Problem 1. Consider the Cournot competition game from the last problem set (PS8, Problem 2). Now suppose this game is infinitely repeated with common discount factor $\delta < 1$.

(a) What is the set of feasible and strictly individually rational payoffs in this game? In particular, what is the greatest symmetric feasible payoff vector?

(b) Show that, using trigger strategies which involve switching to the static Nash equilibrium $\left(q_1 = \frac{1}{3}, q_2 = \frac{1}{3}\right)$, the players can attain their greatest symmetric feasible payoffs in a SPE whenever $\delta \geq \frac{9}{17}$.

(c) We will now show that the set of feasible and individually rational payoff vectors for the Cournot game includes some where the players receive *lower* payoffs than they would if they repeatedly played the static Nash equilibrium. Use the one-shot deviation principle to show that the symmetric strategy profile described below is a SPE if $\delta \geq \frac{9}{40}$, and show that the per-period payoffs players receive from it corresponds to a payoff vector in which each player receives less than the static Nash equilibrium profit.

$$q_i(h^t) = \begin{cases} 3/8 & \text{if } t = 0 \\ 1/3 & \text{if } t > 0 \text{ and both players followed strategy } q \text{ in the previous period} \\ 3/8 & \text{otherwise} \end{cases}$$

(Note that the "punishment" of $q_1 = q_2 = \frac{3}{8}$ lasts for only one period after a deviation.)

(d) Construct a SPE using the strategies in part (c) as punishments where the players attain their greatest symmetric feasible payoffs in a SPE for some $\delta < \frac{9}{17}$.

Problem 2. In Lectures 19-20 we saw that, in the repeated prisoners' dilemma with anonymous random matching, for any fixed N, there exists $\bar{\delta}$ such that, if $\delta > \bar{\delta}$, *Always Cooperate* is a Nash equilibrium outcome. Prove that, for any fixed δ, there exists $\bar{N}$ such that, if $N > \bar{N}$, the unique Nash equilibrium outcome is *Always Defect*. So, with anonymous random matching, is cooperation possible in a large group of patient players, or isn't it?