

[SQUEAKING]

[RUSTLING]

[CLICKING]

**JONATHAN  
GRUBER:**

All right. OK, let's get started. So today we're going to continue our discussion of factor markets. But really, what we're going to focus on today is essentially taking the insights for capital markets and thinking about how individuals and firms make decisions over time.

Last time we talked about the interest rate and its role in equilibrating capital markets. We talked about the demand for capital being driven by the marginal revenue product of capital. But then the more interesting side was the supply of capital being driven by individual decisions on how much to save. That savings went into either the bank or into corporate bonds or stocks. And that's the money that firms borrow to make investments.

I wanted-- this lecture is basically going to dive into that and think about in the real world, how do we think about these savings and investment decisions-- decisions that will be so important to you as individuals and maybe someday to you as business owners or executives? And I want to start by discussing the key concept of present value. One of the most important concepts in both economics and finance is the concept of present value.

The basic idea of present value is quite simple, which is that a dollar in the future is worth less than a dollar now. And the reason a dollar in the future is worth less than a dollar now is because you had that dollar now, you could save it and make interest. So delaying getting money till the future has an opportunity cost. You've missed the opportunity to earn interest on that money today.

So essentially, the idea is you can't just add up money that comes in different periods. You can't say, if I have a dollar today and a dollar in five years, a dollar in 10 years, that's \$3. You can't make that statement. Those are not the same thing.

That'd be like saying, I have a pound of gold, and a pound of apples, and a pound of butter. So I have 3 pounds. You wouldn't say that. You'd say, well, those are worth very different things. I can't just add them up and put them in one unit, which is pounds.

Likewise, you can't just add up dollars today and dollars in the future and put them on group and just say they're dollars. You need to actually think about how to put these into common values. So with apples and gold and butter, we might use dollars. With money in different periods, we're going to use the concept of present value.

Essentially, what we're going to do is we're going say take all future dollar flows and translate them into today's terms. The present value of a dollar at any point in time is what it's worth in the present, what it's worth today.

So suppose, for example, that the interest rate is 10%, and you start-- and that you want to have \$100 next year. You would like to have \$100 next year.

Well, so in other words, you know that if you put some amount  $y$  into the bank, at the end of the year what do we have? We have  $y$  times  $1 + i$ . That's how much you'll have at the end of the year. You'll get your money back plus the interest. And you want that to equal 100.

That's the equation you want to solve. If you want to have \$100 at the end of next year, that means that  $y$  equals 90.9. Ah, that's messy. 90.9 is  $y$ . That's the amount you have to put in the bank today to have \$100 dollars in one year.

Inverting that, that means that more generally, the present value of any amount of money is its future value over  $1$  plus the interest rate to the  $t$ , where  $t$  is how many periods in the future you get the money.

So if you get the money in one year, the present value of a dollar, a dollar in one year is  $1$  over  $1$  plus the interest rate. A dollar in two years is  $1$  over  $1$  plus the interest rate squared and so on. Essentially, this is a set of weights that allows you to value dollars in all periods in today's terms by putting them in present value.

So suppose, for example, you come to me and say, look, John, I want to borrow \$30. I'm going to pay you back \$10 each of the next three years. And let's say the interest rate is 10%

I would say to you, well, that's a bad deal for me. Because what's the value of the stream of payments you're going to give me? You're going to give me \$10. You're going to give me \$10 next year. You're going to give me \$10 next year. So that's worth  $\$10$  over  $1.1$  to me. Because if I had it today, I could have already-- it would have been \$11 next year.

You give me \$10 in two years, well, that's worth  $10$  over  $1.1$  squared. And you're giving me \$10 in three years. Well, that's worth  $10$  over  $1.1$  cubed. So, in other words, the \$10 you give me for the next three years is only worth \$24.87.

I can't just add it up and say, well, you're going to give me \$30, so I'll give you \$30 today. It's not the same thing. Because you give it to me today, it's worth more than if you give it to me in the future. Likewise, you give it to me in the future, it's worth less than if you give it to me today.

And the reason is simply that I can translate a given amount of dollars today into more dollars in the future through savings. Since I can do that, since I can translate that, money is worth less in the future.

So more generally, if we write a formula for the present value of any stream of payments, any stream of payments, OK, it's going to be how much you're going to pay. Let's imagine if I had a fixed stream of payments. Let's call that  $f$ . Let's call  $f$  the amount you're going to pay me back.

It was \$10 in our example times  $f$  times  $1$  over  $1$  plus  $i$  to the first plus  $1$  over  $1$  plus  $i$  to the second plus dot, dot, dot, all the way to  $1$  over  $1$  plus  $i$  to the  $t$ , where  $t$  is the last period you're going to pay me back.

So you're paying me back over 10 years. There's 10 terms in this equation--  $1$  over  $1.1$ ,  $1$  over  $1.1$  squared, all the way to  $1$  over  $1.1$  to the 10th. That is the equation for the present value of a stream of payments. That's a very messy equation to evaluate once the number of periods gets large.

And that's why we typically use a shortcut, which is we consider the case of a perpetuity, a payment-- a fixed payment you'll get forever. If you do an infinite sum of that equation, some of you will know that that will yield you, that the present value of  $f$  forever is simply  $f$  over  $i$ . That's the infinite sum of that equation. So there was \$10 forever, and a 10% interest rate is worth \$100.

It's the value of the payment over the interest rate I'm earning. And that's a convenient formula. Now, stuff never goes on forever. But if you think about  $t$  getting large, then you could converge to using that formula as a shortcut.

So we'll typically give you examples in this class with small numbers of periods or incredibly large numbers of periods to allow you to use that shortcut because having you evaluate this for 12 terms is just a pain in the ass. So that's a shortcut we can use as the number of periods gets large. Questions about that?

Now, you might also want to know about the-- you might want to invert it and say-- you might want to know about the future value of a given stream of payments. You might say, look, what happens if I save \$10 every year? What am I going to have in the future?

So, for example, if I save \$10 this year, next year I'll have \$11. I'm always assuming the rate is 10% for now. But then let's say I save that-- I leave that in the bank. Well, I have the next year. Well, the next year I'll have 12.10.

Why will the increment from 0 to 1 be small and the increment from 1 to 2? Why will it be smaller? Because I've not only saved the \$10, I've saved the extra dollar. And that extra dollar itself earns interest. And, indeed, at the end of  $t$  years, you will have that the future value of a stream of savings is  $y$  times  $1 + i$  to the  $t$  if you save for  $t$  years. And this illustrates the beauty of compounding.

The beauty of compounding is that when you put money in a savings account and let it sit, you not only earn interest on what you initially put in, you earn interest on the interest. And over time, that actually becomes a larger part of what you save. Over time, the big issue is not what you put in. The issue is all the interest you're earning and the interest you're earning on that interest.

So basically, let's-- to sort of work that out, let's consider an example. Let's consider you making a retirement savings decision. You might think retirement? Jeez, I'm not even out of college yet. Like, take it easy, old man. I'll worry about that in the future.

Well, actually, I'm going to tell you why you shouldn't worry about it in the future, why you should worry about it now. I'm going to illustrate that with a simple example.

Imagine you plan to work from age 22 to age 70. Imagine-- that's not a crazy guess for your working life. You work from 22 to 70. And let's say that the interest rate is 7%-- so you're going to work 22 to 70. And let's say the interest rate is 7%. That's our assumptions.

And I'm going to give you two choices for retirement savings plan. I'm going to give you two options. Option one for your retirement savings plan is you're going to save \$3,000 every year for-- right away, you're going to start saving \$3,000 every year for 15 years, and then you're never going to save again.

So from age, 22 to 37, you're going to save \$15. You save \$3,000 a year and then never save again. So it's \$3,000 a year for 15 years-- for 15 years. Well, what will you have?

Well, after the first 15 years of doing this, by our formula, you will have 75,387. You won't have \$45,000. You might be tempted to multiply 3,000 by 15. Say, we'll have 15 years, we'll have 45,000. That's wrong because you missed compounding. You miss the fact that all the savings is also getting interest on it. So the thing is, you already have much more than just multiplying those two.

But that's not all. You then let that sit there until you retire. So you let this money sit there for the next 33 years. After 33 years, this turns into 75,387 times 1.07 to the 33. So by the time you retire, you have \$703,000. By saving \$3,000 a year for 15 years, by the time you retire, you have over \$700,000.

Let's do an alternative savings plan. What most people do, which is, hell, I'm not going to save for retirement when I'm 22. That's nuts. Let's say what you do instead is you don't save anything until you're 37. You don't save anything for the first 15 years, and then you start to save. Then you start to save \$3,000 a year.

So 0 for 15, and then 3,000 for 33. If you follow that savings plan, by the time you retire, you are going to have \$356,800. You're going to have half of what you had with the first plan. Why? Because of compounding.

The way compounding works, the more you can get in the bank earlier, the more money you'll end up with because you earn interest on your interest. How many of you have been to the science museum here in Boston? OK, a few of you.

Well, you probably haven't been to the little kid part, but I have. And the little kid part in the science museum has these cool three ramps. And one ramp is flat and then steep. One ramp is steep and then flat. And one ramp is in between. And you have kids put the marbles on and see which goes faster.

And, of course, the one that goes fast is the one that's steep and then flat because you take advantage of the acceleration over the flat part, relative to one that goes flat and then steep. That's just compounding. Compounding is simply like getting the velocity up early. You're going to get there faster.

So this is an important lesson not just for mathematics, but for you for thinking about your decisions. You, when you get a job, most of you are going to get a job in a good firm that is going to offer you an option of retirement plan. You are going to think, why should I bother? I'm superhuman. I'm never going to retire.

The answer is you're wrong. You are going to retire. And every dollar you put in now will be worth so much when you retire that it's actually worth deferring consumption now.

Remember, capital is deferred consumption. So the decision you'll face when you join your firm is, should I defer today's consumption to have money for the future? And the answer is you should defer money today to have consumption for the future.

Moreover, it's even better than that because most of you will work for firms which offer a retirement plan called a 401(k), which is tax preferred. What that means is if you take the money home today, you pay tax on it. If you put it in your 401(k), you don't pay tax on it until you retire.

Now, why do you care if you're going to pay tax anyway? Why do you care if you pay tax today versus paying tax when you retire? Well, the answer is just like a dollar in the future is worth less than a dollar today, paying taxes in the future is effectively cheaper than paying taxes today because you're paying taxes with future dollars that are worth less than today's dollars.

So by deferring your taxes-- just like I don't want to defer my getting repayment from you, by deferring your taxes, you save money. It's even better than that because most employers will match what you save in your retirement plan. So when you go to a firm, and you start a new job, many employers will say, well, the first \$3,000 we'll match you dollar for dollar.

This is an unbelievably good deal. You're going to get the compounding for the next 50-plus years. You're going to get a tax break, and your employer is going to help you out. So you should absolutely save as much as you possibly can as early as you possibly can for retirement because of the beauty of compounding.

Let's think of a real-world example. How many of you have heard of Bobby Bonilla? Anybody hear about Bobby Bonilla? I got a couple head-- I got a few head nods-- three, four.

Bobby Bonilla was a good, but not great-- well, at peak, great, but mostly good-- baseball player. He played for the Mets at the end of his career. But by the end of his career his talent wasn't worth the salary they were paying him. He fell off a cliff pretty quickly, and wasn't worth the money they were paying him.

So they offered to buy him out. They said in 1999, we'll give you 6 million bucks to just bag your contract and leave. You won't have to play baseball anymore. You can retire-- pay you \$6 million out the door.

He said. that's a great idea. I'm happy to do it, but I don't really need the money right now. How about if we change the deal? How about if we said we defer the money from 1999 at an 8% interest rate. And instead, I'll collect it over the 2011 to 2035 period? So deferred for 20 years at an interest rate, and then collect it over a time period.

The way that worked out for him is that by the time his money started being paid out in 2011, his \$6 million was worth \$30 million. So he basically gets \$1 million a year till he's 72 years old. Indeed, there's Bobby Bonilla Day, which is the day the Mets pay Bobby Bonilla \$1 million.

So basically, this is a guy who understood compounding, understood the value of deferring, and now he gets a million bucks a year until he's 72 instead of just getting \$6 million at once back when he was young and didn't need it as much. So this is the value of deferring the value of compounding, which says save early and save often. Questions about that? Yeah?

**AUDIENCE:** How did they calculate the 75,000, the [INAUDIBLE].

**JONATHAN GRUBER:** This one? Oh, I simply use this formula. I mean, simply-- someone did- I did it in a spreadsheet, but-- yeah?

**AUDIENCE:** [INAUDIBLE] over a long period of time. But what exactly constitutes as a long time?

**JONATHAN GRUBER:** Well, that's a great question. Basically, it's approximately infinity. We will let you know in a problem whether you can use that shortcut or not. But roughly speaking, we're not going give you a problem if you [? count this ?] over 10 periods. We'll leave you a short number of periods, or we'll give you a huge number of periods, which is effectively infinite. But we'll let you know. OK?

Technically, in the real world, not an exam, you have to do it this way until you get to infinity. This is just an approximation. But, in practice, it's a pretty good approximation once you get out 30 periods. It depends on the interest rate. The higher the interest rate is, the fewer periods you need to go Out For that to be a good approximation. The lower the interest rate is, the more periods you have to go out for that to be a good approximation. OK. Other questions?

All right. So now-- yeah?

**AUDIENCE:** When you say that paying tax now are cheaper-- having paying tax-- [INAUDIBLE] cheaper than paying tax now. But isn't [INAUDIBLE] of the present value also [INAUDIBLE] 30% of future value when you get the return?

**JONATHAN GRUBER:** You're talking about the tax piece?

**AUDIENCE:** Yeah.

**JONATHAN GRUBER:** But the point is with the tax-- the point is-- think of it this way. If I pay taxes today-- let's say, I pay a dollar of taxes today versus a dollar of taxes in the future. The dollar of taxes today that I pay to the government, they get to earn interest on. Whereas, if I hold it, I get to earn interest on it.

So as long as the tax rate is less than 100%, I get to make the-- if the government holds the money, they make the money. If I hold the money, I make the money. Better for me to make the money. Does that make sense? Good question. Other questions?

Now, some of you may be thinking this is all well and good, but shit's going to cost more in the future. Bobby Bonilla making a million bucks in 2030 is buying much more expensive stuff than Bobby Bonilla was buying in 1999. What about that? What about the role of inflation? So let's talk about inflation.

In other words, in my discussions so far, I assumed prices didn't change. But, in fact, prices do change. They go up. We typically measure that by something called the consumer price index. The consumer price index is essentially a measure of what a fixed basket of goods costs over time.

So essentially, define a basket of goods that a typical person would buy. And they go and measure every quarter-- they literally go to stores and say, what does this cost? So a fixed basket of goods includes one men's shirt. OK, let's go see what the average price of men's shirt is. It includes one women's shirt. Let's see what the average price of the women's shirt is. It includes 67% of a new-- or 12% of a new computer. Let's see what a new computer costs, et cetera.

So they basically have a set of weights that are determined by a consumption bundle. Say, for that set of good weights, what does it cost to buy the same set of things over time? And that gives you a new index value. And that they normalize it to 1 at the year 1982 for some arbitrary reason.

So basically, the CPI says, what does stuff cost relative to what it cost in 1982? And you can see that in figure 17-1. Figure 17-1 is a graph of the CPI. What you can see is basically it's identically 100 in 1982, and it's rising before and after 1982.

You can see that the rise is relatively rapid in the 1970s in percentage terms. That was a lot of inflation in the 1970s into the 1980s, and then it flattened out. Inflation has been slower since. So this is how we measure inflation.

The fact there's inflation means that everything I've taught you is wrong in one critical respect, which is everything I've taught you is about  $i$ , The Interest rate. But that's not what you should care about. You don't care about what the interest rate is, what the nominal interest rate is. You don't care about how much money you have in the future. You care about how much goods you can consume in the future. Money is just a means of exchange.

If I tell you in 20 years you'll have \$100, you don't know what's that worth without knowing what stuff's going to cost in 20 years. What you really care about is your purchasing power. What you really care about is your purchasing power.

So, for example, suppose today I'm going to save \$100 at a 10% interest rate-- \$100 at a 10% interest rate. What I care about is not how much money I have next year, but how many goods I can buy next year.

So, let's say, I'm like Marshawn Lynch, and all I care about is Skittles. That's all I care about. And let's suppose that Skittles today are \$1 a bag. So my \$100 today could about 100 bags of Skittles. So I could buy 100 bags today.

Now let's say the price of Skittles does not change, but I earn 10% interest. Then next year I can buy 110 bags-- next year. The price of Skittles is still \$1. I save my \$100 at 10%. I have \$110. I'm going to buy 110 bags.

But, let's say, the price of Skittles has also gone up by 10% How many bags can I buy next year? Someone tell me. If the price of Skittles has gone up from \$1.00 to \$1.10? Yeah. Back to 100. Back to 100 bags.

So basically, what I can buy is a function of the interest rate in prices. So what I care about is not the nominal interest rate, but what we call the real interest rate, which is the nominal interest rate minus inflation, which for some reason we denote by  $\pi$ , even though  $\pi$  is also profits. I don't why we do that. We're a difficult profession.

So the real interest rate is the nominal interest rate minus inflation. And that's what I care about. That's what I can buy is not the nominal interest rate, but the real interest rate determines what I can buy.

Now, this is a big topic of discussion in macroeconomics. In particular, a really fascinating point about this is that really this should be the expected inflation rate because I don't know what the inflation rate is going to be.

When I put my money in the bank today, I know  $i$ , but I don't know what inflation is going to be. I know what it's been. But I don't know it's going to be. So if I say what's \$100 worth in 20 years, you have to give me your expectation of inflation.

So it's actually pretty complicated, and it goes back to things we've talked about-- how people make decisions-- and things we will talk about-- how people think about these things, like how do you form expectations of inflation rate?

It turns out there is a near perfect predictor of people's inflation rate expectations, which is what's happening to gas prices. Most people look at what's happened to gas prices, and based on that, they decide what inflation is going to be.

But the bottom line is, in theory, it's a very sophisticated-- there are people-- there are actually hundreds of thousands of people whose entire job, mostly work in financial sectors, but some working for the Federal Reserve, whose entire job is to form good expectations of this. That's their whole life, is to figure out how can I-- what's inflation going to be, and how does that affect decisions we're going to make? And it turns out to be a very hard and complicated calculation.

So that determines to be very, very important in thinking about what the real interest rate is going to be. OK? Questions about that?

Now, I'm going to go back, and sometimes I'll use  $r$ . Sometimes I'll use  $i$ . The bottom line is what matters for real decision making is  $r$ . Sometimes we use  $i$  because it's more convenient. But if ever I use  $i$ , it means I'm assuming inflation is zero. So whenever I use  $i$ , I'm assuming inflation is zero I really should use  $r$  everywhere. Sometimes we slip back and forth. But basically, we really should use  $r$ . That's a relevant decision-making tool. OK? Questions? Yeah?

**AUDIENCE:** So you mentioned that the CPI is normalized, like 1982. If there were goods that were invented between 1982--

**JONATHAN GRUBER:** Awesome. Great question. And a whole field of research is, what do you do about innovation? What do you do?

So here's a perfect example. Imagine that there's a good that in 19-- here's a great example. Let's take the laptop, which in 1990 was incredibly expensive and is now much, much cheaper. So basically you would say, well, basically, we have-- life is cheaper now.

But, in fact, laptops do a ton more. How do we account for that? How do we-- we really want to think about the price per unit of power. Not the price of a laptop, but the price per unit of doing stuff. That's gone down a ton more.

The average laptop today is maybe half the price of average laptop in 1990, but the average value of a laptop today is 100 times the average value of a laptop in 1990. We can do so many more things with it. The computing power is so much larger. That's not reflected in the typical CPI.

So many people think we need to adjust the CPI for essentially what's happening to productivity-- what's happening not just to the goods we buy, but what we can do with them. And that's a very difficult and interesting field.

The clear conclusion is the CPI is too large. The growth rate of prices you hear every year is too large because we're missing the underlying innovations that for the same price you're getting a better good. You're missing that.

And so, as a result, when we say, gee, inflation went up 3%, you think, wow, it's 3% more expensive to live in America. That's not quite true. It's 3% to live in a-- more expensive to live in America and buy the same stuff. But you're getting better stuff, and that should be accounted for. It's a good question. Other questions? Yeah?

**AUDIENCE:** So I guess I have a question with [? macro. ?] I've been told sometimes that inflation is sometimes just a result of a healthy economy, and that if the economy is growing, there's going to be inflation, that people [INAUDIBLE] better.

**JONATHAN GRUBER:** Yeah. So now this is why you all should take 1402, especially now. Right now in the last year, you've heard a lot about interest rates and inflation and things like that. So basically, I'm going to give you a two-minute primer, and we'll leave the rest to 1402.

So two-minute primer is go back to last lecture. Think about the market for capital. You've got the supply of capital on the x-axis, the price of capital, which is really we now know the real interest rate on the y-axis. You've got the supply of capital and the demand for capital and some equilibrium interest rate  $r^*$ , and some equilibrium capital on that  $k^*$ .



now, Where does the demand for capital come from? Well, let's start with supply. Where does supply of capital come from? Comes from people's savings. So as the interest rate goes up, people save more. What do they do less of? They consume less. So by raising the interest rate, you lower the amount of demand for spending, or you cool off the economy.

Likewise, what do firms do as the interest rate goes up? Well, as the interest rate goes up, they spend less money building machines. There's less demand for investment. So you cool off the economy.

So the idea is by rising interest rate cools off the economy by lowering both the demand and supply of capital. And that that's basically essentially why the Fed uses interest rate as a tool to fight that. That is something you'll learn a lot more and a lot richer in 1402. But that's the basic deal for those of you who want to understand the news at that level.

And obviously, the tricky thing is how much do you raise the interest rate? And that depends on this, which is why there are hundreds of people, thousands of people probably, employed at Feds around the country trying to figure out what this is because that determines what they should do to set the right  $r^*$ .

Now, let's go back to micro, and let's talk about choices over time. Let's talk about how you make, how individuals and firms, make choices over time. We'll have three-- choices over time.

OK. So we're now equipped to ask the question, what do you do when you face decisions that have different payouts at different periods of time? And the answer is, you just pick the payout with the highest present value. It doesn't matter when the money comes, as long as you put it in the same terms.

So, for example, consider a professional athlete who's comparing two possible contracts. Contract A is \$1 million today. Contract B is \$500,000 today and \$1.5 million in 10 years.

Well, despite the Bobby Bonilla fun example, it's not obvious which of these you should choose because they're paid at different times. You add up. You say, well, this is \$2 million. This is \$1 million. You should clearly choose B, but that's not right because this is worth less because it's in the future. What you want to do is compare the present value of these two contracts.

Well, you know the present value of this contract is \$1 million that you had today. What's the present value of this contract? Well, the present value is going to be \$500,000-- \$500 k-- plus \$1.5 million. But you get it in 10 years, so you have to discount it. You put it over  $1 + r$  to the 10th. That's the value that you'll have.

So, for example, if the interest rate was 7%, if  $r$  equals 7%, then the value of this equation is \$1.3 million. So at a 7% interest rate, you should choose option B. At a 14% interest rate, then it's worth \$0.9 million. So the 14%, you should choose option A. What's going on? Why does option B become less attractive as the interest rate goes up? Somebody explain the intuition to me. You see the math there. Why does option B become-- yeah?

**AUDIENCE:** It's money now [INAUDIBLE].

**JONATHAN GRUBER:** Exactly. At a 14% interest rate, if I had this money now, I'd get a ton of money 10 years from now. Here, I'm waiting 10 years to get the money. Money is worth a lot less in the future at a 14% interest rate. It's all about opportunity cost. The higher the interest rate, the higher the opportunity cost of getting money in the future relative to getting it today.

So the key is basically you want money up front when there is a-- you want money up front when you have a higher interest rate. You're happy to defer money when there's a lower interest rate.

So, for example, let's come back to sports again. Let's talk about Max Scherzer. Max Scherzer is a pitcher with the Washington Nationals. In 2015, he signed a seven-year, \$210-million contract-- the 10th biggest contract in history at the time.

But it turned out the way the contract was signed, it was a seven-year contract, but it was paid out over 14 years. He only had to play for seven years, but they would pay him-- instead of paying him \$30 million a year, they're paying him \$15 million for 14 years, even though he only had to play for seven. So the present value of that contract was not \$210, but more like \$166 million.

Now, why did they do that? Well, for Max Scherzer's ego because if you look at the number \$210 million-- that was the second highest contract ever for a pitcher. If you look at the present value of \$166 million, it was like the fifth highest contract ever for a pitcher. So Max Scherzer couldn't go bragging he got the second highest contract ever for a pitcher if they just give him \$166 million. So that's how it can be confused.

Or more generally, let's think about the lottery. You see the Mega Millions is \$1 billion. You think, this is great. \$1 billion, but you're not getting \$1 billion. What you're getting is something like \$33 million over the next 30 years. So the present value of Mega Millions is less than half of what's advertised.

They don't hand you \$1 billion today. They say, we're going to give you \$33 million over the next 30 years-- 30 year-- the next 30 years. It's still not bad. I mean, I'd take it. But a lot less than you think because it's paid out over time.

So this determines the numbers you hear. When you hear a number from now on, whenever your number people make the mistake of adding up numbers in the future period. But today, you've got to recognize that's wrong, and you should be using the present value, which was everything in today's terms, rather than falsely adding up future dollars with today's dollars.

This doesn't just affect you. This affects firms' investment decisions-- firms' investment decisions. So, for example, firms have a particularly interesting choice. Because firms have decisions where they put aside money today to get money in the future. That's what investment is.

So what firms consider when they make an investment decision is they consider what's the Net Present Value of that investment-- the NPV. What's the net present value?

So, for example, consider a firm that a stream of payments where you're going to get revenues in each period of  $R_t$  and cost of  $C_t$ . So you have a project where you're going to get revenues of  $R_t$  and cost of  $C_t$  in every period.

The net present value of that stream of payments is equal to  $R_0$  minus  $C_0$ , plus  $R_1$  minus  $C_1$  over  $1 + R$ , plus  $R_2$  minus  $C_2$  over  $1 + R$  squared and so on. That's the net present value.

This matters because cash flow is typically negative in the early years of an investment. Typical investment means  $R_0$  is less than  $C_0$ . You're going to buy this big machine, you don't make that much money right away. But eventually, you make the money.

If you simply add it up and don't discount, investment will always look great. But if you recognize that investing a lot today might yield money in 20 years when it's worth less, it might not look as good.

So basically, imagine, for example, you have a big upfront investment of \$100 in year one with no revenues. So you have something where in year one you get 100-- it's \$100 cost and no revenues. In year two, it's \$200 in revenues and no cost.

Well, you might think, gee-- I'm sorry-- year two you get \$150 of revenue and no cost. So year one,  $C_0$  is 100.  $C_1$  equals 0.  $R_0$  equals 0.  $R_1$  equals 150.

Well, if you add these up you'll say, gee, this is a great deal. It costs \$100. I make \$150. That's a great deal. But you know now that's not the right way to think about it. If the interest rate is 10%, then this formula-- at a 10% interest rate, this formula will say the NPV of this investment is \$36.36. That'll be the straight 10%.

But as the interest rate goes up, eventually that will go negative. Why? Because there's an opportunity cost of investing in the machine today, which is you could have put the money in the bank.

Firms, just like people, have a decision to make about whether to invest today in the future. As described a minute ago, talk about the macro stuff, the higher the interest rate, the less attractive investment today is. Why? Because they could put it in the bank instead.

Firms always have the option, just like we do, of putting the money in the bank. The interest rate is 20%. In the bank, that would mean a pretty fucking good investment opportunity to give up 20% in the bank to go build a machine-- to buy a machine. Because I'm giving up a huge rate of return on the money sitting in the bank.

So essentially, what drives firms' investment decisions is, is the NPV greater than zero? Firms invest if the NPV is greater than zero. That's it. Simple rule. And all they need to know is what the revenues and costs will be, but also, what the interest rate is going to be.

Now, obviously, you need to know future interest rate. So they need to know their expectations of interest rates. But, in principle, in a world of perfect information, all they need to know is what the revenues will be, the costs will be, and the interest rates will be. And they can ask, then, is my money better off sitting in the bank or being invested in this good-- in this machine?

But that's the key point. Once you come back to the macro [INAUDIBLE] minute ago, the higher our interest rates, the less investment because the more NPV is going to be driven below zero by the upfront cost being today, and the revenues you earn being in the future. OK? Questions about that?

So the bottom line is both consumers and firms face this. Indeed, consumers make investment decisions. Let me give a simple example. About 20 years ago, I had to decide whether to insulate my house. It's a very old house.

At that point, I had heating bills of \$2,000 a year. I had \$2,000 a year heating bills on my house. And I decide-- and the company that gives-- that provides insulation said that in insulating my house, I could lower this by 25% forever. I could lower my insulation cost by-- heating cost by 25% forever if I insulated. But the cost of doing so would be \$4,000.

So I just did the NPV. I said, look, should I do it? What's the NPV? It's minus \$4,000. That's my initial investment. Plus, I save \$500 the first year-- 25% of \$2,000-- plus \$500, plus \$500 over the interest rate because I'm going to save \$500 forever.

That's the calculation. Negative in the first year, and then I'm going to get \$500 forever from doing this. So what's the answer? The answer is as long as the interest rate was less than 12.5%, it made sense to do this. You can show that math to yourself. As long as the rate was less than 12.5%, it made sense to make this investment. But if it's above 12.5%, it no longer made sense to make this investment.

So the bottom line is that-- the bottom line is that it all depends on what else I could do with that \$4,000. That's going to determine whether I should make this investment or not and what my expectation of the interest rates are. So we all face these decisions all the time. Anytime we face a decision with a short-run cost and a long-run benefit, we should be doing the NPV calculation.

The flip side is anytime we face a decision with a short-run benefit and a long-term cost, we also do this. So when you decide whether to smoke-- we'll talk about this in a few lectures in more detail-- you should think about the short-term benefit, which is you feel better, maybe, if you're a smoker. And the long-term cost is you die sooner. But you die sooner off in the future. So you got to decide what's that worth? You have to discount it.

This is true-- so every example says you have to weight the present and the future differently to add them up and think about decisions you make today. Questions about that? Yeah?

**AUDIENCE:** When you use  $i$  rather than  $r$ , what does that accomplish?

**JONATHAN GRUBER:** It should always be  $r$ . Did I use-- where do I use  $i$ ?

**AUDIENCE:** On the last one [INAUDIBLE].

**JONATHAN GRUBER:** Oh, yeah. You're right. It should be  $r$ . Sorry. OK? For those of you who are going to check my math at home, actually, the 12.5% comes from example we don't get-- where we don't get benefits the first year.

So the example gave me 12.5%. It makes the math easy. I assume the installation doesn't start working until year two. Just so you-- in case you're going to check my math, but the intuition is the same.

Now, let's go to one last example. You guys-- it's like, jeez, I'm thinking about professional athletes and retirement, insulating my house. Like, this is so irrelevant to me. Let's come to example that is totally relevant to you, or at least was to you four or five years ago. Should you go to college? It's a pretty relevant decision to a lot of people in America.

Well, this is a decision, an investment decision, over what we call-- we've talked so far about machines and investing in capital. This is what we call investment decision in terms of human capital. All the time, we make decisions about how much to invest in ourselves. Every time we decide to go to college or decide to get any other training or learn a new thing, we are investing in our human capital. We are building our productivity.

If the new thing's sports, you're not building your productivity, as my wife constantly tells me. But if the new thing is something that's actually productive for your job, you are building your productivity. You're investing in your human capital.

So, for example, let's imagine-- let's go to figure 72. We're going to do an example. Imagine that when you graduate-- your decision when you graduate from high school is to go to college or not. If you don't go to college, you work from age 18 to age 70. If you do go to college, you work from age 22 to age 70.

Now, if you go to college, there's two costs that you get if you don't go to college, that you add relative to not going to college. First of all, you got to pay for college. So during the four years that you're in college, that green area, you're paying tuition. That's why it's negative.

But there's a second piece. What's the other opportunity cost of college? Yeah?

**AUDIENCE:** The wages that you're not making.

**JONATHAN GRUBER:** You could be working. That's a real cost, just like tuition. You all are-- you all could be out there earning money right now. So you're sitting here. The opportunity cost of this lecture is the money you all be out making-- real money. There's an opportunity cost.

You are, for those four years, not only paying tuition. You're foregoing the opportunity cost. That is a huge benefit to not going to college.

However, if you graduate, once you graduate college-- not if. Once you graduate college, you earn more. So you get more money for the rest of your life by being a college graduate. That is shown after year four in the yellow part of the graph.

Andrew, I think we can draw this a little more neatly next time. It kind of looks like a preschooler drew it. It looks-- or me. So we can draw this a little more neatly next time.

Basically, the idea is beyond year four, there's some net benefit. The net benefit is how much you get from earning more because you got a college degree relative to earning-- relative to just having a high school degree. And that is the yellow area.

So if you decide whether to go to college, you basically need to think about, what is the cost and benefits? Well, that's going to depend on the interest rate. So, for example, if the interest rate is zero, then the NPV of college, as shown in the table at the bottom, the NPV of college is way above the NPV of just dropping-- and going to high school-- should definitely go to college.

But once the interest rate gets above 3.5, then-- I'm sorry. If the interest rate is-- I don't know. That 3.5 is weird. Yeah, ignore that 3.5. I don't know what that's doing there. That's weird. That's just-- I think that should be maybe 1 or something. I don't know what's going on there, but we'll fix that. Ignore the 3.5.

At 2%, you should still go to college. At 4%, you should still go to college. At 6%, you should still go to college. But once you get to 8%, it breaks even. At an interest rate of 8%, you are literally financially indifferent between going to college and not going to college. And what's above 8%? You lose money by going to college. It's phenomenal.

College graduates earn a ton more than high school graduates. And yet, with the power of compounding, you actually lose money by going to college once interest rates are above 8% It's pretty phenomenal.

That is why one of the major interventions the US government does to try to increase college attendance is student loans at a low-- at a subsidized interest rate. One of the reasons-- one of the ways we try to encourage people to go to college is by giving them subsidized loans at a lower rate to make this a better deal. And that's why that can be a very powerful tool to get people to go to college. Questions about that?

All right. We'll stop there, and we'll come back next time and talk about uncertainty.