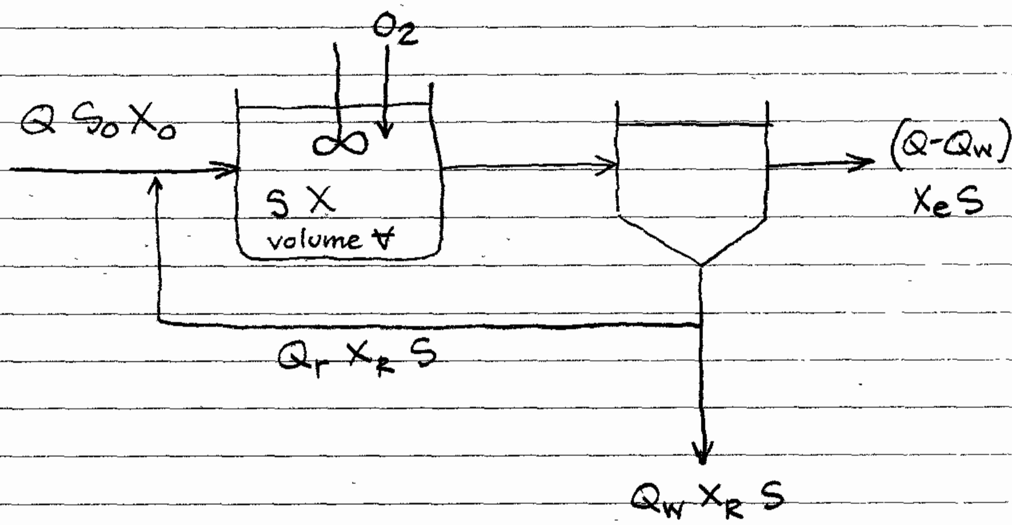
Lecture 19 - Reactor Modeling and Activated Sludge Treatment

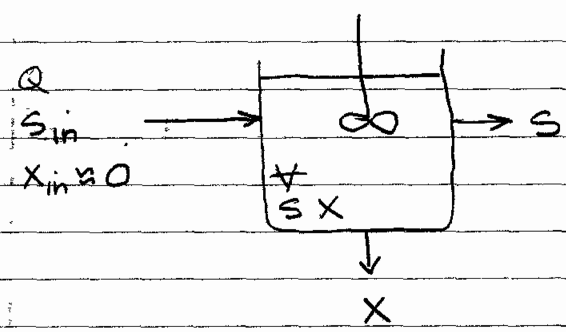
Why are we interested in modeling substrate and microbiota?

To construct good mass balances over treatment systems for design and operational analysis

Example: activated sludge treatment



Consider first a simpler system: FMT with Monod kinetics:



$$t_R = \frac{\forall}{Q}$$

remember $r_{su} < 0$

$$\frac{dS}{dt} = \frac{Q}{\forall}(S_{in} - S) + r_{su} \quad (1)$$

$$= \frac{Q}{\forall}(S_{in} - S) - \frac{\mu_g}{Y} X \quad (2)$$

$$= \frac{1}{t_R}(S_{in} - S) - \frac{\mu_g}{Y} X \quad (3)$$

$$\frac{dX}{dt} = -\frac{Q}{\forall}X + r_g = -\frac{X}{t_R} + \mu X$$

$$= -\frac{X}{t_R} + \mu_g X - K_d X \qquad (4)$$

Consider steady state and to simplify, assume $K_d = 0$

$$\frac{dS}{dt} = 0 \longrightarrow \frac{S_{in} - S}{t_R} = \frac{\mu_g}{Y} X \qquad (5)$$

$$\frac{dX}{dt} = 0 \longrightarrow \frac{X}{t_R} = \mu_g X \qquad (6)$$

Divide by X: $\mu_g = \frac{1}{t_R}$ (7)

Substitute μ_g into Eq. 5

$$Y(S_{in} - S) = X \qquad (8)$$

$$\mu_g = \mu_{max}\left(\frac{S}{K_s + S}\right) = \frac{1}{t_R} \qquad (9)$$

Rearrange:

$$S = \frac{K_s}{\mu_{max} t_R - 1} \qquad (10)$$

Sub into (8):

$$X = Y\left(S_{in} - \frac{K_s}{\mu_{max} t_R - 1}\right) \qquad (11)$$

Treatment efficiency $Eff = \frac{S_{in} - S}{S_{in}} = \frac{X}{Y S_{in}}$

$$= 1 - \frac{K_s / S_{in}}{\mu_{max} t_R - 1} \qquad (12)$$

Example: BOD $K_s = 100$ mg/L

$S_{in} = 300$ mg/L

$\mu_{max} = 5 \text{ day}^{-1}$

For $t_R = 8$ hr, Eff = 50%

16 hr, Eff = 86%

Efficiency is highly dependent on residence time

Efficiency increased, dependence on t_R decreased if X is increased by sludge recycling

Note what happens if $t_R = \frac{K_s + S_{in}}{\mu_{max} S_{in}} = t_W$?

$$X = Y\left(S_{in} - \frac{K_s}{\mu_{max} t_R - 1}\right) \quad \text{(11 from above)}$$

$$= Y\left(S_{in} - \frac{K_s}{\cancel{\mu_{max}}\left(\frac{K_s + S_{in}}{\cancel{\mu_{max}} S_{in}}\right) - 1}\right)$$

$$= Y\left(S_{in} - \frac{\cancel{K_s}}{\frac{\cancel{K_s}}{S_{in}} + \frac{\cancel{S_{in}}}{\cancel{S_{in}}} - \cancel{1}}\right) = 0$$

$S = S_i$

"Wash out" condition – bugs are washed out of reactor before they can grow

Note $t_W = \frac{1}{\mu_{max}}\left(\frac{K_s}{S_{in}} + 1\right) \approx \frac{1}{\mu_{max}}$ if $S_{in} \gg K_s$

Slightly more complicated model - do not assume $k_d = 0$
For FMT with $X_{in} = 0$:

$$\frac{dS}{dt} = \frac{Q}{\forall}(S_{in} - S) - \frac{\mu_g}{Y} X = 0 \text{ for steady state} \qquad (13)$$

$$\frac{dX}{dt} = -\frac{Q}{\forall} X + \mu_g X - k_d X = 0 \qquad (14)$$

from Equation 14, solution is either $X = 0$ (boring!)

$$\text{or} \quad -\frac{Q}{\forall} + \mu_g - k_d = 0 \qquad (15)$$

$$\mu_g = \mu_{max}\left(\frac{S}{K_s + S}\right) = \frac{1}{t_R} + k_d \qquad (16)$$

This yields steady-state solution for S:

$$S = \frac{(1 + k_d t_R) K_s}{\mu_{max} t_R - 1 - k_d t_R} \qquad (17)$$

Note that soln is independent of S_{in}

μ_{max}, k_d, K_s are a function of the "bugs" and not design variables

t_R is only design variable available

substitute into $\frac{dS}{dt}$ equation to solve for X:

$$X = Y\left[S_{in} - \frac{K_s}{(\mu_{max} t_R - 1)}\right]\frac{1}{(1 + k_d t_R)} \qquad (18)$$

Solution depends on S_{in}

Efficiency $\qquad \dfrac{S_{in} - S}{S_{in}} = \dfrac{(1 + K_d t_R) K_s / S_{in}}{\mu_{max} t_R - 1 - K_d t_R}$ $\qquad (19)$

Wash-out condition $\quad (X \to 0)$

Set Eq 18 to zero:

$$S_{in} = \frac{K_s}{(\mu_{max} t_R - 1)} \qquad (20)$$

$$t_R = t_W = \frac{S_{in} + K_s}{S_{in} \mu_{max}} \qquad (21)$$

$$= \frac{1}{\mu_{max}} \left(1 + \frac{K_s}{S_{in}} \right)$$

Graph on page 6 shows S, X, eff. vs. t_R for typical values:

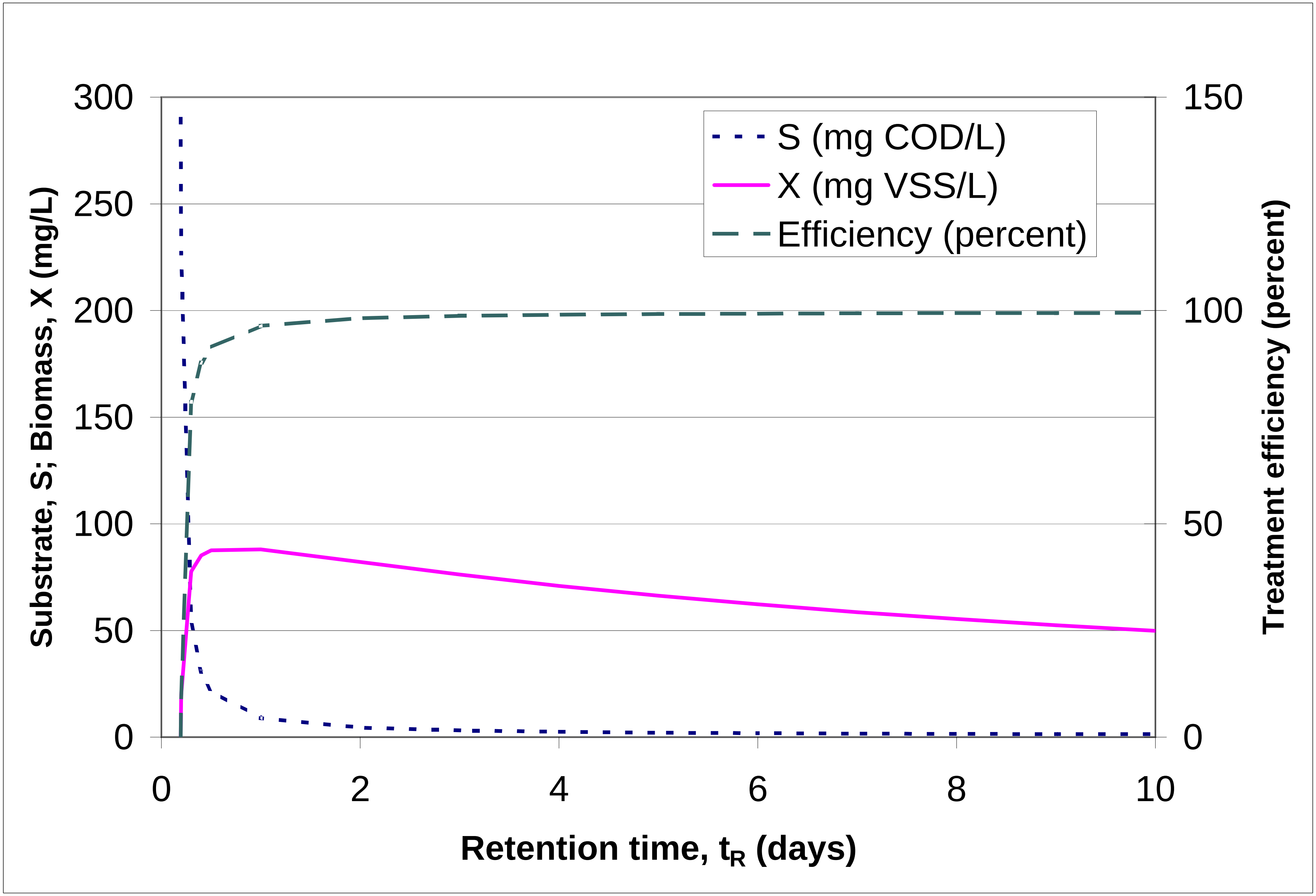
K_s = 40 mg COD/L
K_d = 0.1 day^{-1}
μ_{max} = 6 day^{-1}
S_I = 250 mg COD/L
Y = 0.4 mg VSS/mg COD

Observations –

t_W = 0.19 days

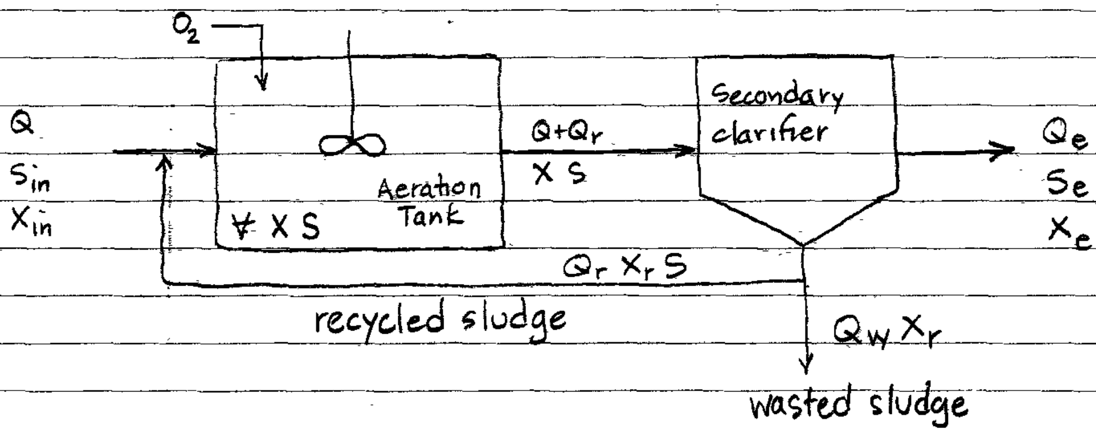
X has maximum
and then declines

Efficiency continues to increase with t_R but with marginal improvement after $t_R \approx 1$ day



S (mg COD/L)
X (mg VSS/L)
Efficiency (percent)
Substrate, S; Biomass, X (mg/L)
Treatment efficiency (percent)
Retention time, t_R (days)
300
250
200
150
100
50
0
150
100
50
0
0
2
4
6
8
10

What if cells get recycled?



X = cell concentration (e.g. mg VSS/L)
S = substrate conc. (e.g. mg COD/L)

Above conceptual model assumes that only cells X are settled in secondary clarifier - whatever substrate S is left is soluble and does not settle

Overall mass balances:

$$\forall \frac{dX}{dt} = \underbrace{QX_{in}}_{\text{inflow}} - \underbrace{Q_e X_e}_{\text{outflow}} - \underbrace{Q_W X_W}_{\text{wasted}} + \underbrace{\forall \mu_g X}_{\text{growth}} - \underbrace{\forall k_d X}_{\text{death}} \tag{22}$$

At steady state with $X_{in} = 0$

$$\forall (\mu_g - k_d) X = Q_e X_e + Q_W X_W = \underbrace{P}_{\text{sludge production}} \tag{23}$$

We can define two parameters for design or evaluating system operation:

1. Food : microorganism ratio, F/M
(also called substrate removal velocity, U)

$$U = \frac{F}{M} = \frac{\text{substrate used for cell growth / unit time}}{\text{unit mass of cells}}$$

$$= \frac{Q(S_{in} - S)}{\forall X} = \frac{S_{in} - S}{t_R X} \qquad (24)$$

Or alternatively,

$$U = \frac{\forall \frac{1}{Y} \mu_{max} \left(\frac{S}{K_s + S}\right) X}{\forall X}$$

$$= \frac{\mu_{max}}{Y} \left(\frac{S}{K_s + S}\right) = \frac{\mu_g}{Y} \qquad (25)$$

Units for $U = \left[\frac{M \text{ substrate}}{M \text{ cells} \cdot T}\right]$ e.g. $\frac{g\ COD}{g\ VSS \cdot day}$

2. Solids retention time SRT θ_c
(also call mean cell residence time or sludge age)

$$\theta_c = \frac{\text{mass of cells}}{\text{change in mass of cells / unit time}}$$

$$= \frac{\forall X}{Q_w X_r + Q_e X_e} \qquad (26)$$

or alternatively,

$$\theta_c = \frac{\forall X}{\forall \left(\mu_{max} \frac{S}{K_s + S} - k_d \right) X}$$

$$= \frac{K_s + S}{\mu_{max} S - k_d (K_s + S)} \qquad (27)$$

$$\frac{1}{\theta_c} = \mu_{max} \frac{S}{K_s + S} - k_d \qquad (28)$$

Based on Eq 25:

$$\frac{1}{\theta_c} = YU - k_d \qquad (29)$$

If we rearrange Eq 27, we get:

$$K_s + S = \mu_{max} \theta_c S - k_d \theta_c (K_s + S)$$

$$S \left(1 - \mu_{max} \theta_c + k_d \theta_c \right) = -K_s - k_d K_s \theta_c$$

$$S = \frac{K_s (1 + k_d \theta_c)}{\mu_{max} \theta_c - k_d \theta_c - 1} \qquad (30)$$

This is the same solution for S as for the FMT (Eq 17) <u>except</u> θ_c has replaced t_R

→ Effect of sludge recycle is to increase effective residence time for treatment

As with FMT, S is not a function of S_{in}

Consider mass balance over secondary clarifier only:

$$(Q+Q_r)X = Q_e X_e + Q_W X_r + Q_r X_r \quad (31)$$

Define $R = \dfrac{Q_r}{Q}$ = Recycle ratio

$$\frac{Q_e X_e + Q_W X_r}{Q} = (1+R)X - RX_r \quad (32)$$

Since from Eq 26 $\theta_c = \dfrac{\forall X}{Q_W X_r + Q_e X_e}$

$$\theta_c = \frac{\forall X}{Q\left[(1+R)X - RX_r\right]}$$

$$= \frac{t_R}{1+R-R(X_r/X)} \quad (33)$$

By managing R and X_r/X, we can get $\theta_c > t_R$ and achieve greater treatment than simple FMT

(Note: V&H textbook uses θ for t_R)

t_R is residence time in aeration tank only

Washout condition

As with FMT, there is a minimum θ_c that will cause a washout condition

At washout, $S = S_{in}$

Plug $S = S_{in}$ into Eq 28:

$$\frac{1}{\theta_{cw}} = \mu_{max} \frac{S_{in}}{K_s + S_{in}} - K_d \qquad (34)$$

At washout $S_{in} >> K_s$

$$\frac{1}{\theta_{cw}} \cong \mu_{max} - K_d \qquad (35)$$

This defines the minimum value of θ_c for treatment

Plants typically operate with safety factor:

$$SF = \frac{\theta_c}{\theta_{cw}} = 2 \text{ to } 20$$

Other limit is very long θ_c

Consider equation 30 for large θ_c:

$$S = \frac{K_s(1 + K_d\theta_c)}{\mu_{max}\theta_c - K_d\theta_c - 1}$$

$$= \frac{K_s\left(\frac{1}{\theta_c} + K_d\right)}{\mu_{max} - K_d - \frac{1}{\theta_c}} \cong \frac{K_d K_s}{\mu_{max} - K_d} \qquad (36)$$