12.810 Problem set 3

Need help?: Office hours Thursdays 1.30-2.30pm in 54-1712.

Collaboration is allowed, but write up the solution on your own. Show all work. Give units for all numerical results. Put axis labels and units on any graphs.

1. It has been argued that polar amplified warming has slowed down the jet stream and the eastward propagation of Rossby waves, and that this has increased the duration of extreme events. One estimate suggests that the zonal-mean zonal wind at 500hPa averaged over 30-70N decreased in speed by roughly 0.2 m/s/decade over the period 1980-2011 for the October-November-December (OND) season. In this question, we will use the shallow-water QG Rossby-wave dispersion relation to investigate possible changes in the phase speeds of Rossby waves as a result of this change in zonal wind.

   (a) Calculate the Rossby wave phase speed in the zonal direction in m/s and the intrinsic period in days (based on the intrinsic frequency) for a wave at 45N, with equivalent depth 10km, and with 'zonal wavenumber' of 5, and assuming a basic state zonal wind of 20 m/s. Note that when a zonal wavenumber is given as a dimensionless integer m, this means that there are m wavelengths in a latitude circle at that latitude. The zonal wavenumber k considered in class is different and has units of m^{-1}. You should assume that the wave has the same wavelength in the x and y directions such that $k = l$.

   (b) Make an estimate for the trend in the zonal phase speed based on the observed trend in the mean zonal wind for OND that was mentioned above. Don't forget to include changes in the meridional PV gradient. Is this a large trend as a percentage of the phase speed?

2. In class, we derived the Rossby-wave dispersion relation for a plane wave in the shallow-water quasigeostrophic equations on the midlatitude beta plane for waves of small amplitude. In this question you will extend the analysis to waves that are not of small amplitude.

   (a) Starting with the evolution equation for the shallow-water QGPV as derived in class ($\frac{Dq}{Dt} = 0$), derive an equation for $\frac{\partial q'}{\partial t}$ that includes all nonlinear terms and does not assume small amplitude perturbations. Here $q'(x, y, t)$ is the perturbation QGPV, $q_0(y)$ is the basic state QGPV, and the total QGPV is $q = q_0 + q'$. The horizontal wind in the basic state is $(u_0, 0)$, where u_0 is a constant.

   (b) Suppose that the perturbation streamfunction takes the form

   $$\psi' = A\cos(kx + ly - \omega t) + B\sin(kx + ly - \omega t)$$

   where A and B are real constants. Show that this plane wave is actually a solution of the full nonlinear equation. Explain in words why this is the case in terms of contours of the streamfunction and QGPV.

   (c) Is the sum of two plane waves with *different* wavenumbers a solution of the nonlinear equation?

3. Suppose there is a spherical *anticyclonic* anomaly of PV in the middle troposphere in the northern hemisphere. Assume that the background state is statically stable and motionless. Make a sketch in the x-z plane that illustrates the anomaly, the induced flow into and out of the page, and the contours of constant potential temperature. Make sure that your sketch shows the effect of the anomaly at all levels. Mark on the sketch whether there is increased or decreased static stability, and positive or negative relative vorticity in the following locations: in the anomaly, to the east and west of the anomaly, and above and below the anomaly. *Explain your reasoning in each case.*