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**JONATHAN  
GRUBER:**

So today, we're going to continue our discussion of producer theory. Remember, the first few lectures we developed consumer theory and where the demand curve came from. Now, we're turning to producer theory and where the supply curve comes from, drawing on a lot of parallels from the math and techniques that we use for consumer theory.

So we're going to start today by talking about the key construct, a key construct of producer theory, which is short-run cost curves. Remember, the short run is the period of time over which some inputs are fixed, in our case, capital, and other inputs are variable, in our case labor.

So basically we last time, developed a firm production function. Now I want to translate that production function into costs. And why do we want to do that? Because remember, the way firms maximize profits is by minimizing costs, by producing goods as efficiently as possible at minimum costs.

Remember, our production function from last time, was little  $q$  equals the square root of  $L$  times  $K$ . That was our production function. Now, what we want to do today, is write down a new function, which is, what does it cost to produce little  $q$ . For any given  $L$  and  $K$ , what's its cost?

Well, the cost of producing little  $q$ , which is a function of  $q$ , is equal to the amount of capital you have, which is fixed, which is-- I'm sorry, which is fixed times the rental rate of capital. Remember, that's the price of machines is their rental rate, plus the amount of labor, which is a function of  $q$  times the wage. So basically, your choice variable is how many workers you want to have. To decide how much  $q$  you want to produce, you can decide how many hours of labor or workers you want to employ.

By the way, I will talk interchangeably between hours of labor and workers. There's no right measure of  $L$ . It depends on the context, et cetera. So don't feel like-- it's just two different ways of measuring the  $L$  input. So it matters less when I say hours of work versus workers. If it's an important distinction, I'll tell you. But right now, I'm using two different ways of measuring  $L$ . The bottom line is it's a labor input, OK.

So now we want to say what is the explicit expression for the cost function? Well, in fact, if we know a production function, and we know  $W$  and  $R$ , we can write down a cost function. So we know this production function. And for this case, let's assume  $W$  equals 5, and  $R$  equals 10.

Where do those come from? I'll teach you that in a few lectures. Remember, we peel the onion in this class. We're revealing the mystery slowly. Right now, just take those as given. In a few lectures, I'll tell you where they come from.

So we know the production function is  $q$  equals square root of  $L$  times  $K$ . So we can take that from that production function. We can just flip it and say  $L$  equals  $q$  squared over  $K$  bar, just squaring that, moving to the other side, OK. So then we could say that we can rewrite the cost function as  $C$  equals  $10K$  bar, because 10 is the rental rate, plus  $5q$  squared over  $K$  bar, plugging in from this equation. 5 is the wage rate

OK, now,  $K$  bar is a fixed number. To make life easy, let's just assume  $K$  bar equals 1. It could be any fixed number in the short run. Let's assume  $K$  bar is one to make life easy. So that says that the cost function, we could simply write as  $10$  plus  $5q$  squared. That's our cost function.

To get that cost function, which is a relationship between the costs of the firm and the quantity they produce, the cost function relates to the quantity produced to the cost, all we do is start with the production function, plug-in these parameters, and we end up with the cost function. So that's how you translate production to cost.

Now, let's look at this cost function for a second. And I want to give you some key definitions that we'll be using throughout this course. We are going to call fixed costs-- fixed costs are going to be costs which are not variable in the short run. Fixed costs are costs which are not variable in the short run. Here, it's 10. The fixed costs in this example is 10. There's nothing you can do in the short run to change your fixed costs because capital is fixed.

Variable costs are the costs that can be varied in the short run. In our case, variable costs are  $5q$  squared. So your variable costs are  $5q$  squared. That's the part of costs that you can move in the short run. And total costs is the sum of those two. That's expressed there. Total cost is the sum of fixed and variable costs.

Now, the key, the single key that will drive production theory, is what we're going to call marginal costs. Marginal cost is the differential of total costs with respect to quantity. So marginal cost is  $dc/dq$ , which, given that fixed costs are fixed in the short run, it's the same as saying.  $d$  variable cost  $dq$ .

In the short run,  $d$  total cost  $dq$  is the same as  $d$  variable cost  $dq$  because fixed costs are fixed. So marginal costs are the marginal cost of producing the next unit. What is the cost to produce the next unit? It's the change in variable costs for the next unit produced.

And then finally, we will define average costs as what they sound like, which is  $c$  over  $q$ . Total cost over quantity produced, that's average cost. We can graph these various cost curves in figure 6-1. So figure 6-1 in your handout, shows the cost curves for our sample cost function, which remember, is really our example production function, given an  $r_w$  and  $k$  bar.

What you see here is the marginal cost, that is, the variable cost  $dq$  is equal in our example, to  $10q$ , just differentiating that with respect to  $q$ , is  $10q$ . Our marginal costs are the line with the equation  $10q$ . Our average costs are this expression over  $q$ . So it's  $10$  over  $q$  plus  $5q$ .  $10$  over  $q$  plus  $5q$  is average costs. OK, well, how did I get that? I just divided the cost by  $q$ .

Also on this graph, you can see average variable cost and average fixed cost. So you can see that the average cost line is a combination of two pieces. An average variable cost, which is everywhere upward sloping, and an average fixed cost, which is everywhere is everywhere falling.

So average costs themselves, first fall, then rise. Average costs first fall, then rise. Why is that? Well, that's because at first, you have these big fixed costs you have to pay off. But over time, the fixed costs get smaller relative to the variable costs, over time, it rises.

So average costs, so the first fall because the first couple of units, your costs are very high. Your costs are super high at the start, because you have all these fixed costs. Each unit, those fixed costs per unit fall and fall and fall. That's the AFC line. Fixed cost per unit fall and fall and fall. Whereas, variable costs per unit rise and rise and rise.

And the minimum of average costs is where it crosses the marginal-cost curve. It's just mathematical. If you take a function, the minimum of the average is going to be where it intersects the margin. Because wherever the marginal cost is above the average cost, the average cost is rising. Wherever the marginal cost is below the average cost, it's falling. Therefore, marginal cost intersects average cost at the minimum.

In other words, if you're to the point to the left of 1.5, if you're using less than 1.5 units, marginal costs is below average cost. So by producing more, you drive average costs down because the marginal cost is less than the average cost. So as you increase production from one unit to one and 1/2 unit, you drive average cost down.

But once marginal cost is above average cost, increasing production drives average costs up. And that's why marginal cost hits average cost at the minimum, OK. Questions about those relationships or those graphs?

So now let's go back and note a key relationship between marginal cost and something we discussed last time, the marginal product of labor. Remember, cost equals  $k$  bar times  $r$  plus  $l$  times  $w$ . If you differentiate this with respect to  $q$ , to get the marginal cost, you get marginal cost equals  $w$  times  $dl/dq$ . Because this term drops out. And so you just differentiate by the law of-- you get  $w$  times  $dl/dq$ .

So the marginal cost of producing the next unit, is the price per unit of labor times how many units of labor it takes to make the next unit. This is the price per unit of labor,  $w$ . This is how much labor it takes to make the next unit. So the marginal cost of the next unit is the product of those.

But we also know from last lecture, we know from last lecture, that  $dq/dl$ . We had a name for that. We called that the marginal product of labor. We defined that last lecture. How productive the next unit of labor is was  $dq/dl$ . So we can substitute in to write-- we can substitute that in to write that marginal cost equals  $w$  over the marginal product of labor.

That is, the marginal cost of producing the next unit of the good, is higher the more you have to pay each worker and lower, the more productive each worker is. So as the marginal product of labor is very high, unless wage is very high, marginal cost is going to be low. So basically, what's going to matter is the marginal product of each unit of labor and the wage you have to pay for that unit.

Now, last point I want to make here on definitions, I want to add something called sunk costs. I'm going to already contradict something I said last lecture. Last lecture, I said in the long run, all inputs are variable, in the short run, only some inputs are variable.

We have some costs which in the long run are not variable. Those are called sunk costs. Those are costs where once you make them, you can never get them back. A fixed cost is something like a machine. If you buy a machine, and you want to get rid of it, you can sell that machine or a building. If you buy a building, and you want rid of it, you can sell that building.

A sunk cost is something you can't ever get rid of or sell. The best example would be med school. If you want to be a doctor, you have to go to med school. Having gone to med school, you can't ever un-go to med school. You have paid that cost of time and money, and you're done. It's sunk.

A sunk cost is essentially a long-run fixed cost. It's something where once you make that investment, you never get it-- there's no way to get it back. Now, you can get benefits from it. I'm not saying it's not a good idea. I'm just saying unlike a building which you could sell or a machine which you could sell, med school, you can't sell your med school experience.

And this is very important because it leads to a very counterintuitive difference between sunk costs and fixed costs, something which trips people up all the time, which is that sunk costs are irrelevant to decision making once they're paid. Once you've paid your sunk costs, they are irrelevant because you can't get them back anyway it's done. Or as the economists say, sunk costs are sunk. Doesn't matter. It's irrelevant.

Now, here's why that matters in real life. Let me talk about a simple example. Many of you have heard of the band Journey, classic 80s band. A few years ago, Journey was touring, and they have a new lead singer, this guy they found actually through YouTube. He's from the Philippines, sounds just like the old lead singer. It's kind of cool. So they're playing down in Mansfield, Mass. And I bought a couple of tickets for my wife and I for \$125 each, good seats.

Then I looked at the setlist, and I thought, I don't actually like that many Journey songs. I'm going to sit through this hour and a half concert for the three songs I like. It doesn't really seem worth it. I don't think I want to go. I think I'm going to try to sell these tickets on the secondary market. So I can go to StubHub and sell the tickets.

The question is, I had paid \$250 for those tickets. How should that impact the price I set on StubHub? The money I paid for those tickets, how should money-- forget everything else, just the money I paid. How should it impact the price I set on StubHub? Yeah?

**AUDIENCE:** It shouldn't.

**JONATHAN GRUBER:** It shouldn't. Why not?

**AUDIENCE:** Because it's a sunk cost.

**JONATHAN GRUBER:** It's a sunk cost. I paid it already. It's irrelevant. It's done. I already paid the 250. It's gone. I might feel sad, but it's gone. How should I decide what to put the tickets on sale for at StubHub? Well, it's a function of two factors.

First is I can look at what things are selling for and try to use the market. That's one thing. But let's say I had no information. Let's say I had no information on the market. Leave that aside. It's just an internal decision on what I'm willing to sell tickets for. How should I decide? Yeah?

**AUDIENCE:** Think about how much [INAUDIBLE]

**JONATHAN GRUBER:** Well, that's one way to think about it. You're not willing to pay to buy the tickets. That's one way. But that's a confusing way. There's another way to think about it. What's another way to think about it? It's related to that, sort of the flip of what you said. How should I think about how much I'm willing to say-- how much am I willing?

Forget the price that I set, which is a function of what I think the market is. My willingness, how much should I be willing to sell those tickets for? Well, think about it. If I sell the tickets, what am I not doing? I'm not going to the concert. So how much should I be willing to sell the tickets for? Yeah, yeah, go ahead, white shirt, you had your hand up, yeah.

**AUDIENCE:** The opportunity cost.

**JONATHAN GRUBER:** Yeah, basically, I should sell them for how much I'm willing to pay to go to the show. If literally, I don't want to go, then I should be willing to sell them for zero. I should be willing to give them away.

I'm not saying I will sell them for zero because I might make money on it. But there's a my underlying opportunity cost at zero because I literally would rather sit at home. If I say, yeah, I don't want to pay 250. But if you said you'd get Journey tickets for \$50 bucks each, I'd go, then I should set the price of \$50 each.

If someone will pay me more than 50, I sell them. If they'll pay me less than 50, I go. So the amount I should be willing to get should be determined by the opportunity cost of going, not by what I paid for them. That is a very confusing logic. Yeah?

**AUDIENCE:** But I know it's sunk costs. I feel like I would think about trying to recover those sunk costs.

**JONATHAN GRUBER:** Yeah, you would because that's the way humans think. But it's wrong.

**AUDIENCE:** It's not a real thing.

**JONATHAN GRUBER:** It's not a real thing.

**AUDIENCE:** But you could get it back. You wouldn't have lost anything.

**JONATHAN GRUBER:** But it doesn't matter. They're already gone. So once again, let me think of it this way. Let's say that the fact you've paid the costs makes you sad. You've paid those costs. You made them sad. But right now, whatever you sell them for, that's all you can get back.

So let me put it this way. Let's say I could go online and sell them for \$500 each. Should I only sell them for 250 because that's what I paid for them? No, sell them 500. So it's irrelevant that I paid 250.

If I go sell them for 50, I'd sell them for 50. If I can sell them for 500, I sell them for 500. What I paid is some amount I paid in the past. I may be sad about it. And look, let me be clear. The way you're thinking about it is the way that the world thinks about it, and it's wrong.

I got chastised by my advisor in graduate school because I had plane tickets that I was going to cancel, and I was suffering from the sunk cost fallacy. I was going to say, well, I paid this much for them And he said, it doesn't matter what you paid for them. What matters is the value to you of flying home. The sunk costs are gone. Yeah?

**AUDIENCE:** So what's the difference between selling the tickets and selling [INAUDIBLE].

**JONATHAN GRUBER:** Instead of selling what?

**AUDIENCE:** Like, the building [INAUDIBLE]

**JONATHAN GRUBER:** Oh, that's a great point. The difference is with the building, when I-- The difference is with the building, I buy it as an investment that I intend to use for a while and then sell. With the tickets, I just bought them. I wasn't thinking about selling them. I was thinking about buying them.

I mean, for example, I could have bought the Journey tickets as an investment, like a building. I could have bought the Journey tickets and said, well, look, I'm going to pay this for them because I can get that for my plan to sell them. But I wasn't planning to sell them. I just bought them. I thought I was done. Then things changed.

So from my new perspective, they were sunk. That's a good question, good questions. Yeah?

**AUDIENCE:** Sunk costs are basically goods that you didn't want to sell in the first place?

**JONATHAN GRUBER:** Sunk costs are things, investments you make. You decide, I want to go to med school. I want to go to Journey. And you make those decisions, and then things change, or they don't change. But the point is, whatever you want to do next, it's irrelevant that you made that decision. You already made it. Now the question is, what to do next?

It's not an easy concept, but it's an important one, yeah?

**AUDIENCE:** [INAUDIBLE] like a type of investment where you can use [INAUDIBLE]

**JONATHAN GRUBER:** Oh, you use it. Buying Journey tickets was a kind of investment that I would have used to go to Journey. The difference if I decide I don't want to be a doctor-- let's say being a doctor is composed of three things. Going to med school, buying an office, and working every day. If I decide I want to be a doctor, I could stop working every day. I can sell the office, but I can't un-go to med school. It's done.

So my decision tomorrow, do I want to stay being a doctor? It's irrelevant if I went to med school. I just say, well, how much do I like it, and how much money I'm making relative to the cost of the office and the cost of my time and working every day, OK.

Guys, these are great questions because this is exactly the kind of concept that I want you to be able to think differently about coming out of this course. yeah?

**AUDIENCE:** Wait until that concert, at least for sunk cost, does intent matter?

**JONATHAN GRUBER:** What's that?

**AUDIENCE:** Does intent matter?

**JONATHAN GRUBER:** Well, intent matters in whether we label it fixed or sunk. That was the question here. Intent matters-- if you intend to eventually use it and sell it, then it's fixed. If it can't be sold, or you don't intend to sell it, then it's sunk. But that's a fuzzy line.

**AUDIENCE:** OK.

**JONATHAN  
GRUBER:**

All right, great discussion, let's go on. So now, let's talk about the long run versus the short run, the long run versus the short run. The difference in the long run-- let's put sunk costs aside now. Having introduced, let's put them aside, OK. That's more just a fun thing for you to be thinking about in your life,

But for most of this class, we'll talk about costs being fixed or variable, not sunk. So let's go back to our just fixed and variable costs. And we want to ask, how do costs differ in the long run to short run? And here, there's a relatively simple but important intuition, which is that long-run costs will always be at the lower bound of short-run costs.

The cost of production and long run are always the same or lower than the cost of production in the short run. To see this, let's go to figure 6-2. This is a hard intuition. Let's go. Figure 6-2.

In this figure, there's a firm that has three possible scales of production. Think of this as three different  $k$  bars they can choose. And the way production work, is in the short run, you choose a  $k$  bar, you then decide how much  $l$  to use, and that's what you produce. But in the long run you can change your  $k$  bar.

So think of a firm who has three different  $k$  bar options, and those three different  $k$  bar options give three different average cost curves. Because you can see, if I change  $k$  bar from 1 to 2, my average cost would change. My average cost would no longer be-- would no longer be  $10 \text{ over } q \text{ plus } 5q$ . My average costs would be  $20 \text{ over } q \text{ plus } 5q$  if  $k$  bar changed.

So think of these three curves as three different  $k$  bars. Now, the firm, in the short run, has to choose which one it wants. So for example, SRAC 1, that's Short Run Aggregate Cost curve 1. That would be the average cost curve they choose. If they think they're not going to produce that much. They think they don't have much demand for their good.

As opposed to SRAC 3, that's the cost curve they'd choose if they think there's going to be a lot of production. They think there will be a lot of demand for the good. So think of this about how big you build the plant.

If you think people aren't going to want much of your good, you build a small plant. You think they're going to want a lot of your good, you build a big plant. and SRAC 2 is in between. So in other words, if the firm wants to produce the amount little  $q_1$ , the most efficient  $k$  bar to choose is the  $k$  bar that yields SRAC 1. If the firm wants to produce little  $q_3$ , the most efficient  $k$  bar to choose is the  $k$  bar that yields SRAC 3. But in the long run, no matter what, since it can re-optimize over  $k$ , it can always do better than the short run, or at least no worse.

So if it turns out then the long run demand is  $q_1$ , and you chose SRAC 1, great, the long-run costs is the same as the short-run costs. But if by mistake you chose SRAC 3, you can do better in the long run by re-optimizing your capital. It's just like any general scientific rule, the more variables you have to toy with, the more you can optimize. OK. Here, you can toy with two variables, labor and capital, so you can do better in terms of minimizing costs.

This is not just a theoretical concept. Let's talk about one of the most important companies in America today, which is Tesla. When Tesla was founded in 2013, Elon Musk had to decide how large a battery production plant to build. He built a plant which was the size such that he would sell 20,000 cars in 2017. He figured his demand would be 20,000 cars in 2017.

So he built a plant, let's call it SRAC 1 because he expected  $q$  little 1. What happened was he got  $q$  little 3. By 2017, there was a 200,000 person waiting list for Teslas because they couldn't produce the batteries fast enough.

So Elon Musk realized he'd built too small a plant. So he then has built the largest electric battery plant in the world in Nevada. He's gone from SRAC 1 to SRAC 3. This plant can produce 500,000 cars a year.

But what if people decide Elon Musk is an idiot, and they don't want to buy Tesla's anymore? And all of a sudden, Tesla sales fall. Well, that one could be too big. And maybe next time he'll sell that plant and build one in between. The point is, he can always re-optimize, depending on how demand turns out to be, so he can always do better.

Elon Musk did not produce most efficiently in the short run, so he built a bigger plant. Will he now produce most efficiently? It depends on whether the demands is up at  $q_3$  or somewhere else. OK, questions about that?

So basically the point is, in the long run, it's just like the short run, except you get to choose  $k$  bar, which means you get to take multiple shots at what average-- but having chosen  $k$  bar, you're back in the short run. All you can do is vary the number of workers.

So think of the long run as a set of short-run games, and each short run, you choose  $k$  bar, you produce as efficiently as possible. If it turns out you've chosen the wrong  $k$  bar, you then choose a different one the next period, the next long-run period, OK.

So now let's talk about long-run costs. Let's talk about how we think about firm production decisions and costs in a world where both labor and capital is variable. So remember last time, we developed an isoquant. The isoquant was the choice of all  $l$  and  $k$  that give the same little  $q$ , the choice of all  $l$  and  $k$  that give the same little  $q$  was the isoquant.

And you could choose. And basically the point was, along any given isoquant, there are different combinations of  $l$  and  $k$  that can give you little  $q$ , just like along an indifference curve, there are many combinations of pizza and cookies that make you equally happy.

So the question is, how do you decide which combination to use? Well to do so, we create-- just like in consumer theory, we solve the problem by creating a budget constraint. In producer theory, we solve the problem by creating an isocost line.

The isocost line, which is basically the firm's budget constraint, if you will. Isocost line is the line, which gives you the cost of labor and capital that help you choose which combination you'd like to choose. So I'm not saying it well in words. Let's go to the graphs.

OK, here's some isocost lines. Remember, I assumed that the wage was 5 and the rental rate of capital was 10. So basically, these lines are combinations of capital and labor that yield the same total level of costs. The combination of capital and labor that yields the same total level of costs.

So you see the \$50 isocost, says if you want to spend \$50, you could either have 10 units of labor and no capital, 5 units of capital and no labor, or some combination in between. But if you want to spend \$150, you could then have 30 units of labor and no capital, or 15 units of capital and no labor, or some combination in between.



So it's basically the combination of capital and labor that yields the same total cost. And what is the slope of the isocost line? Well, the slope is minus  $w$  over  $r$ , minus the ratio of the price of labor to the price of capital. What does this say? This says that you have to give up half a unit of capital to get a unit of labor. It's back to opportunity costs.

For a given total cost of production, if you want one more worker, you're going to have to give up half a machine, for a given total cost of production. That's the slope of the isocost line. So then we say the firm, to minimize costs will use the efficient decent combination of labor and capital to produce any level of goods.

And how do we get that? We get that by the tangency between the isoquant and the isocost. The tangent between the isoquant and the isocost, gives us the efficient combination of capital and labor for any level of production little  $q$ .

So for example, keeping these isocost lines and going back to our production function, and our therefore, cost function, we can draw the isoquant for production. So we start with the production function. We start with our production function  $q$  equals square root of  $L$  times  $K$ .

We know that the slope of the isoquant, the isoquant slope we defined last time, is minus  $MPL$  over  $MPK$ ,  $MPL$  over  $MPK$ . So at the tangency we set these two things equal. We set  $MPL$  over  $MPK$  equal to  $w$  over  $r$ .

That is, in the optimum, with this example, you set these things equal, you always set the slope of the isocost line equal to the slope of the isoquant. But in this example, that is such that you want to use half as much capital as labor. You want to basically use-- because basically, if you look at  $MPL$ , if you use our example, we defined last time, setting these equal says, remember from last time, the  $MPL$  was minus  $L$  over  $6$ . Here, we're saying, till equal to minus  $w$  over  $r$ .

If you remember last time, we solved for the marginal, the  $MRTS$ . I'm sorry,  $K$  over  $L$ ,  $K$  over  $L$ ,  $K$  over  $L$ . So minus  $K$  over  $L$  equals minus  $w$  over  $r$ . Minus  $K$  over  $L$ , we defined last time, as the marginal rate of technical substitution, which is generally this formula, but in our specific case, it's that. Minus  $w/r$  is the price ratio.

The marginal rate of technical substitution is the slope of the isoquant. Minus  $w$  over  $r$  is the slope of the price ratio. Here, in our example, we know that equals minus one half. So in this example, you want to set  $K$  over  $L$  equal to minus one half. You want half as much capital as labor.

Why is that? Well, same intuition is with consumers. The market is telling you it's going to cost half as much to have a worker as a machine. Your technology is telling you that you're basically indifferent, that basically at the point you're at, you're basically-- the trade-off between workers and machines is  $K$  over  $L$ .

So you set those equal, and that gives you the rule that you want to set in our particular example,  $K$  over  $L$  is equal to minus  $w$  over  $2$ . So let me review again. A, the isoquant curve comes from the production function and delivers the marginal rate of technical substitution, which is this, step one.

Step two, the isocost has a slope of this. Step three, you set them equal, so you get minus  $K$  over  $L$  which is what this is, equals minus one half, which is what that is. So you use capital and labor until using half as much capital as labor.

And once again, you can use the same kind of examples we use for consumer theory to show you why. If you weren't using half as much capital as labor, you'd be getting it wrong. You saw yourself at home-- remember last time, we had points where the indifference curve wasn't tangent to the budget constraint, we showed why they were wrong?

You can do the same thing here. You can draw points where the isoquant is not tangent to the isocost and show you why that's inefficient. Why it's inefficient production. Now, questions about that? Yeah?

**AUDIENCE:** What are the substance for M?

**JONATHAN GRUBER:** Oh that's a K, oh, Jesus. Let me make sure I get the inversion right. It's MPL over MPK, sorry about that. That's not any better, MPK, MPL over MPK. Thank you for asking. Once again, 70% of the class had no idea what those were, so you did a good service. Yeah, go ahead.

**AUDIENCE:** [INAUDIBLE] set MRTS has only for this situation?

**JONATHAN GRUBER:** For this example, yes. The general formula is you set the MRTS equal to the input price ratio. That's the general formula. So the general formula is you set minus MPL over MPK equal to minus w over r. That's the general formula. In our particular case, that amounts to K over L equals one half. Yeah?

**AUDIENCE:** So [INAUDIBLE] you're restructuring what cost and what quantity you're producing. Is there a version of the infinite substitution [INAUDIBLE]

**JONATHAN GRUBER:** That's a really interesting question that we won't really get into. I think the difference is-- actually, I will teach you why I won't get into it. It's a good segue to why firms are not the same as consumers, and I'll talk about that in a minute. Other questions?

OK, well, that's a great segue, which is, when we did consumer theory, we were done here. We said, well, I just find the point where this is true, that's the optimal bundle. And then in come the substitution effects and all that, as was pointed out.

This is why producer theory is harder because we're not done. Why aren't we done? Because I didn't tell you the right isocost. With consumer theory, I gave you a budget constraint. I'm not giving you an isocost here. I'm just saying there are multiple possible isocosts. Therefore, there are multiple possible tangencies.

So with consumer theory, I gave you a budget constraint, and therefore, I told you two equations, two unknowns gives you the answer. Here, two equations, two unknowns does not give you the answer. It just gives you a formula. But how you apply that formula is going to depend on how big you want your firm to be, which isocost you want to choose. Well, how the hell are you going to choose that? That's where you have to add one more step. That's where producer theory gets one step harder.

To see that, let's look at figure 6-5a. Figure 6-5a, shows for different isocosts, how much capital and labor we want to use, for our example. So the previous example, we were at point a. We were choosing to spend \$50. That allowed us to produce square root of 12.5 units at a tangency of 5 units of labor and 2 and 1/2 units of capital. You flip back and forth to figure 6-4, you'll see that's where we were, right?

However, I could have said instead, no, I want to be bigger. I want to make more. I'm going to choose to spend 100. Then I would be at point b, where I could have 20 units. Not 100, square root of 50, it's four times-- yeah, yeah, I'm going to spend 200.

So basically here, if I spend 200, I can have 20 workers and 10 machines. If I spend 220 workers at 10 machines, and then I'll produce square root of 50. Which is right? I don't know. I haven't told you yet. How do you choose as a boss?

Remember, capital is fixed. This is not the Elon Musk example. It's not capital. This is within the short run. This is how many workers do you hire? Well, you don't know. You could hire 10 workers, 20 workers, 30 workers. It depends how much you want to produce.

How much do you want to produce? That is the extra step we have to add for producer theory. We call this the long-run expansion path. This is essentially the combinations of labor and capital, at every single  $q$  that you would choose.

In other words, you can map out-- I haven't told you which  $q$  you're going to choose, but I've told you how to map out, for each  $q$ , how much  $L$  and  $K$  you want to use. If you decide the  $q$  you want is square root of 12.5, you choose 10 workers and 5 machines. If you decide it's a square root 112.5, you choose 30 workers and 15 machines. Ultimately, the question is, how much you want to produce?

Now, in this example, the long-run expansion path is linear, but it doesn't have to be. For example, consider figure 6-5.b. This is a long-run expansion path where capital is becoming less productive as you produce more. OK, in 6-5a, no matter how much you produce, you always want to use half as much capital as labor. That ratio never changes.

In 6-5b, it does change. That means it's a different production function. I haven't written down the underlying production function for 6-5b, but it's a different production function. The isocosts are the same. The cost of labor and capital, labor is always half the price of capital.

But it's a different production function which has the feature that as you produce more and more capital, becomes less productive. You might think of this as something like a McDonald's long-run expansion path. Once they've got the fry-o-later and the building, then what really matters is how many more workers you have, you employ there, cranking stuff through.

So that's a case where, in some sense, each additional unit of capital, the fry-o-later isn't that productive. It's really labor which is really the thing which you want more and more of as you expand.

Alternatively, you could have something like figure 6-5c, which is an expansion path where capital becomes more productive and labor becomes less productive. So that is, you are choosing, relatively, more and more capital compared to labor.

So let's compare 6-5b to 6-5c. In 6-5b, when you go from point a to point c, you go from 4 workers to 15 workers but only 3 machines to 7 and 1/2 machines. OK, go from point a to c in 6-5b, The number of workers goes up almost four times. The number of machines only goes up 2 and 1/2 times.

On the other hand, with figure 6-5c, the number of workers going from a to c goes from 7 to 16, which is a little more than twice. But our machines goes from 1.5 to 7, which is almost five times as much. Capital is becoming more productive, workers are becoming less productive in 6-5c. This might be something you might have for a heavy machinery plant, where over time, you want more and more machines, and you need fewer and fewer workers to run them.

The point of this is not to say that I can tell you what these specific examples are, and each of these obviously have different production functions under them. It's just to show you that long-run expansion paths can vary in the real world. It's basically the question of, as you want to produce more, does labor and capital become equally more productive? OK, questions about that? Yeah?

**AUDIENCE:** So let's say around about 80% of startups that fail. Do I choose mine shutdown if I can't jump from 80%

**JONATHAN GRUBER:** I haven't talked about shutdown yet. You're a bit ahead of me. That is related to how you're going to decide on your  $q$ . And I'm going to get to that next lecture.

So I haven't told you how you choose  $q$ , but next lecture, I will. I'm going to introduce the one extra step that lets you choose  $q$ . But before I do, I want to ask an existential question, which is how do we actually measure costs in the real world?

Step 4 measuring costs, and this is where I get to tell you that economics is just cooler than accounting. And let me explain why. Accounting only considers the cash-flow costs that you have when you start a firm. Economics more appropriately considers opportunity costs, as well.

So profits is revenue minus cost. We're going to focus on the costs here. Costs in accounting are literally what you spend. In economics, they're what your opportunity cost is. And those could differ.

So let me give you a simple example. You're going to start a website design firm when you graduate. It's kind out of date but whatever. OK, you start a website design firm when you graduate. And that involves you working full time, plus some slave programmer you can hire for \$40,000 a year. So you're going to work full time and hire the programmer for \$40,000 a year.

Moreover, you don't even need to buy a computer for the programmer because you have an extra computer that's got one more year before it becomes useless. So you can give that computer to the programmer, and they can use that for the year.

So you can have a one-year company. You're going to work full time. You're going to pay your programmer \$40,000, and you give your programmer this computer you have that has one more year to work.

And let's say your goal is, by the end of year one I want to be making a profit. My parents will be mad at me if I take more than one year at this. By the end of year one, I want to make a profit.

And let's say at the end of year one, your revenues are \$60,000. We can say to your parents, who are accountants, well, gee, I earned \$60,000. I paid my programmer \$40,000. That's my cost. So I made profit of \$20,000. See, my company is great.

Your accounting parents-- let's say you have an accounting mom and an economics dad. The accounting mom is fooled, but the economics dad is not. Because what does economics dad point out? Yeah?

**AUDIENCE:** [INAUDIBLE]

**JONATHAN GRUBER:** Well, go on, let's talk about your time.

**AUDIENCE:** [INAUDIBLE] even though [INAUDIBLE]

**JONATHAN GRUBER:** There's an opportunity cost of the year you spent with this firm, and you're an MIT graduate, for God's sake. Who knows? You could have made-- I don't know, 100K, 200K. Who knows what you guys make these days?

OK, let's say you could have made 100K. You did not make 100K by starting this firm. That is a true economic cost. It's not an accounting cost, but it's a true economic cost. It's an opportunity cost. You gave up 100K to do this.

Moreover, that m instead of one year left, you could have sold it for something. There's an opportunity cost to giving it to this guy. You could have sold it. Let's say you could have sold it for \$1,000. So your opportunity costs are 101,000 plus the 40,000, Your total costs are 141,000. This was a terrible idea. Your total cost of 141,000, you've lost \$81,000.

OK, opportunity costs are real costs. Opportunity costs are the value of the best-- the next best forgone alternative. Remember, that's why we call economics the dismal science. We point out nothing is free. Your time wasn't free. The time you dedicate to this firm meant you could have been doing something else. And that's not free.

And as a result, you are not making money in this company, even if you think you are. It's not saying we shouldn't do startups that lose money in the first year. Obviously, most startups lose money in the first year. We'll get to that later in the semester, about how you think about companies that lose money in the first year, make money in the second year. We'll come back to that.

But in this simple example, you shouldn't fool yourself to think that you actually made money because of the opportunity cost. OK, questions about that? All right, let's stop there. Thanks again for all the great questions today. And we'll come back Wednesday and figure out how we pick our q.