10.40 Thermodynamics **Fall 2003**

Problem Set 2

Problem 4.18 Text

(Image removed due to copyright considerations.)

Solution:

(a)

Assumptions:

Ideal gas

$C_p, C_v \neq f(T)$

Isothermal process (Diathermal boundaries)

Quasi-static process (air is well mixed)

This problem uses the theory and equations discussed in Section 4.8 of the book, and follows Example 4.6 closely. The major difference is that rather than having a constant outlet pressure from the compressor, both the inlet AND outlet compressor pressures change as the compressor moves air from one tank to the other. We have assumed that the compressor moves air from tank B to tank A, so that the final pressure of tank A is 3 bar. From Eq. (4.66)…

$$\boxed{\frac{\delta W}{\delta N_{eng}} = +\int_{P_B}^{P_A} VdP = +RT \ln \frac{P_A}{P_B}}$$

The key to this problem is to realize that the number of moles going through the compressor is equal to the moles that enter A, which equals the number of moles that exit B.

$$\delta N_{eng} = dN_A = (\underline{V}_A / RT)\, dP_A = -dN_B = -(\underline{V}_B / RT)\, dP_B$$

Therefore,

$$\delta W = +[\underline{V}_A \text{ In } P_A\, dP_A + \underline{V}_B \text{ In } P_B\, dP_B$$

by a material balance $P_A\underline{V}_A + P_B\underline{V}_B = P_{A_o}\underline{V}_A + P_{B_o}\underline{V}_B$

since $\underline{V}_A = \underline{V}_B$, $P_B = P_{A_o} + P_{B_o} - P_A$. Thus

$$W = +\underline{V}\left[\int_2^3 \text{In } P_A\, dP_A + \int_2^1 \text{In } P_B\, dP_B\right]$$

$$= +\underline{V}\left[(P_A \text{ In } P_A - P_A)\Big|_2^3 + (P_B \text{ In } P_B - P_B)\Big|_2^1\right]$$

$$= +\,0.1 \times 10^5\,[3 \text{ In } 3 - 3 - 2 \text{ In } 2 + 2 + 1 \text{ In } 1 - 1 - 2 \text{ In } 2 + 2]$$

$$W = +\,5.23 \times 10^3 \text{ J}$$

(b)

There are several ways to do part (b), but the simplest is a 1st Law Energy Balance using the combined gases in tanks A and B as the system

For an ideal gas, $\Delta\underline{U} = 0$ (T = const.) as $W = -Q$.

$$Q = -5.23 \times 10^3 \text{ J}$$