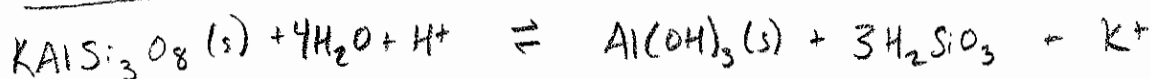
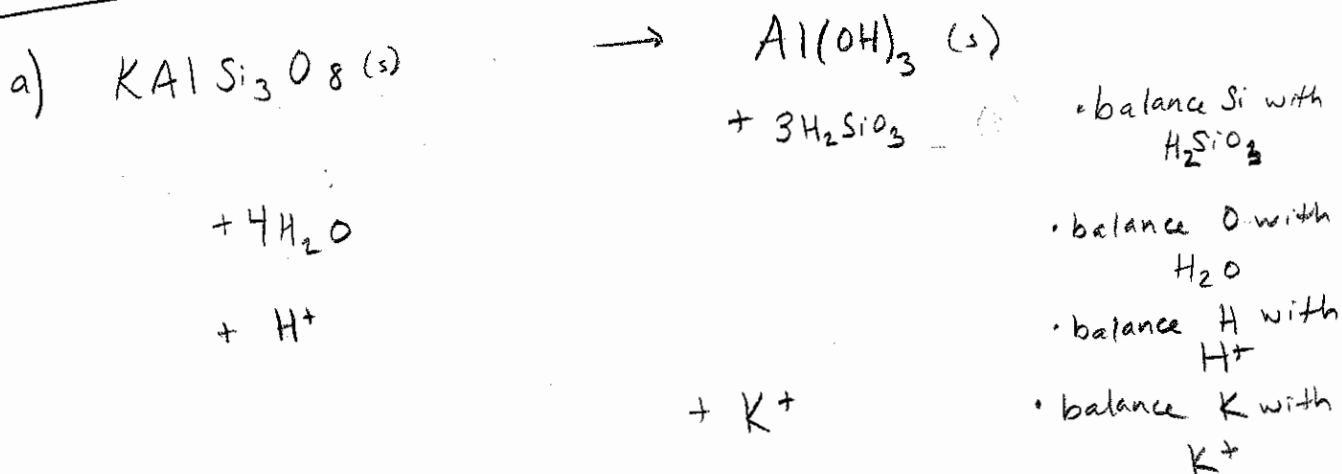


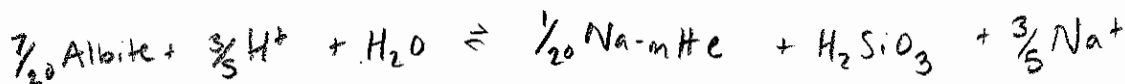
PROBLEM SET #4

1.76 - Aquatic

PROBLEM 1

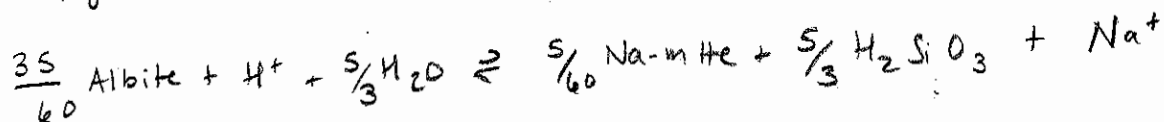


b) Book rxn:

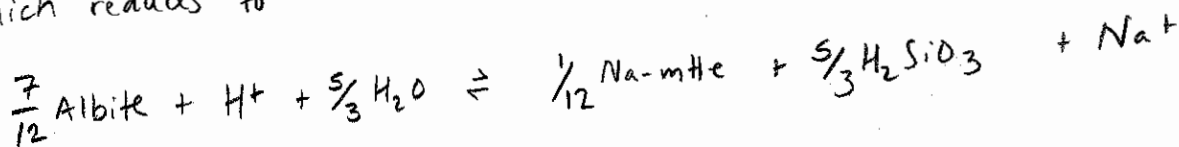


Rewritten so Na^+ , H^+ have coefficients of 1:

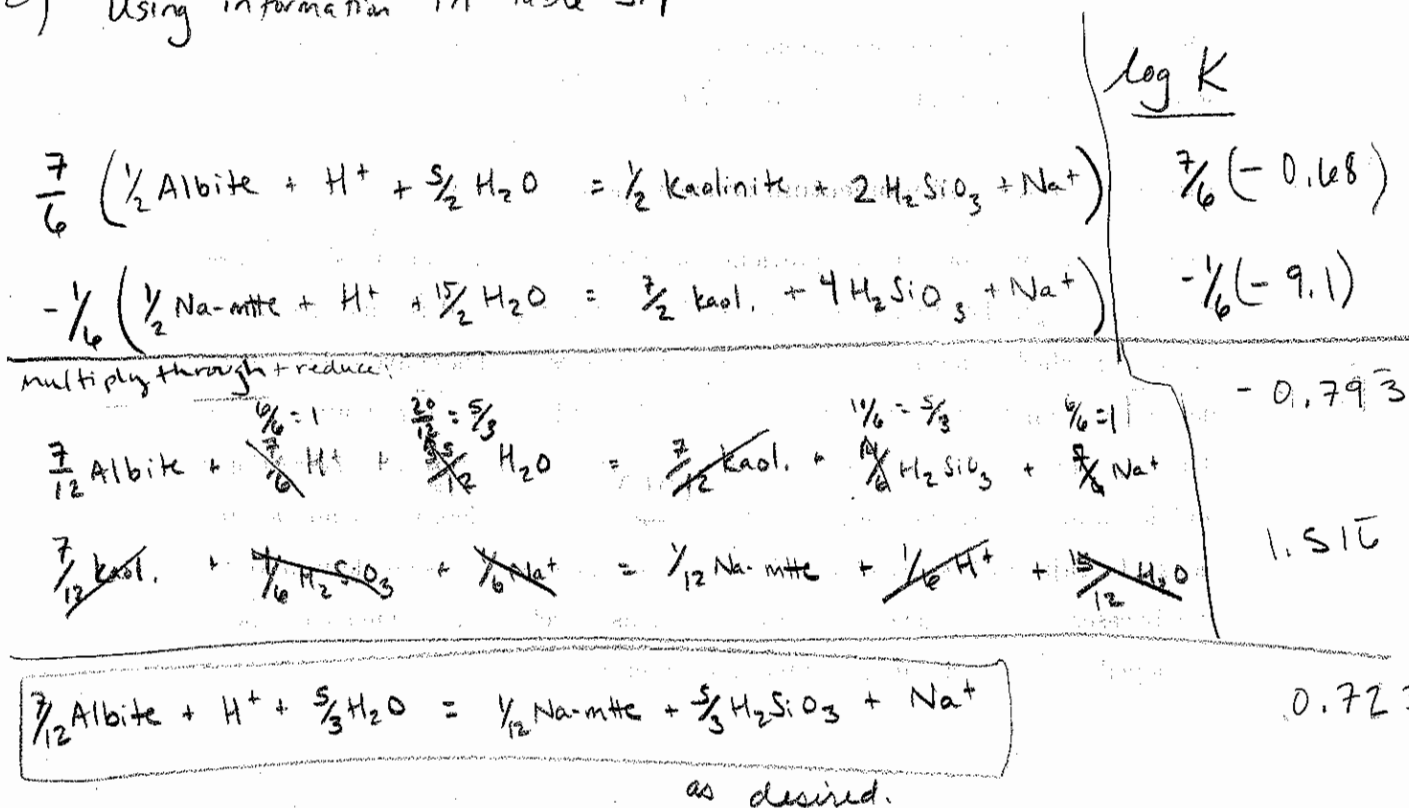
Multiply above rxn by $\frac{5}{3}$



Which reduces to



c) Using information in Table S.4



$$K_{eq} = 10^{0.723} = \frac{[\text{H}_2\text{SiO}_3]^{5/3} [\text{Na}^+]}{[\text{H}^+]}$$

d) Assume $[\text{Na}^+]_T = [\text{Na}^+] = 10^{-2} \text{ M}$

Given $[\text{H}_2\text{SiO}_3]_T = [\text{H}_2\text{SiO}_3] + [\text{HSiO}_3^-] + [\text{SiO}_3^{2-}]$

$$K_{a1} = \frac{[\text{HSiO}_3^-][\text{H}^+]}{[\text{H}_2\text{SiO}_3]} = 10^{-9.86}$$

$$K_{a2} = \frac{[\text{SiO}_3^{2-}][\text{H}^+]}{[\text{HSiO}_3^-]} = 10^{-13.1}$$

$$[\text{HSiO}_3^-] = (10^{-9.86}) [\text{H}_2\text{SiO}_3] / [\text{H}^+]$$

$$[\text{SiO}_3^{2-}] = (10^{-13.1}) [\text{HSiO}_3^-] / [\text{H}^+] = (10^{-13.1})(10^{-9.86}) [\text{H}_2\text{SiO}_3] / [\text{H}^+]^2$$

$$[H_2SiO_3]_T = [H_2SiO_3] + [HSiO_3^-] + [SiO_3^{2-}]$$

$$= [H_2SiO_3] \left(1 + 10^{-9.86} / [H^+] + (10^{-9.86})(10^{-13.1}) / [H^+]^2 \right)$$

$$[H_2SiO_3] = [H_2SiO_3]_T / \left(1 + 10^{-9.86 + pH} + 10^{-22.96 + 2pH} \right) \quad (\text{Eqn 1})$$

Using $K_{eq} = \frac{[H_2SiO_3]^{5/3} [Na^+]^2}{[H^+]}$ and Eqn 1,

We can solve for H^+ by trial + error.
 $\Rightarrow$ but the system won't converge!!

(Here, you can see why a coefficient 1 on H_2SiO_3 makes the problem simpler to solve by hand...)

Assume ① $[H_2SiO_3]_T = [H_2SiO_3]$, ② $[Na^+]_T = [Na^+]$:

$$[H^+] = \frac{[H_2SiO_3]_T^{5/3} [Na^+]_T^2}{K_{eq}} = \frac{(10^{-4.0})^{5/3} (10^{-2})}{10^{0.723}}$$

$$[H^+] = 4.08 \times 10^{-10} M$$

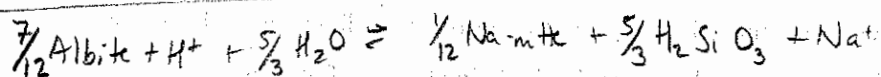
$$pH = 9.39$$

so assumption 1 isn't valid
 (pretend it is... there was an error when the problem was written - the given $[H_2SiO_3]_T$ & $[Na^+]_T$ concentrations are not possible if you are in equilibrium with Albite + Na-mtte).
 ~sorry~

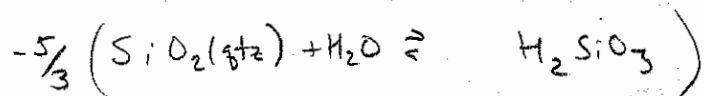
e)

Run from b + info in Tab. 5.4

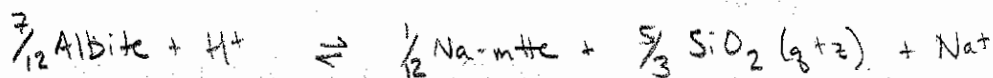
log K



0.723



$-\frac{5}{3}(-4.00)$



7.39

$$K_{\text{eq}} = \frac{[\text{Na}^+]}{[\text{H}^+]} = 10^{7.39}$$

$$[\text{H}^+] = [\text{Na}^+] / K_{\text{eq}} = \frac{10^{-2}}{10^{7.39}} = 10^{-9.39}$$

$$\boxed{\text{pH} = 9.39}$$

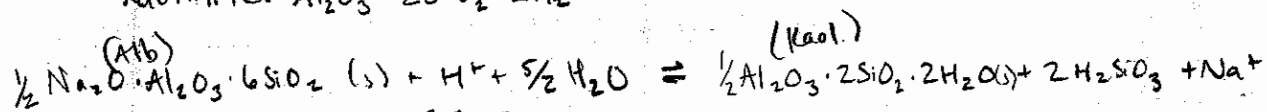
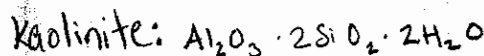
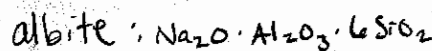
The same pH as before!!

→ fluid was in equilibrium with quartz also.

PROBLEM 2

$$p\text{CO}_2 = 10^{-1.5} \text{ atm}$$

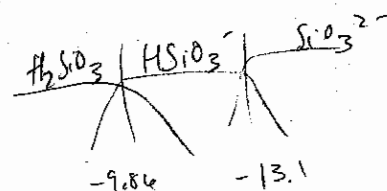
$$\text{Calcite: } \text{CaCO}_3 \quad 10^{-8.35} = K_{sp}$$



$$10^{-0.68} = K_{eq} = \frac{[\text{H}_2\text{SiO}_3]^2 [\text{Na}^+]}{[\text{H}^+]}$$

$$\frac{[\text{H}^+][\text{HSiO}_3^-]}{[\text{H}_2\text{SiO}_3]} = 10^{-9.86} = K_{a1}$$

$$\frac{[\text{SiO}_3^{2-}][\text{H}^+]}{[\text{HSiO}_3^-]} = 10^{-13.1} = K_{a2}$$



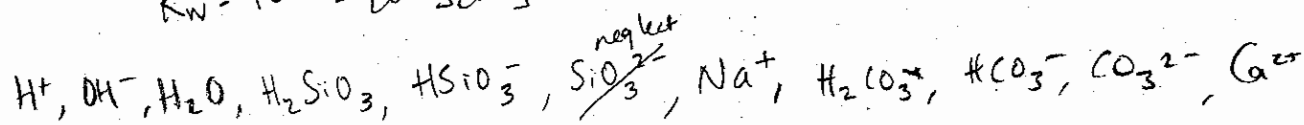
$$[\text{Ca}^{2+}][\text{CO}_3^{2-}] = 10^{-8.35}$$

$$[\text{H}_2\text{CO}_3^*] = p\text{CO}_2 K_H = 10^{-1.5} 10^{-1.46} = 10^{-2.96} \text{ M}$$

$$\frac{[\text{H}^+][\text{CO}_3^{2-}]}{[\text{HCO}_3^-]} = 10^{-10.33}$$

$$\frac{[\text{H}^+][\text{HCO}_3^-]}{[\text{H}_2\text{CO}_3^*]} = 10^{-6.35}$$

$$K_w = 10^{-14} = [\text{OH}^-][\text{H}^+]$$



$$(\text{H}_2\text{SiO}_3 = \frac{1}{2} \text{Alb} + \text{H}^+ + \frac{5}{2} \text{H}_2\text{O} - \frac{1}{2} \text{Kaol.} - \text{Na}^+)^{1/2}$$

	H ₂ O	H ⁺	H ₂ CO ₃ [*]	CaCO ₃ (s)	Albite (s)	Kaol. (s)	Na ⁺
H ₂ O	1						
H ⁺		1					
OH ⁻	1	-1					
H ₂ CO ₃ [*]			1				
HCO ₃ ⁻	-1	-1	1				
CO ₃ ²⁻		-2	1				
Ca ²⁺		2	-1	1			
H ₂ SiO ₃	5/4	1/2			1/4	-1/4	-1/2
HSiO ₃ ⁻	5/4	-1/2			1/4	-1/4	-1/2
Na ⁺							1

$$\text{TOT H}^+: [\text{H}^+] - [\text{OH}^-] + \frac{1}{2}[\text{H}_2\text{SiO}_3] - \frac{1}{2}[\text{HSiO}_3^-] - [\text{HCO}_3^-] - 2[\text{CO}_3^{2-}] + 2[\text{Ca}^{2+}] = 0$$

$$\text{TOT Na}^+: -\frac{1}{2}[\text{H}_2\text{SiO}_3] - \frac{1}{2}[\text{HSiO}_3^-] + [\text{Na}^+] = 0$$

From TOT H:

$$[\text{H}^+] - \frac{10^{-14}}{[\text{H}^+]} + \frac{1}{2} \left(\frac{(10^{-0.168}) [\text{H}^+]}{[\text{Na}^+]} \right)^{1/2} - \frac{1}{2} \left(\frac{(10^{-9.86})}{[\text{H}^+]^{1/2}} \left(\frac{10^{-0.168}}{[\text{Na}^+]} \right)^{1/2} \right)$$

$$- \frac{10^{-6.39}}{[\text{H}^+]} - \frac{10^{-2.96}}{[\text{H}^+]} - 2 \left(\frac{10^{-8.35}}{[\text{Ca}^{2+}]} \right) + 2[\text{Ca}^{2+}] = 0$$

$$[Ca^{2+}] = 10^{-8.35} / [CO_3^{2-}]$$

$$[CO_3^{2-}] = \frac{(10^{-10.33})[HCO_3^-]}{[H^+]} = \frac{(10^{-6.35})(10^{-10.33})(10^{-1.5})(10^{-1.46})}{[H^+]^2} = \frac{10^{-19.64}}{[H^+]^2}$$

$$[Ca^{2+}] = \frac{10^{-8.35}}{(10^{-19.64} / [H^+]^2)} = \frac{10^{-8.35} [H^+]^2}{10^{-19.64}} = 10^{11.29} [H^+]^2$$

To replace $[Na^+]$ in TOTH, use TDT Na°.

$$-\frac{1}{2} \left(\frac{10^{-0.68} [H^+]}{[Na^+]} \right)^{1/2} - \frac{1}{2} \left(\frac{10^{-9.86}}{[H^+]^{1/2}} \left(\frac{10^{-0.68}}{[Na^+]} \right)^{1/2} \right) + [Na^+] = 0$$

$$-\frac{1}{2} \left(\frac{10^{-0.68}}{[Na^+]} \right)^{1/2} \left[[H^+]^{0.5} + \frac{10^{-9.86}}{[H^+]^{0.5}} \right] = -[Na^+]$$

(put $^{-1/2}$ into exponent)

$$10^{-0.64} \left([H^+]^{0.5} + \frac{10^{-9.86}}{[H^+]^{0.5}} \right) = (-[Na^+]) (-[Na^+]^{1/2}) = [Na^+]^{3/2}$$

(take the $2/3$ root of both sides)

$$[Na^+] = \left(10^{-0.64} \left([H^+]^{0.5} + 10^{-9.86} [H^+]^{-0.5} \right) \right)^{2/3}$$

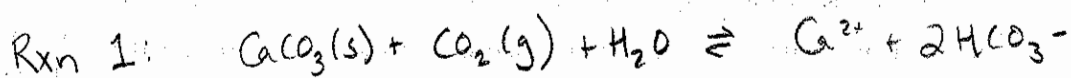
Choose a pH, solve for $[Na^+]$, $[Ca^{2+}]$, $[H_2SiO_3]$, $[HSiO_3^-]$, $[CO_3^{2-}]$, $[HCO_3^-]$, plug in to TOTH, find pH where TOTH = 0.

pH	[H ⁺]	Ca ²⁺	Na ⁺	H ₂ SiO ₃	HSiO ₃ -	TOTH
7.0230	9.48418E-08	0.001753881	1.71E-03	0.003405044	4.95590E-06	3.8537E-05
7.0240	9.46237E-08	0.001745822	1.71E-03	0.003402428	4.96351E-06	9.1799E-06
7.0250	9.44061E-08	0.001737801	1.71E-03	0.003399814	4.97113E-06	-2.0130E-05
7.0260	9.41890E-08	0.001729816	1.71E-03	0.003397202	4.97876E-06	-4.9393E-05
7.0270	9.39723E-08	0.001721869	1.70E-03	0.003394591	4.98640E-06	-7.8608E-05
7.0280	9.37562E-08	0.001713957	1.70E-03	0.003391983	4.99406E-06	-1.0778E-04
7.0290	9.35406E-08	0.001706082	1.70E-03	0.003389377	5.00173E-06	-1.3690E-04
7.0300	9.33254E-08	0.001698244	1.70E-03	0.003386773	5.00940E-06	-1.6598E-04
7.0310	9.31108E-08	0.001690441	1.70E-03	0.00338417	5.01709E-06	-1.9501E-04
7.0320	9.28966E-08	0.001682674	1.70E-03	0.00338157	5.02480E-06	-2.2400E-04
7.0330	9.26830E-08	0.001674943	1.70E-03	0.003378972	5.03251E-06	-2.5295E-04
7.0340	9.24698E-08	0.001667247	1.69E-03	0.003376375	5.04023E-06	-2.8194E-04
LOG (spp)	-7.02E+00	-2.76E+00	-2.77E+00	-2.47E+00	-5.30E+00	

TOTH
 = 0
 Somewhat
 in this
 pH range,
 pH = 7.02

$[H^+] = 10^{-7.02} \quad M$
 $[OH^-] = 10^{-6.98} \quad M$
 $[Ca^{2+}] = 10^{-2.76} \quad M$
 $[Na^+] = 10^{-2.77} \quad M$
 $[H_2SiO_3] = 10^{-2.47} \quad M$
 $[HSiO_3^-] = 10^{-5.30} \quad M$
 $[H_2CO_3^*] = 10^{-2.96} \quad M$
 $[HCO_3^-] = 10^{-2.29} \quad M$
 $[CO_3^{2-}] = 10^{-5.40} \quad M$

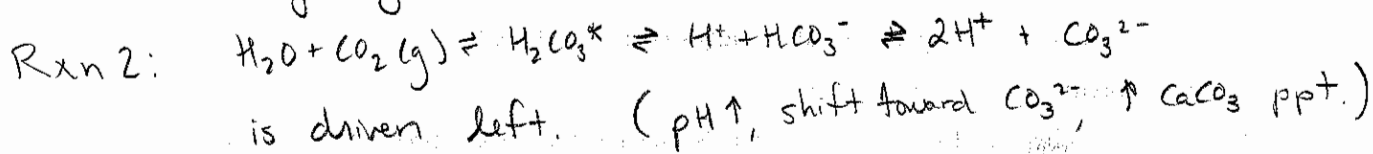
PROBLEM #3



As $\text{CO}_2(\text{g})$ escapes, $\text{CaCO}_3(\text{s})$ ppt (rxn goes left, $\uparrow \text{pH}$, $\uparrow \text{CaCO}_3$ preservation).

The outgassing of CO_2 will have no effect on the alkalinity ($\Delta \text{Alk} = 0 (\Delta \text{CO}_2(\text{g}))$).

The outgassing of CO_2 will increase the pH, as this rxn



C_T will begin to decrease, as $C_T = \text{H}_2\text{CO}_3^* + \text{HCO}_3^- + \text{CO}_3^{2-}$ and we are decreasing H_2CO_3^* ($\uparrow \text{HCO}_3^-$ & $\uparrow \text{CO}_3^{2-}$, pulling Rxn 2 to the left).

According to rxn 1, $\text{CaCO}_3(\text{s})$ will begin to ppt, and for every CaCO_3 removed, $\text{Alk} \downarrow$ by 2 equivalents.

$$(\Delta \text{Alk} = \Delta(-\text{TDTH}) = \Delta \text{HCO}_3^- = -2 \text{ mol Alk/mol CaCO}_3)$$

C_T will continue to decrease as more CO_2 outgasses + $\text{CaCO}_3(\text{s})$ ppts, since you are removing CO_3^{2-} and H_2CO_3^* .

The pH should increase, as rxn 1 is going left, $\downarrow \text{HCO}_3^-$ ($\text{HCO}_3^- \rightleftharpoons \text{H}^+ + \text{CO}_3^{2-}$), thereby $\downarrow \text{H}^+$, and $\uparrow \text{pH}$.

$$\Delta C_T = -1(\text{CaCO}_3 \text{ ppt}) - 1(\text{CO}_2 \text{ escaped})$$

Freshwater - ignore ionic strength effects

$$p\text{CO}_2 = \left(\frac{350 \times 10^{-6} \text{ ml}}{\text{ml air}} \right) \left(\frac{\text{L}}{1000 \text{ mL CO}_2} \right) \left(\frac{\text{mol}}{22.4 \text{ L}} \right) \left(\frac{1000 \text{ mL}}{\text{L air}} \right) (298 \text{ K}) \left(\frac{8.2 \times 10^{-2} \text{ L} \cdot \text{atm}}{\text{mol} \cdot \text{K}} \right)$$

$$= 3.82 \times 10^{-4} \text{ atm} = 10^{-3.4} \text{ atm} = \text{typical atmospheric } p\text{CO}_2$$

As $10^{-3.4} = p\text{CO}_{2,\text{atm}} < p\text{CO}_{2,\text{gdwater}} = 10^{-1.5} \text{ atm}$, our water will be super saturated with respect to CO_2 .

Assume: • we are no longer in eq. w/ albite or kaolinite
• there are no sources/sinks of Na^+ or H_2SiO_3

Solve using charge balance (why not try something new?):

Charged spps: H^+ , OH^- , HSiO_3^- , SiO_3^{2-} ^{neglect $p\text{H} < 12.9$} , HCO_3^- , CO_3^{2-} , Na^+ , Ca^{2+}

Charge Balance:

$$0 = [\text{H}^+] + [\text{Na}^+] + 2[\text{Ca}^{2+}] - [\text{OH}^-] - [\text{HSiO}_3^-] - 2[\text{CO}_3^{2-}] + [\text{HCO}_3^-]$$

We will rewrite species in terms of $[\text{H}^+]$ and solve by trial & error using a spreadsheet.

$$[\text{Na}^+] = 10^{-2.77} \text{ M}, \text{ as in Problem 2}$$

$$[\text{HSiO}_3^-] = \frac{[\text{H}_2\text{SiO}_3]_{\text{T}}}{(1 + [\text{H}^+]/10^{-9.86})}; \quad [\text{H}_2\text{SiO}_3]_{\text{T}} = 10^{-2.47} \text{ M}, \text{ as in Prob 2.}$$

$$[\text{Ca}^{2+}] = \frac{K_{\text{sp}, \text{CaCO}_3}}{[\text{CO}_3^{2-}]} = \frac{10^{-8.35}}{[\text{CO}_3^{2-}]}$$

$$[HCO_3^-] = \frac{[H_2CO_3^*] K_{a1}}{[H^+]} = \frac{(pCO_2)(K_H) K_{a1}}{[H^+]} = \frac{(10^{-3.4})(10^{-1.46})(10^{-6.35})}{10^{-pH}}$$

$$[CO_3^{2-}] = \frac{[HCO_3^-] K_{a2}}{[H^+]} = \frac{(pCO_2)(K_H)(K_{a1})(K_{a2})}{[H^+]} = \frac{(10^{-3.4})(10^{-1.46})(10^{-6.35})(10^{-11.33})}{10^{-2pH}}$$

$$[OH^-] = \frac{K_w}{[H^+]} = \frac{10^{-14}}{10^{-pH}} = 10^{-14+pH}$$

$$pH = 8.48$$

$$C_T = 10^{-2.72} M$$

$$Alk = 10^{-2.77} \text{ eq/l}$$

$$Ca^{2+} = 10^{-3.76} M$$

Alk Tableau

	H ₂ O	H ⁺	CaCO ₃	H ₂ CO ₃ [*]	H ₂ SiO ₃	Na ⁺
H ₂ O	1					
H ⁺		1				
OH ⁻		-1				
H ₂ CO ₃ [*]				1		
HCO ₃ ⁻				-1		
CO ₃ ²⁻				-2		
Ca ²⁺			1	-1		
H ₂ SiO ₃					1	
HSiO ₃ ⁻					-1	
Na ⁺						1

$$Alk = -TOTM = -[H^+] + [OH^-] + [HCO_3^-] + 2[CO_3^{2-}] - 2[Ca^{2+}] + [HSiO_3^-]$$

PROBLEM #4

a) $[Fe^{2+}] = 10^{-6} M$

complex	log β
$FeOH^+$	4.5
$Fe(OH)_2$	7.4
$Fe(OH)_3^-$	11.0

$$[Fe(II)]_T = [Fe^{2+}] + [FeOH^+] + [Fe(OH)_2] + [Fe(OH)_3^-]$$

$$= [Fe^{2+}] + \beta_{FeOH^+} [Fe^{2+}][OH^-] + \beta_{Fe(OH)_2} [Fe^{2+}][OH^-]^2 + \beta_{Fe(OH)_3^-} [Fe^{2+}][OH^-]^3$$

$$= [Fe^{2+}] (1 + \beta_{FeOH^+} [OH^-] + \beta_{Fe(OH)_2} [OH^-]^2 + \beta_{Fe(OH)_3^-} [OH^-]^3)$$

$$[Fe^{2+}] = \frac{[Fe(II)]_T}{\left(1 + 10^{4.5} \frac{K_w}{[H^+]} + 10^{7.4} \left(\frac{K_w}{[H^+]}\right)^2 + 10^{11.0} \left(\frac{K_w}{[H^+]}\right)^3\right)} \quad (\text{Eqn 1})$$

$$[FeOH^+] = \beta_{FeOH^+} [Fe^{2+}][OH^-] = 10^{4.5} \frac{K_w}{[H^+]} [Fe^{2+}] \quad (\text{Eqn 2})$$

$$[Fe(OH)_2] = \beta_{Fe(OH)_2} [Fe^{2+}][OH^-]^2 = 10^{7.4} \left(\frac{K_w}{[H^+]}\right)^2 [Fe^{2+}] \quad (\text{Eqn 3})$$

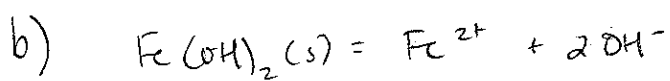
$$[Fe(OH)_3^-] = \beta_{Fe(OH)_3^-} [Fe^{2+}][OH^-]^3 = 10^{11.0} \left(\frac{K_w}{[H^+]}\right)^3 [Fe^{2+}] \quad (\text{Eqn 4})$$

$$[H^+] = 10^{-pH}$$

$$[OH^-] = \frac{K_w}{[H^+]} = 10^{-14+pH}$$

$$K_w = 10^{-14}$$

Equations 1-4 used to generate plot.



$$K_{sp} = 10^{-15.1} = [\text{Fe}^{2+}][\text{OH}^-]^2$$

$$[\text{Fe}^{2+}]_{\text{eq. w/ Fe}(\text{OH})_2(\text{s})} = K_{sp} / [\text{OH}^-]^2 = K_{sp} \left(\frac{[\text{H}^+]}{K_w} \right)^2 = \frac{(10^{-15.1})(10^{-2\text{pH}})}{10^{-28}}$$

$\text{Fe}(\text{OH})_2(\text{s})$ will ppt when

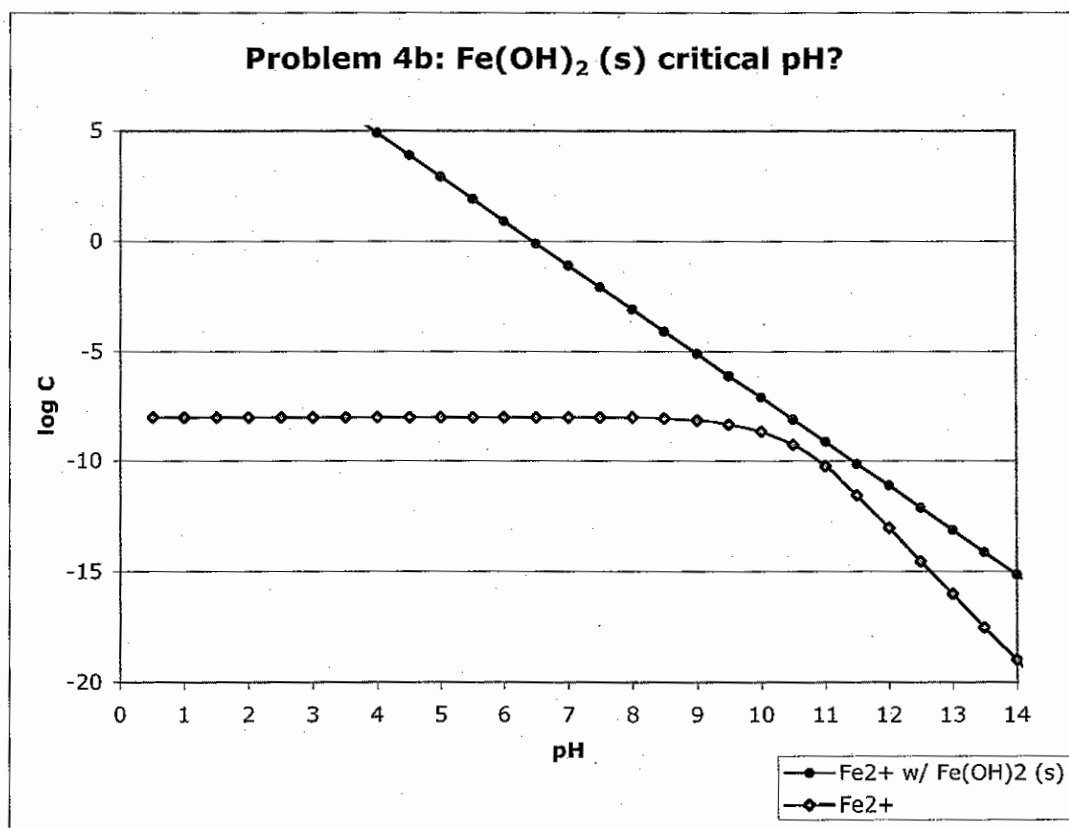
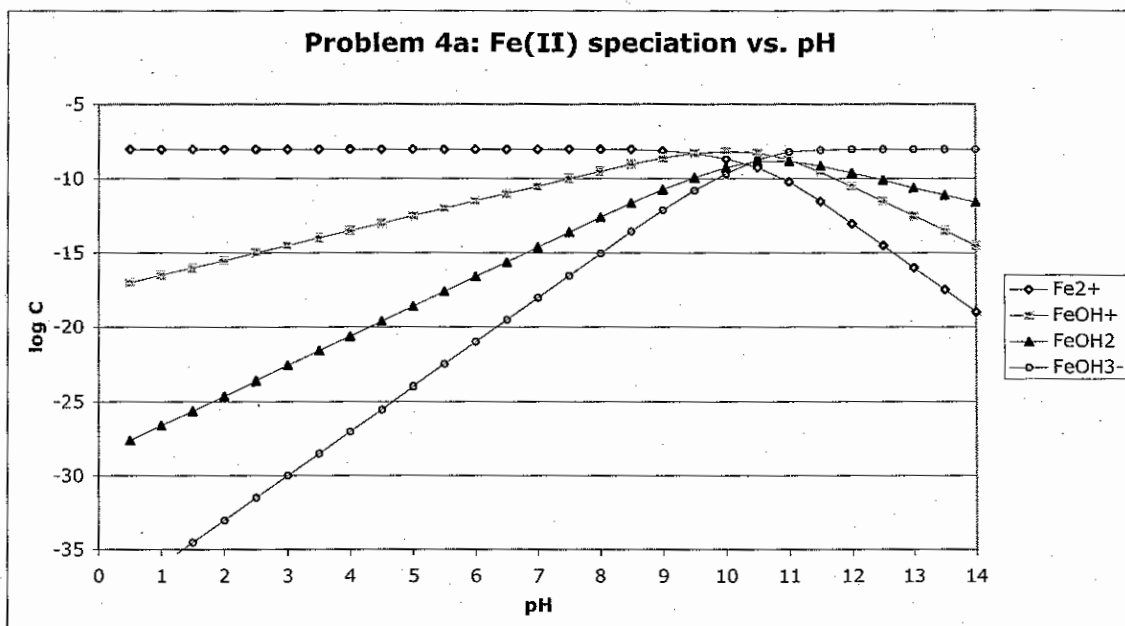
$$[\text{Fe}^{2+}]_{\text{eq. w/ Fe}(\text{OH})_2(\text{s})} \leq [\text{Fe}^{2+}]_{\text{free}}$$

Plot $[\text{Fe}^{2+}]_{\text{free}} ([\text{Fe}^{2+}])$ and $[\text{Fe}^{2+}]_{\text{eq. w/ Fe}(\text{OH})_2(\text{s})}$,

solve for pH where

$$[\text{Fe}^{2+}] = [\text{Fe}^{2+}]_{\text{eq. w/ Fe}(\text{OH})_2(\text{s})}$$

From the plot, we see that $[\text{Fe}^{2+}] < [\text{Fe}^{2+}]_{\text{eq. w/ Fe}(\text{OH})_2(\text{s})}$ for all pH, so $\text{Fe}(\text{OH})_2$ does not ppt.



$$c) [H_2S]_T = [H_2S] + [HS^-] + [S^{2-}] = 10^{-5} M$$

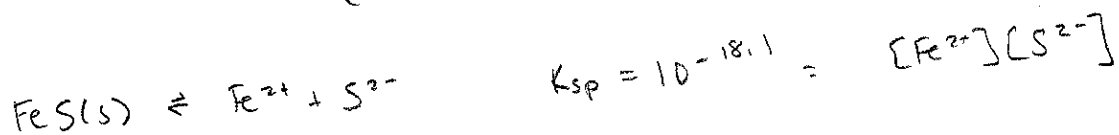
$$K_{a1} = \frac{[HS^-][H^+]}{[H_2S]} = 10^{-7.02} \Rightarrow [H_2S] = \frac{[HS^-][H^+]}{10^{-7.02}} = \frac{[S^{2-}][H^+]^2}{(10^{-7.02})(10^{-13.9})}$$

$$K_{a2} = \frac{[S^{2-}][H^+]}{[HS^-]} = 10^{-13.9} \Rightarrow [HS^-] = \frac{[S^{2-}][H^+]}{10^{-13.9}}$$

$$[H_2S]_T = \frac{[S^{2-}][H^+]^2}{(10^{-7.02})(10^{-13.9})} + \frac{[S^{2-}][H^+]}{10^{-13.9}} + [S^{2-}]$$

$$[S^{2-}] = [H_2S]_T / \left(1 + \frac{[H^+]}{10^{-13.9}} + \frac{[H^+]^2}{(10^{-7.02})(10^{-13.9})} \right)$$

$$[S^{2-}] = [H_2S]_T / \left(1 + 10^{-pH+13.9} + 10^{-2pH+20.92} \right) \quad (\text{Eqn 1})$$



$$[Fe^{2+}]_{eq. w/ FeS(s)} = 10^{-18.1} / [S^{2-}] \quad (\text{Eqn 2})$$

Plug Eqn 1 into Eqn 2 to generate $[Fe^{2+}]_{eq. w/ FeS(s)}$, plot with $[Fe^{2+}]_{free}$, $[Fe^{2+}]_{eq. w/ Fe(OH)_2(s)}$.

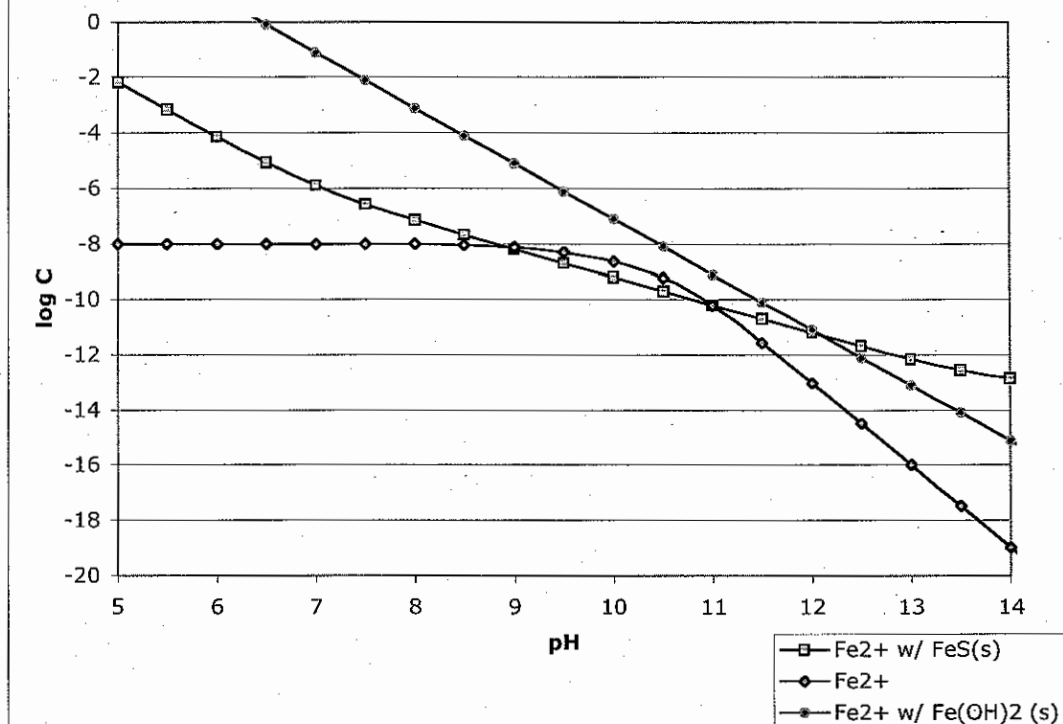
The critical pH is the pH where

$$[Fe^{2+}]_{eq. w/ FeS(s)} \leq [Fe^{2+}]_{free}, [Fe^{2+}]_{eq. w/ Fe(OH)_2(s)}.$$

$$pH = 8.8$$

(note: used solver, did not read off plot)

Problem 4c: FeS (s) critical pH?



d) $\text{FeS}(s)$ is more stable at pH values < 12

(as $[\text{Fe}^{2+}]_{\text{in eq. w/ FeS}(s)} < [\text{Fe}^{2+}]_{\text{in eq. w/ Fe(OH)}_2}$ for all pH values < 12 .)

e) Ignoring complexing $[\text{Fe}^{2+}] = [\text{Fe}^{2+}]_T$,
and $\text{Fe(OH)}_2(s)$ will ppt when $[\text{Fe}^{2+}][\text{OH}^-]^2 \geq K_{sp} = 10^{-15.1}$

$$[\text{OH}^-] = \sqrt{[\text{OH}^-]^2} = \sqrt{\frac{K_{sp}}{[\text{Fe}^{2+}]_T}} = \sqrt{\frac{10^{-15.1}}{10^{-8}}} = 10^{-3.55} \text{ M}$$

$$\text{pOH} = 3.55$$

$$14 - \text{pOH} = \text{pH} = 10.45$$

Aqueous complexing increases the solubility of $\text{Fe(OH)}_2(s)$
(but does not change K_{sp}), by lowering the
 $[\text{Fe}^{2+}]_{\text{free}}$.