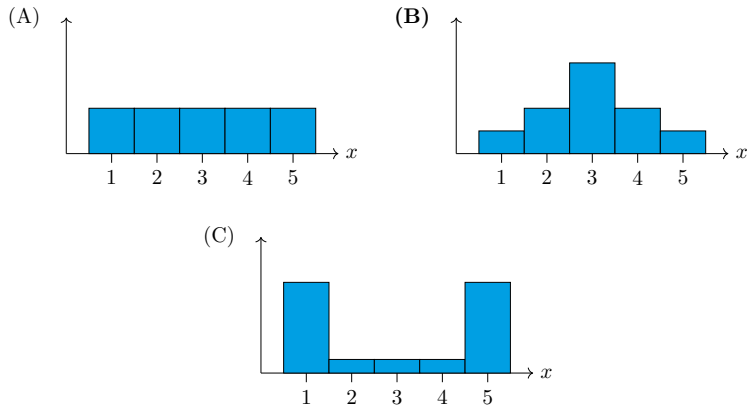
Class 5 in-class problems, 18.05, Spring 2022

Concept questions

Concept question 1. Order the variance

The graphs below give the pmf for 3 random variables.



Order them by size of standard deviation from biggest to smallest. (Assume x has the same units in all three.)

1. ABC 2. ACB 3. BAC 4. BCA 5. CAB 6. CBA

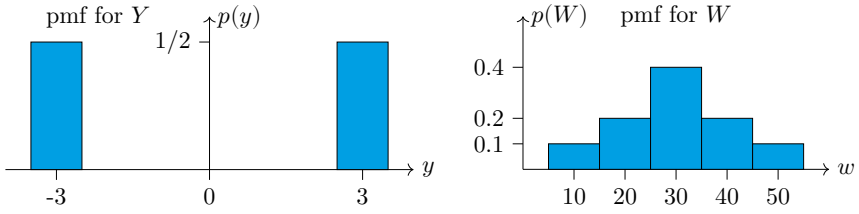
Concept question 2. Zero variance

Suppose X is a discrete random variable,

True or False: If $\text{Var}(X) = 0$ then X is constant.

Concept question 3. Standard deviation

Make an intuitive guess: Which pmf has the bigger standard deviation? (Assume w and y have the same units.)



1. Y 2. W

Concept question 4.

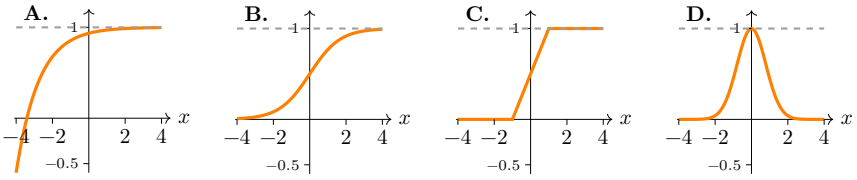
Suppose X is a continuous random variable.

(a) If the pdf of X is $f(x)$ can there be an x where $f(x) = 10$?

(b) What is $P(X = a)$?

(c) Does $P(X = a) = 0$ mean X never equals a?

Concept question 5.
Which of the following are graphs of valid cumulative distribution functions?



Board questions

Problem 1.
(a) Let $X \sim \text{Bernoulli}(p)$. Compute $\text{Var}(X)$.

(b) Let $Y \sim \text{Bin}(n, p)$. Show $\text{Var}(Y) = n\,p(1-p)$.

(c) Suppose $X_1, X_2, \dots, X_n$ are independent and all have the same standard deviation $\sigma = 2$. Let $\overline{X}$ be the average of $X_1, \dots, X_n$.

What is the standard deviation of $\overline{X}$?

Problem 2.
Suppose X has range $[0, 2]$ and pdf $f(x) = c\,x^2$.

(a) What is the value of c?

(b) Compute the cdf $F(x)$.

(c) Compute $P(1 \le X \le 2)$.

(d) Plot the pdf and use it to illustrate part (c).

Problem 3.
Suppose Y has range $[0, b]$ and cdf $F(y) = y^2/9$.

(a) What is b?

(b) Find the pdf of Y.

Problem 4.
I've noticed that taxis drive past 77 Mass. Ave. on the average of once every 10 minutes.

Suppose time spent waiting for a taxi is modeled by an exponential random variable

$$X \sim \text{Exponential}(1/10); \qquad f(x) = \frac{1}{10}\mathrm{e}^{-x/10}$$

(a) Sketch the pdf of this distribution

(b) Shade the region which represents the probability of waiting between 3 and 7 minutes

(c) Compute the probability of waiting between between 3 and 7 minutes for a taxi

(d) Compute and sketch the cdf.

In class examples and discussion

Example. Computation from tables
Compute the variance and standard deviation of X.

values x	1	2	3	4	5
pmf $p(x)$	1/10	2/10	4/10	2/10	1/10

Example. A very useful formula
Recompute the previous example using the very useful formula for variance

$$\text{Var}(X) = E[X^2] - E[X]^2 = \left(\sum_{i=1}^{n} p(x_i)x_i^2\right) - \mu^2.$$

Extra problems

Extra 1. Let X take value 1, with equal probability on $\{1, 2, 3, 4, 5\}$ (X is a uniform random variable). Compute $\text{Var}(X)$.

Let Y be uniform on $\{7, 8, 9, 10, 11\}$. What is the variance of Y?

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