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RICARDO CABALLERO: So after doing IS-LM in the first part of the course and where we took prices completely sticky and output was fully determined by aggregate demand, we said, well, that dominates in the very, very short run. But over time, at some point the supply side starts showing up. There are constraints. The labor market gets very tight, and so on.

And so we added a block that started from wage determination. And then we look at the impact of wages on prices. And then we related inflation rate. You start to relate the inflation rate to economic activity. So output above or below the potential output, or the natural level of output and things of that kind.

So remember, the starting point was a wage demand equation, so what workers demand for a wage. This period depends on what is the price level they expect for the period. Because they set the wage today and they have to live through the year or two, whatever is the contracting period, with that nominal wage.

So naturally, if they expect higher price level in the future, they're going to demand a higher nominal wage today. And then we said that's a function that is also going to be decreasing in the level of unemployment because obviously that weakens bargaining power for workers or makes actually becoming unemployed or not having a job more costly because it's very difficult to exit out of unemployment.

And then we made normalization, this function, also an increasing function on this variable z, which captures a bunch of labor market institutions, including wage labor bargaining power. So more bargaining power means that for any given level of unemployment, workers would tend to demand a higher wage.

So that's what the z variable was all about. Then we wanted to go from wages to prices, because the ultimate goal was to bring inflation into the picture. And for that, we had to produce-- we introduce a production function because in particular, we made output a function of employment. And that, very naturally, will connect wage pressure to price pressure because you need labor to produce output. So that the labor market is very tight, that means also there's going to be more expensive to produce output.

And we simplify this production function a lot. We made it output equal to employment. And that meant also that one unit of labor, in order to produce one extra unit of output, you need one extra unit of labor, which means you need to pay a wage, one unit of the wage.

And so then we said, suppose that the price setting from the side of the firms simply takes this cost, which is the wage, and adds a markup to it to pay for a bunch of other things that we haven't introduced in this model. So the price charged by firms is equal to the wage times 1 plus some positive numbers, 8.2 or something like that, so 1.2.

And we can rewrite this price setting equation as a wage, a real wage the firms are willing to offer. And it's just equal to that. So when the markup goes up, that means the real wage firms are willing to offer is lower than otherwise.

So that took us to the concept of the natural rate of unemployment. And what I said, no, there's nothing natural about the natural rate of unemployment. It's simply a definition that says that's unemployment that results when the price, expected price, is equal to the actual price. That's all that is.

And when we have that condition, then we can think of the real wage demanded by workers because I can replace the expected price for actual price and divide both sides by price. So the actual wage demanded by workers is equal to a function of the natural rate of unemployment. And I stick the end there precisely because I replace expected price or P, for no other reason.

But now we have two equations for the real wage, the real wage that firms are willing to pay and the real wage and workers need to demand. And we can make them both equal. And that determines the natural rate of unemployment.

OK, so remember this from the point of view of the firm. This is equal to 1 over 1 plus the markup. The only endogenous variable in the markup is a constant z. This is also a parameter is exogenous. And so from here, we can solve the natural rate of unemployment, 1 over 1 plus m. And we can solve the natural rate of unemployment. And if you do the algebra right, you're going to get to a point like that. That pins down the natura rate of unemployment.

Again, there is nothing natural about the natural rate of unemployment. It depends on a bunch of parameters, which, for example, it clearly depends on the markup. It depends on things that we took as constant here as given here, all the things that were in z. Those are part of that.

And so then we look at things that change. And that's just done with equations. We look at things that change the natural rate of unemployment. That's one example. If bargaining power by workers goes up, they're going to demand a higher wage at the initial natural rate of unemployment. Well, that obviously, that higher wage is inconsistent with what firms are willing to pay.

The only way equilibrium can be restored in this model, that's the medium run equilibrium, is for the natural rate of unemployment to rise to UN prime. There you have it. Nothing natural. Natural rate of unemployment is not constant. It depends on institutional parameters, such as bargaining power.

Another example is mark ups. It depends on mark ups, as well, the degree of competition, if you will, in the goods market. If we are in some equilibrium like this one and now suddenly firms, for whatever reason, choose or need to charge a higher markup, that means that at this level of unemployment, the wage that workers would demand is higher than the wage that firms are willing to pay, the real wage. And the only thing that can clear the market in this case here, in the medium run, is for the natural rate of unemployment to rise.

So here, we go two experiments where we move some parameter-- in one, the bargaining power of workers and the other one the markup of the firms. And both increase the natural rate of unemployment. Good. The next step was to look at things that happen outside the natural rate of unemployment, in particular, what happens to prices there.

So what we did is we took the-- we went back to the model with the expected price here. That means that unemployment that comes out from this equilibrium is not going to be necessarily the natural rate of unemployment. That will be the case only if P happens to be equal to P. Then we simplify this function, F, here for something linear, like this. Very simple, but again, decreasing in unemployment, increasing in this institutional parameter, z.

We replace this wage here from this expression here and rearrange. So we got this here. And the next step was just to go from here to the rate of inflation. And we did it through several steps and approximations. And we ended up with what is known as the Phillips curve.

So this says that inflation is increasing and expected inflation on this institutional parameters. As the markups go up, that will tend to increase inflation. If bargaining power by workers go up, then that is the same. But most importantly, it's negatively related to unemployment. And that's the reason that today, nowadays, you know, there's lots of discussion about the tightness in the labor market and whether that's really necessary.

Do we need to cause a recession, a situation where unemployment goes up a lot, in order to really finally bring down inflation? Yeah, there was a question.

AUDIENCE: [INAUDIBLE]

RICARDO CABALLERO: Yeah. Remember that I made up this function. We said this function is decreasing unemployment. I just replace that function for that. So it's the sensitivity of wage demand by workers to their employment rate. Alpha is very high, it means that wage demand is very sensitive, very responsive to unemployment.

AUDIENCE: What would be the situation for an expected price? Like, could you connect that back to like, I don't know, some sort of a commodity or something?

RICARDO CABALLERO: So what is the intuition for this?

AUDIENCE: Yeah, just like a price feels tactile. But an expected price, I don't--

RICARDO CABALLERO: Well, I mean, imagine that workers and firms bargain for a wage that will live through the year. You're bargaining for the wage, nominal wage today. You don't set a real wage. You set the nominal wage, say $100, whatever.

Well, the wage demand will depend a lot on what they expect inflation to be during this period. If I expect inflation to be 10%, you're very likely to demand a higher nominal wage because you have to live, on average, with higher prices. So that's the role of that, is the price.

I mean, I would prefer-- and there are countries where that's done-- to set my wage in real terms so I don't need to worry about that. But in practice, in economies with low inflation like the US, you don't do that. You get a nominal wage and you have to live for the year or until the next negotiation for your wage contract with that level of wages.

AUDIENCE: [INAUDIBLE] be dependent on time, [INAUDIBLE] with the interest rate or with the inflation rate, whereas I guess the regular price is defined by the wage is dependent on the market?

RICARDO CABALLERO: No, they're both the same. But the only thing is that this price here is not the current. It's what you really expect the price to be during the year. If this is here just because at the moment in which you set the wage, you don't know the price that you're going to face as a worker. But it's the price.

So you don't know the price that you're going to actually face. So the best you can do is calculate, well, I think inflation is going to be 10%. So give me what I would have had in mind with inflation equal to 0 plus 5%, so on average, I'm about right. That's the logic.

But this expected price is meant to be your best proxy you have at the moment in which you're bargaining for your wage for what the actual price will be during the life of that particular wage.

So we end up with that Phillips curve here. Importantly, this is a decreasing function of unemployment. And then we made different assumptions about expectations. If expected inflation, for example, is a constant, that's when we say expected inflation is very well anchored, then you get a Phillips curve that looks like this, in which inflation has a constant here and a decreasing on the rate of unemployment.

And during the '60s, that relationship sort of held fairly well. It was a downward sloping relationship. It got to be steeper and steeper as we moved into higher and higher inflation levels. And then I said, but in the '70s, the whole thing broke loose. There's nothing like a downward sloping curve here. That happened for two reasons. There were some cost push shocks. You can think of lots of shocks to them.

But more interestingly, expected inflation became an anchor. And then we changed the expected inflation model for rather than being a constant, being some weighted average like this. And we said, look, during the '70s essentially that theta was equal to 1. So expected inflation was really-- whatever was inflation last year, people expected that level of inflation to stay in the next year, rather than going back to that whatever was the constant or inflation target or historical constant pi.

And that meant that during that period, really the Phillips curve looked more like a relationship of the change in the inflation rate as a decreasing function of unemployment. So that means that when you increase unemployment here, you reduce the rate at which unemployment, inflation is rising. That's the goal of the situation in a case in which expected inflation is an anchor.

And the last step we had there is we replace-- we noticed, we said, well, what happens if we stick in here the natural rate of unemployment? Then that will give us-- that will happen only when expected price is equal to actual prices. So that means that when inflation is equal to expected inflation, from here we can solve the natural rate of unemployment as a function of these structural parameters.

And once we have that, we could go back to our Phillips curve and rewrite it in this way. So you can think of the Phillips curve in this way. And this is the way we typically write it down, in which it says inflation is decreasing in the unemployment gap.

So if the unemployment is above the natural rate of unemployment, that means inflation will tend to be below expected inflation. If expected inflation happens to be equal to log inflation, that means if unemployment is above the natural rate of unemployment, then inflation will be falling. Any questions? Good. You need to know this, how to derive these things. I mean, not so much-- Yeah, you should know how to derive. But you need to understand this relationship between the unemployment gap and inflation relative to expected inflation. Yep.

AUDIENCE: Could you talk about uncut versus de-uncut inflation.

RICARDO CABALLERO: Expected inflation. It's just a statement about a what is the model we have for expected inflation. So suppose we have the following model for expected inflation-- 1 minus theta-- theta is some number between 0 and 1-- times a constant inflation plus something that is a function of theta times whatever is previous inflation.

Central banks try to set a target for the inflation rate. In the US it's around 2%. And ideally, people will tend to believe-- they may see an inflation that is above, say, 2%. But as long as people expect that to be undone in the next period, then inflation will say they are very well anchored. So that's a case in which very well anchored means theta equal to 0 here. And you always stick in there, in the case of the US, at 2%.

And that's the way the economy is behaving right now. Inflation today is 5%. But if you ask people, what do you expect inflation to be two years from now? People tell you, look around 2%, 2.5% or so. An anchor expectation is when you don't have that anchor, that 2% that the Fed told you is whatever it was the previous inflation, that's what people extrapolate will be inflation for next period. And that's a lot harder when you get into an inflationary episode in that context.

It's very difficult because you have 5%. People are still expecting 5% for next year. So you need to-- it's much harder to bring inflation down. You need to create much more unemployment to bring inflation back to the 2% target. That's what it means to anchor. So anchor means theta very close to 0. And anchor theta very close to 1. That's a formal definition.

We then move to what I think is probably the most important model you'll see in this course, which is the IS-LM-PC, which is just the IS-LM plus the Phillips curve. And that allows us to talk about the short run, which is what we did in the IS-LM and then all the way to the medium run. A medium run understood, that's when you go back to the natural rate of unemployment, natural level of output, and so on.

We got a banking crisis there. But that's-- This you may find useful. Here I was trying to explain the banking crisis. And I said, we have a model for that already. Remember, we had this x spreads in the investment function and said, well, you can think of a negative financial shock, something like a credit spread shock, as an increase in x, and that will shift IS to the left. OK, just saying. Good.

IS-LM-PC model was just going back to the IS-LM model. We're going to simplify things by just assuming that the central bank sets the real interest rate and the real interest rate is that. And to that, we added a Phillips curve. But we didn't like that Phillips curve because we have everything is a function of output here and interest rate. And now we have inflation and the unemployment rate and so on-- yet another variable to carry around.

So we went from the output gap to unemployment gap to an output gap. And we did that just by noticing that the output is equal to the labor force times 1 minus the unemployment rate. Similarly, you can define potential output or the natural output level as employment times 1 minus the natural rate of unemployment.

Subtract these 2 and you get that the output gap is equal to minus L times the unemployment gap. And so we replace this for that expression divided by L and we end up with the Phillips curve written in the form of an increasing function of the output gap. So when the output gap is positive, then inflation will exceed expected inflation.

If expected inflation is an anchor, that is, expected inflation is equal to log inflation, then that means that a positive output gap leads to an increase in inflation, in inflation rate. We look at an example here. This is the type of-- now we're going to have the real interest rate here. Just makes it simpler to think about central monetary policy in terms of the real interest rate. Otherwise, too many things move at once.

So this is what we have done for quiz 1. Here you have some particular equilibrium, IS-LM. With this real interest rate, we got some equilibrium output equal to y. The contribution of this block of the course is that now we need to also check whether this is y is consistent with potential output or not, or with natural level of output. And that for that we need to see whether this level of output is, again, is above or below the natural rate of output.

And for that, we need to look at the Phillips curve. And in this particular case, that's not the case because output is above the natural rate of output. You put now, given that observation, you draw here the Phillips curve. You know that because output is above the natural rate of output, the natural level of output, that means inflation is above expected inflation. If expected inflation happens to be an anchor equal to pi minus one, that means that at this output gap, that there is an inflation that is rising.

Now, inflation rising means the central bank will have to react. And so you have to do something up here. You need to bring output down. And how can you bring output down? So this economy is engaging in an inflationary spiral, actually, given this model of expectation. How do you stop that?

AUDIENCE: Raise interest rates.

RICARDO CABALLERO: If you are the Fed, and you when-- you raise interest rates, no? Because you need to bring them back. So the equilibrium level of output, you increase the real rate up to a point in which the equilibrium level of output is equal to the natural rate of output. And you may have to do more than that.

If inflation was an anchor and you find yourself with 5% inflation, you may have to temporarily actually, to bring inflation back down to 2%, you may have to overshoot, raise interest rates a lot, generate a negative output gap for a while. And then once you reach the level of inflation you like, the 2%, then you can go back to the natural level of output. OK, so that's the reason central banks worry a lot about unanchored expectations because then they know that they find themselves on inflation above their target, it's not going to be enough to bring the output gap to 0. They're going to have to overshoot in the way down in order to reanchor expectations-- well, in order to bring inflation back down to the target of 2%.

But in any event, even if inflation expectation's well anchored, you still have to bring output down because at the very least, you need to close this positive output gap. And that, if you are the Fed in the US or any central bank, you do it by increasing the real interest rate. Now, in practice, central banks really don't control the real interest rate, control the nominal interest rate. So there is a little fight there between inflation and what they do to the nominal interest rate. But let's ignore that complication for now.

Now, suppose that the Fed is in vacation and so somebody someone else decides in the government decides that, no, we cannot have this very high level of inflation. So what else could you do? And you're not the Fed. The Fed is in a vacation. Who else can make policy? The government, the central government, the Treasury, and so on. What is the instrument they have? What do they need to do? The problem they have is output is too high. And that's what is leading to lots of inflation. So what do you think they should do?

AUDIENCE: Cut government spending.

RICARDO CABALLERO: Cut government spending, raise taxes, something of that kind. But you need a fiscal contraction because that will bring the IS down. And so the equilibrium output will be lower. So that's an alternative you have. You should know this.

And here I just did what we just discussed, just in steps. These things happen slowly. The Fed doesn't hike interest rates in one shot and so on. It takes a while before you get to the final equilibrium.

I showed you the deflationary spiral. I said, sometimes things can get very complicated because you may hit the 0 lower bound. The Fed can bring the nominal interest rate to 0. But if inflation is already low, that may not give you the real interest rate you need in order to get output equal to the natural rate of output. Here was one example in which you need a negative real interest rate to get output to be equal to the natural rate of output. But that may not happen because you hit the 0 lower bound.

And so at that point, the problem you have is that-- and that was a tragedy of Japan for so long-- is that not only you cannot bring the interest rate, the nominal interest rate below 0, but you start getting into deflationary. Inflation below expectation, and the expectation goes to number very close to 0 because of an anchor deflationary expectations.

Then you start getting negative expected inflation. And when you get negative expected inflation, even if you are at the 0 lower bound in the nominal interest rate, that means a positive real interest rate. So effectively, you are increasing interest rate at the same time. And that can be a very complicated thing to get out of. Again, that's what happened to Japan for a long time.

What would you do as a government if you fall into a situation like that? And Japan did a lot of that. Well, you can do lots of things. But in particular, the kind of things you know, what would you do? If you are in a situation like this in which the 0 lower bound is binding and inflation is actually falling? Here I had a benign case in which inflation expectation was well anchored. That's not what happened to Japan. After they experience a long period of deflationary forces, then people began to expect more deflation, more deflation, and so on. So what else can you do?

Let me give you a hint. Japan is one of the countries at the highest levels of public debt. How do you accumulate public debt? Yeah, you need to borrow a lot. You have big fiscal deficit. So that's the way you can fight this. You can shift IS to the right by having an expansionary fiscal policy. That's the only tool really have. You lose the power of monetary policy against the 0 lower bound. But you still have fiscal policy. And they did a lot of fiscal policy.

No, not interesting. This is interesting. This is a different kind of shock. Suppose you are at your medium-run equilibrium and then all of a sudden mark ups go up, perhaps, for example, because the price of oil went up a lot and something like that. So then that's a different kind of shock from the previous one, from any fiscal shock or anything like that. That's an aggregate demand.

This is an aggregate supply problem because the first thing I know of a permanent at least change in m is that the natural rate of unemployment has to rise. If the natural rate of unemployment has to rise, that means my Phillips curve will shift now. In that particular case, I know the Phillips curve will shift to the left. How do I know that?

Well, because I know that the natural rate of unemployment went up, which means that the natural rate, natural level of output has to come down. And the natural level of output coming down means simply that the level at which expected inflation and inflation are equal happens at a lower level of output. So the Phillips curve moves to the left.

So suppose you were in this equilibrium. Here I'm doing for the case of an anchor expectations. But the same logic goes for the case of anchor expectations. So suppose you were at some equilibrium like this. What's your medium run equilibrium? But now the price of energy goes up a lot and you expect that to last for a while.

That means the Phillips curve moves up. So that means that if output, with output at this level, now you have a problem because you start getting inflation out of this, because this level of output is too high relative to the new level of the natural-- the new natural level of output. So you have a positive output gap.

Positive output gap means inflation above expected inflation. If you have an anchor expectations, means inflation starts rising up. So that means the Fed now needs to react to that and needs to tighten interest rate in order to go to the new level of, natural level of output. And that's the response to that.

But if the Fed does not react-- and a little bit of this is what happened. We had some supply shocks and so on that were considered to be temporary, well, they weren't as temporary. So there was no reaction. But it turns out that they lasted a lot longer than the Fed expected. And so now they had to catch up.

That was part of the reason we got into a high inflation episode. That was the main reason in Europe. The US is a mixture of aggregate demand, lots of fiscal policy and so on. And supply side in Europe was very much a story of this kind.

Well, a financial panic, you need to offset it with a decline in real interest rates. And a little bit of that has been happening. It's not the Fed that has cut the rates, but the markets have anticipated the Fed will not raise interest rates as much as they expected before we got into this banking mess.

So we had already studied the short run, the medium run. And now we want to look at the long run. And that's what economic growth is about-- economic growth theory and facts and so on. Let me go to-- So one of the things I highlighted is that we tend to see, among countries that are fairly similar along the education and variables like that-- systems, economic systems and political systems and so on-- you tend to find relationships like this, that is countries with a lower per capita income at the beginning of the sample tend to grow faster in the sample. And that captures very much the idea that there is a convergence, there is a force towards convergence of income per capita, if you will.

That's another illustration of that phenomenon. Lots of dispersion here. 70 years later, a lot less dispersion. But we also said that some countries do not match that. And but we focus most of what we did in growth on understanding this process, the process of convergence and how it happened without technological progress and so on. And then we spent a little bit of a lecture, say at most 10 points worth in a quiz, or seven points, talking about anomalies and things like that-- 5 points or something like that.

So the key object here, one of the key objects-- there are a couple-- but one of them was, well, now we need to be a little bit more serious about the production function, we said. Because for the short run, it's OK to take capital as given and just worry about most of the fluctuations in output will come from fluctuations in employment. That's not so over long periods of time. Capital accumulation plays a huge role.

And so we need to be explicit about the fact that capital matters a lot for production. And so we postulated a production function like this, output as increasing function of capital and labor. And now we said for this part of the course we're not going to worry about unemployment. And so on, employment, labor force population, they're are all the same for us here, for this part of the course.

And then they said this production function has some important properties. One, it has constant returns to scale. Things change quite a bit. If you don't have constant returns to scale. So we have constant returns to scale, which means-- you should know this-- that if you scale all output, all inputs, all the factors of production by the same factor, output also rises by the same factor.

OK, so our production function we use a lot was Cobb-Douglas, output equals square root of k, times square root of n. Well, the sum of those exponents is 1. So that's a production function with constant returns to scale. Anything that has the exponents add up to 1, then that's a constant returns to scale technology.

But importantly, it also has decreasing returns with each of its factors of production. That means rather than moving both factors of production up, you move only one. Well, you're going to increase output, but just keep increasing that factor alone, you're going to increase output by less and less and less and less because essentially, it has fewer and fewer of the other factor of production to work with.

And so that's decreasing returns to capital or labor. I mean, if you fix the other factor of production, you move up. It's going to increase at a decreasing rate. So one normalization that we started with was, well, a scaling factor could be population, 1 over population. That's a scaling factor.

And if I do that, I multiply everything by 1 over n. We go into output per person is an increasing function of-- an increasing function, but at a decreasing rate-- of capital per person. We plot that function here. And this is increasing, but it's concave. That shows the decreasing returns of capital.

And we got that function there. So when you move in this, you can increase output per person, per person by simply increasing capital per person. And the more you increase capital per person, output will increase more and more but at a decreasing rate. You can see that moving the distance between A and B is the same as the distance between C and D. However, the increase in output, when you go from A to B, is enormous when compared with the increase in output that you get from the increase in capital over per person from C to D, decreasing returns.

There is another way of increasing output per person, which is with technological progress, with the function, F, is shifting up over time. And we split the two main lectures in growth into part one, which shut down the second channel. And then the second important lecture here, we focus on this channel. So that's what we.

So let's go to when I shut down this channel for now and focus on the case without technological progress first. So we put things together. We said this comes from the previous lecture. We can write output per person as an increasing function of capital per person. Second key equation is-- well, this is a property and it has to be, if you are in a closed economy, no government expenditure or anything. We could add that. But it's not important for the message.

Then investment has to be equal. So investment is going to be very important here because what will make the capital stock grow. But there has to be funding for that. And the funding come from savings. And we simplify things by assuming that the saving function is just proportional to the level of output, which is reasonable when you think about long run. All these things scale up.

When you're thinking about very short term, no, we have some constants and so on floating around. But over the long run, things do scale up. And so we can write investment in equilibrium. Investment has to be equal to savings. Saving is proportional to output.

So we get that investment in this economy is increasing in output. This is a constant somewhere between 0 and 1. And the last key equation here is the capital accumulation equation. The capital accumulation equation says that capital, T plus 1, is equal to capital today, minus the depreciation-- some fraction of the machines break down in every period-- but plus the new investment, plus It.

And we rewrote things and replaced the saving function and so on. And we end up in an expression like this that says capital per person here grows with investment, which is funded by savings, which is an increasing function of output, which in turn is an increasing function of capital per person, minus whatever is the depreciation. And what we did in the start diagram in the other model is, we plot this function and that function. We know that the steady state is when these two things are equal. That pins down the k star over, k over n star of this economy.

But we also know that to the left of that point in capital space, this term is greater than that. And therefore, the capital stock is rising. To the right we get that this term is greater than that. And therefore the capital stock is falling. And that's what we have in this diagram. So that's the steady state of the economy, when the depreciation per worker, which is the required--

The minimum investment you need to keep the capital stock constant is whatever is depreciation per worker. Anything else you do, it will grow the stock of capital. Anything less you do, you're not maintaining enough of your capital stock. It's declining. And that's exactly what happens here. That's the steady state. If your capital below that, then a capital stock will be rising because you have lots of savings and therefore lots of investment relative to what you need in order to maintain the stock, the small stock of capital you have, until you reach steady state.

And this model alone can explain really the pattern we have that the poorest economies tended to grow faster than the richer economies. If you think of poorer economies as economies that are otherwise similar, but that have a low stock of capital to start with, well, those economies are going to be to the left of the steady state, and therefore they're going to tend to grow at whatever is the steady state rate of growth, plus this catching up growth. And so this is a very powerful little model. It can explain a lot of those convergence, the convergence that we saw in the data.

Do you understand this? This is important. Then we did some experiments. What happens if you increase the savings rate? At the time when Solow was writing this model, many people say that what was behind growth was saving. Well, in this model we show that, indeed, it is a saving rate rises. Then at any given level of capital-- suppose that was the oldest save state. If now the saving rate rises, that will increase investment above what you need to maintain that level of the stock of capital.

So the stock of capital is going to start growing. When that happens, output per capita will grow faster than in a steady state because you're going to be going from here to there. But eventually you'll converge to the same old rate of growth. So the point here is that the saving rate can not change, per se, does not change the rate of growth in the long run. But it gives you transitional growth.

And a lot of the Asian miracle of the very fast rates of growth from the '60s and '70s and '80s has to do with this kind of thing-- very sudden increase in saving rates plus other institutional changes and so on. But high increase in the savings rate also led to very fast growth. Again, this little model can explain a lot, as well.

It can explain when you see those growth miracles, often it's associated to-- for some reason, it varies a lot across different scenarios-- the saving rate went up quite a bit. But point is, that gives you very fast growth in the short term, but eventually peters out.

The next thing-- All that we did for a fixed population and said, well, suppose now the population is growing. I said the diagram we had before would be very unpleasant because all these curves would be shifting. So what we need to do is divide not by a constant. We need to divide by whatever is the population at that point in time. And that will give us the same diagram we had with one little twist.

So I went through a little algebra here to arrive to a capital accumulation equation, capital per person equation, an equation for the change in the capital per person, which is very similar to what we had. The only difference is that the required capital, investment required to maintain the stock of capital per person, has an extra term here, gN. And I said that gN-- so let's think about this term.

So delta, think of this as the required investment in order to maintain the stock of capital where it was. If this delta comes from the fact that, well, you have a stock of capital. You lose a fraction of that. Well, you need an investment equal to that fraction that you lost in order to maintain the stock of capital the same. That's clear.

But that's not enough to maintain the capital per person constant if per person is rising, population is rising. Because even if you maintain the stock of capital constant, the denominator is rising at the rate of population growth. So in order to maintain the capital per person constant, you need to deal with the growth of the denominator, as well. And that means you need a little-- you need also investment to match the increase in population so they can keep the capital per person constant.

So that's a modification. So in terms of our diagram, all that happened here, have technological progress, as well. Set it to 0 for now. All that happened relative to the previous diagram is that now I rotated this curve up, upward a little bit. But then you conduct analysis exactly the same way.

The only thing that is different now is that in the previous model, we had that the rate of growth, the steady state rate of growth was equal to 0. And the steady state rate of growth of output per person was also equal to 0. Population was not growing. Output was not growing. The ratio wasn't growing either.

Here it's still the case that in a steady state, output per person is not growing. But that also means since population is growing at the rate of gN or N, I don't know how we call it here, that means output must be also growing at the rate gN. That's what will keep output per person not growing.

So for the output itself, a very important factor in growth is population growth. And if you look at rates of growth in general in the world, certainly in the developed world, they are falling for a variety of reasons. And one of them is because population growth is falling. But per person, that doesn't make a difference. But for the level of the rate of growth, it does.

And then we added technological progress, which we modeled as effective as enhancing labor. So having a better technology means is you had more workers. So for any given level of workers, having a better technology, we modeled it as having more workers. And you can model it exactly that way. You can use exactly the same diagram we had before. But now we divide our scaling factor rather than being 1 over the population, it's going to be 1 over effective population, effective workers, 1 over AN.

And you conduct exactly the same analysis. You do exactly the same approximations I did before. But the difference now is that here rather than have gN, you have AN. Why is that? Well, because if I want to maintain the capital per effective worker constant, then I need to first make up for the depreciation of the stock of capital. Thus, I have to stabilize the numerator.

But then I have to take into account that the denominator is growing for two reasons-- because population is growing and because technology is growing. And in order to maintain the ratio constant, I'm going to have to add investment so to maintain that ratio constant. And that's the reason. Now we have this line here, rotates even further. And we get delta plus gA plus gN.

And you should play with these things. What happened in this diagram if I increase gA or stuff like that. And notice that here, now still you have a steady state. But it's a steady state in the space of output per effective worker and capital per effective worker. That means, for example, that these quantities are not growing in the steady state. But output will be growing at which rate in the steady state? In any state, state here, what is the rate of growth of output?

If output over AN is constant in the steady state, how can that happen? Output has to be growing at which rate? At the same rate as the denominator. So it's gA plus gN. That's a trickier question. What happens to output per person in this steady state? What rate is it growing at, output per person?

Sorry, somebody said the right thing. gA, exactly. I want to keep this ratio constant. I'm asking the question of which space does this need to rise in order to maintain this constant, while at the same rate as A is growing. Good.

Here we also did, we ask the question, well, could it be that here, if we change the saving rate, we get some extra kick in the long run? And the answer is no for the same logic as we had before. You're going to get transitional growth. But eventually you're going to convert to a steady state. And the rate of growth in the long run is not going to be a function of the saving rate. It's going to be equal to gA plus gN.

I do here. Measuring technological progress, blah, blah. I told you the story of China. Good. We run out of time. So let me just say the last thing that I want to say.

So the last thing we discussed, you say, well, what happen if you add education to this and try to-- [INAUDIBLE]. I expanded the model a little bit. I had education and said, does it change conclusions a lot? I said, no, not really. I mean, it doesn't change the conclusion with respect to the long run. It affects the level of output per capita if you have more education. But you won't achieve effective rate of growth in the long run.

And the last point I made is that, look, in this model, if you expand it and you try to assume that technology is the same and the rate of technological progress is the same across the world, you stick those parameters in the model-- population growth, the education levels and all that-- then you don't explain the amount of inequality we see in the world. The world will look a lot flatter if it was just a differences in population growth, deprivation, education levels, and things like that, but with the same technology.

So if you want to account for-- so this model, the model essentially doesn't produce enough inequality in the world. You need to add something else that explains that we have some countries in Africa that are not growing or growing at a very low level of rate. And we said that something else technology, for whatever reason, it happens that there is a pocket of countries that seem to have a lower, sort of permanently lower level of technology at both level and growth rate. And that's what explains sort of a subset of countries that are sort of stacked, that are not consistent with this convergence type thing.

So that's the reason it's called conditional convergence. Those countries themselves are converging to something. But they're converging to something much lower and with much lower rate of growth than most of the rest of the world. But the final lesson is for the average country, on average, it's clear that poorer countries grow faster than richer countries. That's a dominant force. But you need a little more if you want to explain certain pockets of the world. OK.