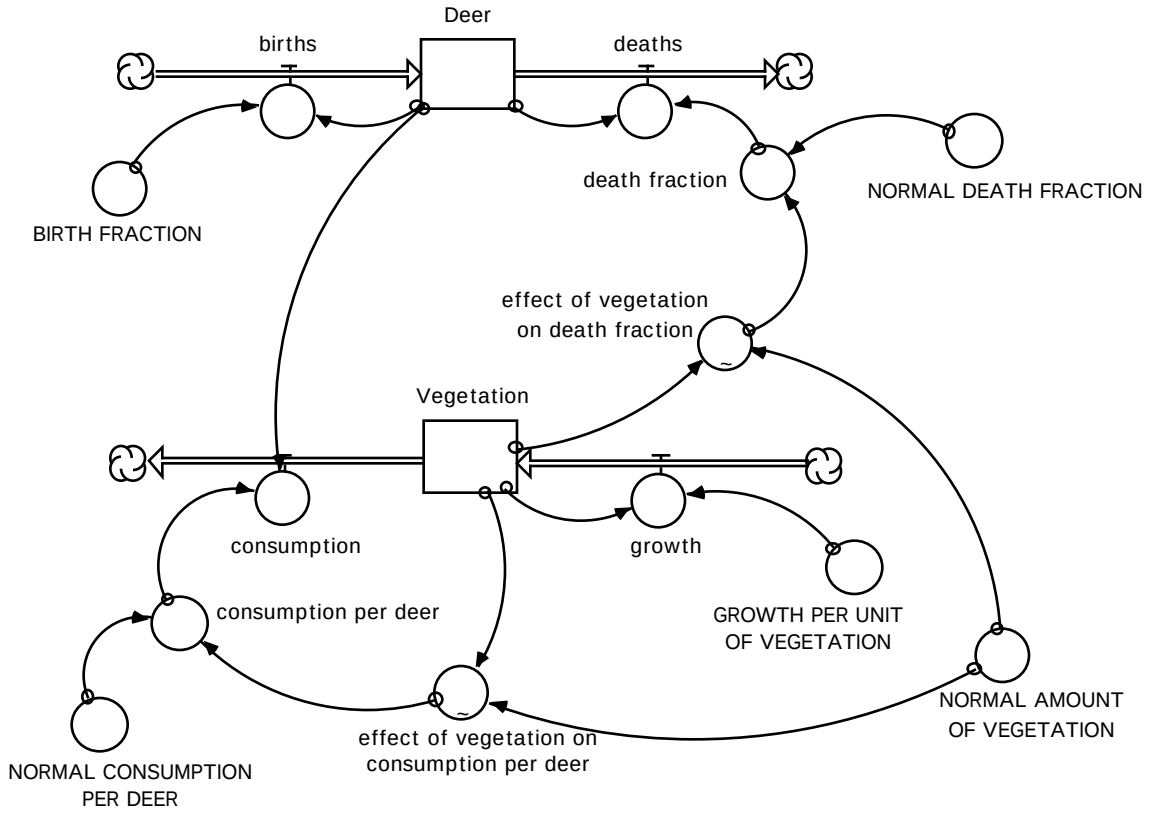


Figure 4: Model of the deer population

Notice that the structure of the model is similar to the structure of the petroleum extraction model from section 2.1 with only a few differences. Petroleum is a nonrenewable resource: it has no inflow, only an outflow. On the other hand, the "Vegetation" in the deer population model is able to regenerate and grow. The stock of "Vegetation" is increased by "growth" and decreased by "consumption." The inflow of

[4] Petroleum pumps take advantage of the pressure difference between the bottom of the well and the section of the rock deep underground that contains the petroleum. As petroleum is extracted, the pressure difference decreases. Typically, by the time 25% of the reserves have been extracted, the pressure on both ends of the pump equalizes, and any further extraction would require expensive artificial means of maintaining the pressure difference.

"growth" is determined by the amount of "Vegetation" and by a constant fraction of "GROWTH PER UNIT OF VEGETATION."

Another difference between the two models is that the "drilling fraction" in Example 1 depends on the amount of "Petroleum Reserves" remaining, while the "birth fraction" in the deer population model is assumed to be independent of the amount of "Vegetation" left.

A comparison of the two models shows that the underlying structure of the two systems is similar. Likewise, one can expect the models to produce similar behaviors. The deer population model generates the behavior shown in Figure 5.

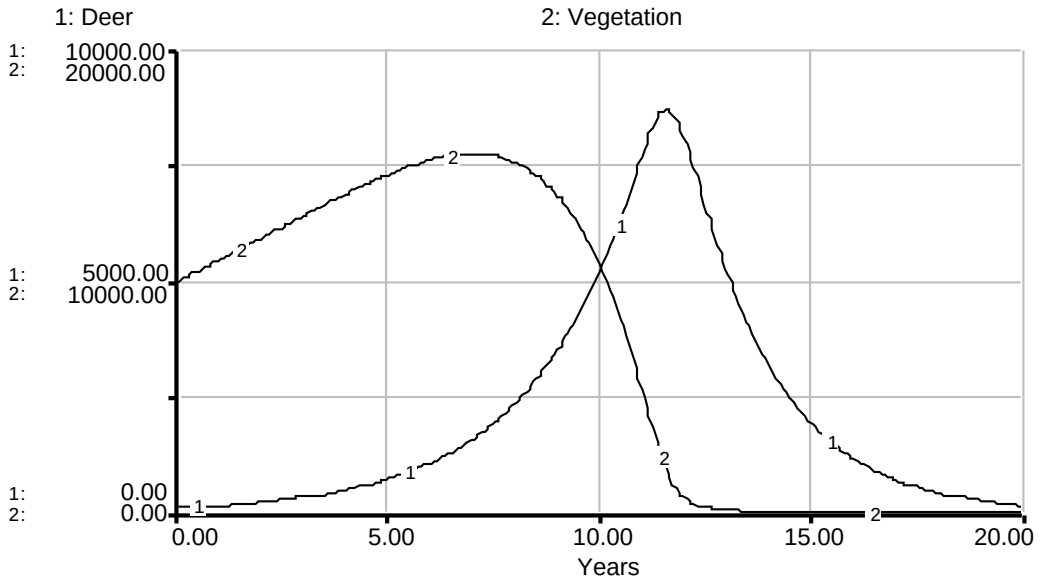


Figure 5: Behavior of "Vegetation" and "Deer"

The behavior of the deer population model is similar to the behavior of the petroleum extraction model. Initially, the amount of "Vegetation" is sufficient to support all of the "Deer." The number of "Deer" is so low that the "growth" of "Vegetation" is higher than "consumption," so "Vegetation" increases. The increase in "Vegetation" is possible because, unlike "Petroleum Reserves," the stock of "Vegetation" has an inflow. Consequently, the "effect of vegetation on death fraction," and hence the "death fraction" is low, so deer "deaths" are fewer than "births." The population of "Deer" increases exponentially. The increase in "Deer," however, means more demand for food, and soon the "consumption" rate becomes higher than the natural rate of "growth," so "Vegetation" starts to decline. A lower amount of "Vegetation" means less food per deer, which, through the "effect of vegetation on death fraction," increases the "death fraction." The "death fraction" becomes equal to the "BIRTH FRACTION" around year 12. The "Deer" population no longer grows. Thus, after a period of exponential growth,

the population peaks and starts declining. Because the number of “Deer” is still high, “consumption” is also high, so “Vegetation” keeps decreasing. The depletion of “Vegetation" continues as long as “consumption” by the “Deer” is higher than “growth.” The “Deer” population, just like the number of “Wells” in the previous example, collapses.

A similar behavior may occur repeatedly within a system. After most of the deer die, “consumption” becomes small, and the natural rate of “growth” will cause the “Vegetation” to start growing again. If at least a few deer managed to survive the temporary shortage of food, the population might start growing again, and the behavior of overshoot and collapse could repeat in the future.

3. THE GENERIC STRUCTURE

3.1 Model Diagram

The two models discussed in the examples from section 2 share the same basic structure that generates their similar behavior. Figure 6 shows the generic structure which produces overshoot and collapse.

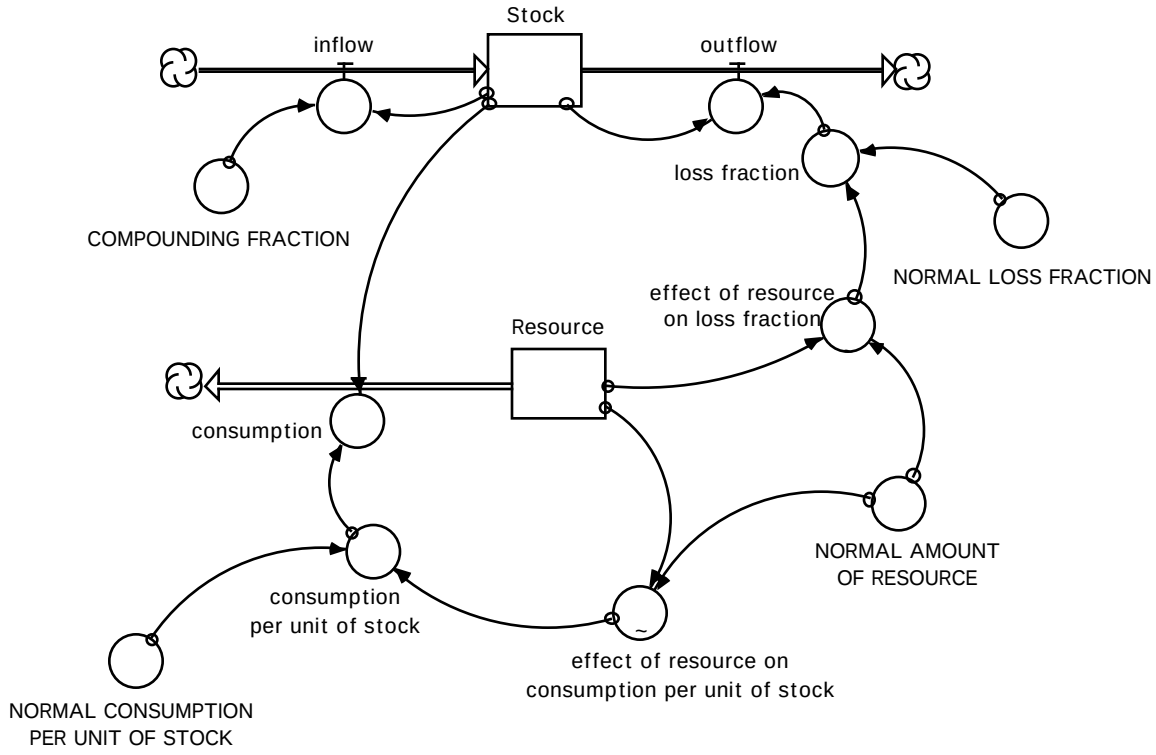


Figure 6: Generic structure producing overshoot and collapse (with a nonrenewable resource)

The structure is composed of several feedback loops. The only positive feedback loop in the system is the loop that joins the "Stock" to the "inflow": an increase in the "Stock" causes and increase in the "inflow," which in turn causes a further increase in the "Stock." As a counterpart to the positive feedback loop, the system also contains a simple negative feedback loop between the "Stock" and the "outflow": as the "Stock" increases, the "outflow" increases, causing a decrease in the "Stock."

The system also contains negative feedback loops that control the growth of the "Stock." One of the loops shows a negative relationship between the amount of the "Stock" and the amount of the "Resource." An increase in the "Stock" increases

"consumption," thus decreasing the "Resource." A decrease in the "Resource," however, increases the "loss fraction" through the "effect of resource on loss fraction," and thus the "outflow" increases and the "Stock" decreases. In addition, if the amount of "Resource" increases,[5] the "consumption per unit of stock" also increases through the "effect of resource on consumption per unit of stock." Thus, the total "consumption" increases, causing a decrease in the "Resource."

The structure contains two stock variables. The first one is referred to as "Stock" because it is the stock of interest to the modeler. In the two examples in section 2, the number of "Wells" and the population of "Deer" correspond to the "Stock." The second stock variable is called "Resource" because it is a resource necessary for the existence of the "Stock." The "Resource" corresponds to "Petroleum Reserves" and "Vegetation" in the two examples.

The "Stock" increases by an "inflow," the product of the "Stock" and a usually constant "COMPOUNDING FRACTION." Examples of an "inflow" include "drilling wells" and "births" in the preceding examples. The "outflow" from the "Stock" is determined by the amount of the "Stock" and a variable "loss fraction." In the petroleum extraction model, the "outflow" corresponds to "closing wells," and in the deer population example, the "outflow" corresponds to "deaths." The "loss fraction," the equivalent of the "closing fraction" and the "death fraction," is the product of a "NORMAL LOSS FRACTION" and an "effect of resource on loss fraction" multiplier. The multiplier is a table function expressing the negative relationship between the "loss fraction" and the fraction of the "Resource" remaining, which is the ratio of the "Resource" to the "NORMAL AMOUNT OF RESOURCE." As the ratio declines, the "Resource" becomes less available to the "Stock," so the "loss fraction" increases.

The "Resource" is depleted by "consumption," the product of the number of units of the "Stock" and the "consumption per unit of stock." In the generic structure, "consumption" corresponds to "extraction" of petroleum and "consumption" of vegetation in the examples from section 2. The "consumption per unit of stock" depends on the "NORMAL CONSUMPTION PER UNIT OF STOCK" and on an "effect of resource on consumption per unit of stock." The "effect of resource on consumption per unit of stock" is a table function showing that as the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE" decreases, the "Resource" is less available to the "Stock," so "consumption per unit of stock" also decreases.

---

[5] Note that an increase in the amount of "Resource" is impossible if the "Resource" is nonrenewable. The analysis of the feedback loop, however, is simpler with the assumption of an initial increase in the "Resource."

In some cases, as in the deer population model in section 2.2, the "Resource" may be increased by regeneration, as "Vegetation" is increased by "growth." The addition of an inflow to the "Resource" affects the behavior of the model to some degree. In the deer population model, "Vegetation" was growing initially, and thus the population of "Deer" was able to grow to a larger number than it would have grown without the "growth" of "Vegetation." The overall behavior of overshoot and collapse, however, was still present in the system. As long as the regeneration inflow to the "Resource" is not significantly stronger than the outflow of "consumption," regeneration will not have a considerable effect on the qualitative behavior of the system.

3.2 Model Equations

The following are the equations for the generic structure producing overshoot and collapse. The constant numerical values used to run the simulation were chosen arbitrarily and do not carry any meaning.

☐ **Resource(t)** = Resource(t - dt) + (- consumption) * dt

INIT Resource = *a constant*

DOCUMENT: The resource that the stock consumes. It corresponds to the "Vegetation" and the "Petroleum Reserves" in the examples above. It can either be nonrenewable, as "Petroleum Reserves," or renewable, as "Vegetation." Note that the addition of a flow renewing the resource is a structural change to the model.

Units: units of resource

☞ OUTFLOWS:

consumption = Stock * consumption_per_unit_of_stock

DOCUMENT: The rate at which the resource is consumed or depleted. It corresponds to "extraction" and "consumption" in the examples above.

Units: units of resource/unit of time

☐ **Stock(t)** = Stock(t - dt) + (inflow - outflow) * dt

INIT Stock = *a constant*

DOCUMENT: The stock that consumes the resource. It corresponds to "Wells" and "Deer" in the examples above.

Units: units of stock

- INFLOWS:

  inflow = Stock * COMPOUNDING_FRACTION

  DOCUMENT: The inflow is the fraction of the stock that flows into the system per unit of time. It corresponds to “drilling wells” and “births” in the examples above.

  Units: units of stock/unit of time

- OUTFLOWS:

  outflow = Stock * loss_fraction

  DOCUMENT: The outflow is the fraction of the stock that flows out of the system per unit of time. It corresponds to “closing wells” and “deaths” in the examples above.

  Units: units of stock/unit of time

- **COMPOUNDING_FRACTION** = *a constant*

  DOCUMENT: The “COMPOUNDING FRACTION” along with the “Stock” determines the “inflow” to the “Stock.” It corresponds to the “NORMAL DRILLING FRACTION” and “BIRTH FRACTION” in the examples above. It can either be constant, as the “BIRTH FRACTION” in the deer population model, or can be affected by the percentage of “Resource” remaining, as the “NORMAL DRILLING FRACTION” in the petroleum extraction model.

  Units: 1/unit of time

- **consumption_per_unit_of_stock** = NORMAL_CONSUMPTION_PER_UNIT_OF_ STOCK * effect_of_resource_on_consumption_per_unit_of_stock

  DOCUMENT: Consumption of the resource by the stock per unit of time. It depends on the “NORMAL CONSUMPTION PER UNIT OF STOCK” as well as on the “effect of resource on consumption per unit of stock.” It corresponds to the “extraction per well” and “consumption per deer” in the examples above.

  Units: (units of resource/units of stock)/unit of time

- **NORMAL_AMOUNT_OF_RESOURCE** = *a constant*

  DOCUMENT: The normal amount of resource. It corresponds to the “INITIAL PETROLEUM RESERVES” and “NORMAL AMOUNT OF VEGETATION” in the examples above.

  Units: unit of resource

○ **loss_fraction** = NORMAL_LOSS_FRACTION * effect_of_resource_on_loss_fraction

DOCUMENT: The fraction of the stock that is lost per unit of time. It depends on the "NORMAL LOSS FRACTION" and on the "effect of resource on loss fraction." It corresponds to the "closing fraction" and "death fraction" in the examples above.

Units: 1/unit of time

○ **NORMAL_CONSUMPTION_PER_UNIT_OF_STOCK** = *a constant*

DOCUMENT: The amount of resource that a unit of stock consumes per unit of time while the resource is readily available. It corresponds to the "MAXIMUM EXTRACTION PER WELL" and "NORMAL CONSUMPTION PER DEER" in the examples above.

Units: (units of resource/units of stock)/unit of time

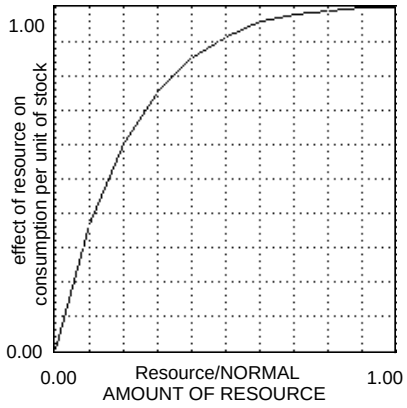
○ **NORMAL_LOSS_FRACTION** = *a constant*

DOCUMENT: The fraction of the stock that is lost given a "NORMAL AMOUNT OF RESOURCE." It corresponds to the "NORMAL CLOSING FRACTION" and "NORMAL DEATH FRACTION" in the examples above.

Units: 1/unit of time

⊘ **effect_of_resource_on_consumption_per_unit_of_stock** =

GRAPH (Resource/NORMAL_AMOUNT_OF_RESOURCE)
(0.00, 0.00), (0.1, 0.365), (0.2, 0.6), (0.3, 0.75), (0.4, 0.855), (0.5, 0.91), (0.6, 0.955), (0.7, 0.98), (0.8, 0.99), (0.9, 1.00), (1, 1.00)



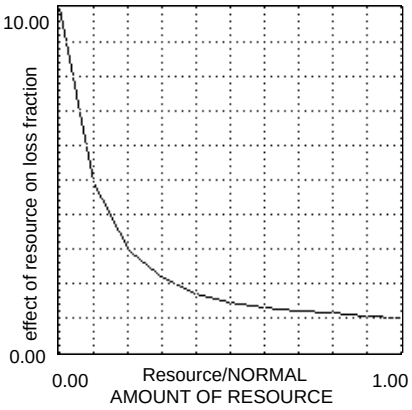
DOCUMENT: The effect of the remaining percentage of the resource on the consumption per unit of stock. It corresponds to the "effect of reserves on

extraction per well" and "effect of vegetation on consumption per deer" in the examples above.[6]

Units: dimensionless

⊘ **effect_of_resource_on_loss_fraction** =

GRAPH(Resource/NORMAL_AMOUNT_OF_RESOURCE)
(0.00, 10.0), (0.1, 4.90), (0.2, 3.00), (0.3, 2.20), (0.4, 1.70), (0.5, 1.45), (0.6, 1.30), (0.7, 1.20), (0.8, 1.12), (0.9, 1.05), (1, 1.00)



DOCUMENT: The effect of the remaining percentage of the resource on the loss fraction. It corresponds to the "effect of reserves on closing fraction" and "effect of vegetation on death fraction" in the examples above.[7]

Units: dimensionless

[6] The shape and assumptions of the "effect of resource on consumption per unit of stock" table function will be explained in section 4.1.

[7] The shape and assumptions of the "effect of resource on loss fraction" table function will be explained in section 4.2.

3.3 Model Behavior

Figure 7 shows the behavior produced by the generic structure.

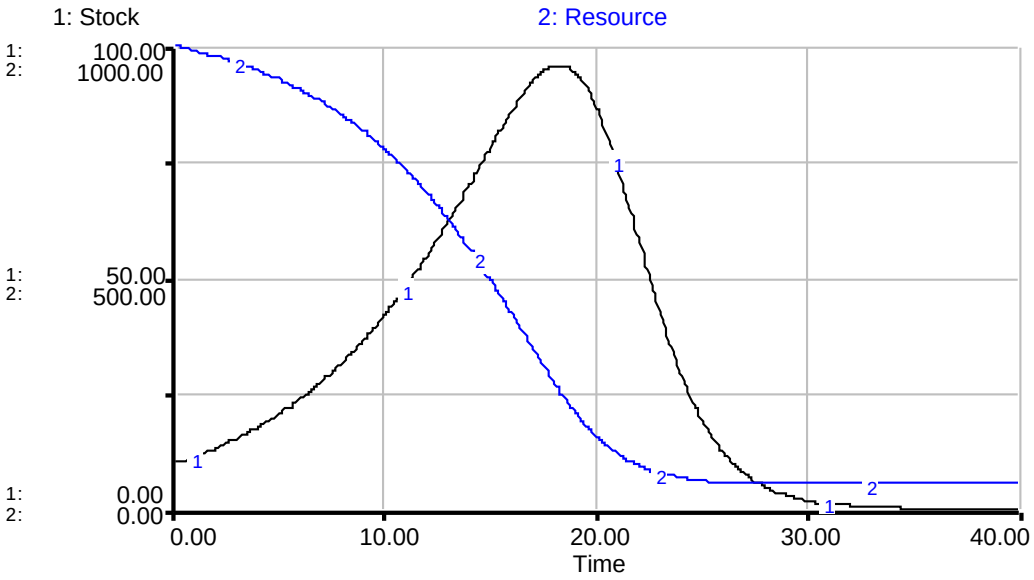


Figure 7: Overshoot and collapse behavior produced by the generic structure

Initially, the positive feedback loop composed of the "inflow" and the "Stock" drives the exponential growth of the "Stock." The exponential increase in the "Stock" causes "consumption" to also increase exponentially. Consequently, the "Resource" decreases exponentially, causing the "loss fraction" to increase exponentially. The *slope* of the net flow[8] into the "Stock" begins to decrease to zero.

Soon, the increasing "loss fraction" makes the "outflow" increase faster than the "inflow," but the "inflow" is still larger than the "outflow." The slope of the net flow is then negative: the net flow decreases but is still positive. A graph showing the behavior of the net flow is shown in Figure 8.

[8] The net flow is defined as "inflow" - "outflow." The net flow is not part of the model; however, for better understanding of the structure, it is useful to create a converter called "net flow" in the model, and to observe its behavior on a graph.

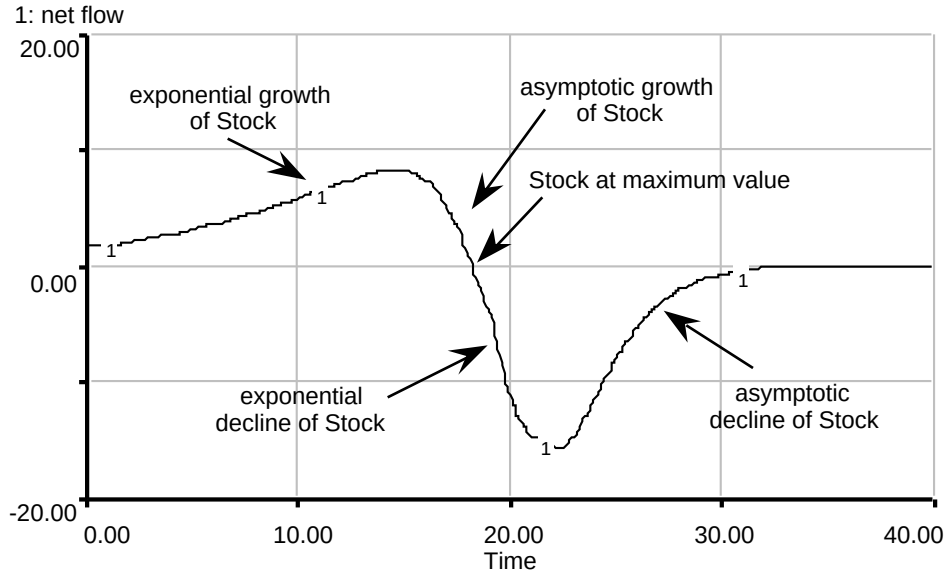


Figure 8: Net flow into the "Stock" of the generic structure

Meanwhile, as the growth rate slows because of the decreasing net flow, the "Stock" stops growing exponentially; its growth becomes asymptotic. The "Resource" continues being depleted through "consumption," and the "loss fraction" continues to grow. Then, the value of the "loss fraction" reaches the value of the "gain fraction," and the "outflow" reaches the "inflow." Thus, the net flow is zero (it is still decreasing), and the "Stock" peaks. In Figure 9, the dashed line at time 18 indicates that the "Stock" reaches its maximum when the "outflow" equals the "inflow" (net flow is equal to 0).

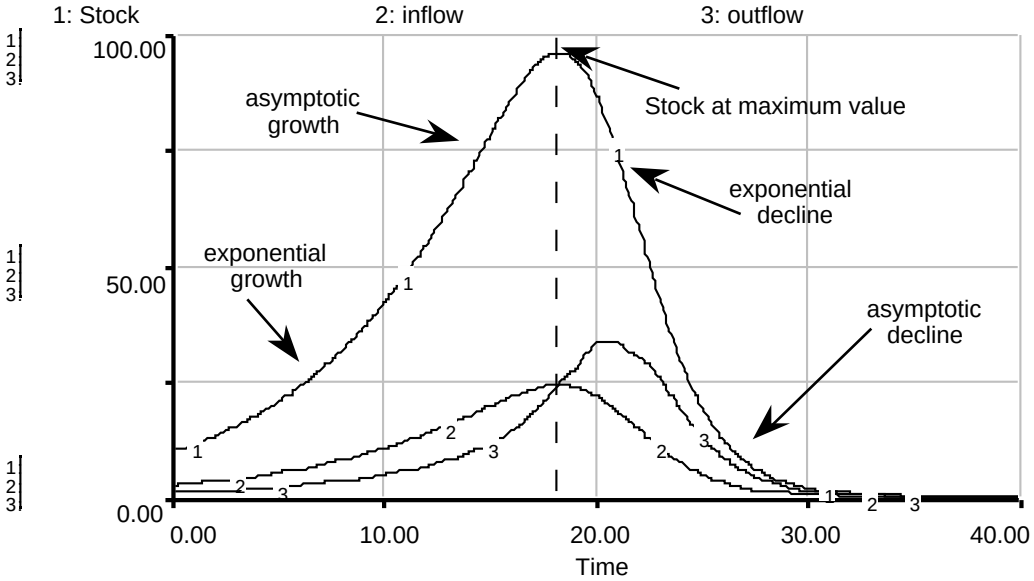


Figure 9: Behavior of "Stock," "inflow," and "outflow" of the generic structure

The lack of "Resource" causes the "loss fraction" to keep increasing, and the "outflow" exceeds the "inflow" (the net flow becomes negative). As soon as the net flow becomes negative, the "Stock" reverses direction and starts decreasing. The "outflow" still increases because of the growing "loss fraction," while the "inflow" has started to decrease as a result of the decrease in the "Stock." The slope of the "Stock," which is equal to the net flow into the "Stock," becomes increasingly negative. The "Stock" is declining exponentially.

However, as the "Stock" starts decreasing, the decrease in "Resource" is no longer exponential, but asymptotic. As a result, the "loss fraction" also stops increasing exponentially and switches to asymptotic growth. Consequently, the "outflow" starts decreasing because of the decrease in "Stock," and the net flow starts increasing again. The net flow still stays negative, however, causing a further, although slower, asymptotic decline of the "Stock." The net flow eventually approaches zero, and the "Stock" stabilizes at zero.

It is crucial to realize that the overshoot and collapse behavior occurs because the "loss fraction" is not constant but varies with the amount of "Resource." As the "Resource" is being consumed, the "loss fraction" increases and so does the "outflow." As soon as the "outflow" exceeds the "inflow," the collapse of the "Stock" begins and continues until the "Stock" reaches zero.

The goal value of the "Stock" and the only possible equilibrium in the generic structure is zero. The "Resource" cannot reach equilibrium as long as the "Stock" is greater than zero. If the initial amount of "Resource" is sufficiently large to permit growth of the "Stock," then even a slight initial disturbance of the "Stock" away from zero starts off the "consumption" of the "Resource." The slight initial disturbance produces the overshoot and collapse behavior that leads the "Stock" to stabilize at zero again.

4. TABLE FUNCTIONS IN THE GENERIC STRUCTURE

The generic structure producing overshoot and collapse presented in section 3 contains two table functions: the "effect of resource on consumption per unit of stock"

and the "effect of resource on loss fraction." [9] This section explains the shape of the table functions and their effect on model behavior.

4.1 "Effect of resource on consumption per unit of stock" table function

The "effect of resource on consumption per unit of stock" table function is an increasing curve with a decreasing slope, with values ranging from 0 to 1 on both axes. Figure 10 shows the graph of the table function.

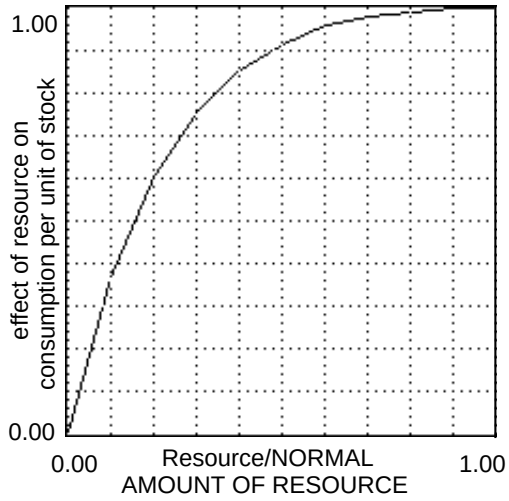


Figure 10: The "effect of resource on consumption per unit of stock" table function

The slope of the function is positive because of the positive relation between the "consumption per unit of stock" and the fraction of the "Resource" remaining, which is the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE." The higher the ratio, the higher the "consumption per unit of stock," so the table function gives a high value. As the ratio decreases, so does "consumption per unit of stock," so the table function outputs a lower value.

In addition, the curve of the table function increases with a decreasing slope. If the "Resource" is still relatively high as a fraction of the "NORMAL AMOUNT OF RESOURCE," "consumption per unit of stock" is not greatly affected. The "effect of

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[9] In some models, additional table functions might be necessary to represent the feedback mechanisms of the system. For example, the petroleum extraction model from section 2.1 contains a table function of the "effect of reserves on drilling fraction" because it can be argued that the remaining amount of "Petroleum Reserves" not only affects the rate of "closing wells," but also the rate of "drilling wells." For simplicity, however, the table function of the "effect of resource on compounding fraction" was omitted from the generic structure.

resource on consumption per unit of stock" is then close to 1. In Figure 10, for example, the table function outputs values greater than 0.9 for values of the ratio greater than 0.5. As the "Resource" decreases further, however, the "effect of resource on consumption per unit of stock" becomes stronger, so the table function declines more steeply.

The table function ranges from 0 to 1 for both the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE" and for the "effect of resource on consumption per unit of stock." When the "Resource" is equal to the "NORMAL AMOUNT OF RESOURCE" (the ratio is equal to 1), "consumption per unit of stock" is equal to the "NORMAL CONSUMPTION PER UNIT OF STOCK," and the table function has a value of 1. As the ratio decreases, however, the "Resource" becomes more difficult to obtain, so "consumption per unit of stock" becomes smaller than the "NORMAL CONSUMPTION PER UNIT OF STOCK." The table function then outputs a value between 0 and 1. When the "Resource" is completely depleted (the ratio is equal to 0 because "Resource" is equal to 0), "consumption per unit of stock" must also be equal to 0, so the table function outputs the value of 0. If the amount of "Resource" is greater than the "NORMAL AMOUNT OF RESOURCE" (the ratio is greater than 1), the table function will output the value of 1 by default because it is assumed that "consumption per unit of stock" is never greater than the "NORMAL CONSUMPTION PER UNIT OF STOCK," even if the "Resource" is overabundant.

The reader may try to change the shape of the table function to see that, as long as the table function is increasing, the exact shape does not determine the behavior of the model. Even if the table function is linear or has an increasing slope, the model still exhibits overshoot and collapse.

4.2 "Effect of resource on loss fraction" table function

The "effect of resource on loss fraction" table function is a curve whose negative slope decreases in magnitude. The curve ranges from 0 to 1 for the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE," and from 1 to a higher value for the "effect of resource on loss fraction." Figure 11 shows the graph of the table function.

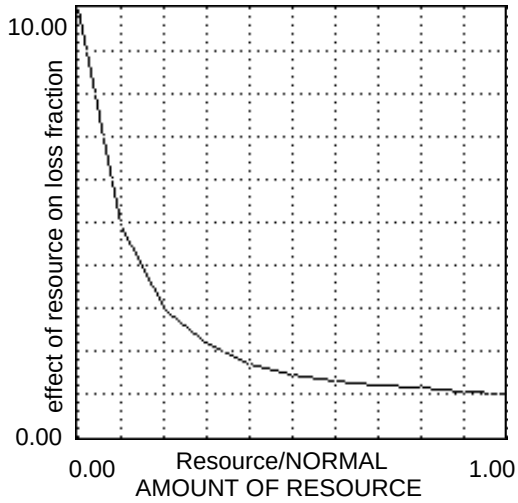


Figure 11: The "effect of resource on loss fraction" table function

The slope of the function is negative because of the negative relationship between the "loss fraction" and the fraction of "Resource" remaining, which is the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE." The higher the fraction of the "Resource" remaining, the closer the "loss fraction" to the "NORMAL LOSS FRACTION," so the table function outputs a value close to 1. As the ratio decreases, the "loss fraction" becomes more significant, so the table function outputs a value larger than 1.

The "effect of resource on loss fraction" table function has a negative slope that decreases in magnitude. While the fraction of the "Resource" remaining is still high, the depletion of the "Resource" will not have a strong impact on the "loss fraction." For example, in the table function in Figure 11, the "effect of resource on loss fraction" is lower than 2 when the fraction of "Resource" remaining is greater than 0.4. As the "Resource" declines further, however, its effect on the "loss fraction" becomes stronger, so the curve is steeper as the ratio decreases.

The table function ranges from 0 to 1 for the fraction of the "Resource" remaining. When the "Resource" is equal to (or even greater than) the "NORMAL AMOUNT OF RESOURCE," the "loss fraction" is equal to the "NORMAL LOSS FRACTION." The table function then outputs a value of 1.[10] The minimum value of the "effect of resource on loss fraction" is therefore 1. As the remaining fraction of "Resource" decreases, the "loss fraction" becomes higher than the "NORMAL LOSS FRACTION," so the table function outputs values greater than 1.

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[10] Assuming that an overabundance of the "Resource" will not reduce the "loss fraction" below the value of the "NORMAL LOSS FRACTION."

The maximum value of the "effect of resource on loss fraction" can significantly affect the behavior of the model. In the generic structure, the maximum value is set to 10. Thus, the "loss fraction" becomes ten times the "NORMAL LOSS FRACTION" when the "Resource" is completely depleted. If the value of the "COMPOUNDING FRACTION" is less than ten times the "NORMAL LOSS FRACTION," then the "loss fraction" will eventually become greater than the "COMPOUNDING FRACTION." When the "loss fraction" is greater than the "COMPOUNDING FRACTION," the "outflow" will be greater than the "inflow," and the "Stock" will decline.

As soon as the table function outputs a value that makes the "loss fraction" greater than the "COMPOUNDING FRACTION," the "Stock" will start to decline. With the values used to produce the simulation of the generic structure ("COMPOUNDING FRACTION" = 0.25, "NORMAL LOSS FRACTION" = 0.1), the "outflow" reaches the "inflow" when the table function outputs the value of 2.5. Because the table function ranges up to 10, the "outflow" will significantly exceed the "inflow" when the "Resource" is almost depleted, and thus the "Stock" collapses.

5. SUMMARY OF IMPORTANT CHARACTERISTICS

- **Structure**

The structure generating overshoot and collapse is a second-order negative feedback loop. The "Stock" feeds on the "Resource" whose depletion eventually causes the "Stock" to collapse. The "loss fraction" that determines the "outflow" from the "Stock" depends on the remaining amount of "Resource." The "Resource" does not have to be nonrenewable but should at least be slow to regenerate.

- **Behavior**

Initially, the "Stock" undergoes exponential growth. Because of the accumulating "Stock," the "Resource" is depleted. The negative feedback from the "Resource" to the "loss fraction" slows the growth of the "Stock," until the "outflow" becomes greater than the "inflow," which causes a reversal of the behavior: the negative feedback loop of the "outflow" becomes very strong and the "Stock" collapses.

6. USING INSIGHTS GAINED FROM THE GENERIC STRUCTURE

Generic structures are useful because they help a modeler to transfer knowledge among different systems. The following exercises will emphasize the concept of transferability of structure.

6.1 Exercise 1: Health Deterioration

This exercise discusses the effects of too much work on a person's health. Bryant, a graduate student, just started working on a new project. At the beginning, he is excited and enthusiastic about the project, so he starts spending more time working on the project than the usual eight hours a day. As he spends more and more time on the project, he starts getting tired and becomes ill—his health deteriorates. Eventually, by the time Bryant works more than 15 hours a day, his health becomes so weak that he has to quickly start cutting back on the number of hours he works, until he stops working completely.

Using the generic structure from section 3, build a model showing how the number of hours Bryant spends on the project exhausts his health. Additional variables or parameters may need to be added to the generic structure.

First choose the variables corresponding to the "Stock" and the "Resource." For Bryant, the number of hours he spends on the project every day depletes his health. The level of the resource of health can be measured on a scale either from 0 to 1, or from 0 to 100, representing the percentage of Bryant's health remaining. Assume that initially Bryant is perfectly healthy. Also assume that, as Bryant's health declines, the fraction of his health that deteriorates for every additional hour he works becomes smaller. Note that the time horizon of the model (approximately a month) is too short for Bryant's health to recover significantly.

6.2 Solution to Exercise 1

Figure 16 shows a possible health deterioration model. Documented equations are in section 7.3 of the Appendix.

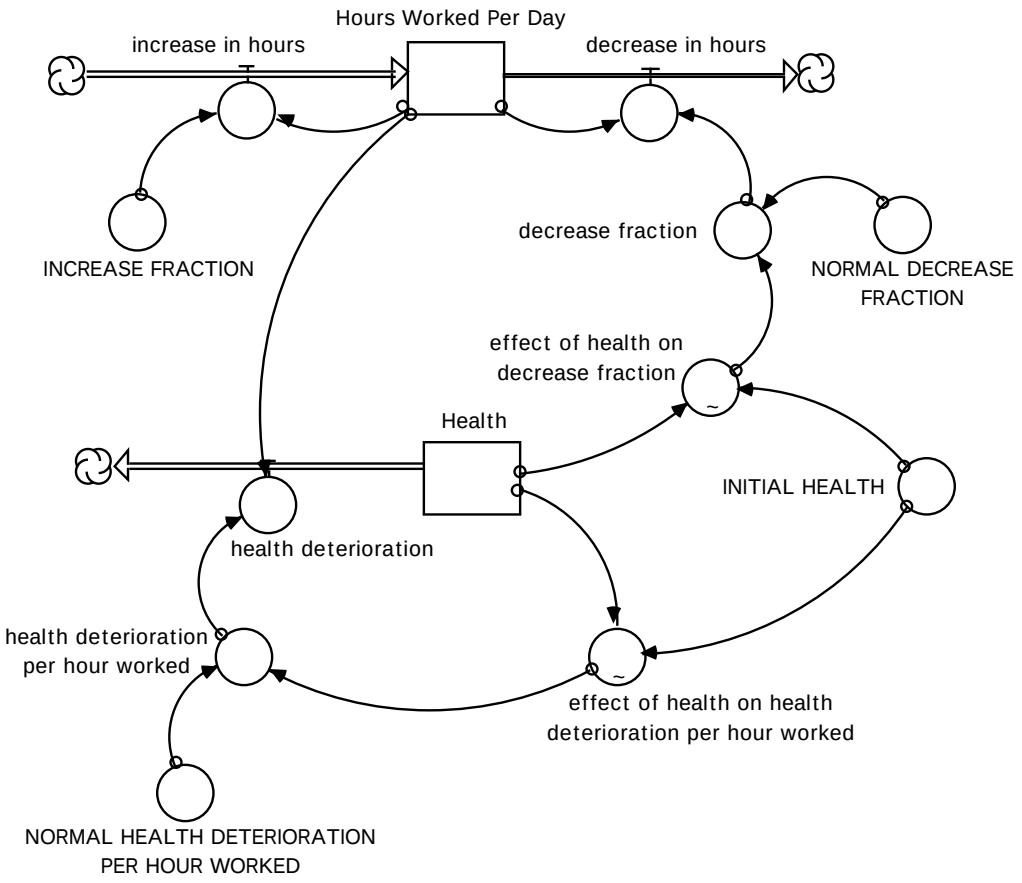


Figure 12: Model of Bryant's health

One can build a model of Bryant's health that has the same structure as the generic structure presented in section 3. The stock of "Hours Worked Per Day" has an inflow of "increase in hours," determined by the number of "Hours Worked Per Day" and a constant "INCREASE FRACTION." The "increase in hours" positive feedback loop shows that the more Bryant works on the project, the more he likes it, causing him to work even more. The number of "Hours Worked Per Day" decreases through a "decrease in hours," which is the product of the "Hours Worked Per Day" and a "decrease fraction." The "decrease fraction" depends on a "NORMAL DECREASE FRACTION" and an "effect of health on decrease fraction" multiplier. The multiplier reflects a negative relationship between "Health" and the "decrease fraction": as the ratio of "Health" to "INITIAL HEALTH" decreases, the "decrease fraction" increases. While

his “Health” is still relatively strong, the “effect of health on decrease fraction” will be quite small. As his “Health” declines further, the multiplier outputs a higher value, until it reaches the value of 100, which occurs when Bryant’s “Health” has dropped 70 percent. Such a high value for the multiplier emphasizes the fact that Bryant would start cutting back on his hours quite drastically if he started to become seriously ill.

Bryant’s “Health” becomes worse as he works more hours every day. His “health deterioration” is the product of the number of “Hours Worked Per Day” and a fraction called “health deterioration per hour worked.” The fraction is determined by a “NORMAL HEALTH DETERIORATION PER HOUR WORKED” and a multiplier called “effect of health on health deterioration per hour worked.” The multiplier shows that as the ratio of Bryant’s “Health” to his “INITIAL HEALTH” decreases, “health deterioration per hour worked” becomes smaller. Over a one-month time horizon, Bryant’s “Health” will not be able to regenerate significantly, so “Health” does not have an inflow.

Figure 17 shows the behavior of the model.

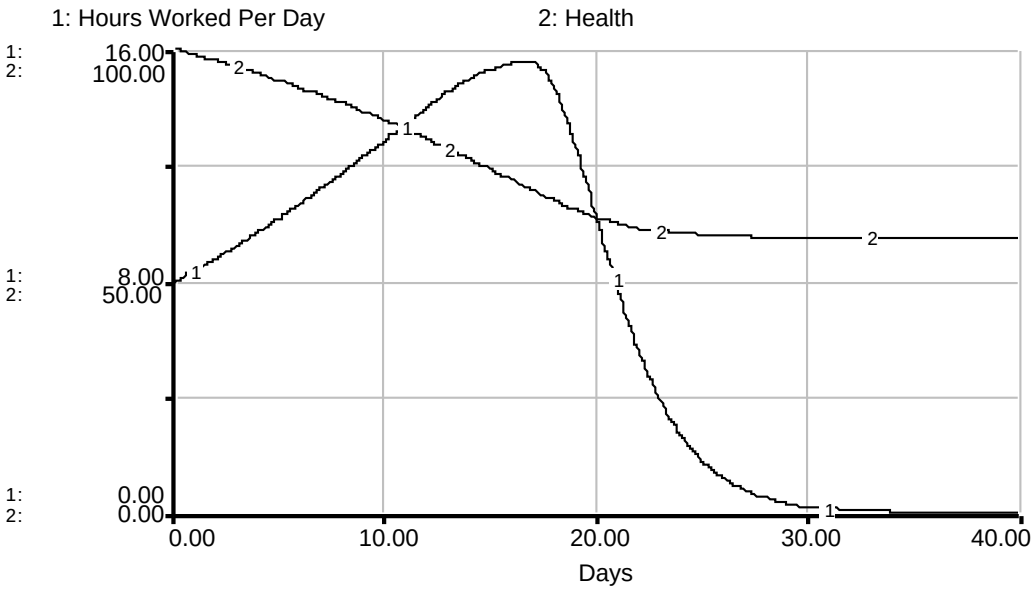


Figure 13: Behavior of the health deterioration model

As Bryant starts spending more hours working on the project every day, his “Health” starts deteriorating slowly. After approximately 16 days of intensive work, when he works more that 15 hours every day, Bryant realizes the bad effects of too much work on his “Health,” so he starts working less and less. Eventually, after day 30, Bryant decides not to do any work at all, so his “Health” stops deteriorating.

6.3 Exercise 2: Mayan Population

The second exercise examines a possible cause of the decline of the Mayan empire. Please read carefully the following excerpts from an article entitled "Study Depicts Mayan Decline."

> One of the great mysteries of human history has been the sudden collapse of one of the main centers of Mayan civilization in Central America at a time when it was apparently at a peak of culture, architecture and population around A.D. 800. ... Just before the final cataclysm, the new research suggests, the population in one area ranged from about 200 to 500 persons per square kilometer. ... Within two to four Mayan generations, which probably meant less than 100 years, the population dropped back to what it had been almost 2,000 years before—20 or less per square kilometer. ... [New] research ... showed there was an exponential growth in Mayan population during at least 1,700 years... Human numbers doubled every 408 years... [The population] grew at a steadily increasing pace, but, for many centuries, the growth was too slow for any single generation to see what was happening. ... Gradually-intensified agriculture seems to have done severe cumulative damage to an originally verdant environment. To this was added the impact of increases in human dwellings and other major architectural works on the land. Essential nutrients washed, slid and were moved downhill to be lost in the lakes, diminishing the fertility of agricultural land. ... [The new research gives] documented estimates for the growth pattern of Mayan population and correlate[s] this growth with the damage to the environment that went with it. [11]

Based on the generic structure from section 3, build a model showing the collapse of the Mayan population after its environment is destroyed. Use the numerical values provided in the article to calculate approximate values for the model parameters. The model does not have to reproduce the generic structure exactly; additional converters or table functions may be needed to represent the system more realistically. Here are some additional hints for building the model:

First, determine which variables correspond to the "Stock" and "Resource" of the generic structure. The "Stock" will most likely be the Mayan population, while the "Resource" could be an aggregate variable representing the state of the environment in which the population lives ranging from 0 to 100 percent of the environment capacity. Assume that initially the environment is undamaged. The time horizon of the simulation should be at least 2000 years, as specified in the article, and should include A.D. 800.

Notice that it is not the actual level of the Mayan population that damages the environment, but rather the density of the population. Therefore, the model should

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[11] From Harold M. Schmeck, Jr. "Study Depicts Mayan Decline." *New York Times*, October 23, 1979.

contain a converter representing density, the ratio of the population to a constant land area. The size of land area is somewhat arbitrary; it should, however, be set such that the initial density is approximately 20 people per square kilometer, as indicated in the article. The density should then be compared to a normal value of density, to determine the effect of population density on the fraction of the environment that is damaged. The effect will in turn multiply a normal damage fraction to determine the actual fractional damage to the environment.

In addition, the model should contain an inflow into the resource of the environment because the environment is renewable over a time horizon of 2000 years. Depending on the assumptions of the modeler, the time over which the environment regenerates may be either constant or may vary as a function of the environment.

6.4 Solution to Exercise 2

Figure 18 shows a possible model of the Mayan population based on the description from section 6.3. Documented equations are in section 7.4 of the Appendix.

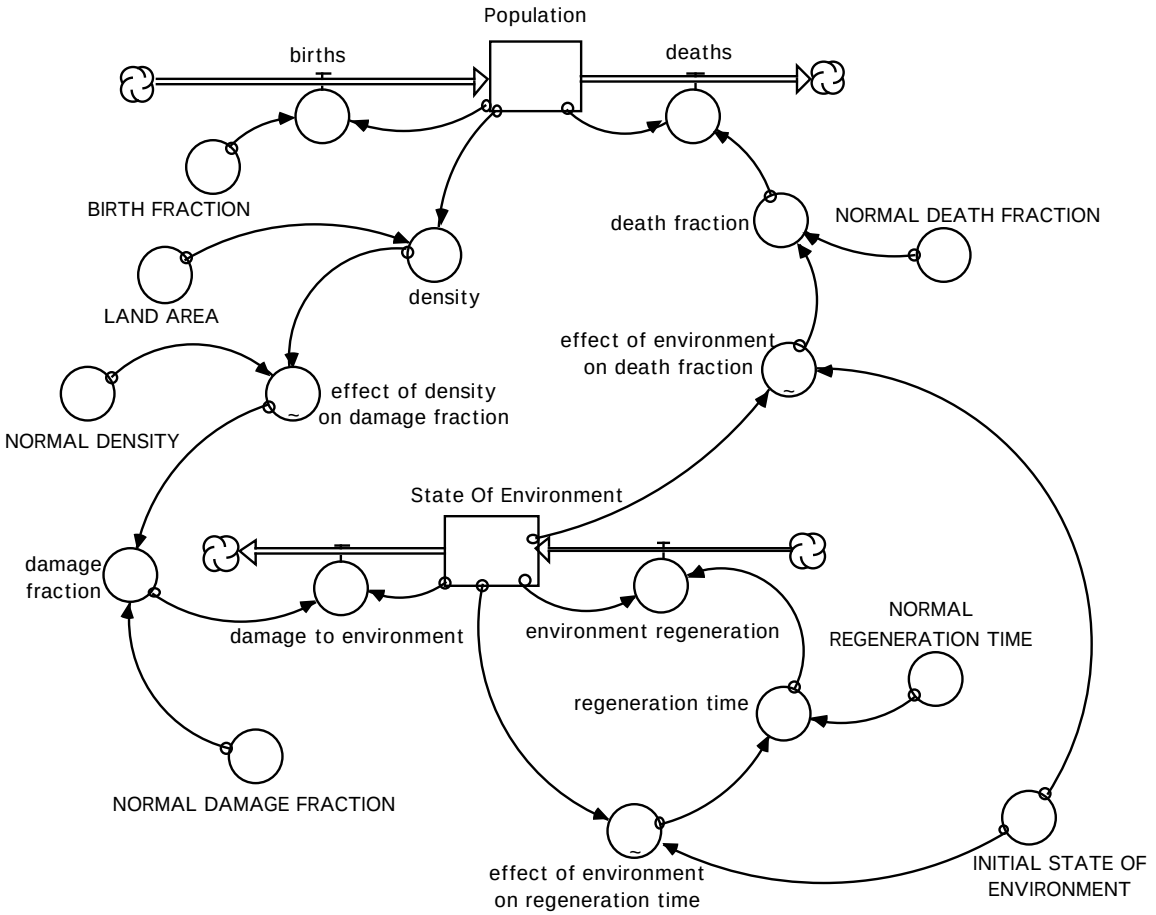


Figure 14: Model of the Mayan population

The two stocks in the model, corresponding to "Stock" and "Resource" of the generic structure, are "Population" and "State of Environment," respectively.

"Population" increases by "births," the product of the "Population" and a constant "BIRTH FRACTION." The outflow from "Population" is "deaths," determined by the level of "Population" and a "death fraction." The "death fraction" is the product of a "NORMAL DEATH FRACTION" and a multiplier called "effect of environment on death fraction." The multiplier captures a negative relationship between the "State of Environment" and the "death fraction": as the "State of Environment" declines, the "death fraction" rises.

The "State of Environment" resource is depleted through an outflow of "damage to environment." The "damage to environment" is the "damage fraction" of the "State of

Environment." The "damage fraction" is the product of a "NORMAL DAMAGE FRACTION" and a multiplier called "effect of density on damage fraction." The multiplier shows that as the "density" increases, the "damage fraction" becomes larger. The "effect of density on damage fraction" is a function of the ratio of the actual "density" to a "NORMAL DENSITY." The "density" is the ratio of "Population" to a constant "LAND AREA."

Because environment is a renewable resource, "State of Environment" increases through an inflow of "environment regeneration." The "environment regeneration" is determined by the "State of Environment" and a "regeneration time." Assuming that the "regeneration time" varies with "State of Environment," "regeneration time" is the product of a "NORMAL REGENERATION TIME" and an "effect of environment on regeneration time" multiplier. The multiplier shows that as the ratio of "State of Environment" to the "INITIAL STATE OF ENVIRONMENT" decreases, it takes longer for the environment to regenerate itself, so the "regeneration time" increases.

Before running the model to see whether it demonstrates the expected behavior of overshoot and collapse of the Mayan population, the modeler should determine an appropriate time horizon for the model and decide on the time unit to be used in the simulation. As the article indicates, the Mayan population collapsed around A.D. 800 after growing exponentially for at least 1700 years. With such a long time horizon over which the behavior occurred, running the simulation in units of years would be very slow. Thus, it might be more convenient to choose "Century" as a time unit, and to run the simulation from the 10th century B.C. to the 10th century A.D.

The modeler must also specify values for constant parameters in the model, as well as determine the functional relationships captured by the table functions. To find the value of the "BIRTH FRACTION" and "NORMAL DEATH FRACTION," remember that the article mentioned a population doubling time of 408 years, or 4.08 centuries, corresponding to a net growth fraction of approximately 0.18 per century.[12] The net growth fraction is the difference between the "BIRTH FRACTION" and the "NORMAL DEATH FRACTION." Thus, the modeler could pick the value of 2 per century for "BIRTH FRACTION," and 2 – 0.18, or 1.82 per century, for "NORMAL DEATH FRACTION," corresponding to an average lifetime of about 55 years.[13]

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[12] Remember that the doubling time of exponential growth is approximately 0.7/compounding fraction. Thus the compounding fraction is 0.7/doubling time. For the Mayan population, the net growth fraction is 0.7 / 4.08, which is approximately 0.18.

[13] The average lifetime is calculated as 1/"NORMAL DEATH FRACTION."

The value of "LAND AREA" can be chosen by the modeler, but should have a specific relation to the initial value of the "Population" to give the historically observed initial value for "density" as described in the article, 20 people per square kilometer. For example, with a "LAND AREA" of 10,000 square kilometers, the initial value of "Population" would be 200,000 people.

Considering that the "State of Environment" ranges on a scale from 0 to 100, it seems reasonable to choose the value of the "NORMAL DAMAGE FRACTION" to be, for example, 1 per century. Also, a reasonable value for "NORMAL REGENERATION TIME" could be 1 century.[14]

The "effect of environment on death fraction" table function should have the same general shape as the "effect of resource on loss fraction" described in section 4. The multiplier takes on values close to 1 while the ratio of "State of Environment" to "INITIAL STATE OF ENVIRONMENT" is relatively high, and the multiplier only starts increasing significantly when the ratio decreases to approximately 0.25. When the ratio reaches 0, the multiplier has the value of 5.

The "effect of density on damage fraction" table function represents the relationship between the "damage fraction" and the ratio of "density" to "NORMAL DENSITY." According to the article, the "density" of the Mayan population in some areas reached up to 500 persons per square kilometer, leading to a value of 25 for the ratio (assuming that "NORMAL DENSITY" is the initial density of the population, 20 persons per square kilometer). To have a more robust model, let the ratio range from 0 up to, say, 35. As the ratio increases, the "effect of density on damage fraction" also increases. The table function, however, gives relatively small values when the ratio is less than 20. Then, even a small increase in the ratio leads to a large increase in the multiplier, so the slope of the table function will be large. The slope, however, flattens out as the ratio exceeds the value of 30. The "effect of density on damage fraction" eventually reaches the value of 50 as the ratio of "density" to "NORMAL DENSITY" reaches 35.

Finally, the "effect of environment on regeneration time" multiplier shows the relationship between the environment "regeneration time" and the ratio of "State of Environment" to "INITIAL STATE OF ENVIRONMENT." The multiplier stays close

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[14] The exact value of the parameters will usually not have a significant effect on the overall behavior mode of the system. See Lucia Breierova, 1996. *An Introduction to Sensitivity Analysis* (D-4526), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 6, 40 pp.

to 1 for values of the ratio larger than 0.5, and then starts increasing faster up to the value of 2 when the ratio becomes 0.

Figure 19 shows the behavior of the model.

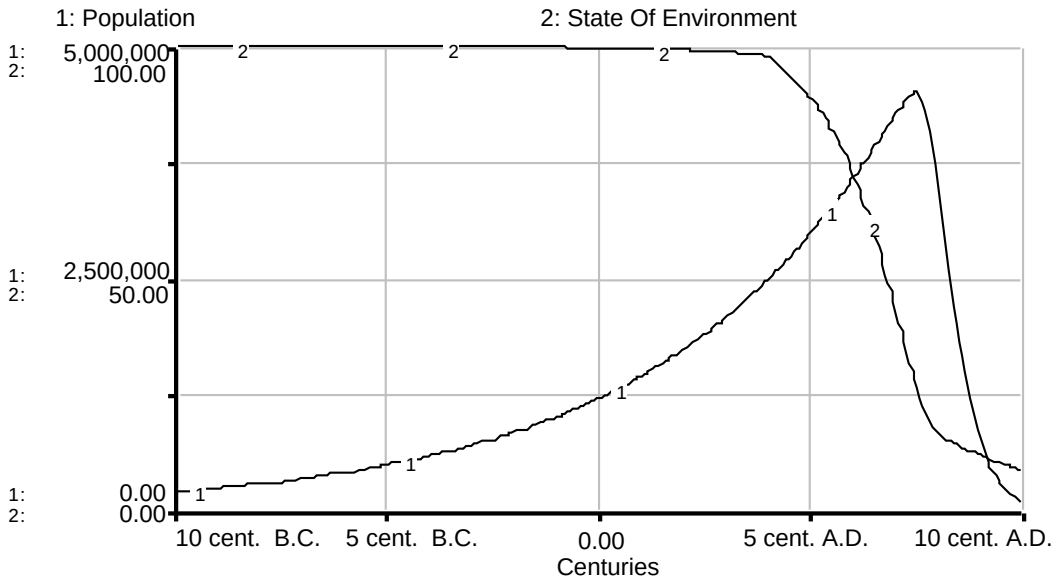


Figure 15: Behavior of the Mayan population model

As expected, the model exhibits the overshoot and collapse behavior mode. The Mayan population grows exponentially from the 10th century B.C. to approximately the 7th century A.D., when it peaks sharply and quickly collapses within about one century.

The decline of the "State of Environment" is almost unnoticeable until around the 4th century A.D. Then, as the "Population" rises, the "density" also increases, so the "effect of density on damage fraction" increases, causing a faster decline of the "State of Environment." As the "State of Environment" declines, the "effect of environment on death fraction" multiplier rises sharply. Hence, the "death fraction" eventually exceeds the "BIRTH FRACTION," causing a quick collapse of the "Population."

Therefore, the model closely replicates the behavior described in the article. As long as the reader's model exhibits overshoot and collapse, it is not necessary to try to replicate the described behavior precisely. A closer fit to the historically observed behavior may be achieved by changing some parameter values or modifying the table functions.

7. APPENDIX: MODEL DOCUMENTATION

7.1 Petroleum Extraction Model

☐ **Petroleum_Reserves(t)** = Petroleum_Reserves(t - dt) + (- extraction) * dt
INIT Petroleum_Reserves = 10^9
DOCUMENT: The estimated number of barrels of petroleum under the field is initially a billion, or 10^9.
Units: Barrels

☞ OUTFLOWS:
extraction = Wells * extraction_per_well
DOCUMENT: Number of barrels of petroleum extracted each year.
Units: Barrels/Year

☐ **Wells(t)** = Wells(t - dt) + (drilling_wells - closing_wells) * dt
INIT Wells = 1
DOCUMENT: Number of wells on the field extracting petroleum.
Units: Wells

☞ INFLOWS:
drilling_wells = Wells * drilling_fraction
DOCUMENT: Number of wells drilled each year.
Units: Wells/Year

☞ OUTFLOWS:
closing_wells = Wells * closing_fraction
DOCUMENT: Number of wells closing each year.
Units: Wells/Year

○ **closing_fraction** = NORMAL_CLOSING_FRACTION * effect_of_extraction_on_closing_fraction
DOCUMENT: The fraction of wells closing each year.
Units: 1/Year

○ **drilling_fraction** = NORMAL_DRILLING_FRACTION * effect_of_reserves_on_drilling_fraction
DOCUMENT: The fraction of wells drilled each year.
Units: 1/Year

○ **extraction_per_well** = MAXIMUM_EXTRACTION_PER_WELL * effect_of_reserves_on_extraction_per_well
DOCUMENT: The amount of petroleum extracted per well each year.
Units: (Barrels/Well)/Year

○ **INITIAL_PETROLEUM_RESERVES** = 10^9
DOCUMENT: The initial amount of petroleum in the field is estimated to be a billion, or 10^9.
Units: Barrels

○ **MAXIMUM_EXTRACTION_PER_WELL** = 1000000
DOCUMENT: The amount of petroleum that a well can extract each year at full capacity when there is ample petroleum left.
Units: (Barrels/Well)/Year

○ **NORMAL_CLOSING_FRACTION** = 0.05
DOCUMENT: The normal fraction of wells that close each year.
Units: 1/Year

○ **NORMAL_DRILLING_FRACTION** = 0.25
DOCUMENT: The normal fraction of wells drilled each year.
Units: 1/Year

⊘ **effect_of_extraction_on_closing_fraction** =
GRAPH(extraction_per_well/MAXIMUM_EXTRACTION_PER_WELL)
(0.00, 20.0), (0.1, 19.9), (0.2, 19.8), (0.3, 18.8), (0.4, 11.1), (0.5, 4.60), (0.6, 2.30), (0.7, 1.40), (0.8, 1.05), (0.9, 1.00), (1, 1.00)
DOCUMENT: The effect of the remaining percentage of petroleum reserves on the closing fraction.
Units: dimensionless

⊘ **effect_of_reserves_on_drilling_fraction** =
GRAPH(Petroleum_Reserves/INITIAL_PETROLEUM_RESERVES)
(0.00, 0.00), (0.1, 0.00), (0.2, 0.00), (0.3, 0.00), (0.4, 0.41), (0.5, 0.695), (0.6, 0.84), (0.7, 0.935), (0.8, 0.98), (0.9, 0.995), (1, 1.00)
DOCUMENT: The effect of the remaining percentage of petroleum reserves on the drilling fraction.
Units: dimensionless

⊘ **effect_of_reserves_on_extraction_per_well** =
GRAPH(Petroleum_Reserves/INITIAL_PETROLEUM_RESERVES)
(0.00, 0.00), (0.05, 0.00), (0.1, 0.00), (0.15, 0.00), (0.2, 0.00), (0.25, 0.00), (0.3, 0.00), (0.35, 0.00), (0.4, 0.00), (0.45, 0.00), (0.5, 0.00), (0.55, 0.00), (0.6, 0.005), (0.65, 0.045), (0.7, 0.15), (0.75, 0.37), (0.8, 0.725), (0.85, 0.92), (0.9, 0.985), (0.95, 0.995), (1.00, 1.00)
DOCUMENT: The effect of the remaining percentage of petroleum reserves on the extraction per well.
Units: dimensionless

7.2 Deer Population Model

□ **Deer(t)** = Deer(t - dt) + (births - deaths) * dt
INIT Deer = 100
DOCUMENT: The population of deer.
Units: Deer

⤳ INFLOWS:
births = Deer * BIRTH_FRACTION
DOCUMENT: Number of deer born every year.
Units: Deer/Year

OUTFLOWS:
deaths = Deer * death_fraction
DOCUMENT: Number of deer dying every year.
Units: Deer/Year

Vegetation(t) = Vegetation(t - dt) + (growth - consumption) * dt
INIT Vegetation = 10000
DOCUMENT: Amount of vegetation present in the area.
Units: Unit of vegetation

INFLOWS:
growth = Vegetation * GROWTH_PER_UNIT_OF_VEGETATION
DOCUMENT: Rate of regeneration of the vegetation.
Units: Units of vegetation/Year

OUTFLOWS:
consumption = Deer * consumption_per_deer
DOCUMENT: Rate of consumption of the vegetation.
Units: Units of vegetation/Year

BIRTH_FRACTION = 0.5
DOCUMENT: Number of deer born per deer every year.
Units: 1/Year

consumption_per_deer = NORMAL_CONSUMPTION_PER_DEER * effect_of_ vegetation_on_consumption_per_deer
DOCUMENT: Amount of vegetation consumed per deer per year.
Units: (Units of vegetation/Deer)/Year

death_fraction = NORMAL_DEATH_FRACTION * effect_of_vegetation_on_ death_fraction
DOCUMENT: Fraction of deer dying each year.
Units: 1/Year

GROWTH_PER_UNIT_OF_VEGETATION = 0.1
DOCUMENT: Units of vegetation regenerated per unit of vegetation every year. Every year, the amount of vegetation in the area grows by approximately 10 percent.
Units: 1/Year

NORMAL_AMOUNT_OF_VEGETATION = 10000
DOCUMENT: The normal amount of vegetation in the area.
Units: Unit of vegetation

NORMAL_CONSUMPTION_PER_DEER = 1
DOCUMENT: Number of units of vegetation that a deer consumes per year if there is enough vegetation available.
Units: (Units of vegetation/Deer)/Year

NORMAL_DEATH_FRACTION = 0.1
DOCUMENT: Fraction of deer dying per deer every year when there is enough vegetation available.
Units: 1/Year

⊘ **effect_of_vegetation_on_consumption_per_deer** =
GRAPH(Vegetation/NORMAL_AMOUNT_OF_VEGETATION)
(0.00, 0.00), (0.1, 0.305), (0.2, 0.545), (0.3, 0.72), (0.4, 0.835), (0.5, 0.905), (0.6, 0.945), (0.7, 0.97), (0.8, 0.985), (0.9, 1.00), (1, 1.00)
DOCUMENT: Effect of the availability of vegetation on consumption per deer.
Units: dimensionless

⊘ **effect_of_vegetation_on_death_fraction** =
GRAPH(Vegetation/NORMAL_AMOUNT_OF_VEGETATION)
(0.00, 10.0), (0.1, 7.15), (0.2, 5.05), (0.3, 3.15), (0.4, 2.15), (0.5, 1.60), (0.6, 1.35), (0.7, 1.15), (0.8, 1.05), (0.9, 1.00), (1, 1.00)
DOCUMENT: Effect of the availability of vegetation on the deer death fraction.
Units: dimensionless

7.3 Health Deterioration Model

☐ **Health(t)** = Health(t - dt) + (- health_deterioration) * dt
INIT Health = 100
DOCUMENT: State of Bryant's health. A perfect health is defined to have the value of 100, as in 100%.
Units: units of health.

OUTFLOWS:
health_deterioration = Hours_Worked_Per_Day * health_deterioration_per_hour_worked
DOCUMENT: Bryant's health gets worse as he works more.
Units: units of health/day

☐ **Hours_Worked_Per_Day(t)** = Hours_Worked_Per_Day(t - dt) + (increase_in_hours - decrease_in_hours) * dt
INIT Hours_Worked_Per_Day = 8
DOCUMENT: Number of hours that Bryant works per day.
Units: hours per day

INFLOWS:
increase_in_hours = Hours_Worked_Per_Day * INCREASE_FRACTION
DOCUMENT: The number of hours by which Bryant daily increases the number of hours he works per day.
Units: hours per day/day

OUTFLOWS:
decrease_in_hours = Hours_Worked_Per_Day * decrease_fraction
DOCUMENT: The number of hours by which Bryant daily decreases the number of hours he works per day.
Units: hours per day/day

○ **decrease_fraction** = NORMAL_DECREASE_FRACTION * effect_of_health_on_decrease_fraction
DOCUMENT: The fraction of an hour by which Bryant decreases the number of hours he works per day.
Units: 1/day

○ **health_deterioration_per_hour_worked** =
NORMAL_HEALTH_DETERIORATION_PER_HOUR_WORKED * effect_of_health_on_health_deterioration_per_hour_worked
DOCUMENT: The fraction by which Bryant's health deteriorates each day for every hour he works.
Units: (units of health/hours per day)/day

○ **INCREASE_FRACTION** = 0.1
DOCUMENT: The fraction by which Bryant increases the number of hours he works each day. As Bryant spends more hours working on the project, he likes it more and becomes addicted to his work, so he spends even more hours working.
Units: 1/day

○ **INITIAL_HEALTH** = 100
DOCUMENT: The initial state of Bryant's health. Assume Bryant is initially perfectly healthy, so his health is at 100%.
Units: units of health

○ **NORMAL_DECREASE_FRACTION** = 0.05
DOCUMENT: The fraction by which Bryant decreases the number of hours he works each day while his health is perfect.
Units: 1/week

○ **NORMAL_HEALTH_DETERIORATION_PER_HOUR_WORKED** = 0.15
DOCUMENT: The fraction by which Bryant's health deteriorates with every hour he works.
Units: (units of health/hours per day)/day

⊘ **effect_of_health_on_decrease_fraction** = GRAPH(Health/INITIAL_HEALTH)
(0.00, 100), (0.1, 100), (0.2, 100), (0.3, 100), (0.4, 50.0), (0.5, 26.0), (0.6, 10.0), (0.7, 2.00), (0.8, 1.20), (0.9, 1.05), (1, 1.00)
DOCUMENT: The effect of Bryant's health on the "decrease fraction." As his health deteriorates, Bryant realizes that his has to stop working so many hours every day, so the "decrease fraction" grows.
Units: dimensionless

⊘ **effect_of_health_on_health_deterioration_per_hour_worked** =
GRAPH(Health/INITIAL_HEALTH)
(0.00, 0.00), (0.1, 0.435), (0.2, 0.645), (0.3, 0.79), (0.4, 0.87), (0.5, 0.915), (0.6, 0.945), (0.7, 0.97), (0.8, 0.985), (0.9, 0.995), (1, 1.00)
DOCUMENT: The effect of Bryant's health on the "health deterioration per hour." As Bryant's health gets worse, the fraction of his health that deteriorates for every hour he works becomes smaller.
Units: dimensionless

7.4 Mayan Population Model

□ **Population(t)** = Population(t - dt) + (births - deaths) * dt
INIT Population = 200000
DOCUMENT: The size of the Mayan population.
Units: People

INFLOWS:
births = Population * BIRTH_FRACTION
DOCUMENT: Number of people born per century.
Units: People/Century

OUTFLOWS:
deaths = Population * death_fraction
DOCUMENT: Number of people dying per century.
Units: People/Century

State_Of_Environment(t) = State_Of_Environment(t - dt) +
(environment_regeneration - damage_to_environment) * dt
INIT State_Of_Environment = INITIAL_STATE_OF_ENVIRONMENT
DOCUMENT: The state of the environment in which the Mayan population lives. The initial value is 100, as in 100%.
Units: Environment units

INFLOWS:
environment_regeneration = State_Of_Environment/regeneration_time
DOCUMENT: Regeneration of the environment per century.
Units: Environment units/Century

OUTFLOWS:
damage_to_environment = State_Of_Environment * damage_fraction
DOCUMENT: Environment damage per century.
Units: Environment units/Century

BIRTH_FRACTION = 2
DOCUMENT: Number of people born per person per century.
Units: 1/Century

damage_fraction = NORMAL_DAMAGE_FRACTION * effect_of_density_on_
damage_fraction
DOCUMENT: Fraction of the environment that is damaged per century.
Units: 1/Century

death_fraction = NORMAL_DEATH_FRACTION * effect_of_environment_on_
death_fraction
DOCUMENT: Number of people dying per person per century.
Units: 1/Century

density = Population/LAND_AREA
DOCUMENT: Density of the Mayan population.
Units: People/square kilometer

INITIAL_STATE_OF_ENVIRONMENT = 100
DOCUMENT: Initial state of the environment.
Units: Environment units

LAND_AREA = 10000
DOCUMENT: Area on which the Mayan population lived.
Units: square kilometers

○ **NORMAL_DAMAGE_FRACTION** = 1
DOCUMENT: Fraction of the environment that is damaged per century when density is at its initial value.
Units: 1/Century

○ **NORMAL_DEATH_FRACTION** = 1.82
DOCUMENT: Number of people dying per person per century when the environment has no negative impact on the lifetime.
Units: 1/Century

○ **NORMAL_DENSITY** = 20
DOCUMENT: Density of the Mayan population that does not have any damaging effect on the environment.
Units: People/square kilometer

○ **NORMAL_REGENERATION_TIME** = 1
DOCUMENT: Time necessary for the environment to regenerate itself when no substantial damage has been done yet.
Units: Century

○ **regeneration_time** = NORMAL_REGENERATION_TIME * effect_of_ environment_on_regeneration_time
DOCUMENT: Time necessary for the environment to regenerate itself.
Units: Century

⊘ **effect_of_density_on_damage_fraction** = GRAPH(density/NORMAL_DENSITY)
(0.00, 1.00), (1.00, 1.00), (2.00, 1.00), (3.00, 1.00), (4.00, 1.00), (5.00, 1.00), (6.00, 1.00), (7.00, 1.00), (8.00, 1.00), (9.00, 1.00), (10.0, 1.00), (11.0, 1.00), (12.0, 1.00), (13.0, 1.10), (14.0, 1.10), (15.0, 1.10), (16.0, 1.20), (17.0, 1.20), (18.0, 1.35), (19.0, 1.35), (20.0, 1.60), (21.0, 1.60), (22.0, 1.90), (23.0, 1.90), (24.0, 2.30), (25.0, 2.80), (26.0, 3.40), (27.0, 4.25), (28.0, 6.25), (29.0, 26.8),
DOCUMENT: Effect of the density as compared to the normal density on the damage fraction.
Units: dimensionless

⊘ **effect_of_environment_on_death_fraction** =
GRAPH(State_Of_Environment/INITIAL_STATE_OF_ENVIRONMENT)
(0.00, 5.00), (0.1, 2.33), (0.2, 1.33), (0.3, 1.03), (0.4, 1.02), (0.5, 1.00), (0.6, 1.00), (0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
DOCUMENT: Effect of the environment as compared to the initial state of the environment on the death fraction.
Units: dimensionless

⊘ **effect_of_environment_on_regeneration_time** =
GRAPH(State_Of_Environment/INITIAL_STATE_OF_ENVIRONMENT)
(0.00, 2.00), (0.1, 1.50), (0.2, 1.32), (0.3, 1.18), (0.4, 1.09), (0.5, 1.04), (0.6, 1.01), (0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
DOCUMENT: Effect of the environment as compared to the initial state of the environment on the time necessary for the environment to regenerate itself.
Units: dimensionless

Graphical Integration Exercises Part Five: Qualitative Graphical Integration

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Prof. Jay W. Forrester

by
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1. ABSTRACT

Stocks accumulate the difference between the inflow and outflow from the stock over time. The value of a stock at any time can be calculated by integrating (adding continuously) the flows affecting the stock. Graphical integration is the process of estimating the behavior of a stock by looking at graphs of the flows into and out of the stock.

This paper studies various positive and negative flows and the effect of the flows on stocks: positive flows (inflows to a stock) increase the value of the stock, while negative flows (outflows from a stock) decrease the value of the stock. The slope of the flow graph determines how rapidly the value of the stock changes. Thus, flows that are increasing in magnitude and flows that are decreasing in magnitude affect stocks in different ways. This paper helps the reader to develop an intuitive understanding of how stocks are affected by flows. The focus of this paper is to estimate the general behaviors of stocks and not to calculate the exact value of a stock at a particular point in time.

2. INTRODUCTION

Graphical integration is the process of estimating the behavior or value of a stock by studying the flows into and out of the stock. Previous Road Maps chapters introduced graphical integration of linear flows in a quantitative manner. After completing Graphical Integration Exercises Part 1 through 3,[1] the reader should be able to perform simple arithmetic operations to calculate the value of the stock at any time from a graph of the linear flow behavior. If more than one flow affect a stock, the behavior of the stock depends on the sum of the effects of each of the flows: the effect of the net flow. Precise quantitative measurements of flows, however, may not be readily available. A system dynamicist should be able to simply look at the shape of a flow graph and qualitatively determine the behavior of the stock. Often, the behavior pattern of a stock is more important in determining the behavior of the system than the exact value of the stock. Readers should also learn to estimate the flows from looking at the stock diagrams. The process of reverse graphical integration, introduced in Graphical Integration Exercises Part Four: Reverse Graphical Integration,[2] is helpful in understanding the behavior of systems.

This paper generalizes some of the behaviors of linear flows seen in the first three papers in the Graphical Integration series. The reader will better understand the fundamental concepts of graphical integration and will be able to estimate the behavior of stocks easily and without precise calculations. This paper includes a discussion of fundamental properties of linear flows, followed by a set of exercises with detailed solutions.

3. GRAPHICAL INTEGRATION OF LINEAR FLOWS

The linear flows discussed so far in Road Maps have been functions that can be graphed as straight lines. Among the topics covered in this paper are constant, linearly

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[1] Alice Oh, 1995. *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), December 1, 19 pp;
Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571), May 29, 25 pp;
Kevin Agatstein and Lucia Breierova, 1996. *Graphical Integration Exercises Part Three: Combining Flows* (D-4596), March 14, 32 pp.
All papers written for the System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

[2] Laughton Stanley, 1996. *Graphical Integration Exercises Part Four: Reverse Graphical Integration* (D-4603), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, August 20, 25 pp.

increasing, and linearly decreasing flows. A general set of steps for performing graphical integration qualitatively is presented.

The reader should be able to combine flows to find the net flow. The net flow is simply the arithmetic sum of all the individual flows over time. The reader should understand certain mathematical terms such as slope, magnitude, and value. The concept of the slope of a line and methods to calculate the slope are discussed in Graphical Integration Exercises Part 2: Ramp Functions.[3] The slope of a line measures how rapidly the y-axis value of the line changes with respect to the x-axis value. All graphs in this paper have "time" on the x-axis; hence, the slope measures how rapidly the y-axis value of the line changes over time. The magnitude of a number is also called the absolute value of the number. The magnitude measures the numeric part of a number, ignoring the sign of the number. For example, the magnitude of "-6" is "6," the magnitude of "-17" is "17," and the magnitude of "+3" is "3." The value of a number essentially shows where the number appears on the number line. The value is definited by the magnitude and the sign of the number. For example, the value of "-10" is "-10," and the value of "+14" is "+14." Thus, for a positive function, the value and magnitude are the same, so an increase in the magnitude also means an increase in value: the function moves away from zero. For negative functions, however, increasing value means that the function is moving closer to zero, and thus has decreasing magnitude. Thus, a negative function with increasing value has decreasing magnitude. A negative function with decreasing value grows more and more negative with time (moves away from zero) and thus has increasing magnitude.

3.1 Types of Linear Flows

Flows can be positive, zero, or negative. A positive flow is an inflow to a stock, while a negative flow is an outflow from the stock. A flow with a value of zero means that nothing flows into or out of the stock. A biflow may be positive and negative over different time periods, but at any given time, a biflow has either a positive or negative value.

This paper studies linear flows that can be graphed as straight lines. Such flows may be constant, increasing, or decreasing in magnitude. A constant flow means that in each time period, a fixed amount is added to or removed from the stock. Linearly increasing or decreasing flows are called ramp functions.[4] The value of the flow changes

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[3] Kevin Agatstein and Lucia Breierova, D-4571.

[4] System dynamics modeling software applications use "RAMP" in the list of built-in functions to plot linearly increasing and decreasing functions.

by a fixed amount during each time interval. Thus, the amount by which the stock changes in each time period is not constant over time because the value of the flow is changing.

Therefore, based on whether the flow is positive, zero, or negative, and whether the magnitude of the flow is constant, increasing, or decreasing, linear flows studied in this paper can be classified into seven types:

1. zero
2. positive and constant
3. positive and increasing in magnitude
4. positive and decreasing in magnitude
5. negative and constant
6. negative and increasing in magnitude
7. negative and decreasing in magnitude

The paper studies the effect of each type of flow on the stock.

3.2 Zero Flow

A zero flow means that the sum of all the flows into the stock equals the sum of all the flows out of the stock. Thus, a zero net flow causes no change in the value of the stock, and the stock remains at the initial value, irrespective of whether the initial value is positive, zero, or negative. Figure 1 shows a zero flow and the stock the flow affects. The stock has an initial value equal to “initial value.”

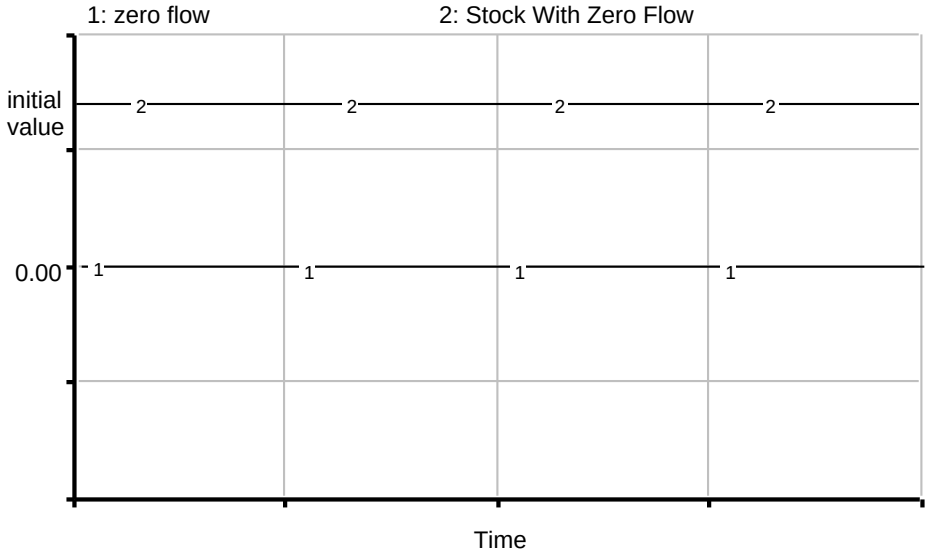


Figure 1: Net zero flow and stock affected by a net zero flow. The stock remains at the initial value equal to "initial value."

3.3 Positive Flows

Positive flows correspond to flows into a stock. A positive flow increases the value of the stock. A positive flow may be constant, increasing, or decreasing in magnitude.

3.3.1 Constant Positive Flows

The graph in Figure 2 shows a constant positive flow and the stock affected by the flow. The initial value of the stock is "initial value." The value of the stock increases by a constant amount each time period because the slope of the stock graph, which is equal to the value of the flow, is constant. The reader will be unable to calculate the slope of the stock graph because the value of the stock at any time is not given.

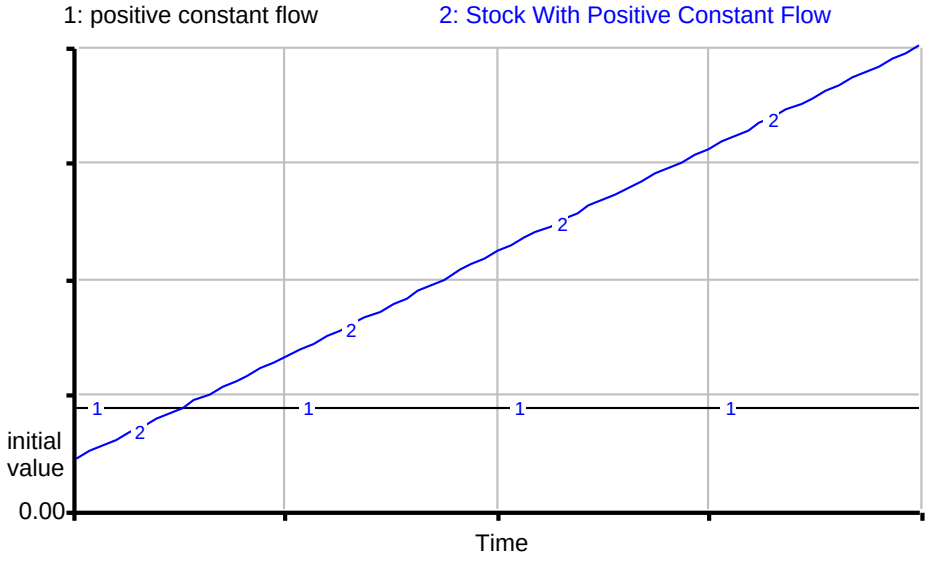


Figure 2: Positive constant flow and the stock affected by the flow

3.3.2 Positive Flows Increasing in Magnitude

A positive flow increasing in magnitude adds a larger amount to the stock in each successive time period. Thus, the value of the stock grows at an increasing rate over time.

Figure 3 shows a positive linearly increasing flow and the stock affected by the flow. The stock has a negative initial value equal to "initial value."

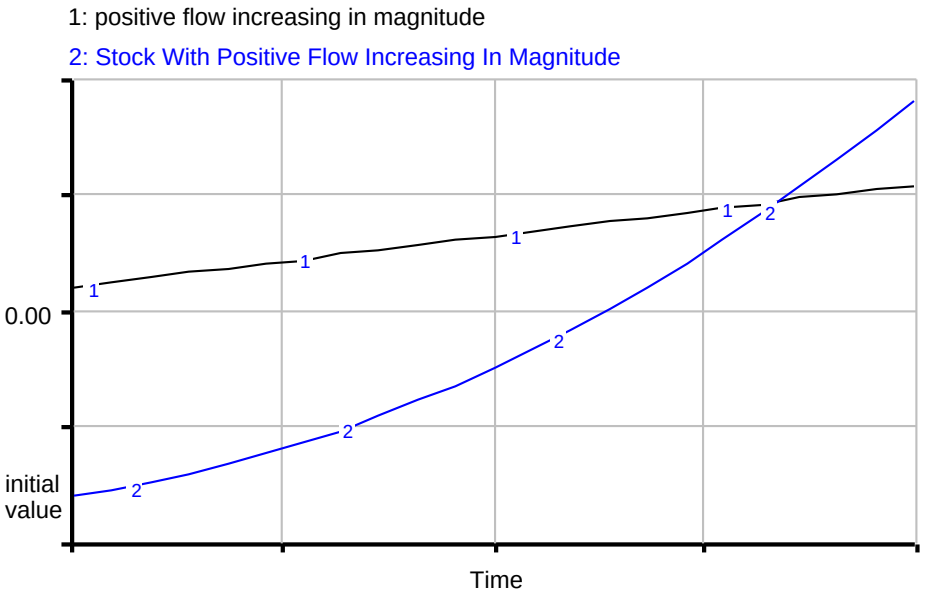


Figure 3: Positive flow increasing in magnitude and the stock affected by the flow

As seen in Figure 3, the slope of the stock graph increases as time passes. The slope of the stock graph at any time equals the value of the flow at that time. Because the magnitude of the flow is increasing with time, the stock grows more and more rapidly as time passes. The initial value of the stock has no effect on the behavior of the stock. A stock with any initial value will grow at a faster and faster rate with time if the net flow into the stock is positive and increasing in magnitude.

3.3.3 Positive Flows Decreasing in Magnitude

If the magnitude of a positive flow is decreasing over time, the stock will increase by a smaller and smaller amount as time passes. As the value of the flow approaches zero, the stock approaches a certain value. When the flow reaches and stays zero, the stock stops changing and attains an equilibrium value.

Figure 4 shows a linearly decreasing positive flow and the stock affected by the flow.

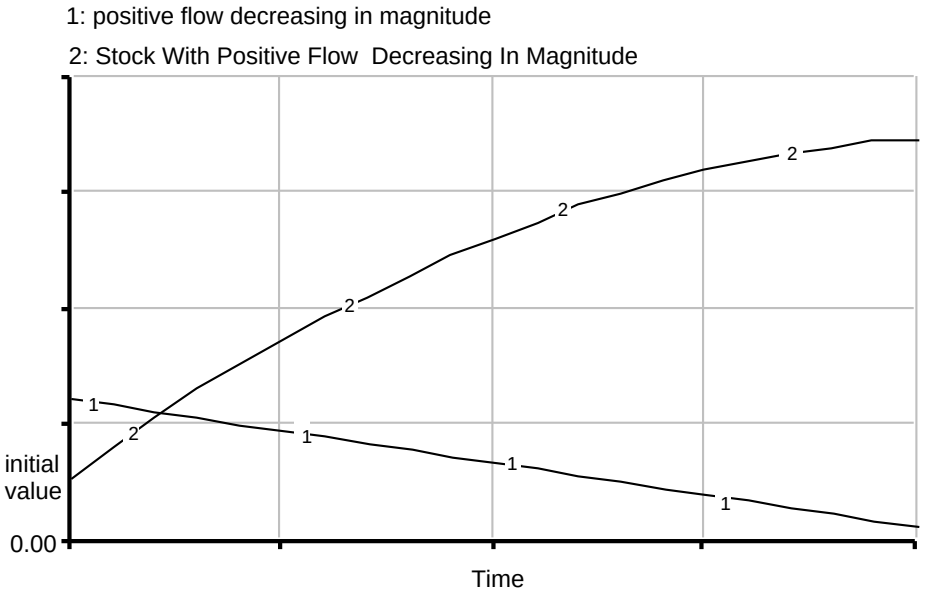


Figure 4: Positive flow decreasing in magnitude and the stock affected by the flow

Figure 4 shows that even if the positive flow is decreasing in magnitude, the value of the stock increases because as long as the flow is positive, the contents of the flow are being added to the stock. As time passes, the stock increases by a smaller and smaller amount, approaching an equilibrium value as the flow approaches zero.

Thus, a positive flow always increases the value of the stock irrespective of the initial value of the stock. The stock may have an initial value greater than, equal to, or less than zero, but the value of a stock with a positive flow increases with time because the contents of the flow are continuously being added to the stock. The rate at which the value of the stock increases depends on the magnitude of the positive flow. The behavior of the stock over time depends on whether the positive flow is constant, increasing, or decreasing over time.

3.4 Negative Flows

Negative flows correspond to outflows from the stock. A negative flow decreases the value of the stock. A negative flow may be constant, increasing, or decreasing in magnitude.

3.4.1 Constant Negative Flows

The graph in Figure 5 shows a stock with a constant negative flow. The value of the stock decreases from the initial value, irrespective of whether the initial value of the stock is positive, zero, or negative. Figure 5 shows the stock with an initial value equal to "initial value." The amount by which the stock decreases in each time period is constant because the outflow (negative flow) from the stock is constant.

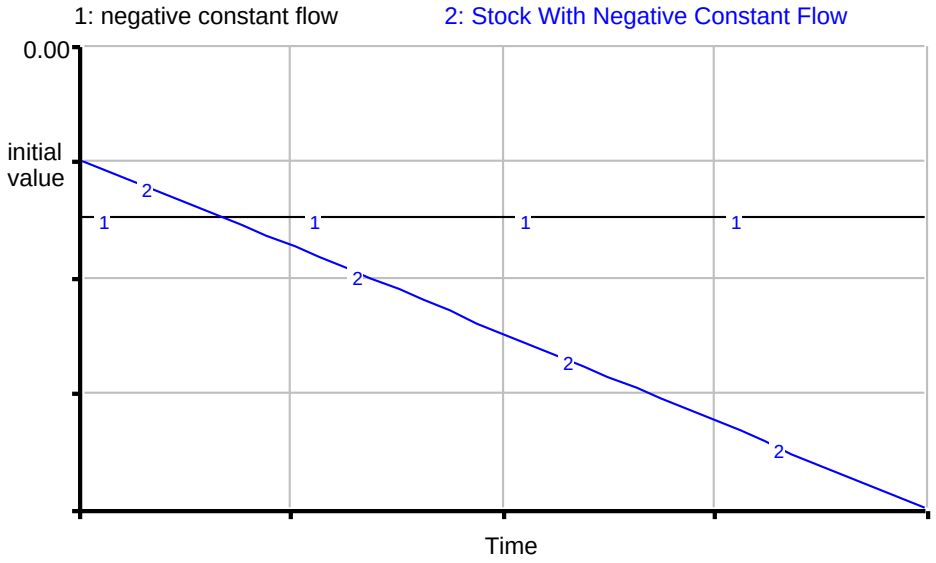


Figure 5: Negative constant flow and the stock affected by the flow

3.4.2 Negative Flows Increasing in Magnitude

A negative flow increasing in magnitude will remove the contents of the stock from the stock at an increasing rate as time passes because the magnitude of the flow increases with time. Figure 6 shows a negative flow increasing in magnitude and the stock behavior produced by the flow.

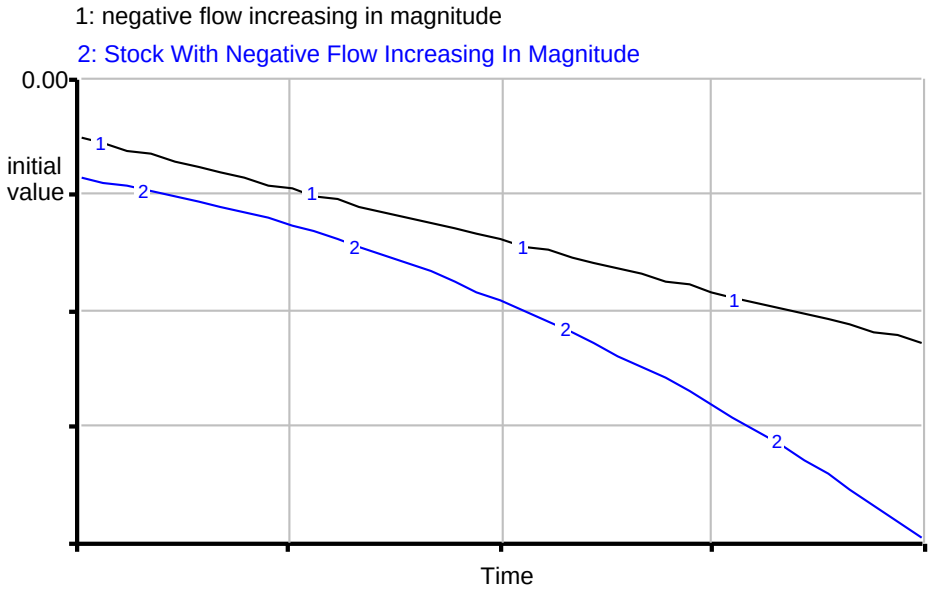


Figure 6: Negative flow increasing in magnitude and the stock affected by the flow

In Figure 6, the value of the stock decreases more and more rapidly over time because the flow becomes more and more negative. The magnitude of the flow, however, increases over time; therefore, the flow is a negative flow increasing in magnitude. Because the flow is negative, the value of the stock decreases over time.

3.4.3 Negative Flows Decreasing in Magnitude

A negative flow linearly decreasing in magnitude reduces the value of the stock by a smaller and smaller amount in each successive time period. Figure 7 shows a negative flow decreasing in magnitude and the stock affected by the flow. The stock in Figure 7 has an initial value of zero, but the behavior of the stock would be the same as in Figure 7 even if the initial value was not equal to zero.

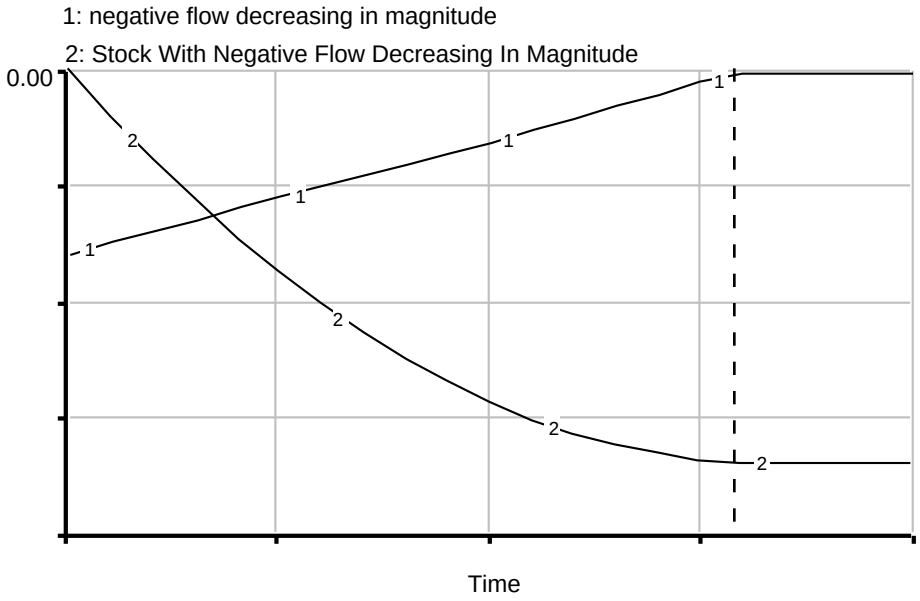


Figure 7: Negative flow decreasing in magnitude and the stock affected by the flow

Figure 4 and Figure 7 both show flows that are decreasing in magnitude. In Figure 4, the flow is positive, and the stock approaches an equilibrium value. The flow in Figure 4 does not reach zero, and so the stock does not reach an equilibrium value, at least in the time period shown in the graph. In Figure 7, however, the negative flow decreasing in magnitude reaches zero after a certain time period and then remains zero. The stock reaches an equilibrium value when the flow reaches zero, and stays at the equilibrium value because the flow remains zero (the value of a stock cannot change if the net flow into the stock is zero).

A negative flow, being an outflow, always decreases the value of a stock over time. The initial value of the stock does not affect the behavior of the stock. Therefore, a stock may have a positive, zero, or negative initial value, but if the net flow out of the stock is negative, the value of the stock will decrease over time.

4. EXAMPLES

4.1 Example 1: Positive and Negative Flows: Crossing the Time Axis[5]

Often, a flow is initially positive but decreases linearly and becomes negative, or an initially negative flow increases linearly and becomes positive. Such a flow is continuously decreasing or increasing in value, but not in magnitude.

An initially positive flow that decreases linearly and becomes negative has a negative slope and a constantly decreasing value. The magnitude of the flow, however, first decreases as the flow approaches zero. When the flow becomes negative, the magnitude of the flow starts increasing. Thus, the flow is a combination of a positive flow with decreasing magnitude and a negative flow with increasing magnitude. Therefore, the behavior of the stock changes when the flow crosses the time axis. Figure 8 shows an initially positive flow with decreasing magnitude that crosses the time axis and becomes negative with increasing magnitude, and the stock behavior produced by the flow.

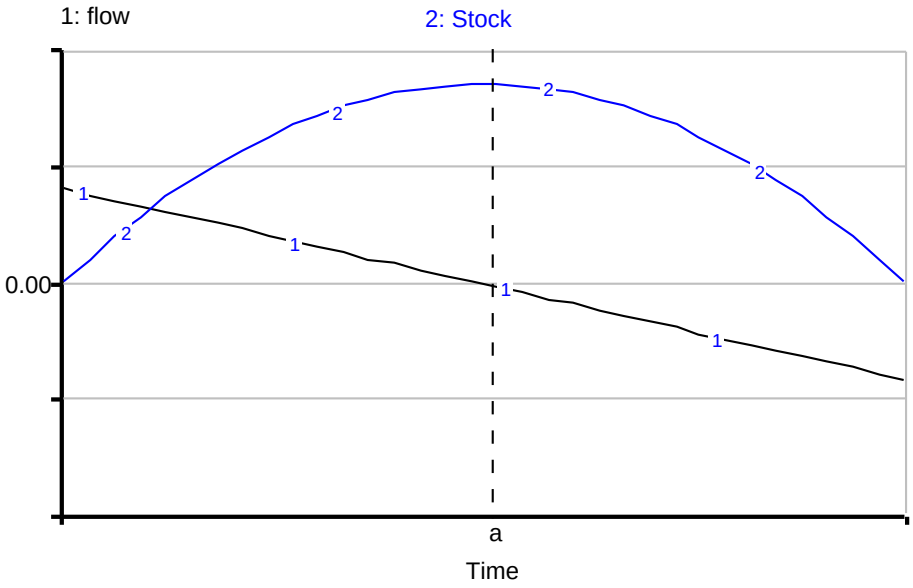


Figure 8: System with a linear flow that crosses the time axis

In Figure 8, the flow has a positive value at time zero. From time 0 to 'a,' the flow is positive and decreasing in magnitude. The stock increases in value because the flow is

[5] In Figure 8, the flow graph does not appear to cross the time axis. Consider the time axis to be the line where 'y = 0.' If the line 'y = 0' is the time axis, then clearly the flow graph crosses the time axis at time 'a.'

positive, but grows at a slower and slower rate because the magnitude of the flow is decreasing. The flow decreases linearly and equals zero at time ‘a.’ After time ‘a,’ however, the flow is negative and increasing in magnitude, so the stock decreases faster and faster.

Two observations can thus be made from the graph in Figure 8:

1. As long as the flow is positive, the value of the stock increases, irrespective of the magnitude of the flow. A positive flow is an inflow; therefore, a positive flow always increases the value of the stock. Similarly, a negative flow always decreases the value of the stock, irrespective of the magnitude of the flow or whether the flow is constant, increasing, or decreasing in magnitude.
2. The shape of the stock graph depends on the rate of change of the flow magnitude. If the flow is decreasing in magnitude, the stock grows or declines more and more slowly as time passes, as in the time between zero and ‘a.’ If the flow is increasing in magnitude, the stock grows or declines more and more rapidly as time passes. Note that the rate at which the stock grows until time ‘a’ and the rate at which the stock declines after time ‘a’ are equal in magnitude because the slope of the flow is constant. The stock behaviors before and after time ‘a’ are mirror images of each other. If the slope of the flow changes, the shape of the stock also changes.

Figure 9 shows a system where the slope of the flow increases in magnitude after time ‘b.’ [6] As a result, the stock decays faster after time ‘b’ than the stock grows before time ‘b.’

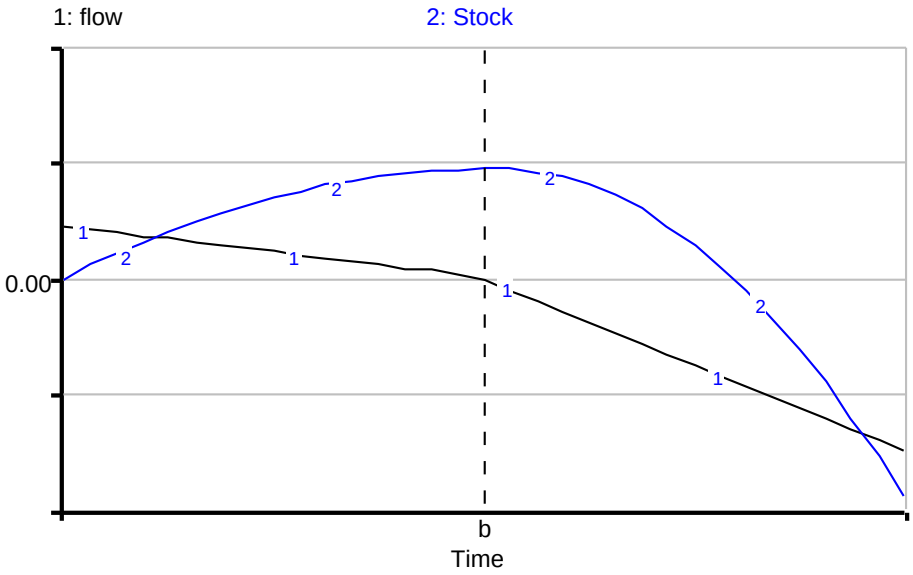


Figure 9: Flow with changing slope that crosses the time axis

[6] The flow in Figure 9 has a negative slope. When the slope becomes more negative after time ‘b,’ the value of the slope decreases because the value of the slope becomes more negative, but the magnitude of the slope increases.

4.2 Example 2: Constant and Linearly Changing Flow

Figure 10 shows a flow that changes characteristics several times.

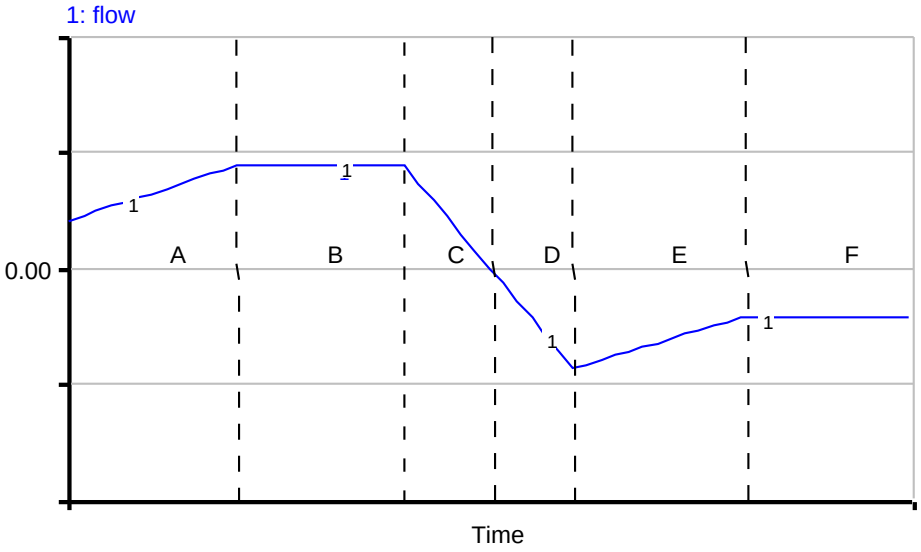


Figure 10: A changing linear flow

The flow can be divided into six segments:

- During segment A, the flow is positive and increasing in magnitude.
- During segment B, the flow is positive and constant.
- During segment C, the flow is positive and decreasing in magnitude.
- During segment D, the flow is negative and increasing in magnitude.
- During segment E, the flow is negative and decreasing in magnitude.
- During segment F, the flow is constant and negative.

Finally, note that the slope magnitudes in segments C and D are greater than the slopes of segments A and E. Therefore, the stock changes more rapidly during segments C and D than in segments A and E. The reader should be able to intuitively estimate the stock behavior, shown in Figure 11. Assume that the initial value of the stock is zero.

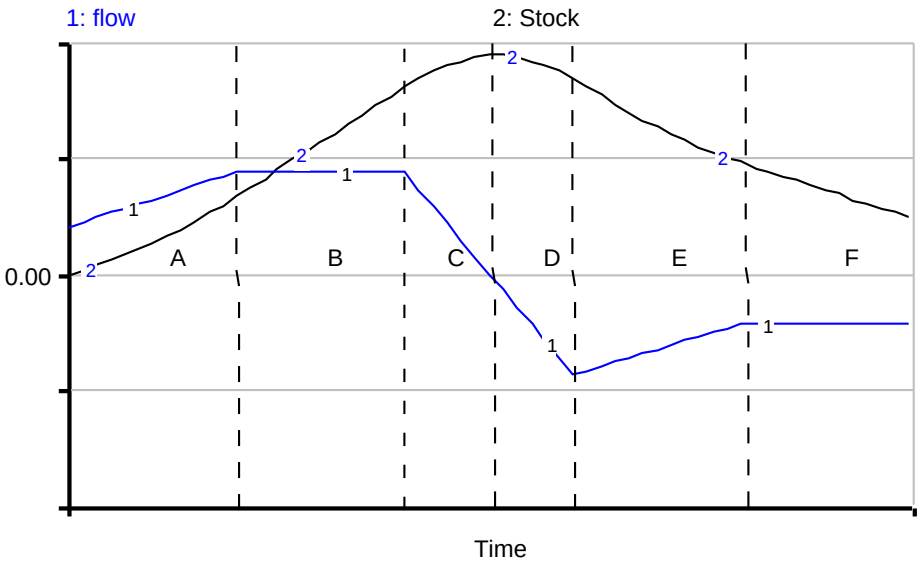


Figure 11: Stock corresponding to the changing linear flow

Estimating the stock behavior:

- During segment A, the flow is positive and increasing: the stock grows faster and faster with time.
- During segment B, the flow is constant: the stock graph is a straight line with a positive slope.
- During segment C, the flow is positive and decreasing: the stock grows at a decreasing rate. At the end of segment C, the stock stops growing because the value of the flow reaches zero.
- The point of time between segment C and segment D is important because the stock behavior changes from positive and decreasing in magnitude to negative and increasing in magnitude.
- During segment D, the flow is negative and increasing in magnitude: the stock decreases faster and faster.
- During segment E, the flow is negative and decreasing in magnitude: the stock continues to decrease but at a slower and slower rate.
- During segment F, the flow is negative and constant: the stock graph is a straight line with a negative slope.

4.3 Example 3: Graphical Integration of Combined Flows

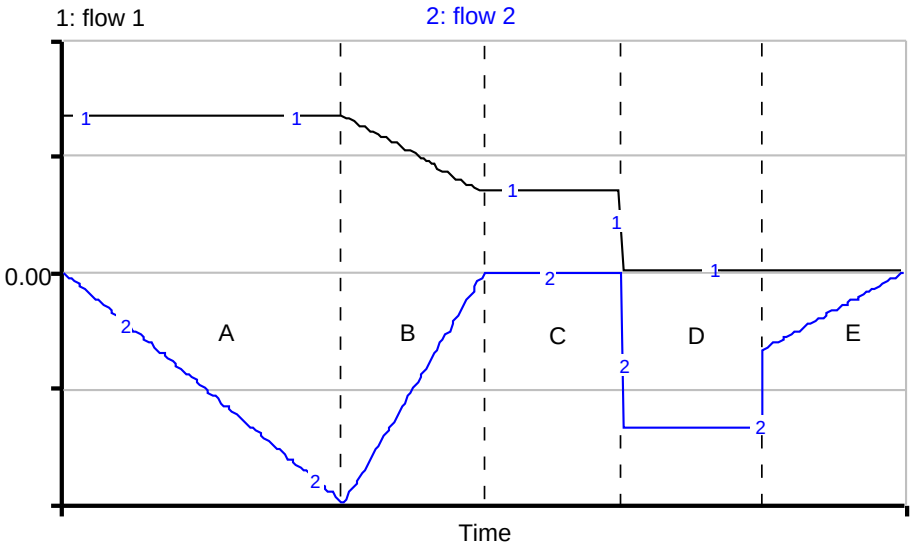


Figure 12: Two flows into the same stock

Figure 12 shows two flows going into the same stock. To estimate the behavior of the stock, the flows in each segment have to be added. The net flow can be calculated by arithmetically adding the values of the flows at every point of time. Figure 13 shows the net flow, the sum of flow 1 and flow 2 in Figure 12.

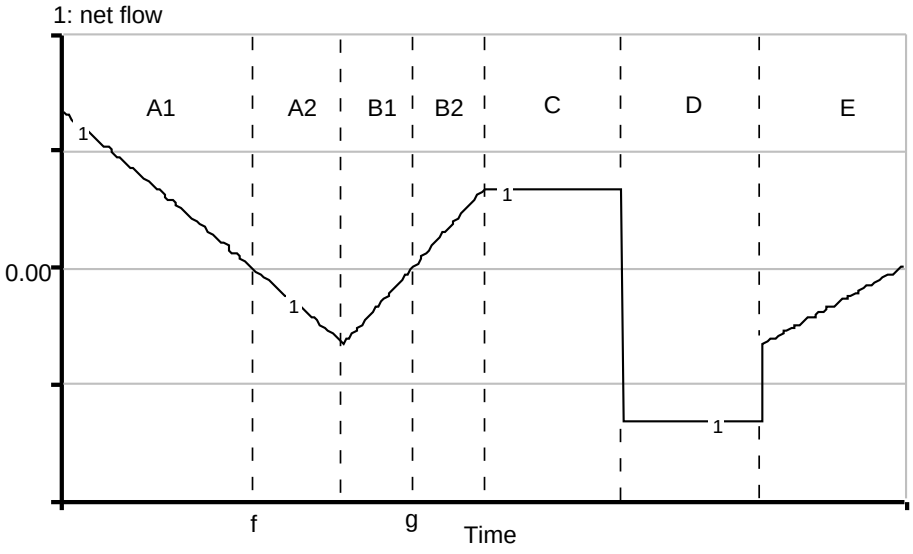


Figure 13: The net flow

The next step is to graphically integrate the net flow. Note that the net flow is divided into seven rather than five segments. At time 'f,' the net flow changes from positive and decreasing in magnitude to negative and increasing in magnitude. Segment A is divided into A1 and A2 because while the slope of the flow is constant in segment A, the characteristics[7] of the flow change at time 'f.' At time 'g,' the flow changes from negative and decreasing in magnitude to positive and increasing in magnitude. Segment B is divided into B1 and B2 because the characteristics of the flow change at time 'g.' The graphical integration of the net flow in Figure 13 is the first flow in this paper that requires the graphical integration of a step function.[8] A segment-by-segment graphical integration yields the stock behavior shown in Figure 14. Assume the initial value of the stock is zero.

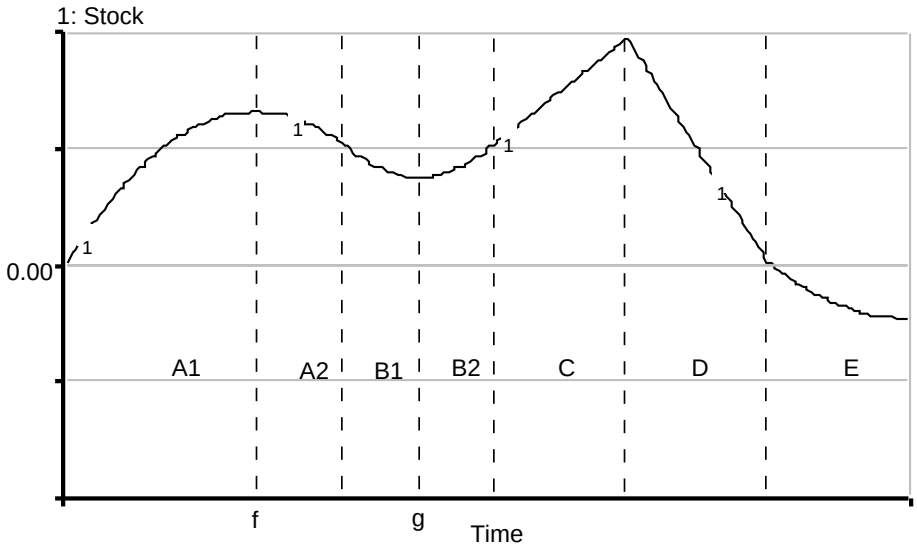


Figure 14: Stock corresponding to the net flow in Figure 13

Estimating the stock behavior:

- During segment A1, the net flow is positive and decreasing in magnitude: the stock grows but at a decreasing rate.
- During segment A2, the net flow is negative and increasing in magnitude: the stock declines at an increasing rate.
- During segment B1, the net flow is negative and decreasing in magnitude: the stock declines at a decreasing rate.

---

[7] The sign of the flow changes from positive to negative, and the flow changes from being decreasing in magnitude to increasing in magnitude. Note, however, that even though the sign of the flow changes, the value of the flow is constantly decreasing.

[8] The "STEP" function is a built-in function in system dynamics simulation softwares. The STEP function is used to model an instantaneous change in the value of a variable.

- During segment B2, the net flow is positive and increasing in magnitude: the stock grows at an increasing rate.
- During segment C, the net flow is positive and constant: the stock grows linearly.
- At the time point between segment C and segment D, the net flow steps down to a negative value.
- During segment D, the net flow is negative and constant: the value of the stock declines linearly.
- At the time point between segment D and segment E, the flow steps up. The flow is still negative, but has a smaller magnitude.
- During segment E, the net flow is negative and decreasing in magnitude: the stock declines in value but at a decreasing rate.

At the time point between segment C and segment D, the net flow steps down and changes instantaneously from a positive to a negative value. In Figure 14, the stock responds by decreasing linearly for the time period that the flow is negative and constant, segment D. Thus, even though the flow changes instantaneously, the stock changes gradually. When the type of linear flow changes, the stock graph changes gradually depending on the characteristics of the flow. The change in the slope of the stock graph is also continuous and gradual. A step function, however, creates a certain point at which a sudden change occurs in the slope of the stock. The slope of the stock graph equals the value of the flow. Thus, when the value of the flow suddenly changes, the slope of the stock graph changes suddenly too. It is important to keep in mind that the value of a stock always changes gradually. There are no sudden jumps or discontinuities in the value of a stock; a stock changes continuously over time, even though flows may change values instantaneously.

5. GENERAL STEPS FOR PERFORMING GRAPHICAL INTEGRATION

The behavior of a stock affected by linear flows can be determined by applying the following general steps:

1) Divide the flow into segments. A segment is a section of the flow diagram during which the characteristics of the flow do not change. Often a flow will be a combination of more than one of the types of flows mentioned in section 3 of this paper: the flow changes from one type to another after a certain time interval. Step 2 below lists the various types of linear flows studied in this paper. Each time the type of flow changes, it marks the beginning of a new segment. Usually, a new segment starts when the slope of the linear flow changes. Each time the flow changes from positive to negative or negative to positive, the slope of the flow may not change but the type of flow does change.

2) Determine the type of linear flow in each segment. Flows can be:

1. zero
2. constant and positive
3. positive and increasing in magnitude
4. positive and decreasing in magnitude
5. constant and negative
6. negative and increasing in magnitude
7. negative and decreasing in magnitude

3) After identifying the different segments of the flow, use the following table to determine the behavior of the stock in each segment:

#	Sign of Flow	Type of Flow	Stock Behavior
1	zero	constant	no change in the stock
2	positive	constant	increasing in a straight line
3	positive	increasing in magnitude	increasing faster and faster
4	positive	decreasing in magnitude	increasing slower and slower
5	negative	constant	decreasing in a straight line
6	negative	increasing in magnitude	decreasing faster and faster
7	negative	decreasing in magnitude	decreasing slower and slower

Table 1: Stock behavior corresponding to different types of linear flows

4) After determining the stock behavior in each segment, combine the behavior of the stock in different segments to estimate the stock behavior over the entire time range being studied.

6. CONCLUSION

Section 3 studied various types of linear flows. Two characteristics of linear flows determine the behavior of the stock:

- the sign of the flow (zero, positive, or negative)
- the magnitude of the flow (constant, increasing, or decreasing)

The exact value or equation of the flow at any time is not crucial in determining the dynamics of the stock. The value of the flow is required, however, to calculate the exact value of a stock at any time. Estimating the behavior of a stock is often more useful than the ability to calculate the exact value of the stock.

When the type of linear flow changes over time, one must divide the flow into distinct segments for each type of behavior and integrate each section separately. The behavior of a stock with more than one flow can be estimated by graphically integrating the sum of all the flows, the net flow.

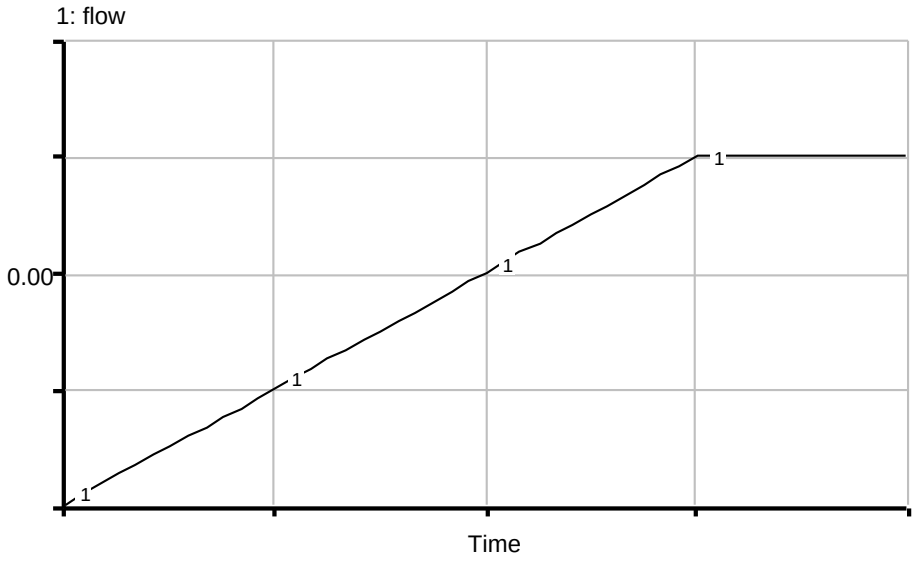
7. EXERCISES

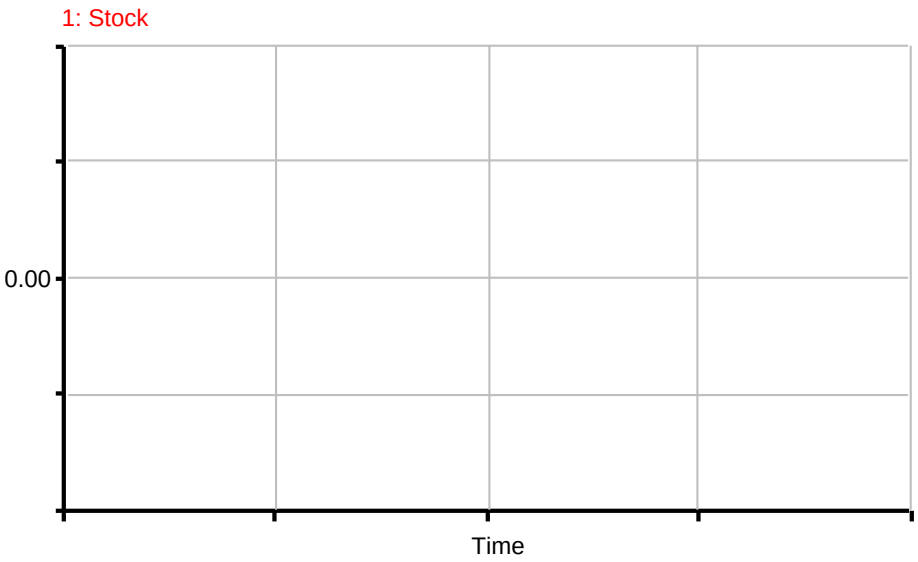
Complete the following exercises, then refer to the solutions in section 8.

7.1 Exercise 1

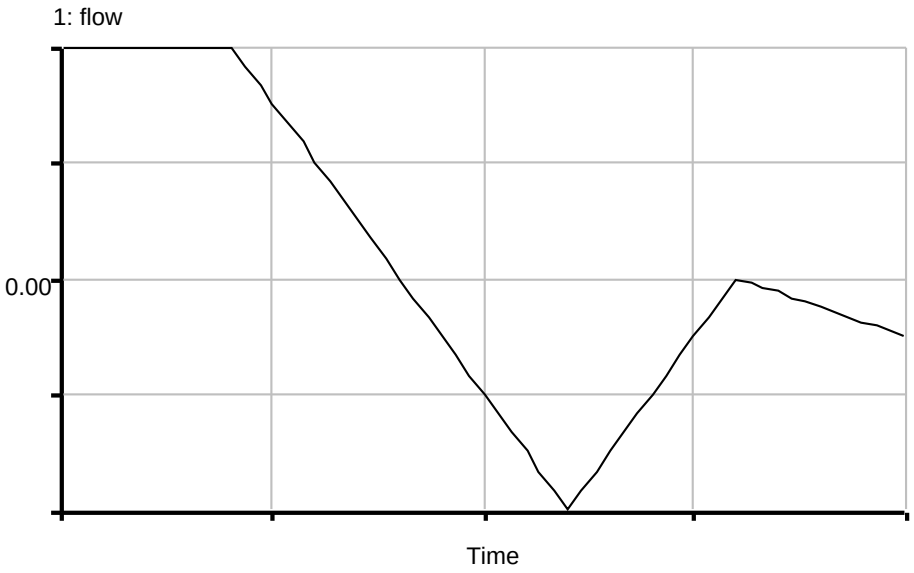
Divide each of the flows in the exercise into appropriate time segments for integration. Draw the corresponding stock behavior on the blank graphs. Assume that the initial values of the stocks are zero (the stock behavior will be the same irrespective of the initial value chosen).

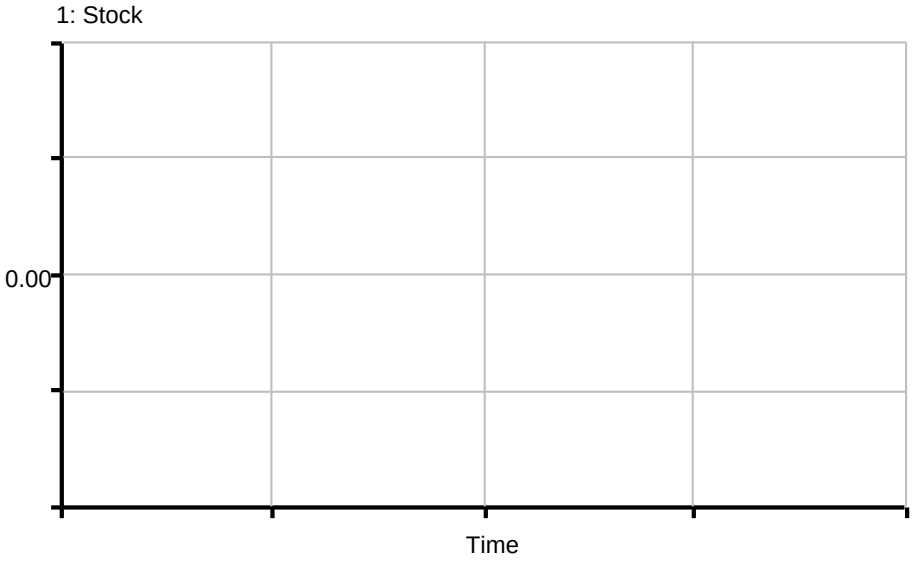
A.





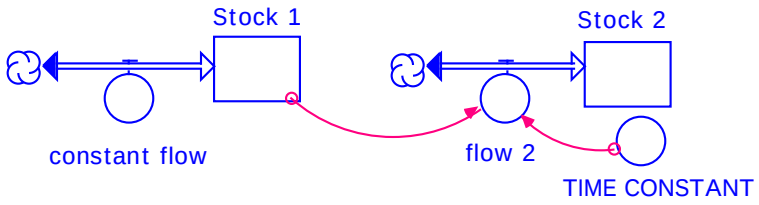
B.

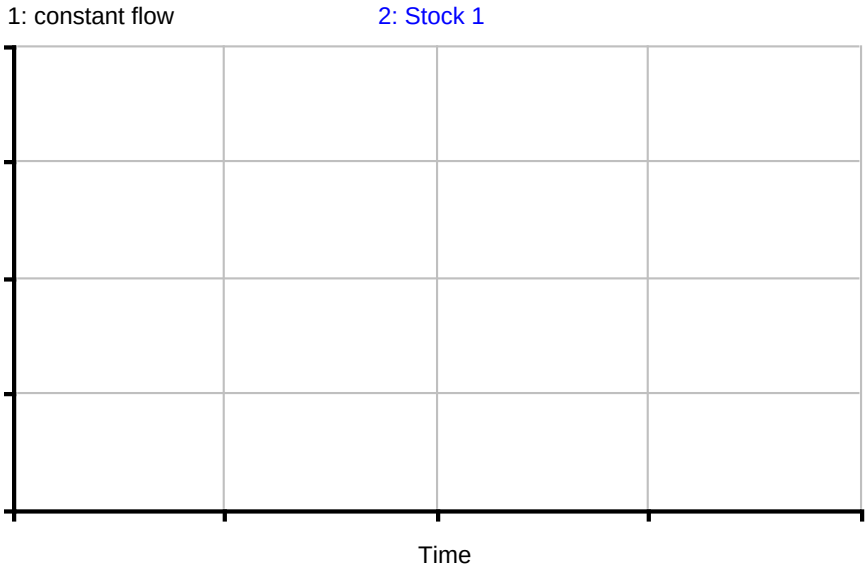




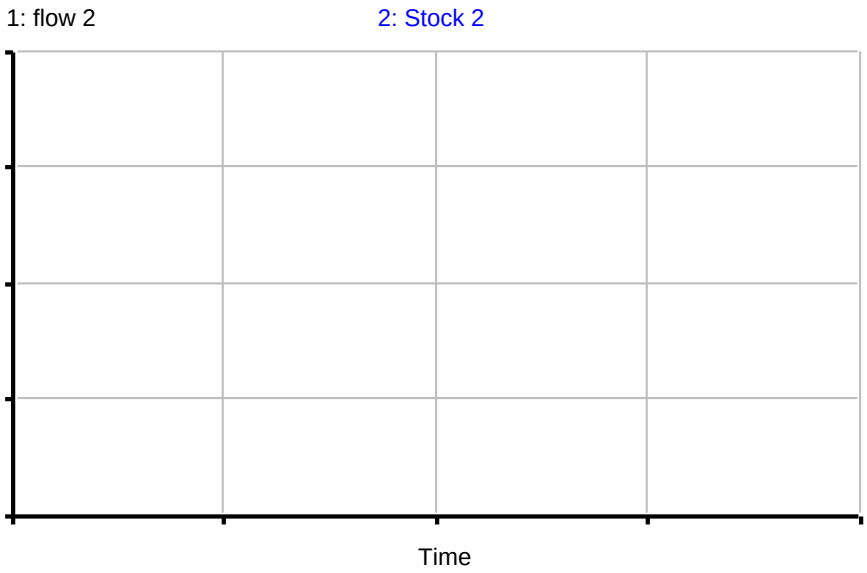
7.2 Exercise 2

The flow into "Stock 1," "constant flow," has a constant positive value. If "flow 2" is equal to "Stock 1" for all time, estimate the behavior of "Stock 1" and "Stock 2" over time.[9] Assume that the initial value of both stocks is zero.



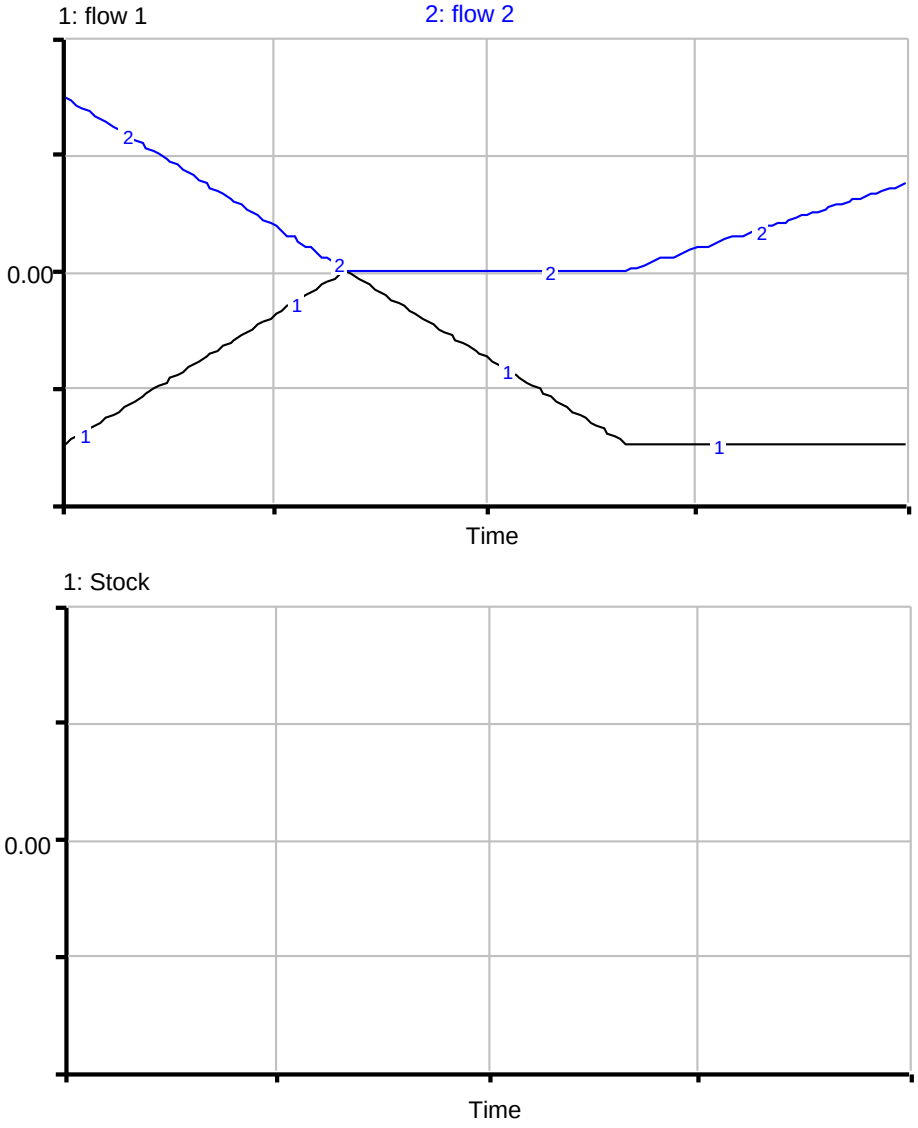


[9] The variable "TIME CONSTANT" has been introduced for dimensional consistency of the model. The graph of "flow 2" will have the same shape as the graph of "Stock 1." The slope of the graph of "flow 2" will equal the slope of the graph of "Stock 1" if the "TIME CONSTANT" is equal to one. Assume that the value of the "TIME CONSTANT" is equal to one.


1: flow 2
2: Stock 2
Time

7.3 Exercise 3

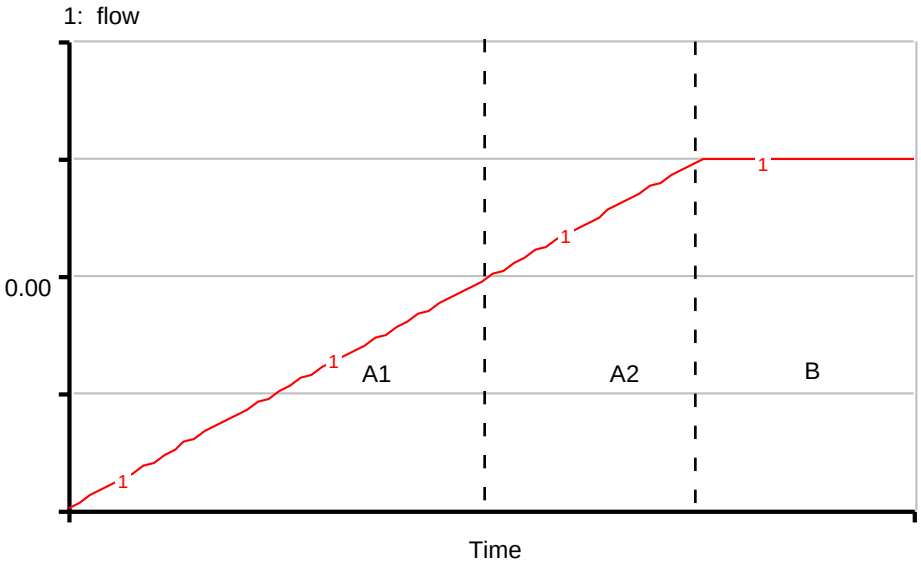
Estimate the behavior of a stock with two flows as shown below. Assume that the initial value of the stock is zero. For convenience, plot the net flow on the graph with the flows.



8. SOLUTIONS

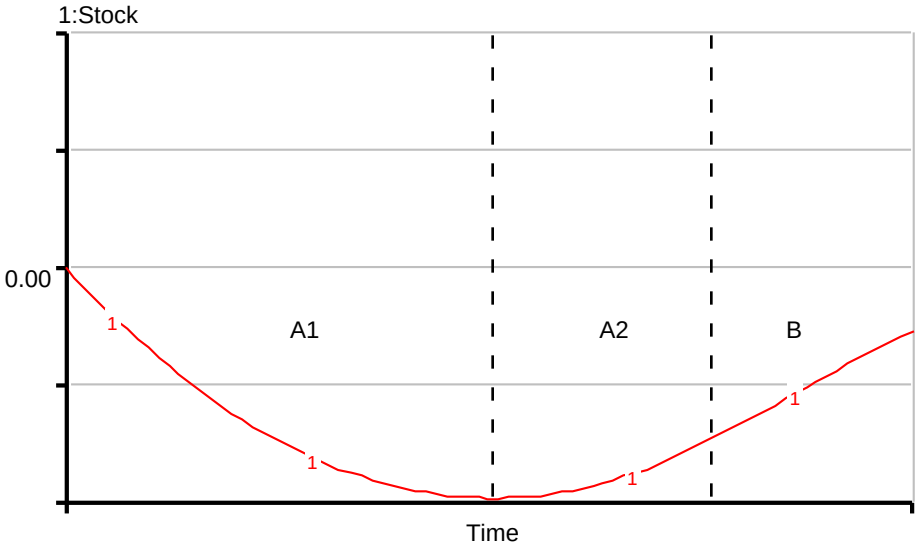
8.1 Solutions to Exercise 1

A. The correct segmentation is shown in the graph below.

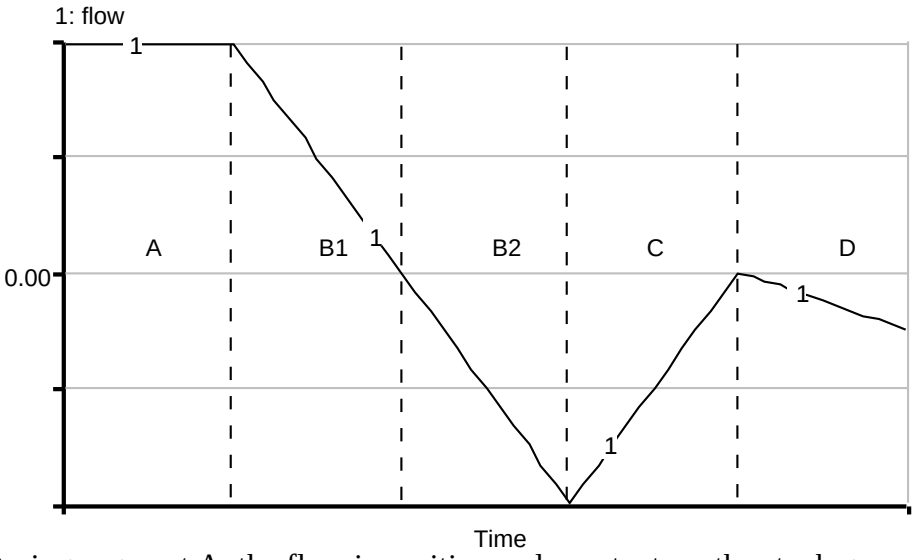


During segments A1 and A2, the slope of the flow does not change, but the sign of the flow does change from negative to positive.

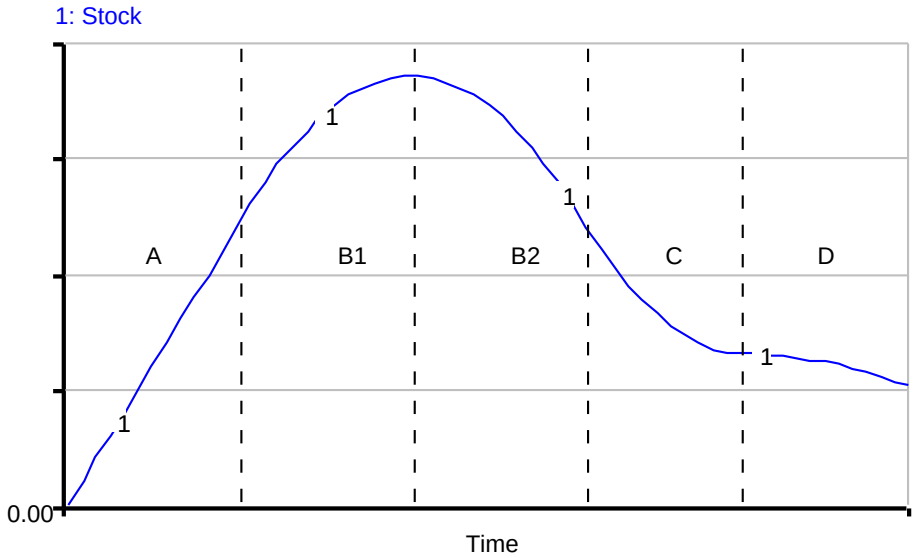
During time A1, the flow is negative and decreasing in magnitude; thus, the value of the stock falls below the initial value of zero, but decreases slower and slower as time passes. As the flow becomes positive and the magnitude of the flow increases in period A2, the stock starts growing at an increasing rate. During time B, the flow is positive and constant, causing linear stock growth. The overall stock behavior is shown in the following graph.



B. The flow in this exercise can be segmented at the obvious turning points. Notice, however, that the flow also crosses the time axis and should be segmented at the point of crossing the time axis also. The correct segmentation is shown below.

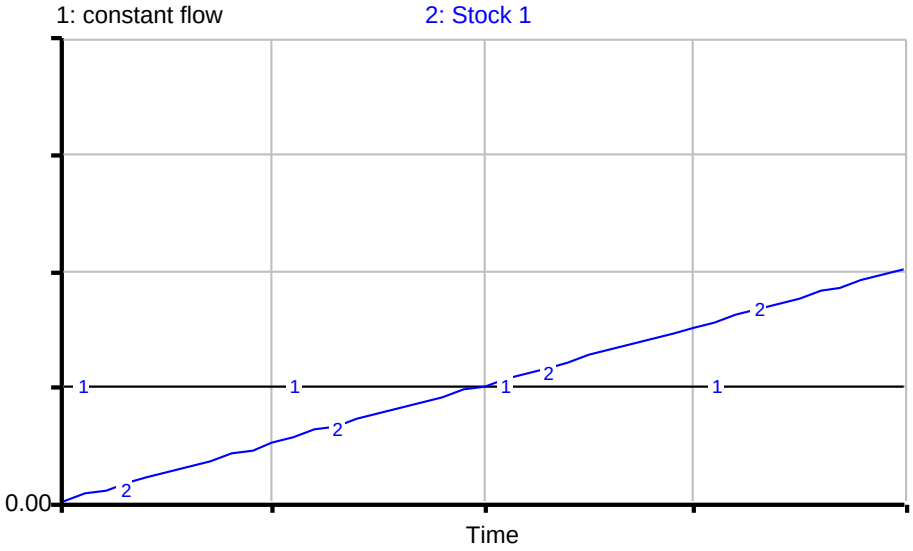


During segment A, the flow is positive and constant, so the stock grows linearly. During segment B1, the flow is positive but decreasing in magnitude, driving the stock to increase in value but at a slower and slower rate. After crossing the time axis, during segment B2, the flow is negative but now increasing in magnitude. The stock starts decreasing faster and faster during segment B2. During segment C, the flow is still negative but decreasing in magnitude, so the stock is still decreasing but slower and slower. During segment D, the flow is similar to the flow during segment B2, but the slope is smaller in magnitude. Thus, the stock decreases at an increasing rate until the end of the period shown on the graph, but does not decrease as rapidly as in section B2.

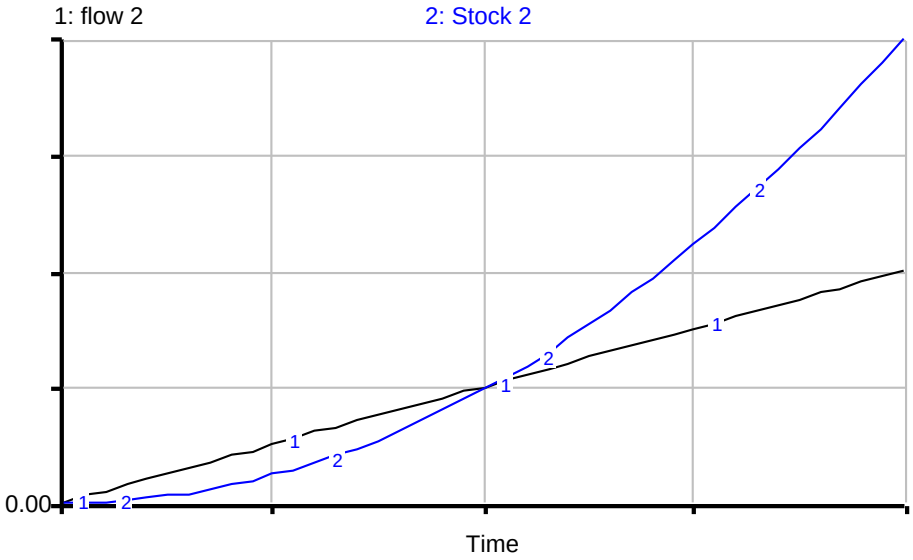


8.2 Solutions to Exercise 2

Solve the problem one step at a time. First, graphically integrate "constant flow." Because "constant flow" is positive and constant, "Stock 1" will grow at a constant rate from the initial value of zero. The rate at which "Stock 1" grows (the slope of the "Stock 1" curve) depends on the magnitude of "constant flow." The following graph shows the behavior of "constant flow" and "Stock 1."

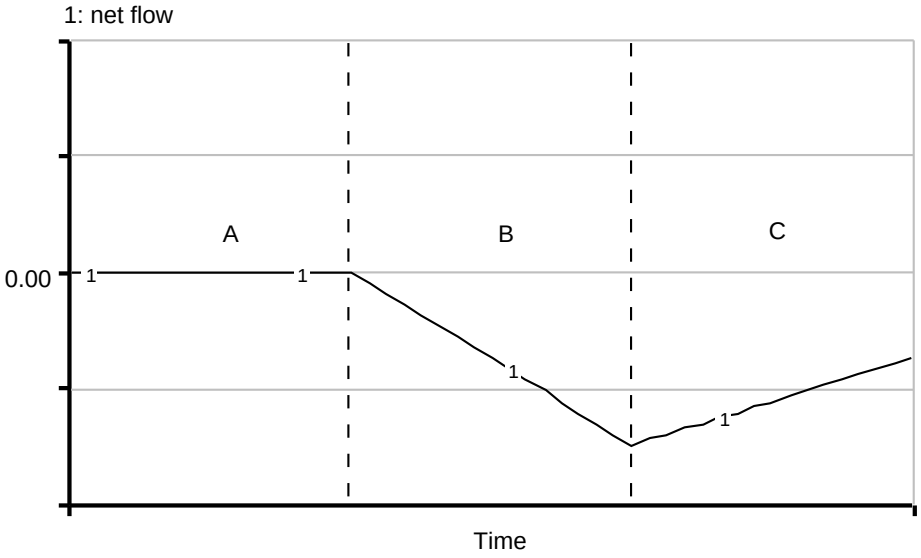


Because "flow 2" is equal to "Stock 1," "flow 2" will behave exactly the same way as "Stock 1." Thus, "flow 2" will be a positive flow increasing in magnitude. A positive flow increasing in magnitude causes the stock to grow faster and faster as time passes. Thus, "Stock 2" grows at an increasing rate, as the next graph shows.

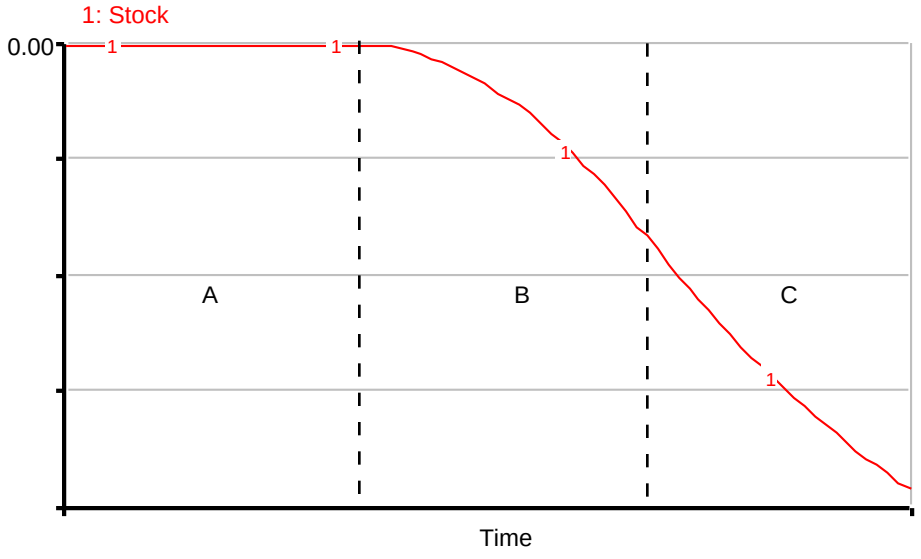


8.3 Solutions to Exercise 3

First, add flow 1 and flow 2. The following graph shows the net flow.



The next step is to segment and then graphically integrate the graph of the net flow. Because the graph is always negative or zero, the flow has to be segmented at only the points where the slope of the flow graph changes. With a constant net flow of zero, as in segment A, the stock remains at the initial value. When the flow becomes negative and increasing in magnitude during segment B, the stock decreases faster and faster. As the direction of the flow graph changes and the flow becomes negative and decreasing in magnitude in segment C, the stock is still decreasing, but slower and slower. The stock behavior is shown in the next graph.



3

Road Maps Glossary[1]

accumulation: The collection of some quantity over time. Examples of accumulation include water in a bathtub, savings in a bank account, *inventory*. In the *STELLA* modeling software, the accumulator is also referred to as a *Stock* or *Level*.

aggregation: The incorporation of numerous distinct system components into one variable. Aggregation is done for simplicity when combination generates the same behavior of interest as representing the components separately..

amplification: A system response that is greater than is seemingly implied by input causes. Amplification occurs in information-feedback systems when policies adjust levels to values that change with varying *flow* rates. It is associated with *delays*, order/inventory processes, forecasting, etc.

asymptotic growth/decay: *Goal-seeking behavior* produced by *negative feedback*. The *stock* of the system moves towards the goal, slowing down as it approaches the goal.

auxiliary equation/variable: An equation that takes the present value of variables (levels, constants, or other auxiliary variables) to compute the present value of an auxiliary variable. Auxiliary variables are part of a rate equation connecting a level to a rate. Auxiliary equations embody unit consistency and are symbolized by a circle in a *flow equation*.

balancing loop: A loop that seeks *equilibrium* — it tries to bring stocks to a desired state and keep them there. It limits and constrains, and is also called a *negative loop*. When a balancing process has a long *delay*, the response may overcorrect, often leading to instability, the opposite of what is intended.

The Beer Game: A "laboratory replica" of a real organization that involves a production/distribution system. The players at each position make decisions about placing orders, and find from practice the problems that originate from traditional ways of thinking and interaction.

behavior over time diagram: A *system dynamics* tool that shows how certain variables change over time. Several variables can appear on the same graph for comparison. and Time is shown on the horizontal axis.

boom and bust cycle: See *overshoot and collapse*

boundary: Border enclosing only the parts of system structure needed to generate the behavior of interest. In other words, the system boundary excludes all components not relevant to the problem behavior of the system. Also called *system boundary*.

business cycle: See *boom and bust cycle, oscillation*

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[1] Italicized words represent terms that are defined in this glossary

causal loop diagram: Diagram representing a closed loop of cause-effect linkages (causal links) which is intended to capture how the variables interrelate. The five steps of constructing a causal loop diagram are: 1. Choose variables; 2. Link variables with arrows pointing from cause to effect; 3. Assign a direction of effect (same or opposite) to each link; 4. Indicate delays; 5. Assign *polarity* to overall loop.

closed system: A system that functions without any *exogenous variables* generated outside the model. The system internally generates the values of all the variables through time by their interactions.

cloud: An origin or ending place of a flow that is outside the *boundary* of the system. In other words, the cloud represents an input or output of the system that is inconsequential to its behavior of interest. In a population system, clouds might represent origin of the inflow "birth rate" and destination of death rate that determine a "population" *stock*.. Where babies come from and where dead bodies go may not be dynamically important and thus represented by clouds.

compounding: Phenomenon that occurs when the *stock* serves as the basis for generating its own inflow, producing *exponential growth.* The behavior is also said to be self-reinforcing.

computation interval: See *solution interval*

computer model: A set of descriptions that tell the computer how each part of the system acts. A good computer model captures the dynamic essence of the system it represents. It explicitly contains the assumptions being made about the system.

connector: The building blocks that carry information from one element in a model to another element. "Information" may be a constant, an algebraic relationship, a graphical relationship (contained by *converters* or table functions), or a quantity (e.g. how many dollars in your savings account). "Information" flows through connectors to converters (*auxiliary variable*) or *flows* (rates), but not to *stock*s.

conserved flow : A flow that moves a quantity of material between two distinct stocks so that the total amount of material in that part of the system is unchanged. There is only so much material and it is divided between the two stocks. Non-conserved flows, on the other hand, flow from or to a cloud that has no limit in the quantity it can supply or absorb.

converter: A term used in the *STELLA* software. More generally, known as *auxiliary variables*. They are usually represented in diagrams by circles. Converters do not accumulate flows and do not have memory, but rather are recalculated from scratch each time calculations are performed. Three types of converters define constants, algebra, or graphs.

conveyor: A type of stock that represents a space into which material flows and stays for a fixed amount of time, then exits. Its associated parameters determine transit time which represents how long material stays in the conveyor. Material that flows in at a given time is not mixed with material that had flowed in earlier — whatever entered first will also leave first.

counterintuitive behavior: A surprising result of policies devised to remedy a problem. Often the presumed "solutions" result in counter-productivity. Thus as troubles

increase, efforts are intensified which actually worsen the problem. See *policy resistance, fixes that fail*.

cyclical behavior: See *oscillation, overshoot and collapse*

decision function: Also known as a rate equation. It is a policy statement that determines how the levels are related to the decisions (rates).

delay: A phenomenon where the effect of one variable on another does not occur immediately. Delays result from decisions often require a long period of time to be effective. Delays can result in *overshoot* or *oscillation*.

dimensional analysis: A process that checks for unit consistency in equations. The saying "one can't add apples and oranges" shows why dimensional analysis is necessary. Dimensional inconsistency is a sign of an incorrect equation.

disaggregation: The opposite of *aggregation*. Disaggregation is done to separate variables that do not have similar effects on system behavior.

doubling time: Length of time it takes an exponentially growing quantity to double in size. If a rate flowing into a level equals a multiplier times the level, then doubling time equals approximately 0.7 divided by the multiplier. Associated with *exponential growth*.

dynamic model: A model that deals with relationships that vary with time.

DYNAMO: An equation-based simulation language for system dynamics modeling on MS Windows computers.

endogenous variable/view: Opposite of *exogenous*, meaning internal. An endogenous view approaches a problem searching for its causes and cures within some *boundary*. Endogenous variables affect and are affected by the rest of the system.

equilibrium: A situation in a dynamic system where the inflows and corresponding outflows balance, and the levels cease to change.

exogenous variable/view: The opposite of *endogenous*.. An exogenous view considers systems under the influence of outside events that are not part of the internal dynamics of the system. An exogenous variable is an outside variable that affects but is not affected by the behavior of the system.

exponential growth/decay: Behavior that occurs when the rate of growth depends on the size of the *stock* at that point in time. As the stock gets larger, its growth gets progressively faster. Or, for decay, as the stock gets smaller, the decay gets progressively slower. Exponential growth/decay has a *doubling time*. Associated with *positive feedback,* or a *half-life* associated with decay.

feedback system: A closed system influenced by its past behavior. Feedback systems have feedback loop structure that consist of closed paths of cause and effect. They are self-regulating and can be either a *positive feedback* system or a *negative feedback* system.

first order system: See *order*

Fish Banks Game: A management flight simulator. The Fish Banks Game consists of a group of players acting as competing fishermen trying to maximize profits. The game passes on to its participants the lessons of *tragedy of commons, S-shaped growth,* and *limits to growth*..

fixes that fail: A situation where a "fix" has immediate positive results but eventually makes the problem worse. The underlying structure of the system that caused the problem in the first place does not change. Similar to *policy resistance* and *counterintuitive behavior*.

flow: Movement of a quantity from one level to another.

generic structure: A structure that can be applied across different settings due to fundamentally same underlying structures and relationships.

goal-seeking behavior: Behavior that results from a *balancing loop* which drives a system toward a specified goal. The farther the system from the goal, the quicker it changes towards that goal, the equilibrium *homeostasis* point, and as it approaches the goal, the growth/decay slows down. Associated with *negative feedback*.

graphical function diagram: A diagram that relates movement in one variable to movement in another. The input and output variables are placed on the axes of a graph, and the relation plotted.

half-life: The amount of time it takes for a stock to go halfway towards its goal. It is the converse of *doubling time* in *positive feedback*. Half-life is approximated by 0.7 times the time constant, or 0.7 divided by the decay fraction. Associated with *goal-seeking behavior.*

homeostasis: Control through the operation of *negative feedback* loops — homeostasis is reached when the goal is attained and stable *equilibrium* achieved.

impulse: Theoretically, a signal of zero width but finite area. Practically, in models, a signal (rate of flow) of specified area lasting for one solution interval and occurring at a specified time.

integration: See *accumulation, stock, level*

inventory: A frequently-used stock in modeling economic systems. When production is greater than consumption, the excess production goes into inventory.

ithink: The business-oriented application version of *STELLA*, by High Performance Systems.

learner-centered learning: Innovative learning style in which the teacher is no longer a dispenser of knowledge but rather a colleague. The teacher is a facilitating participant who sets directions and introduces opportunities. Students working in small groups acquire their own knowledge without having it spoon-fed to them. Such classrooms make acquisition of facts and information a consequence of needing inputs to reach an objective.

level: An *accumulation* in a system. It represents the accumulated difference between inflow and outflow rates, illustrating the results of action within the system over time. Levels are conserved quantities that can be changed only by moving contents in and out.

leverage point: A policy which can yield large changes in a system.

limits to growth: A resource constraint, an external or internal response to growth. A growth caused by reinforcing feedback processes begins to slow and eventually come to a halt at the limit, and may even reverse itself and collapse. For instance, population growth is limited by space, water and resources essential for survival.

linear system: A system in which different behavior models can be superimposed without interacting with one another.

loop dominance: A system in which one loop is stronger. In a system with multiple loops, magnitudes and algebraic signs of variables determine what kind of behavior, positive or negative feedback, is dominant at any given time. If the system exhibits *exponential growth*, then the positive loop is dominant. If *asymptotic behavior* is evidenced, the negative loop has dominance. *S-shaped growth* is a common behavior of a system in which loop dominance shifts with time.

mental model: A model representing the relationships and assumptions about a system held in a person's mind. Mental models are often correct in system structure, but frequently draw wrong conclusions about system behavior.

negative feedback: Feedback that works to cancel deviations from a goal. It exhibits *goal-seeking behavior*. The control decision attempts to adjust a system level to a value given by a goal introduced from outside the loop.

non-linearity: See *linear behavior*

open-loop thinking: Approaching a problem without applying the concept of feedback.

order: The number of *levels* a system has. For instance, a "second-order" system has two level variables.

oscillation: Behavior exhibited by a second-order or higher-order system in which the stock value moves sinusoidally over time. Three types of oscillation include sustained, where the amplitude is always constant; expanding, where the amplitude increases over time; and dampened, where the amplitude decreases over time.

overshoot and collapse: A system that grows beyond a sustainable condition, reduces the basis for sustained existence, then collapses below the level that might have been sustained. Example: fishing rates that exceed the replenishment rate resulting in a collapse of the fishing population.

parameters: Numerical values that describe relationships in a system and are considered constant, at least during the computation span of one model run. Parameters are shown by a circular *converter* in *STELLA* and by other symbols in other software.

period of oscillation: The time duration of each cycle in oscillatory behavior from the analogous part of one wave to another.

polarity: Positive or negative sign of a *causal loop*. Positive loops are called reinforcing, and negative loops are called balancing.

policy analysis: Analysis employed to evaluate the causes of undesirable behavior in a system. It allows the model-builder to compare how a system would react to different policies through simulation.

policy resistance: See *fixes that fail*

positive feedback: Feedback that contains *reinforcing loops* which produce *exponential change*. Change in one direction results in more and more change in the same direction. Positive feedback produces growth that would be out of control if it weren't for *limits to growth*.

POWERSIM: MS Windows-based modeling software package for system dynamics models. POWERSIM is equipped with advanced functions such as interactive simulation, On-Line Help, expandable function library, and data exporting.

queue: Waiting lines, an explicit line build-up as opposed to a pile-up implied by a *reservoir*. In the *STELLA* software, they are used in conjunction with a *conveyor* to represent batches of material waiting to enter a process or activity.

ramp function: A way to designate the value of a variable to change linearly over time.

rate: Definition of the present, instantaneous *flow* to and from a *level* in a system. Rates represent activity, while levels represent the state to which the system has been brought by the activity. Rates are determined by the levels of a system according to rules defined by the *decision functions*. In turn, rates cause levels to change.

reinforcing loop: Loop that occurs when an action causes change that accelerates that action, reinforcing the effect of the original action. Associated with *positive feedback.*

reservoir: The type of stock that is analogous to a tub with water flowing in and draining out. In many ways it is the simplest of stocks. When something flows into a tub, it is mixed into any "stuff" that is already in the tub so that it can not be distinguished from earlier or later inflows.

sensitivity analysis: Analysis used to determine how "sensitive" a model is to changes in specific parameters, or policies, or structures. If the behavior of a model changes drastically, that suggests a critically important factor, or high sensitivity. Conversely, if a large change results in little change in behavior, that factor is not likely to be central to the dynamics in question, that is, it shows low sensitivity.

shifting the burden to the intervenor: A system behavior where an intervenor acts to improve a system and the effort fails because the system lacks incentive to continue its own effort to take the action.

simulation: Conducting dynamic experiments on a model instead of on the real system.

smoothing: Filtering out the superimposed short-term fluctuations in order to detect underlying, significant changes in data. There is formal (numerical processing of data into averages) and intuitive smoothing.

solution interval: The amount of time elapsed between successive computer calculations of *flows* accumulating into *stocks*. Also known as *computation interval* or *DT* (for Delta Time). It is measured in units of time and must be short enough so that its value does not affect the computed results, but also long enough to avoid unnecessary waste of computation time.

S-shaped growth: Growth that exhibits behavior in the shape of the letter "S." It expands rapidly at first, then slows down as stock approaches its maximum value. S-shaped growth is caused by a shift in *loop dominance* from positive to negative feedback.

stability: Behavior exhibited by a system that returns to its initial condition after being disturbed. In an unstable system, a disturbance is amplified, leading increased growth or *oscillation*. A stable oscillation is one at a constant amplitude, as in a clock pendulum.

state variable: See *stock, level*

steady-state: A behavior pattern that is repetitive with time and in which the behavior in one time period is of the same nature as any other period.

STELLA: Visual diagram-based simulation software for system dynamics models. It is the education-geared version of the *ithink* software.

step input: An input (usually for testing purposes) that suddenly changes and then remains at the new value.

stock: An element of a system that is accumulating or draining over time. Stocks are the memory of a system and are only affected by flows. Also known as *levels*, they are signified by rectangles in system dynamics diagrams.

structural diagram: Diagram that gives an overview of the structural elements that produce system behavior by showing the *stocks* and *flows*. Also known as *flow diagrams*.

structure: The building blocks and interval connections of a system. It is the way in which system elements are organized or interrelated.

system: A collection of parts that interact to function as a whole. A system is almost always defined with respect to a specific purpose. Systems often contain circular patterns of cause and effect called *feedback loops*.

system boundary: See *boundary*

system dynamics: A field for understanding how things change through time. System dynamics deals with how the internal *feedback*-loops within the structure of a system create behavior. Computer simulation models are used to achieve better understanding of system behavior over time. With a better comprehension of systems, one can redesign structure or policies to improve the behavior. The field of system dynamics was created by Jay Forrester beginning in 1956.

systems archetypes: A *system dynamics* structure that is common to many systems. See also *generic structures.*

systems thinking: Thought process that involves 1)seeing interrelationships (*feedback loops*) instead of linear cause-effect chains, and 2)seeking processes of change over time rather than snapshots. Systems thinking involves understanding many concepts of *system dynamics*, most notably, *feedback*. It helps thinkers see things on three levels: events, patterns of behavior, and system *structure*.

table function: Feature used to formulate a non-linear relationship when it is necessary to use an *auxiliary variable* that is not a simple algebraic function of other variables. The function is a graph that can be sketched to capture the relationship at hand. A table function can be represented with a *graphical function diagram*.

tragedy of commons: The structure in which each person pursues action beneficial only to oneself, then eventually the system cannot support all the activity and everyone experiences diminishing results. The commons is the resources everyone calls upon, but does not take care to replenish. It is a *systems archetype* that surfaces in many socioeconomic systems. The *Fish Banks Game* is an excellent example of tragedy of commons behavior.

transferable structure: See *generic structure*

transient: A dynamic response that does not persist.

unit-conversion: A modeling function that allows for units of a certain variable to change as that variable is altered. Example: cloth (square meters) is made into clothing (articles).

validity: Judgment of a model's suitability for a particular purpose. A model is valid when it accomplishes what is expected of it.

VENSIM: Modeling software for system dynamics. VENSIM offers a "Workbench" featuring simulation tools for tracing sources of behavior in a model.

vicious cycle: Reinforcing, amplifying process that yields undesirable results.

virtuous cycle: Process that reinforces in desired directions.

D-4697-2

Formulating Models of Simple Systems

using

Vensim PLE

version 3.0B

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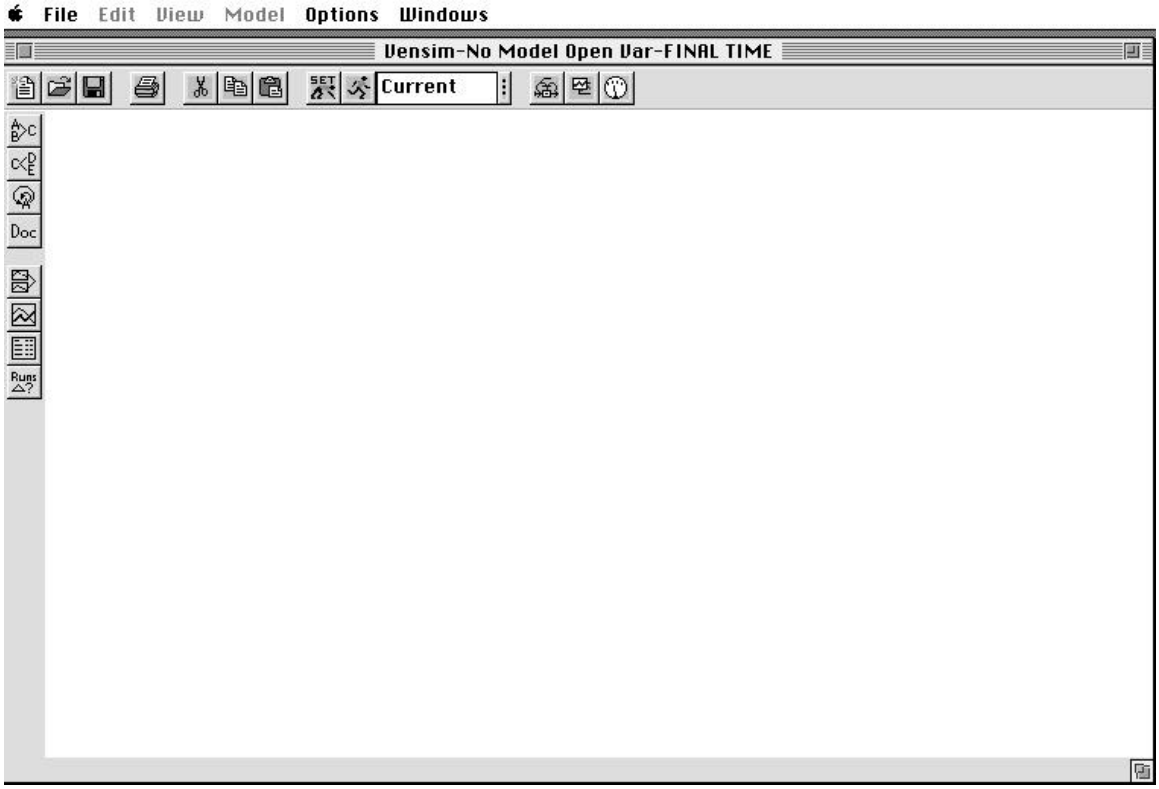
Edited by Laura Black, Farzana S. Mohamed, and students in the System Dynamics in Education Project, April 1998.

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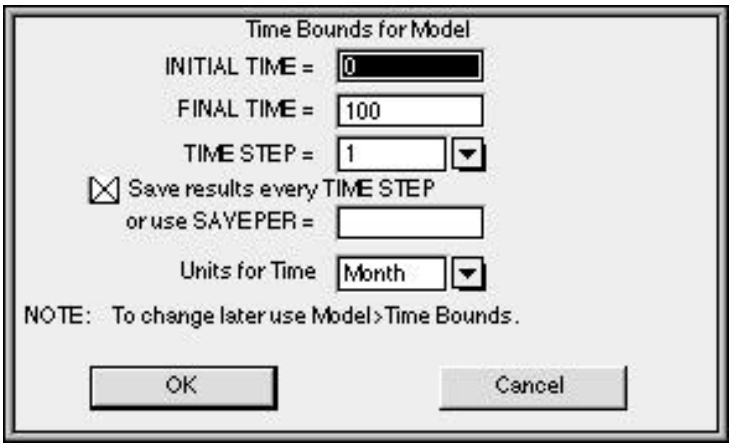
I. Introduction and Getting Started

The purpose of this tutorial is to help you develop some familiarity with building and analyzing system dynamics models using the Vensim PLE software. In order to become familiar with Vensim PLE, you are going to build a simple model of the federal deficit.

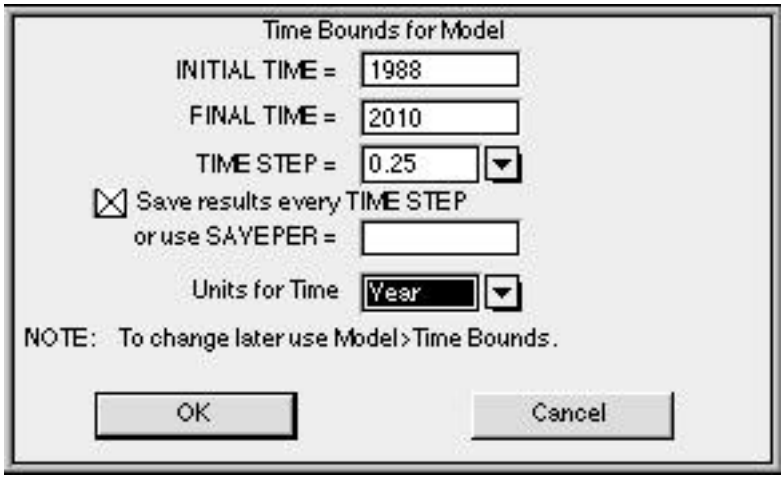
To begin you need to get Vensim PLE ready for modeling. This tutorial makes use of the Macintosh version on Vensim PLE; the IBM-Compatible version should work similarly, but some of the screens may look different. When you first open Vensim PLE on your computer, the screen should look like this:



To start working on a new model go to the **File** menu and select **New Model**. Vensim PLE will return the following dialog box:



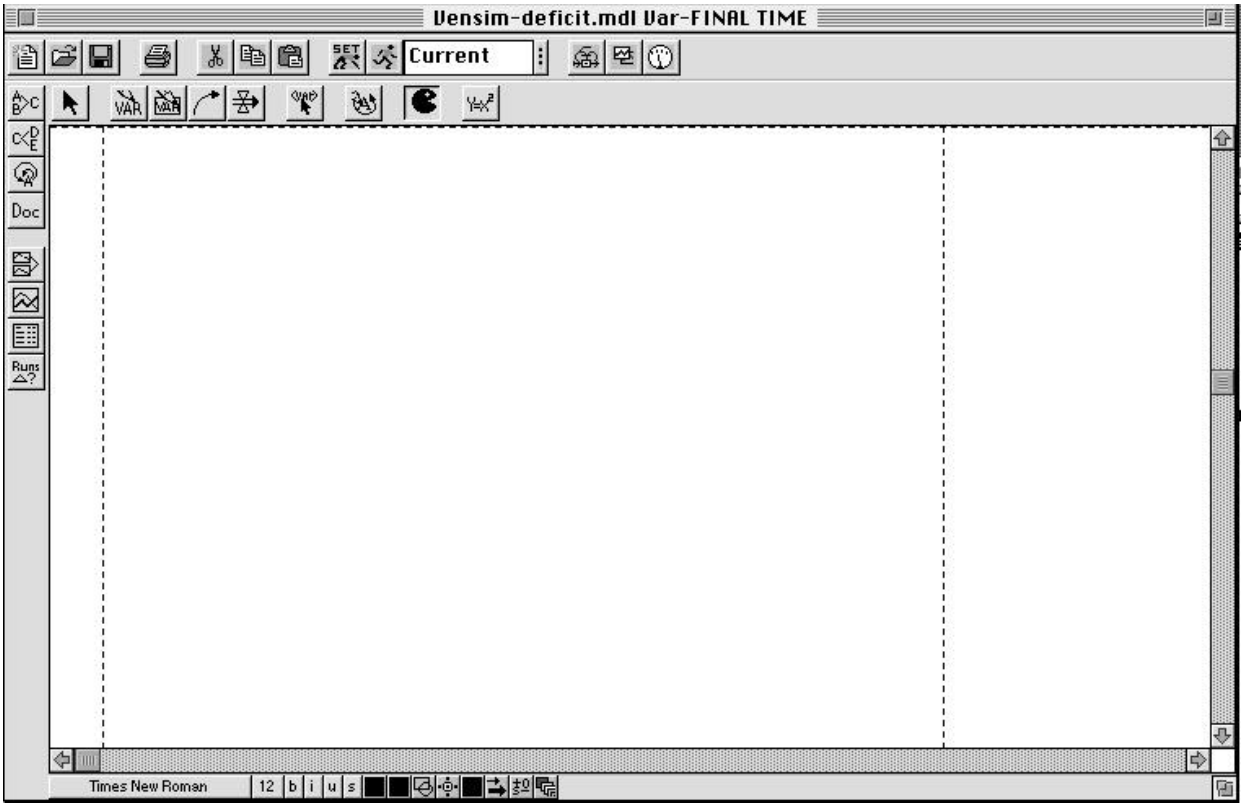
To begin your effort you must choose the time horizon of your model (when your simulation will start and finish), the appropriate time step (how accurately you wish to simulate your model), and the units of time. Start your model of the deficit in 1988 (enter 1988 in the **INITIAL TIME** box) and simulate it through the year 2010. Select a **time step** of 0.25 years. Finally, change the **units of time** from **Month** to **Year**. Your dialog box should now look like this:



Click on **OK** or hit return. To give your model a name, choose the **Save As...** command from the **File** menu and enter the desired name in the text field and click on OK. (Vensim PLE should automatically supply the **.mdl** extension. If you are working with a different version of Vensim and see a *Show all of type* option on the right side of the dialog box, make sure that the **.mdl Fmt Models** extension is selected. This allows Vensim PLE to save the model in a format that can be used by both Macintosh and IBM-compatible computers.)[□]

[□] Vensim saves every simulation run and custom graph you produce as a separate file. It supplies a **.vdf** extension for simulation runs. These files cannot be opened from outside the Vensim application; they can be opened from inside Vensim through the **Datasets / Simulate Model...** and **Control / Custom Graphs** dialog boxes.

Your screen should now look like this:



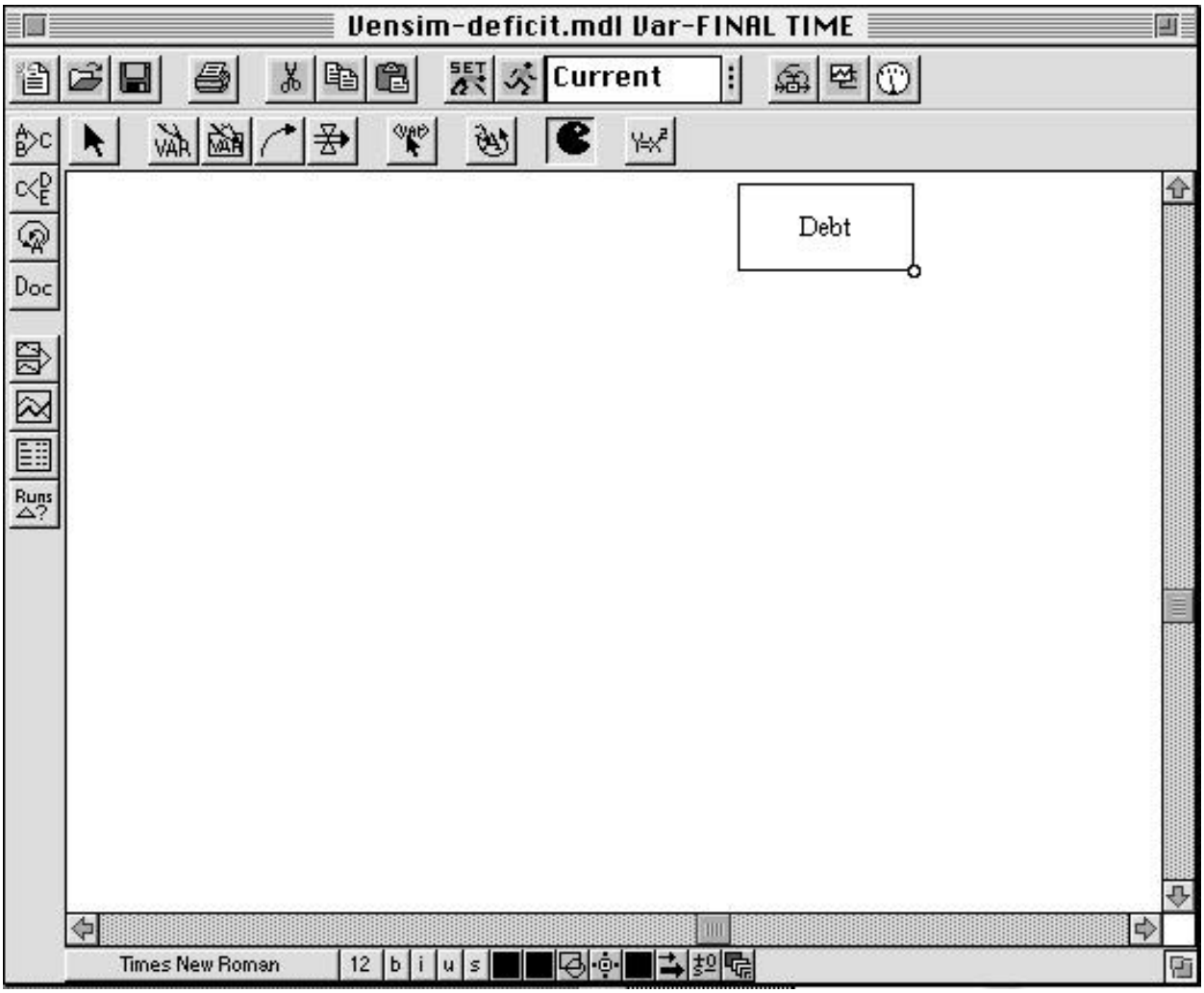
You are ready to start building your model.

II. Developing the Stock, Flow, and Feedback Structure

The Vensim PLE software is designed using the metaphor of a "work bench." The large blank area in the middle of the screen is your work area, where you actually develop and analyze your model. The different buttons on the border of the work area represent the different "tools" available as you work on your model. The upper toolbar consists of the Title Bar, a Menu, a Main Toolbar, and Sketch Tools. The Main Toolbar comprises two sets of tools: file operation tools that control standard file functions—opening, closing and saving files, printing, cutting, copying, and pasting—as well as simulation and graphing tools that will allow you to set up and run simulations, and set up display graphs. The sketch tools allow you to build in model components. The tools on the Status Bar (the bottom of the window) allow you to change the formatting of the diagram. The Analysis Tools on the left on the window are tools that you will use to analyze your model to understand its behavior. You will become familiar with many of these tools as you build the deficit model.

To begin, add a stock representing the outstanding federal debt to your model. Click on the  button (the one with the box in it) on the sketch tools bar and then click in the right center of the screen. You use this tool whenever you want to add a stock (also known as a level) variable to your model. Vensim PLE then returns an empty text box and a blinking cursor. Type the word **Debt** and then hit the return key.

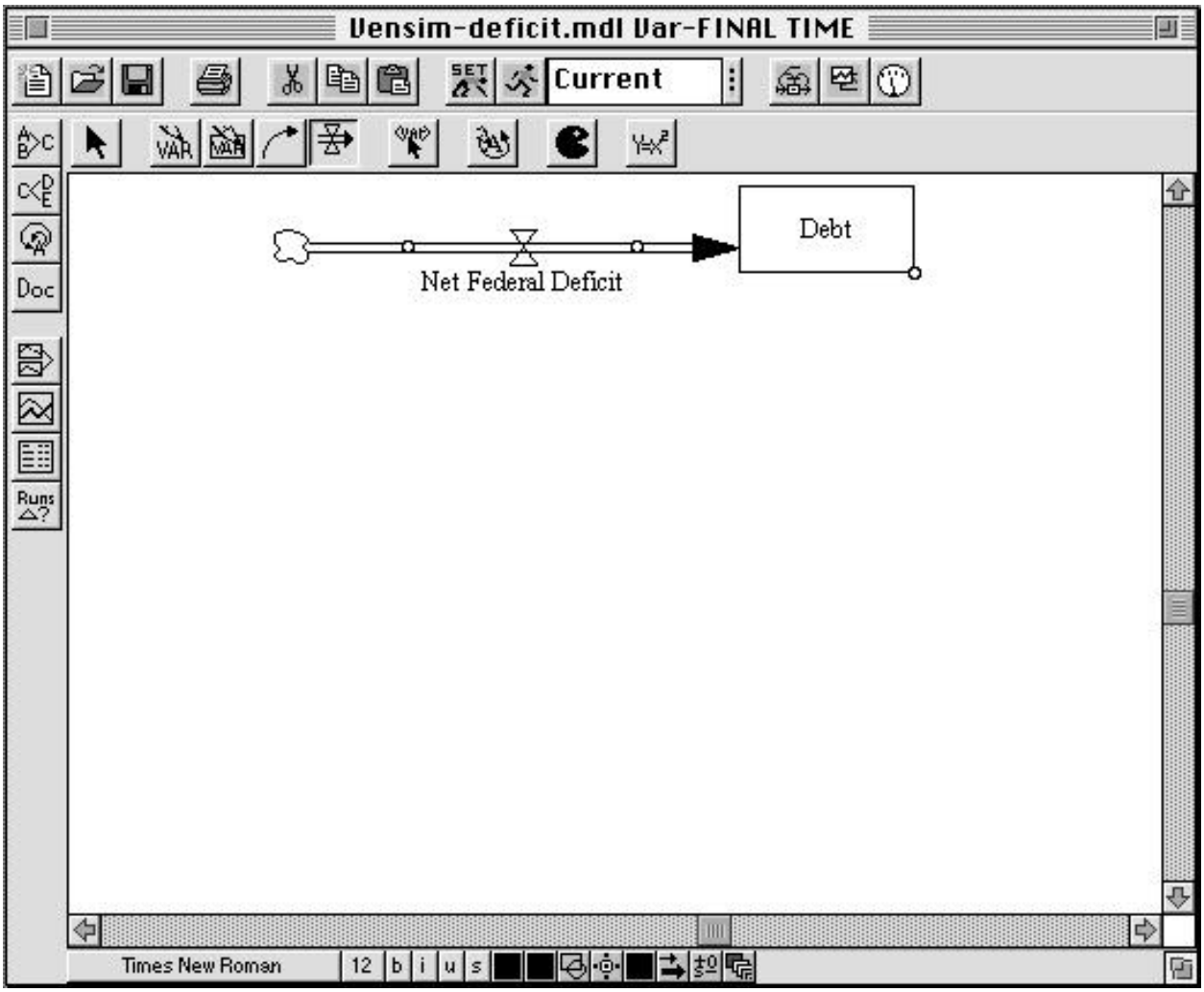
Your screen should now look like this:



You have just created the first variable in your model, the stock of money that constitutes the federal debt.

Now, add the inflow to the stock of **Debt**. Click on the  button on the sketch tools bar. Now, click and release once to the left of the **Debt** stock, move the cursor so that it sits *inside* the **Debt** stock, (the stock should "blacken" if you do this correctly) and click and release again. Vensim PLE then gives you an empty text box and a blinking cursor. Type **Net Federal Deficit** and hit the return key.

Your screen should now look like this:



Note: The ⟲ icon (which is supposed to resemble a cloud) represents the boundary of your model. In this case the "cloud" on the left side of the flow signifies that you do not, at the moment, care about where the deficit comes from — you are not keeping track of the stock that is being drained by the deficit flow. You do care, however, where the deficit goes: hence, you are accumulating the deficit in the **Debt** stock. If your deficit flow has "clouds" on both ends, then you have not hooked the flow to the stock correctly. To fix this problem, click on the tool and then click on the flow valve. This action will remove the flow from the model and let you start over again.

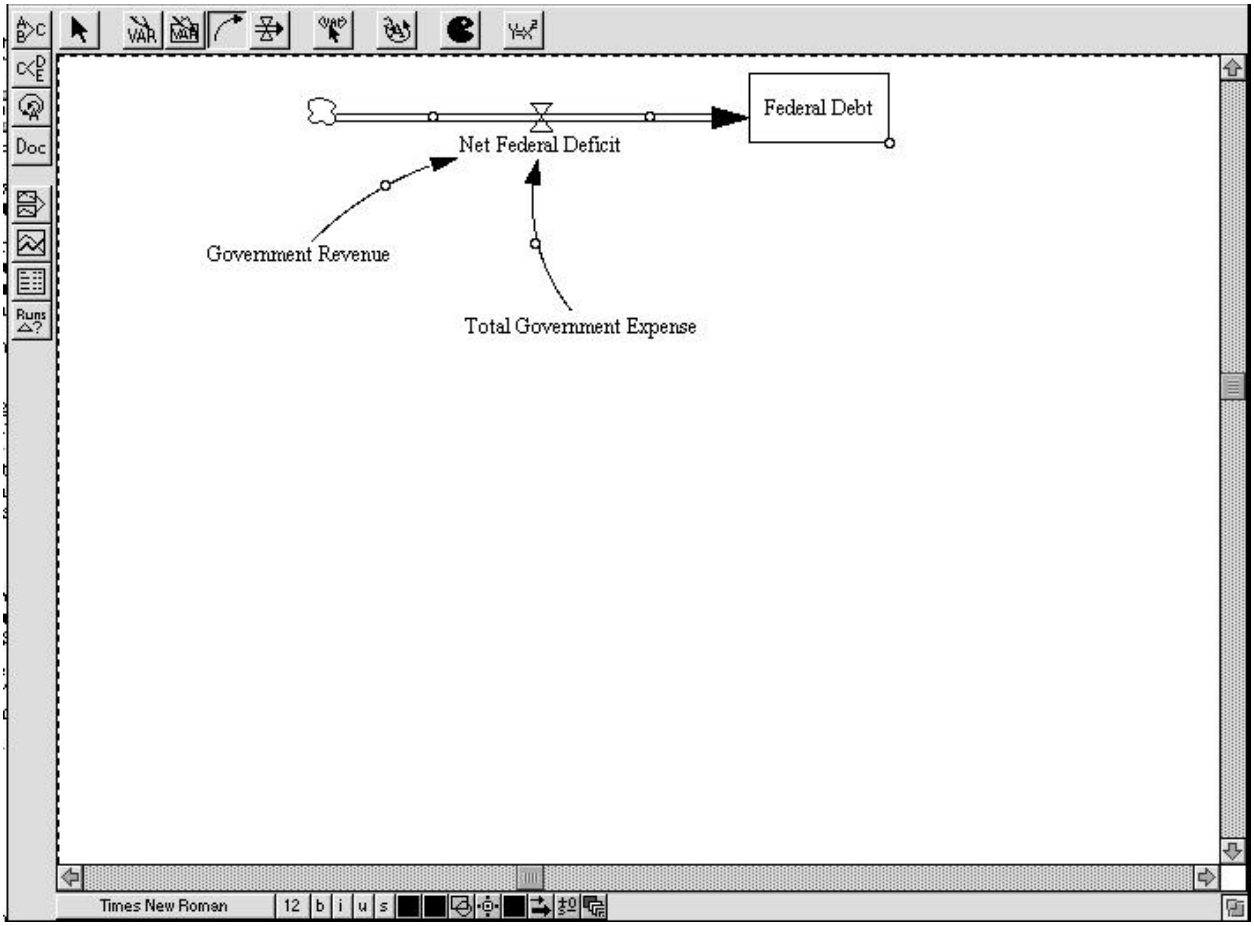
You have now created the flow, **Net Federal Deficit**, which increases the stock of **Debt**. At this point you may you wish to change the name of the stock variable from **Debt** to **Federal Debt**. Click on the VAR button (the one without the box in it) on the sketch tool bar and then click on the **Debt** stock. Vensim PLE gives you a text box with **Debt** already written. You can now edit the text in any way you choose. Click in front of the **D,** add the word **Federal**, and hit the return key.

Now you need to create the variables needed to determine the **Net Federal Deficit**. Assume the **Net Federal Deficit** is determined by two variables, **Government Revenues** and **Total Government Expenses**. Click on the  button again and add these two variables to your diagram (click and release on the work area, type the variable name, hit return, and then click and release again, on a different part of the work area, to add the next variable). Unlike the previous two variables — **Federal Debt**, a stock, and **Net Federal Deficit**, a flow — the new variables are not attached to a valve or a box. These variables are called *auxiliary variables*.

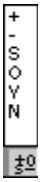
To show pictorially that the **Net Federal Deficit** is determined by **Government Revenues** and **Total Government Expenses**, we connect them with causal arrows. First click on the  button to select the causal arrow tool. Now, click and release on the variable **Government Revenue** and then click and release again on **Net Federal Deficit**. Do the same for **Total Government Expenses**. Make sure your causal arrows actually end on the words **Net Federal Deficit**. They should *not* be attached to the cloud, the stock, or directly to the valve.

You can delete arrows using the  tool.

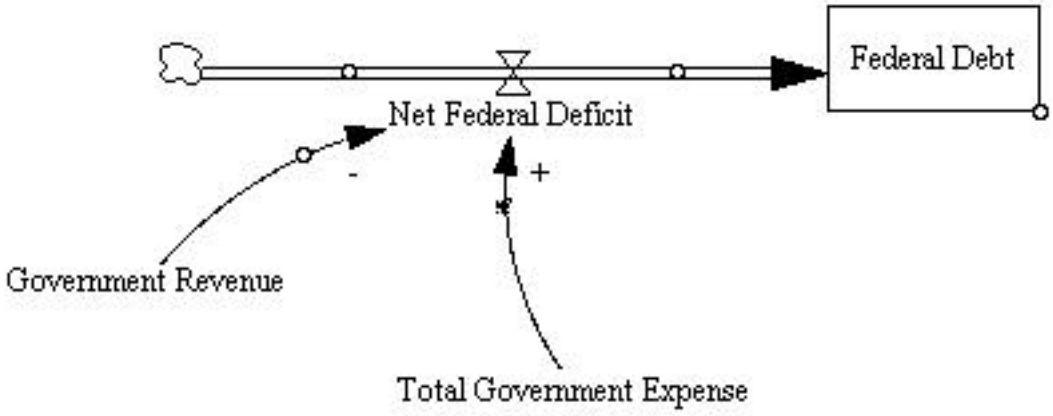
Clicking on the  button allows you to select the variables you have created and move them to different places on the screen. To move variables, place the arrow cursor over the variable you wish to move, hold down the mouse button, move the variable to the desired place, and then release the mouse button. You can also select the "handles" of the causal arrows (the small circles in the middle of the arrow) and change the curvature of the arrow. Arrange your variables and arrows so that your diagram looks approximately like this:



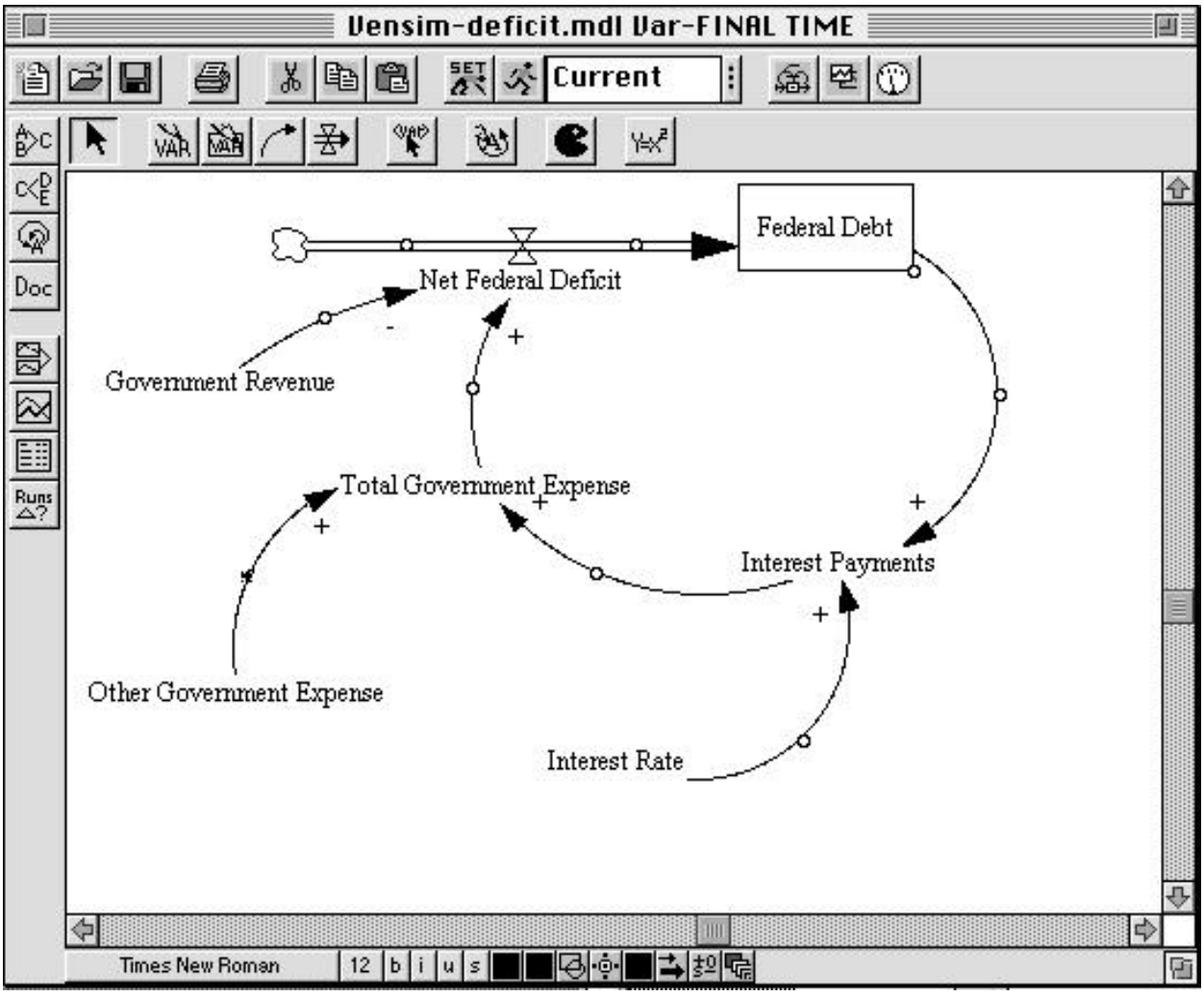
Now, you may want to update your diagram by labeling the arrows to show that **Government Revenue** and **Total Government Expense** affect the **Net Federal Deficit** in different ways. Specifically, an *increase* in revenue causes the deficit to *decrease*, while an *increase* in expenses causes the deficit to *increase.* To do this, first click on the  button. Then select the "handle" of the arrow you wish to label by clicking and releasing on the small circle in the middle of the arrow (the handle darkens when selected). Now, with the handle selected, click and release the  button on the bottom horizontal toolbar. You then see a pop-up menu that looks like this:



Click and release on the desired label, and it will show up in the diagram. Label your two causal arrows so your diagram looks like this:

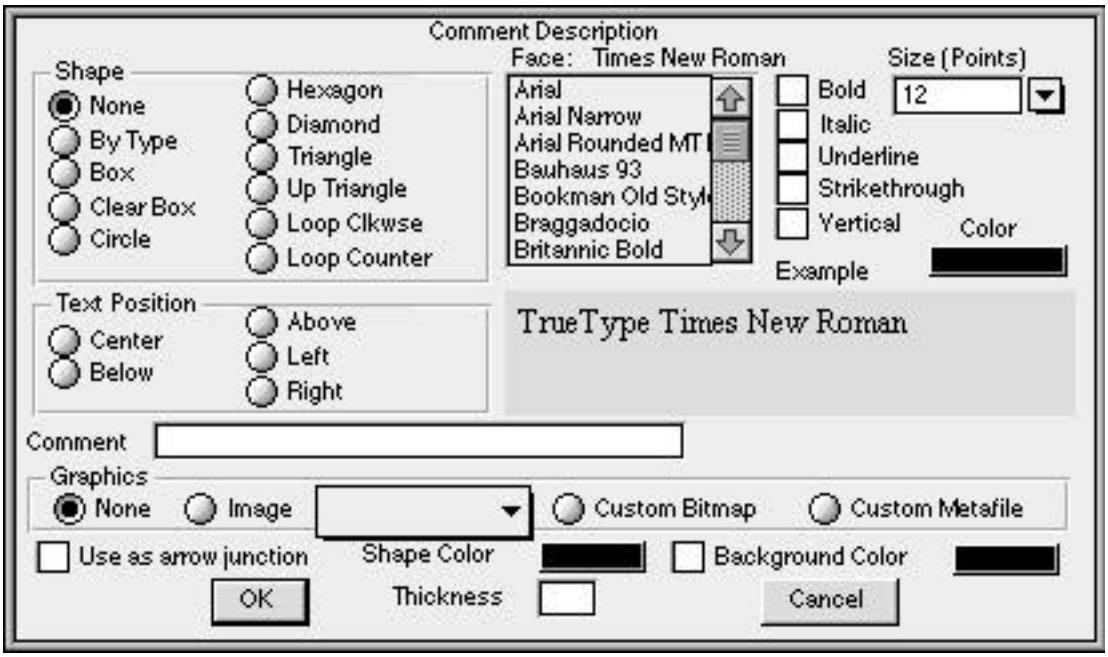


Now, using the same steps discussed above, complete the stock, flow and feedback so your diagram looks like this:

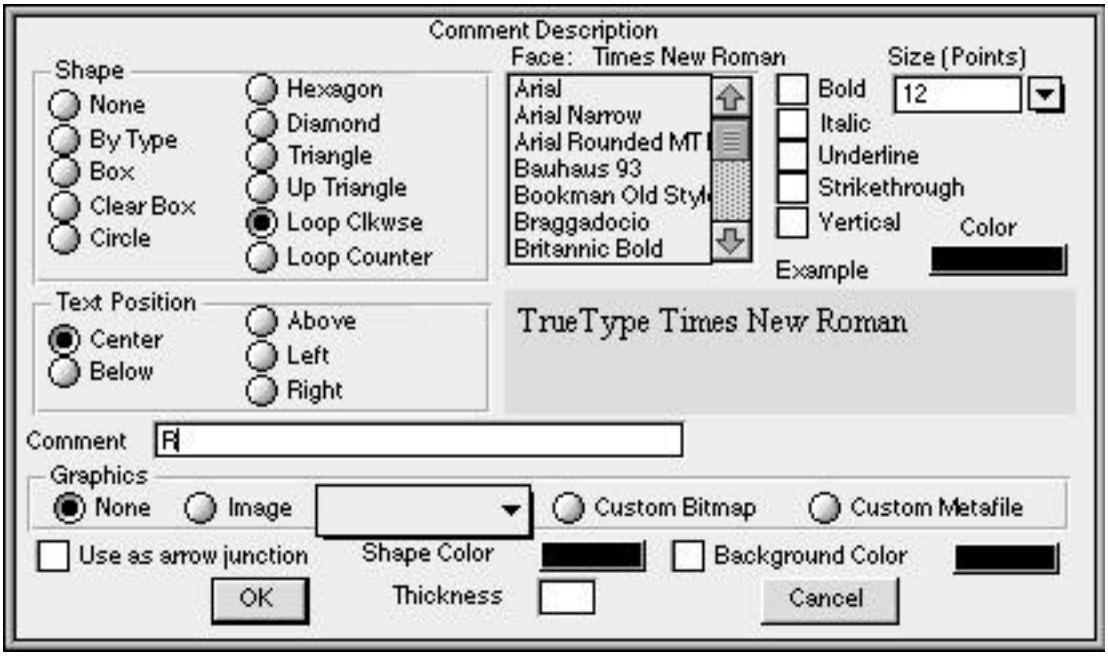


You may want to slide the handle of each arrow close to its arrowhead, so each label is clearly associated with its causal arrow.

Finally, you may wish to label the positive feedback loop you have just created. Click on the  button and then click in the center of the feedback loop. You can use this tool to create comments that, while having no structural use, can greatly help someone else to understand your model diagram. After clicking in the center of the loop, you should see the following dialog box:

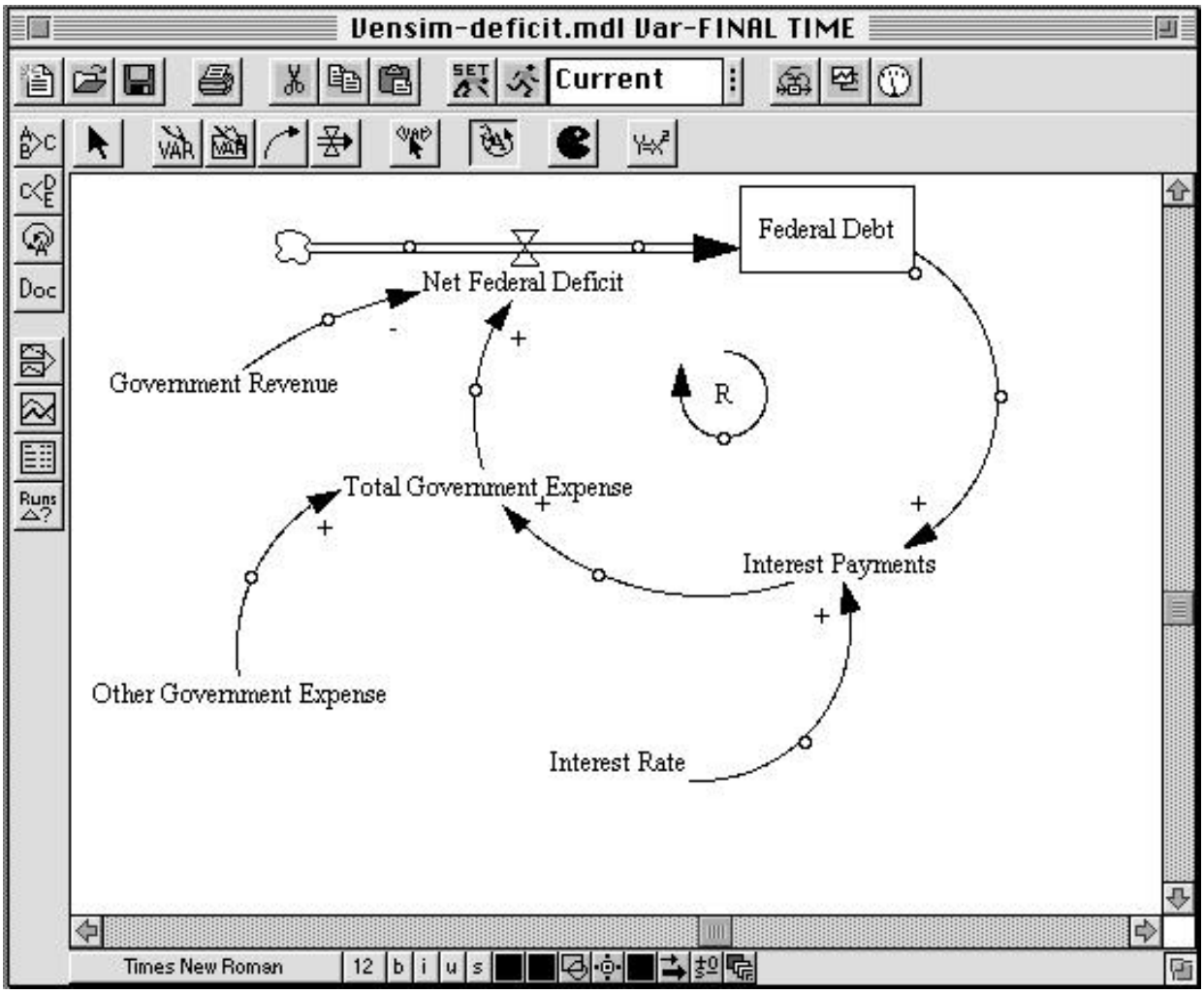


Click on the **Loop Clkwse** button in the **Shape** box; click on **Center** in the **Text Position** box; and type **R**, for **reinforcing**, in the **Comment** box. You may also type **+** or **P** to denote a positive feedback, also known as a reinforcing, loop. Your screen should now look like this:



Click on the **OK** button or hit return.

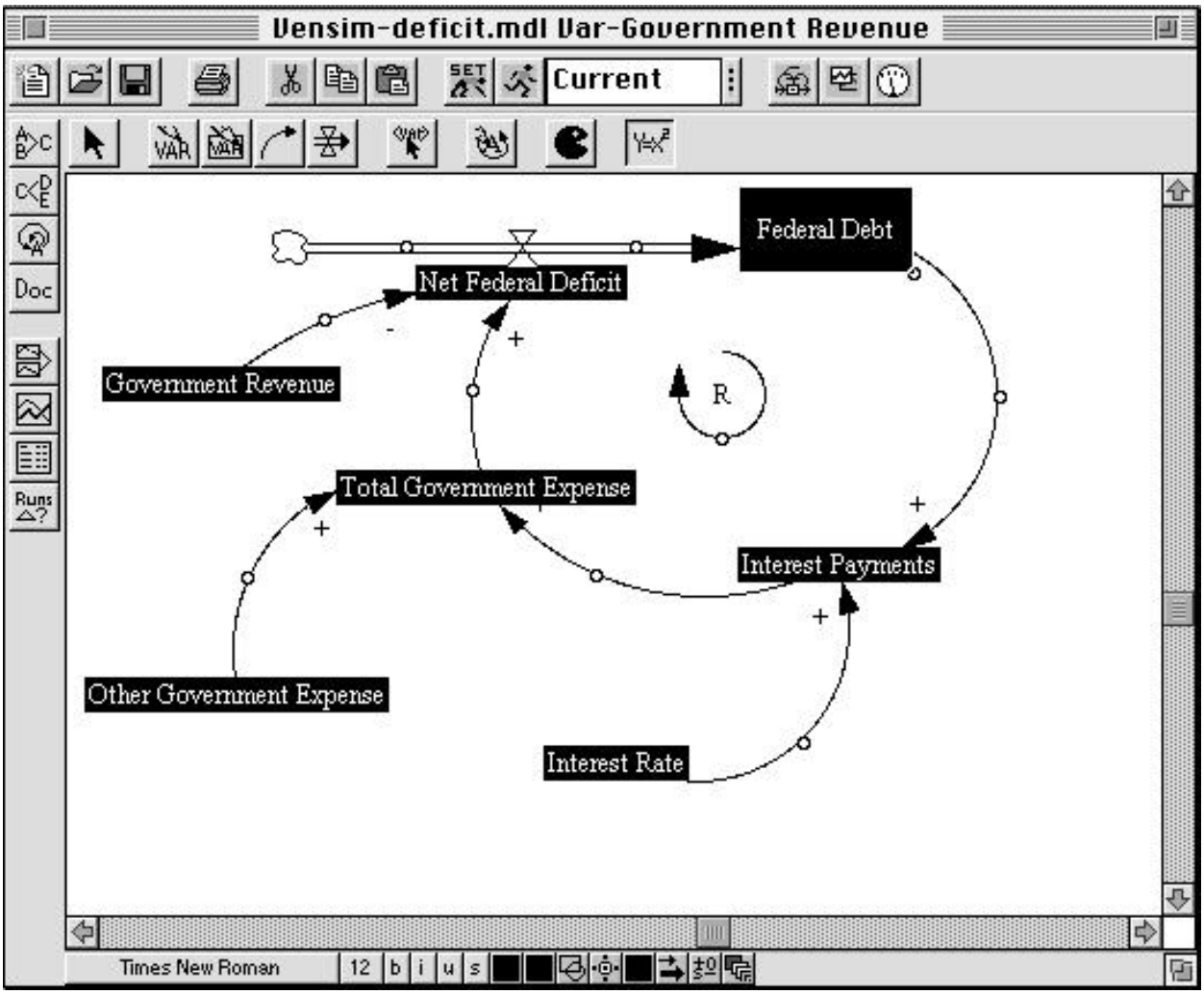
Your screen should now appear as:



III. Specifying Equations for Your Model

Now that you have developed a complete stock, flow, and feedback representation of the deficit, you need to write equations for each of the variables. Equation formulation is a critical step in the process of model building and is a key part of the process of developing a rigorous understanding of the problem at hand.

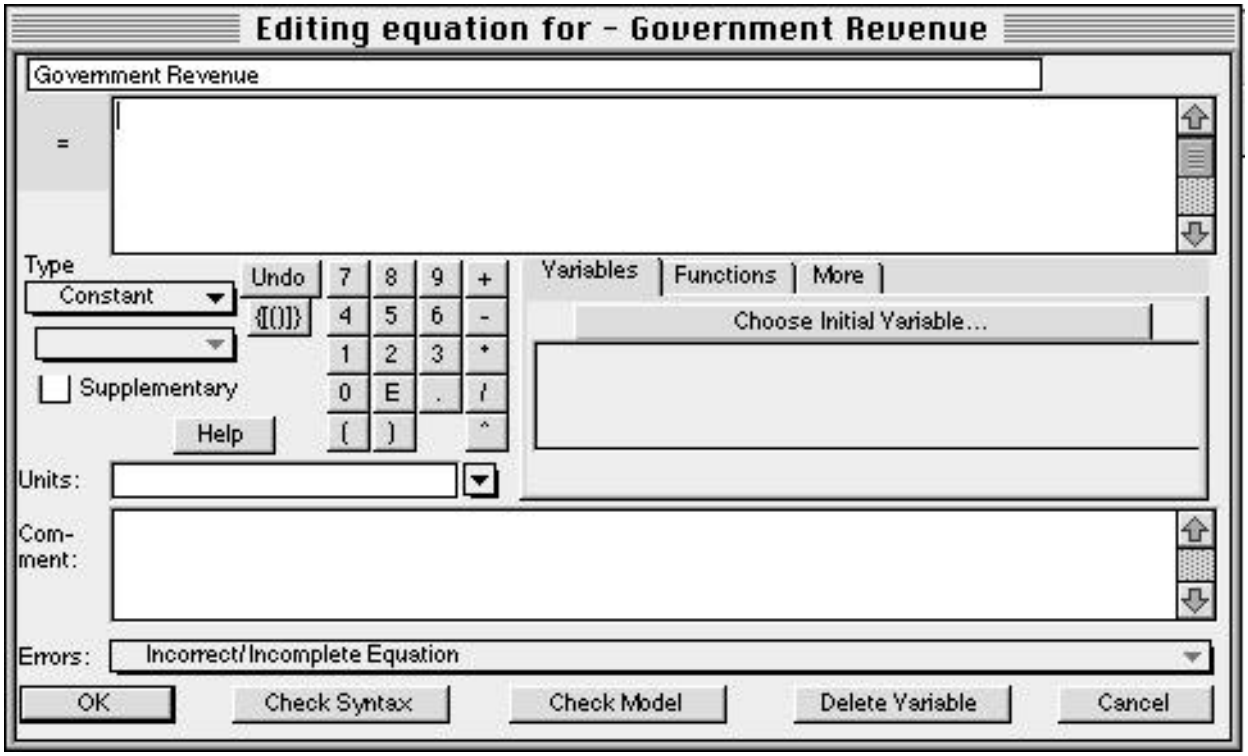
To begin writing equations, click on the  button on the sketch tool bar. The variables in your diagram become highlighted.



A highlighted variable indicates that the equation for that variable is incomplete.

Variables in system dynamics models are classified as either exogenous or endogenous. Exogenous variables are those that are not part of a feedback loop, while endogenous variables are members of at least one feedback loop. Your deficit model has three exogenous variables—**Government Revenue**, **Other Government Expenses**, and the **Interest Rate**—and four endogenous variables—**Interest Payments**, **Total Government Expense**, **Net Federal Deficit**, and the **Federal Debt**.

Start by writing the equations for the exogenous variables. To begin, click on the highlighted variable **Government Revenue**. You then see the following dialog box:



Good modeling practice requires that each equation in a model have three elements: the equation itself, specified units of measure, and complete documentation. You enter the equation in the box to the right of the **=** sign. You enter the unit of measure in the text field to the right of the word **Units**. Equation documentation or "comment" is entered in the box to the right of the word **Comment**.

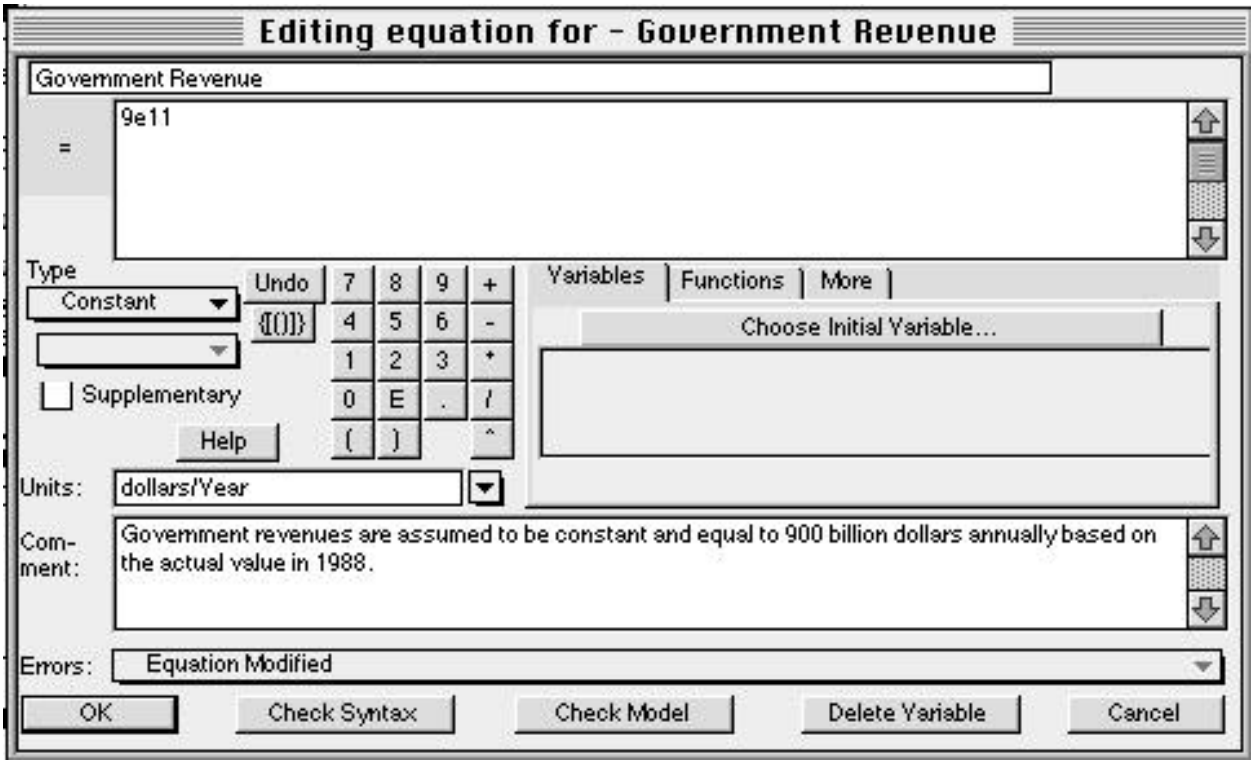
To write an equation for Government Revenue, click in the box to the right of the **=** sign. Assume that government revenue is constant, so that all you need to do is enter the appropriate number for government revenue. In 1988, government revenue was about 900 billion dollars annually, so type 900000000000 in the box. Alternatively, you can write 9e11, which is Vensim PLE shorthand for $9 * 10^{11}$.

Now, fill in the units. Revenue is a flow variable, so the appropriate unit of measure for this equation is dollars/time unit. Because you already chose to run the model in time steps of 1 year, the appropriate unit is dollars/year. Type **dollars/year** in the units field. (The next time you specify the units for a variable in this model, dollars/year will appear in the units pull-down menu. You can click on the arrowhead to the right of the units field to see units already specified for other variables in the model, and then use the mouse to select the units from that list when appropriate.)

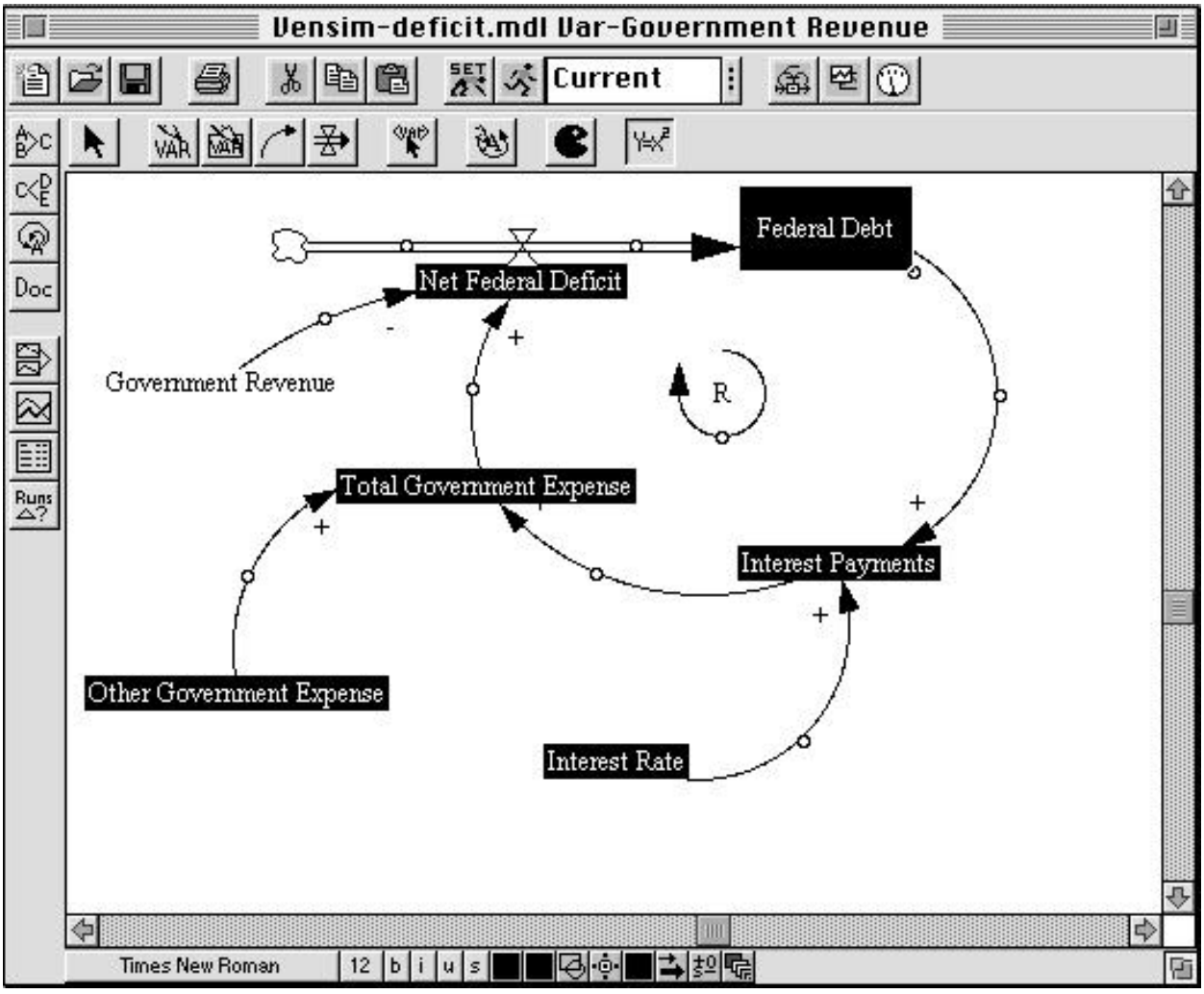
Finally, provide a description of this equation in the comment field. A good comment will be brief, but it will also give the reader the logic behind the equation as well as state the key assumptions. For example, one might write for this equation:

> **Government revenues are assumed to be constant and equal to 900 billion dollars annually based on the actual value in 1988.**

Your dialog box should now look like this:



Click on **OK** or hit return and your diagram will look like this:



Government Revenue is no longer highlighted because you have just specified its equation.

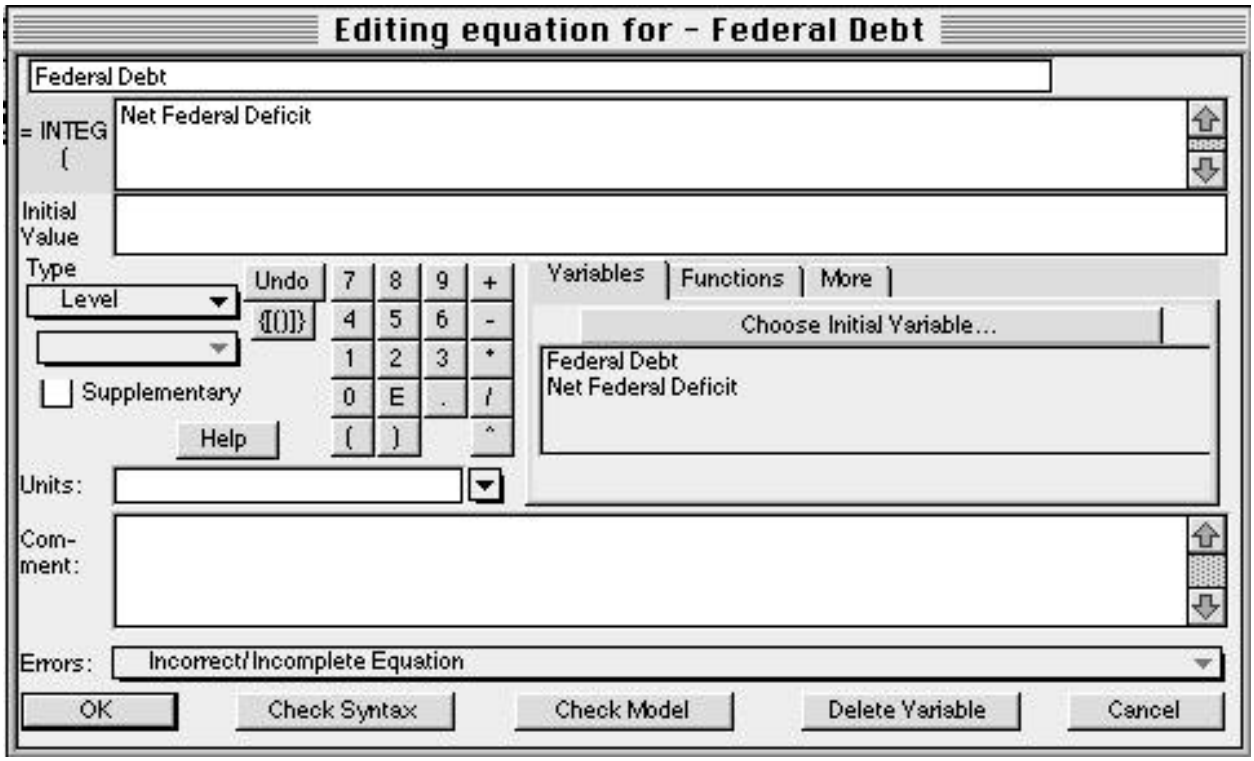
Following the process above, write equations for the two other exogenous variables, **Interest Rate** and **Other Government Expenses**. Use the following information:

- Government expenses, excluding interest on the debt, were approximately 900 billion dollars in 1988.
- The interest rate paid on the national debt in 1988 was around 7%/year.

Now that the equations for the exogenous variables are formulated, turn your attention to the endogenous variables. Writing equations for the stocks and the flows is a little different, so let's do an example of each. First we formulate the equation for the stock, **Federal Debt.**

Again, click on the  button on the sketch tool bar and then click on the stock, **Federal Debt**.

The following dialog box will be displayed:



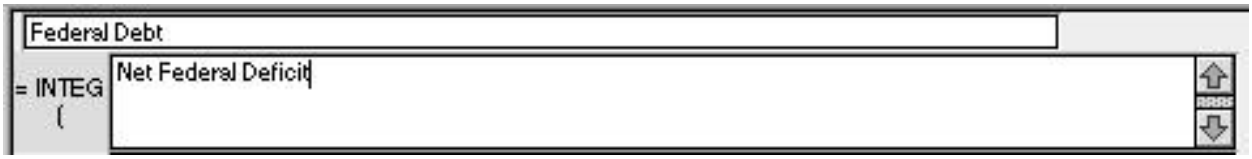
Unlike flows and constants, a stock requires that an additional element be specified in its formulation; after you specify the equation, you need to select an initial or starting value.

You enter the equation for the stock in the box to the right of the word **Integ**. **Integ** stands for "integrate" and simply means that the stock at any moment in time is equal to the sum of all the inflows minus the sum of all the outflows plus the initial value.

When you created the stock, flow, and feedback diagram, you connected the flow **Net Federal Deficit** to the stock **Federal Debt**. Vensim PLE captures this stock-flow dependency by providing a list of the required **Variables** to the stock **Federal Debt** on the right side of the equation dialog box. (The variable we are formulating, **Federal Debt**, itself also appears in the **Variables** box, but we focus on the input **Net Federal Deficit.** In general, you will never want to have the same variable on both the left and right sides of an equation.)

Because the model diagram shows the flow **Net Federal Deficit** feeding into the stock **Federal Debt**, Vensim has anticipated that the flow is an input to the stock equation and has placed the **Net Federal Deficit** variable name in the box to the right of **Integ**. If this is not the case in your version of Vensim PLE, then simply click in the box to the right of the **Integ** and then click on the variable **Net Federal Deficit** in the **Variables** box to write the equation for the change in **Federal Debt**. (Note: If **Net Federal Deficit** is *not* in the **Variables** box, then your model diagram is incorrect and needs to be changed—make sure the flow is attached to the stock).

The **Integ** box should now look like this:



Below the **Integ** box is the **Initial Value** box. Here you enter the initial condition or starting point for the stock. In 1988, the outstanding federal debt was approximately 2.5 trillion dollars, so enter 2500000000000 in the initial value box (alternatively you can write 2.5e12, which is Vensim PLE shorthand for 2.5×10^{12}). The **Initial Value** box should look like this:

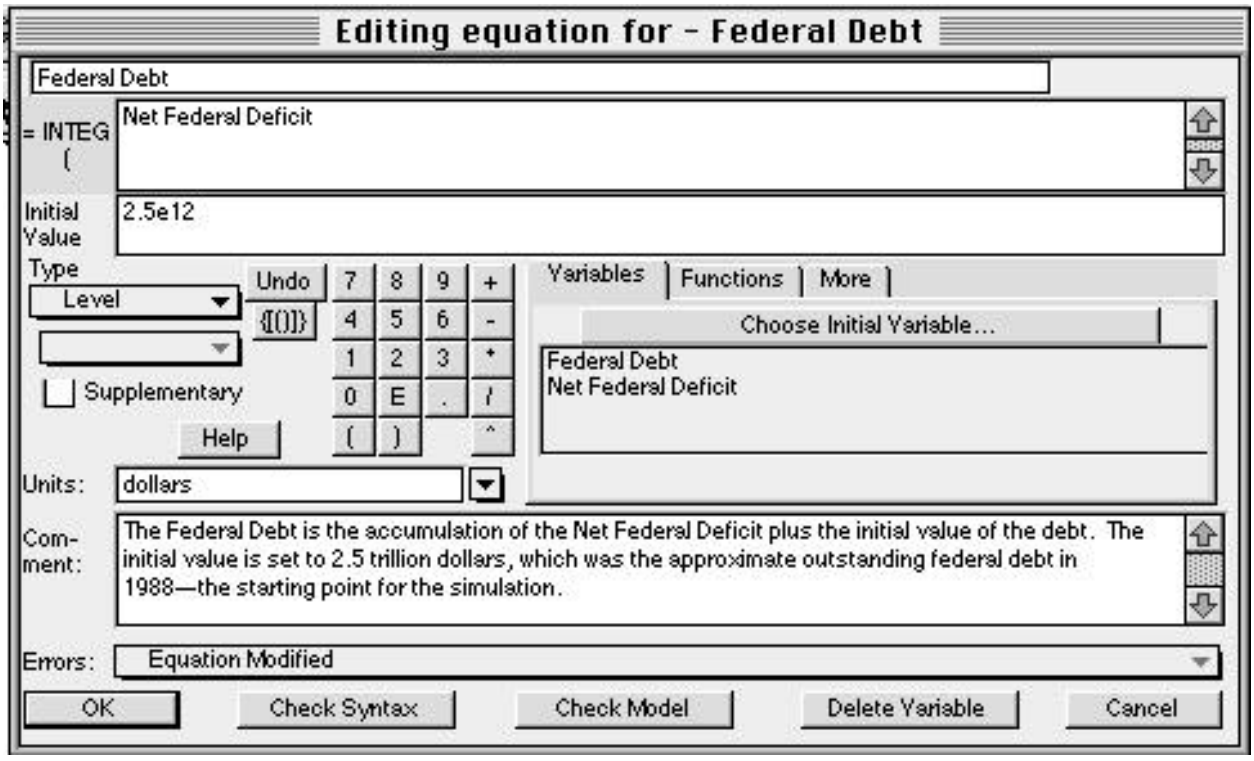


Now the equation specification for the **Federal Debt** stock is complete. Your equation indicates that the federal debt is simply the accumulation of the **Net Federal Deficit** since 1988 added to the initial value.

You still need to specify the unit of measure and document your equation in the comment field. The units should be fairly straightforward. The **Federal Debt** is a stock and its units are dollars. Useful comments briefly explain the structure of the equation and highlight the key assumptions made. A sample comment for **Federal Debt** is:

> **The Federal Debt is the accumulation of the Net Federal Deficit plus the initial value of the debt. The initial value is set to 2.5 trillion dollars, which was the approximate outstanding federal debt in 1988—the starting point for this simulation.**

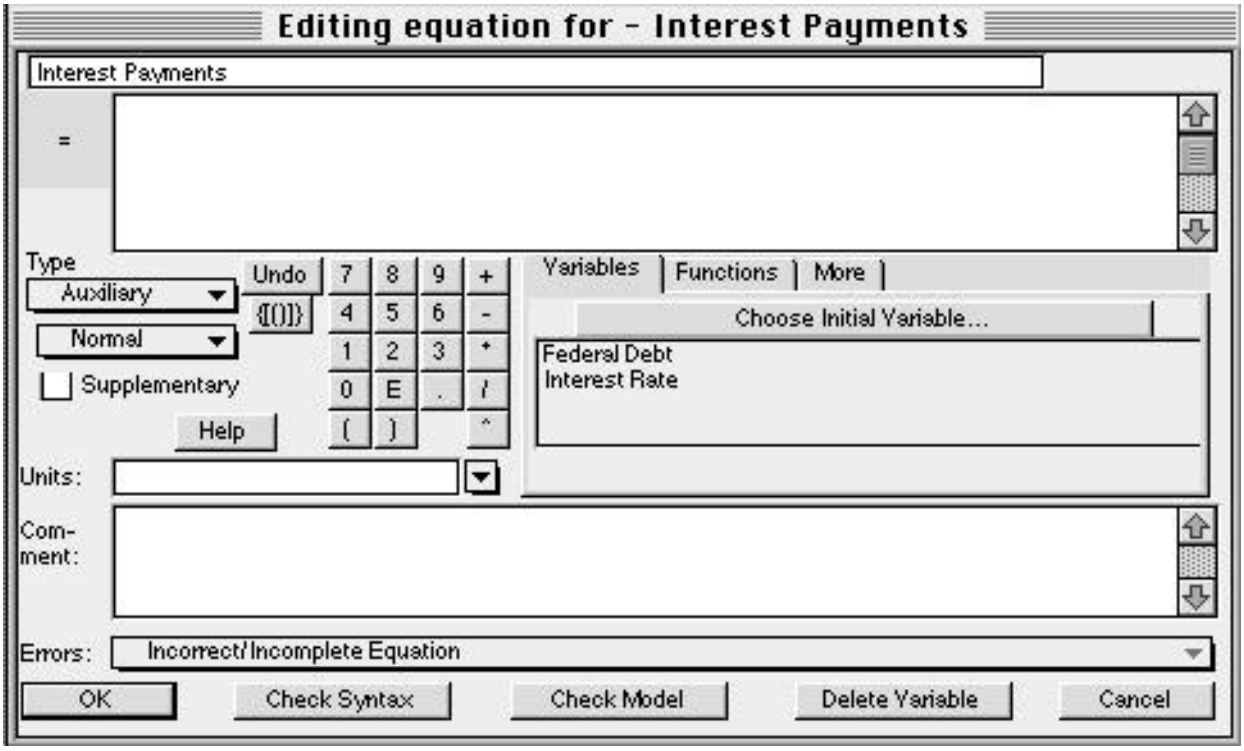
Your dialog box should now look like this:



Click on **OK** or press return.

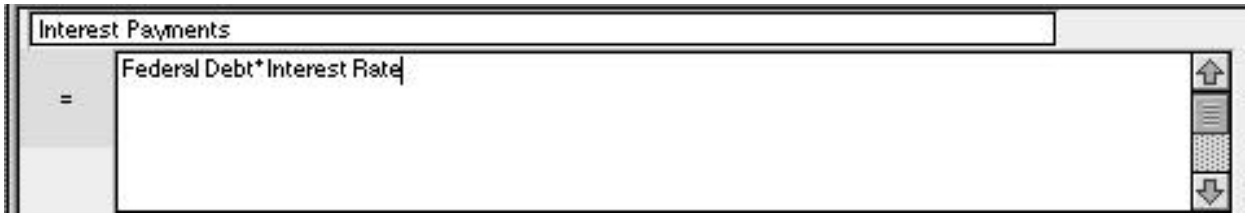
Now you need to specify the equations for the auxiliary variables and the flow.

Using the  tool on the sketch tool bar, click on the **Interest Payments** variable. You should see a dialog box that looks like this:



This box is identical to those used to specify the exogenous variables, but, in this case, there are two other variables in the **Variables** box; you are required to use these variables in the equation. When you developed the stock, flow, and feedback diagram, you drew causal arrows connecting the variable **Federal Debt** and constant **Interest Rate** to the variable **Interest Payments**. Vensim PLE has conveniently recognized this fact and has provided a list of the required inputs to your equation based on the picture you have already created. In fact, if you try to write your equation without using the two required variables, Vensim PLE will give you an error message.

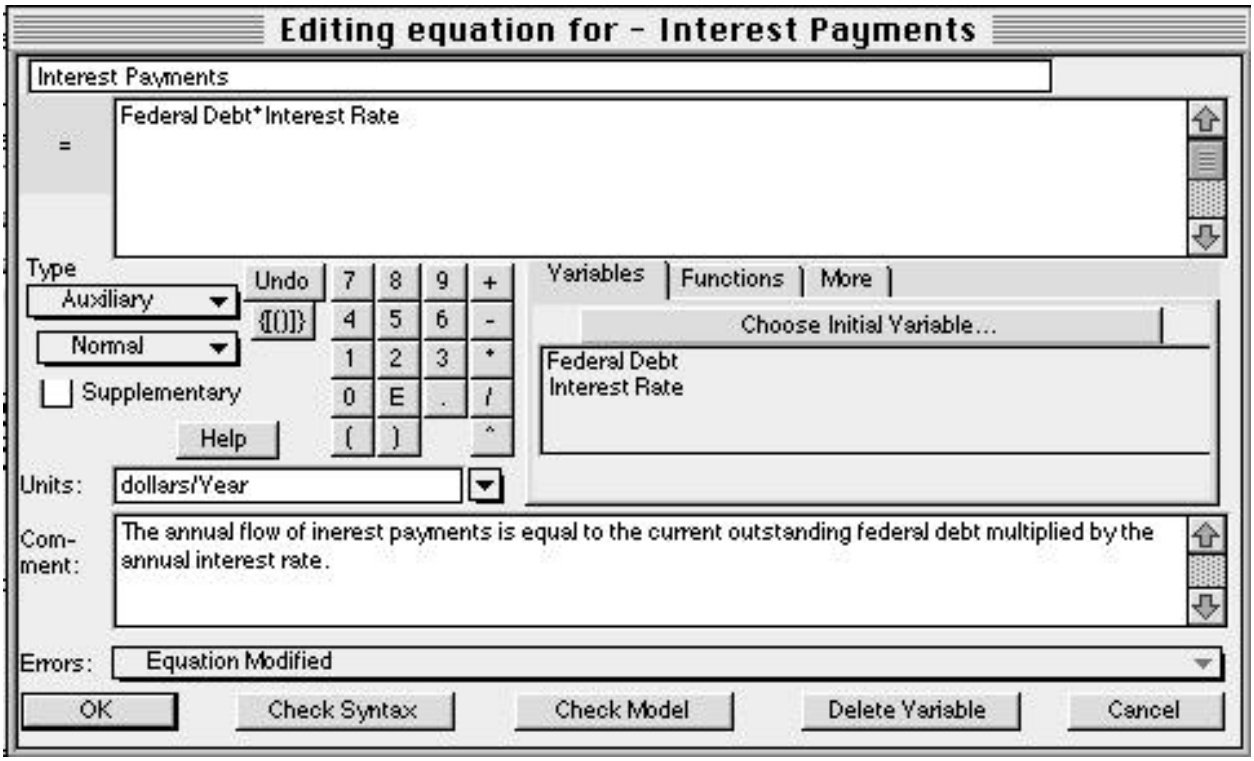
The rate of interest payment is simply equal to the current debt stock multiplied by the interest rate. To enter this equation, first click on the **Federal Debt** variable in the **Variables** box. It now appears in the equation box. Now type * (alternatively you can click on the  button), and then click on the **Interest Rate** variable in the **Variables** box. Your equation box should now look like this:



To complete the equation, you need to specify the units, dollars/year, and document your equation in the comment field. An appropriate comment might look like the following:

> **The annual flow of interest payments is equal to the current outstanding federal debt multiplied by the annual interest rate.**

The dialog box for the variable **Interest Payments** should now look like this:



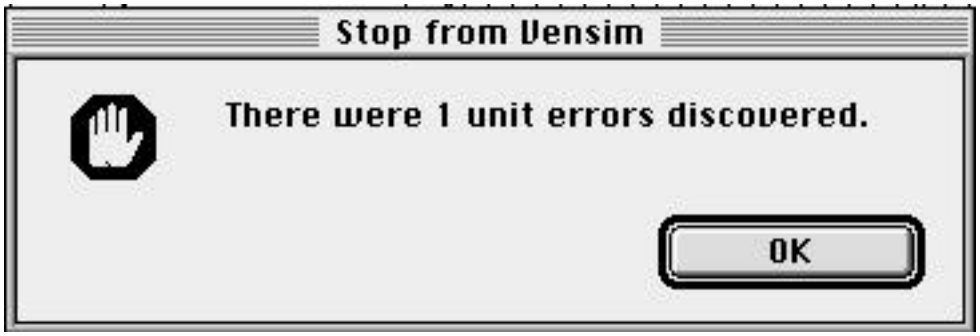
Following a similar process to the one outlined above, you should now be able to complete your model.

IV. Using the Model Structure Analysis Tools

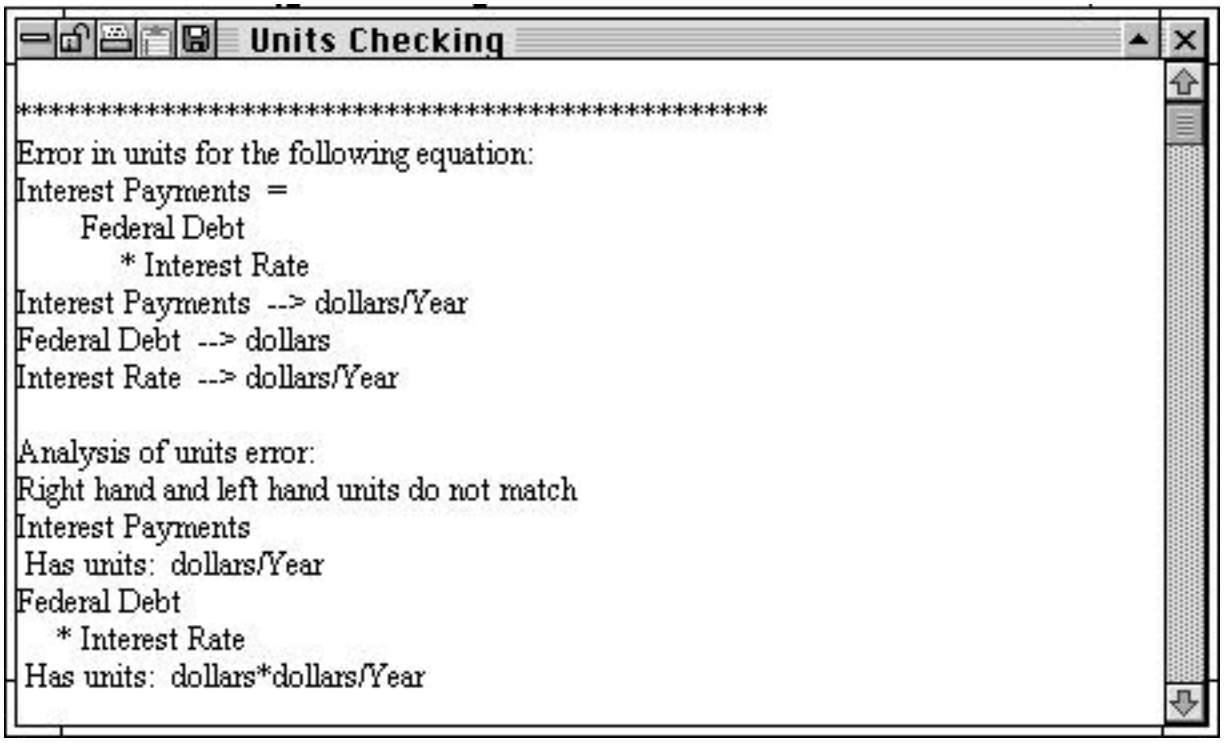
Vensim PLE provides five tools for analyzing and understanding the structure of your model.

By far the most important of these is the unit-checking tool.

An important feature of any system dynamics equation is *dimensional consistency*, which is just a fancy way of saying that the units of measure must be the same on both the left and right sides of the equation. As an example, suppose you had chosen the units of the **Federal Debt** stock to be dollars and the units of the **Interest Rate** to be dollars/year. If so, then pressing the *apple* key and *u* (alternatively, you could select the **Model** menu, then select **Units Check**) simultaneously would yield the following message:



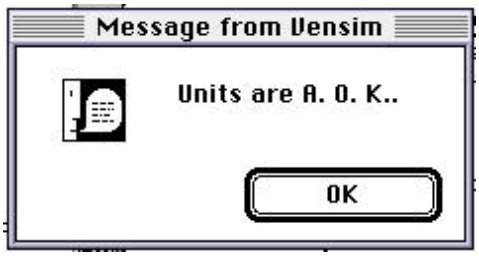
Followed by:



The problem is that, in this example, the equation for **Interest Payments** is not dimensionally consistent: the right and left sides of the equation have different units. The flow **Interest Payments** is measured in dollars per year. The **Federal Debt**, because it is a stock, is measured

in dollars. Multiplying **Federal Debt** by something with units in dollars/year results in a quantity that has units in dollars^2/year—hence the error.

The cause of the problem is that the unit of measure for **Interest Rate** is incorrect. The interest rate is not measured in dollars per year. An easy way to think about this fact is to recognize that the interest rate really has nothing to do with dollars. It could easily apply to any other currency or any other type of measurement unit. In fact, the interest rate has no unit of measure; it is dimensionless. Although it has no unit of measure, it is, nevertheless, time-dependent; an annual interest rate is not equivalent to a monthly one. As a result, the appropriate unit of measure for **Interest Rate** is 1/year. If you enter 1/year into the unit field of the interest rate variable and simultaneously press the *apple* key and u, you should receive the following message:



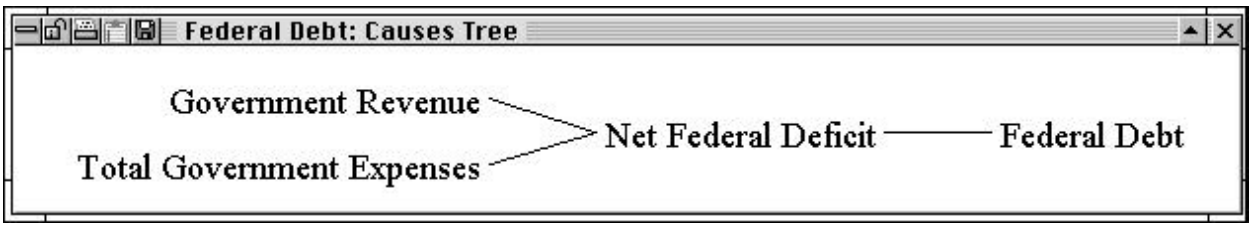
The units in your model now balance.

In this example, the unit-checking tool identified an incorrect assumption in a common mental model of the interest rate. Dimensional consistency is an important feature of system dynamics models, and Vensim PLE's unit-checking feature often helps you to identify serious flaws in both your understanding of the system under consideration and your resulting model formulations. Always make sure that the units in your model balance!

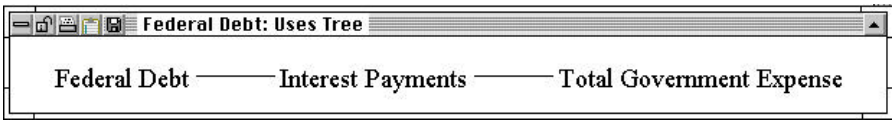
The other analysis tools that Vensim PLE provides can also be useful. The  and the  buttons create "causes" and "uses" trees for a variable. To use these tools, you need to first "select" a variable. To select a variable, first click on the  button and then double-click on the variable you wish to select. You can tell what variable is selected by looking at the top border of the Vensim PLE window. If you select the variable **Federal Debt,** the top border will look like this:



Having selected **Federal Debt**, clicking on the two "causes" and "uses" buttons in sequence gives you:



and



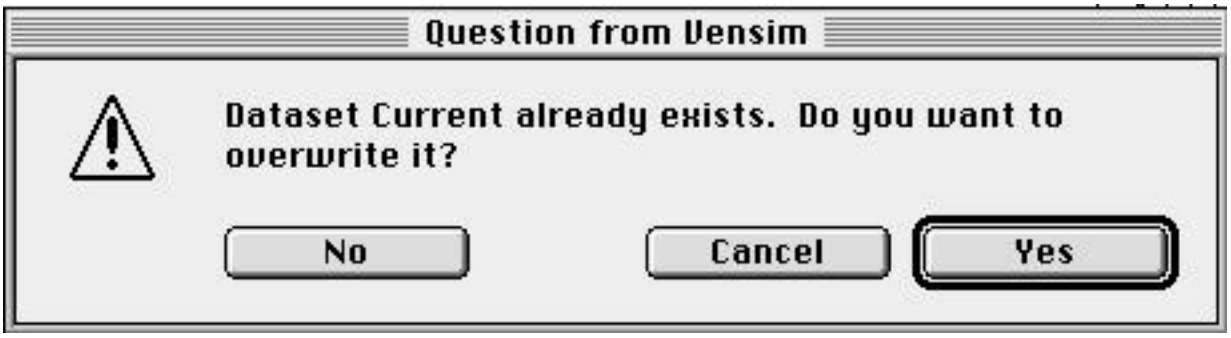
The 

button on the analysis tool bar provides you with a complete listing of the equations in your model. The  tool on the analysis tool bar identifies all the feedback loops of which the selected variable is a member.

V. Simulating Your Model

Vensim PLE also has tools to help you analyze the behavior of your model. Before analyzing the behavior, however, you must actually simulate the model so that you have some behavior to analyze.

To run a simulation, you first need to click on the running man  icon on the top toolbar. Vensim may display the following dialog box.



Clicking on yes will overwrite the "current" dataset displayed in the box to the right of the running man icon. Selecting "No" will allow you to create a different dataset. It is helpful to choose names that suggest some idea of what is being tested rather than simply using name like SIM1, SIM2, etc. Because this run is the base case run for your model, you might choose to call the run BASE.*

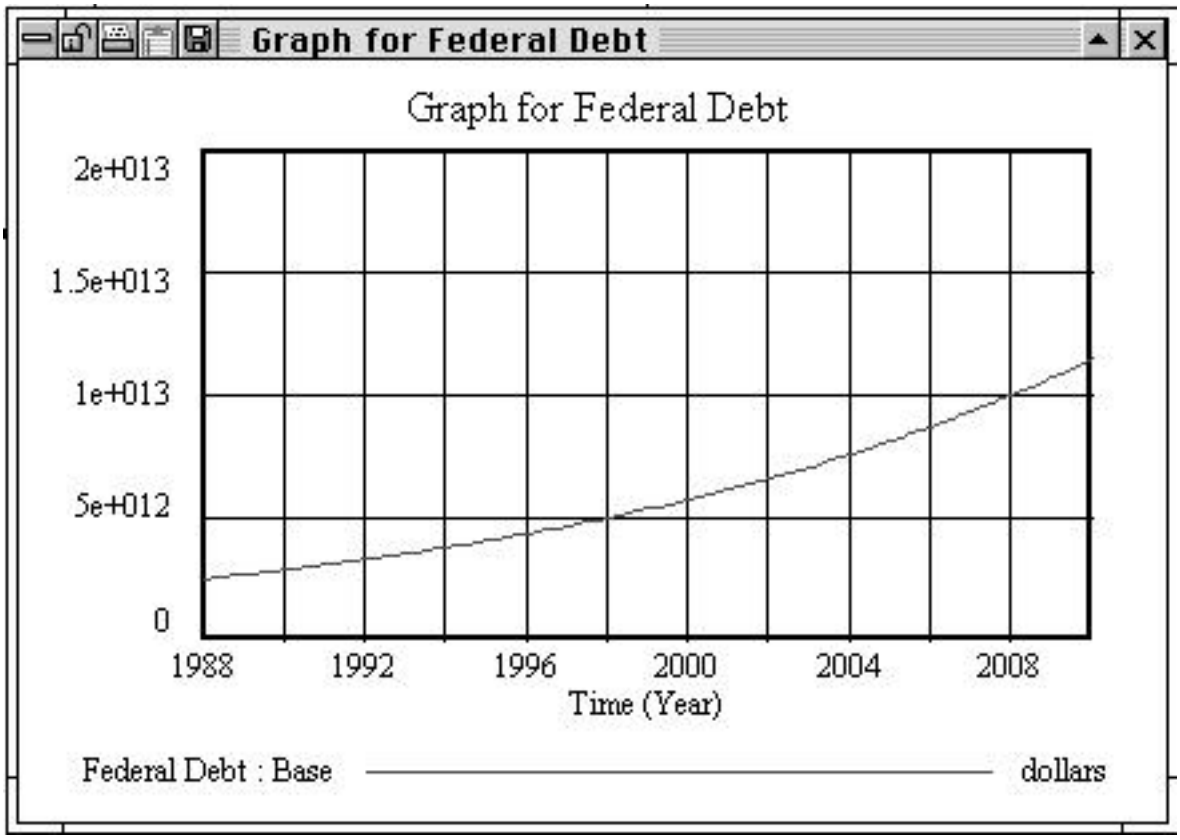
Click on **No**, type in BASE as your new dataset name, click on **Save** or hit return, and your model will start simulating.

Once the simulation run is completed, you can look at the results of your simulation. Vensim PLE provides many tools with which to view simulation output. The most basic, and often the most useful, of these tools is the strip graph. To create a graph of the **Federal Debt**, first click on the  tool and then double-click on the stock to select the variable **Federal Debt**.

To see a strip graph, click on the  button on the analysis tool bar.

* Advanced Tip: Vensim PLE also offers you the choice of two numerical integration methods, Euler and Runge-Kutta 4. Runge-Kutta 4 is a more accurate integration method, but it is also more computationally intensive. Generally it is better to use the Euler method and only change if you believe you are seeing integration error.

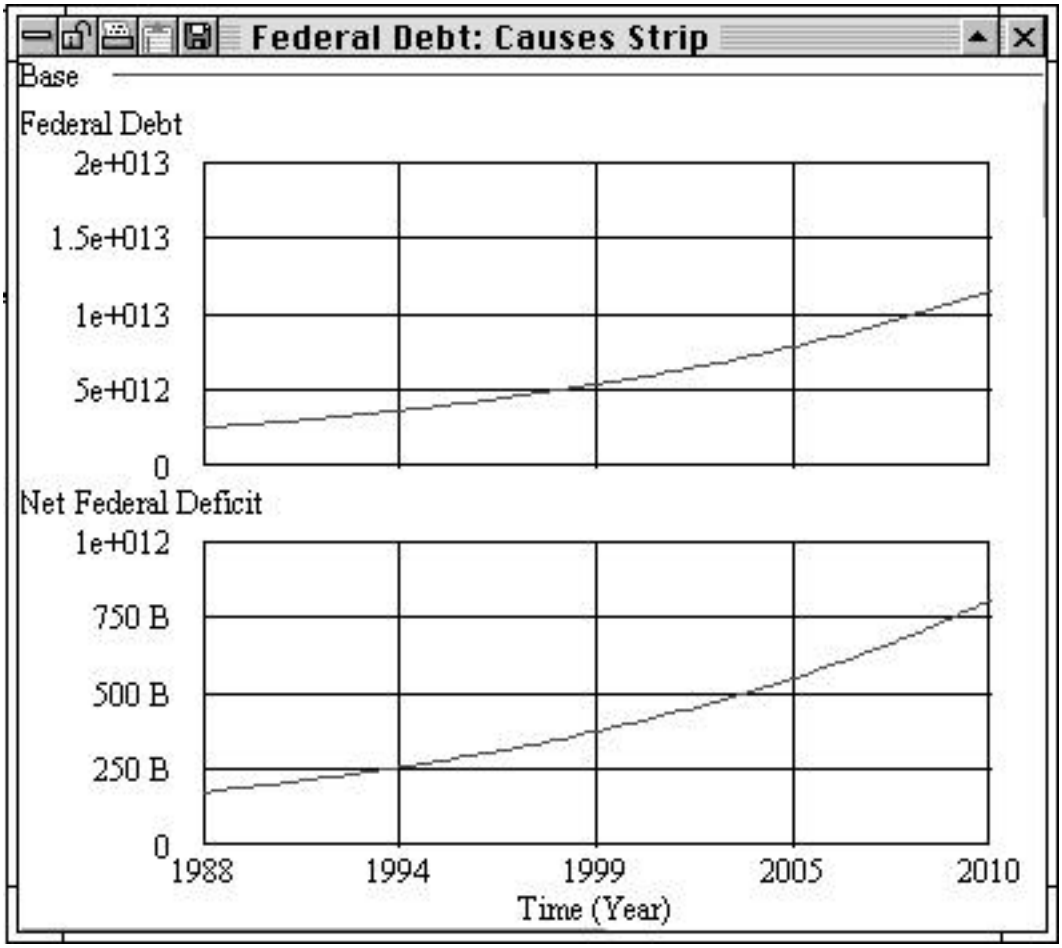
You then see:



By the year 2010, given the current assumptions in the model, the federal debt will grow to more than 10 trillion dollars, four times its value in 1988.

Besides the strip graph, Vensim PLE provides many other ways to examine simulation output.

The  button displays a strip graph of the currently selected variable, along with graphs of all the variables that determine the value of the selected variable (the causes). Clicking on the button gives you:

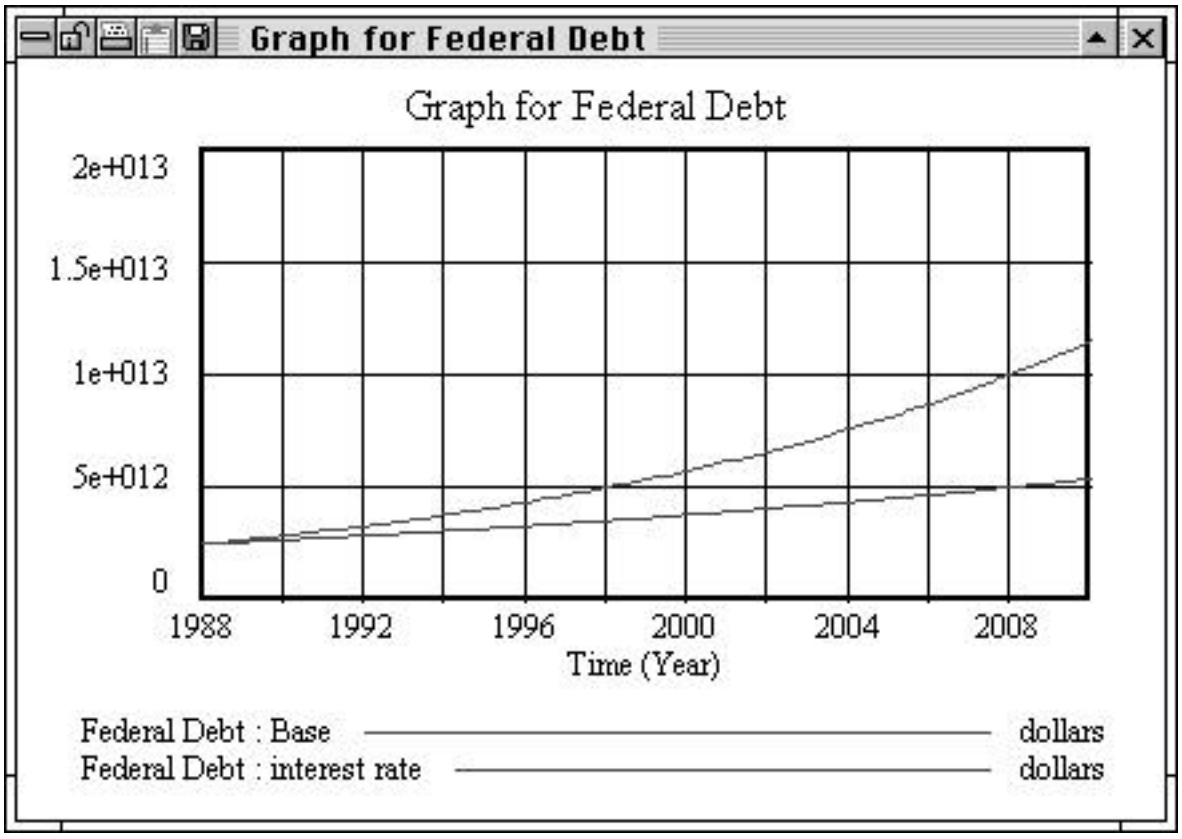


Vensim PLE also can present the output in the form of a table rather than a graph. To see a table of the selected variable simply click on the  button.

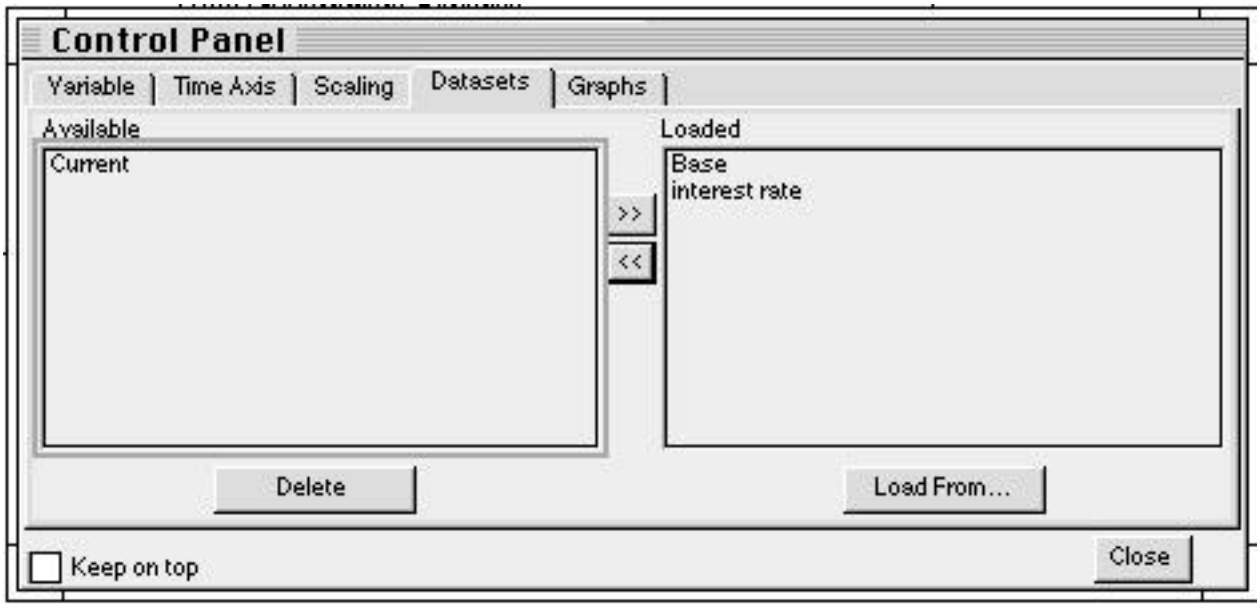
Having analyzed this simulation, you may wish to run additional simulations under different assumptions. For example, what might happen if the prevailing interest rate were 3.5% rather than 7%?

One way to change the parameter is to change the model equation in the original model. With the  button selected, click on **Interest Rate.** A dialog box will appear. In the constant box change the interest rate from 7% to 3.5%. Again, run this new simulation but do not overwrite the simulation named **Base**. Instead, name it **interest rate**.

Your new graph should look like this:



If you do not wish to see the previous run (base) displayed with the new simulation run, click on the  button on the upper toolbar. Your screen should look like this:



A dialog box appears and shows on the left side the two simulation runs you have created so far. Double-click on the name of the simulation run you wish to *remove* from the graph (or highlight it and click on the **<<** button to remove it from the right side of the dialog box). Close the **Datasets** window and close and re-display the strip graph. Now, only the new simulation run should appear.

You may also wish to run the model for a longer period of time. In this case, select **Time Bounds...** from the **Model** menu. You then see the same dialog box that you saw when you first started to develop your model.

You can extend your simulation by setting a new date for your final time. Run your model out to the year 2075.

VENSIM CONVERSION GUIDE

Aaron Diamond

December 2000

I. Introduction

Since the Road Maps series was begun in 1992, a number of software programs for system dynamics modeling have come to market. Because many system dynamics modelers use Vensim software to create their models, we are adding new Vensim sections to the current Road Maps papers which were written using STELLA examples. Vensim software is sold by Ventana Systems Inc. at http://www.vensim.com. The Personal Learning Edition (PLE) can be downloaded from Ventana free and offers a less complete (but adequate for Road Maps models) version of Vensim. (Vensim PLE Plus, or Vensim PLE32, is a slightly updated version of Vensim PLE and was used for the creation of Vensim additions found in Road Maps chapters 2-9.) Beginning with Chapter 10, we expect to use only Vensim examples and exercises.

In this "Vensim Conversion Guide to Road Maps," we are addressing differences between STELLA and Vensim, Vensim PLE and Vensim PLE Plus in system dynamics modeling. The individual appendices are not intended to give step by step instructions for how to create the models, but provide correct models created according to the specifications in Road Maps' text. To access explicit instructions for creating models using Vensim Software, the following references are extremely helpful:

** D-4697, Road Maps 2, "Formulating Models of Simple Simple Systems Using Vensim PLE," Leslie Martin, MIT,*

from the world wide web:

** Vensim's PLE Guide: http://vensim.com/venple.html#PLE Guide,*

** "Vensim PLE Quick Reference and Tutorial": http://www.public.asu.edu/~kirkwood/sysdyn/VenPLE.pdf.*

II. Differences between STELLA models and the Vensim models in the appendices

In this section, we will address some of the differences between modeling with Vensim and STELLA:

1. **Naming standard:** All models should follow what is called the Vensim Naming Standard. The Standard universalizes the way in which variables in models are named:

* Stocks are written with the first letter of each word capitalized
* CONSTANTS or FRACTIONS--variables whose value is not affected by other variables within the model--are written in all capital letters;
* flows or rates, as well as endogenous variables are written in all lower-case letters.

You will notice that the STELLA models in Road Maps do not follow a naming standard, whereas the Vensim Conversion Guides follow the Vensim Naming Standard.

2. **Documentation:** Contrary to Stella's stock/flow ordering, Vensim PLE documentation is alphabetized (Vensim Plus allows the modeler to choose stock/flow ordering or alphabetized ordering). The documentation attached to models in the Vensim Conversion Appendices is the documentation Vensim gives after each equation is entered

in the model. It does not show how to enter the data, but after reviewing the Vensim user guides, the documentation accompanying the models will be quite helpful.

3. **Lookup Functions:** Entering lookup functions in Vensim models is significantly different from entering lookup functions in Stella models. The most effective method for entering a lookup function in a Vensim model is the following:

a) The LOOKUP is entered as a constant, connected usually to what is an "effect" variable. The LOOKUP contains the actual lookup graph (or coordinates), whereas the "effect" variable contains the LOOKUP as a function of another variable in the model. The "effect" variable's equation is entered as …LOOKUP ("variable"), where quotations are not used. For example: land/availability multiplier=LAND AVAILABILITY LOOKUP(land fraction occupied). In this case, "land fraction occupied" is an input to the LOOKUP, and the LOOKUP outputs the corresponding value as dictated by the LOOKUP graph or coordinates.

b) To enter the LOOKUP function, first access the equation editor to open the "equation" box for the LOOKUP. To the left, just below the equation box, there is a small rectangle with the name "Auxiliary."

c) Click and hold on the box marked "Auxiliary" (on the downward pointing triangle). Slide down to "Lookup" and let go of the mouse button.

d) You will see, just below what should now be a "Lookup" rectangle, a small rectangle with the words "As Graph" appears. Click on the "As Graph" rectangle.

e) A graph will appear, where you can either type in the points on the left or click on the graph where you want the points to be. Using the equation for the Lookup (which, for Road Maps models is found in the Vensim Example at the end of each paper), enter

the input/output for each point. You should see the shape of the curve on the graph to the right.

f) Click "OK" once the values are entered. If you have any trouble with Lookup Functions in Vensim, the third suggested Vensim user guide, found at http://www.public.asu.edu/~kirkwood/sysdyn/VenPLE.pdf provides a very useful tutorial.

4. **Initial Variables:** Every Vensim model has an additional input for each stock, called "INITIAL 'STOCK NAME.'" Unlike the STELLA models currently in Road Maps, where the initial value of the stock is numerically entered into the Initial Value "equation" box, in Vensim models, the name "INITIAL 'STOCK NAME'" is typed into the Initial Value box for the Stock, and the numeric value is entered in the equation of the "INITIAL 'STOCK NAME.'" For Vensim Version 3.0 (or lower), the "INITIAL 'STOCK NAME'" cannot be attached to the stock, and must be "floating." For Vensim PLE Plus and the newer versions of Vensim, the "INITIAL 'STOCK NAME'" can be attached using an arrow (the arrow will either become smaller, turn blue, or shadow). This convention makes it considerably more convenient to change initial conditions while running simulations.

5. **Special Functions:** Entering SMOOTH, PULSE, STEP, MIN/MAX, and RAMP functions in Vensim is slightly different from Stella. When entering an equation for a variable, simply click on the rectangle to the right of the "Variables" rectangle in the equation editor and the list will appear. Once a function is selected, the equation box will instruct on what variables to enter in the equation. If you need further help, the PLE Guide will instruct you on how to enter special functions.

6. **Units:** Vensim has a units check. Because the earlier versions of Stella do not, there are several units errors in some of the Stella models in Road Maps. The units for the models in all the Vensim Conversion Appendices, however, are consistent. Some Vensim models contain extra variables, or "Normalizing Variables" that ensure that the units work out. To check units using Vensim, type ctrl-u, and Vensim will tell you if "Units are A.O.K." or if there are any units errors.

7. **Restrictions on embedded constants:** Vensim only allows numeric entries to be used in the equations for constants. Because of this restriction, a variable cannot have a constant embedded in its equation, as it must have the name of the constant entered instead. Because some of the original Stella models had embedded constants entered in equations, the corresponding Vensim models have extra variables and equations representing those constants. Placing restrictions on embedded constants makes it considerably easier to change constants while running simulations.

III. Conclusion

Once you are able to understand the differences between the STELLA and Vensim models and apply the Vensim methods to your models, your modeling skills will be greatly enhanced. The Vensim examples in the Road Maps series can serve as a guide for building Road Maps models. Each of the models has been run according to the specifications in the text, and the graphs are supplied. Also, each model is created using the proper modeling conventions and the documentation contains consistent units.

We hope you find the Vensim Conversion Appendices to be useful while developing your modeling skills. Please e-mail your comments or suggestions on the Vensim Guide or any of the appendices to <rm-help@sysdyn.mit.edu>.

System Dynamics Model Correctness Checklist

SYSTEM DYNAMICS IN EDUCATION PROJECT
SLOAN SCHOOL OF MANAGEMENT
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
By: Danny Lai and Rebekah Wahba

"Perfect models" are rare in Systems Dynamics because the "correctness" of model is relative to its purpose and varies widely, depending on the modeler, users, and modeling conventions. Below are some pointers one should use when building and simulating models. They include some accepted modeling standards, and also tips to check if your model "works".

1. **Units check** – Vensim and some other programs (Stella, iThink etc.) have built-in units feature that will check all equations for consistency in units. (That is to make sure the left and right side of all equations have the same units.) This feature is a valuable tool that can be used to check that valid dimensions, constants and equations are used. One must check models for dimensional consistency before simulating either using the software feature or manually.
2. **Naming variables** – A general element naming convention has been proposed by Ventana, the makers of Vensim. The first letter of Stock names should be capitalized; CONSTANTS should be in all capitals, and names of all other variables, including flows are all lower case. Use names that clearly explain what a model element represents. In general, the word "rate" is reserved for flows. For example, do not confuse birth fraction, with birth rate.
3. **No constants embedded in equations** – It is often tempting to simplify equations by using numeric constants embedded in equations. One must not do so! A good model will show all constants explicitly as individual elements. One must be able to recognize all model elements at a glance. Name the constant and use the constant name in the equation. This allows constants to be recognized and changed easily in future simulations, without changing any equations in the model.
4. **Do not mention parameter values in the documentation** – The documentation complements the equations and should merely describe what the equations mean, and contain special notes. Frequently, constants are changed and sensitivity analysis is performed on models, and parameter values are changed in the equations (in the case of constants). If parameter values are mentioned in the documentation, they will have to be changed every time the equation is changed. This can be tedious and confusing as a reader will see two different numbers if the documentation is not updated, (See #3). Not mentioning parameter values in the documentation keeps it robust and avoids potential confusion.
5. **Choose appropriately small time steps** – Choose the time step to be about one-eighth the value of the smallest time constant in the model (the time constant is the reciprocal of a growth or decay fraction). Doing so will increase the frequency at which the software solves the model equations, improving the approximations of continuous time and avoiding some mathematical errors.
6. **Stock values can be changed only by flows** – The only model elements with direct connections to stocks are flows. No constants or auxiliary variables should directly enter the stock equation, except for the initial values of the stock, (See #11).

7. **Every flow should be connected to a stock** – A flow only increases or decreases a stock; it cannot be used as a source of information in a model as it cannot be measured, (See #8). A flow unattached to a stock serves no purpose in the model, as it does not affect anything.
8. **Flows should not be linked to auxiliary variables or to other flows –** Flows are instantaneous and cannot be measured in real-time. In fact, flows can only be measured by calculating the change in stock value per time unit. Furthermore, it takes time for information to move from one flow to another. So a flow theoretically cannot give another flow a value equal to itself in the same time period. Besides, as flows cannot be measured, clearly one flow cannot pass information to another flow. Therefore, one must not use a flow to provide information to an auxiliary variable. If two flows are defined by the same structure, then one should use the same structure and equation to define both flows (and be a little redundant) instead of simply connecting the two flows.
9. **Stocks should not be linked to stocks** – A stock is the integral of a flow, (See #6). To show information transfer between two stocks, connect the first stock to the flow of the second stock. Be sure to check the units.
10. **Using IF THEN ELSE, MIN/MAX and other logic statements** – Almost no real-life situations behave according to IF THEN ELSE or MIN/MAX statements. Change is almost always gradual and not sudden like such functions suggest. One must use table functions to avoid discontinuities introduced by such statements.
11. **Use of Initial Values –** When initial values are used in a model, they should be clearly specified and connected to the model. The newest versions of most of the popular modelling software enforce this practice. There are occasions though, where the software does not allow for these connections, another way to explicitly connect the initial values (e.g. Word) should be used. This modelling convention makes it considerably easier to change initial conditions while running simulations.
12. **Curving connectors** – This issue deals with aesthetics. The connectors that link one variable to another should be curved as a model with curved connectors looks nicer, and the feedback loops are easier to trace.

A TABLE FUNCTION EXAMPLE

By Lei Lei and Nathaniel Choge
January 2001

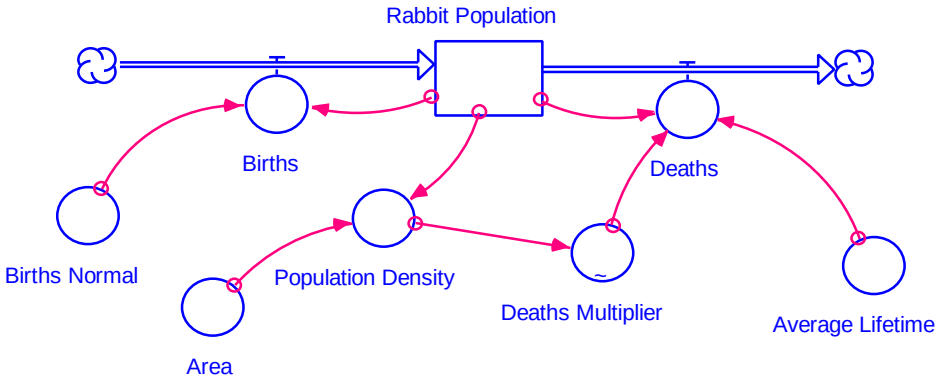
I. Introduction

While building a system dynamics model, the modeler may be faced with a situation where there is a relationship between two variables but no simple algebraic equation to define the relationship. For example, the death rate of a population of rabbits in a fixed area increases rapidly as the area gets more crowded. In cases where the relationship between two variables is not linear, table functions can be used to represent the relationship.

Table functions are used in preference to complicated equations because the modeler can control the shape, slopes and saturation points to accurately represent the non-linear relationship between two variables. Table functions are also easier to interpret and visualize than complex algebraic equations.

II. Using Table Functions in Vensim

The example below, shows how to translate table functions from STELLA to Vensim using the Rabbit Populations model found in "Generic Structures: S-Shaped Growth I" (D-4432-2) in Road Maps 5.



STELLA Version: S-Shaped Growth Structure I

The lookup function for **deaths multiplier** has a dimension in our STELLA example which is not good modeling practice because it violates dimensional consistency

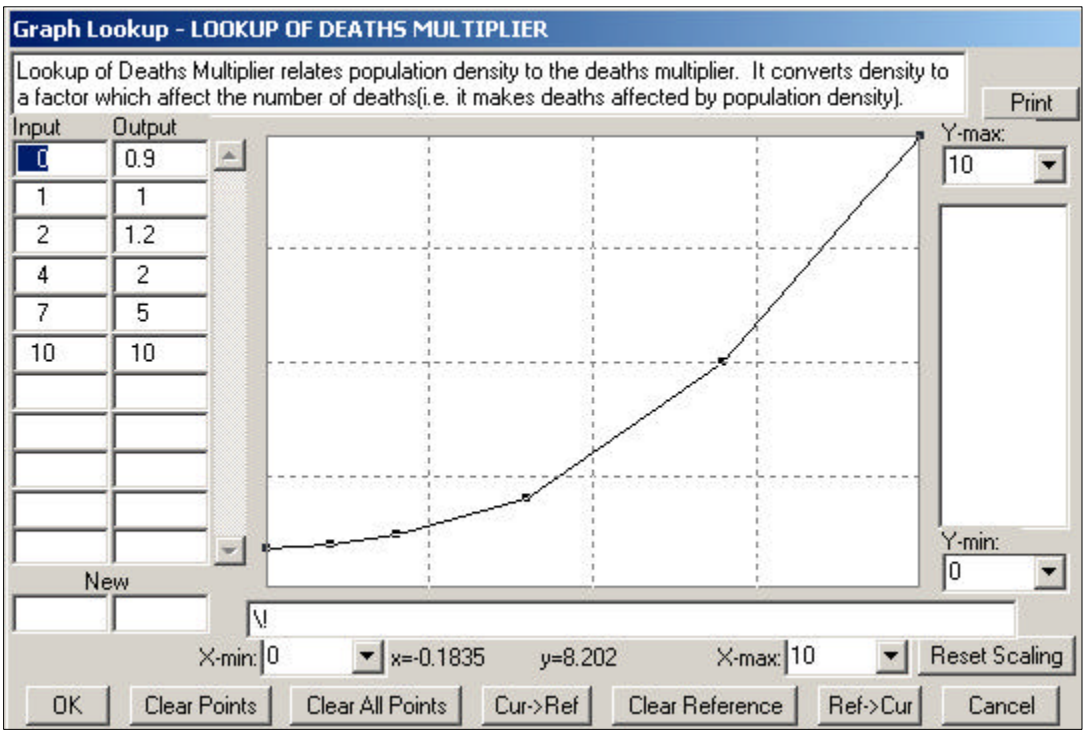
in the model. It is better practice to have dimensionless inputs and outputs to all table functions in system dynamics models.

A variable called "normal" is used to divide the input variable, which in the example is population density, to produce a non-dimensional input to the lookup table. Normals are constants that act as reference points for dynamic behavior e.g. in the example above, the "normal" is the **population density** at which the **deaths multiplier** is 1 and the "deaths" is determined only by the "average lifetime," "Rabbit Population" and "deaths multiplier."

In converting the example from STELLA to Vensim, first create two new constants: **NORMAL POPULATION DENSITY** and **lookup of deaths multiplier**. The constant, **NORMAL POPULATION DENSITY** divides **population density** to produce the **relative population density**, a non-dimensional input to the lookup table.

The relationship **lookup of deaths multiplier** will contain all the data points representing the non-linear relationship between the two variables: **relative population density** and **deaths multiplier**.

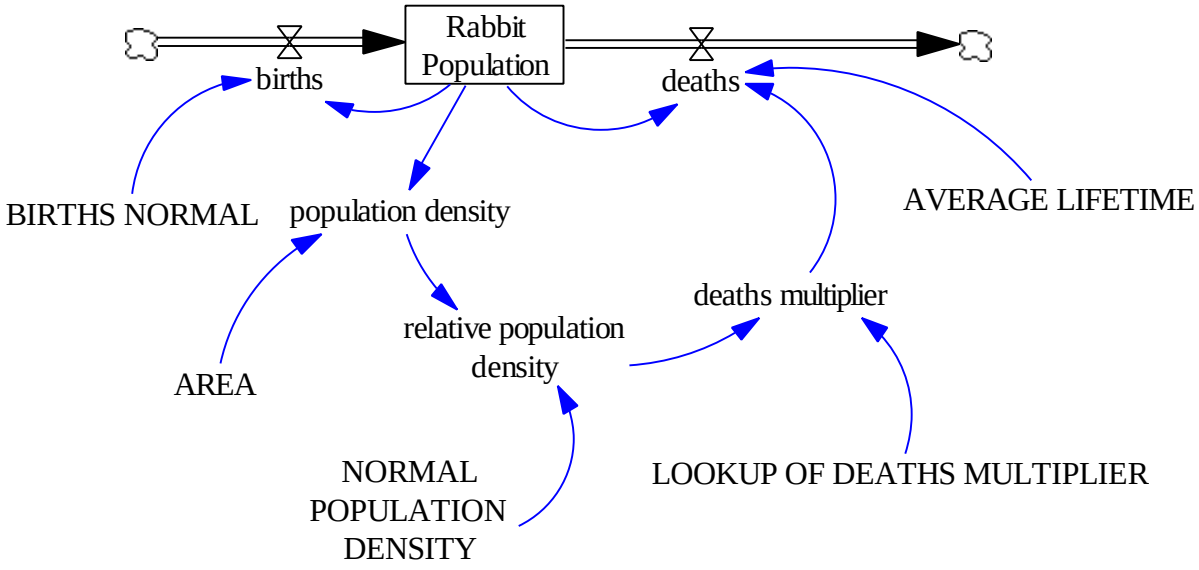
To input data points for the lookup of deaths multiplier, go to the equation editor and under "type," select "Lookup," then "As Graph" and you will be taken to a screen where you can enter the data points in the form of a graph:



The equation that relates the table function, normal and multiplier is:

deaths multiplier = lookup of deaths multiplier (relative population density)

The complete Vensim version of the example is shown below:



Vensim Version: S-Shaped Growth Structure I

Documentation for Rabbit Population Model

(01) AREA = 1
Units: acre
The amount of land the rabbit population is living on.

(02) AVERAGE LIFETIME = 4
Units: year
The number of rabbits that die per rabbit in the population per year.

(03) births = Rabbit Population*BIRTHS NORMAL
Units: Rabbits/year
The number of rabbits being born each year.

(04) BIRTHS NORMAL = 1.5
Units: 1/year
The number of rabbits born into the population per rabbit in the population each year.

(05) deaths = (Rabbit Population/AVERAGE LIFETIME)*deaths multiplier
Units: Rabbits/year
The number of rabbits that die each year.

(06) deaths multiplier = LOOKUP OF DEATHS MULTIPLIER (relative population density)
Units: dmnl
deaths multiplier gives the effect of population density on the rate of deaths of rabbits.

(07) FINAL TIME = 12
Units: year
The final time for the simulation.

(08) INITIAL TIME = 0
Units: year
The initial time for the simulation.

(09) LOOKUP OF DEATHS MULTIPLIER([(0,0)-0,10)],(0,0.9),(1,1),(2,1.2),(4,2),(7,5),(10,10))
Units: dmnl
Lookup of Deaths Multiplier relates population density to the deaths multiplier. It converts density to a factor which affect the number of deaths(i.e. it makes deaths affected by population density).

(10) NORMAL POPULATION DENSITY = 1
Units: Rabbits/acre

(11) population density = Rabbit Population/AREA
Units: Rabbits/acre
The number of rabbits per unit of area.

(12) Rabbit Population= INTEG (births-deaths,1)
Units: Rabbits
The number of rabbits in the rabbit population.

(13) relative population density = population density/NORMAL POPULATION DENSITY
Units: dmnl
relative population density gives the ratio of population density to NORMAL POPULATION DENSITY to provide a dimensionless input to the lookup table.

(14) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.

(15) TIME STEP = 0.0625
Units: year
The time step for the simulation.

To see the complete STELLA version of the example refer to “Generic Structures: S-Shaped Growth I” (D-4432-2) in Chapter 5 of Road Maps. A detailed discussion of table functions, normals and multipliers can be found in “Mistakes and Misunderstandings: Table Functions” (D-4653) in Chapter 9 of Road Maps.

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