Massachusetts Institute of Technology
14.41 Public Finance & Public Policy – Problem Set 2

QUESTION 1: [70 points]

Public goods are typically discussed in the contexts of governmental projects and policies; however, the public goods framing isn't restricted only to governments. Let's explore this topic using a simplified model of a labor union, the United Widget Workers (UWW), interacting with their employer, Widget Co.

Widget Co.'s employees consist of 20 researchers, 35 salespeople, and 45 machinists (with incomes Y_r, Y_s, and Y_m, respectively), who are all represented by the UWW and vote in its elections. Suppose UWW's budget is given by B, which is funded by equal flat union dues d (conceptually the same as a tax) imposed on workers, so that $B = d * k$, where k is the number of workers paying dues. In particular, the UWW must decide how to split its budget of B between funding worker safety improvements S (e.g. installing ventilation or fire proofing) and career training programs T (e.g. digital literacy training or for specialized certifications), so that $B = S + T$.

PART I: [35 points]

1. (4 points) What must be true about the worker safety improvements and training programs for them to be considered pure public goods from the workers' point of view? Argue whether or not this seems reasonable in the real world.

> **Solution:** Worker safety improvements and training programs must be non-rivalrous and non-excludable to be considered pure public goods. These seem reasonable for many worker safety improvements, like installing ventilation or fire proofing: Me as a worker having ventilated air or fire proofing doesn't mean you don't get them (non-rivalrous), and I can't prevent you from getting them (non-excludable). The training programs may only be considered public goods if everyone has access to them.
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> **Grading notes:** 2 points for naming non-rivalrous and non-excludable. 2 points for reasonable argument for whether it is applicable in the real world.

Suppose Widget Co. is located in a so-called Right-to-Work state, which means workers can opt-out of paying union dues if they choose. Workers care about only two things: benefits from the union and all other consumption. Each dues-paying worker's budget constraint is $Y = C + d$, where C is consumption of goods, and each worker's utility is of the form:

$$U = \ln(B) + \ln(C)$$

2. (4 points) Assume k workers are currently paying dues. Worker i knows this, and is considering whether to start paying dues. When should worker i pay dues? Set up the inequality in terms of k, d, and y_i.

Solution: Worker i pays dues if: $U_i^{pay\ dues} > U_i^{don't\ pay\ dues} \rightarrow \ln((k+1)*d)+\ln(y_i-d) > \ln(k*d)+\ln(y_i)$. Worker i doesn't pay otherwise. Note that if the utilities are exactly equal the worker is indifferent between paying or not.

Grading notes: 4 points for setting up the inequality correctly. Indifferent workers can either pay or not, so it doesn't matter whether the answer classifies them as paying or not.

Let's find how many workers decide to not pay dues in equilibrium. For now, assume $y_r > y_m = y_s$.

3. (4 points) Explain why we should expect the machinists and salespeople to always have higher utility incentives to *not* pay dues compared to researchers. You may either answer intuitively or mathematically.

Solution: Worker i's incentives to not pay dues are given by $U_i^{don't\ pay\ dues} - U_i^{pay\ dues} = \ln(k*d)+\ln(y_i) - \ln((k+1)*d) - \ln(y_i-d)$. Algebraic manipulation gets us $\ln(\frac{kdy_i}{(k+1)*d*(y_i-d)})$. It can be shown that $\frac{d}{dy_i}$ of this value is strictly negative, so the incentive to not pay dues is decreasing in y_i; alternatively, the incentives to pay dues is increasing in y_i. Intuitively, this makes sense as the fixed union dues are a larger portion of the lower-income workers' income, and lower-income workers are able to free ride off the higher income ones.

Grading notes: 4 points for correct mathematical or intuitive answer. Give full credit if answer explains why researchers have lower incentives to not pay dues (or, equivalently, why researchers have higher incentives to pay dues).

4. (6 points) Suppose all researchers pay their dues. Solve for the equilibrium number of machinists and salespeople who ultimately decide to pay dues. Denote this value as k^*_{ms}, which should be in terms of y_m (or equivalently y_s) and d. (Hint: Consider a dues-paying worker deciding whether to continue paying dues or to opt-out of paying. In equilibrium, that worker is indifferent from paying dues and not.)

Solution: There are two ways of solving this, either from the perspective of a dues-paying worker considering opting-out, or an opted-out worker considering opting-in. The cases yield slightly different results, depending on whether the "marginal" worker is counted in k or not:

1) Considering opting-out perspective: In equilibrium, a dues-paying machinist/salesperson deciding whether to opt-out is indifferent between the two decisions: $U^{pay\ dues} = U^{don't\ pay\ dues} \rightarrow \ln(20d+k_{ms}*d)+\ln(y_m - d) = \ln(20d+(k_{ms}-1)*d)+\ln(y_m) \rightarrow k^*_{ms} = \frac{y_m}{d} - 20$.

2) Considering opting-in perspective: In equilibrium, an opted-out machinist/salesperson deciding whether to opt-in is indifferent between the two decisions: $U^{pay\ dues} = U^{don't\ pay\ dues} \rightarrow \ln(20d+(k_{ms}+1)*d) + \ln(y_m-d) = \ln(20d+k_{ms}*d)+\ln(y_m) \rightarrow k^*_{ms} = \frac{y_m}{d} - 21$.

The hint pushes towards the opt-out perspective, but the opt-in perspective is also a valid equilibrium.

Grading notes: 4 point for setting up utility equality of marginal worker correctly. 2 point for correct k^* value. No points deducted for opting-in perspective.

5. (6 points) Let $d = 1$, $y_m = y_s = 40$, $y_r = 70$, and k^*_{ms} be the number found in the previous question. Calculate society's (i.e. the sum of all the workers') utility.

> **Solution:** With $k^*_{ms} = 20$ (using the opting-out perspective), 20 researchers and 20 machinists/salespeople pay dues, so $B = 40$. Each of the dues-paying researchers have utility $\ln(40) + \ln(70 - 1) = 7.923$. Each of the dues-paying machinists/salespeople have utility $\ln(40) + \ln(40 - 1) = 7.352$. Each of the opted-out machinists/salespeople have utility $\ln(40) + \ln(40) = 7.378$. The social welfare in this setup is $20 * 7.923 + 20 * 7.352 + 60 * 7.378 = 748.18$.
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> With $k^*_{ms} = 19$ (using the opting-in perspective), 20 researchers and 19 machinists/salespeople pay dues, so $B = 39$. Each of the dues-paying researchers have utility $\ln(39)+\ln(70-1) = 7.90$. Each of the dues-paying machinists/salespeople have utility $\ln(39)+\ln(40-1) = 7.327$. Each of the opted-out machinists/salespeople have utility $\ln(40)+\ln(40) = 7.352$. The social welfare in this setup is $20*7.90+19*7.327+61*7.352 = 745.69$.
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> **Grading notes:** 2 point for finding correct utility of dues-paying researchers. 2 point for finding correct utility of dues-paying and opted-out machinists/salespeople. 2 points for correct social welfare calculation summing all workers' utility. Do not deduct points if answer from previous part carried over into this question.

6. (6 points) Now suppose the union behaves as a benevolent planner that seeks to maximize its workers' social welfare, and obliges all its workers to pay equal flat dues d each, no matter the worker type. With $y_m = y_s = 40$ and $y_r = 70$, what is the optimal d^* it chooses? (Use a calculator!)

> **Solution:** The social welfare function is given by $SWF = 100\ln(100d) + 80\ln(40 - d) + 20\ln(70 - d)$ with $B = 100 * d$, $y_m = y_s = 40$ for each of the 80 machinists/salespeople, and $y_r = 70$ for each of the 20 researchers. The FOC is given by $\frac{100*100}{100*d} = \frac{80}{40-d} + \frac{20}{70-d}$. The optimal $d^* = 21.31$.
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> **Grading notes:** 2 point for setting up SWF correctly. 2 point for setting up FOC correctly. 2 point for correct d^*.

7. (5 points) How does the optimal d^* found in part 6 compare to the d that generated an equilibrium in part 5? How do the social welfare values compare? Provide intuition as to where this difference is coming from.

> **Solution:** The optimal $d^* = 21.31$ is much more than the $d = 1$ that generated an equilibrium in part 6. The optimal social welfare in this setup is $100*\ln(100*21.31)+80*\ln(40-21.31)+20*\ln(70-21.31) = 1078.38$, which is much larger than the equilibrium case. This is a standard free rider problem where workers opt-out because they can still benefit from the public goods funded by everyone else.
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> **Grading notes:** 2 point for correct comparison that d^* and SWF is much larger in social optimum. 3 point for correct intuition of free rider problem.

PART II: [35 points]

Now, suppose Widget Co. is not in a right-to-work state, so all workers pay equal union dues $d = \frac{B}{100}$ so that the UWW has a fixed budget B. Additionally, let's now assume that all workers have the same income $y_r = y_m = y_s = 40$ but each worker type has different preferences about how to allocate B. Let X be the fraction of spending that goes towards safety improvements; the remaining $1 - X$ goes towards training programs. (Since X is a fraction, the union cannot choose a value of X outside the range $[0, 1]$). Assume now that each worker has the following preferences:

Researchers have preferences

$$U_r(y, X, B, d) = \ln(B)\left(1 - (X - 0.1)^2\right) + \ln(y - d)$$

while machinists have preferences

$$U_m(y, X, B, d) = \ln(B)\left(1 - (X - 0.4)^2\right) + \ln(y - d)$$

and salespeople have preferences

$$U_s(y, X, B, d) = \ln(B)\left(1 - (X - 0.8)^2\right) + \ln(y - d)$$

.

With this utility functional form, workers have different preferences about how public goods spending is split between worker safety improvements and training programs, and they get more utility from public goods spending the closer the split of spending matches their preferences.

1. (2 points) For each worker type, calculate their ideal choice of X, given that B is fixed at $\bar{B}$. Label these X_r^*, X_m^*, X_s^* respectively.

> **Solution:** $X_r^* = 0.1, X_m^* = 0.4, X_s^* = 0.8$
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> **Grading notes:** 1 point for setting up FOCs correctly but with some mathematical mistakes. Full credit if they just identify the correct values from the functional form without setting up the maximisation problem.

2. (3 points) Are each worker's preferences single-peaked? Explain why or why not.

> **Solution:**
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> Yes: quadratic loss function in X.
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> **Grading notes:** People can make the argument either intuitively by considering the functional form, or my looking for concavity of the function / some other mathematical condition that guarantees single-peakedness. No points for the wrong answer.

Suppose the union holds a series of votes between each of these three ideal points. That is, it first holds a vote between X_r^* and X_m^*, then one between X_r^* and X_s^*, then one between X_m^* and X_s^*.

3. (4 points) Identify the majority winner of each of the three votes. Is there a consistent winner (one that beats both of the other two alternatives)? How does this relate to your answer to part 2?

> **Solution:** 0.4 beats 0.1 and 0.8, 0.1 beats 0.8, so 0.4 is the Condorcet winner. Single-peaked preferences ensure that there is a consistent winner at the median by Median Voter Theorem.
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> **Grading notes:** Up to three points for correctly identifying the winners of each of the votes and the Condorcet winner; one point for realising that single-peaked preferences imply that the MVT holds.

Now suppose that the union holds a vote over both X and B. It does this in two stages. First, it holds a vote over the level of B. Then it holds votes about the choice of X as described in part 3, taking B as fixed by the first vote.

4. (2 points) What value of X will be chosen in the second stage? Denote this value $\bar{X}$. Note that this will not depend on the level of B.

> **Solution:** $\bar{X} = 0.4$ as argued above.
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> **Grading notes:** Full credit for an answer that's incorrect but consistent with the answer to part 3.

Now consider the first-stage vote. Suppose that each worker knows that whatever value of B is chosen, the second-stage vote will result in a fraction $\bar{X}$ being spent on safety improvements.

5. (6 points) Calculate the ideal value of B for each worker type, based on your answer to part 4. Label these values B_r^*, B_m^*, B_s^* respectively.

> **Solution:** B_r^* maximises $\ln(B)\left(1-(0.1-0.4)^2\right) - \ln(y - \frac{B}{100}) \iff \frac{1}{B}\left(1-(0.1-0.4)^2\right) - \frac{1}{100*(40-\frac{B}{100})} = 0 \iff B_r^* = 1905.76$. Similar calculations for m and s give $B_m^* = 2000, B_s^* = 1826.09$.
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> **Grading notes:** Full credit for an answer that is correct except for being based on plugging in an incorrect answer to part 4. 3 points for setting up the FOCs correctly but making mathematical mistakes in solving for B^*.

6. (4 points) Suppose the union holds majority votes between B_r^* and B_m^*, B_r^* and B_s^*, and B_m^* and B_s^*. Which outcome will be the consistent winner?

> **Solution:** $B_r^* = 1905.70$ is the Condorcet winner (can apply median voter theorem or look for the winner of each pairwise vote).
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> **Grading notes:** 1 point for answer that uses right process (median voter theorem or looking at winner of pairwise votes) but gets the winner wrong.

7. (3 points) You should find that the group that gets its ideal choice of X is different from the group that gets its ideal choice of B. Why is this?

> **Solution:** Machinists are the median voter in the second stage, but that means they get their ideal split of public goods, so they get the most value from each unit of public goods spending. This means they must be on the extreme in the vote over B as they want more public goods spending than either of the other groups. Thus they aren't the median voter over B and so don't get their ideal choice of B; instead the group that is closest to them, the researchers, get their ideal.
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> **Grading notes:** 2 point for observing that the median voter over B is different from the median voter over X, 1 point for relating this to the fact that machinists get their ideal X which means it gets the most value from government spending and thus is not the median voter over B.

Now, we'll compare the democratic outcome to the outcome if the union knew the voters' preferences and was able to directly choose the utilitarian socially optimal outcome for its workers.

8. (2 points) Write down the workers' social welfare function, which is the sum of each worker's utilities.

Solution:

$$\begin{aligned}
SWF(B,X) =& 20\left[\ln(B)\left(1-(X-0.1)^2\right)+\ln(y-\frac{B}{100})\right] \\
&+45\left[\ln(B)\left(1-(X-0.4)^2\right)+\ln(y-\frac{B}{100})\right] \\
&+35\left[\ln(B)\left(1-(X-0.8)^2\right)+\ln(y-\frac{B}{100})\right] \\
&=\ln(B)\left(100-20(X-0.1)^2-45(X-0.4)^2-35(X-0.8)^2\right)+100*\ln(y-\frac{B}{100})
\end{aligned}$$

Grading notes: Make sure to give the point for any equivalent rearrangements of this SWF.

9. (6 points) The utilitarian union's problem is

$$\max_{B,X} SWF \ s.t. \ B \geq 0, d=\frac{B}{100}, 0 \leq X \leq 1$$

i.e. to choose B and X simultaneously to maximise the social welfare function from part 8, subject to the constraints that $B \geq 0$ and $0 \leq X \leq 1$. (Note that once the union chooses B, each worker's dues are fixed at $d=\frac{B}{100}$ because the budget must be balanced). Calculate the values of B and X that maximise this social welfare function. (Hint: first calculate the optimal value of X)

Solution: Differentiating SWF with respect to X gives

$$-40(X-0.1)-90(X-0.4)-70(X-0.8)=0 \iff X^*=0.48$$

Substituting this into the SWF gives us

$$SWF(B,X^*)=\ln(B)\left(100-20(0.48-0.1)^2-45(0.48-0.4)^2-35(0.48-0.8)^2\right)+100*\ln(40-\frac{B}{100})$$

which implies $B^*=1930.04$.

Grading notes: 2 points for the right process (calculate X, substitute back into SWF and calculate optimal B) even if there are arithmetic mistakes. 3 points for having the right FOCs. If X^* is wrong but B^* is right given X^*, don't remove additional points.

10. (3 points) Intuitively, why does the utilitarian social optimum (which you calculated in part 9) differ from the democratic outcome (which you calculated in parts 4–6)?

Solution: The utilitarian social optimum is able to take into account the strength of preferences, as well as preferences of people away from the median. There is a large group of people who would like substantially higher X (salespeople), but because the group is smaller than the median, their preferences are not represented democratically whereas they are represented by the utilitarian government.

Grading notes: 1 point for making a point about strength / intensity of preferences, 1 for observing that in this setting X is higher because salespeople is large and has a strong preference for higher X, 1 for making some argument about what this implies for B.

QUESTION 2: [30 points]

Suppose that the city of Cambridge is planning to remodel and improve all 4 middle schools in the city. Each middle school has 400 students and 50 teachers. Teachers earn on average $81,000 per year and work on average 50 hours per week for 45 weeks a year (assume there are no distortions and this reflects their per-hour valuation of time outside of work).

The construction will require $10 million in construction materials per school per year and 1 million hours (total for all construction workers) of construction labor per school per year. Construction workers earn an equilibrium wage of $20 per hour.

Remodeling each school will take 3 years. Each year, they will start to remodel one new school. Each school will require $0.5 million in maintenance costs per year, starting in the year that it is finished. While each school is being remodeled, the students who would otherwise attend that school are sent to other schools (including outside of Cambridge). This increases transport times for the students by 1 hour per day, and takes away from the time they can spend in extra-curricular activities or doing homework. Assume that each hour outside of school is worth $20 to a student (due to enjoyment, immediate positive effects, and the long-run discounted value of an increase in the likelihood in being admitted to a top college from doing extra-curriculars or more careful homework). Teachers also have to travel to these other schools and travel for an extra hour a day. (You can assume that the construction is isolated on each school campus and doesn't affect anyone else's commute times). A school year has 180 days.

When finished, each school will have a new library and modern playground, and the school will be a beautiful place to be. This is projected to improve student attendance rates and test scores, and these changes are expected to increase student lifetime earnings by $40,000 each year, starting 15 years after the start of the project. Assume that they remain in Cambridge for the rest of their lives. Finally, higher-income families are expected to move to Cambridge and send their children to the public schools. The city's total income tax revenue is expected to increase by $40M per year, starting in the year when all of the schools are finished. Conditional on the maintenance costs, all of these benefits are expected to go on forever.

Assume that the private-market alternative to funding this project would be a financial investment that returned 8% per year. Assume the Cambridge income tax is a flat 5% tax on gross earnings, and the federal income tax is a flat 25% tax on gross earnings (e.g. a worker making $100 pays $5 to Cambridge and $25 to the federal government).

1. (8 points) **Economic costs:** Calculate each of the economic costs associated with the project. Then compute the total cost of the project. *Throughout, feel free to round to the nearest million dollars.*

Solution: Note: in year 0 there is 1 school under construction, in year 1 there are 2 schools, in year 2 there are 3 schools, in year 3 there are 3 schools, in year 4 there are 2 schools, and in year 5 there is 1 school under construction. And, throughout, $r = 0.08$.

- Construction materials: $10M per year per school under construction

$$\sum_{t=0}^{t=2} \frac{\$10M}{(1+r)^t} + \sum_{t=1}^{t=3} \frac{\$10M}{(1+r)^t} + \sum_{t=2}^{t=4} \frac{\$10M}{(1+r)^t} + \sum_{t=3}^{t=5} \frac{\$10M}{(1+r)^t}$$

$$= \frac{\$10M}{(1+r)^0} + 2 * \frac{\$10M}{(1+r)^1} + 3 * \frac{\$10M}{(1+r)^2} + 3 * \frac{\$10M}{(1+r)^3} + 2 * \frac{\$10M}{(1+r)^4} + \frac{\$10M}{(1+r)^5}$$

$$= \$100M$$

- Construction labor costs: $20M (equilibrium wage*number of workers) per year per school under construction. So, plug in $20M wherever there is $10M above: $= \$200M$

- Maintenance costs: $0.5M per year per school once it is finished. One school is done in year 3, two in year 4, etc. until all are done in year 6 and require maintenance forever

$$\frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \sum_{t=6}^{\infty} 4 * \frac{\$0.5M}{(1+r)^t}$$

$$= \frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \frac{1}{(1+r)^6} \sum_{t=0}^{\infty} 4 * \frac{\$0.5M}{(1+r)^t}$$

$$= \frac{\$0.5M}{(1+r)^3} + 2 * \frac{\$0.5M}{(1+r)^4} + 3 * \frac{\$0.5M}{(1+r)^5} + \frac{1}{(1+r)^5} * 4 * \frac{\$0.5M}{r}$$

$$= \$19M$$

- Teacher travel costs: Teachers earn 81000/(50*45) = $36 per hour, which we will use as their valuation of time spent (working and not working). Per year, they lose 180 hours when their school is under construction. With 50 teachers per school, that's $324,000 per school per year. Altogether:

$$= \frac{324,000}{(1+r)^0} + 2 * \frac{324,000}{(1+r)^1} + 3 * \frac{324,000}{(1+r)^2} + 3 * \frac{324,000}{(1+r)^3} + 2 * \frac{324,000}{(1+r)^4} + \frac{324,000}{(1+r)^5}$$

$$= \$3M$$

- Student travel costs: 400 students per school, 180 hours per student per year, $20 per student per hour is $1,440,000 per school per year. Altogether:

$$= \frac{1,440,000}{(1+r)^0} + 2 * \frac{1,440,000}{(1+r)^1} + 3 * \frac{1,440,000}{(1+r)^2} + 3 * \frac{1,440,000}{(1+r)^3} + 2 * \frac{1,440,000}{(1+r)^4} + \frac{1,440,000}{(1+r)^5}$$

$$= \$14M$$

Summing it up, the total economic cost of the project is:

$$\$100M + \$200M + \$19M + \$3M + \$14M = \$336M$$

Grading notes: 1 point for construction materials, 1 point for labor costs, 1 point for maintenance costs, 1 point for teacher and travel costs, 1 point for student travel costs, 1 point for summing it up. (2 points of extra credit if they mention anything about why we might want to use a different valuation of teacher time).

2. **Economic benefits:**

(a) (8 points) Calculate each of the economic benefits associated with the project for *the city of Cambridge*. Then compute the total benefit of the project to the city.

Solution:

We want to include the benefit of the net increase in student incomes, not just their tax revenue, to be consistent with including student and teacher time costs (though you could also argue that those have fiscal repercussions on the government budget constraint). Then, the benefits are:

- Increase in student lifetime earnings net of federal taxes (we don't include Cambridge taxes since it is just a wealth transfer in the aggregate benefits): $\$40,000 * (1 - 0.25) = \$30,000$ per kid per year, and 1600 kids per year:

$$\sum_{t=15}^{\infty} \frac{30000 * 1600}{(1+r)^t}$$
$$= \frac{1}{(1+r)^{14}} \sum_{t=1}^{\infty} \frac{30000 * 1600}{(1+r)^t}$$
$$= \frac{1}{(1+r)^{14}} * \frac{30000 * 1600}{r}$$
$$\approx \$204M$$

- Tax revenue from higher-income families:

$$\sum_{t=6}^{\infty} \frac{\$40M}{(1+r)^t}$$
$$= \frac{1}{(1+r)^5} \sum_{t=1}^{\infty} \frac{\$40M}{(1+r)^t}$$
$$= \frac{1}{(1+r)^5} * \frac{\$40M}{r}$$
$$= \$340M$$

Summing up this alternative, the total economic benefit of the project is:

$$\$204M + \$340M = \$544M$$

Some answers might argue that economic benefits only consists of tax revenue generated for Cambridge. See grading notes. The solution with this approach:

- Tax revenue from student lifetime earnings: $2,000 per kid per year, and 1600 kids per year:

$$\sum_{t=15}^{\infty} \frac{2000 * 1600}{(1+r)^t}$$

$$= \frac{1}{(1+r)^{14}} \sum_{t=1}^{\infty} \frac{2000 * 1600}{(1+r)^t}$$

$$= \frac{1}{(1+r)^{14}} * \frac{2000 * 1600}{r}$$

$$\approx \$14M$$

- Tax revenue from higher-income families:

$$\sum_{t=6}^{\infty} \frac{\$40M}{(1+r)^t}$$

$$= \frac{1}{(1+r)^5} \sum_{t=1}^{\infty} \frac{\$40M}{(1+r)^t}$$

$$= \frac{1}{(1+r)^5} * \frac{\$40M}{r}$$

$$= \$340M$$

Summing it up, the total economic benefit of the project is:

$$\$14M + \$340M = \$354M$$

Grading notes: 2 point for student lifetime earnings, 2 point for tax revenues from higher-income families, 1 point for summing it all up consistently. +1 EC point for answers that use tax revenue from student lifetime earnings.

Now assume you work for the federal Department of Education, which behaves to maximize benefit for *the nation as a whole*. You are deciding whether to provide a grant to the city of Cambridge to remodel their middle schools.

(b) (6 points) Calculate the total benefit for *the nation as a whole* associated with the project.

Solution: Since the federal government is looking to maximize the nation's welfare, we only want to consider the effect on student lifetime earnings and not the increase in the tax base from higher-income families moving into Cambridge, since those families are leaving some other city's tax base. The increase in student lifetime earnings is recalculated as we did previously, except using the full $40,000, yielding a benefit of $272M. Notice we do not subtract this increase in lifetime earnings by neither federal nor Cambridge taxes, if we think of the "the nation as a whole" as including the students, city, and federal government, since the taxes are just wealth transfers but do not affect the nation's overall welfare.

Alternatively, if the federal government cared about tax revenues from student lifetime earnings, it would be $\$40,000 * (0.25) = \$10,000$ per kid per year, and 1600 kids per year. Using the formula used to calculate tax revenues in the previous part, the federal government would receive $68M in increased tax revenue as an economic benefit.

Grading notes: 3 points for excluding higher tax base, 2 points for the right conclusion based on their assumptions. Give credit for reasonable answers that outlined how they defined as "the nation as a whole".

(c) (3 points) Would the city of Cambridge want to embark on this project? Should the federal government give them a grant to do so? Why or why not?

Solution: Regardless of whether the city is using total societal benefits and costs, or just government budget benefits and costs, the city of Cambridge would like to do the project because their benefit ($544M or $354M, depending on your assumptions) is greater than the total cost ($336M or $322M, depending on your assumptions). (Note that technically, if you are not counting the increase in student incomes as a benefit, you should also not count the time costs, but we tried to give credit for any reasonable assumptions made).

If the federal government is basing their benefit calculation on the increase in student incomes (assuming this is an increase in productivity, and not coming at the expense of others outside Cambridge), they should offer the grant because the benefits add to $272M compared to the cost of $17M from increased commute times by teachers and students (note that the dollars spent in construction/maintenance are part of "the nation as a whole"). Alternatively, if the federal government was purely interested in tax revenues in making its decision, it might not want to. It receives $68M in increased tax revenue, compared to $336M in costs.

Grading notes: 1 point for each conclusion. If they made a mistake with the numbers and but draw the correct conclusion from their numbers, full credit.

3. (3 points) Now, imagine an ordinance passed and the city must pay all contractors at least $30 per hour. How does this change the cost-benefit analysis, and why?

Solution: This doesn't affect the cost-benefit analysis, because it only affects the accounting cost of the project. The cost-benefit analysis uses the equilibrium wage regardless of the wage paid because it is concerned with opportunity cost, and the equilibrium wage is still $20 per hour. The additional $10 is a transfer from the government to the construction workers.

Grading notes: 1 points for not changing the cost-benefit analysis, 1 points for mentioning the equilibrium wage being what matters.

4. (3 points) Imagine that the same exact project (with the same exact costs and benefits) was proposed in 4 middle schools in the Boston Public School system. Average household incomes of public school students in Boston are much lower than in Cambridge. Why might the federal DOE decide to give the grant to Boston Public Schools instead of Cambridge Public Schools?

Solution: BPS serves more students who are below the poverty line and/or receive food assistance; on average BPS students come from families that earn half the annual income of the average CPS student. The grant administrator may weight the benefits more highly for a district that serves a less advantaged population.

Grading notes: 3 points for any explanation related to distributional effects. 1 point for only discussing less uncertainty over the benefits in BPS; that's not really applicable here but is one reason you might prefer one project proposal to another.

5. (4 points) The DOE decided to provide a fixed \$335M grant to Boston Public Schools on the condition that they use it to remodel their four oldest middle schools.

   (a) (2 points) What type of grant is the DOE providing to BPS?

   Solution: This is a conditional block grant: a fixed amount of money earmarked for a particular purpose.

   Grading notes: 2 points for conditional block grant. 1 point if they only mention a block grant.

   (b) (2 points) Why do you think the DOE is providing this grant instead of some other type of transfer?

   Solution: An unconditional block grant would not necessarily lead to increased spending in education, and certainly may not lead to the financing of this whole project. A matching grant changes the substitution patterns between paying for this project and other educational spending – and the DOE doesn't want to incentivize over-spending on this project relative to other educational investments.

   Grading notes: 1 point for why this is superior to a matching grant, 1 point for why this is superior to an unconditional block grant.