Problem 1. Consider the model of cooperation on networks: each period, each player i chooses an effort level $x_i \geq 0$ and receives payoff $u_i(x) = \sum_{j \neq i} f(x_j) - x_i$, and each player observes only her neighbors actions. Assume f is the square root function: $f(x_i) = \sqrt{x_i}$.

(a) Suppose the network is an n-player clique. Prove that the maximum equilibrium cooperation level for each player is the same number $w > 0$, given by

$$w = \delta(n-1)\sqrt{w}.$$

(b) Suppose the network is a $n+1$-player star. Prove that the maximum equilibrium cooperation level of the center player and the maximum cooperation level of each periphery player are given by $y, z > 0$, respectively, where y and z solve the system of equations

$$\begin{aligned} y &= \delta n \sqrt{z} \\ z &= \delta \sqrt{y} + \delta^2 (n-1) \sqrt{z}. \end{aligned}$$

(c) Let $n = 5$. Numerically, find one discount factor δ for which $w > y$, and find another discount factor δ' for which $w < y$. Which is larger, δ or δ'? Explain intuitively why one discount factor leads to more cooperation in the clique and the other leads to more cooperation in the star.

Problem 2. Alice and Bob are trying to meet for lunch. They can each go to the Cafe or the Diner. Alice's office is near the Cafe, so she knows the exact length of time w it would take to wait in line at the cafe. Bob's office is far from the Cafe, so all he knows is that w is distributed $U[0,2]$. All else equal, Alice would be equally happy eating at the Cafe and the Diner, but Bob prefers eating at the Cafe by an amount b that varies from day to day: assume that Bob knows the exactly value of b, while Alice knows only that b is distributed $U[0,3]$, independently of w. In addition, Alice and Bob get a benefit of 1 from having lunch together. Summarizing, with Alice as player 1 and Bob as player 2 the payoff matrix is

	C	D
C	$1-w, 1-w+b$	$-w, 0$
D	$0, -w+b$	$1, 1$

(a) Formally model this situation as an incomplete information game.

(b) Find a BNE, and prove that it is unique. How often do Alice and Bob have lunch together?

Problem 3. Consider a seller who must sell a single good. There are two potential buyers, each with a valuation for the good that is drawn independently and uniformly from the interval $[0, 1]$. The seller will offer the good using a second-price sealed-bid auction, but he can set a "reserve price" of $r \geq 0$ that modifies the rules of the auction as follows: If both bids are below r then neither bidder obtains the good and it is destroyed. If both bids are at or above r then the regular auction rules prevail. If only one bid is at or above r then that bidder obtains the good and pays r to the seller.

(a) Compute the seller's expected revenue as a function of r.

(b) What is the optimal value of r for the seller?

(c) Intuitively, why does the seller benefit from setting a non-zero reserve price?