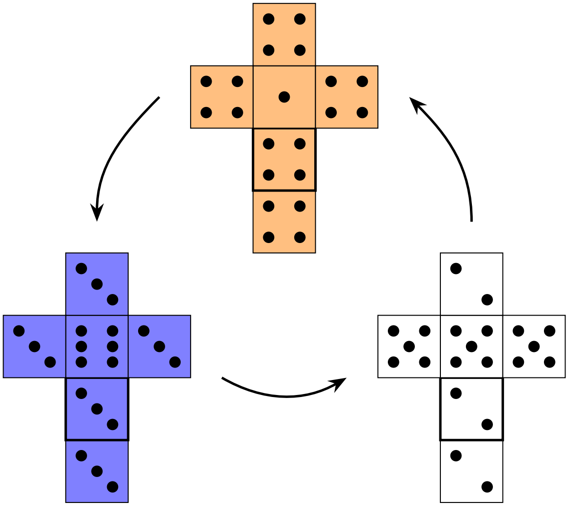
Probability: Terminology and Examples
18.05 Spring 2022



Announcements/Agenda

Announcemnets

For tomorrow:

- Do the R tutorial on our MITx site (20-30 minutes).
- Bring your laptop to class
- The studio packet will be available on MITx after class today. It has detailed instructions.
- In class we can clear up any confusing instructions.

Agenda

- Probability basics
  - sample space
  - events
  - probability function
  - experiments

Board Question 1

Deck of 52 cards

- 13 *ranks*: 2, 3, ... , 9, 10, J, Q, K, A
- 4 *suits*: $\heartsuit, \spadesuit, \diamondsuit, \clubsuit$,

Poker hands

- Consists of 5 cards
- A *one-pair* hand consists of two cards having one rank and the remaining three cards having three other ranks
- Example: $\{2\heartsuit, 2\spadesuit, 5\heartsuit, 8\clubsuit, \text{K}\diamondsuit\}$

Question

(a) How many different 5 card hands have exactly one pair?

Hint: practice with how many 2 card hands have exactly one pair.
Hint for hint: use the rule of product.

(b) What is the probability of getting a one pair poker hand?

Probability Cast

- Experiment: a repeatable procedure
- Sample space: set of all possible outcomes S (or Ω).
- Event: a subset of the sample space.
- Probability function, $P(\omega)$: gives the probability for each outcome $\omega \in S$
    1. Probability is between 0 and 1
    2. Total probability of all possible outcomes is 1.

Example (from the reading)

Coin tossing experiment

One trial: toss a fair coin, report heads or tails.

Sample space: $S = \{H, T\}$.

Probability function: $P(H) = 0.5, \quad P(T) = 0.5$.

Use tables to summarize:

Outcomes	H	T
Probability	1/2	1/2

(Tables can really help in complicated examples)

Discrete sample space

Discrete = listable

Examples of discrete sample spaces

{a, b, c, d} (finite)

{0, 1, 2, ...} (infinite)

{sun,cloud,rain,snow,fog}

{patient cured, unchanged, died}

Events

Events are sets of outcomes:

- Can describe in words
- Can describe in notation
- Can describe with Venn diagrams

Example. Experiment: toss a coin 3 times.

Event:
You get 2 or more heads = { HHH, HHT, HTH, THH}

Clicker Test

CQ: Can you connect to respond to clicker questions?

1. No
2. yes

Concept question 1: What's the event?
(Connecting words and set notation.)

Experiment: toss a coin 3 times.

Which of following equals the event "exactly two heads"?

$A = \{THH, HTH, HHT, HHH\}$
$B = \{THH, HTH, HHT\}$
$C = \{HTH, THH\}$

(1) A (2) B (3) C (4) B or C

Concept question 2: Describe the event
(Connecting words and set notation.)

Experiment: toss a coin 3 times.

Which of the following describes the event
$\{THH, HTH, HHT\}$?

(1) "exactly one head"
(2) "exactly one tail"
(3) "at most one tail"
(4) none of the above

Concept question 3: Are they disjoint?
(Connecting words and set notation.)

Experiment: toss a coin 3 times.

The events "exactly 2 heads" and "exactly 2 tails" are disjoint is.

(1) True (2) False

Events, sets and words

Concept question 4: Does A imply B?
(Connecting words and set notation)

Consider two events: A and B.

Are the words "A implies B" equivalent to $A \subseteq B$?

(1) True (2) False

Probability rules in mathematical notation

Sample space: $S = \{\omega_1, \omega_2, \dots, \omega_n\}$

Outcome: $\omega \in S$

Probability between 0 and 1: $\quad 0 \le P(\omega) \le 1$

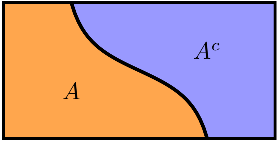
Total probability is 1: $\displaystyle\sum_{j=1}^{n} P(\omega_j) = 1, \quad \sum_{\omega \in S} P(\omega) = 1$

Event A: $\displaystyle P(A) = \sum_{\omega \in A} P(\omega)$

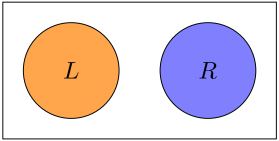
Probability and set operations on events

Events A, L, R

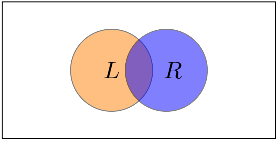
1. Complements: $P(A^c) = 1 - P(A)$.
2. Disjoint events: If L and R are disjoint then $P(L \cup R) = P(L) + P(R)$.
3. Inclusion-exclusion principle: For any L and R: $P(L \cup R) = P(L) + P(R) - P(L \cap R)$.



$\Omega = A \cup A^c$, no overlap



$L \cup R$, no overlap



$L \cup R$, overlap $= L \cap R$

Table question

- Class has 50 students
- 20 male (M), 25 brown-eyed (B)

For a randomly chosen student, what is the range of possible values for $p = P(M \cup B)$?

(a) $p \leq 0.4$
(b) $0.4 \leq p \leq 0.5$
(c) $0.4 \leq p \leq 0.9$
(d) $0.5 \leq p \leq 0.9$
(e) $0.5 \leq p$

Table Question

Experiment:

1. Your table should make 9 rolls of a 20-sided die (one each if the table is full).
2. Check if all rolls at your table are distinct.

Repeat the experiment five times and record the results.

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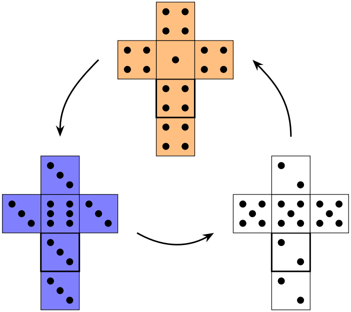
Repeat the experiment five times and record the results.

For this experiment, how would you define the sample space, probability function, and event?

Compute the true probability that all rolls (in one trial) are distinct and compare with your experimental result.

Preamble: Jon's dice

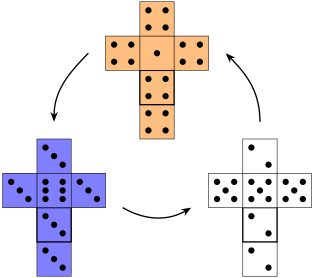
Jon has three six-sided dice with unusual numbering.



A game consists of two players each choosing a die. They roll once and the highest number wins.

Which die would you choose?

Board Question: Jon's dice



1. Make probabilitiy tables for the the blue and white dice.

2. Make a probability table for the product sample space of blue and white.

3. Use the table to compute the probability that blue beats white.

4. Pair up with another group. Have one group compare blue vs. orange and the other compare orange vs. white. Based on the three comparisons, rank the dice from best to worst.

Concept Question

Lucky Lucy has a coin that you're quite sure is not fair.

- They will flip the coin twice
- It's your job to say whether it more probable that the tosses are the same, i.e. HH or TT, or different, i.e. HT or TH.

Which should you choose?

1. More probable they are the same
2. More probable they are different
3. Doesn't matter

4. It depends on the actual probabilities of getting heads or tails.

Board Question

Lucky Lucy has a coin that you're quite sure is not fair.

- They will flip the coin twice
- Let A be the event the tosses are the same, i.e. $\{HH, TT\}$
- Let B be the event the tosses are the different, i.e. $\{HT, TH\}$

Let p be the probability of heads. Compute and compare $P(A)$ and $P(B)$.
(If you don't see the symbolic algebra try p = 0.2, p=0.5)

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