Midterm 1

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Please answer all four questions. There are 80 points in total. You have 80mins to complete the exam.

1 Short Answer (30 pts)

Answer the following questions in no more than one paragraph each. True and False answers need to have explanations to receive full credit.

a) (6 pts) In a utility maximization problem with two goods X_1, X_2, suppose we have found a maximum that is a corner solution where $X_1 = 0$. What do we know about the relationship between $MRS = MU_1/MU_2$ relative to the price ratio p_1/p_2?

Solution: At the corner solution $X_1 = 0$, then either you have a tangency at zero or you would prefer to have an even lower X_1 without the non-negativity constraint. Therefore $MU_1/MU_2 \leq p_1/p_2$.

b) (6 pts) True or False: You can consume two goods X_1, X_2 and have a fixed budget I. Prices for the two goods are p_1, p_2. If p_2 increases, it is always the case that consumption of X_1 will increase since it is now relatively cheaper.

Solution: Not always true. The substitution effect would lead you to want to consume more X_1, but if X_1 is a normal good, there is an offsetting income effect from higher prices that would lead you to want to consume less X_1. Which one dominates depend on the utility function. The utility function is not specified here, so X_1 or X_2 could be inferior goods, or there may not be any substitution effects (e.g. in the case of perfect complements). One specific example of why the statement may fail is needed for full credit.

c) (6 pts) Define "Pareto-optimal allocation" and "feasible allocation."

Solution: A feasible allocation satisfies the technology and endowment constraints. An allocation is Pareto-optimal iff it is feasible and there is no other feasible allocation that is weakly preferred by all and strictly preferred by some.

d) (6 pts) There is a feasible allocation $\mathbf{x}$ and some other feasible allocation $\mathbf{x}'$ such that for some i, $\mathbf{x}'_i \succ_i \mathbf{x}_i$ but for some other j, $\mathbf{x}_j \succ_j \mathbf{x}'_j$. Does that tell us anything about whether $\mathbf{x}$ is a Pareto-optimal allocation or not?

Solution: This does not tell us about whether $\mathbf{x}$ is Pareto-optimal or not. If $\mathbf{x}'_j \succeq_j \mathbf{x}_j$ for all j and $\mathbf{x}'_j \succ_j \mathbf{x}_j$ for some j, then we would know that

x is not Pareto-optimal. However, there is not enough information in the given statement to say whether **x** is Pareto-optimal or not.

e) (6 pts) Describe the experiment Jensen and Miller performed with poor households in China. Summarize the evidence for and against Giffen behavior.

Solution: They experimentally subsidize the price of several staple food items, and find Giffen good behavior with rice and weakly with wheat purchases. Lowering the price for these staple goods reduced household demand for them. These results are stronger in Hunan because households have fewer substitution possibilities and are much poorer, as predicted by theory.

2 Production Functions (10 pts)

In this section, we will ask about returns to scale for production functions.

a) (4 pts) How is homogeneity of degree one related to returns to scale?

Solution: Homogeneity is the same as constant returns to scale.

Now using your answer to part (a) or any other techniques, show whether the following production functions are decreasing, constant, or increasing returns to scale.

b) (3 pts) $F(X, Y) = X^{1/3}Y^{2/3}$

Solution: This is constant returns to scale

c) (3 pts) $F(X, Y) = 2X^{1/2} + 2Y^{1/2}$

Solution: This is decreasing returns to scale

3 Expected Utility, Risk, and Insurance (20 pts)

An agent has a CRRA utility utility function

$$u(c) = \frac{c^{1-\theta}}{1-\theta}$$

An agent has poor health, and there are two possible states s they could be in (1) $s = H$ for healthy and (2) $s = S$ for sick, each with probability $1/2$. Suppose that consumption when healthy is $c = 2$ and consumption when sick is $c = 1$. Recall from lecture that expected utility would be

$$E[u(c)] = \sum_{s \in \{H,S\}} \pi(s)u(c; s)$$

Here $\pi(s) = 1/2$ for $s = H, S$. The agent can pay for insurance which will lead consumption to be $2 - p$ in both states, eliminating all risk. We can think of p as the cost of insurance.

a) (10 pts) What is the expected utility of the agent without insurance and with insurance?

Solution: Expected utility without insurance is

$$E[u(c)] = \frac{1}{2}\left[\frac{1}{1-\theta} + \frac{2^{1-\theta}}{1-\theta}\right]$$

Expected utility with insurance is

$$E[u(c)] = \frac{(2-p)^{1-\theta}}{1-\theta}$$

b) (10 pts) Solve for the highest amount the agent would be willing to pay for insurance p^*.

Solution: The highest the agent would be willing to pay is to set the two expected utilities equal.

$$\frac{1}{2}\left[\frac{1}{1-\theta} + \frac{2^{1-\theta}}{1-\theta}\right] = \frac{(2-p)^{1-\theta}}{1-\theta}$$

With some rearrangement and simplication, the solution is

$$p^* = 2 - (\frac{1}{2} + 2^{-\theta})^{\frac{1}{1-\theta}}$$

4 Dynamic Consumption and Saving (20 pts)

Suppose that a village plants crops in $t = 0$ and the crops will produce output in $t = 1, 2, 3$. Output is decreasing over time where $R_1 = 4$, $R_2 = 1$, $R_3 = 0$. The village only consumes in periods $t = 1, 2, 3$, so the world ends in $t = 3$.

The village can store crop output across time periods **without any depreciation or uncertainty**. There is **no discounting of future periods**. The utility function from consumption is $u(C_t) = \sqrt{C_t}$. If the village stores S_t crops in period $t-1$, then it can consume up to S_t crops plus any output R_t in period t, but minus any storage for the next period S_{t+1}.

a) (6 pts) Set up the village's maximization problem to find the optimal consumption and storage plan. Be clear about the objective function and constraints.

Solution:

$$\begin{aligned} \max_{C_t, S_t} \quad & \sum_{t=1}^{3} \sqrt{C_t} \\ \text{s.t.} \quad & C_1 = R_1 - S_2 \\ & C_2 = R_2 + S_2 - S_3 \\ & C_3 = R_3 + S_3 \end{aligned}$$

b) (4 pts) Notice that the utility function is strictly concave and there is no discounting over time. Do you have any conjectures about what the optimal consumption plan looks like?

Solution: Since utility is strictly concave, we do not want extreme amounts of consumption in each period. Since there is no discounting, we value consumption in each time period equally, and therefore utility is maximized by setting consumption equal across time.

c) (6 pts) Write the FOCs for the optimal consumption and storage amounts C_t for $t = 1, 2, 3$ and S_t for $t = 2, 3$

Solution: We can rewrite the maximization problem by subbing in R_1, R_2, plugging C_t into the objective so we have a function in S_2, S_3

$$\max \sqrt{4 - S_2} + \sqrt{1 + S_2 - S_3} + \sqrt{S_3}$$

leading to the FOCs

$$-\frac{1}{2}(4 - S_2)^{-1/2} + \frac{1}{2}(1 + S_2 - S_3)^{-1/2} = 0$$
$$-\frac{1}{2}(1 + S_2 - S_3)^{-1/2} + \frac{1}{2}(S_3)^{-1/2} = 0$$

d) (4 pts) Solve for optimal consumption and storage C_t, S_t

Solution: Rearranging the FOCs, we find that $S_2 = 7/3$, and $S_3 = 5/3$. We can plug these back into the consumption constraints to get $C_1 = C_2 = C_3 = 5/3$. Clearly, we can see that it is optimal to smooth consumption over time since the utility function is concave and there is no discounting.