**12.010 HW03**

***Evaluation of Bessel functions:***

The Bessel functions of the first kind $\mathrm{J_n(x)}$ are defined as the solutions to the Bessel differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^{2-n^2}\right)y=0$$

where $n$ is referred to as the order. For this problem set, n will be $\geq 0$, and $x$ will be real and $\geq 0$. Bessel functions are often encountered in problems involving cylindrical coordinates.

Bessel functions can be computed from the series expansion:

$$J_n(x)=\sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,(n+m)!}\left(\frac{x}{2}\right)^{(n+2m)}$$

The functions can also be solved recursively using the recursive relationship

$$J_{n+1}(x)=\frac{2n}{x}J_n(x)-J_{n-1}(x)$$

so that once $J_0(x)$ and $\mathrm{J_1(x)}$ have been generated, all higher order terms can be generated.

The functions can also be computed from the following integral:

$$J_n(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta-n\theta)\,d\theta$$

Equations from: https://mathworld.wolfram.com/BesselFunctionoftheFirstKind.html and https://www.math.colostate.edu/~shipman/47/volume2a2010/Sekeljik.pdf

Questions: Each part is worth 20-pts.

(1) In this problem, you will compute the Bessel functions of the first kind by using the SciPy special function and through the solution to the the series expansion, the recursive relationship, the $\theta$ integral and differential equation. You will compute these functions for *n*=0:5 and *x*=0:0.5:10. Looking at the ranges over which the function will be computed, explain the issues you see that need to be considered in computing the functions to 6 significant digits. Answer this question in a markdown cell with equations. This part can be answered in a Markdown cell at the start of the notebook. As a reference set of values, compute the Bessel function using the SciPy package and show results in a table and as a plot. (Bessel functions of the first kind for n=0:5 and x from 0:0.5:10). The table should be in increments of 0.5, while the plot should be generated at a finer resolution. Look at tabulate for creating the table.

(2) Create the table and plot as in part (1) using the series expansion above taking care of the limits on the summation given that x/2>1. The table should be given with six significant digits for $J_n(x)$ and the plot should be at a finer resolution than the 0.5 spacing in the table.

(3) Repeat (2), but use the recursion relationship and ensure your results are accurate to 6 significant digits. $J_0(x)$ and $J_1(x)$ can be computed with SciPy special function or using the function created in part (2).

(4) Repeat (2), but use the $\theta$ integral and ensure your results are accurate to 6 significant digits.

(5) Repeat (2), but solve the differential equation and ensure your results are accurate to 6 significant digits. Python ordinary differential equation solution codes can be used. The initial conditions for the differential equation ($J_n(x_0)$ and ${dJ_n(x_0)}/{dx}$) can be computed using the SciPy package.

All parts should generate the same results. *Optional:* Look at the differences between parts (2)-(5) and part (1) (The SciPy generated are likely the most accurate result). All of your results should be accurate to at least six significant digits.

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