

9 Uncertainty

9.1 Lecture 20: Uncertainty

9.1.1 Expected utility and expected value

- **Expected value** is equal to the probability of each outcome times the value of that outcome.

- If a random variable X can take the values $x_1, x_2, \dots, x_k$ and each value occurs with probability $p_1, p_2, \dots, p_k$. Then the expected value of X is

$$E[X] = x_1 \cdot p_1 + x_2 \cdot p_2 + \dots + x_k \cdot p_k.$$

- A fair gamble means zero expected value.

- **Expected utility** is the probability weighted average of utility.

$$EU[X] = u(x_1) \cdot p_1 + u(x_2) \cdot p_2 + \dots + u(x_k) \cdot p_k.$$

- In a coin flipping game,

$$EU[X] = Pr(lose)U(lose) + Pr(win)U(win).$$

- Different than utility of expected value, since utility functions usually concave (due to diminishing marginal utility of income). Diminishing marginal utility of income means that the next dollar is worth less to you than the last one was in terms of happiness you gain.

9.1.2 Risk preferences

- **Risk averse**: concave utility, diminishing marginal utility income
 - For example, $U(C) = \sqrt{C}$.
- **Risk neutral**: linear utility, constant marginal utility income, when an agent only cares about expected value
 - For example, $U(C) = C$.
- **Risk loving**: convex utility, increasing marginal utility income
 - For example, $U(C) = C^2$.

9.1.3 Applications

- Insurance
 - Risk averse people will pay money to turn a gamble into a certain payoff since they get higher utility from certain income than from a gamble with the same expected value.

- Maximum amount they're willing to pay for this is their **risk premium**. The risk premium rises as the size of the loss rises (holding other variables constant). The risk premium falls as income rises (because loss is closer to linear).
- Lottery behavior is a puzzle – maybe risk averse at low incomes and risk loving at high incomes.

9.1.4 TO KNOW – Conceptual Understanding

- Explain why there is less risk aversion for small gambles

9.1.5 TO KNOW – Graphical and Math Understanding

- Given a utility function, be able to determine whether the agent is risk neutral, risk averse, or risk loving
- Calculate the expected value and expected utility from a gamble, given a utility function and a description of the gamble
- Calculate the risk premium for insurance, given a utility function and a description of the relevant risks

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