Problem Set 6

1. The Rössler system is given by

$$\dot{x} = -y - z \tag{1}$$
$$\dot{y} = x + ay \tag{2}$$
$$\dot{z} = b + z(x - c). \tag{3}$$

We take the parameters $a = b = 0.2, c = 5.7$.

(a) Use `roessler` to simulate the time evolution of these equations, using the provided initial condition. Plot time series[1] of $x(t), y(t), z(t)$, and comment on the similarities and differences.

(b) Calculate power spectra[2] of x, y, z (again log-log scales help), and comment on the similarities and differences. Are the spectra sparse or dense? What does this mean?

(c) Using the time series data you just obtained, make a Poincaré section in the plane $x = 0$, and plot your result. You should see some order emerge out of the chaos.

Feel free to do this any way you like, but here is one suggestion: make a new array that contains only the points where $|x(t)| < 0.1$ (because our numerical integration uses discrete timesteps, the data points themselves won't intersect $x = 0$ exactly). You can do this in Python by writing something like

```
x_intersect = x[np.abs(x[:,0])<0.1,:],
```

or in Matlab by writing

```
x_intersect = x(abs(x(:,1))<0.1,:).
```

This gives you a sequence of (x, y, z) values for where the flow intersects with $x = 0$.

(d) Now assume that $x(t)$ is experimental data, and that you know nothing else about the system (i.e. you don't know $y(t), z(t)$, or the underlying equations). Use the method of delays[3] to reconstruct the geometry of the attractor in 3D phase space.

[1] We suggest using an end time t_{end} of order 1000.

[2] Use your power spectrum code from last week. Please don't hesitate to ask for help if you need it!

[3] i.e. plotting $x(t), x(t + \tau), x(t + 2\tau)$ for some constant τ large enough for "independence".